Optimal Information and Security Design*

Nicolas Inostroza[†] Anton Tsoy[‡]

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Abstract

An asset owner designs an asset-backed security and a signal about its value. After experiencing a liquidity shock and privately observing the signal, he sells the security to a monopolistic buyer. Within double-monotone securites, asset sale is uniquely optimal, which corresponds to the most informationally sensitive security. Debt is a constrained optimum under external regulatory liquidity requirements on securities. Thus, the "folk intuition" behind optimality of debt due to its low informational sensitivity holds only under additional restrictions on security or information design. Within monotone securities, a live-or-die security is optimal, whereas additional-tier-1 debt is optimal under the regulatory liquidity requirements.

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[†]University of Toronto and FTG. Email: nicolas.inostroza@rotman.utoronto.ca
[‡]University of Toronto and FTG. Email: anton.tsoy@utoronto.ca.

1 Introduction

Corporations routinely raise funds by selling assets and asset-backed securities when experiencing liquidity needs. Asymmetric information is a major impediment to such sales: at the time of sale, the issuer of the security knows more about it than the liquidity supplier, thereby limiting the scope of trade. A central question in corporate finance is how to optimally design securities in such a situation?

Starting from Leland and Pyle (1977), Myers and Majluf (1984), and Myers (1984), the classical corporate finance literature (reviewed below) assumes that the issuer's private information comes from an exogenous signal and focuses on how to optimally design the security payout. Two classical results obtain in this environment. First, retention of cash flows by the issuer is necessary in mitigating the information asymmetry. Second, debt is an optimal form of retention, as its payout is least sensitive to the issuer's private information.

In reality, an important dimension of design (alongside security payout design) is information design – issuers can often control to a certain extent the degree of information asymmetry vis-a-vis the liquidity supplier. For example, security payouts are only one part of financial contracts that generally include many clauses that can curb or exacerbate information asymmetry. To give a specific example, convertible bonds convert into equity when the stock price reaches a specific threshold. Yet, they often include other conversion clauses, such as whether the company made large dividend payments recently or is a takeover target. Such clauses exacerbate information asymmetry, as the management of the company is generally better informed about the likelihood of a takeover or upcoming changes in the dividend policy. Similar logic applies to other financial securities, such as warrants or preferred shares, that often contain clauses about changes in the corporate policy.

Information design can be a part of the asset creation process itself. In the case of mortgage-backed securities (MBS), the issuer can strategically select the underlying pool of mortgages and through this create a greater informational advantage about the pool's performance. This can be done by selecting mortgages from the geographical region or the market segment in which the issuer specializes as a mortgage originator. Alternatively, the issuer can minimize his informational advantage by creating a diverse pool of mortgages or by designing an overly complex MBS that is equally hard for the issuer and outside investors to value. The same idea applies to even more complex collateralized debt obligations and securities backed by credit-card receivables, car loans, and student loans.

Information design can be done through organization design. Consider, for instance, a multi-divisional company divesting one of its divisions. The general management can take a hands-off approach and give the division management lots of autonomy. At the other extreme, the division can be incorporated into the core of the company's business with its operations closely monitored. Different organization structures imply different levels of awareness of the general management about the division prospects, and hence, the extent of information asymmetry during the divestiture. Another example is the sale of stakes by limited partners (LPs) in private equity funds. Even though LPs receive updates about the fund's strategy and performance, they are not directly involved in investment decisions (partially due to lack of sophistication) that are fully delegated to general partners. This organization structure curbs LP's informational advantage vis-a-vis outsiders and allows them to sell their stake in the fund early if they experience a liquidity shock.

Information design can stem from limited information acquisition/processing resources. Corporations have accounting and risk management systems in place that commit them to learn granular information about risks. When resources are scarce, this means that they learn more noisy information about the upside potential, which is also by its nature harder to refine beyond a certain level. For instance, mutual and hedge funds can assume a passive shareholder role in numerous companies, which commits them to have only limited private information about each particular holding and focus their expertise in managing the risk exposure of the portfolio as a whole. At the other extreme, they can adopt concentrated positions and engage in activist campaigns, which exacerbates their informational advantage vis-a-vis outsiders.

All these examples share a common feature: the issuer's ability to influence the level of future asymmetric information. This paper raises the normative question of what is the optimal way for the issuer to jointly design information and securities to raise liquidity? Additionally, it prompts an examination of whether the securities commonly used in practice align with the optimum. This paper addresses these questions and provides answers.

The basic setup is that of DeMarzo and Duffie (1999) and Biais and Mariotti (2005) with the joint information and security design occurring before the private information is revealed to the issuer. This timing is motivated by the common practice of shelf-registration, which allows corporations to quickly react to changing economic conditions by registering securities well in advance of their sale. It is also relevant when the issuer is different from the seller of the security.

Formally, there are three stages: the ex-ante design stage, the trading stage, and the final stage. At the design stage, before getting any private information, the asset owner (the issuer) chooses both the distribution of the private signal about the underlying cash flows from the asset that is revealed to the issuer at the trading stage (information design) and the security payout contingent on the realization of cash flows at the final stage (security design).. We are interested in the optimal joint design of the security and information under weak restrictions on both dimensions of the design. We suppose that the issuer can pick any security satisfying limited liability and monotonicity/double monotonicity (commonly assumed in the security design literature) and can costlessly choose any unbiased signal about the asset.

At the beginning of the trading stage, the issuer observes the signal realization. Due to liquidity costs, he discounts future asset payoffs at a higher rate than the liquidity supplier. This creates gains from trade of the security. However, efficient trade might be impeded by asymmetric information. We suppose that there is a monopolistic liquidity supplier endowed with all the bargaining power, who offers an optimal screening mechanism to the issuer, which in our setting boils down to a posted price. This assumption is realistic in applications where the security is designed to raise liquidity in crisis times when the liquidity supply is scarce and liquidity suppliers have significant market power.

Our analysis builds on two insights. First, we can think of the joint security and information design problem as a sequential process: the issuer first decides on the security, and then picks the signal about cash flows. In this interpretation, (i) the signal about cash flows translates into the signal about the security value, which turns out to be a sufficient statistic for agents' payoffs, and (ii) the choice of the security determines the set of admissible signals about the security value. Then, we can solve this problem backwards. For a fixed security, the optimal signal choice boils down to the information design with interdependent values analyzed in Kartik and Zhong (2023). Their analysis implies that, for any security, any optimal signal distribution satisfies two economic properties: it restricts the highest signal realization to a certain value and ensuresthat the security is always sold. In other words, an optimal signal reveals sufficiently noisy information about high cash flow realizations which mitigates the lemons problems and guarantees the sale of the security.

Our second insights is the novel benefit of informationally sensitive securities when the issuer can flexibly design information. We say that security $\tilde{\varphi}$ is more informationally sensitive than security φ, if fixing the average security payoffs, $\tilde{\varphi}$ crosses φ from below. We establish that, for monotone securities, a more informationally sensitive security has a higher variability of payoffs, which tends to expand the set of admissible signals about the security value thereby leading to better outcomes for the issuer.

This result provides a powerful tool for determining optimal securities. We show that, when the issuer can optimally design information in addition to security, within the class of double monotone securities, it is strictly optimal for the issuer to simply sell the asset rather than issue any security. In other words, any form of cash flow retention is strictly suboptimal. This result is in contrast to the two classical results that retention is generally optimal with exogenous private information, and debt is an optimal form of retention. To see the reason for this, let us recall the folk intuition behind the classical results. Roughly, informationally insensitive securities are valuable because they serve as a commitment device for the issuer not to take advantage of his future private information at the trading stage. A debt security arises as optimal, as it is minimally sensitive to the issuer's private information: it promises a fixed amount (the face value) whenever possible and offers maximal downside protection when cash flows are low. However, it comes at a cost as it limits gains from trade by forcing the issuer to retain cash flows above the face value of debt.

With the added flexibility of optimal information design, the issuer can already curb his informational advantage by properly designing his private signal about the security. In particular, as argued above, this allows him to achieve trade with probability one for any fixed security. Roughly, information design already achieves much of what security design thrives to achieve in models with exogenous private information. On the other hand, informationally sensitive securities hold value as they provide the issuer with greater flexibility in information design. We leverage this intuition and show that selling the asset, which corresponds to the most informationally sensitive security, is strictly optimal.

How does the optimality of the asset sale square with common practices of raising liquidity? Our result explains why in many markets, in which the adverse selection problem is potentially severe, issuers often simply liquidate assets to raise liquidity rather than design complex asset-backed securities. In the examples described above, multi-divisional firms sell entire periphery divisions in times of crisis; there is an active market for limited partners' stakes in private equity funds; and mutual and hedge funds liquidate their holding when facing excessive redemptions. Our analysis stresses that a proper information design – the ability to commit not to learn too positive private information about the asset – is a necessary condition for the optimality of the asset sale. As we argued above, such a commitment can be attained through providing lots of autonomy to periphery divisions, the structure of decision making in private equity funds, or by following passive investment strategies of funds.

At the same time, many securities, such as MBS and other asset-backed securities, are structured as debt securities. The classical view is that this is the optimal way to raise liquidity in the presence of exogenous asymmetric information. In contrast, our result suggests that the prevalence of debt points to the presence of institutional or technological restrictions either on the information or security design. In particular, the existing literature imposes the extreme restriction that no information design is possible.

To reconcile this phenomenon with our theory, we present an alternative explanation for the prevalence of debt in specific markets. We examine the joint design of securities and information while imposing additional external liquidity requirements, where securities must be sold without a substantial discount on their maximum value. These requirements may arise from regulations or shareholder oversight. For instance, banks, pension funds, and insurance companies are mandated to hold sufficient high-quality liquid assets that can be quickly liquidated without significant value loss. Similarly, outside shareholders or boards of directors representing them may be concerned about management selling securities at a significant discount and may block such sales. For these reasons, the issuer may have a strong preference for designing securities that satisfy these external liquidity requirements.

With these external liquidity requirements, we find that debt reemerges as the optimal security within the class of double monotone securities. This implies that debt is influenced by regulations or external oversight rather than being the unconstrained optimal security for raising liquidity. This formalizes the viewpoint often expressed by practitioners that debt arises due to "regulatory arbitrage," where institutional investors demand debt because regulators perceive it as sufficiently safe and liquid.

The underlying intuition for this finding is as follows: The optimal information design restricts the issuer from learning about extremely high security values, resulting in securities generally being sold at a discount to their maximum value. If this discount is substantial, it can violate the liquidity requirements and disqualify certain securities, particularly pure equity. In such a scenario, the informational insensitivity of debt becomes valuable once again, leading to its optimality.

While we view double monotonicity as natural in many environments (and hardly restrictive from the practical standpoint), relaxing this assumption and considering monotone securities yields additional theoretical insights and predictions. We show that, among monotone securities, a “live-or-die security” that pays all the cash flows when they are above a certain level but pays zero when cash flows are below this level is optimal. The reason for this is that, holding the average security payoff fixed, live-or-die securities are most informationally sensitive among monotone securities, which is a valuable property when the issuer can additionally design information.

If we additionally impose external liquidity requirements, then at additional tier-1 (AT1) debt becomes optimal within the class of monotone securities. AT1 debt recently became popular in the banks’ capital structure. It is structured as standard debt in normal times, but becomes junior to other forms of debt and equity if a bank fails to maintain adequate regulatory capital or asset liquidity. Hence, AT1 debt is effectively a live-or-die security capped at the face value of debt. As we argued above, the cap on the payoffs is valuable in the presence of external liquidity requirements, while the high informational sensitivity of the live-or-die part expands the choice of signals about the security value available to the issuer. Thus, it is natural that AT1 debt is optimal in such an environment.

Related Literature. Leland and Pyle (1977), Myers and Majluf (1984), and Myers (1984) first established that in the world of asymmetric information about asset qualities, cash flow retention serves as a credible signal of asset quality and debt arises as optimal among many other securities. The folk intuition is that debt is advantageous, as it is the least sensitive to the issuer’s private information. This work started an extensive literature on optimal security design under adverse selection. Most closely related to our paper are DeMarzo and Duffie (1999) and Biais and Mariotti (2005) who study security design at the ex-ante stage with an exogenous distribution of issuer's private information.[1] Both papers show optimality of debt under general conditions and weak restrictions on the class of securities. Selling the asset is optimal but only as a corner optimum (i.e., debt with face value equal to the highest cash flow realization) when the information asymmetry is not too severe. Other papers showing optimality properties of debt include Nachman and Noe (1994), DeMarzo (2005), DeMarzo et al. (2005), Dang et al. (2013), Daley et al. (2020), Li (2022), Asriyan and Vanasco (Forthcoming), Inostroza and Figueroa (2023) among many others.

We contribute to this literature by solving the joint problem of information and security design in the by now canonical setup of DeMarzo and Duffie (1999) and Biais and Mariotti (2005). We show that, within the class of monotone securities, generally the issuer prefers more informationally sensitive securities, because they provide more flexibility in information design. In contrast to these benchmarks, within the class of double monotone securities, selling the asset is uniquely optimal and retention is strictly suboptimal. We further obtain debt as a constrained solution to the joint design problem, when the security must satisfy external liquidity requirements, and solve this problem in the class of monotone securities.

There is a literature showing that informationally sensitive securities can become optimal when informational sensitivity has additional benefits to the issuer, e.g., it incentivizes information acquisition by investors (Boot and Thakor 1993, Fulghieri and Lukin 2001, Yang and Zeng 2019), it enables the aggregation of information about the optimal scale of project from informed investors (Axelson 2007), or it is complementary to public signals about the asset and allows the issuer to economize on retention (Daley et al. 2023). Our mechanism is different and to the best of our knowledge novel to the literature: informationally sensitive securities are beneficial, because they relax the constraints on the issuer's information design.

Several papers study security design with endogenous information. Yang and Zeng (2018), Yang (2020) allow for flexible information acquisition by the liquidity supplier. In Azarmsa and Cong (2020), Szydlowski (2021), the issuer additionally designs public disclosures to investors. Similarly to ours, these papers impose minimal restrictions on admissible information acquisition or disclosure policies. In contrast to our paper, the optimal security is indeterminate without either positive information acquisition costs or further financing frictions. It is debt when information acquisition is costly in

[1] We discuss in details the relationship to these papers after we present our main results (see in particular Remarks 2 and 4)

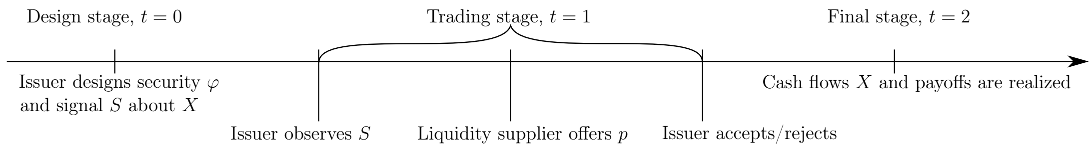


Figure 1: ***Timeline***

Yang (2020) and depends on the kind of additional contracting frictions in Szydlowski (2021) and Azarmsa and Cong (2020). Our study of joint information and security design by the issuer is complementary to this literature.

Our paper is related to the literature on optimal information design in the monopolist screening problem (Bergemann et al. 2015, Roesler and Szentes 2017, Glode et al. 2018).[2] Most closely related is Kartik and Zhong (2023) who study information design with interdependent values. We build on their result to analyze the joint information and security design, and characterize optimal securities that arise in different environments.

The paper is organized as follows. Section 2 presents the model. Section 3 conducts preliminary analysis. Section 4 solves the joint security and information design problem. Section 5 solves the problem under external liquidity requirements. Section 6 considers imperfectly competitive liquidity suppliers. Section 7 discusses positive implications. All omitted proofs are relegated to the Appendix and the Online Appendix.

2 The Model

The basic setup is that of DeMarzo and Duffie (1999) and Biais and Mariotti (2005) with the addition of information design. Figure 1 depicts the timeline. There are three stages $t \in \{0, 1, 2\}$. There is an issuer (he) owning an asset and a liquidity supplier (she). Both parties are risk-neutral. The asset generates cash flows X at the final stage $t = 2$ distributed according to a CDF H on positive support $\mathcal{X}$ with $\underline{x} > 0$ and $\overline{x} < \infty$ being the minimal and maximal elements in $\mathcal{X}$.

At the trading stage $t = 1$, the liquidity supplier's discount factor is normalized to 1 and she values cash flows at X. The issuer discounts future cash flows at a higher

[2]Less related to our paper, Barron et al. (2020), Mahzoon et al. (2022) study interaction of information and contract design in the moral hazard setting.

rate and values them at $\delta X, \delta \in (0,1)$. This captures the issuer's desire to free-up capital to invest in alternative assets/projects, improve his liquidity position in crisis times, raise liquidity to cover redemptions (for investment funds), or focus financial resources on the core business (for multi-divisional companies). There are gains from trade: the liquidity supplier is the efficient asset owner.

At the ex-ante design stage $t = 0$, before receiving any private information, the issuer designs a security to be traded at $t = 1$ and a signal about cash flows to be revealed to him privately at $t = 1$ (before trading). The security payoff $F = \varphi(X)$ is contingent on the realization of X. It is distributed according to the CDF $H^{\varphi} \equiv H \circ \varphi^{-1}$ supported on $[\varphi(\underline{x}), \varphi(\overline{x})]$.[3] Let $\mu^{\varphi} \equiv \mathbb{E}_{H^{\varphi}}[F]$ be the average payoff of $F = \varphi(X)$. The ex-ante design of securities is commonly observed in practice during the shelf-registration, a practice that allows the issuer to register securities in advance to avoid lengthy delays and promtly attend to his liquidity needs.

We assume throughout the paper that security $\varphi(X)$ satisfies *limited liability*: $\varphi(X) \in [0, X]$. Security φ is *monotone*, if it is right-continuous and weakly increasing in X, and it is *double monotone* if in addition $X - \varphi(X)$ is weakly increasing in X. In the security design literature, double monotonicity is often motivated by the "sabotage" argument: if it fails, the party whose payout is non-monotone in X can increase its payout by sabotaging and partially destroying cash flows. An additional justification for monotone securities is that, for non-monotone securities, the issuer profits from artificially boosting cash flows X by either contributing his own funds or borrowing short-term from the market, and this way, reducing the payout to the security holders. Realism of these justifications depends on the application in consideration. We consider a general problem not tailored to any particular application, and so we simply motivate these assumptions by the fact that they are barely restrictive from the practical point, as almost all securities observed in practice, including debt, call options, preferred shares, convertible securities, satisfy double monotonicity. Denote by Φ_1 and Φ_2 the sets of monotone and double-monotone securities, respectively.

At $t = 0$, the issuer can costlessly design any signal S about X. A signal S is described by the probability space $(\mathcal{X} \times \mathcal{S}, \mathscr{X} \times \mathscr{S}, \nu_{X,S})$, where $\mathcal{S}$ is a sufficiently rich Polish space of possible signal realizations (in particular, $\mathcal{X} \subseteq \mathcal{S}$), and $\nu_{X,S}$ is the probability measure on the product of Borel σ-algebras, $\mathscr{X} \times \mathscr{S}$, with the marginal distribution on $\mathcal{X}$ coinciding with the prior distribution of X, H.

[3] We specify $\varphi^{-1}(f) \equiv \sup\{x : \varphi(x) \leq f\}$ as the right-continuous inverse function.

Let $Z = \mathbb{E}[\varphi(X)|S]$ be the expected security value conditional on signal S with the CDF denoted by G^φ. We call G^φ *admissible* for φ if it is generated by some signal S about X, and let $\mathcal{G}^\varphi$ be the set of all admissible distributions. By Strassen theorem, G^φ is admissible if and only if G^φ is a mean-preserving contraction of H^φ (see Lemma 5 in Appendix).[4] That is, $\mathbb{E}_{G^\varphi}[Z] = \mathbb{E}_{H^\varphi}[F]$, and G^φ second-order stochastically dominates H^φ: $\int_{-\infty}^{y} H^\varphi(f)\,\mathrm{d}f \geq \int_{-\infty}^{y} G^\varphi(z)\,\mathrm{d}z$ for all y.[5] As we show shortly, the issuer's information design problem boils down to choosing $G^\varphi \in \mathcal{G}^\varphi$.

At the trading stage $t = 1$, the issuer observes a realization s of signal S and updates his valuation of security to $Z = \mathbb{E}[\varphi(X)|S = s]$. The issuer can obtain liquidity from the liquidity supplier by selling to her the security $\varphi(X)$. We assume a monopolistic liquidity supplier. This assumption is relevant for issuers who have in mind future circumstances in which they sell securities in periods of scarce liquidity when liquidity suppliers have significant monopoly power (e.g., during crisis times) or issuers who anticipate urgent liquidity needs in the future that do not leave sufficient time to solicit competitive bids for their securities. In Section 6, we relax this assumption and show how our results are modified in the extension where the liquidity supplier is competitive in "normal" times, but monopolistic in crisis times.[6]

At $t = 1$, the liquidity supplier offers a posted price p at which she is willing to buy the security, which the issuer accepts or rejects. By Proposition 1 in Biais and Mariotti (2005), posting a price is optimal for the monopolistic liquidity supplier within the general class of incentive-compatible and individually rational mechanisms specifying the quantity of the security traded and the corresponding transfer to the issuer.[7] Conditional on observing signal S, $\delta\mathbb{E}[X|S] - \delta\mathbb{E}[\varphi(X)|S] + p$ is the issuer's expected

[4]A somewhat delicate point here is that the set of admissible distributions, $\mathcal{G}^\varphi$, is generated by signals S about X rather than by signals Z about $\varphi(X)$ (distributed according to H^φ). The Strassen theorem implies that the latter set consists of all mean-preserving contractions of H^φ. We show in Lemma 5 that the two sets in fact coincide.

[5]All integrals in this paper are Lebesgue-Stieltjes integrals for which the integration by parts formula obtains (see Lemma 6 in the Online Appendix).

[6]We believe our insights are applicable in the general setup with a number of imperfectly competitive liquidity suppliers. We leave this extension for future research.

[7]Posted prices need not be optimal when the liquidity supplier is allowed to condition quantities and transfers on the future realization of X. Such mechanisms would clash with the ex-ante security design by the issuer (studied in this paper), as they effectively would give power to the liquidity supplier who can override payoffs $\varphi(X)$ with appropriate quantities and transfers contingent on X. In reality, substantial changes in the security requested by the buyer on the spot require a new registration with the SEC, which defies the whole purpose of the shelf-registration of avoiding regulatory delays in issuance.

payoff from accepting p and $\delta\mathbb{E}[X|S]$ form rejecting it. Hence, $Z = \mathbb{E}[\varphi(X)|S]$ is the sufficient statistic for the issuer's optimal decision, which we call the issuer type.

Given the distribution of Z, G^φ, the liquidity supplier chooses p to maximize her expected profit

$$\pi(p|G^\varphi) \equiv \int_{\varphi(\underline{x})}^{p/\delta} (z-p)\,\mathrm{d}G^\varphi(z).$$

There is a standard adverse selection problem: only issuer types with expected values below p (i.e., $z \leq p/\delta$) accept the price offer, while higher types hold on to the asset. Let $P(G^\varphi) \equiv \arg\max_p \pi(p|G^\varphi)$ be the set of optimal posted prices and $\Pi(G^\varphi) \equiv \max_p \pi(p|G^\varphi)$ – the maximal profit. We suppose that, when indifferent between several $p \in P(G^\varphi)$, the liquidity supplier chooses the most preferred price for the issuer, $p(G^\varphi)$, which is the highest price in $P(G^\varphi)$. Then, given distribution G^φ, the issuer's ex-ante expected payoff equals

$$\mathbb{E}[\delta\mathbb{E}[X|S] + \max\{p(G^\varphi) - \delta Z, 0\}] = \delta\mathbb{E}[X] + v(p(G^\varphi)|G^\varphi),$$

where the equality is by the law of iterated expectations and $v(p|G^\varphi) \equiv \int_{\varphi(\underline{x})}^{p/\delta}(p-\delta z)\,\mathrm{d}G^\varphi(z)$ is the issuer's information rents given price p and distribution G^φ. We denote $V(G^\varphi) \equiv v(p(G^\varphi)|G^\varphi)$.

At the design stage $t = 0$, the issuer optimally chooses a signal S about X and the security design φ. The distribution of S and security design φ enter the issuer's and the liquidity supplier's objective functions V and Π only through the distribution G^φ of Z, which is a signal about $\varphi(X)$. Hence, it is without loss of generality, to suppose that the issuer directly chooses the distribution $G^\varphi \in \mathcal{G}^\varphi$. With a little abuse of terminology, henceforth, we refer to Z (rather than S) as the signal. For a given security design φ, the issuer's optimal information design problem is

$$\boldsymbol{V}(\varphi) \equiv \max_{G^\varphi \in \mathcal{G}^\varphi} V(G^\varphi). \tag{1}$$

For $\Phi \in \{\Phi_1, \Phi_2\}$, the optimal joint security and information design problem is

$$\max_{\varphi\in\Phi} \boldsymbol{V}(\varphi) = \max_{\varphi\in\Phi, G^\varphi\in\mathcal{G}^\varphi} V(G^\varphi). \tag{2}$$

Therefore, the optimal choice of a security $\varphi(X)$ and a signal S about X can be reinterpreted as a sequential choice of, first, the security $\varphi(X)$ that determines the

	$X=\underline{x}$	$X=\overline{x}$
B	$\tau/2$	0
G	$(1-\tau)\,2$	$1/2$

(a) Signal S^I

	$X=\underline{x}$	$X=\overline{x}$
B	$1/2$	$(1-\tau)/2$
G	0	$\tau/2$

(b) Signal S^{II}

Table 1: ***Signal distributions***

Tables describe joint distributions of signals S^I, S^{II} and cash flows X. Parameter τ controls the precision of signals with $\tau = 0$ corresponding to uninformative signals and $\tau = 1$ corresponding to perfectly revealing signals.

set of admissible distributions $\mathcal{G}^{\varphi}$, from which the issuer picks the optimal signal distribution G^{φ}. For brevity, we refer to (2) as the security design problem, and it is implicit that the signal distribution is also chosen optimally.

3 Preliminary Analysis

Simple Example. We start with an illustration of our results in a simple example. Suppose $\delta = 3/4$ and X takes two equally likely values $\underline{x} = 1$ and $\overline{x} = 3$. We impose further restrictions that are not part of our model. First, we focus on debt securities $\varphi(X) = \min\{X, D\}$, $D \in [\underline{x}, \overline{x}]$. Second, we consider one of two binary signals with values G and B described in Table 1. For both signals, we can derive the ex-ante optimal debt face value D and the signal precision τ.

Signal S^I "perfectly reveals bad news:" signal realization B reveals that the cash flows are low, $X = \underline{x}$, while G leads to the posterior probability of $\overline{x}$ equal to $1/(2-\tau)$. Symmetrically, signal S^{II} "perfectly reveals good news:" signal realization G reveals that $X = \overline{x}$, and B leads to the posterior probability of $\underline{x}$ equal to $1/(2-\tau)$. Here, τ captures the signal precision with $\tau = 0$ corresponding to an uninformative signal, and $\tau = 1$ – to a perfectly revealing signal. We can easily find optimal securities for both signals. Under S^I, the issuer's maximal payoff is ≈ 0.41 attained by selling the whole asset and setting $\tau^* \approx 0.71$. Under S^{II}, the issuer's maximal payoff is 0.1875 attained by issuing debt $D^* \approx 1.5$ and $\tau^* = 1$.

This example hints at two of our main insights. First, when the issuer learns noisy information about high cash flows (signal S^I), retention is suboptimal and it is optimal to simply sell the asset (within the class of debt securities). Second, debt becomes optimal when the issuer learns more granular information about high cash flows (signal S^{II}). Third, the issuer prefers the former signal (S^{I}) to the latter (S^{II}). At the same time, this example leaves open the central question of our paper – what is the best way to jointly design security and information.

Information Design. Building on Kartik and Zhong (2023), we first solve the information design problem (1) for a fixed security φ. They introduce *incentive compatible distributions* (henceforth, ICDs) with CDFs $G_{u,\mu}$ parametrized by the upper support boundary u and the mean μ as follows

$$G_{u,\mu}(z) = \begin{cases} 0 & , z < l, \\ (z/u)^{\delta/(1-\delta)} & , z \in [l, u], \\ 1 & , z > u, \end{cases} \text{ where } l = \left(\frac{\mu - \delta u}{1-\delta}\right)^{1-\delta} u^{\delta}. \tag{3}$$

The next result follows from Theorem 2 and Proposition 2 in Kartik and Zhong (2023). Denote by $u(G^{\varphi})$ the highest signal realization under distribution G^{φ}.

Proposition 1. *For any security $\varphi(X)$, let u^{φ} be the solution to*[8]

$$\max\{u : G_{u,\mu^{\varphi}} \in \mathcal{G}^{\varphi}\}. \tag{4}$$

Then, $\boldsymbol{V}(\varphi) = \delta(u^{\varphi} - \mu^{\varphi})$ and a distribution $G^{\varphi} \in \mathcal{G}^{\varphi}$ is optimal for φ if and only if (i) $u(G^{\varphi}) = u^{\varphi}$; (ii) trade occurs with probability one under G^{φ}. Further, $G_{u^{\varphi},\mu^{\varphi}}$ is an optimal signal distribution for φ with support $[l^{\varphi}, u^{\varphi}]$, where $l^{\varphi} \equiv \left(\frac{\mu^{\varphi} - \delta u^{\varphi}}{1-\delta}\right)^{1-\delta} (u^{\varphi})^{\delta}$.

Kartik and Zhong (2023) show that in solving (1) it is without loss of optimality to focus on admissible ICDs, $G_{u,\mu^{\varphi}} \in \mathcal{G}^{\varphi}$. ICDs $G_{u,\mu^{\varphi}}$ are special in that the liquidity supplier is indifferent between any posted price in the support of issuer's expected values of security, $[\delta l, \delta u]$. In particular, she weakly prefers to offer a pooling price δu. This is the most preferred outcome for the issuer under $G_{u,\mu^{\varphi}}$ that gives him payoff of $\delta(u - \mu^{\varphi})$. Hence, the problem (1) boils down to finding an admissible ICD, $G_{u,\mu^{\varphi}} \in \mathcal{G}^{\varphi}$, that results in the maximal price δu.

For our purposes, Proposition 1 has two important implications. First, it allows us to write the constraints of the information design problem in the following analytical form.

[8]Since $G_{\mu^{\varphi},\mu^{\varphi}} \in \mathcal{G}^{\varphi}$ (uninformative signal), $\{u : G_{u,\mu^{\varphi}} \in \mathcal{G}^{\varphi}\}$ is non-empty.

Lemma 1. *The constraint in (4) is equivalent to* $u \leq \varphi(\overline{x})$ *and*

$$\mathcal{L}(y|\varphi, u) \equiv \varphi(\overline{x}) - \delta u - (1-\delta) y^{\frac{1}{1-\delta}} u^{-\frac{\delta}{1-\delta}} - \int_y^{\varphi(\overline{x})} H^\varphi(f)\,\mathrm{d}f \geq 0, y \in [0, u]. \quad (5)$$

Second, Proposition 1 implies that the issuer does not need full flexibility in choosing signal distributions. Any signal distribution G^φ is optimal for security $\varphi(X)$ as long as it satisfies two economic properties. It must ensure *perfectly liquidity*, that is, the full issue of the security $\varphi(X)$ is always sold to the liquidity supplier. Further, the issuer prefers not to learn "too optimistic" information about the security value, i.e., his signal Z is below certain u^φ, which is generally less than the highest payout of the security, $\varphi(\overline{x})$. Importantly, the ICD $G_{u^\varphi, \mu^\varphi}$ is only one optimal distribution, but there are generally many other optimal signals. Practically, this means that the commitment to some optimal signals might not be too demanding, and as we argue in Section 7, in many situations such a commitment might already be in place due to technological restrictions on signals or considerations other than liquidity needs.

Remark 1. *Theorem 2 in Kartik and Zhong (2023) shows that under a natural equilibrium refinement, signal distributions in Proposition 1 are optimal in a richer class of information structures, where the liquidity supplier also gets a private signal that is less informative than the issuer's signal (in the sense that the issuer's signal is a sufficient statistic for the liquidity supplier's signal with respect to the value of security). In particular, this result implies that the issuer does not gain from publicly disclosing information about the security value, and it is without loss of optimality to focus on signals that the issuer privately learns.*

4 Optimal Security Design

We first present our main tool for finding optimal securities. This result formalizes the idea that the issuer weakly prefers more informationally sensitive securities, because they give him "more freedom" in information design. We say that security $\tilde{\varphi}$ is *more informationally sensitive* than φ if there is $x^* \in [\underline{x}, \overline{x}]$ such that $\tilde{\varphi}(x) - \mu^{\tilde{\varphi}} \leq \varphi(x) - \mu^\varphi$ for $x < x^*$ and $\tilde{\varphi}(x) - \mu^{\tilde{\varphi}} \geq \varphi(x) - \mu^\varphi$ for $x > x^*$. In words, once we control for differences in means, $\tilde{\varphi}$ crosses φ from below at some x^*. Thus, informational sensitivity captures differences in the shape of securities. For example, holding the average security payoff fixed, convex securities like call option (i.e., $\varphi(X) = \max\{X - K, 0\}, K \in [0, \overline{x}]$) are more informationally sensitive than standard equity (i.e., $\varphi(X) = \alpha X, \alpha \in [0, 1]$), which in turn is more informationally sensitive than concave securities like debt.

DeMarzo et al. (2005) introduce the "crossing from below" property of security payoffs to capture informational sensitivity of securities in the context of auctions with securities. It is interesting that a similar notion of informational sensitivity plays a key role in our problem of joint information and security design, even though its application is quite different in the two setups. In particular, DeMarzo et al. (2005) additionally imposes strict monotone likelihood ratio property on agents' signals, which is not compatible with flexible information design in our paper. In contrast, our notion of information sensitivity does not impose any additional restrictions on signal distributions, in particular, it is independent of the prior.

Theorem 1. *Suppose that securities $\tilde{\varphi}$ and φ are monotone, $\tilde{\varphi}$ is more informationally sensitive than φ, and $\mu^{\tilde{\varphi}} = \mu^{\varphi}$.*

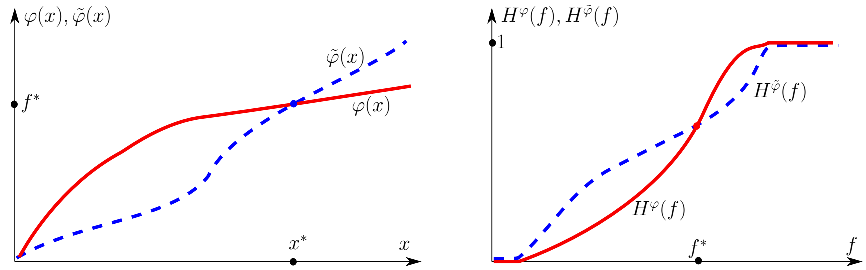
1. *Then, $\mathcal{G}^{\varphi} \subseteq \mathcal{G}^{\tilde{\varphi}}$ and $\boldsymbol{V}(\tilde{\varphi}) \geq \boldsymbol{V}(\varphi)$.*

2. *If there is $\varepsilon > 0$ such that $H^{\tilde{\varphi}}(f) > H^{\varphi}(f)$ for $y \in (\tilde{\varphi}(\underline{x}), \tilde{\varphi}(\underline{x}) + \varepsilon)$ and $H^{\tilde{\varphi}}(f) < H^{\varphi}(f)$ for $y \in (\tilde{\varphi}(\overline{x}) - \varepsilon, \tilde{\varphi}(\overline{x}))$, then $\mathcal{G}^{\varphi} \subset \mathcal{G}^{\tilde{\varphi}}$ and*

$$\int_{-\infty}^{y} H^{\tilde{\varphi}}(f)\,\mathrm{d}f > \int_{-\infty}^{y} H^{\varphi}(f)\,\mathrm{d}f \text{ for all } y \in (\tilde{\varphi}(\underline{x}), \tilde{\varphi}(\overline{x})). \tag{6}$$

   Further, if in addition $[l^{\varphi}, u^{\varphi}] \subset (\tilde{\varphi}(\underline{x}), \tilde{\varphi}(\overline{x}))$, then $\boldsymbol{V}(\tilde{\varphi}) > \boldsymbol{V}(\varphi)$.

Theorem 1 states that holding the average security payoff fixed, the issuer benefits from offering more informationally sensitive securities, because they expand the set of admissible signal distributions. It is related to Theorem 2 in Gershkov et al. (2023), which shows that for any double-monotone φ, debt φ_D, and call option φ_C such that $\mu^{\varphi_D} = \mu^{\varphi_C} = \mu^{\varphi}$, $\mathcal{G}^{\varphi_D} \subseteq \mathcal{G}^{\varphi} \subseteq \mathcal{G}^{\varphi_C}$. We generalize this result in two respects. First, we introduce the relevant notion of informational sensitivity for monotone (not necessarily double-monotone) securities, and show that, holding μ^{φ} fixed, more informationally sensitive securities give weakly more freedom in information design, and so, are weakly preferred by the issuer. Establishing this result for any monotone securities (rather than debt or call option) is crucial for our analysis of the joint information and security design problem under different restrictions on available signals and securities.



(a) Securities φ and $\tilde{\varphi}$ (b) CDFs H^{φ} and $H^{\tilde{\varphi}}$

Figure 2: ***Illustration for Theorem 1***

Second, we provide sufficient conditions for the issuer to strictly prefer a more informationally sensitive security $\tilde{\varphi}$. Roughly, this is the case when higher informational sensitivity expands the set of admissible ICDs. For ICDs, a more appealing distribution has a wider support $[l, u]$, hence, the conditions are on the values of $H^{\tilde{\varphi}}$ close to the extreme security payoffs $\tilde{\varphi}(\underline{x})$ and $\tilde{\varphi}(\overline{x})$.

The proof of Theorem 1 is based on the observation that if $\tilde{\varphi}$ crosses from below φ, its CDF $H^{\tilde{\varphi}}$ crosses from above the CDF H^{φ} (see Figure 2). This implies $\int_{-\infty}^{y} \left(H^{\tilde{\varphi}}(f) - H^{\varphi}(f)\right) \mathrm{d}f \geq 0$ for all y's up to the crossing point f^* of $H^{\tilde{\varphi}}$ and H^{φ}. For $y \in (f^*, \tilde{\varphi}(\overline{x}))$, integrating by parts,

$$\int_{-\infty}^{y} \left(H^{\tilde{\varphi}}(f) - H^{\varphi}(f)\right) \mathrm{d}f = \underbrace{\int_{-\infty}^{\tilde{\varphi}(\overline{x})} \left(H^{\tilde{\varphi}}(f) - H^{\varphi}(f)\right) \mathrm{d}f}_{=\mu^{\varphi}-\mu^{\tilde{\varphi}}=0} - \int_{y}^{\tilde{\varphi}(\overline{x})} \underbrace{\left(H^{\tilde{\varphi}}(f) - H^{\varphi}(f)\right)}_{\leq 0} \mathrm{d}f \geq 0.$$

Note that the conditions in part 2 of Theorem 1 are sufficient to ensure that all inequalities in (6) are indeed strict, which implies that $\mathcal{G}^{\tilde{\varphi}}$ is strictly larger than $\mathcal{G}^{\varphi}$.

In our analysis, we will also use the following lemma.

Lemma 2. *For any security* $\varphi \in \Phi_1$ *and* $\Delta > 0$, *if* $\tilde{\varphi}(X) = \varphi(X) + \Delta \in [0, X]$ *(i.e., satisfies limited liability), then* $\tilde{\varphi} \in \Phi_1$, $\boldsymbol{V}(\tilde{\varphi}) \geq \boldsymbol{V}(\varphi)$, *and* $u^{\tilde{\varphi}} \geq u^{\varphi} + \Delta$.

Lemma 2 shows that adding safe debt is always weakly optimal. Analogous results appear in the security design literature with exogenous private information (DeMarzo and Duffie 1999, Biais and Mariotti 2005). There, pledging a safe payoff of Δ does not give the liquidity supplier extra incentives to screen the issuer. Hence, by switching to security $\tilde{\varphi}$, the issuer gives up Δ of future asset payoff, which he values at $\delta\Delta$, but also increases the security price by $\delta\Delta$. This intuition from models with exogenous private information is carried to our model by noticing that if $G^{\varphi} \in \mathcal{G}^{\varphi}$, then a translation of G^{φ} by Δ belongs to $\mathcal{G}^{\tilde{\varphi}}$.

Optimality of Selling the Asset. We can now solve the security design problem (2) for double-monotone securities Φ_2.

Theorem 2. *Selling the asset (i.e., $\varphi(X) = X$ almost surely) is the unique optimal security within the class of double-monotone securities Φ_2.*

Theorem 2 is in stark contrast to the classical results stressing the role of security design in mitigating information asymmetry. The existing literature described in the Introduction establishes optimality of cash flow retention by the issuer. This literature often obtains debt as the optimal form of retention. Theorem 2 shows that, when the issuer can optimally design his private signal about the security and is restricted to double-monotone securities, security design is not necessary. In fact, any form of retention is strictly suboptimal – the unique optimum is to simply sell the asset and pick one of optimal signal distributions described in Proposition 1.

What is the reason for this difference in predictions? Let us recall the "folk" intuition in models with exogenous private information. There, debt serves as a commitment device for the issuer not to take advantage of his private information when trading with the liquidity supplier. A debt security pays a fixed face value whenever possible and offers maximal downside protection when cash flows are below the face value. In other words, debt is not sensitive to the issuer's private information most of the times, and when it is, the liquidity supplier receives the maximal payout feasible. This insensitivity of debt to private information is crucial in mitigating the lemons' problem and increasing its liquidity, when the private information is exogenous. However, it comes at a cost as it limits gains from trade by forcing the issuer to retain cash flows above the face value of debt.

In contrast, as shown in Proposition 1, the optimal information design already commits the issuer not to learn too optimistic information about the security and guarantees its perfect liquidity, making security design redundant for these purposes. In turn, by Theorem 1, more informationally sensitive securities give the issuer more freedom in information design. Selling the asset gives the liquidity supplier maximal exposure to cash flows, and corresponds to the most informationally sensitive security

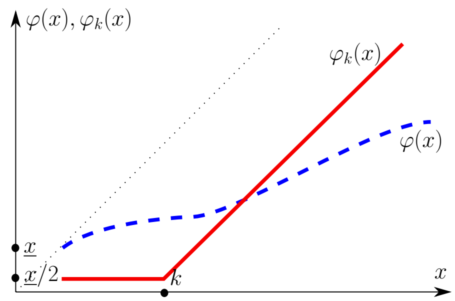


Figure 3: ***Illustration for Theorem 2***

within the class of double-monotone securities. Thus, the issuer prefers it to designing any non-trivial double-monotone security. [9]

More formally, the proof outline goes as follows. By Lemma 2, within the class of double-monotone securities, it is without loss of optimality to consider securities that include safe debt $\underline{x}$, i.e., $\varphi(\underline{x}) = \underline{x}$. Consider security φ_k that combines a safe debt $\underline{x}/2$ and a call option with strike price k such that $\mu^{\varphi} = \mu^{\tilde{\varphi}}$. As illustrated in Figure 3, φ_k is more informationally sensitive than φ. Further, the fact that, by construction, $\varphi_k(\overline{x}) > \varphi(\overline{x})$ and $\varphi_k(\underline{x}) < \varphi(\underline{x})$ implies that the conditions for part 2 of Theorem 1 are satisfied, hence, $\boldsymbol{V}(\varphi_k) > \boldsymbol{V}(\varphi)$. We then get the unique optimality of selling the asset by noticing that only $\varphi(X) = X$ is immune to such an improvement.

The positive implication of Theorem 1 is that raising liquidity with asset sales should prevail in environments where the issuer can commit to learn noisy information about high cash flow realizations and more granular information about lower cash flow realizations (corresponding to optimal signal distributions in Proposition 1). This prediction is in line with the reality that in many situations, corporations simply sell assets to raise liquidity despite potential concern for a high degree of adverse selection. In Section 7, we discuss several specific applications where this is the case and how issuers can commit to an optimal signal distribution.

On the normative side, Theorem 2 provides a counter-point to the established view in the literature that security design serves as a remedy for adverse selection.

[9]Incidentally, in our simple example in Section 3, selling the asset is optimal under the signal technology S^I that is not necessarily optimal but that, similarly to optimal signals, reveals noisy information about high cash flows (we prove this in Proposition 4 in the Online Appendix). These results in total suggest that the key property of the signal distribution making the asset sale optimal is that the signal about high cash flows is sufficiently noisy. Formalizing this result is beyond the scope of this paper and is left for future research.

With sufficient flexibility in information design, it is strictly suboptimal to resort to security design (within the class of double-monotone securities). Thus, security design is relevant only when the issuer cannot for some reason design an optimal signal about the asset. (Proposition 1 suggests that this is the case when the issuer cannot commit not to learn granular information about high cash flows.) Taking Theorem 2 as a benchmark, a justification of a particular security observed in reality must start from identifying what are the realistic restrictions on the information/security design that make the security design relevant in the first place. As an illustration of this approach, we study in the next section how our insights change with the introduction of realistic external liquidity requirements on securities that the issuer can offer.

Remark 2. *We can think of the classical literature as imposing the extreme assumption of no information design. Biais and Mariotti (2005), which is the closest to our framework, assume that the issuer perfectly observes X at $t = 1$ and obtain debt as the optimal security for any distribution H.*

Note that there is no contradiction between this result and Theorem 2. First, since $\mathcal{X}$ is bounded, selling the asset is a special case of debt with face value $\overline{x}$. Second, Biais and Mariotti (2005) assume a very specific form of exogenous private information – learning the cash flows. As Proposition 1 shows, perfectly learning the security payoff is generally suboptimal. An optimal signal distribution instead produces a noisy signal about high valuations.[10] *Relatedly, DeMarzo and Duffie (1999) show optimality of debt under certain conditions on the issuer's private information*[11] *in the model where the issuer signals to the competitive liquidity suppliers the security value by retaining a fraction of it. We show in Section 6 that selling the asset is also optimal in our model when the liquidity supplier is competitive.*

Optimality of Live-or-Die Security. While we view double-monotonicity as a natural restriction of securities that captures relevant agency frictions that are not explicitly modeled, relaxing it provides interesting theoretical insights that we present next. We solve (2) for monotone securities ($\Phi = \Phi_1$). Let us call securities of the form $\varphi(X) = \mathbf{1}\{X \geq L\} X$ *live-or-die securities* – they pay all cash flows above L, but pay nothing ("die") if cash flows fall short of L.

[10]To the best of our knowledge, it is an open question whether in Biais and Mariotti (2005), debt is optimal for more general distributions of exogenous signals about X.

[11]Specifically, the existence of the "uniform worst case" – a condition slightly weaker than the monotone likelihood ratio property.

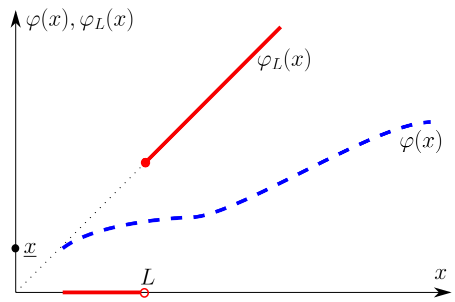


Figure 4: ***Illustration for Theorem 3***

Theorem 3. *Suppose H admits a density on $\mathcal{X}$. A live-or-die security $\varphi^*(X) = \mathbf{1}\{X \geq L^*\} X$ is optimal within the class of monotone securities Φ_1. Further, φ^* strictly dominates selling the asset (in particular, $L^* > \underline{x}$) and gives a payoff of 0 to the liquidity supplier.*

Theorem 3 implies that, for continuous distributions of cash flows, relaxing the double monotonicity assumption in Theorem 2 strictly increases the issuer's payoff: a live-or-die security φ^* that retains cash flows below L^* is optimal among monotone securities. Live-or-die securities are not observed in practice, which suggests that the sabotage incentives of the issuer (or other justifications for double monotonicity) are strong enough to prevent the issuer from selling securities failing double monotonicity. Yet, Theorem 3 reveals interesting theoretical insights about the optimal joint use of information and security design.

In conjunction with Theorem 2, Theorem 3 shows how using the joint security and information design we expand the set of equilibrium payoffs compared to those attainable in the two benchmarks where the issuer either (i) sells the asset and only uses information design (Kartik and Zhong 2023), or (ii)only uses security design (Biais and Mariotti 2005). By Theorem 2, the expansion of equilibrium payoffs compared to only security design is obtained already within the class of double-monotone securities. Further, Kartik and Zhong (2023) show that by selling the asset in combination with an optimal information design, the issuer can attain a payoff of $\delta\left(u^X - \mu^X\right)$ (where we denote by X the security $\varphi(X) = X$). This outcome also minimizes the liquidity supplier's payoff across all signal distributions, which equals $\mu^X - \delta u^X$. In this outcome, the asset is always sold, and so, there is no tension between maximization of information rents and efficiency.

Theorem 2 shows that, within the class of double monotone securities, no further increase of his payoff is possible despite the added flexibility to design the security. However, by Theorem 3, this is an artifact of the double monotonicity assumption. The optimal live-or-die security allows the issuer to attain a strictly higher payoff of $\delta\left(u^{\varphi^*}-\mu^{\varphi^*}\right)$. This outcome also lowers the minimal equilibrium payoff of the liquidity supplier, who is up against her individual rationality constraint (and gets 0) despite having all the bargaining power.[12] In contrast to Kartik and Zhong (2023), in order to attain this outcome, the issuer needs to sacrifice efficiency – he retains asset cash flows in bad states of the world when $X < L^*$.

The retention that is optimal with information design is the opposite of the optimal retention with exogenous private information. As we discussed above, in the latter case, debt is generally optimal, which makes the issuer retain cash flows in high states when X exceeds the debt face value. With optimal information design, retention of low cash flows increases the information sensitivity of the security, which according to Theorem 1 tends to increase the issuer's freedom in information design. In fact, holding the average payoff fixed, live-or-die securities are the most informationally sensitive monotone securities as illustrated in Figure 4. This implies that there is a live-or-die security that is optimal among all monotone securities.

Theorem 3 shows a stronger result that retention of low cash flows (up to L^*) *strictly dominates* selling the asset (which is a special case of a live-or-die security with $L=\underline{x}$). This result is subtle, because while the retention at the bottom increases the issuer's flexibility in information design, a reduction in the average payoff of the security reduces the gains from trade and may restrict information design. The following lemma establishes that the former effect dominates whenever $\mu^X-\delta u^X>0$, i.e., the liquidity supplier's profit is positive when trading the whole asset. In fact, this result holds more generally for any security (which we use in proving Theorem 5 in Section 5).

Lemma 3. *Suppose H admits density on $\mathcal{X}$. For any $\varphi\in\Phi_1$ satisfying $\mu^{\varphi}-\delta u^{\varphi}>0$, there is a security $\tilde{\varphi}(X)=\varphi(X)\mathbf{1}\left\{X\geq\tilde{L}\right\}$ for some $\tilde{L}>\underline{x}$ such that $\boldsymbol{V}(\tilde{\varphi})>\boldsymbol{V}(\varphi)$.*

Lemma 1 follows from inspection of the constraint $\mathcal{L}(y|\varphi,u^{\varphi})\geq 0, y\in[0,u^{\varphi}]$, of the information design program explicitly stated in (5). Observe that $\mathcal{L}(y|\varphi,u^{\varphi})$ is

[12]In other words, combining the information and security design expands the Pareto frontier of attainable equilibrium payoffs compared to those attainable with only information design in Kartik and Zhong (2023).

affected by the right tail of H^{φ}. Since $\tilde{\varphi}(x) = \varphi(x)\mathbf{1}\left\{x \geq \tilde{L}\right\}$ coincides with $\varphi(x)$ above $\tilde{L}$, it is immediate that $\mathcal{L}(y|\tilde{\varphi}, u^{\varphi}) \geq 0$ for $y \geq \varphi\left(\tilde{L}\right)$. Since $\tilde{\varphi}(x) = 0$ for $x < \tilde{L}$, $H^{\tilde{\varphi}}(f) = H^{\varphi}\left(\varphi\left(\tilde{L}\right)\right)$ for $f < \varphi\left(\tilde{L}\right)$, and so,

$$\mathcal{L}(y|\tilde{\varphi}, u) = \varphi(\overline{x}) - \delta u - (1-\delta)y^{\frac{1}{1-\delta}}u^{-\frac{\delta}{1-\delta}} - \int_{\varphi(\tilde{L})}^{\varphi(\overline{x})} H^{\varphi}(f)\,\mathrm{d}f - H\left(\tilde{L}\right)\left(\varphi\left(\tilde{L}\right) - y\right)$$

is a strictly concave function on $\left[0, \varphi\left(\tilde{L}\right)\right]$. Thus, $\mathcal{L}(y|\tilde{\varphi}, u^{\varphi})$ attains its minimum at $y = \varphi\left(\tilde{L}\right)$ (which we know is non-negative by the argument above for $y \geq \varphi\left(\tilde{L}\right)$) or at $y = 0$. In turn, $\mathcal{L}(0|\tilde{\varphi}, u^{\varphi}) \geq 0$ is equivalent to $\mu^{\tilde{\varphi}} - \delta u^{\varphi} \geq 0$, which indeed holds for some $\tilde{L} > \underline{x}$ by $\mu^{\varphi} - \delta u^{\varphi} > 0$.[13]

Intuitively, Lemma 3 shows that for any security φ leaving a strictly positive payoff to the liquidity supplier, the issuer can always retain cash flows at the bottom and preserve the optimal price δu^{φ} offered by the liquidity supplier. This is profitable for the issuer who receives the same price but retains a larger fraction of the security's cash flows. At the optimum, the issuer retains as much cash flows at the bottom as possible while respecting the liquidity supplier's individual rationality constraint.

The existence of density of H is important in Theorem 3, as it ensures continuity of μ^{φ} in L for live-or-die securities $\varphi(X) = X\mathbf{1}\{X \geq L\}$, which in turn guarantees that focusing on them is without loss of optimality. When this assumption fails, live-or-die securities need not be optimal. For instance, selling the asset is strictly optimal in the example in Section 3. However, the intuition that retaining cash flows in low states might be optimal is robust even for discrete distributions. To see this, consider the model with two cash flow realizations: $X = \overline{x}$ with probability $\gamma \in (0,1)$ and $X = \underline{x}$ with probability $1-\gamma$. Let u^X be the solution to (4) for $\varphi(X) = X$ and $l^X = \left(\frac{\mu^X/u^X - \delta}{1-\delta}\right)^{1-\delta} u^X$. The following proposition (proven in the Online Appendix) provides a sufficient condition for the optimality of cash flow retention in state $\underline{x}$.[14]

Proposition 2. *In the model with two cash flow realizations, if $l^X > \underline{x}$, then there is a uniquely optimal φ within Φ_1 such that $\varphi(\overline{x}) = \overline{x}$ and $\varphi(\underline{x}) < \underline{x}$.*

[13] Here is where we use that H admits density on $\mathcal{X}$ to ensure continuity of $\mu^{\tilde{\varphi}}$ in $\tilde{L}$.

[14] This condition is necessary but not sufficient. For example, for parameter values $\delta = 3/4$, $\underline{x} = 1$, $\overline{x} = 3/2$, and $\gamma = 0.3$, the uniquely optimal security is given by $\varphi(\underline{x}) \approx 0.85 < \underline{x}$ and $\varphi(\overline{x}) = \overline{x}$, which retains cash flows in the low state, yet, $l^X = \underline{x}$.

5 External Liquidity Requirements

Theorem 2 offers a new perspective on the security design literature. With sufficient flexibility in information design, the issuer does not need to resort to security design at all (at least within the commonly used class of double monotone securities). Thus, a theoretical justification of a particular security should begin with a question: which restrictions on the information design make the security design relevant in the first place. In this section, we take this approach to provide a new microfoundation for the prevalence of debt in practice.

We impose the following restrictions that we call *external liquidity requirements* (or simply, *liquidity requirements*) on the securities that the issuer can offer: (i) the whole security is always sold at $t = 1$; (ii) for a fixed ρ, the security price satisfies $p \geq \rho\varphi(\overline{x})$. Since $\rho > \delta$ is not sustainable,[15] we suppose $\rho \in [0, \delta]$.

Liquidity requirements of this form are often encountered in practice. For example, they naturally arise in the design of mortgage-backed securities (MBSs) or collateralized loan obligations (CLOs). To better fit this application, let us modify the model and make the issuer and the seller of the security separate entities. At $t = 0$, the issuer designs the security and the information that will be privately revealed to the security holder at $t = 1$. He then sells the security to one of many competitive institutional investors that we call for concreteness banks (alternatively, they can be pension funds, insurance companies, or other institutional investors subject to strict regulation on the liquidity of their assets). At $t = 1$, the bank who bought the security observes signal Z about its value and, if hit by a liquidity shock, sells it to the liquidity supplier. Since banks are competitive and there is no information asymmetry at $t = 0$, the issuer extracts all information rents, $\boldsymbol{V}(\varphi)$, from the bank buying the security.

In the context of MBSs and CLOs, the issuer is the underwriter who securitizes mortgages/loans after the origination and sells these securities to competitive institutional investors. Investors in MBSs and CLOs receive proprietary information about the asset pool and its performance from the asset-pool manager and the underwriter.[16] The underwriter specifies what information is contained in these private

[15]Indeed, both $\delta\varphi(\overline{x})$ and $p > \delta\varphi(\overline{x})$ are always accepted, and so, the liquidity supplier strictly prefers $\delta\varphi(\overline{x})$ to p

[16]Under Regulation AB, the SEC imposes disclosure requirements for asset-backed securities offerings (e.g., see https://www.sec.gov/corpfin/divisionscorpfinguidanceregulation-ab-interpshtm).

disclosures when he designs securities, which justifies flexibility of information design at the ex-ante stage. Due to regulatory requirements, institutional investors might have a strong preference for securities satisfying liquidity requirements of the form specified above. For example, Basel III qualifies securities as "high-quality liquid" if they can be liquidated within a short period of time with no significant loss of value. Say, banks should be able to liquidate level-2 assets over a 30-day period with a maximal decline in price of 10%. In the context of our model, this translates into the ability to always sell the security (irrespective of the realization of Z) and $\rho = 90\%$.[17]

Similarly, external liquidity requirements on securities sold by the corporation can also arise due to shareholders' oversight. The corporation's shareholders (or board members representing them) can be concerned that insiders sell securities at a large discount. If they believe that the security price is much lower than the true value, say below $\rho\varphi(x)$, they might block the sale. If shareholders do not have the insiders' private information, they can impose the floor on the price $\rho\varphi(\overline{x})$, which guarantees that the security is never sold below a fraction ρ of its true value.

Optimality of Debt. Let us solve the security design problem under the liquidity requirements. We first show that the liquidity requirements impose non-trivial joint restrictions on the joint security and information design.

Lemma 4. *Security φ satisfies liquidity requirements if and only if $u^{\varphi} \geq (\rho/\delta)\varphi(\overline{x})$.*

Thus, the security design program under liquidity requirements becomes

$$\max_{\varphi \in \Phi} \mathbf{V}(\varphi) \text{ s.t. } u^{\varphi} \geq (\rho/\delta)\varphi(\overline{x}). \tag{7}$$

Theorem 4. *A debt security $\hat{\varphi}(X) \equiv \min\{X, D^*\}$ for some D^* solves the program (7) for $\Phi = \Phi_2$. If in addition $\rho < \delta$, then $\hat{\varphi}$ is uniquely optimal.*

Many securities, such as MBSs and CLOs, are structured as debt securities. As discussed in the Introduction, the classical theory posits that debt is the optimal way to share the liquidity risk under exogenous information. An alternative viewpoint is that debt arises from the "regulatory arbitrage:" regulators view debt as adequately

[17]Requiring $p \geq \rho\varphi(\overline{x})$ is an informationally robust way to ensure compliance. In particular, this guarantees that the maximal haircut on the true value of the security is at most $1 - \rho$ without the regulator knowing the bank's private information about the security or having to trust bank's reporting.

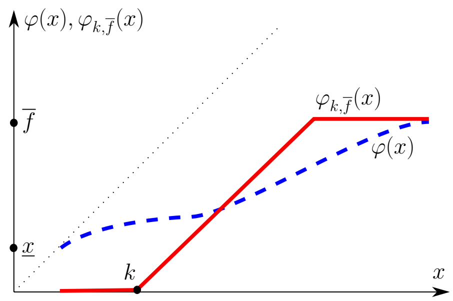


Figure 5: ***Illustration for Theorem 4***

liquid, hence, institutional investors demand it. These two explanations are often presented as contradictory to each other. Theorem 4 reconciles these view points. Similarly to the classical theory, debt is optimal but only under additional constaints imposed by external liquidity requirements. These requirements arise from regulatory (or shareholder) oversight over the securities holders and are similar in nature to the prudential regulation of banks, pension funds, and insurance companies. Importantly and similarly to how they are formulated in practice, our liquidity requirements do not restrict the class of securities, but rather require that adequately liquid securities are sold in a short time without a significant loss of value. That the optimal security is debt comes as a result, not an assumption. In the context of the application to MBS described above, institutional investors who are subject to regulation demand assets that qualify as sufficiently liquid by regulatory standards. The underwriter issues debt because it is the optimal security among those that comply with these regulatory standards.

At the same time, Theorem 4 formalizes the regulatory-arbitrage viewpoint: debt allows financial institutions to optimally address their liquidity needs while complying with regulatory requirements. This result aligns with the fact that MBSs and CLOs are often marketed and held by heavily regulated entities such as banks, insurance companies, and pension funds. In contrast and in line with Theorem 2, less regulated financial institutions, like investment funds, prefer to sell assets to generate liquidity.

The intuition for the optimality of debt under liquidity requirements goes as follows. By Proposition 1, optimal signals for security φ restrict the issuer not to learn too positive information about the security value. In particular, the highest signal u^{φ} for an optimal signal distribution is generally below the highest security payoff $\varphi(\overline{x})$. By Lemma 4, the liquidity requirements act in the opposite direction and "force" the issuer to learn granular information about high values of the security. Intuitively, they put a floor on the security price, and to make this price optimal for the liquidity supplier, it is necessary that the issuer learns information about sufficiently high values of the security. Because of this tension between the optimal information design and the liquidity requirements, certain securities might be disqualified, particularly selling the asset might not be possible. Theorem 4 establishes that, when facing such restrictions on the information design, the issuer finds it optimal to take advantage of the informational insensitivity of debt.

Remark 3. *Biais and Mariotti (2005) show optimality of debt, when the issuer learns X perfectly at $t = 1$. In this case, since the signal is perfect, the issuer is forced to learn high values of the security (in particular, $\varphi(\overline{x})$) whenever they occur. Similarly, in the example in Section 3, the signal technology S^{II}, which makes debt optimal, perfectly reveals high cash flows $\overline{x}$ with positive probability. This suggests a general insight that informational insensitivity of debt is valuable when the private information is more granular about high cash flow realizations.*

More formally, the proof sketch proceeds as follows. Observe that if $\varphi(X)$ satisfies the liquidity requirements so does $\varphi(X) + \Delta, \Delta > 0$. Hence, by Lemma 2, it is without loss of optimality in (7) to focus on securities satisfying $\varphi(\underline{x}) = \underline{x}$. Fix any such φ satisfying the liquidity requirements. Consider security $\varphi_{k,\overline{f}}(X) = \min\left\{\overline{f}, \max\{0, X - k\}\right\}$, where $\overline{f} = \varphi(\overline{x})$ and k is such that $\mu^{\varphi_{k,\bar{f}}} = \mu^{\varphi}$.[18] Security $\varphi_{k,\overline{f}}$ modifies the call option with strike price k by capping its payout at $\overline{f}$ (see Figure 5). Since security $\varphi_{k,\bar{f}}$ is more informationally sensitive than φ, $\boldsymbol{V}\left(\varphi_{k,\bar{f}}\right) \geq \boldsymbol{V}(\varphi)$ (by Theorem 1), and so, $u^{\varphi_{k,\bar{f}}} \geq u^{\varphi}$. At the same time, the cap $\overline{f}$ on the payout ensures that $\varphi_{k,\bar{f}}$ also satisfies the liquidity requirements. Hence, it is without loss of optimality in (7) to restrict attention to $\varphi_{k,\bar{f}}$ and vary parameters k and $\overline{f}$. Observing that $\varphi_{0,\bar{f}}$ is a debt security and also the only such security with $\varphi(\underline{x}) = \underline{x}$, we get that a debt security is optimal.

Note that debt with face value $\overline{x}$ is equivalent to selling the asset, and so, there is no contradiction between Theorems 2 and 4. Debt is optimal when the liquidity requirements are sufficiently stringent (ρ is high), and selling the asset is optimal when they do not bind. As an illustration, Figure 6 depicts the optimal security for different ρ's in the uniform example. For high ρ's, the constraint $u^{\varphi} \geq (\rho/\delta)\varphi(\overline{x})$ is binding and the optimal security is debt with face value D_{ρ} that is weakly decreasing

[18] Such a k exists by continuity of $\mu^{\varphi_{k,\bar{f}}}$ in k and the intermediate value theorem.

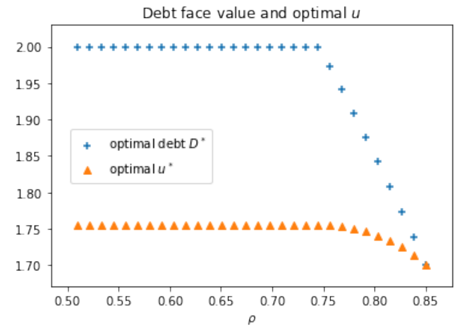


Figure 6: ***Optimal debt face value*** D^* ***and corresponding*** u^* ***as functions of*** ρ
Note: *The distribution of cash flows is* $H(x) = x - 1, x \in [1, 2]$ *and* $\delta = 0.85$.

in ρ. For low ρ's, the constraint is not binding, and it is optimal to sell the asset (that is, $D_\rho = \overline{x}$).

Remark 4. *Optimal debt in Theorem 4 is generally larger than the optimal debt in Biais and Mariotti (2005)'s model where the issuer learns* X *at* $t = 1$. *Indeed, Biais and Mariotti (2005) show that the optimal debt* D^{BM} *in their model is also perfectly liquid, that is, it is traded at price* δD^{BM}. *Hence, debt* D^{BM} *satisfies the liquidity requirements in* (7), *and so,* $D^* \geq D^{BM}$ and $V^* \geq V^{BM}$. *Thus, despite the restrictions imposed by the liquidity requirements, the issuer still gains from the possibility of choosing the signal distribution optimally, which is generally more complex than simply learning cash flows.*

Additionally, our predictions about investors' private information about debt securities differ from Biais and Mariotti (2005). In their paper, the issuer perfectly learn cash flows. That is, he believes that debt is risk-free in most scenarios (when $X \geq D^{BM}$*). In contrast, optimal signals described in Proposition 1 reveal to the issuer an expected debt value consistently lower than its face value, resulting in a generally positive credit spread recorded by investors. This prediction aligns with the industry's standard practice of marking securities to market value, rather than valuing them at face value on the balance sheet.*

Optimality of AT1 Debt. We next relax the double-monotonicity assumption. Let us call securities of the form $\varphi(x) = \mathbf{1}\{X \geq L\} \min\{X, D\}$ *AT1 (additional tier-1) debt securities.* AT1 debt is a fairly new type of security that has become popular in recent years among European banks. In normal times, it is a debt security that promises a fixed payment. Yet, unlike debt, in distress times, it is junior to other debt but also to equity under certain circumstances (in particular, when the bank is taken over by regulators due to inadequate capital or liquidity). Security of the form $\varphi(x) = \mathbf{1}\{X \geq L\} \min\{X, D\}$ captures this by specifying a threshold L above which φ coincides with debt D and below L the security is junior to all other claims and pays 0.

Theorem 5. *Suppose H admits a density on $\mathcal{X}$. An AT1 debt security $\varphi_{D^*,L^*}(X) \equiv \min\{X, D^*\}\mathbf{1}\{X \geq L^*\}$ for some D^* and L^* solves the program (7) for $\Phi = \Phi_1$. Further, φ_{D^*,L^*} strictly dominates standard debt (i.e., $L^* > \underline{x}$) and gives a payoff of 0 to the liquidity supplier.*

AT1 debt was introduced as a quick way to deleverage banks that suffer losses. This is done through write-downs of the principal at regulators' discretion. Theorem 5 provides a complementary liquidity-based microfoundation for AT1 debt. It is optimal in environments where buyers of securities have demand for securities that satisfy external liquidity requirement and the issuer can offer any monotone security. The latter assumption is particularly realistic for banks issuing AT1 debt: due to regulators' scrutiny, banks are less likely to engage in cash flow destruction, and the sabotage argument justifying double-monotone securities is less applicable to them.

The proof of Theorem 5 combines the insights developed in Theorems 3 and 4. Live-or-die securities are optimal among monotone securities due to their high informational sensitivity, while a cap on the security payout allows the issuer to comply with the external liquidity requirements. Taken together, these insights result in optimality of AT1 debt, which is a combination of debt (at high cash flows) and live-or-die security (at low cash flows).

6 Imperfectly Competitive Liquidity Suppliers

In this section, we relax the assumption of a monopolistic liquidity supplier. Suppose there are two states of the world: high-liquidity state ω_H and low-liquidity state ω_L. We suppose that both security and information design can be conditioned on ω, that is, the issuer chooses at $t = 0$ two securities and two signal distributions, $(\varphi_H, G^{\varphi_H})$ and $(\varphi_L, G^{\varphi_L})$. In state ω_L, the liquidity supply is scarce and there is a single monopolistic liquidity supplier as in the baseline model. In state ω_H, there are competitive liquidity suppliers and the issuer chooses the security price to maximize his payoff subject to liquidity suppliers breaking even.[19] Formally, he offers price p in state ω_H solving

$$\max_{p} \int_{\varphi_H(\underline{x})}^{p/\delta} (p - \delta z) \, \mathrm{d}G^{\varphi_H}(z) \ \text{ s.t. } \ \pi\left(p|G^{\varphi_H}\right) \geq 0. \tag{8}$$

The analysis is unchanged in the low-liquidity state. In state ω_H, the maximal surplus from trading security φ^H is $(1-\delta)\,\mu^{\varphi_H}$. If the issuer chooses to be uninformed about the security value and offers price μ^{φ_H}, then the liquidity supplier gets payoff of 0, and so, it satisfies the constraint in (8). This allows the issuer to extract the whole surplus from trade of any security. Therefore, in state ω_H, it is optimal for the issuer to sell the asset and trade it at price μ^X, hence, fully extracting trade surplus.

Proposition 3. *It is optimal to sell the asset in both states ω_H and ω_L. It is optimal for the issuer to choose the signal distribution described in Proposition 1 in state ω_L and to receive no information in state ω_H. The security price is δu^X in state ω_L and μ^X in state ω_H.*

While the issuer sells the asset in both high- and low-liquidity states, the signal distribution differs across states. In the high-liquidity state, the issuer chooses to be ignorant and has no informational advantage over liquidity suppliers. As a result, the liquidity suppliers bid the price up to the ex-ante value of the asset, μ^X, and the issuer captures the whole trade surplus. In the low-liquidity state, the issuer chooses a non-trivial signal distribution, which allows him to capture part of the gains from trade as information rents even though the liquidity supplier has monopolistic power. This result is in contrast to Biais and Mariotti (2005) showing that, when the issuer learns X at $t = 1$, debt is optimal in both competitive and monopolistic setting and the face value of debt is sensitive to the degree of competition.

Proposition 3 establish a connection between the competitive environment and the presence of private information. When liquidity is abundant and liquidity suppliers are competitive, issuers have no incentive to possess private information, as it would hinder liquidity without benefiting them. However, in periods of scarce liquidity when liquidity suppliers hold significant market power, issuers acquire private information

[19]We can also allow cash flow distribution $H(\cdot|\omega)$ to vary across states, which would not change the result.

about the downside of securities, which allows them to capture some information rents while preserving the liquidity of their securities. In essence, it is optimal to remain "ignorant" about asset quality during booms but gain sufficient private information during downturns to maintain market liquidity. As a result, our theory predicts a counter-cyclical pattern of private information among financial institutions.

This prediction aligns with previous theoretical studies that highlight a negative relationship between economic activity and the extent of asymmetric information (e.g., Gorton and Ordonez 2014, Fishman and Parker 2015). Nonetheless, our findings diverge by attributing the correlation to shifts in investors' bargaining power prompted by fluctuations in economic activity, rather than external shocks impacting asset quality.

7 Discussion

In this paper, we address a normative question: how the issuer can best jointly design his private information about cash flows and securities to raise liquidity in crisis times? In this section, we discuss positive implications of our theory.

Our theory predicts two most common ways of raising liquidity in practice – selling assets (as an unconstrained optimum) and debt (as a constrained optimum under the external liquidity requirements).[20] Our microfoundation for these securities differs from that in the classical literature which suggests that retaining assets' cash flows serves as a credible signal of quality in markets with significant information asymmetry. In models with exogenous information, debt is considered the optimal security, and selling the entire asset occurs only in cases when information asymmetry is relatively mild.

However, with optimal information design, retention is generally unnecessary, and selling the entire asset is strictly optimal. As a result, our novel empirical prediction is that, even in environments where information asymmetry is a major concern, investors can raise liquidity by selling assets rather than issuing more complex securities. This requires the seller to commit to having noisy private information about high security valuations and more detailed information about low valuations. In turn,

[20]Here, we discuss positive implications of results under the double-monotonicity assumption. As we discussed above, this assumption is common in the security design literature, captures in reduced form unmodelled agency frictions, and is fairly weak as it encompasses most securities used in practice (and many more).

optimal information design can conflict with external liquidity requirements imposed by regulators or shareholders. In environments with high degree of regulatory or shareholder oversight, we predict that institutions will use debt securities instead of asset sales to raise liquidity. This result explains why MBSs or CLOs that are held by heavily regulated institutional investors are structured as debt securities.

As we discussed in the Introduction, the commitment to an optimal information design can take many forms in practice. For financial securities, the issuer can modulate his informational advantage by adding clauses into the contract describing the security that are more sensitive to his future private information (e.g., the insider's superior knowledge about the future financial/corporate policy of the company). The structure of the underlying asset, such as the pool of mortgages or business loans, can also affect the extent of asymmetric information, and in this way, is a tool of information design. We next discuss several other examples of commitment to the information design. Consistent with our theory, asset sales are common in these examples despite a potentially high degree of information asymmetry.

Commitment through Organization Structure One way to commit to an information design is through the organization structure. Let us give two examples.

First, multidivisional firms generally consist of core and periphery divisions. Under this organization structure, periphery divisions receive a great deal of autonomy in both daily operations and short and medium-term strategic planning. The firm's general management maintains a hands-off approach and only launches thorough investigations when a crisis occurs. This organization design commits the management not to learn granular information about periphery divisions and be more aware of negative news.

Consistent with our theory, despite the potentially high degree of asymmetric information vis-a-vis outsiders, liquidity-constrained multidivisional firms often divest entire divisions to raise funds (Lang, Poulsen and Stulz 1995, Officer 2007) rather than issue securities backed by division cash flows. Further, Kaplan and Weisbach (1992) and Maksimovic and Phillips (2001) find that parent units usually divest periphery, non-core divisions. A key insight of our analysis is that by not monitoring too closely periphery divisions, firms can maintain the liquidity of these assets. Consistent with this prediction, Schlingemann et al. (2002) find that multidivisional firms divest their divisions in highly-liquid markets, and that, perhaps surprisingly, firms are less likely to divest their worst-performing units but rather tend to divest their most liquid divisions.[21]

Another example is private equity funds, where general partners (GPs) oversee investments and secure capital from limited partners (LPs). Despite LPs having access to internal performance reports, their ability to evaluate investment strategies and their involvement in decisions is limited, leading them to delegate decisions to GPs. Our theory suggests that the passive role of LPs enables them to raise liquidity by selling their stakes, whereas GPs face more constraints in this regard. This aligns with the existence of an active secondary-market for LP stakes, where buyers, often funds of funds, provide liquidity to selling LPs impacted by unexpected liquidity needs (Nadauld et al. 2019). Interestingly, there is a segment of collateralized fund obligations issuing highly-rated bonds backed by pools of stakes in private equity funds, but its size remains relatively small. This indicates that the secondary markets for LP stakes are adequately liquid.[22]

Limited Information Acquisition/Processing Resources. An optimal information design can be attained by committing resources to learning about certain aspects of underlying cash flows but not others. The qualitative properties of optimal information designs in Proposition 1 – specifically, the focus on downside risks rather than the upside potential – are in line with accounting principles and risk-management practices. Accounting standard-setters, such as GAAP, recommend the conservatism principle that is widely adopted by investment funds. According to the dictum, financial institutions should record losses as soon as they learn about them, whereas gains are not supposed to be recorded until they are realized (see Ruch and Taylor 2015). Further, standard risk management involves keeping track of market and credit risk exposures of the investment portfolio and the likelihood of potential

[21] Robot maker Boston Dynamics provides an interesting case study. It was bought by Google in 2013 that sold it to Softbank in 2017. In turn, Softbank sold it to Hyundai in 2021 partially in response to a liquidity shock caused by losses in its investment portfolio. Throughout the years, Boston Dynamics maintained a high degree of autonomy by keeping the headquarters in Boston and maintaining its own research team. Because of this autonomy (and in line with our theory), the head companies were able to easily raise liquidity by selling it. Importantly, despite the complex nature of the business, the sale did not involve designing complex securities backed by Boston Dynamics' cash flows. *Heater, Brian. 2021. "Hyundai completes deal for controlling interest in Boston Dynamics." Tech Crunch, June 21. https://techcrunch.com/2021/06/21/hyundai-completes-deal-for-controlling-interest-in-boston-dynamics/.*

[22] *Wiggins, Kaye. 2022. "Collateralised fund obligations: how private equity securitised itself." Financial Times, November 22. https://www.ft.com/content/e4c4fd61-341e-4f5b-9a46-796fc3bdcb03*

losses.

Corporations allocate substantial resources into compliance with accounting standards and a proper risk management to avert catastrophic outcomes or costly lawsuits. In a world where resources for information acquisition/processing are limited, this means fewer resources being directed toward refining the upside projections that are by their nature more challenging to precisely gauge. Thus, the combination of limited resources and the priority of risk management and sound accounting practices leads corporations to have more refined information about risks than the upside potential. Our theory suggests that this combination allows them to also better protect themselves from the liquidity risk by enhancing the liquidity of their assets.

As another illustration, mutual and hedge funds can face large redemptions, which can lead to the liquidation of less liquid assets like private equity or large blocks of public shares in decentralized markets. Although these funds usually hold liquid securities as a safeguard, severe shocks during crises can disrupt this buffer. In such times, buyers in decentralized markets wield considerable market power due to limited liquidity and heightened demand. Except for activist funds with concentrated positions, fund managers oversee numerous firms and have limited knowledge and capabilities to provide effective governance for each company in its portfolio. Consequently, the majority of investment funds tend to be passive, prioritizing liquidity needs in the face of shocks. Consistent with our theory, these funds do not issue securities backed by their holdings, opting instead to raise liquidity through portfolio liquidation.

Reputation Mechanisms. Commitment to an optimal information design can be done through reputation. For example, VCs specialize in early-stage financing of startups with a typical finite life-span of12 years after which the fund must return money to investors. Due to the growth of private equity markets and a recent cooling down of the IPO market, VC-backed startups often prefer to stay private for longer time. This shift makes conventional exit strategies of IPOs or mergers and acquisitions more challenging, leading VCs to liquidate their stakes in startups in an illiquid market for early-stage private equity (Nigro and Stahl 2021, Bian et al. 2022). Given the significant information asymmetry between VCs and external investors, it is somewhat surprising, according to classical theory, that VCs simply sell their entire stakes without developing more intricate securities structures.

Nevertheless, this aligns with our theory that stresses the role of information design. VCs often restrict themselves either contractually or through reputational mechanisms to take a hands-off approach, wherein they provide financial and operational support to the startup but refrain from interference unless the startup fails to meet predetermined milestones. This approach allows VCs to gain more detailed information when the firm performs poorly, prompting them to investigate the underlying causes. Conversely, as long as the startup remains on track, VCs have limited insight into its potential and day-to-day progress, ensuring that they do not set unrealistically high valuation expectations. This hands-off approach, which contrasts with the governance approach involving intensive monitoring of startups, has become prominent in recent years, with many leading VCs maintaining a founder-friendly reputation (Ewens et al. 2018, Lerner and Nanda 2020).

Further, our prediction that cash flow retention can play a limited role squares with recent evidence on the market for syndicated loans. Blickle et al. (2020) report that lead arrangers for syndicated loans, who are arguably the most informed investors in loans due to their prominent role in the underwriting process, often sell their entire loan stake to other investors, e.g., collateralized loan obligations, loan mutual funds, insurance companies, pension funds. They also show that reputational concerns seem to be important: lead arrangers of loans that turned sour tend to subsequently lose the market share. While this evidence contradicts the standard theory that highlights retention by the underwriter as a credible signaling device (e.g., Leland and Pyle 1977), it is consistent with our model. Maintaining reputation for focusing on the downside risk in their due diligence rather than the upside potential enables lead arrangers to offload completely their loan stake to institutional investors.

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Appendix: Omitted Proofs

Lemma 5. $G^{\varphi} \in \mathcal{G}^{\varphi}$ *if and only if* $\mathbb{E}_{G^{\varphi}}[Z] = \mathbb{E}_{H^{\varphi}}[F]$, *and* $\int_{-\infty}^{y} H^{\varphi}(f)\,\mathrm{d}f \geq \int_{-\infty}^{y} G^{\varphi}(z)\,\mathrm{d}z$ *for all* y.

Proof of Lemma 5. Consider a signal Z about $F = \varphi(X)$ described by the probability space $(\mathcal{F} \times \mathcal{Z}, \mathscr{F} \times \mathscr{Z}, \nu_{F,Z})$. Here, $\mathcal{F}$ is the set of possible payoffs of security F (i.e., $\mathcal{F} \equiv \varphi(\mathcal{X})$), $\mathcal{Z}$ is a sufficiently rich Polish space of possible signal realizations (in particular, $\mathcal{F} \subseteq \mathcal{Z}$), and (Z, F) is distributed according to the probability measure $\nu_{F,Z}(f, z)$ on the product of Borel σ-algebras, $\mathscr{F} \times \mathscr{Z}$, such that the marginal distribution of $\nu_{F,Z}$ on $\mathcal{F}$ coincides with the prior distribution of F, H^{φ}. Let $\mathcal{J}^{\varphi}$ be the set of all possible distributions of unbiased signals Z about F (for which $Z = \mathbb{E}[\varphi(X)|Z]$ almost surely). By the Strassen theorem (see Theorem 7.A.1 in Shaked and Shanthikumar 2007), $G^{\varphi} \in \mathcal{J}^{\varphi}$ if and only if $\mathbb{E}_{G^{\varphi}}[Z] = \mathbb{E}_{H^{\varphi}}[F] \equiv \mu^{\varphi}$, and G^{φ} second-order stochastically dominates H^{φ}:$\int_{-\infty}^{y} H^{\varphi}(f)\,\mathrm{d}f \geq \int_{-\infty}^{y} G^{\varphi}(z)\,\mathrm{d}z$ for all y. To prove the lemma, we will show that $\mathcal{J}^{\varphi} = \mathcal{G}^{\varphi}$.

To show $\mathcal{G}^{\varphi} \subseteq \mathcal{J}^{\varphi}$, consider any $G^{\varphi} \in \mathcal{G}^{\varphi}$ and a corresponding signal S about X described by the probability space $(\mathcal{X} \times \mathcal{S}, \mathscr{X} \times \mathscr{S}, \nu_{X,S})$. Let $Z = \mathbb{E}[\varphi(X)|S]$. Then, (X, Z) is distributed according to the joint distribution $\nu_{X,Z}$ on the probability space $(\mathcal{X} \times \mathcal{Z}, \mathscr{X} \times \mathscr{Z}, \nu_{X,Z})$ with $\mathcal{Z} \equiv \mathbb{E}[X|\mathcal{S}]$ and $\nu_{X,Z}(X \leq x, Z \leq z) \equiv \nu_{X,S}(X \leq x, \mathbb{E}[\varphi(X)|S] \leq z)$ for all x and z. By the law of iterated expectations, $\mathbb{E}[\varphi(X)|Z] = \mathbb{E}[\mathbb{E}[\varphi(X)|S]|Z] = Z$ almost surely. Hence, Z is an unbiased signal about F described by the probability space $(\mathcal{F} \times \mathcal{Z}, \mathscr{F} \times \mathscr{Z}, \nu_{F,Z})$, where $\nu_{F,Z}(B_F, B_Z) \equiv \nu_{X,Z}\left(\varphi^{-1}(B_F), B_Z\right)$ for any $(B_F, B_Z) \in \mathscr{F} \times \mathscr{Z}$. Thus, $G^{\varphi} \in \mathcal{J}^{\varphi}$.

To show $\mathcal{J}^{\varphi} \subseteq \mathcal{G}^{\varphi}$, consider any $G^{\varphi} \in \mathcal{J}^{\varphi}$ and a corresponding signal Z about F described by the probability space $(\mathcal{F} \times \mathcal{Z}, \mathscr{F} \times \mathscr{Z}, \nu_{F,Z})$ such that the marginal distribution of $\nu_{F,Z}$ on $\mathcal{Z}$ coincides with G^{φ}. Let $\mathcal{S} = \mathcal{Z}$ and $\mathscr{S} = \mathscr{Z}$. For any x such that $[\underline{x}, x] \in \varphi^{-1}(\mathscr{F})$ and any $B_Z \in \mathscr{Z}$, define $\nu_{X,Z}([\underline{x}, x], B_Z) \equiv \nu_{F,Z}(\varphi([\underline{x}, x]), B_Z)$. Next, consider any x such that $[\underline{x}, x] \notin \varphi^{-1}(\mathscr{F})$, which is the case when φ is flat in some neighborhood of x. Let $[\check{x}, \hat{x}]$ be the largest interval on which φ is constant and equals $\varphi(x)$. Then, $[\underline{x}, \check{x}] \in \varphi^{-1}(\mathscr{F})$ and $[\check{x}, \hat{x}] \in \varphi^{-1}(\mathscr{F})$. For any $B_Z \in \mathscr{Z}$, define $\nu_{X,Z}([\underline{x}, x], B_Z) \equiv \nu_{F,Z}(\varphi([\underline{x}, \check{x}]), B_Z) + \nu_{F,Z}(\varphi([\check{x}, \hat{x}]), B_Z)\,\mathbb{P}_H(X \in [\underline{x}, x] | X \in [\check{x}, \hat{x}])$. We specified a signal Z about X described by $(\mathcal{X} \times \mathcal{Z}, \mathscr{X} \times \mathscr{Z}, \nu_{X,Z})$. Thus, $\mathcal{J}^{\varphi} \in G^{\varphi}$, which completes the proof of $\mathcal{J}^{\varphi} = G^{\varphi}$. $\square$

Proof of Proposition 1. By Proposition 2 in Kartik and Zhong (2023), $G_{u^{\varphi},\mu^{\varphi}}$ minimizes the liquidity supplier's profit over all $G^{\varphi} \in \mathcal{G}^{\varphi}$, which equals $\Pi(G^{\varphi}) = \mu^{\varphi} - \delta u^{\varphi}$. By Theorem 2 in Kartik and Zhong (2023), for any $G^{\varphi} \in \mathcal{G}^{\varphi}$, $V(G^{\varphi}) \leq (1-\delta)\mu^{\varphi} - \Pi(G^{\varphi}) = \delta(u^{\varphi} - \mu^{\varphi})$. Since $G_{u^{\varphi},\mu^{\varphi}}$ attains this upper bound, $\boldsymbol{V}(\varphi) = \delta(u^{\varphi} - \mu^{\varphi})$. For any G^{φ}

satisfying (i) and (ii) in the statement of the proposition, $V(G^{\varphi}) = \boldsymbol{V}(\varphi)$, and so, G^{φ} is optimal for φ. Conversely, if G^{φ} is optimal, then $V(G^{\varphi}) = \delta(u^{\varphi} - \mu^{\varphi})$, and so, inequality $V(G^{\varphi}) \leq (1-\delta)\mu^{\varphi} - \Pi(G^{\varphi})$ cannot be strict, which implies that trade always happens under G^{φ}. This in turn implies that $u(G^{\varphi}) = u^{\varphi}$, otherwise, we get a contradiction to optimality of either G^{φ} or $G_{u^{\varphi},\mu^{\varphi}}$. □

Proof of Lemma 1. The constraint in (4) is $\int_{-\infty}^{y}(H^{\varphi}(z) - G_{u,\mu^{\varphi}}(z))\,\mathrm{d}z \geq 0$, which given equation (3) for $G_{u,\mu^{\varphi}}$ and $l = \left(\frac{\mu^{\varphi}/u-\delta}{1-\delta}\right)^{1-\delta} u$, is equivalent to $\mathcal{L}(y|\varphi,u) \geq 0, y \in [l,u]$, $u \leq \varphi(\overline{x})$, and $l \geq \varphi(\underline{x})$ (the latter is equivalent to $\mathcal{L}(\varphi(\underline{x})|\varphi,u) \geq 0$). Inequalities $\mathcal{L}(y|\varphi,u) \geq 0, y \in \{\varphi(\underline{x})\} \cup [l,u]$ imply that for $y \in [0,l)$,

$$\begin{aligned}\mathcal{L}(y|\varphi,u^{\varphi}) &= \mu^{\varphi} - \delta u^{\varphi} - (1-\delta)\,y^{1/(1-\delta)}\,(u^{\varphi})^{-\delta/(1-\delta)} + \int_{-\infty}^{y} H^{\varphi}(x)\,\mathrm{d}x \\ &> \mu^{\varphi} - \delta u^{\varphi} - (1-\delta)\,l^{1/(1-\delta)}\,(u^{\varphi})^{-\delta/(1-\delta)} + \int_{-\infty}^{y} H^{\varphi}(x)\,\mathrm{d}x = \int_{-\infty}^{y} H^{\varphi}(x)\,\mathrm{d}x \geq 0,\end{aligned}$$

where the first inequality is by $y < l$; the first equality is by integration by parts; the second equality is by $l = \left(\frac{\mu^{\varphi}/u^{\varphi}-\delta}{1-\delta}\right)^{1-\delta} u^{\varphi}$. Thus, the constraint in (4) is also equivalent to (5) and $u \leq \varphi(\overline{x})$, which is the desired conclusion. □

Proof of Theorem 1. 1) Since $\tilde{\varphi}$ is more informationally sensitive than φ, there is $x^{*} \in [\underline{x},\bar{x}]$ such that $\tilde{\varphi}(x) \leq \varphi(x)$ for $x < x^{*}$ and $\tilde{\varphi}(x) \geq \varphi(x)$ for $x > x^{*}$. Let $f^{*} \equiv \varphi(x^{*})$ and $f_{*} \equiv \lim_{x\uparrow x^{*}} \varphi(x)$. By monotonicity of securities, we can choose x^{*} such that $f^{*} = \varphi(x^{*}) \leq \tilde{\varphi}(x^{*})$. Since $\mu^{\tilde{\varphi}} = \mu^{\varphi}$ and $[\varphi(\underline{x}), \varphi(\overline{x})] \subseteq [\tilde{\varphi}(\underline{x}), \tilde{\varphi}(\overline{x})]$, $\int_{-\infty}^{y}\left(H^{\tilde{\varphi}}(f) - H^{\varphi}(f)\right)\mathrm{d}f = 0$ for $y \leq \tilde{\varphi}(\underline{x})$ and $y \geq \tilde{\varphi}(\overline{x})$. Since $H^{\varphi} = H \circ \varphi^{-1}$,

$$H^{\tilde{\varphi}}(f) = H(\sup\{x : \tilde{\varphi}(x) \leq f\}) \geq H(\sup\{x : \varphi(x) \leq f\}) = H^{\varphi}(f), f \in (\tilde{\varphi}(\underline{x}), f_{*}); \quad (9)$$

$$H^{\tilde{\varphi}}(f) = H(\sup\{x : \tilde{\varphi}(x) \leq f\}) = H(\sup\{x : \varphi(x) \leq f\}) = H^{\varphi}(f), f \in [f_{*}, f^{*}); \quad (10)$$

$$H^{\tilde{\varphi}}(f) = H(\sup\{x : \tilde{\varphi}(x) \leq f\}) \leq H(\sup\{x : \varphi(x) \leq f\}) = H^{\varphi}(f), f \in [f^{*}, \tilde{\varphi}(\overline{x})). \quad (11)$$

Hence, for $y \in (\tilde{\varphi}(\underline{x}), f^{*}]$, $\int_{-\infty}^{y}\left(H^{\tilde{\varphi}}(f) - H^{\varphi}(f)\right)\mathrm{d}f \geq 0$. For $y \in (f^{*}, \tilde{\varphi}(\overline{x}))$,

$$\begin{aligned}\int_{-\infty}^{y} H^{\tilde{\varphi}}(f)\,\mathrm{d}f &= \tilde{\varphi}(\overline{x}) - \underbrace{\mu^{\tilde{\varphi}}}_{=\mu^{\varphi}} - \int_{y}^{\tilde{\varphi}(\overline{x})} \underbrace{H^{\tilde{\varphi}}(f)}_{\leq H^{\varphi}(f)}\,\mathrm{d}f \\ &\geq \tilde{\varphi}(\overline{x}) - \mu^{\varphi} - \int_{y}^{\tilde{\varphi}(\overline{x})} H^{\varphi}(f)\,\mathrm{d}f \\ &= \varphi(\overline{x}) - \mu^{\varphi} - \int_{y}^{\varphi(\overline{x})} H^{\varphi}(f)\,\mathrm{d}f = \int_{-\infty}^{y} H^{\varphi}(f)\,\mathrm{d}f.\end{aligned} \quad (12)$$

Thus, $\int_{-\infty}^{y}\left(H^{\tilde{\varphi}}(f)-H^{\varphi}(f)\right)\mathrm{d}f\geq 0$ for all y, and so, $\mathcal{G}^{\varphi}\subseteq\mathcal{G}^{\tilde{\varphi}}$ and $\boldsymbol{V}(\tilde{\varphi})\geq\boldsymbol{V}(\varphi)$.

2) Suppose there is $\varepsilon>0$ such that $H^{\tilde{\varphi}}(f)>H^{\varphi}(f)$ for all $y\in(\tilde{\varphi}(\underline{x}),\tilde{\varphi}(\underline{x})+\varepsilon)$ and $H^{\tilde{\varphi}}(f)<H^{\varphi}(f)$ for all $y\in(\tilde{\varphi}(\overline{x})-\varepsilon,\tilde{\varphi}(\overline{x}))$. Hence, for $y\in(\tilde{\varphi}(\underline{x}),f^{*}]$,

$$\int_{-\infty}^{y}\left(H^{\tilde{\varphi}}(f)-H^{\varphi}(f)\right)\mathrm{d}f\geq\int_{\tilde{\varphi}(\underline{x})}^{\min\{y,\tilde{\varphi}(\underline{x})+\varepsilon\}}\left(H^{\tilde{\varphi}}(f)-H^{\varphi}(f)\right)\mathrm{d}f>0,$$

and for $y\in(f^{*},\tilde{\varphi}(\overline{x}))$,

$$\begin{aligned}\int_{-\infty}^{y}H^{\tilde{\varphi}}(f)\,\mathrm{d}f=&\tilde{\varphi}(\overline{x})-\underbrace{\mu^{\tilde{\varphi}}}_{=\mu^{\varphi}}-\int_{y}^{\tilde{\varphi}(\overline{x})}H^{\tilde{\varphi}}(f)\,\mathrm{d}f\\ \geq&\tilde{\varphi}(\overline{x})-\mu^{\varphi}-\int_{y}^{\max\{\tilde{\varphi}(\overline{x})-\varepsilon,y\}}\underbrace{H^{\tilde{\varphi}}(f)}_{\leq H^{\varphi}(f)}\mathrm{d}f-\int_{\max\{\tilde{\varphi}(\overline{x})-\varepsilon,y\}}^{\tilde{\varphi}(\overline{x})}\underbrace{H^{\tilde{\varphi}}(f)}_{<H^{\varphi}(f)}\mathrm{d}f\\ >&\tilde{\varphi}(\overline{x})-\mu^{\varphi}-\int_{y}^{\tilde{\varphi}(\overline{x})}H^{\varphi}(f)\,\mathrm{d}f=\int_{-\infty}^{y}H^{\varphi}(f)\,\mathrm{d}f,\end{aligned}$$

which proves (6). This together with $\mu^{\tilde{\varphi}}=\mu^{\varphi}$ and $G_{u^{\varphi},\mu^{\varphi}}\in\mathcal{G}^{\varphi}$ implies $\int_{-\infty}^{y}G_{u^{\varphi},\mu^{\tilde{\varphi}}}(z)\,\mathrm{d}z<\int_{-\infty}^{y}H^{\tilde{\varphi}}(f)\,\mathrm{d}f, y\in(\tilde{\varphi}(\underline{x}),\tilde{\varphi}(\overline{x}))$. If in addition $[l^{\varphi},u^{\varphi}]\subset(\tilde{\varphi}(\underline{x}),\tilde{\varphi}(\overline{x}))$, then there is $\varepsilon>0$ such that, for $u^{\varepsilon}=u^{\varphi}+\varepsilon$ and $l^{\varepsilon}=\left(\frac{\mu^{\tilde{\varphi}}/u^{\varepsilon}-\delta}{1-\delta}\right)^{1-\delta}u^{\varepsilon}$, $\int_{-\infty}^{y}G_{u^{\varepsilon},\mu^{\tilde{\varphi}}}(z)\,\mathrm{d}z<\int_{-\infty}^{y}H^{\tilde{\varphi}}(f)\,\mathrm{d}f, y\in(\tilde{\varphi}(\underline{x}),\tilde{\varphi}(\overline{x}))$ and $[l^{\varepsilon},u^{\varepsilon}]\subset(\tilde{\varphi}(\underline{x}),\tilde{\varphi}(\overline{x}))$. Thus, $G_{u^{\varepsilon},\mu^{\tilde{\varphi}}}\in\mathcal{G}^{\tilde{\varphi}}$, and by Proposition 1, $\boldsymbol{V}(\tilde{\varphi})\geq\delta\left(u^{\varepsilon}-\mu^{\tilde{\varphi}}\right)>\delta\left(u^{\varphi}-\mu^{\varphi}\right)=\boldsymbol{V}(\varphi)$. □

Proof of Lemma 2. By Proposition 1, there is an optimal signal distribution for security φ, G^{φ}, and under G^{φ}, the liquidity supplier always trades at price δu^{φ}. Let $G^{\tilde{\varphi}}(z)\equiv G^{\varphi}(z-\Delta)$ for all z. Since $G^{\varphi}\in\mathcal{G}^{\varphi}$, $G^{\tilde{\varphi}}\in\mathcal{G}^{\tilde{\varphi}}$. By Lemma 4 in Biais and Mariotti (2005), under $G^{\tilde{\varphi}}$, the liquidity supplier optimally chooses a screening cutoff type that is weakly greater than $u^{\varphi}+\Delta=u\left(G^{\tilde{\varphi}}\right)$. Hence, the liquidity supplier finds it optimal under $G^{\tilde{\varphi}}$ to offer $\delta u\left(G^{\tilde{\varphi}}\right)=\delta\left(u^{\varphi}+\Delta\right)$ and buy from all types. Thus, $\boldsymbol{V}(\tilde{\varphi})\geq V\left(G^{\tilde{\varphi}}\right)=\delta\left(u^{\varphi}+\Delta-\mu^{\tilde{\varphi}}\right)=\delta\left(u^{\varphi}-\mu^{\varphi}\right)=V\left(G^{\varphi}\right)=\boldsymbol{V}(\varphi)$. By Proposition 1, $\boldsymbol{V}(\tilde{\varphi})=\delta\left(u^{\tilde{\varphi}}-\mu^{\tilde{\varphi}}\right)$, and so, $u^{\tilde{\varphi}}\geq u^{\varphi}+\Delta$. □

Proof of Theorem 2. Suppose φ solves (2) for $\Phi=\Phi_2$. Suppose to contradiction that $\mu^{\varphi}<\mu^{X}$, where $\mu^{X}=\mathbb{E}_H[X]$. The fact that $\varphi\in\Phi_2$ implies that φ is Lipschitz with parameter $K=1$. By Lemma 2, it is without loss of optimality within the class Φ_2 to focus on securities that include safe debt $\underline{x}$: $\varphi(\underline{x})=\underline{x}$. For any such φ, there is a security $\varphi_k(x)=\underline{x}/2+\max\{0,X-k\}$ such that $\mu^{\varphi}=\mu^{\varphi_k}$ and $\varphi_k\in\Phi_2$ (see Figure 3). This follows from the continuity of $\mathbb{E}_H[\varphi_k(X)]$ in k (indeed, $\mathbb{E}_H[\varphi_k(X)-\varphi_{k+\varepsilon}(X)]\in[0,\varepsilon]$ for any $\varepsilon>0$) and $\varphi_{\overline{x}}(x)\leq\varphi(x)\leq\varphi_{\underline{x}/2}(x)$ for all $x\in\mathcal{X}$. Since $\mu^{\varphi}<\mu^{X}$, $k>\underline{x}/2$, and so,

$H^{\varphi_k}(f) > H^{\varphi}(f) = 0$ for $f \in (\underline{x}/2, \underline{x})$. This, in conjunction with $\mu^{\varphi} = \mu^{\varphi_k}$, implies that $\varphi_k(\overline{x}) > \varphi(\overline{x})$, and so, $H^{\varphi_k}(f) < H^{\varphi}(f) = 1$ for $f \in (\varphi(\overline{x}), \varphi_k(\overline{x}))$. By construction, φ_k it is more informationally sensitive than φ. Further, $[l^{\varphi}, u^{\varphi}] \subseteq [\varphi(\underline{x}), \varphi(\overline{x})] \subset (\varphi_k(\underline{x}), \varphi_k(\overline{x}))$. ByTheorem 1, $\boldsymbol{V}(\varphi_k) > \boldsymbol{V}(\varphi)$, which is a contradiction to optimality of φ. Therefore, $\mu^{\varphi} = \mu^X$. □

Proof of Lemma 3. Let L be the maximal $L \geq \underline{x}$ such that $\varphi(X)\mathbf{1}\{X \leq L\} = 0$ with probability 1. Since $\mu^{\varphi} > \delta u^{\varphi} \geq 0$, $L < \overline{x}$. By Lemma 1, for $y \in [0, u^{\varphi}]$, $\mathcal{L}(y|\varphi, u^{\varphi}) = \varphi(\overline{x}) - \delta u^{\varphi} - (1-\delta) y^{1/(1-\delta)} (u^{\varphi})^{-\delta/(1-\delta)} - \int_y^{\varphi(\overline{x})} H^{\varphi}(f)\,\mathrm{d}f \geq 0$. Consider $\tilde{\varphi}(X) = \varphi(X)\mathbf{1}\left\{X \geq \tilde{L}\right\}, \tilde{L} > L$. By $\tilde{L} > L$, $H^{\tilde{\varphi}}(f) = H^{\varphi}(f)$ for $f \geq \varphi\left(\tilde{L}\right)$, and so, $\mathcal{L}(y|\tilde{\varphi}, u^{\varphi}) = \mathcal{L}(y|\varphi, u^{\varphi}) \geq 0$ for $y \in [0, u^{\varphi}] \cap \left[\varphi\left(\tilde{L}\right), \infty\right)$. For $y \in \left[0, \varphi\left(\tilde{L}\right)\right]$, $\mathcal{L}_y(y|\tilde{\varphi}, u^{\varphi}) = -(y/u^{\varphi})^{\delta/(1-\delta)} + H\left(\tilde{L}\right)$ is strictly decreasing in y. Hence, $\mathcal{L}(y|\tilde{\varphi}, u^{\varphi})$ is strictly concave in y on $\left[0, \varphi\left(\tilde{L}\right)\right]$, and it attains its minima on $\left[0, \tilde{L}\right]$ at $y = 0$ or $y = \varphi\left(\tilde{L}\right)$. We showed that $\mathcal{L}\left(\varphi\left(\tilde{L}\right)\middle|\tilde{\varphi}, u^{\varphi}\right) = \mathcal{L}\left(\varphi\left(\tilde{L}\right)\middle|\varphi, u^{\varphi}\right) \geq 0$. Further, $\mathcal{L}(0|\tilde{\varphi}, u^{\varphi}) \geq 0$ is equivalent to $\mu^{\tilde{\varphi}} - \delta u^{\varphi} \geq 0$. Since H admits a density h on $\mathcal{X}$, $\mathbb{E}_H[\tilde{\varphi}(X)] = \int_{\tilde{L}}^{\varphi(\overline{x})} \varphi(x)\,h(x)\,\mathrm{d}x$ is continuous in $\tilde{L}$. This together with $\mu^{\varphi} - \delta u^{\varphi} > 0$ implies that for $\tilde{L}$ sufficiently close to L, $\mu^{\tilde{\varphi}} - \delta u^{\varphi} \geq 0$, and so, $\mathcal{L}(0|\tilde{\varphi}, u^{\varphi}) \geq 0$. Thus, for such $\tilde{L}$, $\mathcal{L}(y|\tilde{\varphi}, u^{\varphi}) \geq 0$ on $y \in [0, u^{\varphi}]$. By Lemma 1, $u^{\tilde{\varphi}} \geq u^{\varphi}$, and so, $\boldsymbol{V}(\tilde{\varphi}) \geq \delta\left(u^{\varphi} - \mu^{\tilde{\varphi}}\right) > \delta(u^{\varphi} - \mu^{\varphi}) = \boldsymbol{V}(\varphi)$, which is the desired conclusion. □

Proof of Theorem 3. Since H admits a density h on $\mathcal{X}$, $\mathbb{E}_H[X\mathbf{1}\{X \geq L\}] = \int_L^{\overline{x}} xh(x)\,\mathrm{d}x$ is continuous in L and ranges from 0 (for $L = \overline{x}$) to μ^X (for $L = \underline{x}$). Hence, for any $\mu \in \left[0, \mu^X\right]$, there is L such that $\mathbb{E}_H[\mathbf{1}\{X \geq L\}X] = \mu$. Further, given the same average payoff, a live-or-die security is more informationally sensitive than any other security in Φ_1. By Theorem 1, there is a live-or-die security that solves (2) for $\Phi = \Phi_1$.

Since H admits a density on $\mathcal{X}$, $\mathcal{L}_y\left(\underline{x}\middle|X, u^X\right) = -\left(\underline{x}/u^X\right)^{1/(1-\delta)} < 0$, and so, $\mathcal{L}\left(\underline{x}\middle|X, u^X\right) > 0$, which implies that $\mu^X - \delta u^X > 0$. By Lemma 3, $\varphi(X) = X(= X\mathbf{1}\{X \geq \underline{x}\})$ cannot be optimal, and so, in the optimum, $L^* > \underline{x}$. By Lemma 3, the optimal φ^* must satisfy $\mu^{\varphi^*} - \delta u^{\varphi^*} = 0$. □

Proof of Lemma 4. The "if" direction is trivial. To prove the "only if" statement, suppose to contradiction that $u^{\varphi} < (\rho/\delta)\varphi(\overline{x})$ but security φ satisfies the liquidity requirements, that is, it is always sold at price $p \geq \rho\varphi(\overline{x})$. The latter implies that the issuer's expected payoff equals $p - \delta\mu^{\varphi} \geq \rho\varphi(\overline{x}) - \delta\mu^{\varphi} > \delta(u^{\varphi} - \mu^{\varphi})$, which contradicts Proposition 1. □

Proof of Theorem 4. Consider securities $\varphi_{k,\overline{f}}(X) = \min\left\{\overline{f}, \max\{0, X-k\}\right\}$ described by two parameters k and $\overline{f}$. For any $k > 0$ and $\varepsilon > 0$, $0 \leq \mathbb{E}_H\left[\varphi_{k,\overline{f}}(X) - \varphi_{k+\varepsilon,\overline{f}}(X)\right] \leq \varepsilon$. Hence, $\mathbb{E}_H\left[\varphi_{k,\overline{f}}(X)\right]$ is continuous in k and takes all values between 0 (for $k = \overline{x}$) and

$\mathbb{E}_H\left[\min\left(X,\overline{f}\right)\right]$ (for $k=0$). Thus, for any $\varphi\in\Phi_2$, there is k such that $\mathbb{E}_H\left[\varphi_{k,\overline{f}}(X)\right]=\mu^{\varphi}$, where $\overline{f}=\varphi(\overline{x})$. Let us argue that $\varphi_{k,\overline{f}}$ dominates φ. By Lemma 2, it is without loss of generality to assume that $\varphi(\underline{x})=\underline{x}$.[23] Since $\varphi(\underline{x})=\underline{x}$ and $\varphi\in\Phi_2$, $\overline{x}-k>\varphi(\overline{x})$, and so, $\varphi_{k,\overline{f}}(x)=\overline{f}$ on $x\in\left[\overline{f}+k,\overline{x}\right]$. Since $\varphi_{k,\overline{f}}$ is more informationally sensitive than φ, $\mathcal{G}^{\varphi}\subseteq\mathcal{G}^{\varphi_{k,\overline{f}}}$ and $\boldsymbol{V}\left(\varphi_{k,\overline{f}}\right)\geq\boldsymbol{V}(\varphi)$ (by Theorem 1). Note that if $u^{\varphi}\geq(\rho/\delta)\overline{f}$, then (by Proposition 1 and $\varphi_{k,\overline{f}}(\overline{x})=\overline{f}$), $u^{\varphi_{k,\overline{f}}}\geq u^{\varphi}\geq(\rho/\delta)\varphi_{k,\overline{f}}(\overline{x})$. To summarize, for any φ satisfying the constraint in (7), there is $\varphi_{k,\bar{f}}$ that dominates φ and also satisfies it. By Lemma 2, $\tilde{\varphi}(X)=\varphi_{k,\overline{f}}(X)+\min\{k,\underline{x}\}$ weakly dominates $\varphi_{k,\overline{f}}$. By the argument in footnote 23, $\tilde{\varphi}$ satisfies the constraint in (7). If $k\leq\underline{x}$, then $\tilde{\varphi}$ is debt. Otherwise, $\mu^{\tilde{\varphi}}=\mu^{\varphi}+\underline{x}$. In this case, we repeatedly apply the same argument to construct $(\tilde{\varphi}_i)_{i=1}^{I}$ such that for all i, $\tilde{\varphi}_i$ weakly dominates $\tilde{\varphi}_{i-1}$ (where $\tilde{\varphi}_0=\tilde{\varphi}$) and $\tilde{\varphi}_i$ satisfies the constraint in (7). Since $\mu^{\tilde{\varphi}_i}=\mu^{\tilde{\varphi}_{i-1}}+\underline{x}$ and $\mu^{\tilde{\varphi}_i}\leq\mu^{X}$, I is finite, and so, $\tilde{\varphi}_I$ must be debt.

To prove the second part, suppose in addition $\rho<\delta$. Consider a non-debt security φ satisfying the constraint in (7) and such that $\varphi(\underline{x})=\underline{x}$, which is without loss of optimality by Lemma 2. We will prove that there is $\hat{\varphi}$ that strictly dominates φ and satisfies the constraint in (7). Thus, the only solution to (7) is a debt security. By the argument above, $\varphi_{k,\overline{f}}$ weakly dominates φ, where $\overline{f}=\varphi(\overline{x})$. Since φ is not debt, $k>0$. Fix $\Delta\in(0,\min\{k,\underline{x}\})$. By Lemma 2, $\tilde{\varphi}(X)=\varphi_{k,\overline{f}}(X)+\Delta$, weakly dominates $\varphi_{k,\overline{f}}$ and $u^{\tilde{\varphi}}\geq u^{\varphi}+\Delta$. By $\rho<\delta$,

$$u^{\tilde{\varphi}}\geq u^{\varphi}+\Delta\geq(\rho/\delta)\,\varphi(\overline{x})+\Delta=(\rho/\delta)\,\tilde{\varphi}(X)+\Delta\,(1-\rho/\delta)\,. \tag{13}$$

For any $\varepsilon\in(0,\Delta)$, consider security $\hat{\varphi}_{\varepsilon,K}(X)=\max\{\tilde{\varphi}(X)-\varepsilon,X-K\}$. Since $0\leq\mathbb{E}_H\left[\hat{\varphi}_{\varepsilon,K}(X)-\hat{\varphi}_{\varepsilon,K+\epsilon}(X)\right]\leq\epsilon$ for $\epsilon>0$, $\mathbb{E}_H\left[\hat{\varphi}_{\varepsilon,K}(X)\right]$ is continuous in K and it takes values between $\mu^{\tilde{\varphi}}-\varepsilon$ (for $K=\overline{x}$) and μ^{X} (for $K=0$). Hence, there is $K(\varepsilon)$ such that $\mathbb{E}_H\left[\hat{\varphi}_{\varepsilon,K(\varepsilon)}(X)\right]=\mu^{\tilde{\varphi}}$. Further, $\hat{\varphi}_{\varepsilon,K(\varepsilon)}$ converges point-wise to $\tilde{\varphi}$ as $\varepsilon\to0$. Hence, for sufficiently small $\varepsilon>0$, $\hat{\varphi}_{\varepsilon,K(\varepsilon)}(\overline{x})<\tilde{\varphi}(\overline{x})+\Delta\,(\delta/\rho-1)$, which combined with (13) implies $u^{\tilde{\varphi}}\geq(\rho/\delta)\,\tilde{\varphi}(X)+\Delta\,(1-\rho/\delta)\geq(\rho/\delta)\,\hat{\varphi}_{\varepsilon,K(\varepsilon)}(\overline{x})$. Choose one such ε and let $\hat{\varphi}=\hat{\varphi}_{\varepsilon,K(\varepsilon)}$.

By construction, $\hat{\varphi}$ is more informationally sensitive than $\tilde{\varphi}$, and $\left[l^{\tilde{\varphi}},u^{\tilde{\varphi}}\right]\subseteq[\tilde{\varphi}(\underline{x}),\tilde{\varphi}(\overline{x})]$. Further, $\tilde{\varphi}(x)=\overline{f}<x-K(\varepsilon)=\hat{\varphi}(x)$ for x in some left neighborhood of $\overline{x}$, and $\tilde{\varphi}(x)=\Delta>\Delta-\varepsilon=\hat{\varphi}(x)$ for x in some right neighborhood of $\underline{x}$. Hence, there is $\epsilon>0$ such that $H^{\hat{\varphi}}(f)<1=H^{\tilde{\varphi}}(f)$ for $f\in(\hat{\varphi}(\overline{x})-\epsilon,\hat{\varphi}(\overline{x}))$, $H^{\hat{\varphi}}(f)>0=H^{\tilde{\varphi}}(f)$ for $f\in(\hat{\varphi}(\underline{x}),\hat{\varphi}(\underline{x})+\epsilon)$, and $\left[l^{\tilde{\varphi}},u^{\tilde{\varphi}}\right]\subseteq[\tilde{\varphi}(\underline{x}),\tilde{\varphi}(\overline{x})]\subset(\hat{\varphi}(\underline{x}),\hat{\varphi}(\overline{x}))$. By Theorem 1, $\boldsymbol{V}(\hat{\varphi})>\boldsymbol{V}(\tilde{\varphi})$, which given the same mean payoffs means that $u^{\hat{\varphi}}>u^{\tilde{\varphi}}$. As we argued above, $u^{\tilde{\varphi}}\geq(\rho/\delta)\,\hat{\varphi}(\overline{x})$, and so, $\hat{\varphi}$ satisfies the constraint in (7) and strictly dominates $\tilde{\varphi}$, which is the desired conclusion. □

[23] Indeed, our argument applies to $\tilde{\varphi}(X)=\varphi(X)+\underline{x}-\varphi(\underline{x})$, which by Lemma 2 weakly dominates φ. Further, $u^{\varphi}\geq(\rho/\delta)\,\varphi(\overline{x})$ implies $u^{\tilde{\varphi}}\geq u^{\varphi}+\underline{x}-\varphi(\underline{x})\geq(\rho/\delta)\,\varphi(\overline{x})+\underline{x}-\varphi(\underline{x})\geq(\rho/\delta)\,\tilde{\varphi}(\overline{x})$.

Proof of Theorem 5. Consider AT1 debt $\varphi_{D,L}$ described by L and D. For a fixed $D \in [0, \overline{x}]$, since H admits a density h on $\mathcal{X}$, $\mathbb{E}_H[\varphi_{D,L}(X)] = \int_L^{\overline{x}} \min\{x, D\} h(x) \, \mathrm{d}x$ is continuous in L and ranges from 0 (for $L = \overline{x}$) to $\mu^D = \mathbb{E}_H[\min\{X, D\}]$ (for $L = \underline{x}$). Hence, for any $\mu \in [0, \mu^D]$, there is L such that $\mathbb{E}_H[\varphi_{D,L}(X)] = \mu$. Further, given the same average payoff, an AT1 debt is more informationally sensitive than any other security $\varphi \in \Phi_1$ satisfying $\varphi(\overline{x}) = D$. By Theorem 1, there is a live-or-die security that solves (7) for $\Phi = \Phi_1$. The proof of $L^* > \underline{x}$ and $\pi(\varphi_{D^*,L^*}) = 0$ follows from the same argument as in Theorem 3. □

Online Appendix (Not for Publication)

Auxiliary Results

In the paper, we use the following integration by parts formula for Lebesgue-Stieltjes integrals.

Lemma 6 (**Integration by parts**). *Suppose F is distributed according to the c.d.f. H with support $\left[\underline{f},\overline{f}\right]$. Then, for any $y \in \left[\underline{f},\overline{f}\right]$, $\int_{\underline{f}}^{y} f \mathrm{d}H(f) = yH(y) - \int_{\underline{f}}^{y} H(f)\,\mathrm{d}f$.*

Proof. Let $H^{-}(f)$ be the left-continuous regularization of H with the convention that $H^{-}\left(\underline{f}\right) = 0$. Then, $\int_{\underline{f}}^{y} f \mathrm{d}H(f) = yH(y) - \int_{\underline{f}}^{y} H^{-}(f)\,\mathrm{d}f = yH(y) - \int_{\underline{f}}^{y} H(f)\,\mathrm{d}f$, where the first equality is by Theorem VI.90 in Dellacherie and Meyer (1982), and the second equality is by the fact that a monotone function can have at most a countable number of discontinuities, and so, $H^{-}(f) = H(f)$ almost everywhere. □

Proof of Proposition 2. We first argue that for any $\varphi \in \Phi_1$ with $\varphi(\overline{x}) < \overline{x}$, there is $\tilde{\varphi} \in \Phi_1$ with $\tilde{\varphi}(\overline{x}) = \overline{x}$ such that $\boldsymbol{V}(\tilde{\varphi}) > \boldsymbol{V}(\varphi)$. By Lemma 2, if $\varphi(\underline{x}) < \underline{x}$, then there is $\varepsilon > 0$ such that $\varphi(X) + \varepsilon$ and $\boldsymbol{V}(\varphi+\varepsilon) \geq \boldsymbol{V}(\varphi)$. Hence, without loss of generality, suppose that $\varphi(\underline{x}) < \underline{x}$. Consider $\tilde{\varphi}$ such that $\tilde{\varphi}(\overline{x}) = \varphi(\overline{x}) + \overline{\varepsilon}$ and $\tilde{\varphi}(\underline{x}) = \varphi(\underline{x}) - \underline{\varepsilon}$ for some $\underline{\varepsilon}, \overline{\varepsilon} > 0$ such that $\tilde{\varphi} \in \Phi_1$ and $\gamma\overline{\varepsilon} = (1-\gamma)\underline{\varepsilon}$. By construction, $\mu^{\tilde{\varphi}} = \mu^{\varphi}$, $\tilde{\varphi}$ is more informationally sensitive than φ, $H^{\tilde{\varphi}}(f) = 1-\gamma > 0 = H^{\varphi}(f)$ for $y \in (\tilde{\varphi}(\underline{x}), \tilde{\varphi}(\underline{x}) + \underline{\varepsilon})$ and $H^{\tilde{\varphi}}(f) = 1-\gamma < 1 = H^{\varphi}(f)$ for all $y \in (\tilde{\varphi}(\overline{x}) - \overline{\varepsilon}, \tilde{\varphi}(\overline{x}))$, and $[l^{\varphi}, u^{\varphi}] \subseteq [\varphi(\underline{x}), \varphi(\overline{x})] \subset (\tilde{\varphi}(\underline{x}), \tilde{\varphi}(\overline{x}))$. By Theorem 1, $\boldsymbol{V}(\tilde{\varphi}) > \boldsymbol{V}(\varphi)$, and so, any optimal security satisfies $\varphi(\overline{x}) = \overline{x}$.

By Lemma 1 and X taking only two values, for any $\varphi \in \Phi_1$, u^{φ} is the largest $u \leq \varphi(\overline{x})$ satisfying

$$\mathcal{L}(y|\varphi, u) = \mu^{\varphi} - \delta u - (1-\delta) y^{1/(1-\delta)} u^{-\delta/(1-\delta)} + \max\{0, y - \varphi(\underline{x})\}(1-\gamma) \geq 0, y \in [0, u].$$

Note that it is sufficient that this inequality holds for $y \in [\varphi(\underline{x}), u]$. Since $\mathcal{L}_y(y|\varphi, u) = 1 - \gamma - (y/u)^{\delta/(1-\delta)}$, $\mathcal{L}(y|\varphi, u)$ is strictly concave in y, hence, it attains its minimum at $y = u$ or $y = \varphi(\underline{x})$. We have that $L(u|\varphi, u) \geq 0$ follows from $u \leq \varphi(\overline{x})$. Thus,

$$u^{\varphi} = \max\left\{u \in [\varphi(\underline{x}), \varphi(\overline{x})] : L(\varphi(\underline{x})|\varphi, u) = \mu^{\varphi} - \delta u - (1-\delta)\varphi(\underline{x})^{\frac{1}{1-\delta}} u^{-\frac{\delta}{1-\delta}} \geq 0\right\}. \tag{14}$$

For φ that is not a safe debt (with $\mu^{\varphi} > \varphi(\underline{x})$), $u^{\varphi} > \varphi(\underline{x})$. Since $\frac{\partial}{\partial u}\mathcal{L}(\varphi(\underline{x})|\varphi, u) = -\delta\left(1 - (\varphi(\underline{x})/u)^{1/(1-\delta)}\right) < 0$ for $u > \varphi(\underline{x})$, if $l^{\varphi} > \varphi(\underline{x})$ (equivalently, $\mathcal{L}(\varphi(\underline{x})|\varphi, u^{\varphi}) > 0$), then $u^{\varphi} = \varphi(\overline{x})$.

	$X=\underline{x}$	$X=\overline{x}$
B	$\tau^I(1-\gamma)$	0
G	$\left(1-\tau^I\right)(1-\gamma)$	γ

(a) Signal S^I

	$X=\underline{x}$	$X=\overline{x}$
B	$1-\gamma$	$(1-\tau^{II})\gamma$
G	0	$\tau^{II}\gamma$

(b) Signal S^{II}

Table 2: ***Signal distributions***

Tables describe joint distributions of signals S^I, S^{II} and cash flows X. Parameter τ controls the precision of signals with $\tau=0$ corresponding to uninformative signals and $\tau=1$ corresponding to perfectly revealing signals.

Suppose $l^X > \underline{x}$. We will show that $\varphi(X) = X$ is suboptimal. By the argument above, if $l^X > \varphi(\underline{x})$, then $\mathcal{L}\left(\varphi(\underline{x})\middle|\varphi, u^X\right) > 0$ and $u^X = \varphi(\overline{x})$. Consider $\tilde{\varphi}$ with $\tilde{\varphi}(\overline{x}) = \overline{x}$ and $\tilde{\varphi}(\underline{x}) = \underline{x} - \varepsilon$ for some $\varepsilon > 0$ such that $\mathcal{L}\left(\tilde{\varphi}(\underline{x})\middle|\tilde{\varphi}, u^X\right) > 0$, which exists by $\mathcal{L}\left(\varphi(\underline{x})\middle|\varphi, u^X\right) > 0$. Thus, $u^{\tilde{\varphi}} = u^X$. By construction, $\mu^{\tilde{\varphi}} < \mu^{\varphi}$. Therefore, $\boldsymbol{V}(\tilde{\varphi}) = \delta\left(u^X - \mu^{\tilde{\varphi}}\right) > \delta\left(u^X - \mu^{\varphi}\right) = \boldsymbol{V}(\varphi)$. □

.1 Model with Two States and Two Signals

Consider the model with two states: $X = \underline{x}$ with probability $1-\gamma$ and $X = \overline{x}$ with probability $\gamma \in (0,1)$. We assume that

$$(1-\delta)\underline{x}(1-\gamma) > \gamma\overline{x} + (1-\gamma)\underline{x} - \delta\overline{x} \tag{15}$$

so that if the issuer perfectly learns X, the liquidity supplier prefers to screen and offers $\delta\underline{x}$ rather than making the pooling offer $\delta\overline{x}$. Consider the two binary signals S^I and S^{II} in Table 2. Signal distribution S^I in Table 2a perfectly reveals "bad news" that $X = \underline{x}$ when $S^I = B$, while signal $S^I = G$ leads to the posterior probability of $\overline{x}$ equal to $\frac{\gamma}{1-(1-\gamma)\tau}$. In turn, signal S^{II} in Table 1b perfectly reveals "good news" that $X = \overline{x}$ when $S^{II} = G$, whereas signal $S^{II} = B$ leads to the posterior probability of $\overline{x}$ equal to $\frac{(1-\tau)\gamma}{(1-\tau)\gamma+1-\gamma}$. The precision of the signal distributions is parameterized by $\tau^i \in [0,1]$, $i \in \{I, II\}$. We solve for the optimal security $\varphi^i \in \Phi_2$ and precision $\tau^i \in [0,1]$ for each type of signals $i \in \{I, II\}$. By Lemma 2, it is without loss to assume that $\varphi^i(\underline{x}) = \underline{x}$, $i \in \{I, II\}$. Thus, the issuer's problem in each case boils down to finding $\left(\varphi^i(\overline{x}), \tau^i\right)$ to maximize his expected payoff.

Proposition 4. *Suppose that inequality (15) holds. Then,*

1. $\varphi^I(\overline{x}) = \overline{x}$, $\tau^I = \frac{1}{1-\gamma}\left(1 - 2\delta\left(1 + \sqrt{1 + \frac{4\delta(1-\delta)\underline{x}}{\gamma(\overline{x}-\underline{x})}}\right)^{-1}\right)$;
2. $\varphi^{II}(\overline{x}) = \delta\underline{x}(1-\gamma)/(\delta-\gamma)$, $\tau^{II} = 1$.

Proof. We solve each case separately.

Perfectly revealing bad news: We first fix the value of $\varphi^I(\overline{x})$ and solve for τ^I. The issuer gets positive information rents only if the liquidity supplier prefers the pooling offer $p = \delta\mathbb{E}[\varphi(X)|\mathrm{G}]$ to the screening offer $p = \delta\underline{x}$ that is only accepted by B-type (with probability $\tau^I(1-\gamma)$). Thus, it is necessary that

$$(1-\delta)\,\underline{x}\tau^I(1-\gamma) \le \mu^{\varphi^I} - \delta\mathbb{E}\left[\varphi^I(X)|\mathrm{G}\right]. \tag{16}$$

In this case, the issuer's expected payoff equals $\delta\left(\mathbb{E}\left[\varphi^I(X)|\mathrm{G}\right] - \mu^{\varphi^I}\right)$. Since $\mathbb{E}\left[\varphi^I(X)|\mathrm{G}\right] = \underline{x} + \left(\varphi^I(\overline{x}) - \underline{x}\right)\frac{\gamma}{1-(1-\gamma)\tau^I}$, making signal S^I more precise (by increasing τ^I) increases the issuer's expected payoff but tightens the constraint (16). Thus, the optimal value τ^I is the highest value that makes (16) bind (unless $\tau^I = 1$). Given $\mathbb{E}\left[\varphi^I(X)|\mathrm{G}\right] = \underline{x} + \frac{\gamma(\varphi^I(\overline{x})-\underline{x})}{1-(1-\gamma)\tau^I}$ and $\mu^{\varphi^I} = \varphi^I(\overline{x})\gamma + \underline{x}(1-\gamma)$, we can re-write (16) as

$$0 \le \left(\varphi^I(\overline{x}) - \underline{x}\right)\left(\gamma - \frac{\gamma\delta}{1-(1-\gamma)\tau^I}\right) + \underline{x}(1-\delta)\left(1-(1-\gamma)\tau^I\right),$$

or equivalently,

$$\left(\varphi^I(\overline{x}) - \underline{x}\right)\left(\frac{\gamma\delta}{\left(1-(1-\gamma)\tau^I\right)^2} - \frac{\gamma}{1-(1-\gamma)\tau^I}\right) - \underline{x}(1-\delta) \le 0.$$

Let us denote $a = \frac{\gamma\left(\varphi^I(\overline{x})-\underline{x}\right)}{1-(1-\gamma)\tau^I}$. Then,

$$\frac{\delta}{\gamma(\varphi^I(\overline{x}) - \underline{x})}a^2 - a - \underline{x}(1-\delta) \le 0.$$

The monotonicity of φ^I implies that the last inequality holds for all $a \in \left[0, a^*\left(\varphi^I(\overline{x})\right)\right]$, where

$$a^*\left(\varphi^I(\overline{x})\right) \equiv \gamma\left(\varphi^I(\overline{x}) - \underline{x}\right)\frac{1+\sqrt{1+\frac{4\delta(1-\delta)\underline{x}}{\gamma(\varphi^I(\overline{x})-\underline{x})}}}{2\delta}.$$

Thus, whenever $\tau^I < 1$,

$$\tau^I = \frac{1}{1-\gamma}\left(1 - 2\delta\left(1+\sqrt{1+\frac{4\delta(1-\delta)\underline{x}}{\gamma\left(\varphi^I(\overline{x}) - \underline{x}\right)}}\right)^{-1}\right). \tag{17}$$

Then, the requirement $\tau^I < 1$ is equivalent to

$$1 + \sqrt{1 + \frac{4\delta(1-\delta)\underline{x}}{\gamma\left(\varphi^I\left(\overline{x}\right) - \underline{x}\right)}} < \frac{2\delta}{\gamma}$$

or given that $\delta > \gamma$,

$$\frac{\varphi^I\left(\overline{x}\right)}{\underline{x}} > \frac{\delta\left(1-\gamma\right)}{\delta-\gamma}.$$

Note that this condition holds for $\varphi^I\left(\overline{x}\right) = \overline{x}$ by (15).

Given the optimal signal precision in (**??**), the issuer's expected payoff equals

$$\begin{aligned}
\delta\left(\mathbb{E}\left[\varphi^I\left(X\right)|\mathrm{G}\right] - \mu^{\varphi}\right) =& \delta\left(\underline{x} + \frac{\gamma(\varphi^I\left(\overline{x}\right) - \underline{x})}{1-(1-\gamma)\tau^I} - (1-\gamma)\underline{x} - \gamma\varphi^I\left(\overline{x}\right)\right) \\
=& \delta\gamma(\varphi^I\left(\overline{x}\right) - \underline{x})\frac{(1-\gamma)\tau^I}{1-(1-\gamma)\tau^I} \\
=& \delta\gamma\left(1 - \frac{2\delta}{1+\sqrt{1+\frac{4\delta(1-\delta)\underline{x}}{\gamma(\varphi^I(\overline{x})-\underline{x})}}}\right)\left(\frac{1+\sqrt{1+\frac{4\delta(1-\delta)\underline{x}}{\gamma(\varphi^I(\overline{x})-\underline{x})}}}{2\delta}\right)\left(\varphi^I\left(\overline{x}\right) - \underline{x}\right) \\
=& \gamma\left(1-2\delta+\sqrt{1+\frac{4\delta(1-\delta)\underline{x}}{\gamma\left(\varphi^I\left(\overline{x}\right)-\underline{x}\right)}}\right)\left(\frac{\varphi^I\left(\overline{x}\right)-\underline{x}}{2}\right).
\end{aligned}$$

The derivative of this function with respect to $\varphi^I\left(\overline{x}\right) - \underline{x}$ equals:

$$\begin{aligned}
&\frac{\gamma}{2}\left(1-2\delta+\sqrt{1+\frac{4\delta(1-\delta)\underline{x}}{\gamma\left(\varphi^I\left(\overline{x}\right)-\underline{x}\right)}} - \frac{\frac{4\delta(1-\delta)\underline{x}}{\gamma(\varphi^I(\overline{x})-\underline{x})}}{2\sqrt{1+\frac{4\delta(1-\delta)\underline{x}}{\gamma(\varphi^I(\overline{x})-\underline{x})}}}\right) \\
=&\frac{\gamma}{2}\left(1-2\delta+\frac{2+2\frac{4\delta(1-\delta)\underline{x}}{\gamma(\varphi^I(\overline{x})-\underline{x})} - \frac{4\delta(1-\delta)\underline{x}}{\gamma(\varphi^I(\overline{x})-\underline{x})}}{2\sqrt{1+\frac{4\delta(1-\delta)\underline{x}}{\gamma(\varphi^I(\overline{x})-\underline{x})}}}\right) \\
=&\frac{\gamma}{2}\left(1-2\delta+\frac{2+\frac{4\delta(1-\delta)\underline{x}}{\gamma(\varphi^I(\overline{x})-\underline{x})}}{2\sqrt{1+\frac{4\delta(1-\delta)\underline{x}}{\gamma(\varphi^I(\overline{x})-\underline{x})}}}\right) \\
=&\frac{\gamma}{4}\left(2\left(1-2\delta\right)+\underbrace{\sqrt{1+\frac{4\delta(1-\delta)\underline{x}}{\gamma\left(\varphi^I\left(\overline{x}\right)-\underline{x}\right)}}+\frac{1}{\sqrt{1+\frac{4\delta(1-\delta)\underline{x}}{\gamma(\varphi^I(\overline{x})-\underline{x})}}}}_{\geq 2}\right) \\
\geq&\gamma\left(1-\delta\right).
\end{aligned}$$

Thus, optimal $\varphi^I\left(\overline{x}\right) = \overline{x}$.

Perfectly revealing good news: Consider now the signal distribution in Table 1b. We first fix the value of $\varphi^{II}(\overline{x})$ and solve for τ^{II}. First, note that the issuer gets positive information rents only if the liquidity supplier makes a pooling offer $p = \delta\varphi^{II}(\overline{x})$ equal to the security value for type G. The liquidity supplier prefers to do so rather than make screening offer $\mathbb{E}\left[\varphi^{II}(X)|\mathrm{B}\right]$ accepted with probability $1-\gamma+\left(1-\tau^{II}\right)\gamma$ if and only if

$$(1-\delta)\,\mathbb{E}\left[\varphi^{II}(X)|\mathrm{B}\right]\left(1-\gamma+\left(1-\tau^{II}\right)\gamma\right) \leq \mu^{\varphi^{II}} - \delta\varphi^{II}(\overline{x}). \tag{18}$$

Then, the issuer's expected payoff is $\delta\left(\varphi^{II}(\overline{x}) - \mu^{\varphi^{II}}\right) = \delta\left(\varphi^{II}(\overline{x}) - \underline{x}\right)(1-\gamma)$ and is independent of the signal precision τ^{II}. Increasing informativeness of signal τ^{II} decreases the payoff from making a screening offer $\mathbb{E}\left[\varphi^{II}(X)|\mathrm{B}\right]$, as it lowers both $\mathbb{E}\left[\varphi^{II}(X)|\mathrm{B}\right]$ and the probability of its acceptance. Hence, $\tau^{II} = 1$ is optimal for any φ^{II}, that is, the issuer perfectly learns the value of security. Plugging it into (18), we get $\varphi^{II}(\overline{x}) \leq \delta\underline{x}(1-\gamma)/(\delta-\gamma)$. Thus, the issuer optimally sets $\varphi^{II}(\overline{x}) = \delta\underline{x}(1-\gamma)/(\delta-\gamma)$ and $\tau^{II} = 1$. Further, inequality (15) implies that $\varphi^{II}(\overline{x}) < \overline{x}$ which means that under the S^{II} the issuer strictly benefits from using retentions. $\square$