# (Unobserved) Heterogeneity in the bank lending channel: Accounting for bank-firm interactions and specialization*


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**Abstract**

Using bank-firm credit data, we develop a framework that estimates the transmission of bank-supply credit shocks on firms accounting for interactions between bank and firm unobserved factors, able to accommodate lending specialization in different margins. Bank shocks can have heterogeneous effects across firm types, where firm-type is unobserved for the econometrician. We decompose credit growth dynamics into time-varying firm and bank-firm-type interaction effects. We uncover significant heterogeneity in the bank lending channel: i)Bank effects vary across our estimated firm types. Observable firm characteristics predict firm-type, and interaction effects correlate with bank specialization. ii)During the Great Recession, exposed banks contracted their supply to some firms but shielded others, iii)We uncover a novel bank-firm matching channel: The transmission of shocks depends crucially on the bank-firm network, with credit growth dropping by up to 20% in a counterfactual random network, iv)Accounting for interactions helps identify the impact of bank shocks on firm outcomes.



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# 1 Introduction

The bank lending channel—the influence of bank credit supply shocks on firms' borrowing capacity, investment, and overall real activity—stands at the center of the macrofinance literature.[1] Due to bank-firm interactions and specialization in lending, the bank lending channel may exhibit heterogeneity across different banks and firms. Consequently, the transmission of shocks might specifically depend on the particular bank-firm match, making the overall impact of the bank lending channel critically reliant on the bank-firm network structure. In this paper, we develop a novel empirical framework capable of assessing the heterogeneous impacts of the bank lending channel and understanding its dependence on the bank-firm network.

Identifying the effect of bank shocks is challenging because credit fluctuations may be driven by demand or supply factors, and events that influence the supply of credit are likely to influence the demand for credit simultaneously. State-of-the-art methodologies exploit matched bank-firm credit data and, by observing multiple connections between firms and banks, are able to control for firm demand factors. The leading contribution of Khwaja and Mian (2008) (KM hereafter) relies on using firm fixed effects to control for firm demand factors when assessing the impact of an observed bank shock, measured as a bank differential exposure to a credit contraction event.[2] This useful methodology has been widely extended to assess the impact of other types of bank shocks in different event studies (see Related Literature). Another influential contribution is Amiti and Weinstein (2018) (AW hereafter) who also exploit multiple connections in the matched bank-firm credit data, and its time series variation, to estimate a model with two-sided unobserved heterogeneity and uncover an unobservable time-varying bank shock after controlling for unobservable firm factors (through time-varying firm- and bank- fixed effects). This strategy has also been employed in several studies. While these methodologies offer a tractable way to study observed or unobserved bank shocks controlling for unobserved firm heterogeneity, they rely on the substantive assumption that the transmission of credit shocks is homogeneous, that is, that shocks to each bank (firm) are propagated equally to all its connected firms (banks), so that the within firm comparison fully absorbs firm-specific changes in credit demand.

[1] Throughout the paper we refer to bank credit supply shocks (or simply banks shocks) as shifts (contractions/expansions) in the supply of bank credit (e.g., driven by bank-specific liquidity shocks, monetary policy shocks affecting banks ability to attract deposits, or changes in capital regulation) that are unrelated to firm demand for credit or creditworthiness.

[2] They explore the effect of a cross-bank liquidity shock using an unanticipated nuclear tests in Pakistan as a natural experiment.

The absence of *interactions* between bank and firm factors in the KM or AW framework then do not allow for the possibility of a heterogeneous transmission of shocks and restrict the specific matching patterns between banks and firms. Bank-firm interactions may be expected when there exists bank specialization or relationship lending. Typically, banks constantly face a portfolio allocation task in which they have to monitor and select their expansion or contraction of credit across borrowers. Such portfolio decision depends on the borrowers' characteristics and on the specific way each bank assesses them, which could differ across banks due to differences in bank information acquisition, lending specialization, or business models.[3] In particular, recent empirical studies provide evidence of lending specialization and an heterogeneous transmission depending on some observable characteristics. Using Peruvian bank-firm credit data, Paravisini et al. (2023) show that bank specialization in export markets is a crucial determinant for the propagation of credit shocks, and Ivashina et al. (2022) show that bank shocks affect differentially firms depending on their type of loan used. Blickle et al. (2023) document loan specialization of large U.S. banks across industries. But, what if the econometrician cannot access such specific data about bank or firm specialization? Or, what if the heterogeneity is based on an unobserved margin or multiple margins? How can we learn about the heterogeneity in the transmission of credit shocks and its implications when there exists interactions between unobserved bank and firm factors?

Our first contribution is methodological; we expand upon the current state-of-the-art framework used to evaluate the bank lending channel by allowing for the presence of bank-firm interactions in *unobserved* factors that may lead to a heterogeneous response to credit shocks, and thus able to accommodate specialization in different margins. This framework is particularly useful in situations where researchers are uncertain about the specific dimensions of lending specialization or when specialization is determined by unobservable firm characteristics, allowing them to test and detect the sources of heterogeneity. We propose an estimator for cases in which the relevant interaction occurs between banks and groups (types) of firms. In addition, we discuss interpretation of the identified interactions and potential concerns that may arise when estimating a model with homogeneous effects. Our second contribution regards the novel empirical findings. We uncover significant heterogeneity in our Peruvian dataset: Banks transmit their credit shocks significantly different across the identified firm groups. Importantly, we show that accounting for such heterogeneity and the bank-firm network structure is crucial to learn about the bank

[3]Theories of relationship lending point towards a special treatment of banks to certain groups of firms (e.g., Petersen and Rajan (1994), Detragiache et al. (2000), Degryse and Ongena (2005).)

lending channel and its real effects.

In our framework, the effect of a credit shock to each specific bank is allowed to vary across unobserved groups or types of firms, while maintaining the assumption of homogeneous transmission *within groups* of firms. The approach relies on the idea that from the perspective of the bank and its relationship with firms, there is a discrete number of types in which firms are classified regarding the relevant characteristics that would lead to a special treatment or an heterogeneous transmission of a credit shock. Besides the heterogeneous transmission across groups, the model allows for firm-specific demand shocks that affect equally all banks. The bank-firm group specific shocks, the firm-specific shocks, and the firms' group membership are left unrestricted as in a "fixed-effects" (or "grouped fixed-effects") fashion and are estimated from the data. Our methodology decomposes credit growth dynamics into time-varying firm effects and time-varying *bank-firm group effects*, minimizing a least-squares criterion across all possible firm groupings.

If firm groups were known by the econometrician, then we could identify bank shocks by applying the KM or AW framework for each group, that is, by measuring the systematic differences in the lending of banks to the same set of firms belonging to the same group. However, since groups are unobserved, we combine this idea with machine learning techniques that help in clustering observations.[4] Typically, clustering techniques classify observations into groups depending on some dissimilarity measure based on observable characteristics. Our goal instead is to cluster firms based on an heterogenous unobserved response to an unobserved credit shock. We follow Bonhomme and Manresa (2015) and propose a "grouped fixed-effects" estimator to our bank lending framework by incorporating the insights from KM and AW. Intuitively, firms whose *differences* in their borrowing patterns from specific banks are most *similar* are grouped together in estimation.

We investigate statistical properties of our proposed bank-firm group effect (BF-GFE) estimator as the number of firms ($N_F$) and banks ($N_B$) tend to infinity. We follow the asymptotic analysis in Bonhomme and Manresa (2015), developed for standard panel data, and accommodate it to our model with multi-year bank-firm bipartite network data. Under well-defined groups and weak spatial dependence in the errors of the model, we show the BF-GFE estimator converges to the least squares estimator under known groups for large values of $N_F$ and $N_B$. However, due to the firms' group classification is learned from their connections with different banks,

[4]E.g., Bonhomme and Manresa (2015), Ando and Bai (2016), Bonhomme et al. (2019), Almagro and Manresa (2021).

there exists a probability of misclassification in panels with small $N_B$, leading to a small sample bias in the BF-GFE estimator.[5] Importantly, the rate of misclassification decreases very rapidly as $N_B$ increases. Through simulations, we illustrate that for moderate values of $N_B$, including calibrations to our dataset, the estimator performs well and is centered around the true value. This finding holds significant relevance given that typical databases often contain a limited number of banks.

We delve into the interpretation of our identified BF-GFE. Such interaction effect may capture heterogeneity in both supply or demand: It could stem from a credit supply shock which banks heterogeneously transmit across firm types, or may capture a group-specific demand shock, which firms transmit heterogeneously across banks, or a combination of both factors. The challenge of interpretation is somewhat mitigated in cases where we can rely on an exogenous credit event that leads to credit supply shifters, as in KW. In such scenarios, we show how to utilize our identified interaction effect to recover a double-heterogeneous (treatment) effect that varies across banks and groups of firms. We underscore the implications of using the standard framework and estimate a homogeneous model in the presence of heterogeneity. Our analysis reveals that under bank-firm endogenous matching based on the heterogeneous effects, the KM estimator fails to provide a consistent estimate of the average bank effect across all banks and groups of firms. Particularly, we find that the estimate of the homogeneous bank effect is a weighted average of the heterogeneous bank effects for each firm group and bank, with weights that may be negative, in a similar fashion to De Chaisemartin and d'Haultfoeuille (2020). Negative weights are specially problematic since one could have that the estimate of the homogeneous bank effect (associated with a negative credit event) appears positive, while all bank effects are negative but heterogeneous. Such concern on the double heterogeneity may be expected in cases of endogenous matching arising due to specialization. Intuitively, a specialized bank heavily exposed to the credit event may shield a certain group of firms relative to a less exposed unspecialized bank. Since, due to specialization, the network of firms used in the estimation will be more heavily composed of the group of firms the specialized bank shields, it could give the impression that exposure to the (negative) credit event is positively associated with lending.

Additionally, for estimating the real effects of bank shocks, we emphasize that

[5] This phenomenon resembles the well-known incidental parameter problem in panel data for small $T$ setups, as documented by Neyman and Scott (1948) and Lancaster (2000). Bonhomme and Manresa (2015) shows that the group fixed effect estimator in a standard panel data suffers from the incidental parameter problem for fixed and small $T$ as the group membership in their framework learns from the time series dimension.

identification may be achievable even without exogenous credit supply shifters, by utilizing our identified groups to control for group-specific demand factors. This identification is feasible as long as there exist idiosyncratic factors influencing the ex-ante bank-firm network structure, ensuring that firms within the same group (affected by identical demand shocks) are differentially exposed to bank shocks.

We apply our estimation framework to credit registry data from Peru from 2005 to 2017. We observe every bank-firm lending relationship for corporate firms. Additionally, we obtain the financial statements information from the Peruvian Stock Exchange to measure firm investment. Combining these sources of information, we are able to study the real effects of bank shocks on firm investment across time.

Our novel framework allows us to uncover new results on the importance of the bank lending channel that cannot be uncovered without a more flexible treatment of unobserved heterogeneity. First, we provide novel evidence on the heterogeneous effect of credit shocks. We show that the lending patterns of firms connected with each bank shows considerably heterogeneity across our identified groups. Interestingly, for some banks, the estimated BF-GFE may have different signs across types of firms: For certain years, there are banks with an estimated positive effect for a group of firms (e.g. expansion of their credit supply) while a negative effect for other group (e.g. contraction in credit). Such heterogeneity in lending patterns between banks and groups of firms leads to sizable statistical gains of our model relative to the standard homogenous effects model: In-sample and out-of-sample mean square errors improve significantly when the model allows for heterogeneous effects.

We show that certain observable characteristics help explain the estimated groups. Interestingly, we observe that firm exports and collateral predict the group membership. This type of sorting is in line with specialization on export destinations markets and lending contracts as emphasized by Paravisini et al. (2023) and Ivashina et al. (2022), respectively. Other relevant characteristics differentiating groups are firms' debt size and risk score. Moreover, building on Paravisini et al. (2023) and Blickle et al. (2023), we measure bank specialization in our firm groups by calculating the relative importance of each firm group in each bank portfolio. We find that the groupings correlate with the specialization measure, indicating that bank shocks lead to a stronger credit expansion for firms in which banks are more specialized in.

Second, we study the transmission of an observed bank shock across our identified firm groups by relying on the credit event studied in Paravisini et al. (2015). The bank shock is measured as the banks' exposure to the foreign funding shortage experienced during the capital flow reversal in Peru in 2008. We find significant differences in the transmission of such credit shock across our identified firm's groups. For instance, the heterogeneous estimates reveal that exposed banks (with high foreign liabilities) decreased (relative to low foreign liability banks) their credit significantly to some group of firms but not to all, there are even some groups of firms to which these banks expand their lending during this episode. Moreover, the estimated average of the heterogenous effect across firm groups and banks is around -0.3, indicating exposed banks reduced credit supply by 30% on average relative to non-exposed banks, while the estimated effect under the homogenous model is -0.18.

Third, the presence of bank-firm interactions and heterogeneity in the impact of credit shocks beg the question whether banks and firms create relationships and sorting patterns that amplify or smooth out the aggregate transmission of credit shocks. Our framework allows us to estimate the entire loan growth distribution corresponding to a counterfactual reallocation of relationships between banks and firms. In particular, we estimate as a counterfactual the aggregate credit growth when bank-firm are randomly matched.[6] With this exercise, our aim is to assess the contribution of the endogenous bank-firm network for the propagation of credit shocks, that is, a *bank-firm matching channel*. We find that, for most years, aggregate credit growth is enhanced by the observed bank-firm credit network relative to a randomly matched network, achieving up to a 20% higher credit growth rate in 2016 and 2017, an a growth rate that is 3% to 10% higher for most years, with the exception of 2008-09 in which the observed network produces a 5% lower growth rate.[7]

Finally, we explore the real effects of bank shocks on firm investment. We use our estimated BF-GFE's to analyze whether firms' investment is sensitive to their bank lenders' shocks. For this exercise, the bank effects, that vary by firm group, are aggregated at the firm level by weighting each bank shock by the bank relative importance in the firm borrowing portfolio in a similar fashion as in AW. As noted above, we control for firm group fixed effects in order to capture group-specific demand shocks that could correlate with our BF-GFE. We find that when bank shocks are estimated from a model with homogeneous effects, we obtain imprecise and insignificant effects of bank shocks on investment. Instead, we find a more precise and significant impact on firm investment when we consider the heterogeneity in the impact of bank shocks across firm groups, highlighting the importance of considering heterogene-

[6] This counterfactual would for example capture credit allocation in an economy with large information frictions preventing firms to find their optimal specialized bank, as represented in the model with specialization described in the Online Appendix.

[7] Under a model with homogenous effects, the transmission of credit shocks is independent of the network, so this exercise would lead to no change in aggregate credit growth by construction.

ity to reveal the real effects of bank shocks. The estimation of the elasticity of bank shocks on investment is about 4 (we get a similar result when the number of groups is chosen to be 2 to 5).

**Related Literature.** Our paper is mainly related to the empirical literature on the bank lending channel. The literature is extensive with many important contributions exploiting matched bank-firm lending data to identify the effects of credit supply shocks, relying on an homogeneous transmission assumption. Some examples include Khwaja and Mian (2008), Jiménez et al. (2012, 2017), Schnabl (2012), Chodorow-Reich (2014), Paravisini et al. (2015), Amiti and Weinstein (2018), Jiménez et al. (2020), Huremovic et al. (2020), Alfaro et al. (2021), Blattner et al. (2023), among others.[8] Most closely, our paper is related to the papers that emphasize *bank-firm interactions* in the transmission of credit shocks. The pioneering contribution of Paravisini et al. (2023) highlights interactions arising due to market specialization and provide an indicator based on export destinations to identify a relevant margin of bank-firm heterogeneity (e.g. banks specialized in US markets treat differently firms exporting to US relative to the treatment of not specialized banks to those types of firms). Gopal (2021) highlights interactions arising due to specialization in collateral choices. These papers correct the estimation by including interactions on the relevant observable margin. Another related paper is Ivashina et al. (2022) which highlights that the propagation of credit shocks may be loan-type specific, inducing a bank-firm interaction when there exists banks specialized in providing certain types of loans, which would bias standard methods assuming transmission homogeneity. They correct the estimation by separately estimating the bank lending channel across each observed type of loan. We show that both the lending specialization model in Paravisini et al. (2023) and the loan-type empirical specification in Ivashina et al. (2022) can lead to our empirical specification with bank-firm group effects. Thus, our paper provides a way of identifying and estimating such types of interactions even when the researcher lacks data access or is uncertain about the specific relevant dimensions, allowing them to test and detect the sources of heterogeneity. Moreover, we extend their discussion on the potential identification concerns that arise when using current state-of-the-art estimation methods assuming homogenous effects. In particular, we formalize that the homogenous bank effect estimate is a weighted av-

[8]Some papers have documented differential transmission of bank shocks across observables (e.g. differences by firm size as in Khwaja and Mian (2008), Chodorow-Reich (2014)). Rather than relying on a particular observed characteristic, we make the margin of heterogeneity an empirical question. Importantly, we emphasize bank-firm interactions that in general point to heterogeneity in the specific bank-firm match that can happen due to two degrees of heterogeneity: The differential effect across firm types can differ by banks, as would be expected with bank specialization.

erage of the heterogeneous bank effects for each firm group and bank, with weights that may be negative, highlighting the potential interpretation concerns of the standard homogenous bank shock estimates. In addition, we highlight the importance of the endogenous creation of bank-firm networks, uncovering and quantifying a novel bank-firm matching channel.

Our econometric approach aligns with the existing literature that estimates interactions between unobserved factors. Specifically, we closely follow the approach outlined in Bonhomme and Manresa (2015) and extend their grouped-fixed effect estimator to our bank lending framework. An important distinction is that our framework, which utilizes network panel data, encompasses three dimensions compared to the standard panel data model, which has two dimensions. Our framework allows for flexibility in the group membership of firms to change over time, which can be particularly relevant in environments with several years in which firms and banks may alter their business models. Additionally, it enables us to incorporate time-varying firm fixed effects alongside the time-varying bank-firm group effect, which is crucial for controlling for time-varying demand shocks.

Our results on the consistency of standard estimators and discussion on the identification concerns arising due to endogenous bank-firm matching and heterogeneous effects relate to the econometric literature that study panel data models with heterogeneous treatment effects (De Chaisemartin and d'Haultfoeuille (2020), Arkhangelsky et al. (2021), Sun and Abraham (2021)).

**Outline.** Section 2 provides an overview of the standard framework used to identify banks shocks exploiting credit registry data, and discusses the identification assumption of homogeneous transmission. Section 3 presents suggestive evidence of heterogeneity in the transmission. Section 4 extends the framework to allow for bank-firm interactions, discusses identification, and describes our estimation algorithm. Section 5 discusses interpretation of the bank-firm interaction effects and concerns when estimating a model with homogenous effects. Section 6 describes the data. We present the results and empirical applications in section 7 and 8. Section 9 concludes.

# 2 Standard framework: a model of homogeneous effects

The empirical model consists of $N_F$ firms and $N_B$ banks that interact in different periods $t = 1....T$. Credit registries contain information about the loan amount $L_{f,b,t}$ of each firm with each bank at time $t$. If firm $f$ has a positive outstanding balance with bank $b$ during period $t$ and $t-1$, we say a network $D_{f,b,t} = 1$ exists; otherwise $D_{f,b,t} = 0$. We denote with $y^*_{f,b,t}$ the potential loan growth rate between bank $b$ and firm $f$ at time $t$ if the network $(f, b, t)$ exists. We observe realizations of $y^*_{f,b,t}$ only when $D_{fbt} = 1$. Let's define the observed loan growth rate as $y_{f,b,t} = \left\{ y^*_{f,b,t} : D_{f,b,t} = 1 \right\}$.

The main methodology used in the literature, which follows KM, relies on a credit event ("natural experiment") occurring at some period $\tau$ that affects differentially banks' balance sheets, and exploit cross-sectional variation around the event to study how such a bank shock is propagated to firms. Thus, the literature typically considers the following linear specification:

$$y^*_{f,b} = \alpha_f + \theta x_b + \epsilon_{f,b}, \tag{1}$$

where $y^*_{f,b} = \ln L_{f,b,\mathrm{Post}(\tau)} - \ln L_{f,b,\mathrm{Pre}(\tau)}$ is the loan growth rate from a window pre-$\tau$ to a window post-$\tau$, $\alpha_f$ captures unobserved firm-specific factors (potentially demand factors linked to the credit event), $x_b$ is a bank-specific observable variable measuring bank exposure to the credit event (e.g., the exposure to a liquidity shock during a natural experiment) and $\theta$ is the constant marginal (treatment) effect of such a bank shock over the loan's growth rate. As KM emphasizes, it is key to consider firm-fixed effects $\alpha_f$ in the regression due to potential correlation with $x_b$. For example, banks more exposed to the credit event may be lending to firms that are also more affected directly by the event.

In another contribution, AW proposes to exploit the time series variation in the data and consider a similar linear specification with two-sided heterogeneity for $y^*_{f,b,t}$:

$$y^*_{f,b,t} = \alpha_{f,t} + \beta_{b,t} + \epsilon_{f,b,t}, \tag{2}$$

where $\alpha_{f,t}$ is time-varying firm-specific unobserved heterogeneity (typically interpreted as credit demand factors), $\beta_{b,t}$ is time-varying bank-specific unobserved heterogeneity (interpreted as credit supply factors), and $\epsilon_{f,b,t}$ is an idiosyncratic unobserved factor that varies across firms, banks and time (capturing any interaction between bank and firm factors).

Crucially, the interpretation of the firm-specific $\alpha_{f,t}$ and bank-specific $\beta_{b,t}$ effects as credit demand and supply factors (or $\theta$ as a credit supply effect), respectively, relies on an assumption of *homogeneous transmission of credit shocks* which is embedded in the linear specification. That is, credit demand shocks to firms—e.g. lending changes due to firm-level productivity shocks, changes in factor costs, firm credit constraints—are transmitted *equally* to all banks, so captured by $\alpha_{f,t}$, and credit supply shocks to banks are transmitted *equally* to all firms. Any bank-firm interactions

$\epsilon_{f,b,t}$, for example, generated due to either firms with bank-specific demand or banks with firm-specific supply cannot be systematic, so they must be uncorrelated with $\alpha_{f,t}$ or $\beta_{b,t}$; moreover, they should be orthogonal to the network structure captured by $D_{f,b,t}$. We state the identification conditions in the following assumption.

**Assumption 1.** ***Exogenous network.*** *Let $D_t$, $\alpha_t$, $\beta_t$ denote the entire vector (at time $t$) of $D_{f,b,t}$, $\alpha_{f,t}$ and $\beta_{b,t}$ for all $f, b$, respectively. Then, $E[\epsilon_{f,b,t}|D_t, \alpha_t, \beta_t] = 0$ in (2) (or $E[\epsilon_{f,b,t}|D_t, \alpha_t, x_t] = 0$ in (1) where $x_t$ is the entire vector of $x_{b,t}$).*

The exogenous network assumption 1 implies the formation of bank-firm networks is exogenous once we condition on the firm-specific unobserved heterogeneity $\alpha_t$ and the bank-specific unobserved heterogeneity $\beta_t$ or observed factor $x_t$. That is, bank-firm interactions $\epsilon_{f,b,t}$ do not affect the formation of relationships. So, there are no systematic differences in $\epsilon_{f,b,t}$'s for different partitions of the network $D_t$.

Assumption 1 allows the network to be endogenous to the factors that we are controlling for in the regression, so it allows for sorting that depends on $\alpha_{f,t}, \beta_{b,t}$ (or $x_{b,t}$). For instance, it allows firms with high $\alpha_{f,t}$ to match more likely with banks with high $\beta_{b,t}$. However, assumption 1 does not allow the network to depends on characteristics that affect loan growth rates that vary at the bank-firm level like specialization or relationship lending. For instance, assumption 1 might fail if matching depends on the specific relation of a bank with a type of firms as highlighted by Paravisini et al. (2023) where firms that export to a particular market are more likely to start a relationship with a bank that specializes in that market.[9]

**Identification.** Given the exogenous network assumption 1, we can estimate model (2) (or (1)) conditioning on the observed network $D$ without the need of jointly modeling the distribution of $\{y^*_{f,b,t}, D_{f,b,t}\}$.

[9]KM and AW raise the identification concerns in the presence of bank-firm interactions. KM emphasize "... perhaps a firm's loan demand is bank-specific and is correlated with shocks to the bank's liquidity. This can happen if, (a) nuclear shocks disproportionately affect export/import demand, (b) firms get "export/import related" loans from banks that specialize in the tradeable sector, or (c) these export/import intensive banks had more dollar deposits and thus suffered a larger liquidity crunch as well"; they proceed to provide some evidence that these are not too relevant for the event they study. AW provide conditions under which bank-firm interactions do not affect the estimation of firm and bank shocks, as their Proposition 1 specifies this happens "as long as the components of the interaction term that vary only at the bank or firm level are defined to be part of the bank and firm shocks." The condition would be violated, for instance, if a group of firms had a bank-specific demand since such demand factor correlating with the bank-fixed effect should not be defined to be part of a bank shock. Also, note that when we exploit the time series, it is easy to include in (2) firm-bank interaction $\eta_{f,b}$ that is constant over time as in Di Giovanni et al. (2022); but this cannot account for interactions or heterogenous effects of time-varying shocks as in the examples explained above.

Multiple bank-firm connections for every $t$ allow to estimate $\alpha_{f,t}$, $\beta_{b,t}$ as time-varying firm and bank fixed effects. By taking differences of equation (2) for the same firm in two banks $b_0, b$ we eliminate the firm-specific factor and get

$$y_{f,b,t} - y_{f,b_0,t} = \left(\beta_{b,t} - \beta_{b_0,t}\right) + \left(\epsilon_{f,b,t} - \epsilon_{f,b_0,t}\right). \tag{3}$$

Assumption 1 plays a key role to separate $\left(\beta_{b,t} - \beta_{b_0,t}\right)$ and $\left(\epsilon_{f,b,t} - \epsilon_{f,b_0,t}\right)$ from equation 3: It implies that $E_{f\in I(b,b_0)}\left[\varepsilon_{f,b,t}\right] = E_{f\in I(b,b_0)}\left[\varepsilon_{f,b_0,t}\right]$, where $I(b, b_0)$ is the set of firms that borrow from both banks $b$ and $b_0$. Therefore, by comparing the average differential borrowing from the same set of firms connected with two different banks we can identify the bank-specific shocks using the following moment condition:

$$E_{f\in I(b,b_0)}\left[y_{f,b,t} - y_{f,b_0,t}\right] = \beta_{b,t} - \beta_{b_0,t}. \tag{4}$$

Such moment condition thus identifies the specific credit supply shock to bank $b$ relative to $b_0$: $\beta_{b,t} - \beta_{b_0,t}$ (or analogously $\theta\left[x_{b,t} - x_{b_0,t}\right]$). Intuitively, under this framework of homogeneous transmission of credit shocks, any systematic difference in the lending of two banks to the *same* set of firms cannot be driven by firm demand, so it is attributed to a bank shock. Similarly, the homogeneity assumption implies that the fixed effect $\alpha_{f,t}$ can be interpreted as a firm demand specific shock.

# 3 Evidence of heterogeneity

If the transmission of credit shocks is homogenous, then all systematic differences in the lending to the same firms across two banks is explained by the (relative) bank shock $\beta_{b,t} - \beta_{b_0,t}$. In particular, this average lending to firms from two banks $b, b_0$ should be the same across different sets/groups of firms, that is, the left hand side in equation (4) should be the same for any grouping of firms $I(b, b_0)$ that borrow from both banks. Figure 1 presents the sample analog of the left hand side of (4) in our data set for different grouping of firms. We consider only firms that borrow from the four main banks in our data set in the year 2017. We show the results for the entire set of common firms and when we split firms in two groups depending on: i) firm age, ii) their posted collateral size, iii) total debt size in previous years.

Panel 1a shows the average loan growth rate (relative to bank 1) across all firms. According to equation (4), such differential borrowing would capture each specific (homogeneous) bank shock $\beta_{b,t} - \beta_{b=1,t}$, so we would conclude that bank 2 experienced a negative bank shock of about -12% (relative to bank 1), while banks 3 and 4 experienced very small banks shocks. However, the next panels show that such differential borrowing across banks changes considerably across the different firm groupings. Panel 1b shows the differential average growth rates for firms above/below the median firm age. We can see that on average firms in both groups borrow less from bank 2 than from bank 1, but the magnitude is considerably different (around -9% vs -16%). More strikingly are the results for bank 3 and 4: firms in the blue (below-median) group experienced an increase in their borrowing from these banks, while firms in the red (above-median) group experienced decrease in their borrowing. Panels 1c and 1b show also differential patterns when splitting firms by their collateral or debt size. Interestingly, 1d shows that bank 3 reduces lending to the blue group and increases it to the red group; while the opposite pattern is shown by bank 4.

All panels indicate there exists heterogeneity in the responses by groups, suggesting that the additive effects of bank and firm shocks implied by the linear specification in section 2 is not satisfied in the data. Moreover, they suggest that heterogeneity may depend on the specific match between banks and firm groups.

# 4 A model with interactions and heterogeneous effects

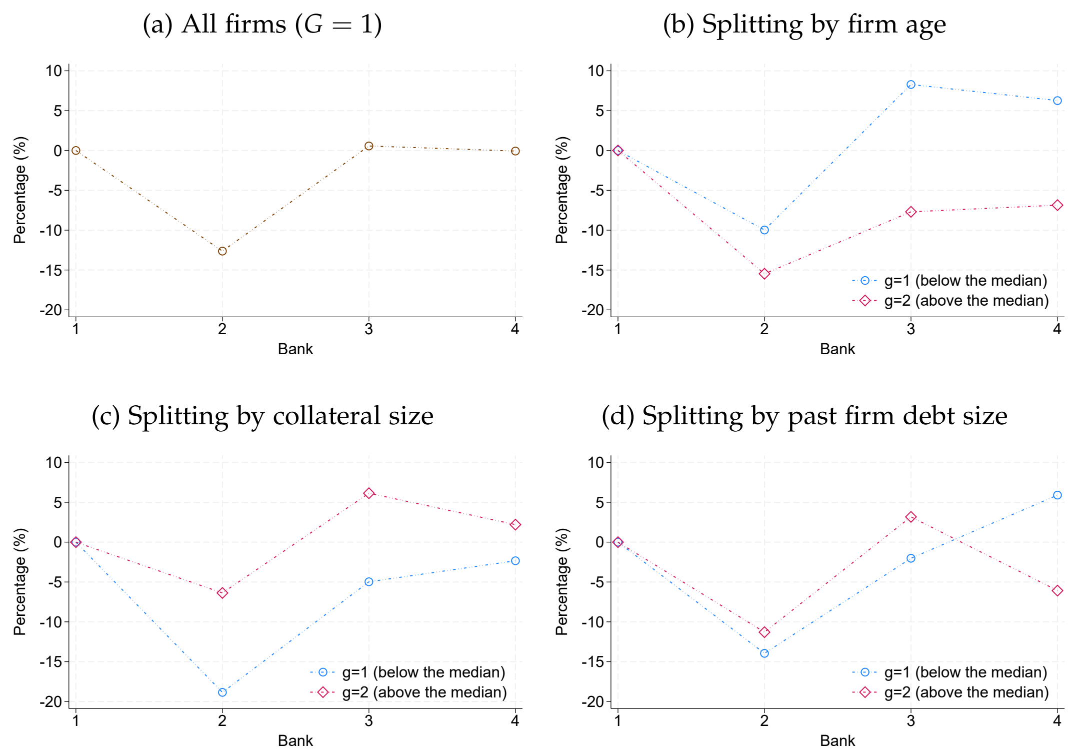
A general framework that accounts for interactions can be written as:

$$y^*_{f,b,t} = \alpha_{f,t} + \beta_{b,t} + \psi_{f,b,t} + \epsilon_{f,b,t}, \tag{5}$$

where $\psi_{f,b,t}$ accounts for any interactive factors that can affect credit growth and, as opposed to the idiosyncratic $\epsilon_{f,b,t}$, can be systematically correlated with $\alpha_{f,t}, \beta_{b,t}$ or the probability of creating a relationship $D_{f,b,t}$. For example, $\psi_{f,b,t}$ could capture a credit supply shock to bank $b$ that propagates heterogeneously to firms, in a context where firms for which the propagation is stronger (higher $\psi_{f,b,t}$) are also firms that are experiencing a larger shock $\alpha_{f,t}$ or that are most likely to be connected to a particular bank, so that $E[\psi_{f,b,t}|D_{f,b,t}] \neq E[\psi_{f,b,t}]$. Another example is that $\psi_{f,b,t}$ could capture a credit demand shock to firm $f$ that propagates heterogeneously to banks, in which the affected firm expands borrowing more from specific banks (higher $\psi_{f,b,t}$) that are also experiencing a larger shock $\beta_{b,t}$ or are more likely to form a connection. Interactions affecting credit growth $y^*_{f,b,t}$ that are not systematically correlated either with $\alpha, \beta$ or the network $D$ are instead accounted by $\epsilon_{f,b,t}$.

Of course, without imposing some structure we cannot identify or estimate specification (5). There are different methods that help uncovering interactions across unobserved factors. We explore two types of specifications. In our preferred approach, we model interactions occurring across a discrete number of firm types or groups, in which heterogeneity is left unrestricted across groups. In a second spec-

Figure 1: Mean differential loan growth rate for $t = 2017$ (relative to bank 1), by firm classification




*Note.* Average credit growth by the four largest banks to the sample of firms connected to all four banks. Panel (a) displays the average credit growth for all firms in the sample across banks. The next panels split the sample in two groups depending on: total firm debt size from all banks in previous years (panel 1d), the firm posted collateral (panel 1c), and firm age (1b).

ification, described in Online Appendix B, we use an interactive fixed effects model that allows for heterogeneity to be continuous but restricted to a factor structure.

**Group interactions.** Let $G$ be the number of groups (which is unknown and fixed), and let $\mathcal{G} = \{g_t(1), ..., g_t(N_F)\}$ be any grouping of firms into the $G$ groups in year $t$. Then, for each $f$, we have $g_t(f) \in \{1, ..., G\}$ which maps firms into groups. Let $y^*_{f,b,t}$ be the potential growth rate of a loan between bank $b$ and firm $f$ that belongs to a group $g_t(f)$ at time $t$ if the link $(f, b, t)$ exists. Let's consider the following specification with bank-firm group specific effects as an extension of KM and AW[10]:

$$y^*_{f,b,t} = \alpha_{f,t} + \beta_{b,g_t(f),t} + \epsilon_{f,b,t}, \tag{6}$$

[10]In Section 5, using our framework, we estimate a model with interactions when we observe an exogenous credit shifter a la Khwaja and Mian (2008).

where $\alpha_{f,t}$ is an unobserved time-varying firm effect (fixed across banks), $\beta_{b,g(f),t}$ is an unobserved time-varying *bank-firm group effect* (fixed across firms within groups), and $\epsilon_{f,b,t}$ is the unit specific error term. The firm effect $\alpha_{f,t}$ and bank-firm group effect $\beta_{b,g_t(f),t}$ are allowed to be arbitrarily correlated: The specification in (6) allows for unrestricted interactions between banks' and firm-types' (groups) characteristics.

**Examples.** Specification (6) can capture heterogeneity in the transmission of credit shocks that arises due to "market-specific" specialization as highlighted in Paravisini et al. (2023) and Blickle et al. (2023) or "loan-type" specialization as emphasized by Ivashina et al. (2022). Paravisini et al. (2023), using data from Peru, show that there is lending specialization based on the firms' export destination market and that the transmission of credit shocks depend on such margin. Blickle et al. (2023), using data from US, show that bank lending specialization occurs at the industry level. Ivashina et al. (2022), using data from Peru, show that bank shocks affect differentially firms depending on their type of loan contract used (e.g. asset based loans vs cash-flow based loans). Thus, firms would be grouped according to their export destination market, their industry, or the type of loans they use, respectively. In Online Appendix A, we describe a model of bank specialization in activities/markets as the one proposed by Paravisini et al. (2023), and show that, in such a model, our empirical specification with bank-firm group effects in (6) arises if firms' type/group is defined by their relative importance of activities.

Importantly, in our framework, the grouping is left unrestricted and treated as an additional unobserved function. This is useful in contexts in which the econometrician cannot access the specific data about bank or firm specialization (lack of data about export destination markets, industries, or loan-type used), or, when the heterogeneity is based on an unobserved margin or multiple margins. As shown below, as long as interactions happen at some (discrete) group level, we will be able to consistently identify the effects in models (6).

We replace assumption 1 for the following one:

**Assumption 2.** ***Exogenous network within groups***. *Let $\beta_t^G$ denote the entire vectors of $\beta_{b,g(f),t}$ for all $f, b$ for given $t$. Then, $E[\epsilon_{f,b,t}|D_t, \alpha_t, \beta_t^G] = 0$ in (6).*

This assumption states that the formation of networks is exogenous once we conditioned on the firm-specific unobserved heterogeneity and the bank-*firm group* unobserved heterogeneity. That is, within-group bank-firm interactions do not affect the formation of relationships between firms and banks. Importantly, as opposed to assumption 1, assumption 2 allows for endogenous matching of firms that depends on interactions or specialization occurring across groups. For example, it allows that firms exporting to particular market (thus belonging to a specific group $g$) have a higher/lower probability to form links with different banks (specialized in different markets), as Paravisini et al. (2023) shows it happens in Peruvian data.

## 4.1 Identification and estimation

**Known groups.** If groups were known, a comparison between the loan growth rate of firms in the same group $g$ borrowing from two banks $b$ and $b_0$, allows us to identify the differential $\beta_{b,g,t}$ from the following moment condition (analogous to (4)):

$$E_{f \in g_t, f \in I(b,b_0)} \left[ y_{f,b,t} - y_{f,b_0,t} \right] = \beta_{b,g_t,t} - \beta_{b_0,g_t,t}, \ \forall g_t \in \{1, ..., G\}. \tag{7}$$

Under a normalization (e.g. $\beta_{b_0,g_t,t} = 0$), we can recover each of the $\beta_{b,g,t}$ from the moment condition in (7). Finally, we can calculate for each firm its loan growth rate relative to the $\beta_{b,g,t}$ and average over all its connected banks, denoted $I(f)$, which from (6) allows to identify the firm-specific factor $\alpha_{f,t}$ as:

$$E_{b \in I(f)} \left[ y_{f,b,t} - \beta_{b,g,t} \right] = \alpha_{f,t}. \tag{8}$$

If groups were known, then a standard estimation of time-varying bank and firm fixed effects for each group would provide consistent estimates of these objects.

**Unknown groups.** We build on Bonhomme and Manresa (2015) and the econometrics literature that develop techniques to group observations (Bai (2009), Ando and Bai (2016), Bonhomme et al. (2019)). Typically, clustering techniques cluster the data into groups depending on some dissimilarity measure based on some observable characteristics. Instead, in our setup, firms are grouped according to their dissimilarities on their unobserved response to unobserved shocks, since the heterogenous credit shocks are unobserved. To implement our framework, we extend the method in Bonhomme and Manresa (2015) that allow for time-varying grouped effects in a standard panel data by combining it with the two-sided heterogeneity framework that exploits bank-firm network data (as in KM or AW).

Lets define the parameter spaces $A$ and $B$ which are subsets of $\mathbb{R}$, and $\alpha_{f,t} \in$ A and $\beta_{b,g_t(f),t} \in B$. Also $\mathcal{G} \in \Gamma_G$ where $\Gamma_G$ is the set of all groupings of $\{1, ..., N_F\}$ into at most $G$ groups. Following Bonhomme and Manresa (2015), for a pre-defined number of groups $G$ we define our estimator, which we refer to as "bank-firm grouped fixed-effects" (BF-GFE), as the solution of the following problem. For each $t = 1, ..., T$,

$$\left(\widehat{\alpha}_t, \widehat{\beta}_t, \widehat{\gamma}_t\right) = \arg \min_{(\alpha,\beta,\gamma)\in A^{N_F}\times B^{N_B G}\times \Gamma_G} \sum_{f=1}^{N_F} \sum_{b=1}^{N_B} \left(y_{f,b,t} - \alpha_{f,t} - \beta_{b,g_t(f),t}\right)^2, \tag{9}$$

where the minimum is taken over all possible groupings $\gamma = \{g_t(1), ..., g_t(N_F)\}$ of the $N_F$ firms into $G$ groups, firm-time effects, and bank-group-time effects.

For given $\alpha_{f,t}$ and $\beta_{b,g_t(f),t}$, the optimal group assignment for each firm is:

$$\widehat{g}\left(f|\alpha_t, \beta_t\right) = \arg\min_g \sum_{b=1}^{N_B} \left(y_{i,t} - \alpha_{f,t} - \beta_{b,g_t(f),t}\right)^2, \tag{10}$$

where we take the minimum $g$ in case of a non-unique solution. The estimator of $\alpha_{f,t}$ and $\beta_{b,g(f),t}$ in (9) can be written as:

$$\left(\widehat{\alpha}_t, \widehat{\beta}_t\right) = \arg \min_{(\alpha,\beta)\in R^{N_F}\times R^{N_B G}} \sum_{f=1}^{N_F} \sum_{b=1}^{N_B} \left(y_{f,b,t} - \alpha_{f,t} - \beta_{b,\widehat{g}_t(f|\alpha_t,\beta_t),t}\right)^2, \tag{11}$$

where $\widehat{g}_t\left(f|\alpha_t, \beta_t\right)$ is given by (10), and the BF-GFE estimate of $g_t(f)$ is $\widehat{g}_t\left(f|\widehat{\alpha}_t, \widehat{\beta}_t\right)$.

A couple of key differences with the empirical model in Bonhomme and Manresa (2015). First, in their model, the individual effects must be fixed in time, our framework allows for both individual and the bank-firm group effects to vary across time by using the multi-year bipartite network. The time-varying individual firm effects are crucial in this framework to account for credit demand shocks. Second, our framework allows for the grouping $\gamma_t$ to change every period, while it must be fixed in time in theirs. We are able to estimate time-varying firm effects and time-varying grouping because we have a panel model with three dimension and we can exploit the multiple connections variation in our data.[11]

The estimation approach in (9) jointly estimates the unobserved grouping structure of the data $g(f)$ and the unobserved time-varying effects $\alpha_{f,t}$ and $\beta_{b,g_t(f),t}$. The

[11]We also estimate the model fixing the grouping in certain time windows, in which case the estimator in (9) is replaced by the following. Let $\mathbb{T} \equiv \{t_1, ..., t_m\}$ be the time window, then for each $t \in \mathbb{T}$:

$$\left(\left\{\widehat{\alpha}_t, \widehat{\beta}_t\right\}_{t\in\mathbb{T}}, \widehat{\gamma}\right) = \arg \min_{(\alpha,\beta,\gamma)\in A^{mN_F}\times B^{mN_B G}\times \Gamma_G} \sum_{t\in\mathbb{T}} \sum_{f=1}^{N_F} \sum_{b=1}^{N_B} \left(y_{f,b,t} - \alpha_{f,t} - \beta_{b,g_t(f),t}\right)^2.$$

problem in (9) can be split into two *unfeasible* estimation problems. The problem in (10) is a classification problem as in the unsupervised learning literature. However, the classification here depends on two unobserved components $\alpha_{f,t}$ and $\beta_{b,g(f),t}$ as opposed to the standard classification problem in unsupervised learning where the classification is based on a dissimilarity measure that is observable. The problem in (11) is a two-way fixed-effects regression for the group classification given by (10). In fact, the estimator in (11) is the sample analogue of the moment conditions in equations (7) and (8) for $g_t(f) = \widehat{g}_t(f|\alpha_t, \beta_t)$ when $N_B = 2$.

The following proposition indicates that the BF-GFE estimator under unknown groups is consistent and asymptotically equivalent to the unfeasible estimator (under known groups). It also implies that $N_B$ can increase polynomially more slowly than $N_F$ which is a crucial condition for empirical applications since standard bank-firm loan-level data contains many more firms than banks. In particular, Online Appendix C discusses the problem of group misclassification for fixed $N_B$, and it shows that the probability of misclassifying tends to zero at an exponential rate which implies that the bias generated by the incidental parameter problem goes to zero very fast as $N_B$ increases, similarly to the group fixed effect estimator of the standard panel model when $T$ increases (see Bonhomme and Manresa (2015)). Moreover, we show in simulations that the performance of our estimator increases very rapidly with $N_B$ and it is centered at the true value for moderate $N_B$.

**Proposition 1.** *(Consistency) Assume a fully connected network with a fixed number of groups $G$ and a grouping function $g_t(f)$ for which assumption 2 holds. Then, under weak cross-sectional dependence in the errors of the model in (6) and well-separated groups, the estimator $\left(\widehat{\alpha}_t, \widehat{\beta}_t\right)$ in (9) provide consistent estimates of $(\alpha_{f,t}, \beta_{b,g_t(f),t})$ as $N_F$ and $N_B$ tend to infinity, and for all $\delta > 0$:*

$$\widehat{\beta}_{b,\widehat{g}_t(f),t} = \widehat{\beta^u}_{b,g_t(f),t} + o_p\left(N_B^{-\delta}\right) \qquad \textit{for all } b, f, t.$$

*where the unfeasible estimator $\widehat{\beta^u}_{b,g_t(f),t}$ is the solution of (11) when the grouping is fixed to its population counterpart rather than being estimated.*

## 4.2 Computation

Following the discussion in subsection 4.1, our proposed algorithm involves decomposing the estimator presented in Equation (9) into two distinct problems. Specifically, we suggest an iterative approach that alternates between addressing the classification problem outlined in Equation (10) and solving the two-way fixed effect estimation problem as described in Equation (11):

**Algorithm** For each $t = 1, ..., T$,

1. Set the number of groups: $G$.
2. Set $s = 0$. Guess initially some group assignment $g^{(s=0)}(f) \in \{1, ..., G\}$.[12]
3. For given $g^{(s)}(f)$, estimate firm and bank-firm-group fixed effects $\hat{\alpha}^{(s)}_{f,t}, \hat{\beta}^{(s)}_{b,g(f),t}$:

$$\left(\hat{\alpha}^{(s)}, \hat{\beta}^{(s)}\right) = \arg \min_{\alpha_{f,t}, \beta_{b,g^{(s)}(f),t}} \sum_{f=1}^{N_F} \sum_{b=1}^{N_B} \left(y_{f,b} - \alpha_{f,t} - \beta_{b,g^{(s)}(f),t}\right)^2. \tag{12}$$

4. For given $\widehat{\alpha}^{(s)}, \widehat{\beta}^{(s)}$, select optimal group assignment: For all $f = 1, ..., N_F$

$$g^{(s+1)}(f) = \arg \min_{g \in \{1,...,G\}} \sum_{b=1}^{N_B} \left(y_{f,b} - \hat{\alpha}^{(s)}_{f,t} - \hat{\beta}^{(s)}_{b,g,t}\right)^2 \tag{13}$$

5. Set $s = s + 1$ and go to Step 3 until numerical convergence.

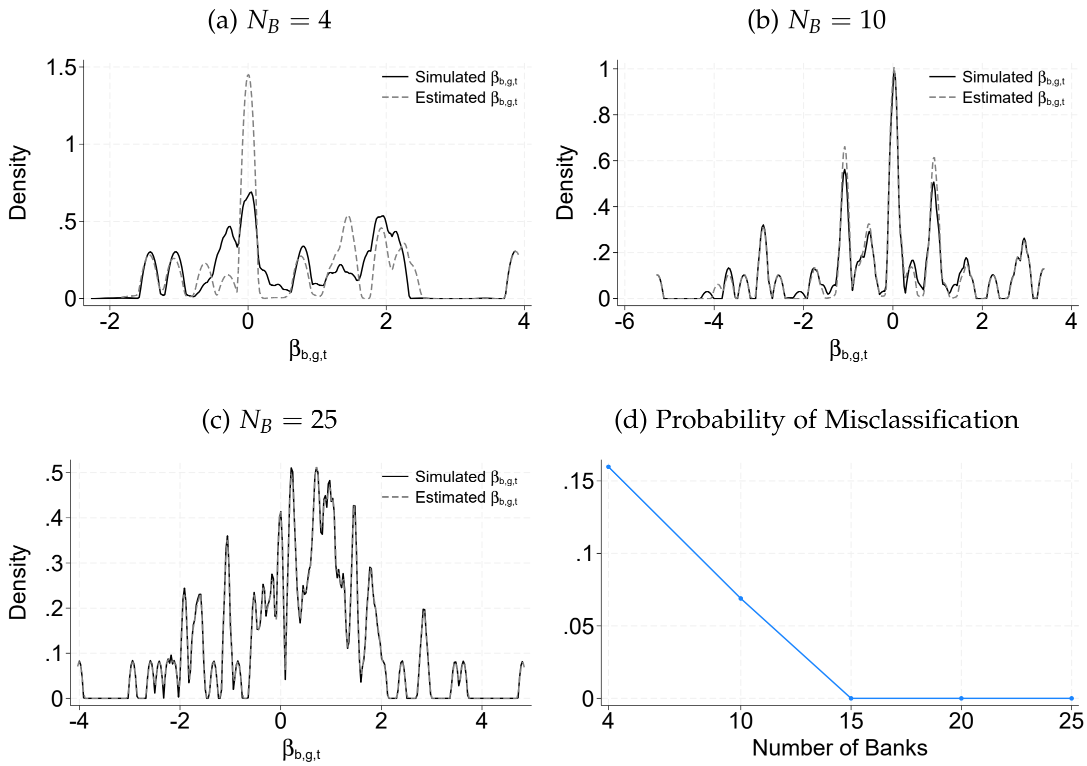
**Properties of our estimator.** To illustrate the performance of our proposed algorithm, we simulate a model based on (6). In an initial simulation, we assume an exogenous network with full connections. The primary objective of this exercise is to emphasize our algorithm's performance across various values of $N_B$. We fixed $G = 4$ and $N_F = 5000$ and conducted 100 replications of $y_{f,b}$ based on (6). For each replication, we estimated $\beta_{b,g(f)}$ using our algorithm. Figure 2 illustrates the estimator accuracy for different values of $N_B$: 4, 10, and 25. Since there are many ($G \times N_B$) credit supply shocks $\beta_{b,g(f)}$ that are simulated, the figure presents a k-density over these $G \times N_B$ generated parameters and the k-density of the estimated ones. Panels (a)-(c) display the density of each of the true parameter $\beta_{b,g(f)}$ (the black solid line) and the density of each of the estimates $\hat{\beta}_{b,g(f)}$ across replications for different values of $N_B$. Panel (d) displays the probability of misclassification for different values of $N_B$. As our asymptotic analysis predicts, increasing the number of banks reduces the probability of misclassifying firms into groups and decreases the bias in the estimator. For a small number of banks ($N_B = 4$), the probability of misclassification is 16% due to the incidental parameter problem inherent in small samples. The latter translate in a

[12] For our initial guess, we estimate homogeneous firm and bank fixed-effects, and then group firms using "k-means" over the estimated $\widehat{\alpha}_{f,t}$'s. We also start with different initial guesses and select the estimator with lowest mean square error.

small sample bias in $\hat{\beta}_{b,g(f)}$. However, the probability of misclassification decreases rapidly as we increase the number of banks. It reduces to 6.89% for $N_B = 10$ and reaches 0 for $N_B = 15$. That is, the bias generated by the incidental parameter problem disappears.

Additional simulation results are presented in section 5 and Online Appendix D. Section 5 presents a simulation with an endogenous network depending on interactions, in which groups are determined by observable characteristics in our dataset. D presents the simulation results for an example specifically calibrated to our dataset. In both cases, our estimator closely tracks the true parameters.

Figure 2: Asymptotic properties of the estimator




*Note.* Illustration of estimator's properties using Monte Carlo simulations. We assume $G = 4$, $N_F = 5000$, and different values of $N_B$. We first draw parameters from the following distributions $\alpha_f \sim N(5,1), \beta_{bg} \sim N(3,1.5)$. For every panel, these parameters are held constant across $S = 100$ replications, for which we simulate $\epsilon_{f,b,(s)} \sim N(0,0.35)$. Panels (a) - (c) show the simulated and estimated parameters for $N_B = 4, 10, 25$. Since there are many $(N_b \times G)$ simulated $\beta_{bg}$'s, we display the k-density across all $\beta_{bg}$'s. Panel (d) displays the probability of misclassification.

**Number of groups.** For most exercises, we provide results for different number of groups $G$ going from 1 to 5. However, in Online Appendix E, we build on Almagro and Manresa (2021) and propose an N-fold cross-validation procedure for our algorithm that helps determining the optimal number of groups based on an out-of-sample forecasting performance.

### 4.3 Opening the black box

The following section uses an example to provide intuition behind the group identification from our algorithm.

Figure 3 presents a simulated example of a model according to specification (6) with five firms ($N_F = 5$). Each circle in the middle denotes each firm, the line between each bank and the firm denotes the credit growth $y_{f,b}$, with the color and width representing the value of $y_{f,b}$. In the example, panel 3a shows the average credit growth from bank $B$ to the five firms is around $-3\%$, while for bank $A$ is close to 0. The standard framework suggests that such difference in average credit growth from the two banks represents the effect of a bank shock $\beta_B - \beta_A$. However, we can visually detect that the difference in credit growth for the top two firms is around 5%, while for the three bottom firms is around $-8\%$, suggesting they belong to different groups. But, by looking at (6) such differential credit could be explained either by a different $\beta_{b,g(f)}$ or by the $\epsilon_{f,b}$. How can we disentangle between these two factors? This is achieved by the algorithm by looking at how other banks treat these firms. For instance, Panel 3b shows the credit growth from an additional bank $C$. If the differences in credit were generated by the idiosyncratic factor $\epsilon_{f,b}$, then the group of firms treated differently by bank $B$ should not predict the group of firms treated differently by bank $C$. However, Panel 3b shows that bank $C$ also treats differently the top two firms relative to the bottom three, suggesting that the two top firms belong to one group, while the three bottom firms belong to another group. In this particular example, bank $C$ lowers credit more to the top firms while bank $B$ expands their credit, which could indicate bank 1 has an special relationship with them.

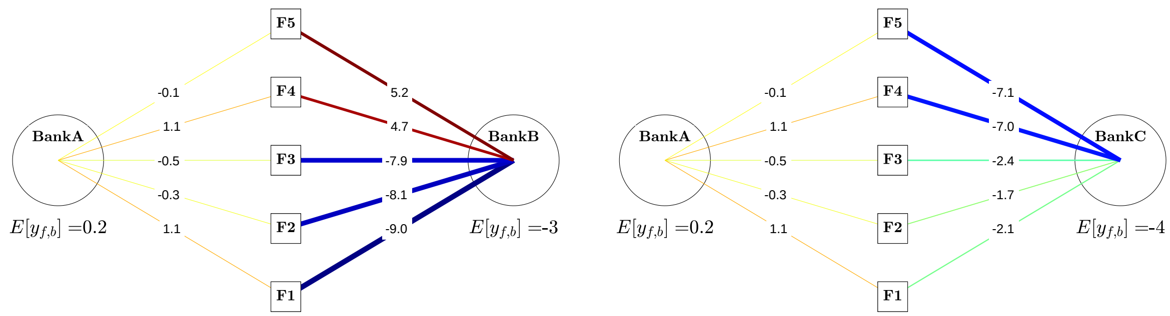
The example illustrates that the algorithm learns the group classification by considering how banks treat systematically different some firms relative to others. This is further discussed in Online Appendix C where we show that the probability of misclassifying tends to zero at an exponential rate as $N_B$ increases.

## 5 Discussion: Interpretation of heterogenous bank-firm group effects and homogenous estimates

Our framework will capture any variation in credit that depend on bank-firm interactions that happens at the bank-firm group level. Any type of bank specialization (e.g. by industry, export destination, type of loan contract) that leads to a differential

Figure 3: Group Identification Illustration: Example with $N_F = 5$

(a) Bank A vs Bank B (b) Bank A vs Bank C




transmission of credit shocks across firm groups can be identified by our method. However, these estimated parameters require careful interpretation.

**Interpretation of *bank-firm group effect*.** The coefficients $\beta_{b,g_t(f),t}$ will capture any factors that vary at the bank-firm group level, but different stories may lead to such bank-firm group variation. A first interpretation is that at $t$ bank $b$ experiences a credit supply shock which is then transmitted differently across each group of firms $g_t(f)$. A second interpretation is that firms in group $g_t(f)$ experience a *common* credit demand shock which leads to a differential borrowing across banks (and therefore not captured by the firm-fixed factor $\alpha_{f,t}$). Third, a combination of both forces. For instance, this can be seen in the model in Online Appendix A, specifically in (A.3).

Even when the true data-generating process includes interactions as in specification (6), the decomposition of credit growth into (homogeneous) firm-specific and bank-specific factors as in equation (2) is indeed possible, but the estimated parameters require careful interpretation. Let us denote the estimate of the homogeneous effect $\beta_{b,t}$ in (2) as $\hat{\beta}^{\text{Homo}}_{b,t}$. Proposition 2 states that if there were no *endogenous matching* on the creation of relationships that depends on such interactions (i.e. $D_{f,b,t}$ is independent of $\beta_{b,g,t}$), then $\hat{\beta}^{\text{Homo}}_{b,t}$ would identify an average of the heterogeneous bank-firm group effects. Under the first interpretation in which $\beta_{b,g_t(f),t}$ captures (heterogeneous) credit supply shocks, the estimate $\hat{\beta}^{\text{Homo}}_{b,t}$ will capture an average credit supply shock to bank $b$, but it will miss the heterogeneity in the transmission of the shock. Under the second interpretation, then the estimate $\hat{\beta}^{\text{Homo}}_{b,t}$ will capture an (average) group demand shock to firms that leads to an heterogeneous borrowing across banks, so in this case a credit supply interpretation would be misleading.

**Proposition 2.** *If bank-firm network $D_{f,b,t}$ is independent of $\beta_{b,g,t}$, then the homogenous estimate $\hat{\beta}^{Homo}_{b,t}$ identifies an average of the heterogeneous bank-firm group effects: $\hat{\beta}^{Homo}_{b,t} \stackrel{a}{=} \sum_{f=1:N_F} \beta_{b,g(f),t}/N_F$.*

Instead, in the case in which there exists endogenous matching based on interactions, so that the probability of creating a relationship $D_{f,b,t}$ depends on the interaction $\beta_{b,g,t}$, then the estimate $\hat{\beta}^{\text{Homo}}_{b,t}$ may not even converge to the average of the heterogeneous effects. For instance, consider the case of two banks $b, b_0$, and let $I_{b,b_0}$ be the joint network; we have that the standard fixed effect estimator (normalizing the bank effect of $b_0$ to zero):

$$\hat{\beta}^{\text{Homo}}_{b,t} \stackrel{a}{=} E\left[y_{f,b,t} - y_{f,b_0,t} \mid f \in I_{b,b_0}\right] = E\left[\beta_{b,g(f)} \mid f \in I_{b,b_0}\right] \neq E\left[\beta_{b,g(f)}\right], \tag{14}$$

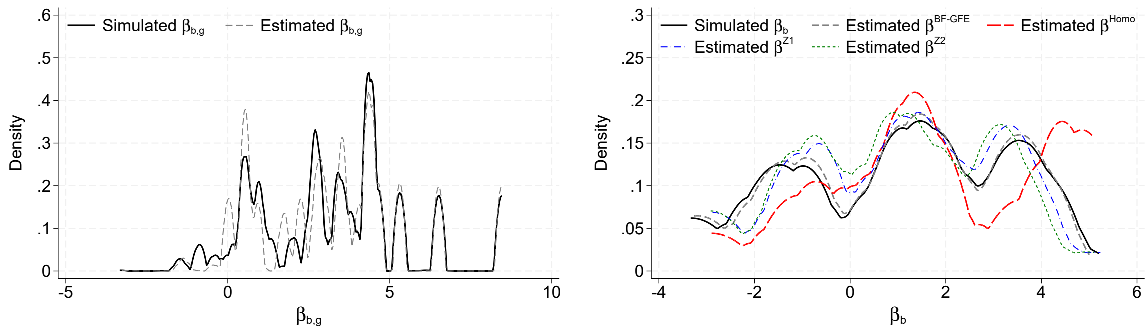
where the first equality follows from (4), the second equality uses (6) and assumption (2), and the third inequality arises because of the conditioning on the joint network. For instance, bank $b$ and $b_0$ may each be specialized in lending to different types of firms, affecting the composition of their joint network.

**Simulation with endogenous matching and groups defined by observables.** To illustrate the bias of $\hat{\beta}^{\text{Homo}}_{b}$ when the network of connections depends on the interaction effect $\beta_{b,g}$, we conduct a Monte Carlo simulation with a matching probability between banks and firms as a function of $\beta_{b,g}$. Following the evidence in section 3, we model the firm group's heterogeneous response as a function of a linear combination of two observable characteristics in the data: firm age ($Z_1$) and collateral ($Z_2$). We simulate 5000 firms distributed into $G = 4$ groups based on the variable $\text{Group}_f$ as the quartile of $\frac{1}{2}Z_1 + \frac{1}{2}Z_2$.[13] We choose $N_B = 10$ and define $\beta_{b,g,t} = \text{Group}_f \times \psi_b$ with $\psi_b \sim N(0,1)$, and set $\alpha_f \sim N(0,1), \varepsilon_{f,b} \sim N(0, 0.05)$. The endogenous matching probability follows $\text{Prob}\left(D_{f,b} = 1 \mid \alpha_f, \beta_{b,g(f)}\right) = \pi\left(\alpha_f + \beta_{b,g_t(f)}\right)$ where $\pi(.)$ is an increasing function. We generate 100 replications of $y_{f,b,t}$ according to (6). Panel (a) of Figure 4 displays the k-density of the $G \times N_B$ interaction effects $\beta_{b,g,t}$ and our estimates $\hat{\beta}_{b,g(f)}$. Similar to the simulations in Figure 2, our BF-GFE estimator closely tracks the true parameters. In Panel (b) of Figure 4, we present estimators of bank-specific *average* effects: $\beta_b = \sum_{g=1}^{G} \frac{N_{b,g}}{N_b} \beta_{b,g(f)}$ for $b = \{1 \ldots 10\}$, where $N_{b,g}$ is the number of firms in group $g$ connected to $b$ and $N_b = \sum_{g=1}^{G} N_{b,g}$. We compare the

[13] Note that $\text{Group}_f$ has four values, and all firms belonging to the same group (the same quartile of $\frac{1}{2}Z_1 + \frac{1}{2}Z_2$) possess the same value of $\text{Group}_f$.

Figure 4: Average estimators using different methods

(a) Estimates of $\beta_{b,g(f)}$ (b) Estimates of $\beta_b$ by method




Note. Illustration of the properties of estimators of $\beta_{b,g(f)}$ and $\beta_b$ using Monte Carlo simulations with endogenous bank-firm matching. Panel (a) displays the individual estimates of $\beta_{b,g(f)}$. Panel (b) shows the estimates of $\beta_b$ under different methods.

distribution across simulations of: $\hat{\beta}_b^{\text{BF-GFE}} = \sum_{g=1}^{G} \frac{N_{b,g}}{N_b} \hat{\beta}_{b,g(f)}$, where $\hat{\beta}_{b,g(f)}$ is the BF-GFE estimator with unknown groups, and $\hat{\beta}_{b,t}^{\text{Homo}}$ the two-way fixed effect estimator (assuming homogeneity). The BF-GFE estimator tracks closely the true average $\beta_b$ for all banks, while the homogeneous estimator is a biased estimator of the bank-specific average effect. Additionally, we show the results for a group estimator defined by using only one of the observable characteristics: $\hat{\beta}_b^{Z_j} = \sum_{g=1}^{G} \frac{N_{b,g}}{N_b} \hat{\beta}_{b,g(f)}^{Z_j}$, where $\hat{\beta}_{b,g(f)}^{Z_j}$ is a within-group estimator (a two-way fixed effect for each group) with groups defined by the observable variable $Z_j$, for $j = \{1, 2\}$. The example shows that controlling heterogeneity using only one observable variable helps but it is insufficient.

## 5.1 Interpretation with credit supply shifter $x_b$.

Typically, the bank lending channel literature rely on a bank-specific variable $x_{b,t}$ associated with a credit supply shock as in the seminal paper by KM. The issue of interpretation is less stringent when we can rely on such credit supply shifters.

In case we observe a variable with bank-specific variation capturing a credit supply shock, consider the following extension to KM model that allows for interactions:

$$y^*_{f,b,t} = \alpha_{f,t} + \theta_{b,g(f),t} x_{b,t} + \epsilon_{f,b,t}, \tag{15}$$

where $\theta_{b,g_t(f),t}$ captures an heterogeneous response of firms to the specific bank shock. In this model, we leave the relationship between $\alpha_{f,t}$, $x_{b,t}$ and $\theta_{b,g(f),t}$ totally unrestricted allowing for correlation between the bank "treatment" shock ($x_{b,t}$) and the bank "treatment" effect ($\theta_{b,g_t(f),t}$), as in the "potential outcome framework" with heterogeneous treatment effects used in the micro-econometric literature. Allowing for unrestricted dependence between $\theta_{b,g_t(f),t}$ and $x_{b,t}$ is important under endogenous matching between banks and firms.

The coefficient $\theta_{b,g_t(f),t}$ can be interpreted as the heterogeneous elasticity to the credit supply shock $x_{b,t}$ as long as any credit demand factor that happens to be correlated with the bank-specific shock $x_{b,t}$ is accounted by the firm-specific factor $\alpha_{f,t}$ (fixed across banks). Thus, any heterogeneous demand factor captured by $\epsilon_{f,b,t}$ must be exogenous as the following assumption specifies.

**Assumption 3.** ***Exogenous network within groups conditional on*** $x_b$. *Let* $\theta_t^G$ *denote the entire vector* $\theta_{b,g(f),t}$ *for all* $f, b$ *for given* $t$. *Then,* $E[\epsilon_{f,b,t}|D_t, \alpha_t, x_t, \theta_t^G] = 0$ *in (15).*

Similar to assumption 2, assumption 3 allows for endogenous matching based on the specific bank-group response $\theta_{b,g_t(f),t}$. Some identification concerns arise in this framework. How can we identify the heterogenous bank (treatment) effects $\theta_{b,g_t(f),t}$? Can we use our estimated interactions $\beta_{b,g_t(f),t}$ to consistently estimate $\theta_{b,g_t(f),t}$? Would the homogeneous bank estimate $\hat{\theta}^{\text{Homo}}$ in (1) identify any interesting *average bank effect*?

Let us start this discussion emphasizing that there are two degrees of heterogeneity in the bank treatment effect of $\theta_{b,g_t(f),t}$. First, the bank effect may differ across firm groups. Second, the effect for each group may be different by banks. This double heterogeneity is what we would expect under bank specialization, since a bank specialized in firms in a particular activity $g$ will treat firms in such a group differently relative to a not specialized bank.

For notational convenience, we eliminate the subindex $t$. Let $x_{b_0} = 0$ denote the case for a bank not exposed to the shock (e.g. a bank with low foreign liabilities in the credit event studied in section 8.1). In the presence of double heterogeneity in effects and varying bank exposures, we define the average bank-specific response of a bank $b$ with bank shock $x_b$ as follows:

$$\Delta_{x_b} = E\left[\sum_{g=1}^{G} \frac{\sum_{f\in I_{b,b_0,g}} \left[y_{f,b} - y_{f,b_0}\right]}{N_{b,b_0}\left[x_b - x_{b_0}\right]} \mid x_b, D\right], \tag{16}$$

where $I_{b,b_0,g} = \{f : g(f) = g\} \cap I_{b,b_0}$, is the set of firms in group $g$ that belongs to $I_{b,b_0}$, $N_{b,b_0,g} = \sum_{f\in I_{b,b_0,g}} 1$ is the number of firms in this set, and $N_{b,b_0} = \sum_{g=1}^{G} N_{b,b_0,g}$ is the total number of firms in $I_{b,b_0}$. Then, given assumption 3

$$\Delta_{x_b} = E\left[\sum_{g=1}^{G} \frac{N_{b,b_0,g}}{N_{b,b_0}} \theta_{b,g(f)} \mid x_b, D\right]. \tag{17}$$

The $\Delta_{x_b}$ represents the average response in credit growth for all groups of firms connected to a bank with exposure $x_b$, relative to the non-exposed bank, in the specific $b, b_0$ joint network, that is, weighted by the number of firms in each group connected to banks $b$ and $b_0$. (17) shows this bank-specific effect is a function of the heterogeneous parameter $\theta_{b,g(f)}$.

Some researchers may be interested in the overall response to the credit event (associated with $x$). For instance, we define the Average Bank Effect ($\theta_{\text{ABE}}$) across banks (relative to the non-exposed) as:

$$\theta_{\text{ABE}} \equiv \sum_{b \neq b_0}^{N_B} \frac{N_{b,b_0}}{N} \Delta_{x_b} = \sum_{b \neq b_0}^{N_B} \sum_{g=1}^{G} \frac{N_{b,b_0,g}}{N} E\left[\theta_{b,g(f)} \mid x_b, D\right],$$

where $N = \sum_{b \neq b_0}^{N_B} N_{b,b_0} = \sum_{b \neq b_0}^{N_B} \sum_{g=1}^{G} N_{b,b_0,g}$. The second equality follows from (17) and shows $\theta_{\text{ABE}}$ is a weighted average across banks and groups of the expected treatment effects $\theta_{b,g(f)}$, where weights capture the importance of each group across the joint networks between exposed and unexposed banks, reflected by $N_{b,b_0,g}$.

Having defined the objects of interest, we discuss how to estimate them consistently. We propose using our BF-GFE estimates $\hat{\beta}_{b,g}$ to construct a consistent estimators of the $\theta_{\text{ABE}}$. We first define estimates of the heterogeneous bank effect as:

$$\hat{\theta}_{b,g} \equiv \frac{\hat{\beta}_{b,g} - \hat{\beta}_{b_0,g}}{x_b}, \ \forall g \in \{1, ..., G\}. \tag{18}$$

and for the bank average effects:

$$\widehat{\theta}_{\text{ABE}} = \sum_{b \neq b_0}^{N_B} \sum_{g=1}^{G} \frac{N_{b,b_0,g}}{N} \hat{\theta}_{b,g}. \tag{19}$$

Using (7), proposition 1 and that $x_{b_0} = 0$, we have that $\hat{\beta}_{b,g} - \hat{\beta}_{b_0,g}$ is a consistent estimator of $E\left[y_{f,b} - y_{f,b_0}\right] = \theta_{b,g(f),t} x_{b,t}$, implying that $\widehat{\theta}_{\text{ABE}}$ is consistent.

A critical question arises regarding the consistency of the estimator of the homogeneous effect in specification (1), where a common parameter is estimated using a firm fixed-effect regression. We refer to such estimate as $\hat{\theta}^{\text{Homo}}$. Proposition 3 demonstrates that the expectation of the fixed effect estimator in (1) is also a weighted average of the heterogeneous effects $\theta_{b,g(f)}$, but with different weights than $\theta_{\text{ABE}}$.

**Proposition 3.** *Assume a bank-firm bipartite network data generated from (15). If assumption 2 holds, then:*

$$E\left[\hat{\theta}^{Homo} \mid x, D\right] = \sum_{b \neq b_0}^{N_B} \sum_{g=1}^{G} \omega_{g(f),b} \frac{N_{b,b_0,g}}{N} E[\theta_{b,g(f)} | x_b, D], \tag{20}$$

*where* $\omega_{g(f),b} = \frac{\sum_{f \in I_{b,b_0,g}}^{N_{b,b_0,g}} v_{f,b} x_b}{\frac{N_{b,b_0,g}}{N} \sum_{b=1}^{N_B} x_b \sum_{g=1}^{G} \sum_{f \in I_{b,b_0,g}}^{N_{b,b_0,g}} v_{f,b}}$ *are weights that add up to one and* $v_{f,b}$ *term represents the residual of a firm fixed effect regression of* $x_b$ *on firm-specific fixed effects, utilizing the network data for firms with connections to at least two banks.*[14]

As opposed to $x_b$, $v_{f,b}$ varies at the firm level since the network is unbalanced so each firm $f$ are connected with different number of banks. Proposition 3 implies that in general the KM estimator $\hat{\theta}^{\text{Homo}}$ is a biased estimator of the $\theta_{\text{ABE}}$. This result is similar to the bias that appears in a typical panel model with random coefficients when the random coefficient is not independent of the regressors of the model (see Chamberlain (1992) and Arellano and Bonhomme (2012)), and the bias that appears in the two-way fixed effect estimator in a dif-dif model with heterogenous treatment effects (see De Chaisemartin and d'Haultfoeuille (2020), Sun and Abraham (2021)).

**Corollary 1.** *If* $E[\theta_{b,g(f)} \mid x_b, D] = E[\theta_{b,g(f)}]$, *then* $\hat{\theta}^{Homo}$ *is a consistent estimator of* $\theta_{ABE}$.

Corollary 1, which its proof is a direct direct consequence of (20), states that only under the absence of endogenous matching in the heterogeneous treatment effect (i.e., when $E[\theta_{b,g(f)} \mid x, D] = E[\theta_{b,g(f)}] = \theta$) or in absence of heterogeneous effects (i.e., when $\theta_{b,g(f)} = \theta$), $\hat{\theta}^{\text{Homo}}$ is a consistent estimator of $\theta_{\text{ABE}}$. This result is analogous to Arkhangelsky et al. (2021) in which a random network in our framework plays a similar role than the random assignment in their dif-in-dif model.

Proposition 3 states that the weights $w_{g,b}$ add up to 1 but they may be negative, in a similar fashion to the result in De Chaisemartin and d'Haultfoeuille (2020). Negative weights make the interpretation of $\hat{\theta}^{\text{Homo}}$ problematic. For instance, one could have that $\hat{\theta}^{\text{Homo}}$ is positive even when all $\theta_{b,g(f)}$ are negative. Endogenous matching

[14]The KM fixed effect estimator uses the sample of firms for which $I_{b,b_0}$ is not null for all $b$.

on the treatment effect and the issue of negative weights may be expected in a context of bank specialization, as the following example illustrates.

To illustrate this result, we present a simulated example with $N_B = 7$ banks, $N_F = 10000$ firms and $G = 2$ groups. We assume three banks ($b = 1, 2, 3$) are specialized in lending to firms in group $g = 1$, other three banks ($b = 4, 5, 6$) are not specialized, while bank 7 is specialized in group $g = 2$. Banks experience a (negative) credit event with the following exposures of $x_{b=1:3} = 1$, $x_{b=4:6} = 0.8$, and $x_{b=7} = 0$. We assume the shock is propagated to firms in group 1 according to $\theta_{b=1:3,g=1} = -0.05, \theta_{b=4:6,g=1} = -0.7$, so that banks 1 : 3 which are specialized protect firms in this group relative to the others. For group 2, we assume $\theta_{b=1:3,g=2} = -0.5, \theta_{b=4:6,g=2} = -0.3$. The likelihood of connecting with firms in each group increases with specialization: we assume Prob $\left(D_{f,b} = 1|g(f) = 1, b\right)$ equals 0.8 for $b = 1 : 3$ due to their specialization in this group, while equals 0.5 for $b = 4 : 6$ and 0.1 for $b = 7$, and Prob $\left(D_{f,b} = 1|g(f) = 2, b\right)$ equals 0.1 for $b = 1 : 3$, equals 0.5 for $b = 4 : 6$, and 0.8 for $b = 7$.

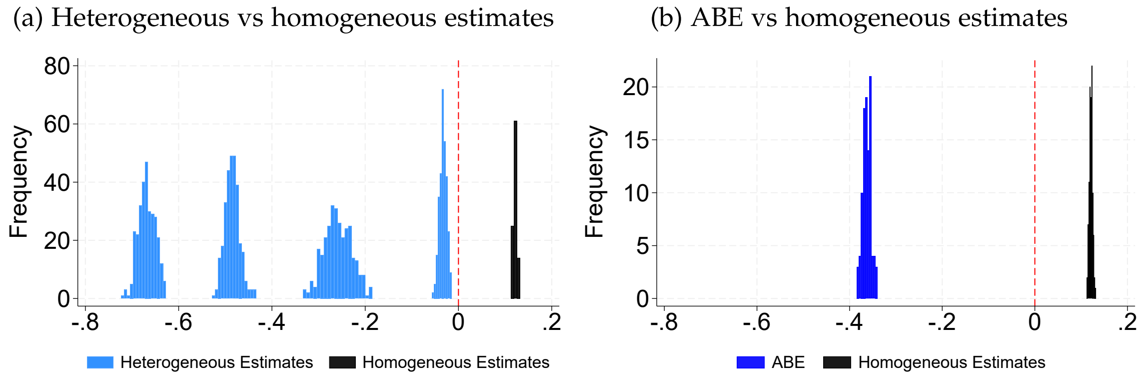
We conduct 100 simulations for this example, estimating $\widehat{\theta}^{\text{Homo}}$ as the KM fixed-effect regression of (1), and using our algorithm to calculate $\widehat{\theta}_{b,g}$ and $\widehat{\theta}_{\text{ABE}}$ according to (18) and (19). Figure 5 shows the results of the simulation. Panel A shows the histogram from the estimates of $\widehat{\theta}_{b,g}$ using our algorithm (light blue) and $\widehat{\theta}^{\text{Homo}}$ (black). Panel B shows the histogram for $\widehat{\theta}_{\text{ABE}}$ (blue) and $\widehat{\theta}^{\text{Homo}}$ (black). We can see that $\widehat{\theta}_{\text{ABE}}$ is negative as it is a weighted average of the estimated $\widehat{\theta}_{b,g}$'s, which are all negative. Finally, we can see that the whole distribution of $\widehat{\theta}^{\text{Homo}}$ is positive. The intuition is as follows. The homogenous estimate compares the credit growth of banks across the common firms to calculate the average effect.[15] In this case, bank 1 protects a certain group of firms, which leads to have the most exposed bank (bank 1) to lend more to them relative to a less exposed bank (bank 2) which is not specialized. Moreover, since bank 1 is specialized, the joint network will be weighted towards firms in this specialized group, aggravating the problem. The homogenous estimate then finds that being more exposed (bank 1) is associated to more credit growth on average.

# 6 Data

**Credit Registry Data:** The source of information is named *Registro de Crédito de Deudores* (RCD) and belongs to the administrative registries of the Peruvian financial regulator, *Superintendencia de Banca, Seguros y AFPs* (SBS). We observe the loan bal-

[15] Note that the fixed-effect estimator $\widehat{\theta}^{\text{Homo}}$ is indeed a linear combination of all the Wald estimators of the first difference (across banks) equations implied by the model.

Figure 5: Comparison of heterogeneous and homogeneous estimates




*Note.* Subfigure (a) plots the distribution of the heterogeneous $\widehat{\theta}_{b,g}$ (light blue) and homogeneous $\widehat{\theta}^{\text{Homo}}$ (black) estimates. Subfigure (b) plots the histogram of $\widehat{\theta}_{\text{ABE}}$ (blue) and $\widehat{\theta}^{\text{Homo}}$ (black).

ance of every corporate firm at a given bank from 2005 to 2018. We consider that a firm is corporate if according to SBS is classified as "*Corporativo*" or "*Gran Empresa*". Considering the definitions of such classifications, our sample of firms correspond to those that obtained at least 5 million dollars in annual sales. Corporate firms represent approximately 55% of the total amount of commercial loans. Additionally to the information of loan balance, we can observe the total amount of loan guarantees, the credit rating, a measure of the size, the type of loan, and loan currency.

**Financial Statements:** We collect information of the financial statements from the Peruvian Stock Exchange, *Superintendencia del Mercado de Valores.* We observe the balance sheet for all the firms that list on the Peruvian Stock Exchange and also a group of firms that does not list, but report voluntarily to such institution. Our main variable of interest from the balance sheet is the value of fixed capital. The information corresponds from 2007-2017.

**Export Value:** We obtained export information from the Peruvian Tax Bureau, *Superintendencia Nacional de Aduanas y de Administración Tributaria.* We observe the total value of exports at the firm level for the period 2004-2012.

# 7 Estimation Results

## 7.1 Evidence of interactions (estimated group heterogeneity)

We start our analysis by examining how credit growth varies across banks for the different grouping estimated by our algorithm. Figure 6a displays a similar plot as

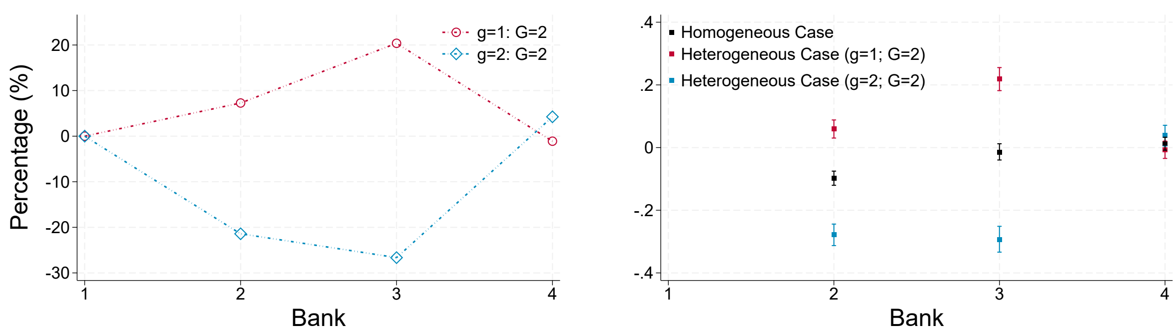
Figure 1, but splitting firms depending on the estimated grouping instead on an arbitrary group choice. The figure thus displays the average credit growth $E\left[y_{f,b,t}|b,t\right]$ of the four main banks splitting firms across the estimated grouping, for year 2017. We can see that there exists significant heterogeneity in the patterns of credit growth across groups. For example, for bank 3 we can see that the average credit growth for group 1 ($g = 1$) is around 20%, while for group 2 ($g = 2$) is close to -25%. For the remaining banks, the differences are also significant. These differences are reflected in the BF-GFE estimator that we describe next.

Figure 6b displays the estimated bank effect for the homogeneous case ($G = 1$, displayed as a confidence interval in black) and the BF-GFE estimator for the heterogeneous case ($G = 2$, displayed as blue and red intervals for each of the two groups). Our visual representations are normalized with respect to bank 1. Similar to the findings in Panel 6a, the estimates vary significantly across groups. For example, for bank 3, we can see that, under the homogeneous case, we find a tiny negative estimated bank shock, while when we allow for heterogeneity, we find the bank shock is positive for group 1 and negative for group 2. Thus, the homogeneity assumption can hide a lot of variation in the transmission of credit shocks, overestimating the bank shocks for some firms and underestimating them for other firms.

Figure 6: Estimated group heterogeneity for $G = 2$ (for $t = 2017$)

(a) Mean credit growth, by estimated group

(b) Estimated bank effects $\widehat{\beta}_{g(f),b,t}$




*Note.* Panel A. Average credit growth, $E\left[y_{f,b,t}|b,t\right]$, among the four largest banks splitting the sample based on our estimated groups for $G = 2$. Panel B. Estimated bank shocks for the four largest banks in our sample (normalizing bank 1 to 0). The black confidence interval shows the estimated bank shock under the homogeneous model, while the red and light blue intervals displays the estimated bank shock by our algorithm for case $G = 2$. The information corresponds to 2017. Standard errors are calculated through a non-parametric bootstrap with 1000 repetitions.

## 7.2 Group characteristics

This section explores observable characteristics behind the identified grouping by our algorithm. We present the results for the case with $G = 2$.

First, we present in Panel A of Table 1 the average values of some firm observable characteristics available in our dataset. The table shows that firms in group 2 (relative to group 1) have lower debt and risk score, and pose less collateral to their loans. Second, we evaluate the ability of a linear combination of these observable characteristics to predict the group selection. In particular, columns (1) and (2) in Panel B of Table 1 present a linear probability model which includes as regressors the observable characteristics in our dataset including firm age, size, and industry as fixed-effects. We can see that the R-square above 0.4, which indicate that observables explain a big part of the firm classification. Third, we allow firms to change groups across time, we estimate a grouping for every year $t$ in the sample. To connect the groupings from two different years, we calculate a transition matrix across the grouping and label as the same group the groupings with higher intersection. Then, we analyze the ability of observables in predicting a firm group change. Columns (3) and (4) in Panel B of Table 1 show that firms that start in group 1 and experienced a decrease in their collateral, debt size, and risk score are more likely to change to group 2 (the group with lower collateral, debt size and risk score).

**Bank specialization.** Paravisini et al. (2023) and Blickle et al. (2023) emphasize that banks specialize in certain markets (based on export destination and industry, respectively) and concentrate their lending disproportionately in those. They further show that their specialization measure is relevant for the transmission of shocks.

Building on these intuitions, we use a measure of specialization similar to theirs to calculate how much banks are specialized in each of our firm groupings. We calculate specialization in a group as the share of bank $b$'s portfolio invested in group $g$ compared to the relative share lent to that group by all banks for a given period. Letting $L_{b.g.t} = \sum_{f:g(f)=g} L_{f,b,t}$ be the total lending of bank $b$ to group $g$, we define:

$$\text{Group Specialization}_{b,g,t} = \frac{\frac{L_{b.g.t}}{\sum_{g'} L_{b.g'.t}}}{\frac{\sum_b L_{b.g.t}}{\sum_b \sum_{g'} L_{b.g'.t}}}.$$

We then compare this measure of specialization with our estimated bank-firm group effect $\beta_{b,g,t}$. Figure 7 indicates a significant positive relationship between $\beta_{b,g,t}$ and the specialization measure. This indicates that banks have a more positive and stronger

Table 1: Observable characteristics and group assignment

Panel A: Summary Statistics

| | Mean $(g=1)$ | Mean $(g=2)$ | $\triangle$Mean $(g_1-g_2)$ |
|---|---|---|---|
| Collateral | 23.49 | 12.40 | 11.09*** |
| Debt Size | 0.24 | 0.18 | 0.06*** |
| Risk Score | 16.71 | 10.28 | 6.43*** |
| Exports | 28.73 | 26.60 | 2.13 |

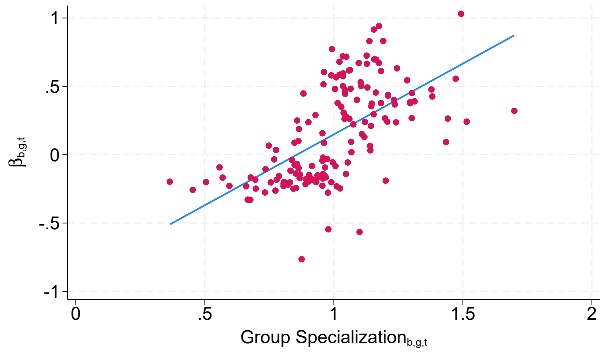
Panel B: Regression Analysis

| Dep. | Prob$(g=2)$ | | Switching Probability | |
|---|---|---|---|---|
| | Full Sample | Sample with Exports | $\Pr(g_t=2\|g_{t-1}=1)$ | $\Pr(g_t=1\|g_{t-1}=2)$ |
| | (1) | (2) | (3) | (4) |
| ln (Collateral) | -0.00300*** | -0.00434*** | | |
| | (-2.77) | (-2.71) | | |
| ln (Debt Size) | -0.0288*** | -0.0197*** | | |
| | (-11.78) | (-4.97) | | |
| Risk Score | -0.0606** | -0.0872** | | |
| | (-2.45) | (-2.33) | | |
| ln (Exports) | | -0.00205 | | |
| | | (-1.12) | | |
| $\triangle$ ln (Collateral$_{t-1}$) | | | -0.000833 | -0.00121 |
| | | | (-0.13) | (-0.57) |
| $\triangle$ ln (Debt Size$_{t-1}$) | | | -0.0708*** | 0.0292*** |
| | | | (-2.98) | (4.37) |
| $\triangle$Risk Score$_{t-1}$ | | | -0.0442* | 0.00633 |
| | | | (-1.78) | (0.47) |
| $\triangle$ ln (Exports$_{t-1}$) | | | 0.00765* | 0.00125 |
| | | | (1.70) | (0.53) |
| Fixed Effects | Yes | Yes | Yes | Yes |
| R-squared | 0.43 | 0.48 | 0.70 | 0.67 |
| *N* | 28,698 | 15,671 | 4,034 | 8,033 |

*Note.* Relationship between observables and group membership (case $G=2$). Panel A: Descriptive statistics across identified firm groups $g$. Panel B: Regression analysis between observables and group membership. Columns (1)-(2) show a firm-level linear probability model with dependent variable equal to 1 if firm belongs to group 2, 0 otherwise. Columns (3)-(4) display linear probability model of group transition with dependent variable $\Pr(g_t=i|g_{t-1}=j)$ equals one if the firm switches from group $j$ to $i$. Collateral denotes one plus the total value of the collateral across all firm loans. Debt size denotes the total value of firm debt across all its loans. Risk score is the average firm credit score, where credit score takes the value from 0 (low risk) to 4 (high risk). Exports is one plus the total value of firm exports. We also include as regressors other observables as fixed effects: firm, sector-year, financial age-year, firm size-year, firm age-year. Firm size is a dummy equal to one if the firm belongs to the group of large corporate firms, 0 otherwise. Financial age are the years of the firm in the banking system. Firm age are the years of the firm since its constitution. Sample with exports goes from 2006 to 2012, while full sample from 2006 to 2017.

propagation of credit shocks specifically for the firms in which they specialize (banks lend relatively more to firms in which they are relatively more invested). Table 3 in Online Appendix F shows the specialization measure significantly predicts the bank-firm interaction effects.

Figure 7: Bank shocks estimates and specialization




*Note.* Figure illustrates relationship between an average of our estimated bank-firm group interaction estimates and measures of bank specialization in each estimated group. The sample corresponds to a balance panel from 2006 to 2017.

## 7.3 Statistical gain of model with interactions

This subsection compares the statistical gain from the model that accounts for interactions in our dataset relative to the standard homogenous model.

**In-sample Fit.** Table 2 compares the R-squared from our model with interactions (with $G = 2$) relative to other specifications including: the homogenous effects model, and models in which interactions depend on observable characteristics (the ones relevant in section 7.2). For the case with observable interactions, we construct three groups (since, for the observable cases, $G = 3$ (terciles) provides a better fit than two groups). Our model with interactions considerable improves the goodness of fit relative to the homogenous case or using interactions based on observables.

**Out-of-sample performance.** We conduct an out-of-sample analysis following the cross-validation procedure described in Online Appendix E, which builds in Almagro and Manresa (2021). The model is estimated in training samples and used to do forecasts on testing samples. Online Appendix E shows that the model with interactions significantly outperforms the model with homogenous effects in the testing sample (not used in the estimation).

Table 2: In-sample performance across specifications

| Group Specification | $R^2$ | Adjusted-$R^2$ |
|---|---|---|
| Interaction with our estimated groups ($G = 2$) | 0.548 | 0.318 |
| Interaction with terciles of collateral | 0.413 | 0.113 |
| Interaction with terciles of risk score | 0.412 | 0.112 |
| Model without interactions | 0.410 | 0.111 |

*Note.* Performance of our estimated grouping versus alternative groups constructed by using observables. We report the $R^2$ and adjusted-$R^2$ of the following specification: $y_{fbt} = \alpha_{ft} + \beta_{Q_t(f)bt} + \epsilon_{fbt}$, where $Q_t(f)$ denotes a given group specification at time $t$. The model without interactions consider one homogenous group. The information spans from 2005 to 2017.

# 8 Applications

Having identified the relevant bank-firm group interaction and the firms' grouping $\mathcal{G}$, we proceed to conduct some exercises that help uncovering the importance of heterogeneity in the bank lending channel. First, we exploit a credit supply shifter following Paravisini et al. (2015): we compare banks with differential exposure to the Global Financial Crisis (GFC) and estimate how this credit supply shock is transmitted across the different identified firm groups. Second, we argue that in the presence of interactions, the network of bank-firm connections becomes crucial for the overall transmission of credit shocks: we quantify a *bank-firm matching channel* by estimating how the overall credit would change under a counterfactual in which banks and firms are randomly matched. Third, we move onto the real effects of the bank lending channel, and estimate how important are bank shocks for firm investment.

## 8.1 Event study: Transmission of bank shocks during GFC

In this subsection, we use our methodology to study heterogeneous responses of credit growth to an observed bank supply shock $x_b$. We follow the empirical setting in Paravisini et al. (2015), which uses a similar approach as KM, and measure bank-specific credit supply shocks $x_b$ with bank-level heterogeneity in the exposure of Peruvian banks to the 2008/09 financial crisis.

Consider the following specification for the log-credit as in Paravisini et al. (2015):

$$\ln L_{f,b,t} = \eta_{f,b} + \tilde{\alpha}_{f,t} + \theta x_b \times Post_t + \epsilon_{f,b,t},$$

where $L_{f,b,t}$ is the average outstanding debt of firm $i$ with bank $b$ during the intervals $t = Pre, Post$, where $Pre$ and $Post$ periods correspond to the 12 months before and after July 2008. $Post$ is a dichotomous variable equal one when $t = Post$. $\eta_{f,b}$ is a time-invariant firm-bank fixed effect that controls for time-invariant unobserved bank-firm characteristics, $\tilde{\alpha}_{f,t}$ is a time-varying firm fixed effect, $x_b$ is an observable variable that varies at the bank level and takes the value of one if a bank $b$ has high foreign liabilities in the pre period and zero otherwise. The linear specification leads to the following model for credit growth:

$$y_{f,b} \equiv \ln L_{f,b,Post} - \ln L_{f,b,Pre} = \alpha_f + \theta x_b + \nu_{f,b}.$$

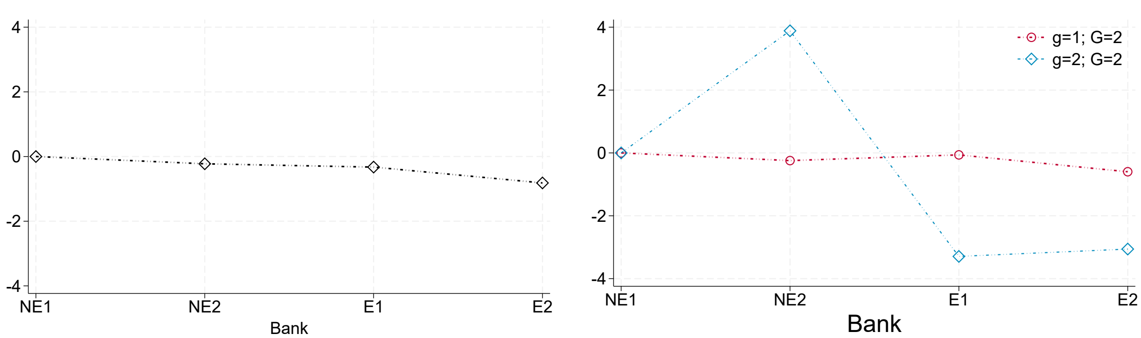
Using our identified groups, we allow for heterogeneous effects, and estimate:

$$y_{f,b} = \alpha_f + \theta_{b,g(f)} x_b + \nu_{f,b}.$$

As in Paravisini et al. (2015), $\theta$ is identified by comparing the differential lending of high foreign liabilities banks ("exposed") and low foreign liabilities banks ("non-exposed") to the same set of firms. For instance, Panel 8b illustrates such differences in the average credit growth for two non-exposed banks (banks NE1 and NE2 in the graph) and two exposed banks (banks E1 and E2) in the data, in the solid-line for all the joint firms and in dashed-lines when splitting firms across the groups estimated by our algorithm (for case $G = 2$). The figure shows that exposed banks are lending less than non-exposed banks to the same set of firms, but there is a differential lending pattern when splitting the firms across the estimated groups: exposed banks lend much less (than non-exposed) to the blue group ($g = 2$) while the difference is smaller for the red group ($g = 1$). Such differential lending will be reflected in the estimates of the treatment effects $\theta_{b,g}$.

Figure 8: Differential loan growth between exposed and non-exposed banks

(a) All firms ($G = 1$) (b) Splitting firms by estimated grouping ($G = 2$)

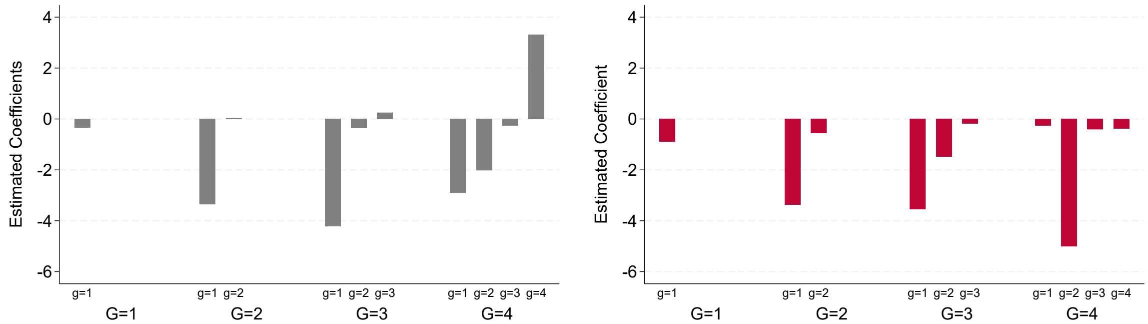



*Note.* This figure displays the average (before and after) credit growth, $\ln L_{f,b,Post} - \ln L_{f,b,Pre}$, for banks with high foreign liabilities banks ("exposed") and low foreign liabilities banks ("non-exposed"). Panel 8b splits the firms by the estimated group by our algorithm, for the case of $G = 2$. The information corresponds to 12 months before and after July 2008.

Figure 9: Differential loan growth between exposed and non-exposed banks

(a) Estimates $\widehat{\theta}_{g,\tilde{b}}$ for exposed bank $\tilde{b}$

(b) Estimates $\widehat{\theta}_{g,\tilde{\tilde{b}}}$ for exposed bank $\tilde{\tilde{b}}$




*Note.* The figure 9a displays the estimates of $\theta_{g,b}$ for each group and two (anonymous) exposed banks: $\tilde{b}$ and $\tilde{\tilde{b}}$.

Figure 9 displays the estimated heterogeneous coefficients for two exposed banks (which we name $\tilde{b}$ and $\tilde{\tilde{b}}$), for cases $G = 1$ to $G = 4$.[16] The results point to considerable heterogeneity in the transmission of this credit shock as it reveals that exposed banks (relative to non-exposed) decreased their credit significantly to some group of firms but not to all. Interestingly, we can see in some cases that each bank treats differently each specific group. For instance, bank $\tilde{b}$ protects certain groups and even increased their lending during this episode, while this is not the case for bank $\tilde{\tilde{b}}$.

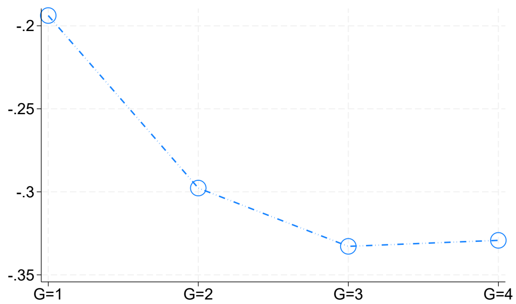
We move into calculating the average effect across all banks and groups of firms, which will be captured by our average bank effect ($\widehat{\theta}_{\text{ABE}}$) defined in (19). Figure 10 illustrates the estimates $\widehat{\theta}_{\text{ABE}}$ for $G > 1$, the case $G = 1$ replicates the homogeneous case as obtained in Paravisini et al. (2015). The results show that, when considering heterogeneity, the estimate of the average bank effect increases in magnitude, implying a more severe negative effect of the bank liquidity shock on firms. The estimated average of the heterogenous effect across firm groups and banks is about -0.33, indicating that on average exposed banks reduced credit supply by 33% relative to non-exposed banks, while the estimate under the homogenous model is -0.18.

## 8.2 *Bank-firm matching channel:* the value of bank-firm networks

When the transmission of credit shocks is heterogeneous, the network of relationships may become key for the overall effect of bank shocks: we should expect a stronger (weaker) effect of a shock, when banks create more relationships with firms for which the transmission is stronger (weaker). In this section, we ask do banks

[16] Table 4 in Online Appendix F shows the number of firms in each group.

Figure 10: Average bank effect $\widehat{\theta}_{\text{ABE}}$




*Note.* The figure displays the average effect across all banks and groups using the $\hat{\theta}_{\text{ABE}}$ in (19). For the homogeneous case, $G = 1$, we follow the same procedure as Paravisini et al. (2015). Data corresponds to 12 months before and after July 2008.

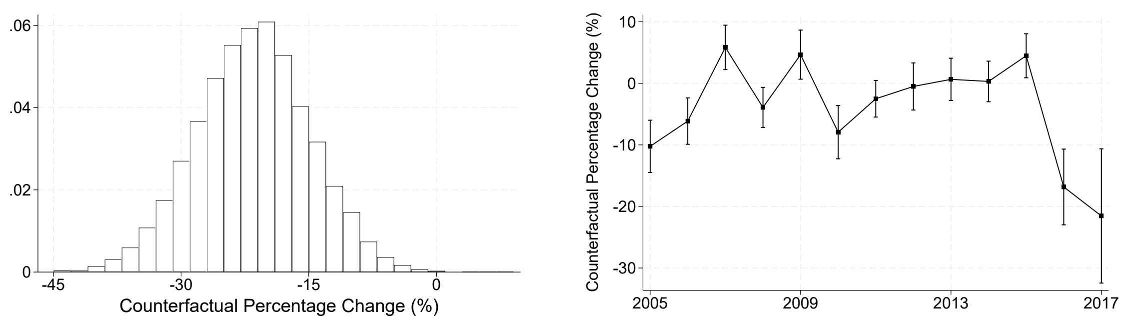
and firms create relationships in a way that amplify or smooth out credit shocks? In order to answer this question, we estimate as a counterfactual the aggregate credit growth when banks and firms are randomly matched. For instance, according to the model in Online Appendix A, this counterfactual represents an economy in which information frictions are so large that firms cannot predict their complementarities with banks arising from specialization, so they end up matching with a random bank instead of their optimal specialized bank. We find that for most of the years aggregate credit growth is enhanced by the observed bank-firm credit network relative to the counterfactual random matched network. We refer to such overall effect from bank-firm complementarities as the *bank-firm matching channel.*

We conduct a random reallocation exercise for every firm-bank observation in our data. Our procedure is the following. First, we obtain from our empirical strategy the decomposition of the average credit growth as the sum of the firm effect $\widehat{\alpha}_{f,t}$ and the heterogeneous bank-firm group effects $\widehat{\beta}_{b,g(f),t}$. Second, keeping the same number of connections of each firm, we randomize without replacement the banks lending to each firm. For example, if a firm is connected with three banks in our data, we keep the three connections but match the firm randomly with three banks. Third, we calculate the credit growth for each of the counterfactual connections using our model, that is, as the sum of the estimated firm effect and the bank-firm group effect associated with the random network. Fourth, we calculate the average credit growth across all the observations. Finally, we repeat this exercise simulating 10,000 counterfactual networks to calculate the distribution of the average credit growth under random matching.[17]

[17]Note that in this counterfactual we are assuming the absence of general equilibrium effects, so that the estimates of $\widehat{\alpha}_{f,t}, \hat{\beta}_{b,g,t}$ do not change under a reallocation of connections.

Figure 11: Percentage change in average credit growth in random network relative to observed network

(a) $t = 2017$ (b) Over different years




*Note.* Percentage change between the average credit growth rate under random networks (simulated 10,000 times) and the observed network. Panel 11a shows the histogram for 2017. Panel 11b shows 90% confidence intervals for all years.

Figure 11 shows the distribution across simulations of the percentage change in average credit growth relative to the observed network. Panel 11a presents it as an histogram for 2017. The median of the distribution suggests that aggregate credit growth falls by 20% under random matching. This results highlight the importance of the value of bank-firm relationships: The observed network of bank-firm relationships enhances credit growth. Note that, under a model with homogeneous effects ($G = 1$), this exercise would lead to no change in credit growth by construction.

Panel 11b explores the bank-firm matching channel across all years in our sample, and present the the confidence interval for the simulated change in credit growth for every year. For most of the years the change is negative, meaning that the endogenous matching in the data enhances credit growth relative to a random network. Only for 2007, 2009 and 2015 we find the opposite result. This could be driven by adverse macroeconomic factors, which were in turn amplified by the network. More specifically, 2009 is influenced by the GFC, indicating that more exposed banks were more connected with firms belonging to the group for which the transmission of the (negative) shock was stronger, while 2015 is also associated with adverse external conditions (large drop in commodity prices and exchange rate depreciation).

## 8.3 The impact of credit supply shocks on real outcomes

In this part, we examine the real effects of credit supply shocks on firm investment. We follow the approach in AW and calculate the credit supply shock at the firm level by weighting the BF-GFEs estimates by the share that each bank represents in the firm borrowing portfolio in the previous period:

$$\text{Supply}_{f,t} = \sum_b \lambda_{f,b,t-1} \hat{\beta}_{b,g(f),t} \text{ with } \lambda_{f,b,t-1} = \frac{L_{f,b,t-1}}{\sum_b L_{f,b,t-1}}, \tag{21}$$

where $L_{f,b,t-1}$ denote borrowing by firm $f$ from bank $b$ at time $t-1$, and where the novelty is that the factor $\hat{\beta}_{b,g(f),t}$ varies at the group level.

We estimate the following regression for investment (measured as the growth rate in total tangible fixed assets plus depreciation):

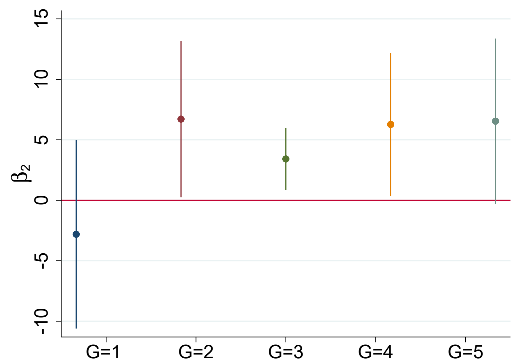
$$\text{Investment}_{f,t} = c + \phi^F_f + \phi^G_{g(f),t} + \beta_1 \hat{\alpha}_{f,t} + \beta_2 \text{Supply}_{f,t} + \epsilon_{f,t}. \tag{22}$$

We control the regression by a set of fixed effects including time-varying fixed effects at the group level. Following Amiti and Weinstein (2018) we also include the estimates of the firm-specific effect.[18]

*Remark.* Including time-varying group effects $\phi^G_{g(f),t}$ is crucial since, recall from section 5, that the BF-GFE may contain information about a demand shock affecting the group of firms, which leads them to borrow differently from banks. The effects of such demand factors on investment are common across firms in the same group, so they can be controlled by the group effect $\phi^G_{g(f),t}$. Importantly, we can identify $\beta_2$ even in the presence of $\phi^G_{g(f),t}$ due to variability in $\lambda_{f,b,t-1}$ across firms within the same group. Our regression compares firms within the same group, thus exposed to the same demand shocks, yet exhibiting different exposures to credit supply shocks owing to variations in their network connections in the previous period (established before the demand shock). Why do firms within the same group have ex-ante different $\lambda_{f,b,t-1}$? Several explanations are plausible. For instance, firms may have varying risk assessments, even within the same group, with more risk-averse firms establishing more relationships, including those with banks not specialized in their type. Alternatively, firms may evolve their business models. For example, some firms within a group exporting to the U.S. at time $t$ may have previously exported to China and maintained relationships with banks specializing in China, thus retaining those connections over time. Also, there could exist idiosyncratic factors influencing the ex-ante bank-firm network structure, such as the information frictions described in the model in Online Appendix A which prevent all firms in the same group to matched only with their optimal specialized bank.

[18]While using $\alpha_{f,t}$ as a regressor could potentially help in controlling for firm specific demand effects, the estimates of $\hat{\alpha}_{f,t}$ may suffer from incidental parameter bias, as this parameter is estimated from the number of connection each firm has, which is typically small.

Figure 12: Estimated $\widehat{\beta}_2$




*Note.* Estimated $\widehat{\beta}_2$in regression (22) for different $G = 1$ to 5. Standard errors are cluster at the main leading bank.

Figure 12 plots the estimated confidence interval for the elasticity of the supply shock on investment ($\widehat{\beta}_2$), under the estimation for different number of groups (as represented in the X-axis). Table 5 in Online Appendix F shows all the estimated coefficients. The results reveal that when banks shocks are estimated under the standard framework with homogeneous effects ($G = 1$), the estimated effect is very imprecise, the point estimate is negative and not statistically significant. Instead, when assuming heterogeneous effects ($G > 1$), the estimated impact of banks shocks on firm investment is statistically significant. The mean estimate for the heterogeneous case is similar for the different number of groups used ($G = 2$ to 5), and suggest that a 1% change in the credit supply increases investment by around 4-6%. We cluster the standard errors at the main leading bank level as in Huremovic et al. (2020).

The results are consistent with the pronounced heterogeneity found in our data set. For instance, in Figure 6a, we observed that, in some cases, the estimated bank effect is positive for firms belonging to one group while it is negative for firms in other group. If credit supply actually matters for investment, one would expect that, investment increases for firms in the former group while decreases for the latter. When $G = 1$ is assumed, the estimated bank effect is homogenous to all of these firms so the correlation with such varying investment levels would be imprecise. Thus, missing the heterogeneity makes hard to uncover the elasticity, even when the average bank effect may be well estimated.

# 9 Conclusions

We develop an empirical framework to identify and estimate heterogeneous effects of bank shocks exploiting bank-firm credit data. Our methodology provides a flexible framework to study the importance of bank-firm interactions and heterogeneity in the bank lending channel. The heterogeneous transmission of credit shocks is particularly relevant in the presence of relationship lending or specialization in markets as stressed by Paravisini et al. (2023) or in lending forms as stressed by Ivashina et al. (2022). To allow for interactions and heterogeneity in a flexible yet parsimonious way, we rely on the idea that from the perspective of the banks and their relationship with firms, there is a discrete number of "types" of firms, and hence, the propagation of shocks depends on the firm's type. These interactions between banks and groups of firms may arise when banks specialize in markets where a group of firms operate.

We combine state-of-the-art panel data techniques that allow for time-varying group fixed effects (Bonhomme and Manresa (2015)) with the two-sided fixed effects framework used to disentangle demand and supply shocks from bank-firm credit data (as in Khwaja and Mian (2008) and Amiti and Weinstein (2018)). We show theoretically and in simulations that our proposed estimator consistently estimates the bank-firm interactions. And, we discuss potential concerns that may arise when estimating a model with homogeneous effects.

We apply our flexible framework to credit registry data from Peru. We uncover significant heterogeneity in the bank lending channel: The patterns of bank lending and the estimated bank-firm group effects show significant differences across our identified firm groups. Importantly, we show that accounting for such heterogeneity and the bank-firm network structure is crucial to learn about the bank lending channel and its real effects.

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# Appendix (Proof of Propositions)

*Proof.* **Proof of Proposition 1.**

Consider the following data generating process for the "bank difference" version of model (6) for all $(f, b, t)$ with $D_{f,b,t} = 1$:

$$y_{f,b,t} - y_{f,b_0,t} = \beta^0_{b,g^0_t(f),t} - \beta^0_{b_0,g^0_t(f),t} + \epsilon_{f,b,t} - \epsilon_{f,b_0,t}$$

where 0-superscripts refer to true parameters. We normalize $\beta_{b_0,g^0_t(f),t} = 0$.

Consider the following additional assumptions. Assumption 4 states $\epsilon_{f,b,t}$ has finite moments and limits the dependence structure of the errors $\epsilon_{f,b,t}$ in the cross section. It is fulfilled when $\epsilon_{f,b,t}$ is independent across units, but also allows for weak spatial dependence in $\epsilon_{f,b,t}$. The time series dependence is left unrestricted so shocks can exhibit strong time series correlation. Assumption 5 implies that the population groups have a large number of firms and the groups are well-separated.

**Assumption 4.** ***(Errors)****: For each $t$, there exist a constant $C_t$ such that $\epsilon_{f,b,t}$ is a strong mixing process in the cross sectional dimension with:*

*i)* $E\left(\epsilon_{f,b,t}\right)=0$, $E\left(\epsilon_{f,b,t}^{4}\right)<C_t$,

*ii)* $\frac{1}{N_F}\sum_{f=1}^{N_F}\sum_{f'=1}^{N_F}\mid \frac{1}{N_B}\sum_{b=1}^{N_B}E\left(\epsilon_{f,b,t}\epsilon_{f',b,t}\right)\mid< C_t$,

*iii)* $\mid \frac{1}{N_F^2 N_B}\sum_{f=1}^{N_F}\sum_{f'=1}^{N_F}\sum_{b=1}^{N_B}\sum_{b'=1}^{N_B}Cov\left(\epsilon_{f,b,t}\epsilon_{f',b,t},\epsilon_{f,b',t}\epsilon_{f',b',t}\right)\mid< C_t$ ,

*iv)* $\mid \frac{1}{N_F N_B}\sum_{f=1}^{N_F}\sum_{b=1}^{N_B}\sum_{b'=1}^{N_B}E\left(\epsilon_{f,b,t}\epsilon_{f,b',t}\right)\mid< C_t$, *and*

*v) the time series dependence of $\epsilon_{f,b,t}$ is left unrestricted.*

**Assumption 5.** ***(Group effects)****: For each $t$:*

*i)* $\forall g_t\in\{1,\ldots,G\}: plim_{N_F\to\infty}\frac{1}{N_F}\sum_{f=1}^{N_F}1\left\{g_t^0(f)=g_t\right\}=\pi_g>0$,

*ii)* $\forall\left(g_t,\tilde{g}_t\right)\in\{1,\ldots,G\}^2$ *s.t.* $g_t\neq\tilde{g}_t$: $plim_{N_B\to\infty}\frac{1}{N_B}\sum_{b=1}^{N_B}\left(\beta_{b,g_t,t}^0-\beta_{b,\tilde{g}_t,t}^0\right)=c_{g_t,\tilde{g}_t}>0$,

*iii) $\beta_{b,g_t,t}^0$ is a strongly mixing process in the cross sectional dimension,*

*iv) the time series dependence of $\beta_{b,g_t,t}^0$ is left unrestricted.*

Assumption 2 implies that $\beta_{b,g_t(f),t}^0$ is contemporaneously uncorrelated with $\epsilon_{f,b,t}$ and $\epsilon_{f,b_0,t}$. Then, using assumption 2, 4, 5 and full connections between bank and firms, we can apply theorems 1 and 2 in Bonhomme and Manresa (2015) (for a model without observable regressors) replacing the time-series dimension in the analysis of Bonhomme and Manresa (2015) for the bank dimension in our bipartite network data. Then, for each $t$ separately and for all $\delta>0$ as $N_F$ and $N_B$ tend to infinity:

$$\frac{1}{N_F N_B}\sum_{f=1}^{N_F}\sum_{b=1}^{N_B}(\widehat{\beta}_{b,\hat{g}_t(f),t}-\beta_{b,g_t^0(f),t}^0)^2\xrightarrow{p}0,$$

$$\text{Prob}\left(\sup_{(f\in\{1,\ldots,N_F\})}\mid\hat{g}_t(f)-g_t^0(f)\mid>0\right)=o(1)+o(N_F N_B^{-\delta}),$$

and $\widehat{\beta}_{b,g,t}=\widehat{\beta^u}_{b,g,t}+o_p\left(N_B^{-\delta}\right)$ for all $b,g$.[19] □

*Proof.* **Proof of Proposition 2**

For the case of $N_B=2$ (and normalization $\beta_{b_0,g_t(f),t}=0$), from (14) we have $\hat{\beta}_{b_1,t}^{\text{Homo}}\overset{a}{=}E\left[y_{f,b_1,t}-y_{f,b_0,t}\mid f\in I_{b_1,b_0}\right]=E\left[\beta_{g_t(f),b_1,t}\mid f\in I_{b_1,b_0}\right]=E\left[\beta_{g_t(f),b_1,t}\right]$ where

[19] Online Appendix C intuitively discusses consistency when shocks are iid normally distributed.

the last equality follows since the network of connections $D_t$ is independent of the interaction effect $\beta_{g_t(f),b,t}$. For the general $N_B$ case, we have that the within-estimator $\hat{\beta}^{\text{Homo}}_{b,t}$ is a linear combination of the Wald estimators exploiting the same moment condition in (4) for all firms in the respective network. Since all $\beta_{g_t(f),b,t}$'s are independent of the whole network structure $D_t$, the same argument follows. □

*Proof.* **Proof of Proposition 3**

From the Frisch-Waugh theorem, the KM fixed effect estimator can be express as:

$$\hat{\theta}^{\text{Homo}} = \frac{\sum_{b=1}^{N_B}\sum_{f=1}^{N_F} D_{f,b} v_{f,b} y_{f,b}}{\sum_{b=1}^{N_B}\sum_{f=1}^{N_F} D_{f,b} v_{f,b} x_{f,b}},$$

where $x_{f,b}$ equals $x_b$ for all the firms in $I_{b,b_0}$. For each firm $f$, $\sum_{b=1}^{N_B} v_{f,b} = 0$ since $v_{f,b} = x_b - \frac{\sum_{b=1}^{N_B} D_{f,b} x_b}{\sum_{b=1}^{N_B} D_{f,b}}$.[20] The expected value conditional on observables is:

$$E\left[\hat{\theta}^{\text{Homo}} \mid x, D\right] = \frac{\sum_{b=1}^{N_B}\sum_{f=1}^{N_F} D_{f,b} v_{f,b} E[\alpha_f + \theta_{b,g(f)} x_b \mid x, D]}{\sum_{b=1}^{N_B}\sum_{f=1}^{N_F} D_{f,b} v_{f,b} x_{f,b}}$$

where this equality comes from the assumption 3. The fixed effect regression implies that $\sum_{f=1}^{N_F}\sum_{b=1}^{N_B} v_{f,b}\alpha_f = 0$, thus

$$E\left[\hat{\theta}^{\text{Homo}} \mid x, D\right] = \frac{\sum_{b=1}^{N_B} x_b \sum_{f=1}^{N_F} D_{f,b} v_{f,b} E[\theta_{b,g(f)} \mid x, D]}{\sum_{b=1}^{N_B}\sum_{f=1}^{N_F} D_{f,b} v_{f,b} x_{f,b}} = \frac{\sum_{b=1}^{N_B}\sum_{g=1}^{G} E[\theta_{b,g(f)} \mid x, D] \sum_{f:g(f)=g} D_{f,b} v_{f,b} x_b}{\sum_{b=1}^{N_B}\sum_{f=1}^{N_F} D_{f,b} v_{f,b} x_{f,b}}.$$

Using $\sum_{f:g(f)=g} D_{f,b} v_{f,b} x_b = \sum_{f\in I_{b,b_0,g}}^{N_{b,b_0,g}} v_{f,b} x_b$ and $\sum_{b=1}^{N_B}\sum_{f=1}^{N_F} D_{f,b} v_{f,b} x_{f,b} = \sum_{b=1}^{N_B} x_b \sum_{f\in I_{b,b_0}}^{N_{b,b_0}} v_{f,b}$:

$$E\left[\hat{\theta}^{\text{Homo}} \mid x, D\right] = \frac{\sum_{b=1}^{N_B}\sum_{g=1}^{G} E[\theta_{b,g(f)} \mid x, D] \sum_{f\in I_{b,b_0,g}}^{N_{b,b_0,g}} v_{f,b} x_b}{\sum_{b=1}^{N_B} x_b \sum_{f\in I_{b,b_0}}^{N_{b,b_0}} v_{f,b}} = \sum_{b=1}^{N_B}\sum_{g=1}^{G} \frac{E[\theta_{b,g(f)} \mid x, D] \sum_{f\in I_{b,b_0,g}}^{N_{b,b_0,g}} v_{f,b} x_b}{\sum_{b=1}^{N_B} x_b \sum_{g=1}^{G}\sum_{f\in I_{b,b_0,g}}^{N_{b,b_0,g}} v_{f,b}}.$$

Defining $\omega_{g(f),b} = \frac{\sum_{f\in I_{b,b_0,g}}^{N_{b,b_0,g}} v_{f,b} x_b}{\frac{N_{b,b_0,g}}{N}\sum_{b=1}^{N_B} x_b \sum_{g=1}^{G}\sum_{f\in I_{b,b_0,g}}^{N_{b,b_0,g}} v_{f,b}}$ and using that $\omega_{b_0} = 0$, we can rewrite: $E\left[\hat{\theta}^{\text{Homo}} \mid x, D\right] = \sum_{b=1}^{N_B}\sum_{g=1}^{G} \omega_{g(f),b} \frac{N_{b,b_0,g}}{N} E[\theta_{b,g(f)} \mid x, D] = \sum_{b\neq b_0}^{N_B}\sum_{g=1}^{G} \omega_{g(f),b} \frac{N_{b,b_0,g}}{N} E[\theta_{b,g(f)} \mid x, D]$.

□

[20] The estimation is for the sample of firms connected to bank $b_0$ and at least one exposed bank $b \neq b_0$ (i.e., firms belonging to $I_{b,b_0}$ for any $b$).

# Online Appendix

## A Model with lending specialization

First, in A.1, we describe a model with bank specialization similar to the one proposed by Paravisini et al. (2023) in which banks are heterogenous (specialized) in their lending capabilities for specific activities (e.g. a bank may have an advantage in lending to firms when exporting to the US). We show that, in such a model, our empirical specification with bank-firm group effects arises if types/groups of firms are defined by the relative importance of activities (e.g. a group of firms have a higher presence in US markets while other group may have higher presence in Europe). Second, in A.2, we show that the empirical specification with loan-type specific shocks considered by Ivashina et al. (2022) leads to our empirical specification with bank-firm group effects, when loan-type specific demand happens at a firm group level.

### A.1 Model.

Consider a two period model $t = 0, 1$ with $N_F$ firms belonging to $G$ groups, and $N_B$ banks. Each firm $f$ is defined by a collection of activities/markets $c = 1, ..., C$ (e.g. export destination country), and the grouping determines how important is the activity for each firm. Firms use credit to fund each of those activities. Each firm chooses the bank $b$ that minimizes the cost of credit for the corresponding activity.

Consider a monopolistic competition environment so that, for each activity $c$, a firm $f$ faces a CES demand structure with elasticity of substitution $\sigma > 1$ and group-activity demand shifter $M^c_{g(f)}$ where $g(f)$ denotes the group to which firm $f$ belongs. $M^c_{g(f)}$ capture market-wide variables exogenous to the firm (e.g. $M^c_{g(f)}$ may characterize the market size of country $c$ for specific products produced by firms specialized in exports to country $c$ or in a certain industry). That is, firm $f$ faces a demand function for activity $c$: $q^c_i = M^c_{g(f)}(p^c_i)^{-\sigma}$. Following Paravisini et al. (2023), we focus on the choice of credit and assume credit is the single factor of production. Specifically, assume a linear production function $q^c_f = A_f L^c_{f,b}$, where $A_f$ is a technology shock and $L^c_{f,b}$ is the amount of credit from a chosen bank $b$.

Banks lend to firms at an activity-specific rate $r^c_b = r_b/\gamma^c_b$, where $\gamma^c_b > 0$ represents bank specialization in activity $c$. For example, the bank may have an ability in securitizing loans associated with a particular activity, so it is able to fund such loans at a lower cost of funds, or the bank may have more information about the particular activity and can assess better the associated risk, or it may also be interpreted as other value added services attached to the issuance of credit than other lenders. We also assume $r_b = \bar{r}e^{\pi_b}$ so that $\bar{r}$ captures the baseline (risk-free) rate common to all banks and $\pi_b$ is a bank-specific risk-premium (related to each bank's balance sheet health).

At $t = 0$, the firm chooses a bank for every activity to minimize the cost of credit but facing some uncertainty about bank abilities and pricing. Specifically, each firm chooses $b$ to minimize

$$\begin{aligned} b &= \min_{b'} \left\{ r_b^c e^{-\mu \nu_f^c} \right\} \\ &= \min_{b} \left\{ (\ln \bar{r} + \pi_b - \ln \gamma_b^c) - \mu \nu_{f,b}^c \right\}, \end{aligned}$$

where $\nu_{f,b}^c$ can be interpreted as information noise experienced by the firm about banks' specialization $\gamma_b^c$, for example, due to marketing, location, or misinformation.[21]

As in Paravisini et al. (2023), we assume $\left\{ \nu_{f,b}^c \right\}_f$ are i.i.d. Gumbel across firms, so the probability of a firm choosing $b$ for activity $c$ is:

$$\tilde{\gamma}_b^c \equiv \frac{\left( \frac{\gamma_b^c}{r_b} \right)^{1/\mu}}{\sum_{b'} \left( \frac{\gamma_{b'}^c}{r_{b'}} \right)^{1/\mu}}. \tag{A.1}$$

Given a chosen bank $b$ for activity $c$, each firm chooses the output level $q_i^c$ and credit amount $L_{f,b}^c$ for every activity to maximize:

$$\max_{q_f^c, L_{f,b}^c} p_f^c q_f^c - r_{b,f}^c L_{f,b}^c$$

subject to their demand $q_f^c = M_{g(f)}^c (p_f^c)^{-\sigma}$ and production function $q_f^c = A_f L_{f,b}^c$, which leads to standard optimal choices on price and quantity:

$$\begin{aligned} p_f^c &= \frac{\sigma}{\sigma - 1} \mathrm{mc}_f^c, \\ q_f &= M_{g(f)}^c \left( \frac{\sigma}{\sigma - 1} \mathrm{mc}_f^c \right)^{-\sigma}, \end{aligned}$$

where marginal cost of production $\mathrm{mc}_f^c = r_{b,f}^c / A_f$. Then, the (potential) optimal

[21] Also, $\nu_{f,b}^c$ could be interpreted as other advantages or value added services unrelated to firm production costs or the credit amount, so that the firm minimizes the cost of credit net of these other services.

amount of credit equals:

$$\ell_{f,b}^{c} = M_{g(f)}^{c} \left( \frac{\sigma}{\sigma - 1} \right)^{-\sigma} \left( A_f \right)^{\sigma-1} \left( r_{b,f}^{c} \right)^{-\sigma}. \tag{A.2}$$

The total amount of credit borrowed by the firm thus depend on the importance of the activity for the firm (measured by $M_{g(f)}^{c}$), the firm technology $A_f$, and the cost of credit $r_{b,f}^{c}$.

From (A.2) and (A.1), we can write the expected total borrowing of a firm $f$ from bank $b$ as:

$$\begin{aligned} E\left[L_{f,b}\right] &= E\left[\sum_c L_{f,b}^{c}\right] = E\left[\sum_c \mathbb{I}_{f,b}^{c} \times \ell_{f,b}^{c}\right] \\ &= \sum_c \tilde{\gamma}_b^{c} \left( M_{g(f)}^{c} \left( \frac{\sigma}{\sigma-1} \right)^{-\sigma} \left(A_f\right)^{\sigma-1} \left(\bar{r} e^{\pi_b}\right)^{-\sigma} \left(\gamma_b^{c}\right)^{\sigma} \right), \end{aligned}$$

where the first line sums the credit from bank $b$ across all activities and $\mathbb{I}_{f,b}^{c}$ is an indicator function equal to one if bank $b$ is chosen for activity $c$ and zero otherwise.

To think about credit growth and the influence of credit supply and demand shocks, we assume that, at $t = 1$, firms and banks may experience different types of shocks: Firm productivity shocks $A_f^{'} = A_f e^{a_f}$, demand shocks $M_{g(f)}^{c'} = M_{g(f)}^{c} e^{d_{g(f)}^{c}}$, interest rate shocks $\bar{r}^{'} = \bar{r} e^{z}$, bank shocks $\pi_b^{'} = \pi_b + z_b$ or bank shocks specific to their lending abilities $\gamma_b^{c'} = \gamma_b^{c} e^{s_b^{c}}$ (e.g. changes in their securitization abilities, risk assessment of specific activities/sectors), where we use primes to denote $t = 1$ variables.

We assume the firm chosen bank persists for both $t = 0$ and $t = 1$. Letting $g_x = x'/x - 1$ be the growth rate of any variable $x$. We have

$$g_{E[L_{f,b}]} \approx \sum_c w_{f,b}^{c} \left( d_{g(f)}^{c} + (\sigma - 1) a_f - \sigma(z + z_b) + \sigma s_b^{c} \right),$$

with weights $w_{f,b}^{c} = \frac{\tilde{\gamma}_b^{c}\left(\ell_{f,b}^{c}\right)}{\sum_c \tilde{\gamma}_b^{c}\left(\ell_{f,b}^{c}\right)}$. Note that the weights capture the relative (expected) importance of the activity $c$ in the bank-firm relationship. Since the importance of the activity depends on the firm specific group, we have that the weights are bank-firm group specific; specifically, we have $w^c_{f,b} = w^c_{g(f),b}$ with

$$w^c_{g(f),b} = \frac{\tilde{\gamma}^c_b \left(M^c_{g(f)} \left(\gamma^c_b\right)^\sigma\right)}{\sum_c \tilde{\gamma}^c_b \left(M^c_{g(f)} \left(\gamma^c_b\right)^\sigma\right)}.$$

Therefore, we can write (up to a growth rate approximation):

$$y_{f,b} = \bar{c} + (\sigma - 1)a_f - \sigma z_b + \sigma \sum_c w^c_{g(f),b} s^c_b + \sum_c w^c_{g(f),b} d^c_{g(f)} + \epsilon_{f,b}, \tag{A.3}$$

where $y_{f,b}$ is the loan growth rate, $\bar{c}$ is a constant capturing common aggregate shocks ($z$) and a Jensen's inequality term, and $\epsilon_{f,b}$ is an idiosyncratic factor.

Comparing (A.3) with our empirical specification (6), we have that $\alpha_f = \bar{c} + (\sigma - 1)a_f$ and $\beta_{b,g(f)} = -\sigma z_b + \sigma \sum_c s^c_b w^c_{g(f),b} + \sum_c d^c_{g(f)} w^c_{g(f),b}$. As discussed in section 5, our interaction factor $\beta_{b,g(f)}$ can be driven either by bank supply shocks $z_b$ or $s^c_b$ heterogeneously propagated to firms (due to differences in importance of the bank specific lending advantage across firm groups) or by a firm-group demand shock $d^c_{g(f)}$ heterogeneously propagated to banks.

## A.2 Empirical model with firm loan-type specific shocks.

Ivashina et al. (2022) argues that the transmission of credit shocks could depend on the specific type of loan contract $l$ used. They consider the following empirical model with firm-loan specific shocks:

$$\Delta \ln L_{f,b,l,t} = \alpha_{f,t} + \eta^l_{f,t} + \beta_{b,t} + \epsilon^l_{f,b,t}, \tag{A.4}$$

where $\eta^l_{f,t}$ captures a loan-type specific shock. The assumption is that conditional on the observed loan type, credit shocks are transmitted homogeneously. So, when loan-type information is available to the econometrician, they can estimate bank and firm fixed effects by including firm-loan specific fixed effects or conditioning the estimation on the specific firm-loan type.

It is common that lending by loan type is not observed by the econometrician, instead some credit registries provide only information about the total lending from each bank $L_{f,b,t} = \sum_l L_{f,b,t,l}$. Estimating the standard fixed-effects specification, as in section (2), ignoring the differential effects by loan-type could lead to bias estimates. For example, this would happen if a firm demand shock is loan-type specific and different banks specialize in providing different types of loans.

We show below that if the loan-type specific demand happens at a firm group level, our empirical framework with bank-firm group interactions arises as an aggregation of the empirical model (A.4) at the bank-firm level.

If we aggregate credit growth at the firm level (dropping subscript $t$ for convenience), we get $y_{f,b} \equiv \Delta \ln L_{f,b} \approx \sum w^l_{f,b} \Delta \ln L_{f,b,l}$, where $w^l_{f,b} = \frac{L^l_{f,b}}{\sum_l L^l_{f,b}}$ denotes the importance of loan-type $l$ in the firm-bank relationship. To capture that banks specialize in providing a certain type of loan, we assume the relative importance of a loan-type across all loans given by the bank $w^l_{f,b} = h(\gamma^l_b)$ is a function of the bank specialization parameter $\gamma^l_b$ (analogous to the model above). Finally, we assume that the firm loan-specific demand depends on the firm group, so that $\eta^l_f = \bar{\eta}^l_{g(f)}$. For example, a group of firms experiencing a liquidity need may demand a short-term loan, firms planning to take a fixed long-term investment may demand long-term loans, or the group of firms wanting to finance equipment purchases may demand asset-based loans. Each type of firm then will have a preference to borrow differently from each bank.

Then, the aggregation of model (A.4) leads to:

$$y_{f,b,t} = \alpha_{f,t} + \sum_l h(\gamma^l_{b,t}) \bar{\eta}^l_{g(f)} + \beta_{b,t} + \epsilon_{f,b,t}, \tag{A.5}$$

with $\epsilon_{f,b,t} = \sum_l h(\gamma^l_{b,t}) \epsilon^l_{f,b,t}$.

Comparing (A.5) with our empirical specification (6), we have that $\beta_{b,g(f)} = \sum_l h(\gamma^l_{b,t}) \bar{\eta}^l_{g(f)} + \beta_{b,t}$.

# B Interactive fixed effects

In this section, we model bank-firm interactions described in (5) by imposing the following interactive specification for the growth rate of loans:

$$y^*_{f,b,t} = \beta_{b,t} + \alpha_{f,t} \gamma_{b,t} + \epsilon_{f,b,t}, \tag{B.1}$$

where $E[\epsilon_{f,b,t} | D_t, \alpha_t, \beta_t, \gamma_t] = 0$ and $D_t$, $\alpha_t$, $\beta_t$ and $\gamma_t$ denote the entire vector (at time $t$) of $D_{f,b,t}$, $\alpha_{f,t}$, $\beta_{b,t}$ and $\gamma_{b,t}$ for all $f, b$, respectively. The observed growth rate of loans is defined by $y_{f,b,t} = y^*_{f,b,t} D_{f,b,t}$.

The model in (B.1) is a interactive fixed effect model in the spirit of Bai (2009) and allows for continuous heterogeneity in the transmission of shocks. In this particular specification $\beta_{b,t}$ could capture bank shocks that affect similarly all the connected firms whereas $\gamma_{b,t}$ can capture bank shocks that affect differently different firms with a loading equal to $\alpha_{f,t}$.

**Identification.** By comparing differences in the amount of loans issued by each bank to different firms ($i$ and j) we have:

$$y_{i,b,t} - y_{j,b,t} = \gamma_{b,t}\left(\alpha_{i,t} - \alpha_{j,t}\right) + \epsilon_{i,b,t} - \epsilon_{j,b,t},$$
$$y_{i,b_0,t} - y_{j,b_0,t} = \gamma_{b_0,t}\left(\alpha_{i,t} - \alpha_{j,t}\right) + \epsilon_{i,b_0,t} - \epsilon_{j,b_0,t}.$$

Lets define $I(b, b_0)$ as a particular group of firms that borrows from both banks $(b, b_0)$ at time $t$ and $J(b, b_0)$ as another (different) group of firms that also borrows from both banks $(b, b_0)$ at time $t$.

Averaging the difference among a group of firms, then we can identify $\gamma_{b,t}/\gamma_{b_0,t}$ by taking the ratio of the following equations:

$$E[y_{i,b,t} \mid i \in I(b,b_0)] - E[y_{j,b,t} \mid j \in J(b,b_0)] = \gamma_{b,t}\left(E[\alpha_{i,t} \mid i \in I(b,b_0)] - E[\alpha_{j,t} \mid j \in J(b,b_0)]\right),$$
$$E[y_{i,b_0,t} \mid i \in I(b,b_0)] - E[y_{j,b_0,t} \mid j \in J(b,b_0)] = \gamma_{b_0,t}\left(E[\alpha_{i,t} \mid i \in I(b,b_0)] - E[\alpha_{j,t} \mid j \in J(b,b_0)]\right),$$

then we have:

$$\frac{\gamma_{b,t}}{\gamma_{b_0,t}} = \frac{E[y_{i,b,t} \mid i \in I(b,b_0)] - E[y_{j,b,t} \mid j \in J(b,b_0)]}{E[y_{i,b_0,t} \mid i \in I(b,b_0)] - E[y_{j,b_0,t} \mid j \in J(b,b_0)]}.$$

A crucial condition for the identification of the bank-factor $\frac{\gamma_{b,t}}{\gamma_{b_0,t}}$ is that $E[\alpha_{i,t} \mid i \in I(b,b_0)] \neq E[\alpha_{j,t} \mid j \in J(b,b_0)]$. This requires a separation between "types of firms" in the sense that the mean of the time-varying firm-specific unobserved heterogeneity is different across the two groups.

As in the standard factor model we require one normalization (e.g. $\gamma_{b_0,t} = 1$). Once $\gamma_{b,t}$ is identified, we can transform the model as:

$$\frac{y_{i,b,t}}{\gamma_{b,t}} = \frac{\beta_{b,t}}{\gamma_{b,t}} + \alpha_{f,t} + \frac{\epsilon_{f,b,t}}{\gamma_{b,t}}$$

Then, we can get rid of $\alpha_{f,t}$ by looking at the difference in the "normalized" growth rate of the same firm $i$ with banks $b$ and $b_0$ (similar to AW and KM):

$$\frac{y_{i,b,t}}{\gamma_{b,t}} - \frac{y_{i,b_0,t}}{\gamma_{b_0,t}} = \frac{\beta_{b,t}}{\gamma_{b,t}} - \frac{\beta_{b_0,t}}{\gamma_{b_0,t}} + \frac{\epsilon_{f,b,t}}{\gamma_{b,t}} - \frac{\epsilon_{f,b_0,t}}{\gamma_{b_0,t}}$$

Averaging over firms in the intersected set of firms $I(b, b_0)$ between bank $b$ and bank $b_0$:

$$\frac{\beta_{b,t}}{\gamma_{b,t}} - \frac{\beta_{b_0,t}}{\gamma_{b_0,t}} = E[\frac{y_{i,b,t}}{\gamma_{b,t}} - \frac{y_{i,b_0,t}}{\gamma_{b_0,t}} \mid i \in I(b,b_0)]$$

which identifies $\beta_{b,t} - \beta_{b_0,t}$. Then, identifying $\alpha_{f,t}$ is straightforward.

# C Discussion: Group-fixed effect estimator under unknown groups

Lets define the unfeasible group fixed effect estimator $\left(\widehat{\alpha^u}, \widehat{\beta^u}\right)$ as the solution of (11) when $g_t(f)$ is fixed to its population counterpart $g_t^0(f)$ rather than be estimated. Under known groups the group fixed effect estimator $\left(\widehat{\alpha^u}, \widehat{\beta^u}\right)$ coincides with the sample analogue of the moment conditions in (7) and (8),[22] which under the normalization $\beta_{b_0,g,t} = 0$:

$$\widehat{\beta^u}_{b,g_t(f),t} = \frac{\sum_{f \in g_t^0, f \in I(b,b_0)} \left[y_{f,b,t} - y_{f,b_0,t}\right]}{\sum_{f \in g_t^0, f \in I(b,b_0)} 1},$$

and

$$\widehat{\alpha^u}_{f,t} = \frac{\sum_{b \in I(f)} \left[y_{f,b,t} - \beta_{b,g,t}\right]}{\sum_{b \in I(f)} 1}.$$

For simplicity, assume full connections and balanced groups so that: $\sum 1_{f \in g, f \in I(b,b_0)} = N_F/G$ and $\sum 1_{b \in I(f)} = N_B$. Therefore, in the case of fixed known groups, we have $\widehat{\beta^u}_{b,g_t(f),t} \xrightarrow{p} \beta_{b,g_t(f),t}$ as $N_f$ tend to infinity and $\widehat{\alpha^u}_{f,t} \xrightarrow{p} \alpha_{f,t}$ as $N_B$ tend to infinity for all $t$. In the case of unknown groups, the group fixed effect estimator will coincide with the unfeasible group fixed effect estimator $\left(\widehat{\alpha^u}, \widehat{\beta^u}\right)$ under correct group classification $\widehat{g}_t\left(f|\widehat{\alpha}, \widehat{\beta}\right) = g_t^0(f)$. However, in finite samples, there is a non-zero probability that estimated and population group membership will not coincide. To illustrate the point, we follow Bonhomme and Manresa (2015) and consider a simple version of the model with just group effects with two groups $(G = 2)$ and normal errors:

$$y_{f,b,t} = \beta^0_{g_t^0(f),t} + \epsilon_{f,b,t}$$

where $\beta^0_{g_t(f),t}$ are the true parameters, $g_t^0(f) \in \{1,2\}$ is the true grouping assignment and $\epsilon_{f,b,t} \sim N(0,\sigma^2)$. From (10), we can see that the probability of wrongly assigning

[22] This is the case for $N_B = 2$, in a similar manner to how the within-group estimator and the first-difference estimator coincide in a standard panel data with $T = 2$. For $N_B > 2$, the unfeasible estimator under known groups will combine all the valid moment conditions across all possible bank first-differences.

into group 2 a firm that belongs to group 1 is:

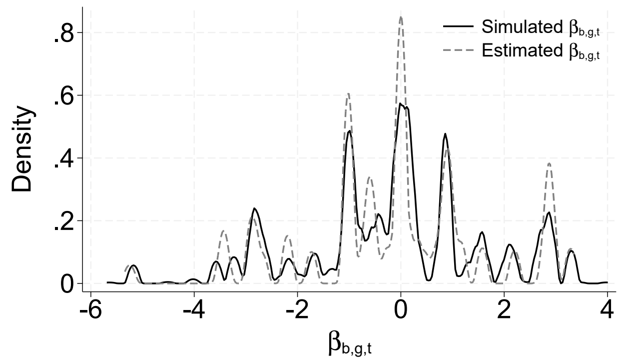
$$\begin{aligned}
\Pr\left(\hat{g}_t(f) = 2 \mid g_t^0(f) = 1\right) &= \Pr\left(\sum_{b=1}^{N_B}\left(\beta_{1,t}^0 + \epsilon_{f,b,t} - \beta_{2,t}^0\right)^2 < \sum_{b=1}^{N_B}\left(\beta_{1,t}^0 + \epsilon_{f,b,t} - \beta_{1,t}^0\right)^2\right) \\
&= \Pr\left(\frac{\sum_{b=1}^{N_B}\epsilon_{f,b,t}}{N_B} > \frac{\beta_{2,t}^0 - \beta_{1,t}^0}{2}\right) \\
&= 1 - \Phi\left(\sqrt{N_B}\left(\frac{\beta_{2,t}^0 - \beta_{1,t}^0}{2\sigma}\right)\right), \text{ for all t,} \qquad \text{(C.1)}
\end{aligned}$$

where $\Phi$ is the standard normal CDF. For fixed (and small) $N_B$, there is a non-negligible probability of misclassifying firms, so $\hat{g}_{(f),t}$ is inconsistent even if $N_F$ tend to infinity. Intuitively, according to (10) the estimator is classifying firms based on their similar responses to different banks, so we need to observe firms connected to several banks to be able to separate the effect $\beta^0_{g_t^0(f),t}$ from the idiosyncratic error $\epsilon_{f,b,t}$. As a consequence, $\widehat{\beta}$ will suffer from an incidental parameter bias and it will be inconsistent for fixed $N_B$ even when $N_F$ tend to infinity. The latter is in contrast to the case when groups are known where $\widehat{\beta}$ is $\sqrt{N_F}$-consistent. Note that the firm fixed effects $\hat{\alpha}$ is always inconsistent for fixed $N_B$ even when groups are known or even when the true model has homogenous effects as in Khwaja and Mian (2008) and Amiti and Weinstein (2018). However, equation C.1 indicates that the probability of misclassifying tends to zero at an exponential rate which implies that the bias generated by the incidental parameter problem goes to zero very fast as $N_B$ increases, similarly to the group fixed effect estimator of the standard panel model when $T$ increases (see Bonhomme and Manresa (2015)). As we show in simulations, the performance of our estimator increase very rapidly with $N_B$ and it is centered at the true value for moderate $N_B$.

# D Calibrated simulation

We evaluate the performance of our estimator in a simulation calibrated to our database. We fix $G = 4$, $N_F = 5000$, and $N_B = 10$ (note that there are 17 banks and 5000 firms in our dataset). We set the standard deviation of the error term to $\sigma_\epsilon = 0.35$ (the one estimated with our data) and we assume partial connections with a probability of matching of 40%. We conduct 100 replications of $y_{f,b,t}$ based on (6); for each replication, we estimate $\beta_{b,g(f)}$ using our algorithm. Figure D.1 displays the density of the true parameter $\beta_{b,g(f)}$ and estimates $\hat{\beta}_{b,g(f)}$ across replications. The figure shows that the estimation closely tracks the true parameters.

Figure D.1: Estimator's properties: calibrated Monte Carlo simulation




Note. Illustration of estimator's properties for a Monte Carlo simulation calibrated to our dataset. We assume $G = 4$, $N_B = 10$, $N_F = 5000$, $\sigma_\epsilon = 0.35$, and matching probability of 40%. The results are generated after 100 replications. The dark solid line displays the k-density across all simulated $\beta_{bg}$'s, while the gray dashed line displays the k-density across estimated $\widehat{\beta}_{bg}$'s.

# E Out-of-sample performance and number of groups

Building on Almagro and Manresa (2021), we propose the following N-fold cross-validation procedure for our algorithm to evaluate the out-of-sample performance of the model with number of groups $G$.

1. For a given year $t$, set the number of groups and repetitions: $G$ and $M$, respectively.

2. For given $G$ and $\forall m \in \{1, ..., M\}$, split the sample in $N$ parts (folds):

$$P_t(G,m) \equiv \{P_{t,1}(G,m), P_{t,2}(G,m), ...P_{t,N}(G,m)\}.$$

3. Take $P_{t,k}(G,m)$ as the testing sample, and the remaining parts of the sample, $P_{t,-k}(G,m)$, as the training sample.

4. Using the training sample, $P_{t,-k}(G,m)$, estimate the group structure and the parameters of interest. After that, compute the out-of-sample mean squared error ($MSE$) for the testing sample, $P_{t,k}(G,m)$, which number of observations is denoted by $J$, as follows:

$$MSE_t(G,m,P_{t,k}) = \frac{1}{J}\sum_{j\in P_k}\left(y_{f,b,t,j} - \widehat{\alpha}_{f,t,-j}(G,m) - \widehat{\beta}_{b,g(f),t,-j}(G,m)\right)^2. \quad \text{(E.1)}$$

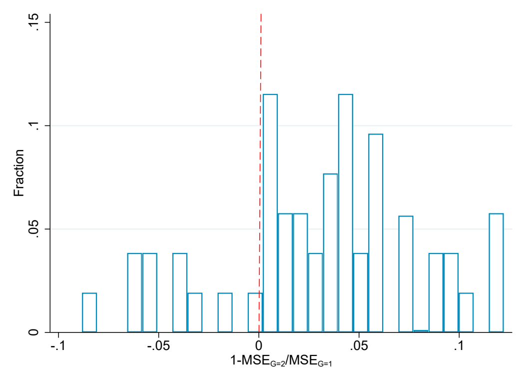
5. We measure the performance of the heterogeneous model $G$ for each of the

repetitions, and take the average for each of the folds and the repetitions in the sample:

$$\overline{MSE}_t(G) = \frac{1}{N}\frac{1}{M}\sum_{m=1}^{M}\sum_{k=1}^{N} MSE_t\left(G, m, P_{t,k}\right).$$

For our data, we find that the optimal number of groups is $G = 2$. In Figure E.1, we display the distribution of the out-of-sample MSE for each of the folds and repetitions for $G = 2$ (heterogenous case) relative to to $G = 1$ (homogenous case). A positive value implies that the heterogeneous case model has a better out-of-sample explanatory power than the homogeneous case. In general, we can observe how the heterogeneous model with $G = 2$ is most likely to perform better than the traditional homogeneous case with $G = 1$.

Figure E.1: Estimated $MSE_{G=2}$ vs $MSE_{G=1}$




Note. Out-of-sample mean squared errors for $G = 2$ vs $G = 1$. A positive value of the measure indicates that the out-of-sample MSE of $G = 2$ is lower than $G = 1$. The red dashed line indicates the case where both measures are equal. The information covers 2005 through 2017.

# F Additional tables

Table 3: Bank specialization and bank shock

| Dep. Variable | $\sum_{f\in g_t} \widehat{\beta}_{b,g_t(f),t}/N_f$ | |
|---|---|---|
| | (1) | (2) |
| Portfolio share in a group$_{t-1}$ | 0.673* | |
| | (1.73) | |
| Group specialization measure$_{t-1}$ | | 0.334* |
| | | (1.73) |
| Fixed Effects: | | |
| Bank-Year | Yes | Yes |
| Group-Year | Yes | Yes |
| R-squared | 0.23 | 0.33 |
| $N$ | 142 | 142 |

*Note.* In this table we estimate the relationship between an average of our estimated bank-group interaction estimates and measures of relevance of each estimated group in the bank portfolio. Portfolio share in a group represents the percentage of the corporate portfolio of the bank assigned to the given group: $\sum_{f\in G} L_{G(f)b,t-1}/\sum_f L_{fb,t-1}$. Group specialization measure denotes a relative portfolio share measure in a given group: $\frac{\sum_{f\in G} L_{G(f)b,t-1}/\sum_f L_{fb,t-1}}{\sum_b \sum_{f\in G} L_{G(f)b,t-1}/\sum_b \sum_f L_{fb,t-1}}$. The sample corresponds to a balance panel from 2006 to 2017.

Table 4: Event study: Number of firms in estimated groups

| Group Assignment | G=2 | G=3 | G=4 |
|---|---|---|---|
| g=1 | 422 | 465 | 382 |
| g=2 | 5,544 | 3,538 | 773 |
| g=3 | | 1,963 | 4,008 |
| g=4 | | | 803 |

*Note.* Number of firms identified for each group $g$ across different number of total groups, $G$.

Table 5: Firm-level investment regression

| Dependent variable: $\frac{\text{Investment}_{f,t}}{\text{Capital}_{f,t-1}}$ | (1) | (2) | (3) | (4) | (5) |
|---|---|---|---|---|---|
| | $G = 1$ | $G = 2$ | $G = 3$ | $G = 4$ | $G = 5$ |
| Bank Shock | -2.809 | 6.711* | 3.419** | 6.271* | 6.539 |
| | (-0.73) | (1.86) | (2.31) | (1.82) | (1.63) |
| Firm FE | -0.145 | 0.810** | 0.804 | 1.161 | 1.289 |
| | (-0.42) | (2.58) | (1.50) | (1.40) | (0.99) |
| Ln(Asset) | 5.937* | 6.737 | 6.354 | 5.380 | 5.947 |
| | (2.18) | (1.54) | (1.49) | (1.07) | (1.50) |
| Fixed Effects: | | | | | |
| Group-Time | Yes | Yes | Yes | Yes | Yes |
| Firm | Yes | Yes | Yes | Yes | Yes |
| R-squared | 0.21 | 0.24 | 0.25 | 0.29 | 0.29 |
| $N$ | 785 | 785 | 785 | 785 | 785 |

*Note.* Standard errors are cluster at the main leading bank. The information spans from 2005 to 2017.