The Cross-section of Subjective Expectations: Understanding Prices and Anomalies

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ABSTRACT

Cross-sectional decompositions using professional forecasts show high price-earnings ratios are accounted for by both low expected returns and overly high expected earnings growth. The magnitudes and timing of the comovements between prices, earnings growth, and returns are consistent with gradual learning rather than expectations being highly sensitive to recent realizations. Earnings growth surprises do not translate 1-1 into one-period returns, but instead are gradually reflected in returns over time. A structural model incorporating constant-gain learning about mean earnings growth, coupled with risk premia linked to cash flow timing, replicates our findings and generates realistic dispersion and persistence in price-earnings ratios.

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It has been known since Basu (1975) and Stattman (1980) that high price ratio stocks (e.g., price-earnings ratios, price-book ratios) earn lower returns than their peers. While the one-month difference between Growth and Value stocks has declined over time (Schwert, 2003; Fama and French, 2020), return differences at longer horizons have remained substantial (De la O, Han, and Myers, 2023)[1] and play a large role in accounting for the level of prices (van Binsbergen et al., 2023; Cho and Polk, 2023). Given that a stock's price is the risk-adjusted value of expected future cash flows, these realized return differences imply that high price ratio stocks have low risk exposure, overly high expected cash flows, or a mix of both.

There is a long-standing debate between research advocating either for risk exposure or incorrect cash flow expectations to explain cross-sectional differences in price ratios and subsequent returns.[2] In this paper, we provide evidence that not only suggests both explanations have merit, but also that the interaction between them is important to generate these cross-sectional differences. Our innovation is twofold.

First, using professional forecasts of both returns and cash flows, we estimate the fraction of cross-sectional dispersion in price ratios that is explained by high price ratio stocks having lower subjective expected returns and the fraction that is explained by high price ratio stocks having overly high subjective cash flow expectations. To the best of our knowledge, we are the first paper to quantify the relative importance of subjective return expectations and subjective cash flow expectations in accounting for cross-sectional dispersion in price ratios. We find that both components play a non-trivial role, however, incorrect cash flow expectations are the quantitatively larger component. Evaluating a number of full-information rational expectations (FIRE) models as well as behavioral models, we surprisingly find that the FIRE models struggle to match the decomposition results for realized cash flows and returns, while the behavioral models struggle to match the results for subjective expected cash flows and

[1]Table I also confirms that long-term return differences are large even for 1999-2020.

[2]See Fama and French (1995) and Daniel and Titman (1997) for early evidence and Lustig and Nieuwerburgh (2005) and Hou, Karolyi, and Kho (2011) for more recent explanations.

returns.

Second, in the words of Brunnermeier et al. (2021), rather than merely rejecting models, "*we need structural models of belief dynamics that can compete with [FIRE] models in explaining asset prices and empirically observed beliefs.*" Therefore, we propose and estimate a structural model of learning about firm-specific cash flow growth. Importantly, the model features risk premia related to cash flow timing, which means that learning about cash flow growth naturally generates variation across firms and across time in risk premia.[3] While we do not use any cross-sectional information when estimating the model, we find that it matches the cross-sectional decomposition results for both realized and expected cash flows and returns. Further, the model matches a number of additional untargeted moments of prices, cash flows, returns, and expectations. Using counterfactuals in which learning or risk premia are removed from the model, we show that the interaction of these two components is crucial for generating dispersion in price ratios that is as large and persistent as we observe in the data.

For our empirical analysis, we utilize a cross-sectional version of the Campbell-Shiller decomposition.[4] Using professional forecasts, we find that 43.3% of dispersion in price-earnings ratios is accounted for by high price ratio firms having higher expected four-year earnings growth and 12.7% of dispersion is accounted for by high price ratio firms having lower expected four-year returns.[5] Thus, both higher expected earnings growth and lower expected returns contribute to high valuation stocks. Interestingly, while Greenwood and Shleifer (2014) and De la O and Myers (2021) show that expected returns are positively correlated with price ratios in the aggregate time series, in the cross-section investors correctly

[3]Throughout the paper, we use "risk premia" to refer to compensation that investors require for expected risk. Note that average realized returns will depend on risk premia as well as deviations from FIRE in investors' cash flow expectations.

[4]Because this decomposition is derived from an identity, it holds even if expectations differ from the objective distribution.

[5]For concision, we shorten "subjective expected earnings growth" and "subjective expected returns" to simply "expected earnings growth" and "expected returns." Any time we refer to FIRE beliefs, we clearly specify that we are using the FIRE-implied distribution.

expect lower returns for high price-ratio firms.[6] The remaining dispersion is explained by expectations of future price-earnings ratios, which reflect expectations of earnings growth and returns beyond four years.

For comparison, realized four-year earnings growth and negative returns account for 9.9% and 32.0% of price-earnings ratio dispersion, respectively. This means that, empirically, high price ratio firms are primarily characterized by lower future returns than their peers, rather than by higher future earnings growth. In other words, investors overestimate the earnings growth of high price ratio firms, which leads to consistent disappointment in earnings growth for these firms. While investors do expect lower returns for high price ratio firms, they understate the magnitude of this relationship. Consistent with the fact that investors are disappointed by realized earnings growth, the realized returns on high price ratios firms are even lower than expected.

Because we are jointly studying return and earnings growth expectations, we also establish important facts on how earnings growth disappointment translates into return disappointment. Comparing the decomposition results at the one-year horizon and the four-year horizon, we find that disappointment in future earnings growth for high price ratio firms is largely concentrated at the one-year horizon. However, this does not immediately lead to large disappointment in one-year returns for these firms. Instead, disappointment in returns gradually accumulates over time as prices slowly decline.

By measuring how future earnings growth expectations are revised in response to earnings growth disappointment, we see why prices are slow to decline. After earnings growth disappointment, expected next year earnings growth initially *increases*, implying that forecasters expect the decline in earnings to be temporary and largely reverse in the following year. In other words, rather than being highly sensitive to recent realizations, expectations of future earnings are relatively "stubborn," in the sense that both positive and negative surprises are largely attributed to temporary shocks.

[6]Dahlquist and Ibert (2023), Bastianello (2023) and Büsing and Mohrschladt (2023) also find evidence that expected returns are negatively related to price ratios.

How do these findings fit with FIRE and non-FIRE models? We find that standard FIRE models struggle to match the magnitude of the empirical relationship between price-earnings ratios and future returns. While risk premia related to growth options or adjustment costs (Berk, Green, and Naik, 1999; Zhang, 2005) can generate return differences between high and low price-earnings ratio stocks, we find that these models predict a relationship that is an order of magnitude smaller than what we observe in the data. In contrast, we find that standard behavioral and learning models struggle to match the subjective expectations data, specifically the timing of earnings growth disappointment and return disappointment. For example, if agents extrapolate from current earnings growth or have diagnostic expectations of earnings growth, then prices should be highly sensitive to recent realized earnings growth, and earnings growth disappointment should translate immediately into large return disappointment.

While we mainly focus on decomposing cross-sectional dispersion in price ratios, we can analogously frame these tests in terms of understanding anomaly returns. Focusing on 20 annual anomalies from Hou, Xue, and Zhang (2015), we show that nearly every anomaly is associated with large return forecast errors. In other words, the realized one-year returns on anomalies are much higher than investors expected.[7] Further, we find that earnings growth surprises are large enough to account for the entirety of these unexpected anomaly returns. In fact, consistent with our finding on earnings growth disappointment and return disappointment for high price ratio stocks, we find that one-year earnings growth surprises are larger than the one-year unexpected returns, again indicating that earnings growth surprises do not immediately translate 1-1 into large one-year returns. Instead, we consistently find that negative (positive) earnings growth surprises increase (decrease) expected next-year earnings growth, in line with our results for high and low price ratio stocks.

To explain these empirical findings, we propose a structural model of belief formation and asset prices. The two key ingredients in the model are learning about firm-specific mean

[7] For a representative combination of the 20 anomalies, a one standard deviation increase in the anomaly variable is associated with a 340 bps increase in unexpected annual returns.

earnings growth and risk premia related to cash flow timing. Specifically, cross-sectional differences in firm earnings depend on each firm's underlying mean earnings growth as well as transitory idiosyncratic shocks. The agent uses constant-gain learning to infer the mean growth based on past realizations. In terms of preferences, the agent's SDF depends on an aggregate shock which is persistent but not permanent. As a result, short horizon cash flows carry higher risk premia, as they are disproportionately exposed to the aggregate shock compared to long horizon cash flows.

Qualitatively, the model aligns with our main empirical findings on subjective expectations and the response to earnings growth disappointment. In the model, firms with high price-earnings ratios have both high subjective expected earnings growth and low subjective expected returns, as beliefs about the underlying mean earnings growth impact both the expected cash flows and the risk premia associated with each firm. Importantly, if the constant-gain parameter is small, then one-period earnings growth disappointment is largely attributed to a temporary shock to the level of earnings rather than information about the underlying mean earnings growth. As a result, one-period earnings growth disappointment leads to an increase in expected next period earnings growth and only a small one-period return disappointment. Rather than being highly sensitive to recent realizations, prices instead adjust gradually over time as agents update their beliefs.

We then estimate and quantitatively test our constant-gain model. We set the constant-gain parameter to match previous studies on constant-gain learning (Milani, 2007; Malmendier and Nagel, 2016; Nagel and Xu, 2022) and estimate the remaining 5 parameters solely using realized earnings growth and average aggregate returns. Despite not using any cross-sectional information, the model successfully replicates our decomposition results, both in terms of magnitudes and timing, outperforming standard FIRE models in matching the realized dynamics of price-earnings ratios, earnings growth, and returns, and outperforming common behavioral models in matching the dynamics of subjective expectations. Further, the model matches several untargeted aggregate and cross-sectional asset pricing moments.

The quantified structural model allows us to extend our empirical results in two ways. First, we can go beyond the four-year horizon to estimate that expected earnings growth and expected returns for all horizons account for two-thirds (65.7%) and one-third (34.3%) of price-earnings ratio dispersion, respectively. This is largely due to errors in earnings growth expectations, which account for half (50.8%) of all price-earnings ratio dispersion.

Second, we can examine the underlying mechanisms which drive expected earnings growth and expected returns, namely constant-gain learning and risk premia related to cash flow timing, to show that the interaction between these two mechanisms is important for generating realistic dispersion and persistence in price-earnings ratios. For example, compared to an economy with no learning and no risk premia, introducing only risk premia has little impact on the dispersion in price-earnings ratios, introducing only learning increases the dispersion by a factor of 2.1, and introducing both increases the dispersion by a factor of 4.5. This highlights the benefit of unifying non-FIRE earnings growth expectations and risk premia related to cash flow timing, as the interaction magnifies the sensitivity of prices to changes in beliefs.

Broadly, this paper contributes to the growing literature using subjective expectations to understand asset prices.[8] In the cross-section, errors in firm-level professional earnings forecasts have been strongly linked to future returns (La Porta, 1996; Frankel and Lee, 1998; Da and Warachka, 2011; So, 2013; van Binsbergen, Han, and Lopez-Lira, 2022) and have been used to study a number of anomalies such as post-earnings announcement drift (Abarbanell and Bernard, 1992), the duration premium (Weber, 2018), and the profitability anomaly (Bouchaud et al., 2019). Moreover, Kozak, Nagel, and Santosh (2018) and Engelberg, Mclean, and Pontiff (2018) find that short legs of multiple long-short anomaly strategies comprise stocks with more optimistic earnings forecasts, whereas Engelberg, McLean, and Pontiff (2020) find that anomaly short legs comprise stocks with more optimistic return

[8] Amromin and Sharpe (2014); Greenwood and Shleifer (2014); Piazzesi, Salomao, and Schneider (2015); Cassella and Gulen (2018); De la O and Myers (2021); Nagel and Xu (2022); and Bordalo et al. (2022) utilize survey expectations for aggregate outcomes such as returns, cash flows and yields.

forecasts.

We differ from these studies in two important ways. First, by utilizing expectations of both earnings growth and returns, we quantify the relative importance of these two expectations in accounting for cross-sectional dispersion in price-earnings ratios and returns.[9] This decomposition sheds light on the relative importance of risk (discount rates) and mispricing in stock prices. It also allows us to quantitatively link unexpected anomaly returns to errors in earnings growth expectations. Second, we use the expectations data to test a structural model of expectation formation, preferences, and asset prices which links our decomposition results to underlying "deep" parameters of learning and risk sensitivity.

In terms of the structural model, our work is closely related to the literature on learning about mean consumption or cash flow growth (Lewellen and Shanken, 2002; Collin-Dufresne, Johannes, and Lochstoer, 2016; Nagel and Xu, 2022) and incorporates risk premia related to cash flow timing, similar in spirit to Lettau and Wachter (2007). We provide new evidence supporting these types of learning models using the cross-sectional dynamics of stocks and show that incorporating learning about temporary shocks to the level of earnings creates distinct qualitative predictions for the timing of earnings growth surprises and returns.[10] We also highlight that learning about cash flow growth naturally complements risk premia related to cash flow timing. Even if the objective timing of cash flows is relatively similar across all firms (Chen, 2017), these risk premia can still play an important role in stock prices so long as investors *believe* there is a large difference in the timing of cash flows. In other words, as argued in Jensen (2023), once we depart from FIRE, the compensation for risk that investors require should be disciplined by data on investors' believed risks, not the objective risks.

The rest of the paper is organized as follows. Sections I and II discuss the decomposition

[9]This differs from the implied cost of capital approach (Chen, Da, and Zhao, 2013; Hommel, Landier, and Thesmar, 2023) in which discount rates are inferred using earnings expectations for observable horizons and assumptions about long-term industry growth or GDP growth.

[10]In models of learning about mean cash flow growth without these temporary shocks to the level, such as Lewellen and Shanken (2002) and Nagel and Xu (2022), disappointing one-period cash flow growth translates more than 1-1 into disappointing realized returns and decreases expected next period cash flow growth.

and data utilized in our empirical exercises. Section III shows the results of the decomposition of price-earnings ratio dispersion, discusses how this relates to several FIRE and non-FIRE models, and extends our results to anomaly returns. Section IV proposes a structural model of constant-gain learning with risk premia related to cash flow timing. Section V quantifies and tests the structural model, calculates the infinite horizon decomposition of price-earnings ratio dispersion, and analyzes the interaction of learning and risk premia in generating realistic cross-sectional asset price moments.

I. Decomposing the cross-section of price ratios

While a large amount of the asset pricing literature has focused on the cross-section of short-term returns, relatively less attention has been paid to the cross-section of prices or price ratios.[11] In particular, we want to understand what can account for the large empirical dispersion in price ratios across stocks, e.g., why do some stocks trade at 50 times earnings while others only trade at 10 times earnings?

To understand dispersion in stock price ratios and how this dispersion relates to subjective cash flow growth expectations and subjective discount rates, we focus on a cross-sectional version of the Campbell-Shiller decomposition. In terms of notation, $E_t^* [\cdot]$ denotes subjective expectations. All other operators use the objective probability distribution. For example, $Var(\cdot)$ and $Cov(\cdot, \cdot)$ denote the observable variance or covariance of variables.

For any stock or portfolio of stocks i, the one-year ahead log return $r_{i,t+1}$ can be approximated in terms of the price-earnings ratio $px_{i,t}$, future earnings growth $\Delta x_{i,t+1}$, and the future price-earnings ratio:

$$r_{i,t+1} \approx \kappa + \Delta x_{i,t+1} + \rho px_{i,t+1} - px_{i,t}, \tag{1}$$

where κ and $\rho < 1$ are constants.[12] To understand cross-sectional dispersion in price-

[11] See Cochrane (2011) for a discussion, "When did our field stop being 'asset pricing' and become 'asset expected returning'?"

[12] Note that this approximation still holds even for non-dividend paying firms. Appendix B discusses the

earnings ratios, let $\tilde{px}_{i,t}$ be the cross-sectionally demeaned price-earnings ratio of portfolio i and let $\Delta\tilde{x}_{i,t+1}$ and $\tilde{r}_{i,t+1}$ be the cross-sectionally demeaned earnings growth and returns. Rearranging equation (1) and applying subjective expectations $E_t^*[\cdot]$, we see that a higher than average price-earnings ratio must be explained by higher than average expected earnings growth, lower than average expected returns, or a higher than average expected future price-earnings ratio,

$$\tilde{px}_{i,t} \approx \sum_{j=1}^{h} \rho^{j-1} E_t^* \left[\Delta\tilde{x}_{i,t+j}\right] - \sum_{j=1}^{h} \rho^{j-1} E_t^* \left[\tilde{r}_{i,t+j}\right] + \rho^h E_t^* \left[\tilde{px}_{i,t+h}\right]. \tag{2}$$

Importantly, equation (2) does not require that expectations are rational. Because this equation is derived from an identity, it holds under any subjective probability distribution.

To measure the relative contribution of subjective cash flow growth expectations and subjective discount rates to the dispersion in price-earnings ratios, we decompose the variance of $\tilde{px}_{i,t}$ into three components:

$$1 \approx \underbrace{\frac{Cov\left(\sum_{j=1}^{h}\rho^{j-1}E_t^*\left[\Delta\tilde{x}_{i,t+j}\right], \tilde{px}_{i,t}\right)}{Var\left(\tilde{px}_{i,t}\right)}}_{CF_h} + \underbrace{\frac{Cov\left(-\sum_{j=1}^{h}\rho^{j-1}E_t^*\left[\tilde{r}_{i,t+j}\right], \tilde{px}_{i,t}\right)}{Var\left(\tilde{px}_{i,t}\right)}}_{DR_h} + \underbrace{\rho^h\frac{Cov\left(E_t^*\left[\tilde{px}_{i,t+h}\right], \tilde{px}_{i,t}\right)}{Var\left(\tilde{px}_{i,t}\right)}}_{FPX_h}. \tag{3}$$

Note that $Var\left(\tilde{px}_{i,t}\right)$ is the average squared cross-sectionally demeaned price-earnings ratio, which means it measures the average cross-sectional dispersion in price-earnings ratios. The coefficients CF_h and DR_h give a quantitative measure of how much dispersion in price-earnings ratios is accounted for by dispersion in earnings growth expectations and how much is accounted for by dispersion in discount rates. Applying the decomposition to multiple horizons h provides information about the timing of expected earnings growth and discount rates. Additionally, the terms in equation (3) can be interpreted as the coefficients from univariate regressions with time fixed effects, e.g., a one unit increase in $px_{i,t}$ is associated

log-linearization in more detail including the role of the payout ratio.

with a CF_1 unit increase in expected one-year earnings growth.

When we estimate equation (3) using professional forecasts, we will use expectations of price growth $E_t^*[\Delta p_{i,t+j}]$ as a proxy for expectations of returns $E_t^*[r_{t+j}]$. Empirically, realized price growth and returns are closely related with a correlation of 0.997 to 0.999 for the $j = 1, ..., 4$ horizons that we study in our analysis. However, to ensure that the use of this proxy does not impact the results, we also estimate an exact decomposition based on price growth in Appendix C.1. Because this alternative decomposition is an exact identity, it also addresses any concerns that cross-sectional differences in payout ratios between high and low price-earnings ratio firms may impact the approximation error in equation (3). As shown in Tables I and AI, the results of this exact decomposition closely match the results from equation (3). Further, De la O, Han, and Myers (2023) show that payout ratios do not account for cross-sectional differences in price-earnings ratios, i.e., high price-earnings ratios are not associated with higher or lower dividend-earnings ratios.

II. Data

The firm-level realized earnings and prices are collected from Compustat and CRSP. The firm-level expected earnings and prices are collected from I/B/E/S (Institutional Brokers' Estimate System) and Value Line. To perform the decomposition from Section I, we sort these firms into the classic Value and Growth portfolios. Specifically, for each month t, we construct five value-weighted portfolios sorted by book-to-market.[13] For these portfolios, we measure the expectations at time t for earnings growth, price growth, and the future price-earnings ratio over the next four years. We also track the realized buy-and-hold future earnings growth, returns, and price-earnings ratios over the next four years. The *main sample*, which contains expectations of both earnings growth and price growth, ranges from 1999-2020. For robustness tests, we also use a *long sample* which ranges from 1982-2020 and

[13]The book-to-market ratio is defined as the market-cap in the portfolio formation month scaled by total book value from the most recent four quarters. To account for potential data errors, we exclude firms with book-to-market ratios over 100 or below 0.01.

contains earnings growth expectations. The subsections below provide more detail on the firm-level variable measurements.

A. Realized data

The sample of stocks consists of all common stocks (share code 10 and 11) listed on NYSE, AMEX, and NASDAQ. We obtain monthly prices, returns, and shares outstanding from the Center for Research in Security Price (CRSP). The firm-level accounting variables are constructed from the quarterly Compustat database. Following Davis, Fama, and French (2000) and Cohen, Polk, and Vuolteenaho (2003), we define book value as stockholders' book equity, plus deferred taxes and investment tax credit if available, minus the book value of preferred stock. If stockholders' book equity is not available at Compustat, we define it as the book value of common equity plus the par value of preferred stock, or the book value of assets minus total liabilities in that order. Depending on availability, we use redemption or par value for the book value of preferred stock. To be consistent with the I/B/E/S's definition of earnings, we define earnings as Compustat net income (item NIq) excluding non-I/B/E/S items, which comprise extraordinary items and discontinued operations (item XIDOq), special items (item SPIq), and non-recurring income taxes (item NRTXTq). This aligns with the measure of earnings proposed in Hillenbrand and McCarthy (2022). At every month, annual earnings at the firm level are defined as the sum of quarterly earnings from the most recent four quarters.[14] The main sample includes all firms which have observable returns $r_{i,t+j}$, earnings growth $\Delta x_{i,t+j}$, and price-earnings ratios $px_{i,t+j}$ in future years $j = 1, 2, 3, 4$. We require a future observation so that we can calculate forecast errors for the subjective expectations. However, for robustness, in Appendix C.4, we drop this requirement and estimate a decomposition using delisting returns to reinvest any delisting firms and find similar results.

[14]To account for possible data errors or extreme outliers, we winsorize annual earnings cross-sectionally at the 1% level.

B. Subjective expectations

The subjective earnings and short-term price expectations are extracted from the I/B/E/S Database. The Summary Statistics of the I/B/E/S Database contains the median forecasts for EPS (earnings per share) since 1976 for shorter horizons and 1982 for longer horizons for U.S. publicly traded companies and the median forecasts for prices at the 12-month horizon since 1999. I/B/E/S gathers their forecasts from hundreds of brokerage and independent analysts who track companies as part of their investment research work. Because the forecasts are not anonymous, analysts have a strong incentive to accurately report their expectations.[15] Furthermore, research on I/B/E/S suggests that financial firms' trades are consistent with their own analysts' forecasts and recommendations, which adds to the evidence that reported forecasts genuinely reflect the beliefs of the firms.[16] More importantly, market participants take seriously these analyst forecasts and trade in line with them, with stock prices increasing (decreasing) shortly after upward (downward) revisions in analyst earnings forecasts (Kothari, So, and Verdi 2016).

The long-term price expectations are obtained from the three-to-five-year price targets from the Value Line Investment Survey. Value Line is an independent investment research and financial publishing firm. The price targets cover approximately 1,700 actively traded U.S. companies every period, approximately 90% of the US publicly listed firms market value.[17] Value Line does not have any investment banking relation with the analyzed firms, nor any other obvious reason for providing biased forecasts. To the best of our knowledge, this is the only widely available survey containing firm-level price forecasts at long horizons.

We construct monthly earnings expectations for every firm in I/B/E/S at different horizons by using the EPS forecasts for up to three Annual Fiscal Periods (FY1-FY3) and the

[15] See Mikhail, Walther, and Willis (1999) and Cooper, Day, and Lewis (2001).

[16] Bradshaw (2004) shows that individual earnings forecasts are correlated to Buy/Sell recommendations, while Chan, Chang, and Wang (2009) show that financial firms' own stock holding changes are significantly positively related to recommendation changes.

[17] Value Line is an industry standard to the extent that it's been documented that a large portion of investment newsletters herds towards Value Line recommendations (Graham, 1999).

Long-Term Growth measure (LTG) meant to forecast earnings growth over the next "three-to-five years." For each month, we first interpolate across the different horizons in the annual fiscal periods to estimate an expectation over the next twelve months. We repeat this procedure to calculate two-year expectations. To estimate the three-year expectations, we use the two-year expectations and compound them with the long-term growth forecasts. We repeat this procedure to get four-year earnings expectations. We exclude from the main sample the following firms: a) firms without a LTG forecast, b) firms that do not have sufficient forecasts to calculate a 12-month interpolated forecast $E_t^* [\Delta x_{i,t+1}]$, and c) firms that do not have sufficient forecasts in the next year to calculate a 12-month interpolated forecast, $E_{t+1}^* [\Delta x_{i,t+2}]$.[18]

To estimate the price expectations, we obtain the one-year price expectations from the price target in I/B/E/S. We then calculate the four-year price expectation as the three-to-five year price targets from Value Line. We exclude from the main sample those firms missing either a one-year or a three-to-five year price forecast. Since analysts update earnings and price forecasts every month, our expectation data are also in monthly frequency. The main sample covers on average 79.7% of the total market size of firms listed for at least four years in CRSP.

III. Empirical Results

Table I shows the results of decomposition (3) applied both in a FIRE (Full Information Rational Expectations) benchmark and using the subjective expectations. The results show the fraction of price-earnings ratio dispersion that is explained by one-year earnings growth expectations and discount rates, as well as the fraction that is explained by four-year earnings growth expectations $\sum_{j=1}^{4} \rho^{j-1} E_t^* [\Delta \tilde{x}_{i,t+j}]$ and discount rates $\sum_{j=1}^{4} \rho^{j-1} E_t^* [\tilde{r}_{i,t+j}]$.

[18]This last point ensures that for every firm in the main sample we can calculate revisions $E_{t+1}^* [\Delta x_{i,t+2}] - E_t^* [\Delta x_{i,t+2}]$. This allows us to study how expectations of future earnings growth are revised after earnings growth surprises.

We first apply the decomposition under the FIRE benchmark using realized values and compare the results with several FIRE models of risk premia. Then, we apply the decomposition using subjective expectations and compare the results with several behavioral and learning models. Overall, we find that the FIRE models struggle to match the decomposition results for realized earnings growth and returns, while the behavioral and learning models struggle to match the decomposition results for subjective earnings growth expectations and return expectations. Finally, in Sections III.C-III.D, we discuss multiple robustness checks and extend our results to study anomaly returns and the timing of forecast errors.

A. *FIRE Benchmark*

Let $E_t^{FIRE}[\cdot]$ denote expectations under FIRE. Because forecast errors $\Delta\tilde{x}_{i,t+j} - E_t^{FIRE}[\Delta\tilde{x}_{i,t+j}]$ are uncorrelated with time t variables under FIRE, we know that $Cov\left(E_t^{FIRE}[\Delta\tilde{x}_{i,t+j}], \tilde{px}_{i,t}\right) = Cov\left(\Delta\tilde{x}_{i,t+j}, \tilde{px}_{i,t}\right)$. The same logic also applies to FIRE expectations of future returns and future price-earnings ratios. Thus, to evaluate the FIRE benchmark, the first and fourth columns of Table I show the estimates of CF_1, DR_1, FPX_1 and CF_4, DR_4, FPX_4 using the covariance of $\tilde{px}_{i,t}$ with realized future earnings growth, returns, and price-earnings ratios. For every coefficient, we report the Driscoll-Kraay standard errors, which account for very general forms of spatial and serial correlation, as well as the block-bootstrap standard errors, following the Martin and Wagner (2019) procedure.

Empirically, high price-earnings ratios are associated with lower future returns and slightly higher future earnings growth. The first column of Table I shows that 10.3% of dispersion in price-earnings ratios is accounted for by differences in one-year future earnings growth and 14.3% is accounted for by differences in one-year future returns. The remaining 74.6% is accounted for by the future price-earnings ratio.[19] At the fourth year horizon, the difference between CF_h and DR_h widens. As shown in the fourth column of Table I, differences

[19]Note that the three coefficients CF_h, DR_h and FPX_h are not mechanically set to equal one. However, the sum of these coefficients is very close to unity, summing 0.992 for the one-year decomposition and 0.969 for the four-year decomposition, which shows that equation (3) holds very tightly, and any potential deviations from the approximation (2) are not correlated with firm's price-earnings ratios.

Table I

Decomposition of dispersion in price-earnings ratios

This table decomposes the variance of price-earnings ratios using equation (3). The FIRE column report the elements CF_h, DR_h and FPX_h of the decomposition using future earnings growth, future negative returns and future price-earning ratios. The Expected column report the elements of the decomposition using expected earnings growth, expected returns and expected price-earning ratios. The Forecast Errors column reports the contribution of the forecast errors of each element. For instance, $CF_1 = Cov\left(\Delta\tilde{x}_{i,t+1}, \tilde{px}_{i,t}\right)/Var\left(\tilde{px}_{i,t}\right)$ is shown in the FIRE column. This component can be split into its expected component $Cov\left(E_t^*\left[\Delta\tilde{x}_{i,t+1}\right], \tilde{px}_{i,t}\right)/Var\left(\tilde{px}_{i,t}\right)$ and its error component $Cov\left(\Delta\tilde{x}_{i,t+1} - E_t^*\left[\Delta\tilde{x}_{i,t+1}\right], \tilde{px}_{i,t}\right)/Var\left(\tilde{px}_{i,t}\right)$. The main sample period is 1999 to 2020. The fourth row shows the element CF_h of the decompositions estimated over the longer sample period of 1982-2020. Driscoll-Kraay standard errors and block-bootstrap standard errors are calculated for each coefficient. In both cases, we use a lag equal to the maximum lag with any significant residual autocorrelation. Superscripts indicate Driscoll-Kraay significance at the 1% (***), 5% (**), and 10% (*) level.

		One-year horizon ($h=1$)			One-to-four year horizon ($h=4$)		
		FIRE	Expected	Forecast errors	FIRE	Expected	Forecast errors
1999-2020	CF_h	0.103^{***}	0.331^{***}	-0.228^{***}	0.099^{*}	0.433^{***}	-0.335^{***}
		[0.037]	[0.024]	[0.032]	[0.054]	[0.020]	[0.053]
		[0.050]	[0.027]	[0.047]	[0.082]	[0.023]	[0.071]
1999-2020	DR_h	0.143^{***}	0.033^{***}	0.110^{**}	0.320^{***}	0.127^{***}	0.192^{**}
		[0.050]	[0.013]	[0.053]	[0.080]	[0.043]	[0.082]
		[0.051]	[0.013]	[0.053]	[0.099]	[0.045]	[0.099]
1999-2020	FPX_h	0.746^{***}	0.620^{***}	0.126^{**}	0.550^{***}	0.385^{***}	0.165^{***}
		[0.05]	[0.019]	[0.056]	[0.057]	[0.027]	[0.062]
		[0.042]	[0.023]	[0.051]	[0.067]	[0.028]	[0.070]
1982-2020	CF_h	0.137^{***}	0.312^{***}	-0.175^{***}	0.147^{***}	0.462^{***}	-0.316^{***}
		[0.026]	[0.021]	[0.027]	[0.040]	[0.027]	[0.034]
		[0.026]	[0.021]	[0.028]	[0.043]	[0.027]	[0.033]

in future earnings growth over the next four years only accounts for 9.9% of dispersion in price-earnings ratios, while differences in future returns account for three times as much of the dispersion (32.0%).[20]

The large role of returns in explaining price dispersion poses a quantitative challenge for traditional FIRE asset pricing models. Even models designed to generate a value premium (i.e., low expected returns for high price ratio stocks) struggle to generate enough dispersion

[20] These results are consistent with De la O, Han, and Myers (2023), who use a longer sample (1963-2020) to show that at least 43.6% of dispersion in price-earnings ratio are reflected in differences in returns after ten years.

in expected returns to match our findings. In Table II, we simulate three FIRE models for the value premium (Berk et al., 1999; Zhang, 2005; Lettau and Wachter, 2007) using their benchmark specifications and calculate the model-implied CF_h and DR_h. As shown by the value of DR_h, in these models, differences in expected returns only account for a small fraction of the dispersion in price-earnings ratios.

Specifically, the three models imply that future returns over the next four years should account for less than 6% of dispersion in price-earnings ratios while, empirically, we find that they account for 32%. In the data and (to some extent) in the models, DR_h increases as we include more horizons. Thus, we also calculate in the model the maximum amount of dispersion that can be explained by returns DR_∞ and find that it is still an order of magnitude smaller than what we observe in the data using just the first four years of realized returns. These results highlight the importance of a quantitative framework. While there are certainly FIRE models in which high price ratio stocks have lower exposure to systematic risk, it is difficult to generate a risk premium that is quantitatively large enough to match the observed relationship between price-earnings ratios and future returns.

Why do these models struggle to generate large DR_h? Broadly, these models cover three distinct mechanisms for generating a value premium. However, all three can be thought of as settings in which agents exhibit preferences for the timing of cash flows. In Berk et al. (1999) and Zhang (2005), existing projects (or capital) cannot be adjusted easily in response to aggregate shocks. Instead, firms primarily adjust their choices about initiating new projects or installing new capital when aggregate shocks occur. Thus, firms whose value primarily comes from future potential projects (capital) rather than existing projects (capital) carry a lower risk premium, as they can more easily respond to aggregate shocks. In Lettau and Wachter (2007), aggregate shocks are partly reversed over time, reducing the exposure of longer-horizon cash flows to aggregate risk. Firms whose value mostly comes from future cash flows rather than current cash flows therefore carry a lower risk premium.

Because agents have rational expectations and know the objective parameters in these

Table II

Decompositions in FIRE Asset Pricing Models

This table calculates the variance decomposition for the price-earnings ratio in different asset pricing models and reports the implied cash flow and discount rate components for one year (CF_1, DR_1) and four years (CF_4, DR_4), as well as the infinite-horizon DR_∞. The first, second, and third rows show the results for models of risk premia. These three models are the model of growth options in Berk et al. (1999), the model of costly reversibility of capital in Zhang (2005), and the model of duration risk in Lettau and Wachter (2007). The last row shows the values measured in the data. All models are solved and estimated using the original author calibrations and simulated over a 20-year sample.

Models	CF_1	CF_4	DR_1	DR_4	DR_∞
Berk, Green, & Naik 1999 (Growth Options)	0.61	0.85	0.01	0.03	0.04
Zhang 2005 (Costly Reversibility of Capital)	−0.31	0.69	−0.01	−0.03	−0.03
Lettau & Wachter 2007 (Duration Premium)	0.03	0.24	0.02	0.06	−0.04
Observed Data (Main Sample)	0.10	0.10	0.14	0.32	n.a.

models, each firm's risk premium is tied to the objective timing of its cash flows. High price ratio stocks can only carry a low risk premium if they objectively have much more backloaded cash flows (i.e., much higher cash flow growth) than their peers. Thus, these models inherently struggle to match the empirical results, in which firms only differ slightly in their objective future cash flow growth but differ substantially in their objective future returns.[21] Appendix E discusses the three models and the simulations in more detail. In Section IV, we propose a model that incorporates this type of preference for the timing of cash flows but, importantly, we do not impose FIRE. In the quantified model, we find that these preferences play an important role in generating cross-sectional dispersion in price ratios because agents believe that firms differ substantially in their future cash flow growth, even though the objective differences in future cash flow growth are minimal.

B. Subjective Expectations

The second and fifth columns of Table I show the results of the decomposition when we use subjective expectations of earnings growth, returns, and future price-earnings ratios rather

[21] In general, these models would require extremely large risk aversion in order for the primary effect of differences in future cash flow growth (i.e., CF_h) to be dominated by the the secondary effect that differences in cash flow growth generate differences in discount rates (i.e., DR_h). These levels of risk aversion would imply that these models would no longer match the aggregate equity premium.

than assuming FIRE. Comparing the subjective results to the FIRE results, there are three important findings.

First, investors substantially overestimate the extent to which high price-earnings ratio stocks will have high future earnings growth. Differences in expected one-year earnings growth account for nearly a third (33.1%) of all dispersion in price-earnings ratios and differences in expected four-year earnings growth account for 43.3% of all price-earnings ratio dispersion. Given that realized one-year and four-year earnings growth only account for 10.3% and 9.9% of the dispersion, respectively, this means that high price-earnings ratios are consistently associated with disappointment in future earnings growth. Rephrased, more than a third of all dispersion in price-earnings ratios is accounted for by the fact that current price-earnings ratios significantly negatively predict future forecast errors (as shown in the "Forecast errors" columns). The final row of Table I shows that our earnings growth results are qualitatively and quantitatively similar over the longer 1982-2020 sample.

Second, investors understand that expensive stocks will have lower returns (i.e., a high price-earnings ratio is associated with lower expected returns), but they underestimate the magnitude of the relationship. As shown in the second row of Table I, differences in expected one-year returns account for 3.3% of dispersion in price-earnings ratios and differences in expected four-year returns account for 12.7%. This contrasts sharply with previous findings for aggregate return expectations, which positively comove with aggregate price ratios (Amromin and Sharpe, 2014; Greenwood and Shleifer, 2014; De la O and Myers, 2021). Consistent with the fact that investors overestimate future earnings growth for high $\tilde{px}_{i,t}$, we find that they consistently overestimate the returns for high $\tilde{px}_{i,t}$. In other words, while investors expect lower returns for high $\tilde{px}_{i,t}$ stocks, the realized returns are even worse than expected.

Combined, these first two findings emphasize that the mistakes in investors' expectations are about magnitudes, not directions. Investors understand that high price-earnings ratios are associated with higher future earnings growth and lower future returns, but they overestimate the magnitude of the earnings growth relationship and underestimate the magnitude of the return relationship. This highlights the benefit of using a quantitative decomposition which captures magnitudes as well as correlations to study these expectations.

The third finding, shown in the third column of Table I, is that the unexpected returns are smaller than the disappointment in earnings growth. While almost one quarter (22.8%) of $\tilde{px}_{i,t}$ dispersion is reflected in one-year earnings growth forecast errors, only 11.0% is reflected in one-year unexpected returns. In other words, the disappointment in earnings growth does not lead to an equally large disappointment in returns. This difference of 11.8% is statistically significant and economically meaningful, as it implies that price-earnings ratio dispersion is more persistent than forecasters expect, as shown in the forecast errors for FPX.

Why is disappointing earnings growth not immediately reflected in unexpected returns? From equation (1), we have that

$$\tilde{r}_{i,t+1} - E_t^* \left[\tilde{r}_{i,t+1}\right] \approx \left(\Delta\tilde{x}_{i,t+1} - E_t^* \left[\Delta\tilde{x}_{i,t+1}\right]\right) + \rho \left(\tilde{px}_{i,t+1} - E_t^* \left[\tilde{px}_{i,t+1}\right]\right). \tag{4}$$

If there is no unexpected change to the price-earnings ratio $\tilde{px}_{i,t+1}$, then disappointment in one-year earnings growth should translate 1-1 into disappointment in one-year returns. If disappointing earnings growth lowers expected future earnings growth, then disappointment in earnings growth will also lower $\tilde{px}_{i,t+1} - E_t^* \left[\tilde{px}_{i,t+1}\right]$, as the change in expected future earnings growth lowers the price-earnings ratio. In this case, we would see that the disappointment in returns is *larger* in magnitude than the disappointment in earnings growth, as returns capture the disappointment in $t+1$ earnings growth and the downward revision to earnings growth for $t+2$ and beyond. Conversely, if disappointment in earnings growth raises expected future earnings growth (e.g., investors expect the disappointment in earnings growth to be partly reversed), then disappointment in earnings growth will be associated with a positive $\tilde{px}_{i,t+1} - E_t^* \left[\tilde{px}_{i,t+1}\right]$ and the disappointment in returns will be smaller than the disappointment in earnings growth. Regressing earnings growth revisions $E_t^* \left[\Delta x_{i,t+1}\right] - E_{t-1}^* \left[\Delta x_{i,t+1}\right]$ on earnings growth surprises $\Delta x_{i,t} - E_{t-1}^* \left[\Delta x_{i,t}\right]$, we indeed find that earnings growth disappointment is expected to be partly reversed (see Table III Panel

B).

How do these findings compare to common behavioral models? Focusing first on return expectations, a well-studied model is return extrapolation (e.g., Barberis et al. 2015 and Jin and Sui 2022). Under these models, a high price ratio is caused by agents having high return expectations and bidding up the stock's price, which subsequently leads to low future realized returns. This is inconsistent with our empirical finding that investors expect *lower* returns for high price ratio stocks.

Focusing on cash flow growth expectations, agents may overstate the persistence of growth (e.g., Hirshleifer, Li, and Yu 2015), or have diagnostic expectations of growth (e.g., Bordalo et al. 2022). We also consider models in which agents are learning about the mean of an i.i.d. growth process (e.g., Lewellen and Shanken 2002 and Nagel and Xu 2022). These mechanisms can all potentially explain our first empirical finding, which is that investors' cash flow growth expectations overstate the objective relationship between current price-earnings ratios and future earnings growth.[22] However, as detailed in Appendix F, these mechanisms all imply that one-year earnings growth disappointment should *lower* expected next-year earnings growth and should translate *more than 1-1* into one-year return disappointment.

If investors believe that earnings growth is highly persistent, then a disappointing earnings growth realization causes investors to negatively revise their future earnings growth expectations. Similarly, if investors are learning the mean of an i.i.d. earnings growth process or have diagnostic expectations that magnify recent shocks, then disappointing earnings growth should induce negative revisions to expected future growth. As mentioned above, this implies that the disappointment in returns is larger in magnitude than the disappointment in earnings growth, as returns reflect both the disappointment in current earnings growth and the revisions to future earnings growth.

In short, our empirical finding that one-year return disappointment is smaller in mag-

[22]In Appendix F, we also discuss the diagnostic expectations model of Bordalo et al. (2019). Because this model features diagnostic expectations about earnings levels, rather than earnings growth, it predicts that high price-earnings ratio stocks have low one-year earning growth expectations, which is not consistent with our first finding.

nitude than the one-year earnings growth disappointment is inconsistent with mechanisms that make prices highly sensitive to the most recent earnings growth realizations. Rather than revising down their expectations of next year growth after a disappointing earnings growth realization, we find that investors expect the disappointment will largely be reversed and that there is only a muted immediate change in prices. It is only after several years that the disappointment in earning growth translates into large negative realized returns. We show in Sections IV and V that having investors use constant-gain learning to infer both mean earnings growth and a temporary shock to the level of earnings successfully matches our empirical findings. This is similar in spirit to learning about the mean of an i.i.d. process, but the inclusion of the temporary shocks qualitatively changes the timing of earnings growth surprises and return surprises.

C. Robustness checks

Given the importance of these decomposition results, we perform a number of robustness checks. First, in addition to the Driscoll-Kraay and block-bootstrap standard errors reported in Table I, we also calculate the significance of our results under a worst-case scenario for overlapping observations. Specifically, in Appendix C.2, we perform Bauer and Hamilton (2018) simulations, which account for trends and potential small-sample bias, and assume a worst-case scenario for overlapping observations in which residuals are MA(47). Note that this substantially overstates the measured persistence of our residuals. With the exception of the coefficient for four-year realized earnings growth, we find that the earnings growth and return coefficients in Table I are all signficant at the 5% level, even under this worst-case scenario.

Second, we estimate an exact decomposition to remove the approximation in equation (3). As shown in Table AI, this exact decomposition gives nearly identical results to Table I.

Third, in Table AIV, we address the concern that dispersion in price-earnings ratios may potentially be driven by fluctations in one-year earnings rather than cross-sectional differences in prices. We find that the dispersion in price-earnings ratios is nearly identical to the dispersion price-to-smoothed-three-year-earnings ratios.[23] Further, we show that the decomposition results are not changed in any noticable way when we repeat the decomposition for price-to-smoothed-three-year-earnings ratios.

Finally, we address potential survivorship bias. For our main estimation, we require that stocks have an observed future price and future earnings, as this allows us to study forecast errors in subjective expectations. However, in Table AV, we remove this requirement and calculate future portfolio outcomes by reinvesting delisted stocks based on the delisting return. We find almost no change in our results.

D. Extending to anomaly returns

The presence of both return and earnings growth expectations can also be used to study cross-sectional anomaly returns using the logic in equation (4). Consider an anomaly variable $\tilde{a}_{i,t}$, such as profitability or investment, which predicts next-period returns. To make comparison across anomalies simple, we normalize $\tilde{a}_{i,t}$ so that it has variance 1 and positively comoves with future returns. From equation (4), we have the identity

$$\begin{aligned} \underbrace{Cov\left(\tilde{r}_{i,t+1} - E_t^*\left[\tilde{r}_{i,t+1}\right], \tilde{a}_{i,t}\right)}_{\sigma_{a,r}} &\approx \underbrace{Cov\left(\Delta\tilde{x}_{i,t+1} - E_t^*\left[\Delta\tilde{x}_{i,t+1}\right], \tilde{a}_{i,t}\right)}_{\sigma_{a,x}} \\ &+ \underbrace{\rho Cov\left(\tilde{px}_{i,t+1} - E_t^*\left[\tilde{px}_{i,t+1}\right], \tilde{a}_{i,t}\right)}_{\sigma_{a,px}}. \end{aligned} \tag{5}$$

For robustness, Appendix C.1 shows an exact decomposition based on price growth, which gives very similar results.

Under full-information rational expectations, we would have $\sigma_{a,r}, \sigma_{a,x}, \sigma_{a,px} = 0$, i.e., any predictable anomaly returns would be fully anticipated and $\tilde{a}_{i,t}$ would not predict forecast errors. For example, a higher $\tilde{a}_{i,t}$ might be related to higher risk exposure and investors

[23]The standard deviation of cross-sectionally demeaned price-earnings ratios is 30.1% and the standard deviation of cross-sectionally demeaned price-to-three-year-earnings ratios is 32.6%. This demonstrates that smoothing the denominator does not reduce the cross-sectional dispersion in price ratios.

Table III

Unexpected anomaly returns

This table measures and decomposes unexpected anomaly returns. The Representative Anomaly is the average ranking of each stock across 20 different anomalies, and $\tilde{px}_{i,t}$ is the demeaned price-earnings ratio. For each anomaly variable, we sort stocks into five equal-value portfolios based on the anomaly variable. Panel A shows the coefficients of regressing each of the dependent variables on a specific anomaly variable. Both anomaly variables are scaled to have unit variance and to positively comove with future returns. The three dependent variables are the unexpected return $\tilde{r}_{i,t+1} - E_t^*\left[\tilde{r}_{i,t+1}\right]$, the earnings growth forecast errors $\Delta\tilde{x}_{i,t+1} - E_t^*\left[\Delta\tilde{x}_{i,t+1}\right]$, and the price-earnings ratio forecast errors $\rho\left(\tilde{px}_{i,t+1} - E_t^*\left[\tilde{px}_{i,t+1}\right]\right)$. The sample period is 1999 to 2020. Panel B shows the effect of earnings growth surprises on revisions. Each column shows the cofficient from regressing the revision in earnings growth $E_{t+1}^*\left[\Delta\tilde{x}_{i,t+2}\right] - E_t^*\left[\Delta\tilde{x}_{i,t+2}\right]$ on the earnings growth surprise $\Delta x_{i,t+1} - E_t^*\left[\Delta x_{i,t+1}\right]$. The first row shows the result of the regressions using the main sample period of 1999 to 2020. The second row shows the result of the regressions using the long sample period of 1982 to 2020. Driscoll-Kraay standard errors and block-bootstrap standard errors are calculated for each coefficient. In both cases, we use a lag equal to the maximum lag with any significant residual autocorrelation. Superscripts indicate Driscoll-Kraay significance at the 1% (***), 5% (**), and 10% (*) level.

Panel A: Anomaly Return Decomposition		
	Representative Anomaly	$\tilde{px}_{i,t}$
$\tilde{r}_{i,t+1} - E_t^*\left[\tilde{r}_{i,t+1}\right]$	0.0340^{***}	0.0331^{**}
	[0.013]	[0.016]
	[0.013]	[0.016]
$\Delta\tilde{x}_{i,t+1} - E_t^*\left[\Delta\tilde{x}_{i,t+1}\right]$	0.0635^{***}	0.0687^{***}
	[0.020]	[0.010]
	[0.020]	[0.014]
$\rho\left(\tilde{px}_{i,t+1} - E_t^*\left[\tilde{px}_{i,t+1}\right]\right)$	-0.0317^{***}	-0.0380^{**}
	[0.09]	[0.017]
	[0.09]	[0.015]
Panel B: Revisions after Surprises		
Main Sample 1999-2020	-0.811^{***}	-0.863^{***}
	[0.49]	[0.075]
	[0.49]	[0.074]
Long Sample 1982-2020	-0.818^{***}	-0.786^{***}
	[0.060]	[0.091]
	[0.062]	[0.088]

would require higher returns on these stocks as compensation. More broadly, positive values of $\sigma_{a,r}$ indicate that investors understate the relationship between $\tilde{a}_{i,t}$ and future returns. In other words, the high returns on high $\tilde{a}_{i,t}$ stocks are not fully anticipated. Negative values for $\sigma_{a,r}$ indicate that investors not only understand that high $\tilde{a}_{i,t}$ stocks have higher returns, but they exaggerate the magnitude of the relationship.

In comparison, the values for $\sigma_{a,x}$ and $\sigma_{a,px}$ indicate how much the predictable return forecast errors are explained by predictable errors in next-year earnings growth expectations and expectations of the future price-earnings ratio. When high $\tilde{a}_{i,t}$ stocks generate unanticipated high next-period returns, these returns can be explained by unexpectedly high next-period earnings growth. Alternatively, $\tilde{a}_{i,t}$ could positively predict forecast errors for the future price-earnings, which would mean that the unanticipated high returns of high $\tilde{a}_{i,t}$ stocks are due to errors in return expectations and earnings growth expectations at longer horizons beyond one period, not because of next-period earnings growth.

We estimate the decomposition in equation (5) using our high and low price-earnings ratio portfolios from the previous section, as well as 20 other annual anomalies from Hou, Xue, and Zhang (2015).[24] For each anomaly, we sort stocks into five equal-value portfolios based on the anomaly variable.[25] We then measure forecast errors for one-year returns, earnings growth, and price-earnings ratios and regress each of the three variables on the anomaly variable. We also calculate a representative anomaly that sorts stocks based on the 20 different variables and uses the average ranking across these variables in the sorting and in the regressions.

Table III shows that the results for the representative anomaly are qualitatively and quantitatively similar to our findings from Section III.B. Note that both the representative anomaly and $\tilde{px}_{i,t}$ have been scaled to have unit variance and to positively comove with future

[24]Beyond the price-earnings ratio, we find 21 anomalies documented in Hou, Xue, and Zhang (2015) which are applicable to annual returns. We then drop the size anomaly because the forecasts are provided primarily for large firms, and are thus, not suited to cover portfolios over this anomaly.

[25]To perform these tests, stocks are required to have one-year expected and realized earnings growth, returns, and price-earnings ratios. We also require that stocks have a future one-year earnings growth expectation $E^*_{t+1}[\Delta\tilde{x}_{i,t+2}]$ for our test of revisions.

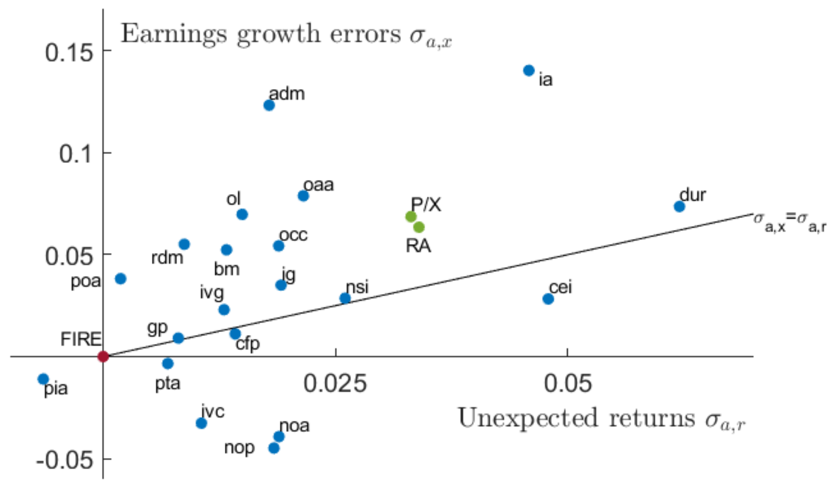

Figure 1. Unexpected anomaly returns. This figure shows the decomposition results $(\sigma_{a,r}, \sigma_{a,x})$ for each anomaly $\tilde{a}_{i,t}$. The x-axis shows $\sigma_{a,r} = Cov\left(\tilde{r}_{i,t+1} - E_t^*\left[\tilde{r}_{i,t+1}\right], \tilde{a}_{i,t}\right)$, which measures how much the anomaly variable predicts unexpected returns. The y-axis shows $\sigma_{a,x} = Cov\left(\Delta\tilde{x}_{i,t+1} - E_t^*\left[\Delta\tilde{x}_{i,t+1}\right], \tilde{a}_{i,t}\right)$, which measures how much the anomaly variable predicts one-year earnings growth forecast errors. The anomalies are shown in blue. In red, we show the FIRE benchmark, which is that $\sigma_{a,r}$ and $\sigma_{a,x}$ should equal 0 for all anomalies. In green, we show a Representative Anomaly (RA) that sorts stocked based on their average ranking across all of the individual anomalies, as well as the results for the portfolios used to study cross-sectional variation in price-earnings ratios (P/X). Each anomaly variable $\tilde{a}_{i,t}$ is scaled to have unit variance and to positively comove with future returns.

returns. Panel A shows that a one standard deviation increase in either of the anomaly variables is associated with a roughly 3p.p. increase in unexpected returns (0.0340 and 0.0331, respectively). This increase in unexpected returns is more than accounted for by the roughly 6p.p. increase in unexpected earnings growth (0.0635 and 0.0687, respectively). Why does the large earnings growth surprise not immediately translate 1-1 into unexpected returns? It is because the earnings growth surprise is expected to be partly reversed by next period earnings growth, as shown in Panel B.

Figures 1 and 2 show the results for each of the 22 anomalies (the 20 individual anomalies, our price-earnings ratio portfolios and the representative anomaly). Starting with Figure 1, we see that for almost every anomaly, we estimate a positive value of $\sigma_{a,r}$, meaning that investors do not fully anticipate the high returns on high $\tilde{a}_{i,t}$ stocks. Further, we find

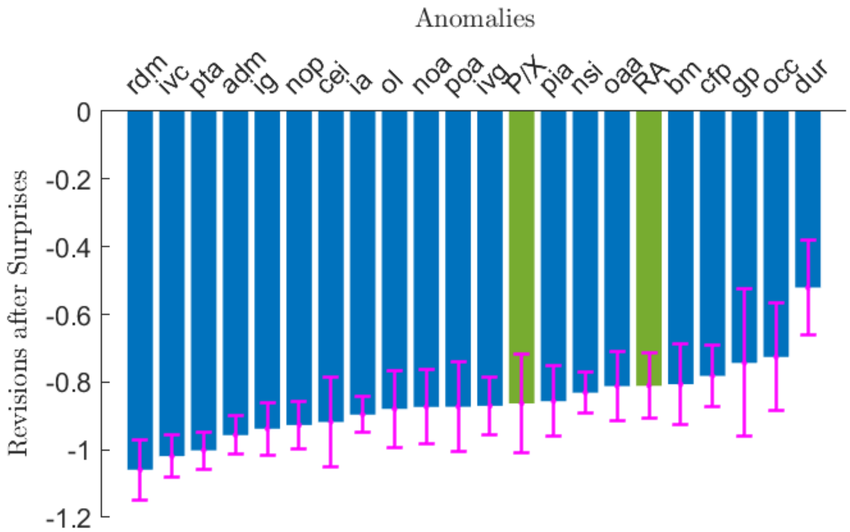

Figure 2. Revisions in anomaly expected earnings growth. This figure shows the effect of earnings growth surprises on revisions for each set of anomaly portfolios. Each bar shows the coefficient from regressing the revision in earnings growth $E^*_{t+1}[\Delta\tilde{x}_{i,t+2}] - E^*_t[\Delta\tilde{x}_{i,t+2}]$ on the earnings growth surprise $\Delta\tilde{x}_{i,t+1} - E^*_t[\Delta\tilde{x}_{i,t+1}]$. The coefficients are shown in ascending order. Individual anomalies are shown in blue. In green, we show the Representative Anomaly (RA) that sorts stocks based on their average ranking across all of the individual anomalies, as well as the results for the portfolios used to study cross-sectional variation in price-earnings ratios (P/X).

that most anomalies (17 out of 22) are associated with large positive one-year earnings growth forecast errors, as shown by the estimates of $\sigma_{a,x}$. Appendix Table AVI show the full decomposition for each anomaly.

Comparing $\sigma_{a,r}$ and $\sigma_{a,x}$ across anomalies, we see that anomalies with higher $\sigma_{a,r}$ generally have higher $\sigma_{a,x}$, i.e., larger unanticipated returns are associated with larger one-year earnings growth forecast errors, and $\sigma_{a,x}$ is generally larger than $\sigma_{a,r}$, i.e., earnings growth surprises translate less than 1-1 into unexpected returns. This means that our findings on the dynamics of earnings growth surprises and unexpected returns from Section III.B also extends to most anomaly portfolios. As shown in Figure 2, we once again find that this pattern is explained by the fact that positive earnings growth surprises $\Delta\tilde{x}_{i,t+1} - E^*_t[\Delta\tilde{x}_{i,t+1}]$ *decrease* expected next period growth $E^*_{t+1}[\Delta\tilde{x}_{i,t+2}]$. For all anomalies in the sample, this coefficient is negative and significant. As this revision test does not necessitate return expectations, we also conduct it over the long sample for robustness and find similar results in Appendix Table AVII.

To summarize, consistent with the results from Section III.B, we find quantitatively large deviations from FIRE in expectations of both returns and earnings growth for anomaly portfolios. The magnitudes of these deviations highlight that not only are unexpected returns positively associated with earnings growth surprises, but also that the relationship is less than 1-1. This evidence, once again, points against models in which unexpected realized returns are highly sensitive to recent earnings growth surprises and highlights the benefit of quantitative decompositions which allow for these types of comparisons. In the next section, we propose a model that can replicate these dynamics of earnings growth surprises and unexpected returns.

IV. Model of cash flow expectations and discount rates

In this section, we introduce a model with slow-moving learning about cash flow growth and risk premia related to cash flow timing. We show in Section V that this model quantitatively replicates our empirical findings. Throughout this section, we use lowercase letters to denote log values, $z \equiv \log(Z)$.

A. *Cash flows and the stochastic discount factor*

For each firm i, the log cash flow $x_{i,t}$ has an aggregate and a firm-level component,

$$x_{i,t} = x_t^{agg} + \tilde{x}_{i,t} \tag{6}$$

$$x_t^{agg} = \phi x_{t-1}^{agg} + u_t \tag{7}$$

$$\tilde{x}_{i,t} = g_i t + v_{i,t}. \tag{8}$$

The aggregate component is an AR(1) process, which can be thought of as business-cycle fluctuations. The firm-level component is a firm-specific trend $g_i t$ plus noise to capture potential cross-sectional differences in growth rates. The shocks $u_t, v_{i,t}$ are uncorrelated and have variances σ_u^2, σ_v^2.

The agent has a log stochastic discount factor

$$m_{t+1} = -r^f - \frac{1}{2}\gamma^2\sigma_u^2 - \gamma u_{t+1} \tag{9}$$

which depends on the aggregate shock u_{t+1}.

B. *Subjective cash flow expectations*

Objectively, the value of g_i is identical across firms, $g_i = \bar{g}$.[26] However, the agent does not know each firm's g_i and forms her subjective expectation $E_t^*[g_i]$ using constant-gain learning,

$$E_t^*[g_i] = E_{t-1}^*[g_i] + \beta\left(\Delta\tilde{x}_{i,t} - E_{t-1}^*[\Delta\tilde{x}_{i,t}]\right) \tag{10}$$

$$E_t^*[v_{i,t}] = (1-\beta)\left(\Delta\tilde{x}_{i,t} - E_{t-1}^*[\Delta\tilde{x}_{i,t}]\right) \tag{11}$$

where β is the constant-gain parameter. Specifically, after observing the surprise $\Delta\tilde{x}_{i,t} - E_{t-1}^*[\Delta\tilde{x}_{i,t}]$, she attributes portion β to firm-specific growth and portion $(1-\beta)$ to the noisy shock $v_{i,t}$. Her expectation for the future growth of the firm-level component is then

$$E_t^*[\Delta\tilde{x}_{i,t+1}] = E_t^*[g_i] - E_t^*[v_{i,t}]. \tag{12}$$

Her expectation for the future level of the firm-level component is

$$E_t^*[\tilde{x}_{i,t+n}] = \tilde{x}_{i,t} + nE_t^*[g_i] - E_t^*[v_{i,t}]. \tag{13}$$

C. *Prices and subjective risk premia*

Sections IV.A and IV.B lay out all of the elements and assumptions of the model. In this subsection, we simply combine the agent's beliefs and the stochastic discount factor to calculate the price for various claims. Appendix A gives the details for all of the equations.

To start, let $P_t^{(n)}$ be the price of an n-period aggregate strip, i.e., a claim that pays X_{t+n}^{agg}

[26]Given that our empirical analysis focuses on price-earnings ratios, we normalize $\bar{g}$ to 0 without loss of generality.

in n periods. The aggregate strip price is

$$
\begin{aligned}
P_t^{(n)} &= E_t^*\left[\left(\prod_{j=1}^{n} M_{t+j}\right) X_{t+n}^{agg}\right] \\
&= \exp\left\{-nr^f - \gamma\sigma_u^2\frac{1-\phi^n}{1-\phi} + \frac{1}{2}\sigma_u^2\frac{1-\phi^{2n}}{1-\phi^2} + \phi^n x_t^{agg}\right\}.
\end{aligned} \tag{14}
$$

The realized return on the strip is

$$
\begin{aligned}
R_{t+1}^{(n)} &= \frac{P_{t+1}^{(n-1)}}{P_t^{(n)}} \\
&= \exp\left\{r^f + \gamma\sigma_u^2\phi^{n-1} - \frac{1}{2}\sigma_u^2\phi^{2(n-1)} + \phi^{n-1}u_{t+1}\right\}
\end{aligned} \tag{15}
$$

and the subjective expected return on the strip is

$$
E_t^*\left[R_{t+1}^{(n)}\right] = \exp\left\{r^f + \gamma\sigma_u^2\phi^{n-1}\right\}. \tag{16}
$$

The first term (r^f) reflects the risk-free rate and the second term ($\gamma\sigma_u^2\phi^{n-1}$) reflects the subjective risk premium, i.e., the compensation agents require for exposure to risk.

Equation (16) shows an important characteristic of this model: longer horizon strips carry a lower annual subjective risk premium $\gamma\sigma_u^2\phi^{n-1}$. Equation (7) shows that aggregate shocks are persistent but not permanent. This means that short horizon cash flows are disproportionately sensitive to the aggregate shock.[27] Because of this, the annual risk premium is higher for short horizon cash flows. This is similar to the mechanism in Lettau and Wachter (2007).

Each firm i can be viewed as a collection of strips. Specifically, since shocks to the firm-level component $v_{i,t}$ are uncorrelated with the aggregate shock, we can express the firm's

[27] Specifically, the covariance of the log cumulative stochastic discount factor $\log\left(\prod_{j=1}^{n} M_{t+j}\right)$ with the aggregate cash flows x_{t+n}^{agg} is $\frac{1-\phi^n}{1-\phi}\gamma\sigma_u^2$, which increases with horizons but not proportionally.

price as

$$
\begin{aligned}
P_{i,t} &= \sum_{n=1}^{\infty} E_t^* \left[\left(\prod_{j=1}^{n} M_{t+j} \right) X_{t+n}^{agg} \tilde{X}_{i,t+n} \right] \\
&= \sum_{n=1}^{\infty} P_t^{(n)} E_t^* \left[\tilde{X}_{i,t+n} \right] \\
&= \sum_{n=1}^{\infty} P_t^{(n)} \exp \left\{ \frac{1}{2} \sigma_v^2 + E_t^* \left[\tilde{x}_{i,t+n} \right] \right\}. \qquad (17)
\end{aligned}
$$

In other words, idiosyncratic firm risk is not priced, so firm prices simply depend on the expected firm-level component $E_t^* \left[\tilde{x}_{i,t+n} \right]$ and the aggregate strip prices $P_t^{(n)}$.

The subjective expected return on firm i is then simply a weighted average of the subjective expected return on the individual strips,

$$
\begin{aligned}
E_t^* \left[R_{i,t+1} \right] &= E_t^* \left[\frac{X_{t+1}^{agg} \tilde{X}_{i,t+1} + P_{i,t+1}}{P_{i,t}} \right] \\
&= \sum_{n=1}^{\infty} w_{i,t,n} E_t^* \left[R_{t+1}^{(n)} \right] \\
&= \sum_{n=1}^{\infty} w_{i,t,n} \exp \left\{ r^f + \gamma \sigma_u^2 \phi^{n-1} \right\} \qquad (18)
\end{aligned}
$$

where the weight $w_{i,t,n} = \frac{\exp\{n E_t^*[g_i]\} P_t^{(n)}}{\sum_{n=1}^{\infty} \exp\{n E_t^*[g_i]\} P_t^{(n)}}$ captures how much of the firm's value in equation (17) comes from its horizon n cash flows.

The realized return for firm i is

$$
\begin{aligned}
R_{i,t+1} &= \frac{X_{t+1}^{agg} \tilde{X}_{i,t+1} + P_{i,t+1}}{P_{i,t}} \\
&= \sum_{n=1}^{\infty} w_{i,t,n} R_{t+1}^{(n)} \frac{E_{t+1}^* \left[\tilde{X}_{i,t+n} \right]}{E_t^* \left[\tilde{X}_{i,t+n} \right]}. \qquad (19)
\end{aligned}
$$

In addition to depending on a weighted average of realized strip returns $R_{t+1}^{(n)}$, the realized firm return also depends on the change in the expected future firm-level component. From equations (10), (11), and (13), this change in expectations can all be expressed entirely in terms of the surprise about one-period growth $\Delta \tilde{x}_{i,t+1} - E_t^* \left[\Delta \tilde{x}_{i,t+1} \right]$, as for $n \geq 2$ we have that

$$\frac{E_{t+1}^{*}\left[\tilde{X}_{i,t+n}\right]}{E_{t}^{*}\left[\tilde{X}_{i,t+n}\right]} = \exp\left\{n\beta\left(\Delta\tilde{x}_{i,t+1} - E_{t}^{*}\left[\Delta\tilde{x}_{i,t+1}\right]\right)\right\}. \tag{20}$$

D. Model implications

Below, we discuss several qualitative implications from the model that are relevant to our empirical findings.

First, increases in $E_t^*[g_i]$ raise the firm's price in two ways: increasing the expected future cash flows and decreasing the subjective risk premium. From equation (17), a higher expected g_i naturally increases the value of the firm by increasing the value of future expected cash flows. What is less straightforward is that raising $E_t^*[g_i]$ lowers the subjective risk premium. As shown in equation (16), longer horizon cash flow strips carry a lower risk premium in this model, as their annualized return is less sensitive to the aggregate shock u_{t+1}. A higher value for $E_t^*[g_i]$ means that more of the firm's value comes from its longer horizon cash flows and therefore the weights $w_{i,t,n}$ in equation (18) are more concentrated on the longer horizon $\exp\left\{r^f + \gamma\sigma_u^2\phi^{n-1}\right\}$. In line with the findings in Table I , this means that both higher expected earnings growth and lower expected returns will help to explain high price-earnings ratios.

Second, if the constant-gain parameter β is small, then a positive surprise $\Delta\tilde{x}_{i,t} - E_{t-1}^*[\Delta\tilde{x}_{i,t}]$ will *decrease* expected next period growth $E_t^*[\Delta\tilde{x}_{i,t+1}]$. After observing a positive surprise $\Delta\tilde{x}_{i,t} - E_{t-1}^*[\Delta\tilde{x}_{i,t}]$, the agent slightly raises her beliefs about g_i but largely attributes the surprise to the noisy shock $v_{i,t}$. Following equations (10)-(12), this lowers her expectations about the following year's earnings growth, as she expects the noisy shock will disappear. This result is in line with the findings in Table III Panel B and highlights how the inclusion of temporary shocks substantially changes the model's predictions relative to the models discussed in Section III.B.

Third, if the constant-gain parameter β is small, then the impact of cash flow surprises

$\Delta\tilde{x}_{i,t+1} - E^*_{t-1}[\Delta\tilde{x}_{i,t+1}]$ on one-period returns will be small. As shown in equations (19)-(20), a positive surprise $\Delta\tilde{x}_{i,t+1} - E^*_t[\Delta\tilde{x}_{i,t+1}]$ will cause the realized return to be higher than the agent's expected return ($E^*_t[R_{i,t+1}]$). However, if β is fairly small, then expectations of g_i will only respond slightly in response to surprises, which means we will not observe a large one-period return. Rephrased, the mapping between cash flow surprises $\Delta\tilde{x}_{i,t+1} - E^*_t[\Delta\tilde{x}_{i,t+1}]$ and realized returns $R_{i,t+1}$ depends on how "stubborn" beliefs are (i.e., how much agents attribute surprises to temporary shocks that do not impact future cash flows). This allows the model to match our empirical finding that one-year earnings growth disappointment does not immediately translate 1-1 into one-year return disappointment. Instead, earnings growth disappointment will be gradually reflected in returns over time, as agents slowly update their beliefs about g_i.

It's important to note that these model implications are the result of two mechanisms: slow updating of future cash flow expectations in response to surprises and a preference for the timing of cash flows. While the model represents these two mechanisms by parameter learning and non-permanent aggregate shocks, there are other mechanisms that could potentially deliver similar implications.

In our model, constant-gain learning about g_i delivers slow updating of future cash flow expectations if the parameter β is small. This choice is motivated by our evidence in Table III Panel B and by previous literature estimating small gain parameters of 0.018 to 0.02 from survey expectations and realized macroeconomic variables (Orphanides and Williams, 2005 and Milani, 2007). Instead of constant-gain learning, one could also consider learning from experience, which Malmendier and Nagel (2016) show can be closely approximated as constant-gain learning with a small gain parameter. Further, one could consider learning about a latent time-varying component of earnings growth rather than a fixed parameter g_i. We consider this alternative specification in Appendix H.3 and find similar qualitative and quantitative results.

The preference for the timing of cash flows in the model is generated by having aggregate shocks that are persistent but not permanent. Because of this, short-term cash flows are disproportionately exposed to aggregate risk and are disproportionately discounted compared to long horizon cash flows. Importantly, the qualitative implications of this model do not rely on short horizon cash flows being objectively riskier. As long as agents believe short horizons are disproportionately riskier than long horizons, an increase in the believed $E_t^*[g_i]$ will lower the subjective risk premium. Using the survey measure of subjective risk constructed in Jensen (2023), we confirm in Appendix G that our high price-earnings ratio portfolios are indeed believed to be riskier than our low price-earnings ratio portfolios. Alternatively, one could consider other mechanisms that generate disporportionate discounting of short-term cash flows such as beta-delta present bias.

Ultimately, we choose to focus on constant-gain parameter learning and differences in perceived risk, as this provides a parsimonious description of these mechanisms and allows us to estimate the model parameters without targeting any of our empirical results from Section III. As shown in Section V, we take the gain β from previous work on constant-gain learning and set risk sensitivity γ to match the aggregate equity premium. As a result, we can fully utilize the empirical decomposition results of Table I to evaluate the quantitative realism of our model.

V. Quantitative model and full decomposition

While the implications of the model are qualitatively consistent with the results in Section III, our findings in Table I also provide important quantitative implications. As shown in Table II, many models qualitatively match the decomposition results but struggle to quantitatively match the magnitudes. To make a fair comparison to the models mentioned in Section III, we do not use any information from the decomposition results to estimate the model parameters. Instead, we set all of the parameters based on time-series moments and previous estimates of the learning gain β, then evaluate how well the model matches the cross-sectional decompositions as well as a number of other moments.

The quantified model fulfills three key purposes. First, quoting Brunnermeier et al. (2021), *"Research focus should be on motivating, building, calibrating, and estimating models with non-RE beliefs rather than on merely rejecting RE models. To make further progress, we need structural models of belief dynamics that can compete with RE models in explaining asset prices and empirically observed beliefs."*[28] This model intends to be a step in this direction. It provides a quantitative model that generates realistic asset pricing moments and outperforms the FIRE models of Table II in matching the empirical decomposition results. Notably, despite using only time-series moments, the model matches both the magnitudes and timing for the cross-sectional dynamics of price-earnings ratios, realized and expected earnings growth, and realized and expected returns. Second, the quantified model allows us to extend the decomposition in equation (3) beyond the four-year horizon to estimate the full role of subjective expected earnings growth and subjective discount rates in accounting for the dispersion in price-earnings ratios. Third, we can analyze the importance of learning, risk sensitivity, and the interaction between learning and risk sensitivity using counterfactuals where one or both of these channels are removed (β and/or γ set to 0).

A. Estimation

The model only has six parameters, which are all shown in Table IV. The parameters for cash flows $(\phi, \sigma_u, \sigma_v)$ are all estimated directly from realized earnings growth for our full sample of 1982-2020. For the aggregate process, the standard deviation and autocorrelation of S&P 500 earnings growth imply a persistence $\phi = 0.83$ and a volatility $\sigma_u = 0.34$. The volatility of individual shocks $\sigma_v = 0.10$ is obtained from the volatility over time of the portfolio-level earnings growth. Appendix H.1 shows the exact formulas mapping these empirical moments to the model parameters. The constant-gain parameter is obtained from Malmendier and

[28] Note that this paper uses RE as a shorthand for full information rational expectations and specifically highlights learning about parameters as a promising form on non-RE models to explore: *"For example, models of Bayesian learning relax the RE assumption that agents know the model of the world and its parameter values".*

Table IV

Model estimation

This table shows the value of the six parameters of the model. The parameters for the aggregate cash flow process (ϕ, σ_u) are derived directly from the autocorrelation and standard deviation of the S&P 500 annual earnings growth. The firm-level volatility σ_v is derived directly from the standard deviation over time of the portfolio-level annual earnings growth. The risk-free rate r^f and risk sensitivity γ are set to match the average one-year Treasury yield and average aggregate equity return during the sample period. The constant-gain learning parameter β is taken from Malmendier and Nagel (2016). All moments are estimated over the full sample period of 1982 to 2020.

Parameter	Value	Moments
	Cash flow process	
ϕ	0.83	$AC(\Delta x_{t+1}^{agg})$
σ_u	0.34	$\sigma\left(\Delta x_{t+1}^{agg}\right)$
σ_v	0.10	$\sigma\left(\Delta x_{i,t+1}\right)$
	SDF	
r_f	4.6%	Risk-free rate
γ	1.61	Average aggregate return
	Learning	
β	1.8%	Constant-gain learning (Malmendier and Nagel, 2016)

Nagel, 2016 as $\beta = 0.018$. Note that this is nearly identical to the gain estimated in Milani (2007) of 0.0183.[29] For the agent's stochastic discount factor, the risk-free rate $r^f = 4.6\%$ and the sensitivity to risk $\gamma = 1.61$ are set to match the average one-year Treasury yield and average aggregate stock return of 10.5% for 1982-2020.[30]

B. Model performance

B.1. Dynamics of prices, cash flows, and returns

After quantifying the model, we now evaluate the joint dynamics of price-earnings ratios, earnings growth, and returns. Figure 3 shows the one- and four-year price-earnings ratio decomposition results from Table I, along with their 95% confidence intervals. For comparison, the black dots show the values implied by our model. Overall, the model successfully

[29] These papers estimate the constant-gain parameter on a quarterly frequency. We show in Appendix H.2 that this estimation is quantitatively very similar to an estimation using an annual frequency. Conceptually, we are simply imposing that all updating occurs at the end of the year rather than allowing for small amounts of updating within the year.

[30] The model is simulated yearly over 500 periods for 300 firms. To avoid being impacted by the initial value of the expectations $E_0^*[g_i]$, we calculate all moments after $t = 150$.

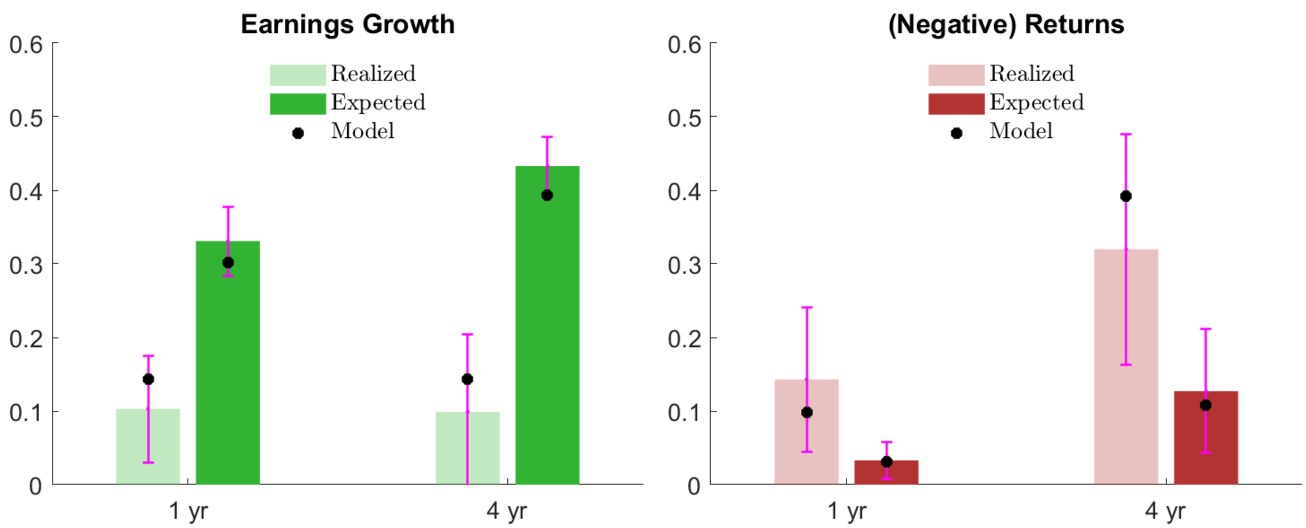

Figure 3. Empirical decomposition and model decomposition. This figure evaluates the one- and four-year decomposition of $\tilde{px}_{i,t}$ dispersion in the model. The light bars show the contribution of realized earnings growth and realized returns to the dispersion of price-earnings ratios obtained in the first and fourth columns of Table I. The dark bars show the contribution of expected earnings growth and expected returns to the dispersion of price-earnings ratios obtained in the second and fifth columns of Table I. Each bar shows Driscoll-Kraay 95% confidence intervals. The black dots show the values of both the realized and expected decomposition implied by the model.

matches both the objective decomposition of price-earnings ratio dispersion (i.e., comovements of price ratios with future earnings growth and future returns) and the subjective decomposition (i.e., comovement of price ratios with expected earnings growth and expected returns).

In the model, high price-earnings ratios are associated with significantly higher expected earnings growth and moderately lower expected returns. Figure 3 shows that a one unit increase in $\tilde{px}_{i,t}$ is associated with a 0.30 (0.39) increase in expected one-year (four-year) earnings growth and a 0.03 (0.11) decrease in expected returns. Because of the temporary shocks to the level of earnings $v_{i,t}$, realized one-year future earnings growth is partly predictable. In line with the data, a one unit increase in the model price-earnings ratio predicts a 0.14 increase in realized one-year earnings growth and this coefficient is unchanged as we increase the horizon to four years. The relationship between price-earnings ratios and expected earnings growth is quantitatively much larger than the relationship between price-earnings ratios and realized future earnings growth, meaning that high price-earnings ratios predict disappointment in future earnings growth. As a result, the relationship between price-earnings ratios and realized negative returns is larger than expected, 0.10 (0.39) at the one-year (four-year) horizon. Overall, this parsimonious model is able to closely match all 8 moments from the decomposition.

The fact that the model matches our decomposition results at multiple horizons highlights its success both in terms of magnitudes and in terms of timing. While the difference in expected and realized one-year earnings growth is large, this does not translate into a large difference between expected and realized one-year negative returns. Instead, agents are slow to adjust their beliefs and the disappointment in earnings growth leads to much lower than expected returns at longer horizons.

Because of this slow adjustment of prices, the model is able to simultaneously match the large one-year earnings growth disappointment shown in Figure 3 and the high empirical persistence of $\tilde{px}_{i,t}$, which is 0.77 in the data and 0.76 in the model. In general, these two facts are difficult to match for models in which growth expectations are sensitive to recent realizations (e.g., overstating the persistence of growth or diagnostic expectations of growth), as disappointing earnings growth for high $\tilde{px}_{i,t}$ firms would cause their price-earnings ratios to quickly fall. Further, our model is still consistent with previous cross-sectional evidence of overreaction. Using the Coibion and Gorodnichenko (2015) regression, we find that revisions in expected long-term growth $E_t^*[g_i] - E_{t-1}^*[g_i]$ negatively predict forecast errors $g_i - E_t^*[g_i]$ with a coefficient of -0.5, which is quantitatively similar to the empirical coefficients estimated in Bordalo et al. (2019) of -0.20 to -0.31. However, the model predicts that these revisions in expectations do not lead to large immediate changes in prices.

Table V

Model evaluation

This table evaluates the model by comparing the untargeted aggregate and cross-sectional moments in the model simulations with those observed in that data. Panel A shows the mean, standard deviation and autocorrelation of the aggregate price-earnings ratio as well as the standard deviation of aggregate stock returns. Panel B shows the cross-sectional standard deviations of price-earnings ratios, future earnings growth and returns, and expected earnings growth and returns. Panel C shows the idiosyncratic volatility across time of price-earnings ratios, future earnings growth and returns, and expected earnings growth and returns. All moments in the table are untargeted, except for idiosyncratic realized earnings growth volatility. Aggregate moments are estimated over the full sample period of 1982 to 2020. The cross-sectional dispersion and idiosyncratic volatility moments are estimated over the main sample of 1999 to 2020 due to data availability.

Panel A: Aggregate value					
	Mean px_t	$\sigma\,(px_t)$	$AC\,(px_t)$	$\sigma\,(r_t)$	
Model	2.31	43.2%	0.81	11.5%	
Data	2.98	42.5%	0.74	15.9%	
Panel B: Cross-sectional standard deviation					
	$\tilde{px}_{i,t}$	$\Delta\tilde{x}_{i,t+1}$	$\tilde{r}_{i,t+1}$	$E_t^*\left[\Delta\tilde{x}_{i,t+1}\right]$	$E_t^*\left[\tilde{r}_{i,t+1}\right]$
Model	20.9%	14.1%	5.6%	11.9%	0.8%
Data	22.6%	12.6%	5.7%	14.0%	2.6%
Panel C: Idiosyncratic volatility					
	$\tilde{px}_{i,t}$	$\Delta\tilde{x}_{i,t+1}$	$\tilde{r}_{i,t+1}$	$E_t^*\left[\Delta\tilde{x}_{i,t+1}\right]$	$E_t^*\left[\tilde{r}_{i,t+1}\right]$
Model	19.7%	14.0%	5.5%	11.7%	0.7%
Data	19.0%	16.6%	6.3%	12.3%	7.4%

B.2. Aggregate and cross-sectional moments

On top of the 8 untargeted moments represented by the decomposition exercises, the model is also able to match several relevant moments from the data. Table V shows a comparison of the untargeted moments in the model and the data. First, despite not using any price information in the estimation other than the average aggregate equity return, the model generates realistic dynamics for the aggregate price-earnings ratio. The unconditional mean, volatility and autocorrelation of the log price-earnings ratio in the model (2.31, 43.2%, and 0.81) are consistent with the observed values (2.98, 42.5%, and 0.74) and the model generates volatile returns.

Second, while no information on cross-sectional dispersion was used in the estimation, the model performs well in matching the empirical dispersion of nearly all of our variables.[31] In

[31] The empirical dispersion is measured as the median cross-sectional standard deviation for each variable.

other words, constant-gain learning with temporary shocks can successfully generate large differences across firms in price-earnings ratios, realized earnings growth, and realized returns, which have model dispersions of 20.9%, 14.1%, and 5.6% respectively. Beyond explaining the realized data, we find that the model also accurately captures the large empirical dispersion in expected earnings growth (11.9%).

We do find that the model understates the cross-sectional dispersion in expected returns. For the sake of parsimony, in the model, subjective discount rates are entirely driven by risk premia related to cash flow timing, see equation (18). Expanding the model to incorporate other risks into discount rates could help to better match this moment. However, as shown in Figure 3, the model still succeeds in matching the covariance of price-earnings ratios with expected returns. In other words, while the model does not capture all cross-sectional differences in expected returns, it does capture the portion that is predictable with price-earnings ratios, i.e., the portion that is useful for generating large differences in price-earnings ratios.

Third, the model replicates the measured portfolio-level volatilities. Note that these volatilities, which reflect variation in a single portfolio across time, are distinct from our estimates of dispersion, which capture cross-sectional variation across portfolios. The only information about portfolio-level volatility utilized in the estimation was the volatility of realized earnings growth, which means that the model provides an accurate mapping of how volatility in earnings growth translates into volatility in price-earnings ratios, returns, and expected earnings growth. Similar to our results for dispersion, we find that the model understates the volatility of expected returns.

In summary, we find that the model not only successfully matches the untargeted decomposition moments, but also generates realistic aggregate stock market moments as well as realistic cross-sectional dispersion and portfolio-level volatility. This demonstrates that a relatively parsimonious structural model of belief formation can feasibly improve upon FIRE models in terms of quantitatively matching the realized data while also matching the dynamics of empirically observed beliefs.

C. Full role of objective cash flows, cash flow mistakes, and discount rates

Using the quantified model, we can measure the full role of cash flow growth expectations and subjective discount rates in accounting for price-earnings ratio dispersion and the persistence of this dispersion. Table VI Panel C shows the decomposition in equation (3) when we extend to the infinite horizon. Specifically, it shows the cross-sectional dispersion $Var\left(\tilde{px}_{i,t}\right)$ and the two components $Cov\left(\sum_{j=1}^{\infty}\rho^{j-1}E_t^*\left[\Delta\tilde{x}_{i,t+j}\right], \tilde{px}_{i,t}\right)$, $Cov\left(-\sum_{j=1}^{\infty}\rho^{j-1}E_t^*\left[\tilde{r}_{i,t+j}\right], \tilde{px}_{i,t}\right)$. Additionally, Panel C shows the persistence of $\tilde{px}_{i,t}$, which measures whether cross-sectional differences in price-earnings ratios are transitory or long-lived.

To start, we focus on the final column, which is our main model parameterization. As shown in the third row of Panel C, the model estimates that differences in expected cash flow growth account for two-thirds (65.7%) of all dispersion in price-earnings ratios.[32] Combined with the aggregate time series findings of De la O and Myers (2021), this means that both time series variation in aggregate price ratios and cross-sectional dispersion in price ratios are both primarily explained by expected cash flow growth. However, unlike the aggregate time series findings, we also estimate a non-trivial role for subjective discount rates in accounting for price-earnings ratio dispersion. The sixth row of Panel C shows that low subjective discount rates for high price-earnings ratio firms accounts for roughly one-third (34.3%) of all dispersion in price-earnings ratios.

Looking at the breakdown of the 65.7% contribution from expected earnings growth, we see that this largely comes from forecast errors. The comovement of price-earnings ratios with realized future earnings growth only accounts for 14.9% of the dispersion, meaning that the remaining 50.8% comes from price-earnings ratios predicting forecast errors for earnings growth. As a result, high price-earnings ratios are largely associated with low future returns,

[32] This is consistent with the empirical results of Table I, where we find that expected earnings growth over just the first four years already accounts for 43.3% of all price-earnings ratio dispersion.

Table VI

Infinite-horizon decomposition and counterfactual analysis

Each column shows the decomposition implied by the constant-gain learning model using different key parameter choices. Panel A shows the parameters which change for each specification. All other parameters are set to the values in the main specification in Table IV. The first column runs a model with no learning or risk sensitivity, $\beta = 0$ and $\gamma = 0$. The second column runs a model with no learning, $\beta = 0$. The third column runs a model with no risk sensitivity, $\gamma = 0$. The main specification is shown in the last column. Models with $\gamma = 0$ are also run with a different risk-free rate $r_f = 10.5\%$ to ensure the average level of equity returns is consistent across all specifications. Panel B shows the magnitudes of the mean aggregate price-earnings ratio and aggregate returns implied by each of the specifications. Panel C shows the decomposition results. The first two rows show the implied persistence and cross-sectional variance of $\tilde{px}_{i,t}$ for each specification. The third and sixth rows of Panel C show the amount of cross-sectional variance in $\tilde{px}_{i,t}$ explained by expected earnings growth $\sum_{j=1}^{\infty} E_t^* [\Delta\tilde{x}_{i,t+j}]$ and subjective discount rates $-\sum_{j=1}^{\infty} \rho^{j-1} E_t^* [\tilde{r}_{i,t+j}]$, estimated through the infinite-horizon version of equation (3):

$$Var\left(\tilde{px}_{i,t}\right) = Cov\left(\sum_{j=1}^{\infty} \rho^{j-1} E_t^* \left[\Delta\tilde{x}_{i,t+j}\right], \tilde{px}_{i,t}\right) + Cov\left(-\sum_{j=1}^{\infty} \rho^{j-1} E_t^* \left[\tilde{r}_{i,t+j}\right], \tilde{px}_{i,t}\right).$$

The fourth and seventh rows of Panel C show the amount of cross-sectional variance in $\tilde{px}_{i,t}$ explained by realized earnings growth $\sum_{j=1}^{\infty} \rho^{j-1} \Delta\tilde{x}_{i,t+j}$ and negative realized returns $-\sum_{j=1}^{\infty} \rho^{j-1} \tilde{r}_{i,t+j}$. Finally, the fifth and eighth rows show the amount of cross-sectional variance in $\tilde{px}_{i,t}$ explained by earnings growth forecast errors $\sum_{j=1}^{\infty} \rho^{j-1}(\Delta\tilde{x}_{i,t+j} - E_t^* [\Delta\tilde{x}_{i,t+j}])$ and return forecast errors $-\sum_{j=1}^{\infty} \rho^{j-1}(\tilde{r}_{i,t+j} - E_t^* [\tilde{r}_{i,t+j}])$. The share of the cross-sectional variance of $\tilde{px}_{i,t}$ is shown in parentheses.

Panel A: Parameter values				
β	0	0	.018	.018
γ	0	1.61	0	1.61
r^f	10.5%	4.6%	10.5%	4.6%
Panel B: Levels				
Mean px_t	2.31	2.31	2.31	2.31
Mean r_{t+1}	10.5%	10.5%	10.5%	10.5%
Panel C: Cross section				
Persistence $\tilde{px}_{i,t}$	0.00	0.00	0.54	0.76
Variance $\tilde{px}_{i,t}$	0.98	0.98	2.07	4.37
Expected earnings growth	0.98	0.98	2.07	2.87
	(100%)	(100%)	(100%)	(65.7%)
Realized earnings growth	0.98	0.98	0.78	0.65
	(100%)	(100%)	(37.6%)	(14.9%)
Forecast errors	0	0	1.29	2.22
	(0%)	(0%)	(62.4%)	(50.8%)
Subjective discount rates	0	0	0	1.50
	(0%)	(0%)	(0%)	(34.3%)
Negative realized returns	0	0	1.29	3.72
	(0%)	(0%)	(62.4%)	(85.1%)
Negative forecast errors	0	0	-1.29	-2.22
	(0%)	(0%)	(-62.4%)	(-50.8%)

with negative realized returns accounting for 85.1% of all price-earnings ratio dispersion. Note that at the infinite horizon, forecast errors for earnings growth and forecast errors for returns are equal (i.e., the forecast error row for earnings growth and negative returns are exactly opposite). While gradual learning affects how quickly earnings growth surprises are reflected in unexpected returns, eventually all unexpected earnings growth will appear as unexpected returns.

Conveniently, we can summarize the relative importance of realized future earnings growth, errors in earnings growth expectations, and subjective discount rates. The model estimates that realized earnings growth accounts for roughly 1/6 (14.9%) of price-earnings ratio dispersion, errors in earnings growth expectations account for 1/2 (50.8%), and subjective discount rates account for 1/3 (34.3%). Additionally, besides decomposing differences in price-earnings ratios, the model also decomposes the low realized returns earned by expensive stocks. The estimation implies that 40.3% (34.3/85.1) of the difference in returns between high and low price-earnings ratio stocks reflects subjective discount rates while 59.7% (50.8/85.1) reflects disappointment in earnings growth.

More broadly, by having a structural model, we can investigate the economic role of learning and risk sensitivity in driving the cross-sectional dispersion in price-earnings ratios. The different columns in Table VI Panel C show the persistence in price-earnings ratios, the dispersion in price-earnings ratios, and the decomposition results when β and/or γ are set to 0, i.e., learning and/or risk sensitivity are turned off. In all cases, the initial expected g_i is set to 0 for all firms. Thus, the two cases where $\beta = 0$ are equivalent to saying that agents know the objective data-generating process and no longer need to learn the parameters. Given that we are interested in cross-sectional dispersion rather than levels, in the two cases where $\gamma = 0$ we also raise the risk-free rate from 4.6% to 10.5%. As shown in Panel B, this ensures that the aggregate level for price-earnings ratios and equity returns are identical across all four cases and it is only the dispersion that changes. Thus, the two cases where $\gamma = 0$ are equivalent to saying that all firms have the same subjective discount rate of 10.5%.

In the first column, both β and γ are set to 0. In this case, the dispersion in price-earnings ratios is less than 1/4 the value in our main specification (0.98 compared to 4.37). The dispersion in price-earnings ratios comes entirely from differences in expected earnings growth, as there are no differences in subjective discount rates. Price-earnings ratios do not predict earnings growth forecast errors. Instead, all differences in expected earnings growth are simply due to the noise shocks $v_{i,t}$. Since these shocks are i.i.d., the autocorrelation in expected earnings growth is zero, which explains why the persistence in $\tilde{px}_{i,t}$ is also zero.

In the second column, the model includes risk sensitivity ($\gamma > 0$) but keeps $\beta = 0$. As shown in Panel C, only including risk sensitivity has no effect on the results relative to the first column. This highlights that, in our model, variation across firms in expected cash flow growth and subjective discount rates are both ultimately related to variation across firms in expected g_i. While agents may be sensitive to risk related to cash flow timing, this only matters if firms are expected to differ in the timing of their cash flows.

In comparison, the third column shows that including learning but keeping $\gamma = 0$ does substantially change the results. The dispersion in price-earnings ratios doubles from 0.98 to 2.07. This largely comes from price-earnings ratios now comoving with future earnings growth forecast errors. However, there is also the interesting result that the comovement of price-earnings ratios with realized earnings growth decreases (0.98 to 0.78). The FIRE expectation for future earnings growth is simply $-v_{i,t}$. With learning, expected earnings growth depends on $E_t^*[g_i]$, which comoves positively with $v_{i,t}$, as a positive shock will tend to increase the guess for g_i. Thus, introducing learning means that the price-earnings ratio, which depends on expected earnings growth, will now be less related to future realized earnings growth due to the muted response to shocks $v_{i,t}$. Further, while objective expected earnings growth has 0 persistence over time, subjective expected earnings growth is persistent when agents are learning about g_i. Because of this, cross-sectional differences in price-earnings ratios are moderately long-lived, with a persistence of 0.54.

Finally, the last column shows the interaction from including both risk sensitivity and learning. While risk sensitivity by itself has no effect, once we incorporate learning, increasing γ from 0 to 1.63 more than doubles the dispersion in price-earnings ratios (2.07 to 4.37) and makes cross-sectional differences in price-earnings ratios more persistent. Specifically, including risk sensitive along with learning increases the persistence of $\tilde{px}_{i,t}$ from 0.54 to 0.76, helping the model match the empirical persistence of 0.77. Looking at the contribution of subjective discount rates (the sixth row of Panel C), we clearly see the interaction between risk sensitivity and learning, as dispersion in subjective discount rates now contributes 1.50 (34.3%) to the total dispersion in price-earnings ratios.

More surprisingly, we also find an important interaction between risk sensitivity and learning for the contribution of earnings growth expectations. Given that γ has no impact on equations (10)-(13), changing γ has no effect on expected earnings growth. Thus, the increase in comovement between price-earnings ratios and expected earnings growth (2.08 to 2.87) is entirely due to changes in the price-earnings ratios. Intuitively, incorporating discount rates that depend on expected cash flow timing increases the sensitivity of price-earnings ratios to $E_t^*[g_i]$ and decreases their sensitivity to transitory shocks $v_{i,t}$. The increased sensitivity to $E_t^*[g_i]$ is reflected in the larger comovement of price-earnings ratios with expected earnings growth, and the reduced sensitivity to $v_{i,t}$ is reflected in an even higher persistence of $\tilde{px}_{i,t}$. This logic extends to any model with preferences for the timing of cash flows and shows that while discount rates may not affect expected earnings growth, they can be quantitatively important for driving the comovement of price ratios with expected earnings growth and earnings growth forecast errors.

Overall, the fact that dispersion in price-earnings ratios for $\beta > 0, \gamma > 0$ is more than twice as large as any of the other counterfactuals highlights the natural interaction between preferences for the timing of cash flows and learning about cash flow growth. We find that this interaction is quantitatively important for matching the large empirical dispersion in price-earnings ratios and the persistence of cross-sectional differences in price-earnings ratios.

VI. Conclusion

We find that subjective expectations have substantial potential to explain the cross-section of stock price ratios and shed light on the relative importance of expected future cash flows and discount rates. Using a variance decomposition, we show that cross-sectional dispersion in price-earnings ratios is primarily explained by predictable errors in subjective expectations of earnings growth. Subjective discount rates play a secondary, but non-trivial role. Disappointment in one-year earnings growth does not immediately lead to an equivalent disappointment in one-year returns. Instead, earnings growth surprises are reflected gradually in future returns over time. To understand these findings, we provide a quantitative model which not only outperforms standard FIRE models in matching the dynamics of prices and realized earnings growth and returns, but also outperforms common behavioral models in matching the dynamics of prices and expectations. The model features constant-gain learning about earnings growth and risk premia related to cash flow timing and emphasizes the importance of slow-moving beliefs in order to match the empirical timing of earnings growth expectations and realized returns.

These findings for the cross-section of stock prices are consistent with the aggregate time-series findings of De la O and Myers (2021, 2023), who emphasize that aggregate stock prices are largely driven by subjective earnings growth expectations and that errors in these expectations play a particularly large role in explaining long-term returns. This harmony between the aggregate time-series and the cross-section indicates that a single mechanism could potentially explain both dimensions of the data and provides a strong motivation for further research understanding how investors form cash flow expectations and discount rates.

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Appendix

A. Model prices and returns

To derive equation (14), we guess and verify a log-affine form for the strip price, $P_t^{(n)} = \exp\left\{A(n) + \phi^n x_t^{agg}\right\}$. The strip price is then pinned down by $P_t^{(0)} = \exp\left\{x_t^{agg}\right\}$ (i.e., $A(0) = 0$) and

$$
\begin{aligned}
P_t^{(n)} &= E_t^*\left[M_{t+1}P_{t+1}^{(n-1)}\right] \\
&= E_t^*\left[\exp\left\{-r^f - \frac{1}{2}\gamma^2\sigma_u^2 - \gamma u_{t+1} + A(n-1) + \phi^n x_t^{agg} + \phi^{n-1}u_{t+1}\right\}\right] \\
&= \exp\left\{-r^f - \frac{1}{2}\gamma^2\sigma_u^2 + A(n-1) + \phi^n x_t^{agg} + \frac{1}{2}\left(\phi^{n-1} - \gamma\right)^2\sigma_u^2\right\}. \qquad \text{(A1)}
\end{aligned}
$$

This gives that

$$
\begin{aligned}
A(n) &= A(n-1) - r^f - \gamma\phi^{n-1}\sigma_u^2 + \frac{1}{2}\phi^{2(n-1)}\sigma_u^2 \\
&= -nr^f - \gamma\sigma_u^2\frac{1-\phi^n}{1-\phi} + \frac{1}{2}\sigma_u^2\frac{1-\phi^{2n}}{1-\phi^2}. \qquad \text{(A2)}
\end{aligned}
$$

The expected and realized strip returns in equations (15)-(16) then simply utilize the formula for $P_t^{(n)}$. The firm price in equation (17) uses the independence of aggregate and idiosyncratic shocks to simplify $E_t^*\left[\left(\prod_{j=1}^n M_{t+j}\right)X_{t+n}^{agg}\tilde{X}_{i,t+n}\right] = E_t^*\left[\left(\prod_{j=1}^n M_{t+j}\right)X_{t+n}^{agg}\right]E_t^*\left[\tilde{X}_{i,t+n}\right] = P_t^{(n)}E_t^*\left[\tilde{X}_{i,t+n}\right]$.

Given that the firm price is simply a collection of strip prices, the return for a firm is

$$
\begin{aligned}
R_{i,t+1} &= \frac{\tilde{X}_{i,t+1}X_{t+1}^{agg} + P_{i,t+1}}{P_{i,t}} = \frac{\sum_{n=1}^{\infty} P_{t+1}^{(n-1)}E_{t+1}^*\left[\tilde{X}_{i,t+n}\right]}{\sum_{n=1}^{\infty} P_t^{(n)}E_t^*\left[\tilde{X}_{i,t+n}\right]} \\
&= \sum_{n=1}^{\infty}\frac{P_t^{(n)}E_t^*\left[\tilde{X}_{i,t+n}\right]}{\sum_{n=1}^{\infty}P_t^{(n)}E_t^*\left[\tilde{X}_{i,t+n}\right]}\frac{P_{t+1}^{(n-1)}}{P_t^{(n)}}\frac{E_{t+1}^*\left[\tilde{X}_{i,t+n}\right]}{E_t^*\left[\tilde{X}_{i,t+n}\right]} \\
&= \sum_{n=1}^{\infty} w_{i,t,n}R_{t+1}^{(n)}\frac{E_{t+1}^*\left[\tilde{X}_{i,t+n}\right]}{E_t^*\left[\tilde{X}_{i,t+n}\right]} \qquad \text{(A3)}
\end{aligned}
$$

where the weight is $w_{i,t,n} = \frac{P_t^{(n)}E_t^*\left[\tilde{X}_{i,t+n}\right]}{\sum_{n=1}^{\infty}P_t^{(n)}E_t^*\left[\tilde{X}_{i,t+n}\right]} = \frac{\exp\{nE_t^*[g_i]\}P_t^{(n)}}{\sum_{n=1}^{\infty}\exp\{nE_t^*[g_i]\}P_t^{(n)}}$ from equation (13).

Applying expectations, we then get equation (18).

B. Connecting returns, earnings growth, and price-earnings ratios

First, we derive the equation for a firm which has zero dividends. For simplicity, we eliminate the index i in this derivation. In this case, the return is equal to the price growth which after log-linearization becomes an exact relationship

$$r_{t+1} \quad = \quad \Delta x_{t+1} - px_t + px_{t+1}. \tag{A4}$$

A high price-earnings ratio px_t must be followed by low future price growth Δp_{t+1} (returns r_{t+1}), high future earnings growth Δx_{t+1}, or a high future price-earnings ratio px_{t+1}.

Now, we consider the case where dividends are non-zero. We start with the one-year return identity of a portfolio

$$R_{t+1} = \frac{P_{t+1} + D_{t+1}}{P_t} = \frac{\left(\frac{P_{t+1}}{D_{t+1}} + 1\right)\frac{D_{t+1}}{D_t}}{\frac{P_t}{D_t}},$$

where P_t and D_t are the current price and dividends. Log-linearizing around $\bar{pd}$, we can represent the price-dividend ratio pd_t in terms of future dividend growth, Δd_{t+1}, future returns, r_{t+1}, and the future price-dividend ratio, pd_{t+1}, all in logs:

$$r_{t+1} \quad \approx \quad \kappa^d + \Delta d_{t+1} - pd_t + \rho pd_{t+1}, \tag{A5}$$

where κ^d is a constant, $\rho = e^{\bar{pd}} / \left(1 + e^{\bar{pd}}\right) < 1$. We can then insert the identity $px_t = pd_t + dx_t$, where dx_t is the log payout ratio, into (A5) to obtain

$$r_{t+1} \quad \approx \quad \kappa + \Delta x_{t+1} - px_t + \rho px_{t+1} \tag{A6}$$

where we approximate $(1 - \rho)\, dx_{t+1}$ as 0 given that $(1 - \rho)$ is very close to 0.[33] Here, $\bar{pd}$ does not need to be the mean price-dividend ratio of this specific stock or portfolio. In order to study cross-sectional variation without resorting to portfolio-specific approximation parameters, we use the average price-dividend ratio of the market for $\bar{pd}$ following Cochrane

[33] The zero dividend relationship in equation (A4) is a special case of equation (A6) as $\bar{pd}$ goes to infinity.

(2011).

While the identity relies on the approximation that $(1-\rho)\,dx_{t+1} \to 0$, empirically equation (A6) holds tightly. For horizons of 1 to 4 years, Table I shows that a one unit increase in px_t is associated with almost exactly a one unit increase in $\sum_{j=1}^{h} \rho^{j-1}\Delta x_{t+j} - \sum_{j=1}^{h} \rho^{j-1} r_{t+j} + \rho^h px_{t+h}$.[34] In other words, the approximation error from ignoring the payout ratio and using a single value for ρ accounts for at most 3.1% of variation in price-earnings ratios in the decomposition of equation (3). For robustness, the next section uses an exact relationship instead of equation (3) to ensure the approximation is not driving our results.

C. Robustness Exercises

C.1. Exact decomposition results

In this section, we derive all the main results using an exact decomposition of price-earnings ratios based on price growth, rather than the approximate decomposition based on returns. For any stock or portfolio of stocks i, the price-earnings ratio $px_{i,t}$ can be expressed in terms of the one-year ahead log price growth $\Delta p_{i,t+1}$, the future earnings growth $\Delta x_{i,t+1}$, and the future price-earnings ratio:

$$px_{i,t} = \Delta x_{i,t+1} + \Delta p_{i,t+1} + px_{i,t+1}. \tag{A7}$$

This equation is exact and does not contain a log-linearization constant ρ. Applying subjective expectations $E_t^*[\cdot]$, we see that a higher than average price-earnings ratio must be explained by higher than average expected earnings growth, lower than average expected price growth, or a higher than average expected future price-earnings ratio,

$$\tilde{px}_{i,t} = \sum_{j=1}^{h} E_t^*\left[\Delta\tilde{x}_{i,t+j}\right] - \sum_{j=1}^{h} E_t^*\left[\Delta\tilde{p}_{i,t+j}\right] + E_t^*\left[\tilde{px}_{i,t+h}\right]. \tag{A8}$$

Just like the main decomposition, this equation holds under any subjective probability

[34]For example, at the one-year horizon, a one unit increase in px_t is associated with a 0.103 increase in Δx_{t+1}, a 0.143 increase in $-r_{t+1}$, and a 0.746 increase in ρpx_{t+1}. At the four-year horizon, a one unit increase in px_t is associated with a with a 0.099 increase in $\sum_{j=1}^{4} \rho^{j-1}\Delta x_{t+j}$, a 0.320 increase in $-\sum_{j=1}^{4} \rho^{j-1} r_{t+j}$, and a 0.550 increase in $\rho^4 px_{t+4}$.

distribution and we can decompose the variance of $\tilde{px}_{i,t}$ into three components:

$$1=\underbrace{\frac{Cov\left(\sum_{j=1}^{h}E_t^*\left[\Delta\tilde{x}_{i,t+j}\right],\tilde{px}_{i,t}\right)}{Var\left(\tilde{px}_{i,t}\right)}}_{CF_h}+\underbrace{\frac{Cov\left(-\sum_{j=1}^{h}E_t^*\left[\Delta\tilde{p}_{i,t+j}\right],\tilde{px}_{i,t}\right)}{Var\left(\tilde{px}_{i,t}\right)}}_{PG_h}+\underbrace{\frac{Cov\left(E_t^*\left[\tilde{px}_{i,t+h}\right],\tilde{px}_{i,t}\right)}{Var\left(\tilde{px}_{i,t}\right)}}_{FPX_h}. \tag{A9}$$

The coefficients CF_h and PG_h give a quantitative measure of how much dispersion in price-earnings ratios is accounted for by dispersion in expected earnings growth and how much is accounted for by dispersion in expected price growth. We can now estimate this equation using the exact expectations of price growth without an approximation.

Table AI shows that the results of this exact decomposition are very similar to the main decomposition results in Table I. We find that 10.3% of dispersion in price-earnings ratios is accounted for by differences in one-year future earnings growth and 13.2% is accounted for by differences in one-year price growth. Just as in the main decomposition, differences in earnings growth are overestimated, with expected earnings growth accounting for nearly a third (33.1%) of all dispersion in price-earnings ratios. Differences in price growth are underestimated, with expected price growth accounting for only 3.3% of all dispersion in price-earnings ratios. A similar pattern can be observed at the four-year horizon. Overall, all the coefficients closely align with those reported in Table I.

We can also estimate an exact version of the unexpected anomaly return decomposition (5). Just as in the main identity, we normalize all anomalies $\tilde{a}_{i,t}$ so that they have variance 1 and positively comove with future price growth. From equation (A7), we have the identity

$$\begin{aligned}\underbrace{Cov\left(\Delta\tilde{p}_{i,t+1}-E_t^*\left[\Delta\tilde{p}_{i,t+1}\right],\tilde{a}_{i,t}\right)}_{\sigma_{a,p}} &= \underbrace{Cov\left(\Delta\tilde{x}_{i,t+1}-E_t^*\left[\Delta\tilde{x}_{i,t+1}\right],\tilde{a}_{i,t}\right)}_{\sigma_{a,x}} \\ &+ \underbrace{Cov\left(\tilde{px}_{i,t+1}-E_t^*\left[\tilde{px}_{i,t+1}\right],\tilde{a}_{i,t}\right)}_{\sigma_{a,px}}.\end{aligned} \tag{A10}$$

Here, the values for $\sigma_{a,x}$ and $\sigma_{a,px}$ indicate how much the predictable price growth forecast

Table AI

Decomposition of dispersion in price-earnings ratios (exact decomposition)

This table decomposes the variance of price-earnings ratios using the exact decomposition (A9). The FIRE column report the elements CF_h, PG_h and FPX_h of the decomposition using future earnings growth, future price growth and future price-earning ratios. The Expected column report the elements of the decomposition using expected earnings growth, expected price growth and expected price-earning ratios. The Forecast Errors column reports the contribution of the forecast errors of each element. For instance, $CF_1 = Cov\left(\Delta\tilde{x}_{i,t+1}, \tilde{px}_{i,t}\right)/Var\left(\tilde{px}_{i,t}\right)$ is shown in the FIRE column. This component can be split into its expected component $Cov\left(E_t^*\left[\Delta\tilde{x}_{i,t+1}\right], \tilde{px}_{i,t}\right)/Var\left(\tilde{px}_{i,t}\right)$ and its error component $Cov\left(\Delta\tilde{x}_{i,t+1} - E_t^*\left[\Delta\tilde{x}_{i,t+1}\right], \tilde{px}_{i,t}\right)/Var\left(\tilde{px}_{i,t}\right)$. The sample period is 1999 to 2020. Driscoll-Kraay standard errors and block-bootstrap standard errors are calculated for each coefficient. In both cases, we use a lag equal to the maximum lag with any significant residual autocorrelation. Superscripts indicate Driscoll-Kraay significance at the 1% (***), 5% (**), and 10% (*) level.

		One-year horizon ($h = 1$)			One-to-four year horizon ($h = 4$)		
		FIRE	Expected	Forecast errors	FIRE	Expected	Forecast errors
1999-2020	CF_h	0.103***	0.331***	-0.228***	0.100*	0.439***	-0.340***
		[0.037]	[0.024]	[0.032]	[0.057]	[0.020]	[0.055]
		[0.051]	[0.026]	[0.050]	[0.073]	[0.023]	[0.075]
1999-2020	PG_h	0.132**	0.033**	0.100*	0.292***	0.135***	0.157*
		[0.052]	[0.013]	[0.054]	[0.088]	[0.046]	[0.089]
		[0.051]	[0.013]	[0.055]	[0.112]	[0.050]	[0.111]
1999-2020	FPX_h	0.765***	0.636***	0.129**	0.608***	0.426***	0.182***
		[0.051]	[0.020]	[0.058]	[0.063]	[0.029]	[0.069]
		[0.045]	[0.024]	[0.055]	[0.074]	[0.032]	[0.083]

Table AII

Unexpected anomaly price growth (exact decomposition)

This table measures and decomposes unexpected anomaly price growth. The Representative Anomaly is the average ranking of each stock across 20 different anomalies, and $\tilde{px}_{i,t}$ is the demeaned price-earnings ratio. For each anomaly variable, we sort stocks into five equal-value portfolios based on the anomaly variable. The table shows the coefficients of regressing each of the dependent variables on a specific anomaly variable. Both anomaly variables are scaled to have unit variance and to positively comove with future price growth. The three dependent variables are the unexpected price growth $\Delta\tilde{p}_{i,t+1} - E_t^*\left[\Delta\tilde{p}_{i,t+1}\right]$, the earnings growth forecast errors $\Delta\tilde{x}_{i,t+1} - E_t^*\left[\Delta\tilde{x}_{i,t+1}\right]$, and the price-earnings ratio forecast errors $\left(\tilde{px}_{i,t+1} - E_t^*\left[\tilde{px}_{i,t+1}\right]\right)$. The sample period is 1999 to 2020. Driscoll-Kraay standard errors and block-bootstrap standard errors are calculated for each coefficient. In both cases, we use a lag equal to the maximum lag with any significant residual autocorrelation. Superscripts indicate Driscoll-Kraay significance at the 1% (***), 5% (**), and 10% (*) level.

Panel A: Anomaly Price Growth Decomposition		
	Representative Anomaly	$\tilde{px}_{i,t}$
$\Delta\tilde{p}_{i,t+1} - E_t^*\left[\Delta\tilde{p}_{i,t+1}\right]$	0.031^{**}	0.030^{*}
	[0.013]	[0.016]
	[0.013]	[0.015]
$\Delta\tilde{x}_{i,t+1} - E_t^*\left[\Delta\tilde{x}_{i,t+1}\right]$	0.064^{***}	0.069^{***}
	[0.020]	[0.010]
	[0.020]	[0.014]
$\left(\tilde{px}_{i,t+1} - E_t^*\left[\tilde{px}_{i,t+1}\right]\right)$	-0.033^{***}	-0.039^{**}
	[0.009]	[0.018]
	[0.009]	[0.015]

errors are explained by predictable errors in next-year earnings growth expectations and expectations of the future price-earnings ratio. Table AII shows the results for the representative anomaly studied in Section III.D. For each anomaly, we estimate a positive value of $\sigma_{a,p}$, meaning that investors do not fully anticipate the high growth on high $\tilde{a}_{i,t}$ stocks. The predictable errors in one-year earnings growth expectations are more than large enough to account for the unexpected one-year price growth (i.e., $\sigma_{a,x}$ is greater than $\sigma_{a,p}$). In Appendix D Table AVIII we show the exact decomposition results for each of the individual anomalies.

C.2. Overlapping observations and Bauer and Hamilton (2018)

Overlapping forecast horizons can increase the persistence of residuals in Table I. Because of this, we use Driscoll-Kraay and block-bootstrap standard errors to account for any autocorrelation. For additional robustness, in this section we also directly calculate the significance of each coefficient under the worst-case scenario for overlapping observations.

We do this following the methodology proposed by Bauer and Hamilton (2018). Specifically, for expected and realized earnings growth and returns, we run simulations to measure how often we spuriously find a coefficient as large as what we observe in the data. For clarity, we discuss the simulation for the regression of earnings forecast errors $\Delta\tilde{x}_{i,t+1} - E_t^*\left[\Delta\tilde{x}_{i,t+1}\right]$ on price-earnings ratios $\tilde{px}_{i,t}$, however, the methodology is identical for the other left hand side variables.

We specify the price-earnings ratio of each portfolio i as an AR(1) process,

$$\tilde{px}_{i,t} \quad = \quad \mu_i + \left(\tilde{px}_{i,t-1} - \mu_i\right) + \sigma_i \varepsilon_{i,t}.$$

The mean, persistence, and variance is set equal to the observed values over our sample. Additionally, the initial value of the simulated price-earnings ratio for portfolio i is set equal to the initial value observed in our data to account for any drift back to the mean which may generate trends in price-earnings ratios over the sample. For example, if the price-earnings ratio for the Growth portfolio is substantially above its mean at the beginning of the sample, then reversion to the mean will create an downward trend in the price-earnings ratio for this portfolio over time. We then simulate one-period forecast errors under the null hypothesis that forecast errors are unpredictable.

If subjective expectations change over time, then there will be little overlap in longer horizon forecast errors. For example, if $E_t^*\left[\Delta\tilde{x}_{i,t+2}\right]$ is very different from $E_{t+1}^*\left[\Delta\tilde{x}_{i,t+2}\right]$, then there is little similarity between the second term of $\sum_{j=1}^{h} \rho^{j-1}\left(\Delta\tilde{x}_{i,t+j} - E_t^*\left[\Delta\tilde{x}_{i,t+j}\right]\right)$ and the first term of $\sum_{j=1}^{h} \rho^{j-1}\left(\Delta\tilde{x}_{i,t+1+j} - E_{t+1}^*\left[\Delta\tilde{x}_{i,t+1+j}\right]\right)$. However, in the worst-case scenario in which expectations do not change at all over time, then $\sum_{j=1}^{h} \rho^{j-1}\left(\Delta\tilde{x}_{i,t+j} - E_t^*\left[\Delta\tilde{x}_{i,t+j}\right]\right)$

will be an MA($h-1$) process. This will cause the four-year forecast errors to be persistent which increases the probability of spuriously finding a large coefficient between price-earnings ratios and four-year forecast errors.

For our simulations, we push this worst-case scenario even further by making each period one month instead of one year. This means that $\sum_{j=1}^{h} \rho^{j-1} \left(\Delta \tilde{x}_{i,t+j} - E_t^* \left[\Delta \tilde{x}_{i,t+j}\right]\right)$ will be MA($12h-1$) which dramatically increases the persistence. For all of the left-hand side variables in Table I, we find that this worst-case scenario substantially overstates the observed variable persistence. We then set the variance of the monthly forecast errors to match the observed variance of $\sum_{j=1}^{h} \rho^{j-1} \left(\Delta \tilde{x}_{i,t+j} - E_t^* \left[\Delta \tilde{x}_{i,t+j}\right]\right)$. We then run 10,000 simulations and report the probability of spuriously finding a coefficient as large as what we observe in the data.

Table AIII shows the coefficients for realized and expected earnings growth and returns, along with their associated p-values from the simulations. With the exception of four-year realized earnings growth, we find that all of the coefficients are statistically significant at the 5% level. Even after accounting for persistence in price-earnings ratios, trends, and a worst-case assumption for overlapping observations, the probability of spuriously generating coefficients as large as what we find in the data is quite low.

C.3. Smoothed earnings

To show that our decomposition results are not influenced by fluctuations in earnings in the denominator of price-earnings ratio, we repeat our analysis normalizing prices with a three-year-smoothed measure of earnings. AIV shows that the result are very similar to the main decomposition results in Table I. We find that 37.6% of dispersion in price–to-smoothed-earnings ratios is accounted for by differences in expected four-year earnings growth and 12.6% is accounted for by differences in four-year returns. Just as our main results, differences in cash flow growth are overestimated, with errors in expected cash flows accounting for nearly a third (31.0%) of all dispersion in price–to-smoothed-earnings ratios,

Table AIII

Worst-case decomposition of dispersion in price-earnings ratios

This table decomposes the variance of price-earnings ratios using equation (3). The FIRE column report the elements CF_h, DR_h and FPX_h of the decomposition using future earnings growth, future negative returns and future price-earning ratios. The Expected column report the elements of the decomposition using expected earnings growth, expected returns and expected price-earning ratios. Worst-case p-values using the Bauer and Hamilton (2018) procedure are reported in parentheses. *p<.1;**p<.05; ***p<.01.

		One-year horizon ($h = 1$)			One-to-four year horizon ($h = 4$)		
		FIRE	Expected	Forecast errors	FIRE	Expected	Forecast errors
1999-2020	CF_h	0.103***	0.331***	−0.228***	0.099	0.433***	−0.335***
		(0.006)	(0.000)	(0.000)	(0.118)	(0.000)	(0.001)
1999-2020	DR_h	0.143***	0.033***	0.110***	0.320***	0.127***	0.192**
		(0.000)	(0.008)	(0.001)	(0.004)	(0.008)	(0.047)

and differences in return are underestimated.

C.4. Delisting firms

As explained in Section III.C, we require in the main analysis that firms have an observed future price and future earnings. This allows us to calculate direct forecast errors for the subjective expectations. To test whether survivorship bias is impacting the results, we repeat our analysis without the requirement that firms must have a future observed price and observed earnings. Instead, when firms exit the sample, we measure the delisting return and reinvest those funds into the remaining firms in the portfolio. We then calculate earnings growth and returns under this reinvestment strategy.

As shown in Table AV, the results are quite close to our main estimation in Table I. We find that 36.3% (7.3%) of dispersion in price-earnings ratios is explained expected (realized) four-year earnings growth and 15.5% (31.1%) is explained by expected (realized) four-year returns.

Table AIV

Decomposition of dispersion in price-earnings ratios using smoothed earnings

This table shows the three components of the left hand side of equation (3) using three-year smoothed earnings instead of annual earnings to form the valuation ratios. Let s_t be the three-year smoothed average of earnings. For each period, we form the price-to-smoothed-earnings ratio $\tilde{ps}_{i,t}$. The FIRE column report the components CF_h, DR_h and FPE_h of equation (3) using future earnings growth, future returns and future price-earning ratios. The Expected column report the elements of the equation (3) using expected earnings growth, expected price growth and expected price-earning ratios. The Forecast Errors column reports the contribution of the forecast errors of each element. For instance, $CF_1 = Cov\left(\Delta\tilde{x}_{i,t+1}, \tilde{ps}_{i,t}\right)/Var\left(\tilde{ps}_{i,t}\right)$ is shown in the FIRE column. This component can be split into its expected component $Cov\left(E_t^*\left[\Delta\tilde{x}_{i,t+1}\right], \tilde{ps}_{i,t}\right)/Var\left(\tilde{ps}_{i,t}\right)$ and its error component $Cov\left(\Delta\tilde{x}_{i,t+1} - E_t^*\left[\Delta\tilde{x}_{i,t+1}\right], \tilde{ps}_{i,t}\right)/Var\left(\tilde{ps}_{i,t}\right)$. The main sample period is 1999 to 2020. The fourth row shows the element CF_h of the decompositions estimated over the longer sample period of 1982-2020. Driscoll-Kraay standard errors and block-bootstrap standard errors are calculated for each coefficient. In both cases, we use a lag equal to the maximum lag with any significant residual autocorrelation. Superscripts indicate Driscoll-Kraay significance at the 1% (***), 5% (**), and 10% (*) level.

		One-year horizon ($h=1$)			One-to-four year horizon ($h=4$)		
		FIRE	Expected	Forecast errors	FIRE	Expected	Forecast errors
1999-2020	CF_h	0.090**	0.290***	-0.200***	0.066	0.376***	-0.310***
		[0.036]	[0.021]	[0.037]	[0.053]	[0.023]	[0.050]
		[0.048]	[0.024]	[0.051]	[0.073]	[0.029]	[0.067]
1999-2020	DR_h	0.131***	0.032***	0.099**	0.291***	0.126***	0.165**
		[0.046]	[0.011]	[0.048]	[0.079]	[0.038]	[0.082]
		[0.043]	[0.012]	[0.048]	[0.097]	[0.038]	[0.097]
1999-2020	FPX_h	0.681***	0.573***	0.108**	0.523***	0.357***	-0.166***
		[0.048]	[0.025]	[0.051]	[0.051]	[0.024]	[0.057]
		[0.048]	[0.032]	[0.048]	[0.059]	[0.026]	[0.066]
1982-2020	CF_h	0.120***	0.264***	-0.143***	0.109***	0.392***	-0.283***
		[0.025]	[0.018]	[0.029]	[0.038]	[0.024]	[0.032]
		[0.026]	[0.018]	[0.028]	[0.038]	[0.025]	[0.030]

Table AV

Decomposition of dispersion in price-earnings ratios including exiting firms

This table decomposes the variance of price-earnings ratios including firms that may exit after portfolio formation. To account for these firms, we reinvest the delisting returns of exiting firms in the corresponding portfolio. The FIRE column report the elements CF_h, DR_h and FPX_h of the decomposition using future earnings growth, future negative returns and future price-earning ratios. The Expected column report the elements of the decomposition using expected earnings growth, expected returns and expected price-earning ratios. The Forecast Errors column reports the contribution of the forecast errors of each element. For instance, $CF_1 = Cov\left(\Delta\tilde{x}_{i,t+1}, \tilde{px}_{i,t}\right)/Var\left(\tilde{px}_{i,t}\right)$ is shown in the FIRE column. This component can be split into its expected component $Cov\left(E_t^*\left[\Delta\tilde{x}_{i,t+1}\right], \tilde{px}_{i,t}\right)/Var\left(\tilde{px}_{i,t}\right)$ and its error component $Cov\left(\Delta\tilde{x}_{i,t+1} - E_t^*\left[\Delta\tilde{x}_{i,t+1}\right], \tilde{px}_{i,t}\right)/Var\left(\tilde{px}_{i,t}\right)$. The sample period is 1999 to 2020. Driscoll-Kraay standard errors and block-bootstrap standard errors are calculated for each coefficient. In both cases, we use a lag equal to the maximum lag with any significant residual autocorrelation. Superscripts indicate Driscoll-Kraay significance at the 1% (***), 5% (**), and 10% (*) level.

		One-year horizon ($h = 1$)			One-to-four year horizon ($h = 4$)		
		FIRE	Expected	Forecast errors	FIRE	Expected	Forecast errors
1999-2020	CF_h	0.126***	0.302***	-0.176***	0.073	0.363***	-0.290***
		[0.040]	[0.028]	[0.042]	[0.068]	[0.025]	[0.065]
		[0.049]	[0.032]	[0.052]	[0.096]	[0.027]	[0.092]
1999-2020	DR_h	0.076	0.046***	0.031	0.311***	0.155***	0.165**
		[0.069]	[0.011]	[0.068]	[0.081]	[0.035]	[0.075]
		[0.068]	[0.012]	[0.066]	[0.110]	[0.038]	[0.097]
1999-2020	FPX_h	0.796***	0.636***	0.160***	0.569***	0.383***	0.194***
		[0.063]	[0.025]	[0.059]	[0.060]	[0.023]	[0.064]
		[0.061]	[0.031]	[0.059]	[0.076]	[0.027]	[0.081]

D. *Extended anomaly results*

In Tables AVI, AVII, and AVIII, we show the detailed results for each of the individual anomalies. Table AVI shows the decomposition of unexpected returns into earnings growth surprises and unexpected future price-earnings ratios. Table AVII shows that positive earnings growth surprises are consistently associated with a decrease in expected next period earnings growth. Table AVIII shows an exact decomposition of anomaly price growth rather than anomaly returns.

E. *FIRE model simulations*

For each model, we simulate the cross-section of firms. We set the number of firms based on the original calculations in each paper. Specifically, we use 50, 5,000, and 200 firms for Berk et al. (1999), Zhang (2005), and Lettau and Wachter (2007), respectively. We set every sample to a length of 20 years and we run 1,000 simulations for each model. All parameter values are taken from the original papers.

For Berk et al. (1999) and Zhang (2005), we sort firms into five portfolios based on their price-book ratios. For Berk et al. (1999), we treat profits as our measure of earnings and for Zhang (2005), we treat profits after the cost of new capital and adjustment costs as our measure of earnings.[35] For Lettau and Wachter (2007), the only firm variables are price and dividends, so we treat dividends as our measure of earnings and sort firms into five portfolios based on their price-dividend ratios. We then estimate the finite-horizon and infinite horizon decomposition in equation (3) for each model.

E.1. Berk, Green, and Naik 1999

Each firm has some existing projects which generate cash flows. Each period, the firm draws a new potential project, which it can pay a fixed cost to undertake. The value of the firm comes from its existing projects as well as the option to undertake future projects ("growth

[35] We find nearly identical results if we use profits as our measure of earnings for Zhang (2005).

Table AVI

Unexpected anomaly returns

This table measures and decomposes unexpected anomaly returns using equation (5). For each anomaly $\tilde{a}_{i,t}$, we sort stocks into five equal-value portfolios based on the anomaly variable. The table shows the coefficients of regressing each of the dependent variables on a specific anomaly variable. The three dependent variables are the unexpected return $\tilde{r}_{i,t+1} - E^*_t[\tilde{r}_{i,t+1}]$, the earnings growth forecast errors $\Delta\tilde{x}_{i,t+1} - E^*_t[\Delta\tilde{x}_{i,t+1}]$, and the price-earnings ratio forecast errors $\rho\left(\tilde{px}_{i,t+1} - E^*_t\left[\tilde{px}_{i,t+1}\right]\right)$. The Representative Anomaly (RA) is the average ranking of each stock across 20 different anomalies. P/X show sthe results for the main portfolios used to study cross-sectional variation in price-earnings ratios. Each anomaly variable $\tilde{a}_{i,t}$ is scaled to have unit variance and to posititvely comove with future returns. The sample period is 1999 to 2020. Driscoll-Kraay standard errors are calculated for each coefficient using a lag equal to the equal to the maximum lag with any significant residual autocorrelation. *p<.1;**p<.05; ***p<.01.

	Decomposition				Decomposition		
$\tilde{a}_{i,t}$	$\sigma_{a,r}$	$\sigma_{a,x}$	$\sigma_{a,px}$	$\tilde{a}_{i,t}$	$\sigma_{a,r}$	$\sigma_{a,x}$	$\sigma_{a,px}$
rdm	0.0087	0.0551***	-0.0437***	noa	0.0189***	-0.0392***	0.0567***
	[0.0058]	[0.0132]	[0.0134]		[0.0069]	[0.0111]	[0.0084]
bm	0.0133	0.0524**	-0.0405***	oaa	0.0215	0.0789***	-0.0585**
	[0.0138]	[0.0234]	[0.0157]		[0.0178]	[0.0292]	[0.0236]
cfp	0.0142	0.0111	-0.0017	ol	0.0150***	0.0698***	-0.0512***
	[0.0144]	[0.0190]	[0.0119]		[0.0053]	[0.0140]	[0.0136]
adm	0.0179**	0.1233***	-0.1038***	pia	-0.0064	-0.0110	0.0068
	[0.0072]	[0.0101]	[0.0104]		[0.0107]	[0.0133]	[0.0139]
nop	0.0184**	-0.0448***	0.0572***	poa	0.0019	0.0382***	-0.0347***
	[0.0082]	[0.0148]	[0.0127]		[0.0067]	[0.0115]	[0.0102]
ia	0.0458***	0.1404***	-0.0955***	pta	0.0070	-0.0034	0.0084
	[0.0157]	[0.0297]	[0.0194]		[0.0072]	[0.0084]	[0.0084]
gp	0.0081	0.0091	0.0016	occ	0.0189**	0.0544***	-0.0337***
	[0.0101]	[0.0156]	[0.0147]		[0.0088]	[0.0162]	[0.0124]
ivc	0.0106	-0.0326***	0.0398***	dur	0.0620***	0.0736**	-0.0126
	[0.0072]	[0.0124]	[0.0094]		[0.0161]	[0.0333]	[0.0202]
ivg	0.0130	0.0230	-0.0115	cei	0.0479**	0.0283	0.0160
	[0.0104]	[0.0145]	[0.0117]		[0.0209]	[0.0311]	[0.0153]
ig	0.0191	0.0351**	-0.0181**	P/E	0.0331**	0.0687***	-0.0380**
	[0.0117]	[0.0141]	[0.0082]		[0.0160]	[0.0096]	[0.0169]
nsi	0.0260**	0.0287	-0.0043	RA	0.0340**	0.0635***	-0.0317***
	[0.0123]	[0.0184]	[0.0090]		[0.0132]	[0.0201]	[0.0088]

Table AVII

Revisions in expectations

This table shows the effect of earnings growth surprises on revisions. For each anomaly $\tilde{a}_{i,t}$, we sort stocks into five equal-value portfolios based on the anomaly variable. Each row shows the coefficient from regressing the revision in earnings growth $E^*_{t+1}[\Delta\tilde{x}_{i,t+2}] - E^*_t[\Delta\tilde{x}_{i,t+2}]$ on the earnings growth surprise $\Delta x_{i,t+1} - E^*_t[\Delta x_{i,t+1}]$. The Representative Anomaly (RA) is the average ranking of each stock across 20 different anomalies. P/X show sthe results for the main portfolios used to study cross-sectional variation in price-earnings ratios. Each anomaly variable $\tilde{a}_{i,t}$ is scaled to have unit variance and to posititvely comove with future returns. The first and third columns show the result of the regressions using the main sample period of 1999 to 2020. The second and fourth columns show the result of the regressions using the long sample period of 1982 to 2020. Driscoll-Kraay standard errors are calculated for each coefficient using a lag equal to the equal to the maximum lag with any significant residual autocorrelation. *p<.1;**p<.05; ***p<.01.

$\tilde{a}_{i,t}$	Main Sample 1999-2020	Long Sample 1982-2020	$\tilde{a}_{i,t}$	Main Sample 1999-2020	Long Sample 1982-2020
rdm	-1.059***	-1.010***	noa	-0.873***	-0.878***
	[0.046]	[0.041]		[0.056]	[0.039]
bm	-0.806***	-0.821***	oaa	-0.812***	-0.833***
	[0.060]	[0.046]		[0.053]	[0.035]
cfp	-0.782***	-0.804***	ol	-0.879***	-0.860***
	[0.046]	[0.039]		[0.058]	[0.037]
adm	-0.957***	-0.917***	pia	-0.857***	-0.887***
	[0.029]	[0.025]		[0.053]	[0.036]
nop	-0.927***	-0.930***	poa	-0.873***	-0.859***
	[0.036]	[0.030]		[0.067]	[0.045]
ia	-0.896***	-0.863***	pta	-1.002***	-0.984***
	[0.027]	[0.024]		[0.028]	[0.024]
gp	-0.743***	-0.777***	occ	-0.726***	-0.749***
	[0.111]	[0.074]		[0.081]	[0.057]
ivc	-1.019***	-0.947***	dur	-0.521***	-0.557***
	[0.032]	[0.031]		[0.071]	[0.063]
ivg	-0.870***	-0.866***	cei	-0.863***	-0.886***
	[0.044]	[0.033]		[0.075]	[0.047]
ig	-0.938***	-0.932***	P/E	-0.918***	-0.786***
	[0.040]	[0.025]		[0.067]	[0.091]
nsi	-0.832***	-0.857***	RA	-0.811***	-0.818***
	[0.030]	[0.031]		[0.049]	[0.060]

Table AVIII

Unexpected anomaly price growth (exact decomposition)

This table measures and decomposes unThis table shows the effect of earnings growth surprises on revisions. For each anomaly, we sort stocks into five equal-value portfolios based on the anomaly variable. Each column shows the coefficient from regressing the revision in earnings growth $E^*_{t+1}[\Delta\tilde{x}_{i,t+2}] - E^*_t[\Delta\tilde{x}_{i,t+2}]$ on the earnings growth surprise $\Delta x_{i,t+1} - E^*_t[\Delta x_{i,t+1}]$. Value is measured using the price-earnings ratio. Profitability is measured using gross profitability. Investment is measured using net stock issuance. The Representative Anomaly is the average ranking of each stock across 22 different anomalies. The first row shows the result of the regressions using the main sample period of 1999 to 2020. The second row shows the result of the regressions using the long sample period of 1982 to 2020. Driscoll-Kraay standard errors are clustered at the portfolio and year level. Superscripts indicate significance at the 1% (***), 5% (**), and 10% (*) level.expected anomaly price growth using equation (A10). For each anomaly $\tilde{a}_{i,t}$, we sort stocks into five equal-value portfolios based on the anomaly variable. The table shows the coefficients of regressing each of the dependent variables on a specific anomaly variable. The three dependent variables are the unexpected price growth $\Delta\tilde{p}_{i,t+1} - E^*_t[\Delta\tilde{p}_{i,t+1}]$, the earnings growth forecast errors $\Delta\tilde{x}_{i,t+1} - E^*_t[\Delta\tilde{x}_{i,t+1}]$, and the price-earnings ratio forecast errors $\left(\tilde{px}_{i,t+1} - E^*_t[\tilde{px}_{i,t+1}]\right)$. The Representative Anomaly (RA) is the average ranking of each stock across 20 different anomalies. P/X show sthe results for the main portfolios used to study cross-sectional variation in price-earnings ratios. Each anomaly variable $\tilde{a}_{i,t}$ is scaled to have unit variance and to posititvely comove with future price growth. The sample period is 1999 to 2020. Driscoll-Kraay standard errors are calculated for each coefficient using a lag equal to the equal to the maximum lag with any significant residual autocorrelation. *p<.1;**p<.05; ***p<.01.

	Decomposition				Decomposition		
$\tilde{a}_{i,t}$	$\sigma_{a,r}$	$\sigma_{a,x}$	$\sigma_{a,px}$	$\tilde{a}_{i,t}$	$\sigma_{a,r}$	$\sigma_{a,x}$	$\sigma_{a,px}$
rdm	0.0102*	0.0551***	-0.0449***	noa	0.0189***	-0.0392***	0.0581***
	[0.0058]	[0.0132]	[0.0138]		[0.0069]	[0.0111]	[0.0086]
bm	0.0108	0.0524**	-0.0416***	oaa	-0.0189	-0.0789***	0.0600**
	[0.0139]	[0.0234]	[0.0161]		[0.0181]	[0.0292]	[0.0242]
cfp	0.0093	0.0111	-0.0018	ol	0.0172***	0.0698***	-0.0525***
	[0.0145]	[0.0190]	[0.0122]		[0.0054]	[0.0140]	[0.0139]
adm	0.0169**	0.1233***	-0.1064***	pia	-0.0040	-0.0110	0.0070
	[0.0072]	[0.0101]	[0.0107]		[0.0109]	[0.0133]	[0.0143]
nop	-0.0138*	0.0448***	-0.0587***	poa	0.0024	0.0382***	-0.0355***
	[0.0080]	[0.0148]	[0.0130]		[0.0067]	[0.0115]	[0.0104]
ia	0.0429***	0.1404***	-0.0980***	pta	0.0054	-0.0034	0.0086
	[0.0165]	[0.0297]	[0.0199]		[0.0070]	[0.0084]	[0.0086]
gp	0.0107	0.0091	0.0016	occ	0.0198**	0.0544***	-0.0346***
	[0.0102]	[0.0156]	[0.0150]		[0.0089]	[0.0162]	[0.0127]
ivc	-0.0081	0.0326***	-0.0408***	dur	0.0615***	0.0736**	-0.0126
	[0.0074]	[0.0124]	[0.0096]		[0.0161]	[0.0333]	[0.0207]
ivg	0.0112	0.0230	-0.0118	cei	0.0447**	0.0283	0.0164
	[0.0105]	[0.0145]	[0.0120]		[0.0212]	[0.0311]	[0.0157]
ig	-0.0165	-0.0351**	0.0186**	P/E	0.0301*	0.0687***	-0.0389**
	[0.0119]	[0.0141]	[0.0084]		[0.0163]	[0.0096]	[0.0175]
nsi	0.0242*	0.0287	-0.0044	RA	0.0309**	0.0635***	-0.0326***
	[0.0124]	[0.0184]	[0.0092]		[0.0132]	[0.0201]	[0.0090]

options"). As the term "growth options" implies, future earnings growth plays a key role in this model. The ratio of the firm's price to its current earnings reflects how much of the firm's value comes from existing projects versus growth options. Firms with high price-earnings ratios derive most of their value from their expected future projects rather than existing projects, and future earnings growth accounts for most dispersion in price-earnings ratios ($CF_4 = 0.84$).

The model features a time-varying risk-free rate which also generates differences in risk premia.[36] Compared to existing projects, the value of growth options is less sensitive to changes in the risk-free rate, as the firm can endogenously change its decision to exercise the option (i.e., it only undertakes the potential project if the risk-free rate is low). Because of this, the agent requires a higher risk premium for firms with low price-earnings ratios. Quantitatively, the difference in risk premia is only a small part of the dispersion in price-earnings ratios ($DR_4 = 0.03$, $DR_\infty = 0.04$).

E.2. Zhang 2005

In this model, firm earnings are

$$X_{i,t} \quad = \quad e^{x_t + z_{i,t} + p_t} k_{i,t}^{\alpha} - f - i_{i,t} - h\left(i_{i,t}, k_{i,t}\right)$$

where x_t is aggregate productivity, $z_{i,t}$ is idiosyncratic productivity, p_t is the aggregate price level, $k_{i,t}$ is firm-level capital, f is a fixed cost, $i_{i,t}$ is investment in capital, and $h\left(i_{i,t}, k_{i,t}\right)$ is an adjustment cost. Differences across firms are due to differences in their sequence of idiosyncratic productivity $\{z_{i,\tau}\}_{\tau=0}^{t}$. Because idiosyncratic productivity is AR(1), future earnings growth is partly predictable and dispersion in price-earnings ratios largely predicts differences in future earnings growth ($CF_4 = 0.69$).

The model also features differences in discount rates. Because of adjustment costs, it is costly for firms to lower their capital to the new optimal level after a negative shock to aggregate productivity x_t. Therefore, the agent requires a higher risk premium for firms

[36] The risk-free rate is closely tied to the agent's stochastic discount factor.

with high capital relative to total firm value, as they are more sensitive to negative aggregate shocks. Quantitatively, these differences in risk premia are small relative to the dispersion in price-earnings ratios ($DR_4 = -0.03$).[37]

In order to calculate CF_h and DR_h, we have to address the issue that model earnings are often negative, even at the portfolio level, which is not compatible with the Campbell-Shiller decomposition.[38] To use the decomposition, we want to think about an investor that makes a one-time payment to buy a claim to the company, never pays anything more in the future, and receives some cash flows in the future. Thus, we will think of an investor that holds some share $\theta_{i,t}$ of the company. When the company has positive cash flows, the investor does not change her share in the company and receives these cash flows. When the company has negative cash flows, we assume the investor sells a part of her stake in the company to cover this. Specifically, this investor receives cash flows $\hat{X}_{i,t} \equiv \theta_{i,t} \max\{X_{i,t}, 0\}$, where $\theta_{i,t} = \theta_{i,t-1}(1 + \min\{X_{i,t}, 0\}/P_{i,t})$. Intuitively, rather than receiving a negative cash flow, this investor dilutes her claim to the future (on average positive) cash flows. This investor receives the same return as someone who owned the entire firm and received the negative cash flows, $\frac{\theta_{i,t}P_{i,t}+\hat{X}_{i,t}}{\theta_{i,t-1}P_{i,t-1}} \equiv \frac{P_{i,t}+X_{i,t}}{P_{i,t-1}}$.

E.3. Lettau and Wachter 2007

In this model, each firm receives some share $s_{i,t}$ of the aggregate earnings. The value of $s_{i,t}$ goes through a fixed cycle, increasing from $\underline{s}$ to a peak value of $\bar{s}$ and then decreasing back to $\underline{s}$. The cross-section of firms is populated with firms at different points in this share cycle. Because all firms receive a share of the same aggregate earnings, the cross-sectionally demeaned log earnings growth $\Delta\tilde{x}_{i,t}$ is simply the log share growth $\log(s_{i,t}) - \log(s_{i,t-1})$.

[37]In the model, high price-earnings ratio firms have *low* price-capital ratios. A 1% increase in $e^{z_{i,t}}$ does not change the current capital ($k_{i,t}$), increases the current earnings by 1%, and increases the current price by less than 1% since the increase in productivity is persistent but not permanent. Thus, an increase in $z_{i,t}$ raises the price-capital ratio and lowers the price-earnings ratio. This is why discount rate news is slightly negative, as the model predicts that high price-capital ratio firms will have lower future returns, which means that high price-earnings ratio firms will have *higher* future returns.

[38]After a large aggregate shock, nearly all firms will substantially change their capital which requires paying large adjustment costs.

In the model, the stochastic discount factor is exposed to shocks that are partly reversed over time, which means that the agent requires a lower risk premium for longer horizon cash flows. Because of this, firms with high price-earnings ratios (i.e., firms with a low current share $s_{i,t}$) earn slightly lower returns for the first few years ($DR_4 = 0.06$). However, the quantitatively larger component is that high price-earnings ratio firms experience higher earnings growth as their share increases ($CF_4 = 0.24$). Over time, the firms with low current $s_{i,t}$ eventually become the firms with high $s_{i,t+h}$ and require a higher risk premium, as their cash flows are now front-loaded. Thus, discount rate news is small and ambiguous in terms of sign at long horizons, $DR_\infty = -0.04$ (0.08).

F. *Behavioral and learning model predictions*

In this section, we discuss how our findings relate to several behavioral and learning models in which subjective discount rates are constant and the comovement of current price ratios with future returns is due to non-FIRE beliefs about cash flow growth. When subjective expected returns are constant, $E_t^*[\tilde{r}_{i,t+j}] = \bar{r}$, equations (2) and (4) imply that realized returns are

$$\tilde{r}_{i,t+1} - \bar{r} \approx (\Delta\tilde{x}_{i,t+1} - E_t^*[\Delta\tilde{x}_{i,t+1}]) + \sum_{j=2}^{\infty} \rho^{j-1} \left(E_{t+1}^*[\Delta\tilde{x}_{i,t+j}] - E_t^*[\Delta\tilde{x}_{i,t+j}]\right).$$

If one-period cash flow growth surprises $(\Delta\tilde{x}_{i,t+1} - E_t^*[\Delta\tilde{x}_{i,t+1}])$ are positively related to revisions in expected future growth, then one-period cash flow growth surprises will impact unexpected returns more than 1-1. Empirically, we find that one-period cash flow growth surprises are negatively related to revisions in expected future growth and thus translate less than 1-1 into unexpected returns.

Consider the case where agents overstate the persistence of growth. Specifically, the true persistence of growth is ϕ but agents believe the persistence is $\phi^* > \phi$. So long as $\phi^* > 0$, then a positive surprise $\Delta\tilde{x}_{i,t+1} - E_t^*[\Delta\tilde{x}_{i,t+1}]$ will raise future expected growth and translate more than 1-1 into realized returns.

Further, we can consider learning about the mean of an i.i.d. growth process, such as Nagel and Xu (2022). In this setting, agents form expectations of growth based on a weighted average of past realized growth. A higher than expected realization for cash flow growth causes the agent to positively revise her beliefs about mean growth. Because there are no temporary shocks to the level of cash flows, this increase in expected mean growth causes the agent to raise her expectations of all future growth. Specifically, Nagel and Xu (2022) show that the realized unexpected return is

$$r_{t+1} - E_t^*\left[r_{t+1}\right] = \left(1 + \frac{\rho v}{1-\rho}\right)\left(\Delta x_{t+1} - \tilde{\mu}_t\right)$$

where $\tilde{\mu}_t$ is the agent's current expectation of growth and v is the learning gain parameter. Given that $v > 0$, it is immediate that cash flow growth surprises translate more than 1-1 into realized returns.

Finally, we discuss the case of diagnostic growth expectations, as in Bordalo et al. (2022). This model proposes that earnings growth is impacted by tangible news τ_{t+1} and intangible news η_t. Specifically, the process for earnings growth is

$$\Delta x_{t+1} = \mu \Delta x_t + \eta_t + \tau_{t+1}.$$

Subjective expectations of growth are

$$\begin{aligned} E_t^*\left[\Delta x_{t+j}\right] &= \mu^{j-1}\left(\mu \Delta x_t + \eta_t\right) + \mu^{j-1}\epsilon_t \\ \epsilon_t &= \phi \epsilon_{t-1} + \theta\left(\mu \tau_t + \eta_t\right) \end{aligned}$$

where ϵ_t captures biases in expectations and ϕ is assumed to be less than μ. As stated in the paper, realized unexpected returns are

$$r_{t+1} - \bar{r} = \Delta x_{t+1} - E_t^*\left[\Delta x_{t+1}\right] + \sum_{j=2}^{\infty} \rho^{j-1}\left(E_{t+1}^*\left[\Delta x_{t+j}\right] - E_t^*\left[\Delta x_{t+j}\right]\right).$$

The covariance of the price-earnings ratio with future earnings growth surprises and future unexpected returns is then

$$\begin{aligned}
Cov\left(px_t, \Delta x_{t+1} - E_t^*\left[\Delta x_{t+1}\right]\right) &= Cov\left(px_t, -\epsilon_t\right) \\
Cov\left(px_t, r_{t+1} - \bar{r}\right) &= Cov\left(px_t, -\left[1 + \frac{\rho}{1-\rho\mu}\left(\mu - \phi\right)\right]\epsilon_t\right) \\
&= \left[1 + \frac{\rho}{1-\rho\mu}\left(\mu - \phi\right)\right] Cov\left(px_t, \Delta x_{t+1} - E_t^*\left[\Delta x_{t+1}\right]\right).
\end{aligned}$$

Given the paper's assumption that $\mu > \phi$, this means that the covariance of price-earnings ratios with unexpected returns must be a magnified version of the covariance of price-earnings ratios with earnings growth surprises. In fact, the model implies that for any time t variable, the comovement of that variable with unexpected returns will be $1 + \frac{\rho}{1-\rho\mu}\left(\mu - \phi\right)$ times the comovement of that variable with earnings growth surprises. Thus, the model cannot match our finding that the comovement of price-earnings ratios with one-year unexpected returns is *smaller* than the comovement of price-earnings ratios with earnings growth surprises.

As an extension, we also consider the model of diagnostic expectations of earnings *levels* in Bordalo et al. (2019). In this model, the difference between subjective and objective expectations of the level of log earnings is

$$E_t^*\left[x_{t+j}\right] - E_t\left[x_{t+j}\right] = a^j \frac{1-(b/a)^j}{1-b/a}\left(\hat{f}_t^\theta - \hat{f}_t\right)$$

where $a, b > 0$, $\hat{f}_t$ is an objective inference of an underlying component of earnings, and $\hat{f}_t^\theta$ is the biased inference of this component. Simulating the model using the paper's parameter values, we find that high price-earnings ratio stocks have *lower* subjective expected one-year earnings growth. Further, we find that price-earnings ratios are negatively related to current $\hat{f}_t^\theta - \hat{f}_t$, meaning that high price-earnings ratio stocks have pessimistic expectations of earnings at all horizons. These predictions do not align with our empirical findings that high price-earnings ratio stocks have high subjective expected one-year earnings growth and that high price-earnings ratios are associated with overoptimism in subjective expected earnings growth.

G. Subjective risk for our portfolios

One of the main components of our model is that investors perceive lower risk for the high $\tilde{px}_{i,t}$ firms. While the relation is supported by our main evidence in Table I that high price-earnings ratios are associated with lower subjective expected returns, we can also look at more direct measures of subjective risk. In this section we explore the relation between our portfolios and two subjective risk measures: the absolute risk index assigned to firms by Value Line, and a cross-sectionally standardized risk index created by Jensen (2023).

The first measure of risk is the "Safety Rank," directly taken from Value Line. This measure ranges from 1 to 5, where 5 denotes a high perceived risk, and it equals the average of the analyst score for price stability and financial strength, two perceived characteristics for each firm. We take a value–weighted average this measure across all firms in each portfolio to obtain our first subjective risk measure. To account for time effects, we also construct a second standardized measure of risk following Jensen (2023). For each firm, we define the subjective risk as the average cross-sectional rank of price stability and financial strength, and then we standardize this measure every period (i.e., we rescale the measure so that the cross-sectional mean and cross-sectional standard deviation are 0 and 1 in every period).

Figure A1 shows that our high $\tilde{px}_{i,t}$ portfolios indeed have lower subjective expected risk using both the direct and the standardized measures of expected risk. The direct measure of risk is on average 2.90 for the lowest $\tilde{px}_{i,t}$ portfolio and 2.69 for the highest $\tilde{px}_{i,t}$ portfolio, while the standardized measure is 0.11 for the lowest $\tilde{px}_{i,t}$ portfolio and -0.12 for the highest $\tilde{px}_{i,t}$ portfolio. As shown by the 95% confidence interval bars, these differences are highly significant for both measures.

H. Model estimation

This section derives the cash flow parameters ϕ, σ_u, and σ_v from the standard deviation and autocorrelation of aggregate earnings growth $\sigma\left(\Delta x_t^{agg}\right)$ and $AC\left(\Delta x_t^{agg}\right)$ and the average across portfolios of the standard deviation over time of earnings growth $\sigma\left(\Delta \tilde{x}_{i,t+1}\right)$. We

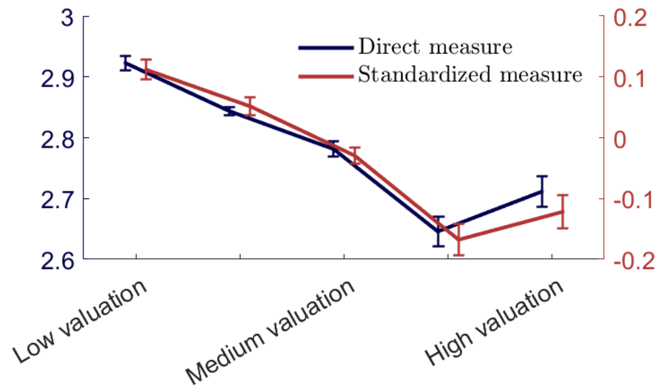

Figure A1. Subjective risk across portfolios. This figure plots the average subjective risk for each of the five main portolios. The direct measure in blue is the "Safety Rank" measure from Value Line. This measure ranges from 1 to 5, where 5 is the highest perceived risk. The standardized measure in red takes the average of the 'Price Stability' and 'Financial Strength' measures and it is cross-sectionally rescaled to have mean zero and unit standard deviation. This measure increases with perceived risk. Each portfolio shows the 95% confidence intervals. The 5 portfolios are shown in ascending order of price-earnings ratios.

also relate the constant-gain learning from annual observations used in our model to the evidence on belief updating from quarterly observations. Finally, we discuss the results from an alternative model in which there is a time-varying latent component to firm earnings growth.

H.1. Parameter values

According to equation (7), we can express aggregate earnings growth as:

$$\Delta x_t^{agg} = \phi \Delta x_{t-1}^{agg} - u_{t-1} + u_t. \tag{A11}$$

Taking covariance of equation (A11) with current earnings growth on both sides results in:

$$\begin{aligned} Cov(\Delta x_t^{agg}, \Delta x_{t-1}^{agg}) &= \phi Var(\Delta x_{t-1}^{agg}) - \sigma_u^2 \\ AC(\Delta x_t^{agg}) &= \phi - \frac{\sigma_u^2}{Var(\Delta x_t^{agg})}. \end{aligned} \tag{A12}$$

Taking the variance of equation (A11) on both sides gives:

$$\begin{aligned} Var(\Delta x_t^{agg}) &= \phi^2 Var(\Delta x_t^{agg}) + 2\sigma_u^2 - 2\phi\sigma_u^2 \\ Var(\Delta x_t^{agg}) &= \frac{2\sigma_u^2}{1+\phi}. \end{aligned} \tag{A13}$$

From equations (A12) and (A13), we have:

$$\begin{aligned}\phi &= 1 + 2AC\left(\Delta x_t^{agg}\right) \\ \sigma_u &= \left(\frac{1+\phi}{2}\right)^{1/2} \sigma\left(\Delta x_t^{agg}\right).\end{aligned}$$

Finally, to estimate the individual variance, we use equation (8) to obtain the value for σ_v in terms of idiosyncratic earnings growth:

$$\sigma_v = \frac{\sigma\left(\Delta \tilde{x}_{i,t}\right)}{\sqrt{2}}.$$

From the empirical values over the 1982-2020 sample of $\sigma\left(\Delta x_t^{agg}\right) = 0.353$, $AC\left(\Delta x_t^{agg}\right) = -0.086$ and a median portfolio volatility of $\sigma\left(\Delta \tilde{x}_{i,t}\right) = 0.140$ we infer $\phi = 0.828$, $\sigma_u = 0.337$ and $\sigma_v = 0.099$

H.2. Constant-gain parameter

We set our constant-gain parameter based on previous estimates of belief updating from Malmendier and Nagel (2016). In this section, we show that for small gains β, annual updating of beliefs based on annual surprises is quite close to quarterly updating of beliefs based on quarterly surprises.

For intuition, first consider the case of semi-annual updating. In this scenario, the agent is attempting to learn the parameter μ from a semi-annual variable x_t using the updating rule

$$E_t^*[\mu] = E_{t-1/2}^*[\mu] + \beta\left(x_t - E_{t-1/2}^*[\mu]\right). \tag{A14}$$

Iterating this equation, we have

$$\begin{aligned}E_t^*[\mu] &= E_{t-1}^*[\mu] + \beta\left(x_t - E_{t-1/2}^*[\mu]\right) + \beta\left(x_{t-1/2} - E_{t-1}^*[\mu]\right) && \text{(A15)} \\ &= E_{t-1}^*[\mu] + \beta\left(x_{t-1/2} + x_t - 2E_{t-1/2}^*[\mu]\right) - \beta^2\left(x_{t-1/2} - E_{t-1}^*[\mu]\right) && \text{(A16)} \\ &\approx E_{t-1}^*[\mu] + \beta\left(x_{t-1/2} + x_t - 2E_{t-1/2}^*[\mu]\right). && \text{(A17)}\end{aligned}$$

Equation (A17) shows that for small values of β, this semi-annual updating rule is closely approximated by an annual updating rule using the same gain β and the annual surprise $x_{t-1/2} + x_t - 2E^*_{t-1/2}[\mu]$. This is because the effect of within-year updating depends on β^2, which in our case would be quite small at 0.0003. In other words, the adjustment for within-year updating is second order compared to the first order change in beliefs $\beta\left(x_{t-1/2} + x_t - 2E^*_{t-1/2}[\mu]\right)$.

Now, we consider the scenario of quarterly updating. The agent is attempting to learn the parameter μ from a quarterly variable x_t. The agent's updating rule is

$$
\begin{aligned}
E^*_t[\mu] &= E^*_{t-1/4}[\mu] + \beta\left(x_t - E^*_{t-1/4}[\mu]\right) && \text{(A18)}\\
&= E^*_{t-1}[\mu] + \beta\left(x_{t-3/4} + x_{t-1/2} + x_{t-1/4} + x_t - 4E^*_{t-1}[\mu]\right)\\
&\quad -\beta^2\left[3\left(x_{t-3/4} - E^*_{t-1}[\mu]\right) + 2\left(x_{t-1/2} - E^*_{t-3/4}[\mu]\right) + x_{t-1/4} - E^*_{t-1/2}[\mu]\right] && \text{(A19)}
\end{aligned}
$$

Once again, this is approximately equal to an annual updating rule based on the annual surprise $x_{t-3/4} + x_{t-1/2} + x_{t-1/4} + x_t - 4E^*_{t-1}[\mu]$. The adjustments for within-year updating are all scaled by β^2.

H.3. Time-varying latent component of growth

In this extension, we consider a model in which each firm's underlying growth is time-varying rather than a fixed parameter. Specifically, the firm-specific component of earnings growth is

$$
\begin{aligned}
\tilde{x}_{i,t} &= z_{i,t} + v_{i,t}\\
\Delta z_{i,t} - \mu &= \phi_z\left(\Delta z_{i,t-1} - \mu\right) + \varepsilon_{i,t}
\end{aligned}
$$

where the shocks are independent and have variances σ^2_v and σ^2_ε, respectively. The agent attempts to infer the underlying $\Delta z_{i,t}$ using constant-gain learning,

$$
\begin{aligned}
E^*_t[\Delta z_{i,t} - \mu] &= \phi_z E^*_{t-1}[\Delta z_{i,t-1} - \mu] + \beta\left(\Delta\tilde{x}_{i,t} - E^*_{t-1}[\Delta\tilde{x}_{i,t}]\right) && \text{(A20)}\\
E^*_t[v_{i,t}] &= (1-\beta)\left(\Delta\tilde{x}_{i,t} - E^*_{t-1}[\Delta\tilde{x}_{i,t}]\right). && \text{(A21)}
\end{aligned}
$$

Her expectation for the future growth of the firm-level component is then

$$E_t^*\left[\Delta \tilde{x}_{i,t+1}\right] \;=\; \mu + \phi_z E_t^*\left[\Delta z_{i,t} - \mu\right] - E_t^*\left[v_{i,t}\right]. \tag{A22}$$

Her expectation for the future level of the firm-level component is

$$E_t^*\left[\tilde{x}_{i,t+n}\right] \;=\; \tilde{x}_{i,t} + n\mu + \phi_z \frac{1-\phi_z^n}{1-\phi_z} E_t^*\left[\Delta z_{i,t} - \mu\right] - E_t^*\left[v_{i,t}\right]. \tag{A23}$$

Since we are considering price-earnings ratios, we will normalize μ to 0 for simplicity. In the case of $\phi_z = 1$ and $\sigma_\varepsilon = 0$, this extended model collapses back to the main model of Section IV.

Note that the introduction of a time-varying component of firm-level growth does not affect the pricing of aggregate strips in equations (14)-(16). Further, given these new definitions for expected cash flows, the description of firm prices, expected returns, and realized returns in equations (17)-(19) still hold.

Qualitatively, this model matches the main implications of our main model in Section IV. A higher expectation of $\Delta z_{i,t}$ will raise a firm's price by increasing the expected future cash flows and by lowering the subjective risk premium. If the constant-gain β is small, then earnings growth disappointment will raise expected next-period earnings growth, as shown by equations (A20)-(A22). Similarly, if the constant-gain β is small. then disappointment in one-year earnings growth will only have a small immediate impact on returns, as shown by equations (19), (A20), (A21), and (A23).

Quantitatively, we find that this extension does not noticably alter our results. Compared to our main model, this extension provides two additional parameters $(\phi_z, \sigma_\varepsilon)$. We estimate these parameters to best match the decomposition targets shown in Figure 3. The result is $\phi_z = 0.9999$ and $\sigma_\varepsilon = 0$, meaning that the extended model is virtually identical to the fixed parameter model of Section IV. In other words, for the purposes of explaining our empirical findings, the main model of Section IV and the extended model with a time-varying latent component of growth perform equally well. Because of this, we focus on the more parsimonious fixed-parameter model for our main analysis.