Optimal Information and Security Design*

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Abstract

An asset owner designs an asset-backed security and a signal about its value. After experiencing a liquidity shock and privately observing the signal, he sells the security to a monopolistic buyer. Within double-monotone securities, asset sale is uniquely optimal, which corresponds to the most informationally sensitive security. Debt is a constrained optimum under external regulatory liquidity requirements on securities. Thus, the "folk intuition" behind optimality of debt due to its low informational sensitivity holds only under additional restrictions on security/information design. Within monotone securities, a live-or-die security is optimal, whereas additional-tier-1 debt is optimal under the regulatory liquidity requirements.

Keywords: security design, asymmetric information, information design

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Corporations facing liquidity needs routinely sell assets and asset-backed securities to raise funds. Asymmetric information is a major impediment to such sales: at the time of sale, the issuer of the security knows more about its value than the liquidity supplier, thereby limiting the scope of trade. A central question in corporate finance is how to optimally design securities to mitigate this friction?

Starting from the seminal papers of Leland and Pyle (1977), Myers and Majluf (1984), and Myers (1984), the classical corporate finance literature studies the optimal design of securities payout assuming a fixed information environment where the issuer's private information originates from an exogenous signal about asset cash flows. Two classical results obtain in this environment. First, retention of cash flows by the issuer is necessary in mitigating the detrimental effects of information asymmetry. Second, debt is the optimal form of retention based on the by-now "folk" intuition that its payout is least sensitive to the issuer's private information.

In reality, the issuer can shape to a certain extent the degree of informational asymmetry vis-à-vis outsiders. For example, with limited resources for information acquisition and processing, the issuer can strategically learn more detailed information about certain aspects of future cash flows but not others. The issuer can design a complex structured product (e.g., mortgage-backed security or collateralized debt obligation), which either amplifies or mitigates his informational advantage depending on his level of sophistication and that of outsiders. Special features in financial securities can similarly tame the informational advantage (e.g., benchmarking the payout to an index) or enhance it (e.g., conversion clauses in bonds that depend on corporate events, such as takeovers or dividend payments, about which insiders are more informed).

In this paper, we take a broader approach to security design and raise a normative question: Which securities are optimal when the issuer can flexibly design both the security payout and his private information? This prompts an examination of whether optimal securities align with those commonly used in practice and whether the kind of private information issuers typically possess corresponds to the characteristics of optimal private information.

We study these question in the parsimonious setup of DeMarzo and Duffie (1999) and Biais and Mariotti (2005) with the security design occurring before the private information about underlying cash flows is revealed to the issuer. This timing is motivated by the common practice of shelf-registration, in which corporations pre-register securities well in advance of their actual sale, which allows them to promptly react to changing economic conditions and avoid lengthy regulatory delays in issuance. We enrich this basic setup by allowing the issuer to also design his private information about the asset cash flows.

Formally, there are three stages: the (ex-ante) design stage, the trading stage, and the final stage. At the design stage, before getting any private information, the asset owner (the issuer) chooses both the signal distribution about the asset's underlying cash flows that is privately revealed to him at the outset of the trading stage and the security payoff contingent on the realization of cash flows at the final stage. We are interested in the optimal joint design of the security and information under weak restrictions on both dimensions. The issuer can pick any security satisfying limited liability and monotonicity/double monotonicity (commonly assumed in the literature) and can costlessly choose any unbiased signal about the asset's cash flows. The flexibility of this approach allows us to uncover which features of securities and information are special in this environment.

At the beginning of the trading stage, the issuer observes the signal realization. Due to liquidity costs, he discounts future asset payoffs at a higher rate than the liquidity supplier. This creates gains from trade of the security. Yet, asymmetric information might impede efficient trade. We suppose that there is a monopolistic liquidity supplier who offers a take-it-or-leave-it price offer to the issuer. This assumption is realistic in applications where the security is designed to raise liquidity in crisis times when liquidity is scarce and liquidity suppliers have significant market power.

Our analysis builds on two insights. First, the joint security and information design problem can be thought of as a sequential process: the issuer first decides on the security offered to the liquidity supplier, and then chooses the signal distribution about cash flows that shape his private information during the trading stage. In this interpretation, the signal about cash flows translates into the signal about the security value, which turns out to be a sufficient statistic for the issuer's and liquidity supplier's payoffs. In turn, the choice of the security determines the set of admissible signals about the security value.

We solve this problem backwards. For a fixed security, the optimal signal choice boils down to an information design with interdependent values (Kartik and Zhong 2023). The analysis implies that, for any security, any optimal signal distribution satisfies two economic properties: it restricts the highest signal realization and ensures that the security is always sold. In other words, an optimal signal reveals sufficiently noisy information about high cash flow realizations to mitigate the lemons problems and guarantees the liquidity of the security.

Our second insight is the novel benefit of informationally sensitive securities when the issuer can flexibly design information. We say that security $\tilde{\varphi}$ is more informationally sensitive than security φ, if fixing the average security payoffs, $\tilde{\varphi}$ crosses φ once from below. For example, holding the average payoff fixed, call option is more informationally sensitive than equity, which in turn is more informationally sensitive than debt. We establish that, for monotone securities, a more informationally sensitive security has a higher variability of payoffs, which tends to expand the set of admissible signals about the security value. In other words, more informationally sensitive securities give the issuer more freedom in moulding his private information about the security value, thereby leading to better outcomes for the issuer.

This result provides a powerful tool to determine optimal securities. We show that, when the issuer can flexibly design information in addition to security payouts, within the class of double monotone securities, it is strictly optimal to simply sell the asset rather than issue any security. In other words, any form of cash flow retention is strictly suboptimal. This result is in stark contrast to the two classical results with exogenous private information mentioned above where retention is generally optimal, and debt is an optimal form of retention.

To see the reason for this difference, it is useful to recall the intuition behind the classical results. Roughly, informationally insensitive securities are valuable because they serve as a commitment device for the issuer not to take advantage of his future private information about the asset at the trading stage. A debt security arises as optimal, as it is minimally sensitive to the issuer's private information: it promises a fixed amount (the face value of debt) whenever possible and offers maximal downside protection when cash flows are low. However, it comes at a cost as it limits gains from trade by forcing the issuer to retain cash flows above the face value of debt.

The added flexibility of optimal information design allows the issuer to curb his informational advantage by properly designing his private signal about the asset. In particular, as argued above, this allows him to achieve trade with probability one for any fixed security. Roughly, information design achieves much of what security design thrives to achieve in models with exogenous private information. On the other hand, informationally sensitive securities hold value, because they provide the issuer with greater flexibility in information design. We leverage this intuition and show that selling the asset, which corresponds to the most informationally sensitive security, is the *unique* optimal security among double monotone securities.

How does the optimality of the asset sale square with common practices of raising liquidity? Our result explains why in many markets where the adverse selection problem is potentially severe, issuers often simply liquidate assets to raise liquidity rather than design complex asset-backed securities. For example, multi-divisional firms sell entire periphery divisions in times of crisis; there is an active market for limited partners' (LPs) stakes in private equity funds; and mutual and hedge funds liquidate their holding when facing excessive redemptions. Our analysis stresses that a proper information design, namely, the issuer's commitment to focus on bad news and not learning too refined positive private information about the asset, makes such asset sales optimal.

Commitment to these features of the information design is realistic in many contexts. Corporations have accounting and risk management systems in place that commit them to learn granular information about risks. When resources are scarce, they learn more noisy information about the upside potential, which is also by its nature harder to refine. In multi-divisional companies, the general management often takes a hands-off approach to peripheral divisions and gives the division management lots of autonomy, unless the division underperforms, which requires intervention and obtaining more granular information about the underlying issues. Mutual and hedge funds often assume a passive shareholder role in numerous companies, which commits them to have only limited private information about each particular holding and focus their expertise in managing the risk exposure of the portfolio as a whole. In private equity funds, even though LPs receive periodic updates about the fund's performance, they are not directly involved in investment decisions (partially due to lack of sophistication) that are fully delegated to general partners. From these reports, LPs learn if the fund underperforms, but it is harder to make precise projections about the upside potential. We stress that these features are in place for other reasons, such as regulatory risk management and accounting requirements, optimal delegation of decision making within a company, or optimal allocation of information processing resources. Nonetheless, as we show, these features also turn out to be instrumental in enhancing the liquidity of the firms' assets, and so, the issuer does not need to reallocate or spend additional resources to acquire information for liquidity management purposes

At the same time, many securities, such as MBS and other asset-backed securities, are structured as debt securities. The classical view is that this is the optimal way to raise liquidity in the presence of exogenous private information. Our results suggest that the prevalence of debt points to the presence of institutional or technological restrictions either on the information or security design. In particular, the existing literature imposes the extreme restriction that no information design is possible. We present an alternative explanation.

We examine the joint design of securities and information while imposing additional external liquidity requirements that demand that securities are sold without a substantial discount on their maximum value (e.g., a maximal haircut for debt securities). We argue that these provisions constrain the issuer's flexibility to design the security and information in a consequential manner. These requirements may arise from regulation or shareholder oversight. For instance, banks, pension funds, and insurance companies are mandated to hold sufficient high-quality liquid assets that can be quickly liquidated without significant value loss. Similarly, outside shareholders or boards of directors representing them may be concerned about management selling securities at a significant discount and may block such sales. For these reasons, the issuer may have a strong preference for designing securities that satisfy these external liquidity requirements. This, however, comes at a cost as the asset sale (which is the unconstrained optimum) might fail to satisfy these requirements.

With these external liquidity requirements, we find that debt reemerges as the optimal security within the class of double monotone securities. This implies that debt is influenced by the regulation or external oversight rather than being the unconstrained optimal security for raising liquidity. This formalizes the viewpoint often expressed by practitioners that debt arises as a response to regulation, where institutional investors demand debt because regulators perceive it as sufficiently safe and liquid. This results also aligns with the fact that debt securities are often placed with more regulated institutional investors, e.g., banks, pension funds, insurance companies, while asset sales in the examples above involve less heavily regulated multi-divisional companies and investment funds.

The underlying intuition for this finding is as follows. The optimal information design restricts the issuer from learning about extremely high security values, resulting in securities generally being sold at a discount to their maximum value. If this discount is substantial, it can violate the liquidity requirements and disqualify certain securities. In particular, simply selling the asset might violate them. The informational insensitivity of debt becomes valuable once again, leading to its optimality.

While we view double monotonicity as natural in many environments (and hardly restrictive from the practical standpoint), relaxing this assumption and considering monotone securities yields additional theoretical insights and predictions. We show that, among monotone securities, a "live-or-die security" that pays all the cash flows when they are above a certain level, but pays zero when cash flows are below this level is optimal and strictly dominates the asset sale. The reason for this is that, holding the average security payoff fixed, live-or-die securities are the most informationally sensitive among monotone securities. At the optimum, the issuer prefers these more informational sensitive securities even though they reduce gains from trade.

Further, if we additionally impose external liquidity requirements, then additional tier-1 (AT1) debt becomes optimal within the class of monotone securities. AT1 debt recently became popular in the banks' capital structure. It is structured as standard debt in normal times, but becomes junior to other forms of debt and equity if a bank fails to maintain adequate regulatory capital or asset liquidity. AT1 debt is effectively a live-or-die security capped at the debt face value. As we argued above, the cap on the payoffs is valuable in the presence of external liquidity requirements, while the high informational sensitivity of the live-or-die part expands the choice of signals about the security value available to the issuer.

Related Literature. Leland and Pyle (1977), Myers and Majluf (1984), and Myers (1984) first established that in the world of asymmetric information about asset qualities, cash flow retention serves as a credible signal of asset quality and issuing debt tends to dominate other securities. The "folk" intuition is that debt is advantageous, as it is the least sensitive to the issuer's private information. This work started an extensive literature on optimal security design under adverse selection. Most closely related to our paper are DeMarzo and Duffie (1999) and Biais and Mariotti (2005) who study security design at the ex-ante stage with an exogenous distribution of issuer's private information (see a detailed discussion in Sections 3 and 4). Both papers show optimality of debt under general conditions and weak restrictions on the class of securities. Selling the asset is optimal but only as a corner optimum (i.e., debt with face value equal to the highest cash flow realization) when the information asymmetry is not too severe. Other papers highlighting optimality properties of debt include Nachman and Noe (1994), DeMarzo (2005), DeMarzo, Kremer and Skrzypacz (2005), Dang, Gorton and Holmström (2013), Daley, Green and Vanasco (2020), Li (2022), Figueroa and Inostroza (2023), Gershkov, Moldovanu, Strack and Zhang (2023b), Asriyan and Vanasco (Forthcoming) among many others. We contribute to this literature by solving the joint problem of information and security design as well as providing a novel microfoundation for standard and AT1 debt.

There is a literature showing that informationally sensitive securities can become optimal when informational sensitivity has additional benefits to the issuer, e.g., it incentivizes information acquisition by investors (Boot and Thakor 1993, Fulghieri and Lukin 2001, Yang and Zeng 2019), it enables the aggregation of information about the optimal scale of project from informed investors (Axelson 2007), or it is complementary to public signals about the asset and allows the issuer to economize on retention (Daley, Green and Vanasco 2023). Our mechanism is novel to the literature: informationally sensitive securities are beneficial, because they relax the constraints on the issuer's information design.

Live-or-die securities (that we show to be optimal within the class of monotone securities) also arise in models of security design with moral hazard (Innes 1990). There, it is optimal for the issuer to retain a live-or-die security, because it incentivizes him to exert the first-best effort in order to maximize the chance of high cash flow realizations. In contrast, in our model, the live-or-die security is sold to the outside investor because it increases the variability of her payout which in turn increases the issuer's flexibility in information design.

Several papers study security design with endogenous information. Yang and Zeng (2019), Yang (2020) allow for flexible information acquisition by the liquidity supplier. In Azarmsa and Cong (2020), Szydlowski (2021), the issuer additionally designs public disclosures to investors. Similarly to ours, these papers impose minimal restrictions on admissible information acquisition or disclosure policies. In contrast to our paper, the optimal security is indeterminate without either positive information acquisition costs or further financing frictions. It is debt when information acquisition is costly in Yang (2020) and depends on the kind of additional contracting frictions in Szydlowski (2021) and Azarmsa and Cong (2020). Our study of joint information and security design by the issuer is complementary to this literature.

Our paper is related to the literature on optimal information design in the monopolist screening problem (Bergemann, Brooks and Morris 2015, Roesler and Szentes 2017, Glode, Opp and Zhang 2018).[1] Most closely related is Kartik and Zhong (2023) who study information design with interdependent values. We build on their result to obtain the solution to the information design problem. Our contribution is in studying the joint information and security design problem and in characterizing optimal securities that arise in different environments (see a detailed discussion in Sections 2 and 3). Our result that more informationally sensitive securities give more freedom in information design is related to the optimality of deductible contracts in the literature on insurance design. Gershkov, Moldovanu, Strack and Zhang (2023a) provide the most general formulation of this result, and we discuss the relationship to their result in Section 3.

The paper is organized as follows. Section 1 presents the model. Section 2 conducts preliminary analysis. Section 3 solves the joint security and information design problem. Section 4 solves this problem under external liquidity requirements. Section 5 discusses robustment of our results. Section 6 presents empirical implications of our analysis. Section 7 concludes. All omitted proofs are relegated to the Appendix and the Online Appendix.

1 The Model

The basic setup is that of DeMarzo and Duffie (1999) and Biais and Mariotti (2005) with the addition of information design. There are three stages $t \in \{0, 1, 2\}$. There is an issuer (he) owning an asset and a liquidity supplier (she). Both parties are risk-neutral. The asset generates cash flows X at the final stage $t = 2$ distributed according to a CDF H on a positive support $\mathcal{X}$ with $\underline{x} > 0$ and $\overline{x} < \infty$ being the minimal and maximal elements in $\mathcal{X}$, respectively. For most of our results, we do not impose any further assumptions on H, which in particular can be discrete or continuous.

At the trading stage $t = 1$, the liquidity supplier's discount factor is normalized to 1 and she values cash flows at X. The issuer discounts future cash flows at a higher rate and values them at $\delta X, \delta \in (0, 1)$, which captures his desire to free-up capital to invest in alternative assets/projects, improve the liquidity position in crisis

[1] Less related to our paper, Barron et al. (2020), Mahzoon et al. (2022), Brooks and Du (2023), Huang et al. (2023) study the interaction of information and mechanism design.

times, raise liquidity to cover redemptions (for investment funds), or focus financial resources on the core business (for multi-divisional companies). Since $\delta < 1$, there are gains from trade: the liquidity supplier is the efficient asset owner.

At the ex-ante design stage $t = 0$, before receiving any private information, the issuer designs a security to be traded at $t = 1$ and a signal about cash flows X to be revealed to him privately at $t = 1$ (before trading starts). The security payoff $F = \varphi(X)$ is contingent on the realization of X. It is distributed according to the CDF $H^{\varphi} \equiv H \circ \varphi^{-1}$ supported on $[\varphi(\underline{x}), \varphi(\overline{x})]$ (where $\varphi^{-1}(f) \equiv \sup\{x : \varphi(x) \leq f\}$ is the right-continuous inverse function). We often refer to security $\varphi(X)$ simply by φ, the function specifying its payoff. Let $\mu^{\varphi} \equiv \mathbb{E}_{H^{\varphi}}[F]$ be the average payoff of security $\varphi(X)$. The ex-ante design of securities is commonly observed in practice during the shelf-registration, a practice where issuers pre-register securities with regulators in advance, which allows them to promptly attend to their liquidity needs by avoiding lengthy regulatory delays.

We assume throughout the paper that security $\varphi(X)$ satisfies *limited liability*: $\varphi(X) \in [0, X]$. Additionally, we make one of two monotonicity assumptions. Security φ is *monotone*, if it is right-continuous and weakly increasing in X, and it is *double monotone* if in addition $X - \varphi(X)$ is weakly increasing in X. Both assumptions are common in the security design literature. Double monotonicity is often motivated by the "sabotage" argument: if it fails, the party whose payout is non-monotone in X can increase its payout by sabotaging and partially destroying cash flows.[2] Realism of this justification depends on the application in consideration. We consider a general problem not tailored to any particular application, and so we simply motivate these assumptions by the fact that they are barely restrictive from the practical point, as almost all securities observed in practice, including debt, call options, common and preferred shares, convertible securities, satisfy double monotonicity. Denote by Φ_1 and Φ_2 the sets of monotone and double-monotone securities, respectively.

At the design stage $t = 0$, the issuer can also costlessly design any signal S about X.[3] A signal S is described by the probability space $(\mathcal{X} \times \mathcal{S}, \mathscr{X} \times \mathscr{S}, \nu_{X,S})$, where $\mathcal{S}$

[2]Double monotonicity also appears in Harris and Raviv (1989), Nachman and Noe (1994), Biais and Mariotti (2005), DeMarzo et al. (2005), Daley et al. (2020), Asriyan and Vanasco (Forthcoming). An additional justification for monotone securities is that, for non-monotone securities, the issuer profits from artificially boosting cash flows X by either contributing his own funds or borrowing short-term from the market, and this way, reducing the payout to the security holders.

[3]We discuss robustness of our results to the flexibility in information design in Section 5.

is a sufficiently rich Polish space of possible signal realizations (in particular, $\mathcal{X} \subseteq \mathcal{S}$), and $\nu_{X,S}$ is the probability measure on the product of Borel σ-algebras, $\mathscr{X} \times \mathscr{S}$, with the marginal distribution on $\mathcal{X}$ coinciding with the prior distribution of X, H. Let $Z = \mathbb{E}[\varphi(X)|S]$ be the expected security value conditional on a signal S about X. Denote its CDF by G^{φ}. We call G^{φ} *admissible* for φ if it is generated by some signal S about X, and let $\mathcal{G}^{\varphi}$ be the set of all admissible distributions. The Strassen theorem implies that G^{φ} is admissible if and only if G^{φ} is a mean-preserving contraction of H^{φ} (Lemma 6 in the Online Appendix). That is, $\mathbb{E}_{G^{\varphi}}[Z] = \mathbb{E}_{H^{\varphi}}[F]$, and G^{φ} second-order stochastically dominates H^{φ}: $\int_{-\infty}^{y} H^{\varphi}(f)\,\mathrm{d}f \geq \int_{-\infty}^{y} G^{\varphi}(z)\,\mathrm{d}z$, for all y.[4] As we demonstrate shortly, the issuer's information design problem boils down to choosing $G^{\varphi} \in \mathcal{G}^{\varphi}$, an admissible signal distribution about the security $\varphi(X)$.

At the trading stage $t = 1$, the issuer observes a realization $S = s$ of signal S and updates his valuation of the security to $z = \mathbb{E}[\varphi(X)|s]$. The issuer can obtain liquidity by selling the security φ to the liquidity supplier. We assume a monopolistic liquidity supplier. This assumption is relevant for issuers who have in mind future circumstances in which they sell securities in periods of scarce liquidity when liquidity suppliers have significant monopoly power (e.g., during crisis times) or issuers who anticipate urgent liquidity needs in the future that do not leave sufficient time to solicit competitive bids for their securities.[5]In Section 5, we relax this assumption and consider an extension where the liquidity supplier is competitive in "normal" times, but monopolistic in crisis times.

At $t = 1$, the liquidity supplier offers a posted price p at which she is willing to buy the security. The issuer then either accepts p or rejects it. By Proposition 1 in Biais and Mariotti (2005), posting a price is optimal for the monopolistic liquidity supplier within the general class of incentive-compatible and individually rational mechanisms specifying the quantity of the security traded and the corresponding transfer to the issuer. Conditional on observing signal S, the issuer's expected payoff from accepting p is $\delta\mathbb{E}[X|S] - \delta\mathbb{E}[\varphi(X)|S] + p$, and his payoff from rejecting it is $\delta\mathbb{E}[X|S]$. Hence, $Z = \mathbb{E}[\varphi(X)|S]$ is the sufficient statistic for the issuer's optimal decision, which we refer to as the issuer's type.

[4]All integrals are Lebesgue-Stieltjes integrals for which the integration by parts formula obtains (see Lemma 5 in the Online Appendix).

[5]Models with a monopolistic outside investor who screens the informed issuer are Biais and Mariotti (2005), Glode et al. (2022), Chaves and Varas (2022). Section 5 discusses the model with competitive liquidity suppliers.

Given the distribution of Z, G^{φ}, the liquidity supplier chooses p to maximize her expected profit $\pi\left(p|G^{\varphi}\right) \equiv \int_{\varphi(\underline{x})}^{p/\delta}\left(z-p\right)\mathrm{d}G^{\varphi}(z)$. There is a standard adverse selection problem: only issuer types with expected values of the security below p (i.e., all types $Z=z$ with $\delta z \leq p$) accept p, while higher types hold on to the security. Let $P\left(G^{\varphi}\right) \equiv \arg\max_{p}\pi\left(p|G^{\varphi}\right)$ be the set of optimal posted prices and $\Pi\left(G^{\varphi}\right) \equiv \max_{p}\pi\left(p|G^{\varphi}\right)$ be the liquidity supplier's maximal profit. We suppose that, when indifferent between several $p \in P\left(G^{\varphi}\right)$, the liquidity supplier chooses the most preferred price for the issuer, denoted $p\left(G^{\varphi}\right)$, which is the highest price in $P\left(G^{\varphi}\right)$. Then, given distribution G^{φ}, the issuer's ex-ante expected payoff equals $\mathbb{E}\left[\delta\mathbb{E}\left[X|S\right]+\max\left\{p\left(G^{\varphi}\right)-\delta Z,0\right\}\right] = \delta\mathbb{E}\left[X\right]+v\left(p\left(G^{\varphi}\right)|G^{\varphi}\right)$, where $v\left(p|G^{\varphi}\right) \equiv \int_{\varphi(\underline{x})}^{p/\delta}\left(p-\delta z\right)\mathrm{d}G^{\varphi}(z)$ is the issuer's information rents and the equality is by the law of iterated expectations. We denote $V\left(G^{\varphi}\right) \equiv v\left(p\left(G^{\varphi}\right)|G^{\varphi}\right)$.

At the design stage $t=0$, the issuer optimally chooses a signal S about X and the security $\varphi\left(X\right)$. The distribution of S and security $\varphi\left(X\right)$ enter the issuer's and the liquidity supplier's objective functions V and Π only through the distribution G^{φ} of Z (which is a signal about $\varphi\left(X\right)$). Intuitively, the signal S about cash flows X is relevant for both agents only to the extent that it provides information about the security value. Hence, it is without loss of generality, to suppose that, for any security $\varphi\left(X\right)$, the issuer directly chooses an admissible distribution $G^{\varphi} \in \mathcal{G}^{\varphi}$ of the signal about $\varphi\left(X\right)$ (rather than a signal S about X). With a little abuse of terminology, henceforth, we refer to Z (rather than S) as the signal. Therefore, the optimal choice of a security $\varphi(X)$ and a signal S about X can be reinterpreted as a sequential choice of first choosing the security $\varphi\left(X\right)$ that then determines the set of admissible distributions $\mathcal{G}^{\varphi}$ from which the issuer picks the optimal signal distribution G^{φ}.

For a given security φ, the issuer's optimal information design problem is

$$\boldsymbol{V}\left(\varphi\right) \equiv \max_{G^{\varphi}\in\mathcal{G}^{\varphi}} V\left(G^{\varphi}\right). \tag{1}$$

For a class of monotone or double-monotone securities $\Phi \in \{\Phi_1, \Phi_2\}$, the optimal joint security and information design problem is

$$\max_{\varphi\in\Phi}\boldsymbol{V}\left(\varphi\right) = \max_{\varphi\in\Phi, G^{\varphi}\in\mathcal{G}^{\varphi}} V\left(G^{\varphi}\right). \tag{2}$$

For brevity, we refer to (2) simply as the security design problem, and it is implicit

	$X=\underline{x}$	$X=\overline{x}$
B	$\tau/2$	0
G	$(1-\tau)/2$	1/2

(a) Signal S^I

	$X=\underline{x}$	$X=\overline{x}$
B	1/2	$(1-\tau)/2$
G	0	$\tau/2$

(b) Signal S^{II}

Table 1: ***Signal distributions***

Tables describe joint distributions of signals S^I, S^{II} and cash flows X. Parameter τ controls the precision of signals with $\tau = 0$ corresponding to uninformative signals and $\tau = 1$ corresponding to perfectly revealing signals.

that the signal distribution is also chosen optimally.

2 Preliminary Analysis

Simple Example. We start with an illustration of our results in a simple example. Suppose $\delta = 3/4$ and X takes two equally likely values $\underline{x} = 1$ and $\overline{x} = 3$. We impose further restrictions that are not part of our model. First, we focus on debt securities $\varphi(X) = \min\{X, D\}$, $D \in [\underline{x}, \overline{x}]$. Second, we consider one of two binary signals with values G and B described in Table 1. Signal S^I perfectly reveals "bad news:" signal realization B reveals that the cash flows are low, $X = \underline{x}$, while G leads to the posterior probability of $\overline{x}$ equal to $1/(2-\tau)$. Symmetrically, signal S^{II} perfectly reveals "good news:" signal realization G reveals that $X = \overline{x}$, and B leads to the posterior probability of $\underline{x}$ equal to $1/(2-\tau)$. As we vary the signal precision τ, we change the signal from uninformative ($\tau = 0$) to perfectly revealing ($\tau = 1$).

Given this additional structure on the problem, it is straightforward to compute the ex-ante optimal debt face value D and the signal precision τ. Under S^I, the issuer's maximal payoff is ≈ 0.41 attained by selling the whole asset and setting $\tau^* \approx 0.71$. Under S^{II}, the issuer's maximal payoff is 0.1875 attained by issuing debt $D^* \approx 1.5$ and setting $\tau^* = 1$. This example hints at some of our main insights. First, when the issuer learns noisy information about high cash flows (signal S^I), retention is suboptimal and it is optimal to simply sell the asset (within the class of debt securities). Second, debt becomes optimal when the issuer learns more granular information about high cash flows (signal S^{II}). Third, the issuer prefers the former signal (S^I) to the latter (S^{II}). However, this example leaves open the central question of our paper: What is the best way to jointly design security and information?

Information Design. We now proceed to the general model. We first fix a security $\varphi(X)$ and solve the information design problem (1). Kartik and Zhong (2023) introduce *incentive compatible distributions* (henceforth, ICDs) with CDFs $G_{u,\mu}$ parametrized by the upper boundary of the support u and the mean μ as follows

$$G_{u,\mu}(z)=\begin{cases}0 & ,z<l,\\ (z/u)^{\delta/(1-\delta)} & ,z\in[l,u]\,,\\ 1 & ,z>u,\end{cases}\quad \text{where } l=\left(\tfrac{\mu-\delta u}{1-\delta}\right)^{1-\delta}u^{\delta}. \tag{3}$$

The next result follows from Theorem 2 and Proposition 2 in Kartik and Zhong (2023). Denote by $u(G^{\varphi})$ the highest signal realization under distribution G^{φ}.

Proposition 1. *For any security $\varphi(X)$, let u^{φ} be the solution to*[6]

$$\max\{u:G_{u,\mu^{\varphi}}\in\mathcal{G}^{\varphi}\}\,. \tag{4}$$

Then, $\boldsymbol{V}(\varphi)=\delta(u^{\varphi}-\mu^{\varphi})$ and $G^{\varphi}\in\mathcal{G}^{\varphi}$ is optimal for $\varphi(X)$ if and only if (i) $u(G^{\varphi})=u^{\varphi}$; (ii) trade occurs with probability one under G^{φ}. Further, $G_{u^{\varphi},\mu^{\varphi}}$ is one optimal signal distribution for $\varphi(X)$ with support $[l^{\varphi},u^{\varphi}]$, where $l^{\varphi}\equiv\left(\frac{\mu^{\varphi}-\delta u^{\varphi}}{1-\delta}\right)^{1-\delta}(u^{\varphi})^{\delta}$.

Kartik and Zhong (2023) show that in solving (1) it is without loss of optimality to focus on admissible ICDs, $G_{u,\mu^{\varphi}}\in\mathcal{G}^{\varphi}$. ICDs $G_{u,\mu^{\varphi}}$ are special in that the liquidity supplier is indifferent between any posted price in the support of the issuer's expected security value, $[\delta l,\delta u]$. In particular, she weakly prefers to offer a pooling price δu. This is the most preferred outcome for the issuer under $G_{u,\mu^{\varphi}}$ that gives him payoff of $\delta(u-\mu^{\varphi})$. Hence, finding the value of the program (1) boils down to finding an admissible ICD, $G_{u,\mu^{\varphi}}\in\mathcal{G}^{\varphi}$, that results in the maximal price of the security, δu.

For our purposes, Proposition 1 has two important implications. First, it allows us to write the constraints of the information design problem more explicitly as follows.

Lemma 1. *The constraint in (4) is equivalent to $u\le\varphi(\overline{x})$ and*

$$\mathcal{L}(y|\varphi,u)\equiv\varphi(\overline{x})-\delta u-(1-\delta)y^{\frac{1}{1-\delta}}u^{-\frac{\delta}{1-\delta}}-\int_{y}^{\varphi(\overline{x})}H^{\varphi}(f)\,\mathrm{d}f\ge0,y\in[0,u]\,. \tag{5}$$

[6]Since $G_{\mu^{\varphi},\mu^{\varphi}}\in\mathcal{G}^{\varphi}$ (uninformative signal), $\{u:G_{u,\mu^{\varphi}}\in\mathcal{G}^{\varphi}\}$ is non-empty.

Second, Proposition 1 implies that the issuer does not need full flexibility in choosing signal distributions. Any signal distribution G^{φ} is optimal for security $\varphi(X)$ as long as it satisfies two economic properties. It must ensure *perfect liquidity*, that is, the full issue of the security $\varphi(X)$ is always sold to the liquidity supplier. Further, the issuer prefers not to learn "too optimistic" information about the security value, i.e., his signal Z is below certain u^{φ}, which is generally less than the highest payout of the security, $\varphi(\overline{x})$. Importantly, the ICD $G_{u^{\varphi},\mu^{\varphi}}$ is only one solution to (1), but there are generally many other optimal signals. Practically, this means that the commitment to some optimal signal might not be too demanding, and as we argue in the Introduction, in many situations such a commitment might already be in place due to considerations other than liquidity needs.

3 Optimal Security Design

In this section, we solve the security design program (2). We first present our main tool for finding optimal securities. This result formalizes the idea that the issuer weakly prefers more informationally sensitive securities, because they give him "more freedom" in information design. We say that security $\tilde{\varphi}$ is *more informationally sensitive* than φ if there is $x^* \in [\underline{x}, \overline{x}]$ such that $\tilde{\varphi}(x) - \mu^{\tilde{\varphi}} \leq \varphi(x) - \mu^{\varphi}$ for $x < x^*$ and $\tilde{\varphi}(x) - \mu^{\tilde{\varphi}} \geq \varphi(x) - \mu^{\varphi}$ for $x > x^*$. In words, once we control for differences in means, $\tilde{\varphi}$ crosses φ once from below at some x^*. Thus, informational sensitivity captures differences in the shape of securities. For example, holding the average security payoff fixed, convex securities like call option (i.e., $\varphi(X) = \max\{X - K, 0\}, K \in [0, \overline{x}]$) are more informationally sensitive than standard equity (i.e., $\varphi(X) = \alpha X, \alpha \in [0, 1]$), which in turn is more informationally sensitive than concave securities like debt (i.e., $\varphi(X) = \min\{X, D\}, D \in [0, \overline{x}]$).

DeMarzo et al. (2005) introduce the "crossing from below" property of security payoffs to capture informational sensitivity of securities in the context of auctions with securities. It is interesting that a similar notion of informational sensitivity plays a key role in our problem , even though the mechanisms are very different in the two setups. In DeMarzo et al. (2005)'s auction setting, it is used in conjunction with the strict monotone likelihood ratio ordering (MLRP) of bidders' signals to show that standard auctions in more informationally sensitive securities to the seller enhance competition between bidders. In contrast, our notion of information sensitivity does not impose any additional restrictions on signal distributions. As we show next, our mechanism is very distinct.

Theorem 1. *Suppose that securities $\tilde{\varphi}$ and φ are monotone, $\tilde{\varphi}$ is more informationally sensitive than φ, and $\mu^{\tilde{\varphi}} = \mu^{\varphi}$.*

1. *Then, $\mathcal{G}^{\varphi} \subseteq \mathcal{G}^{\tilde{\varphi}}$ and $\boldsymbol{V}(\tilde{\varphi}) \geq \boldsymbol{V}(\varphi)$.*

2. *If there is $\varepsilon > 0$ such that $H^{\tilde{\varphi}}(f) > H^{\varphi}(f)$ for $f \in (\tilde{\varphi}(\underline{x}), \tilde{\varphi}(\underline{x}) + \varepsilon)$ and $H^{\tilde{\varphi}}(f) < H^{\varphi}(f)$ for $f \in (\tilde{\varphi}(\overline{x}) - \varepsilon, \tilde{\varphi}(\overline{x}))$, then $\mathcal{G}^{\varphi} \subset \mathcal{G}^{\tilde{\varphi}}$ and*

$$\int_{-\infty}^{y} H^{\tilde{\varphi}}(f)\,\mathrm{d}f > \int_{-\infty}^{y} H^{\varphi}(f)\,\mathrm{d}f \text{ for all } y \in (\tilde{\varphi}(\underline{x}), \tilde{\varphi}(\overline{x})). \tag{6}$$

   Further, if in addition $[l^{\varphi}, u^{\varphi}] \subset (\tilde{\varphi}(\underline{x}), \tilde{\varphi}(\overline{x}))$, then $\boldsymbol{V}(\tilde{\varphi}) > \boldsymbol{V}(\varphi)$.

As we argue in Section 1, the problem of the joint information and security design can be equivalently restated as a sequential problem where the issuer first chooses a security φ, which determines the set of admissible choices of signal distributions $\mathcal{G}^{\varphi}$, and then he chooses a signal distribution from $\mathcal{G}^{\varphi}$ (program (2)). In this formulation, the security φ matters only to the extent that it restricts the set of admissible choices of the signal distribution. Theorem 1 states that, holding the average security payoff μ^{φ} fixed, offering more informationally sensitive securities expands the set of admissible signal distributions, and this way, benefits the issuer.

Theorem 1 is related to Theorem 2 in Gershkov et al. (2023a), which shows that for any double-monotone φ, debt φ_D, and call option φ_C such that $\mu^{\varphi_D} = \mu^{\varphi_C} = \mu^{\varphi}$, $\mathcal{G}^{\varphi_D} \subseteq \mathcal{G}^{\varphi} \subseteq \mathcal{G}^{\varphi_C}$. We generalize this result in two respects. First, we introduce the relevant notion of informational sensitivity for monotone (not necessarily double-monotone) securities, and show that, holding μ^{φ} fixed, more informationally sensitive securities give weakly more freedom in information design, and so, are weakly preferred by the issuer. As we show below, establishing this result for any monotone securities (rather than debt or call option) provides a flexible tool for the analysis of optimal securities under different restrictions on available signals and securities.

Second, we provide sufficient conditions for the issuer to strictly prefer a more informationally sensitive security $\tilde{\varphi}$. Roughly, this is the case when higher informational sensitivity expands the set of admissible ICDs. For ICDs, a more appealing distribution has a wider support $[l, u]$, hence, the conditions in Theorem 1 are on the values of $H^{\tilde{\varphi}}$ that are close to the extreme security payoffs, $\tilde{\varphi}(\underline{x})$ and $\tilde{\varphi}(\overline{x})$.

The proof of Theorem 1 is based on the fact that when $\tilde{\varphi}$ crosses from below φ, its CDF $H^{\tilde{\varphi}}$ crosses from above the CDF H^{φ}. This implies $\int_{-\infty}^{y} \left(H^{\tilde{\varphi}}(f) - H^{\varphi}(f)\right) \mathrm{d}f \geq 0$ for all y's up to the crossing point f^* of $H^{\tilde{\varphi}}$ and H^{φ}. For $y \in (f^*, \tilde{\varphi}(\overline{x}))$, integrating by parts and using $\mu^{\varphi} = \mu^{\tilde{\varphi}}$,

$$\int_{-\infty}^{y} \left(H^{\tilde{\varphi}}(f) - H^{\varphi}(f)\right) \mathrm{d}f = \underbrace{\int_{-\infty}^{\tilde{\varphi}(\overline{x})} \left(H^{\tilde{\varphi}}(f) - H^{\varphi}(f)\right) \mathrm{d}f}_{=\mu^{\varphi}-\mu^{\tilde{\varphi}}=0} - \int_{y}^{\tilde{\varphi}(\overline{x})} \underbrace{\left(H^{\tilde{\varphi}}(f) - H^{\varphi}(f)\right)}_{\leq 0} \mathrm{d}f \geq 0.$$

Note that the conditions in part 2 of Theorem 1 are sufficient to ensure that all inequalities in (6) are indeed strict, which implies that $\mathcal{G}^{\tilde{\varphi}}$ is strictly larger than $\mathcal{G}^{\varphi}$.

In our analysis of optimal securities, we also use the following lemma.

Lemma 2. *For any security $\varphi \in \Phi_1$ and $\Delta > 0$, if $\tilde{\varphi}(X) = \varphi(X) + \Delta$ satisfies limited liability, then $\tilde{\varphi} \in \Phi_1$, $\boldsymbol{V}(\tilde{\varphi}) \geq \boldsymbol{V}(\varphi)$, and $u^{\tilde{\varphi}} \geq u^{\varphi} + \Delta$.*

Lemma 2 shows that adding safe debt is always weakly optimal. Analogous results appear in the security design literature with exogenous private information (e.g., DeMarzo and Duffie 1999, Biais and Mariotti 2005). There, pledging an additional safe payoff of Δ does not give the liquidity supplier extra incentives to screen the issuer. Hence, by switching to security $\tilde{\varphi}$, the issuer gives up Δ of future asset payoff, which he values at $\delta\Delta$, but also increases the security price by $\delta\Delta$. This intuition from models with exogenous private information is carried to our model by noticing that if $G^{\varphi} \in \mathcal{G}^{\varphi}$, then a translation of G^{φ} by Δ belongs to $\mathcal{G}^{\tilde{\varphi}}$.

Optimality of Selling the Asset. We can now solve the security design problem (2). We first solve it for double-monotone securities Φ_2.

Theorem 2. *Selling the asset (i.e., $\varphi(X) = X$ almost surely) is the unique optimal security within the class of double-monotone securities Φ_2.*

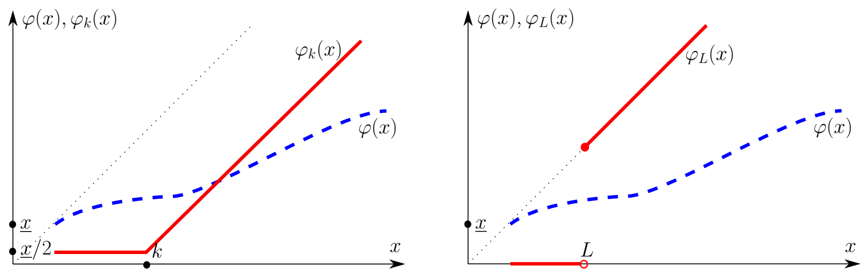
Theorem 2 is in stark contrast to the classical results stressing the role of security design in mitigating information asymmetry. As described in the Introduction, the literature establishes optimality of cash flow retention by the issuer and often obtains debt as the optimal form of retention. Theorem 2 shows that, when the issuer can optimally design his private signal about the cash flows and is restricted to choosing a double-monotone security, security design is not necessary. In fact, any form of retention is strictly suboptimal – the unique optimum is to simply sell the asset and pick one of the optimal signal distributions described in Proposition 1.

What is the reason for this difference in predictions? It is useful to revisit the "folk" intuition in models with exogenous private information. There, debt serves as a commitment device for the issuer not to take advantage of his private information when trading with the liquidity supplier. A debt security pays a fixed face value whenever possible and offers maximal downside protection when cash flows are below the face value. In other words, debt is not sensitive to the issuer's private information most of the times, and when it is, the liquidity supplier receives the maximal payout feasible. This insensitivity of debt to private information is crucial in mitigating the lemons' problem and increasing its liquidity under exogenous private information. However, it comes at a cost as it limits gains from trade by forcing the issuer to retain cash flows above the face value of debt.

In contrast, as shown in Proposition 1, the optimal information design already commits the issuer not to learn too optimistic information about the security and guarantees its perfect liquidity, making security design redundant for these purposes. In turn, by Theorem 1, more informationally sensitive securities give the issuer more freedom in information design. Selling the asset gives the liquidity supplier maximal exposure to cash flows, and corresponds to the most informationally sensitive security within the class of double-monotone securities. Thus, the issuer prefers it to designing any non-trivial double-monotone security. [7]

More formally, the proof outline goes as follows. By Lemma 2, within the class of double-monotone securities, it is without loss of optimality to consider securities that include safe debt $\underline{x}$, i.e., $\varphi(\underline{x}) = \underline{x}$. Consider security φ_k that combines a safe debt $\underline{x}/2$ and a call option with strike price k such that $\mu^{\varphi} = \mu^{\tilde{\varphi}}$. As illustrated in Figure 1a, φ_k is more informationally sensitive than φ. Further, the fact that, by construction, $\varphi_k(\overline{x}) > \varphi(\overline{x})$ and $\varphi_k(\underline{x}) < \varphi(\underline{x})$ implies that the conditions for part 2 of Theorem 1 are satisfied, hence, $\boldsymbol{V}(\varphi_k) > \boldsymbol{V}(\varphi)$. We then get the unique optimality of selling the asset by noticing that, among double-monotone securities with $\varphi(\underline{x}) = \underline{x}$, only $\varphi(X) = X$ is immune to such an improvement.

[7] Incidentally, in our simple example in Section 2, selling the asset is optimal under the signal technology S^I that is not necessarily optimal but that, similarly to optimal signals, reveals noisy information about high cash flows (see Proposition 4 in the Online Appendix). These results in total suggest that the key property of the signal distribution that makes the asset sale optimal is that the signal about high cash flows is sufficiently noisy.



(a) Illustration for Theorem 2 (b) Illustration for Theorem 3

Figure 1: ***Graphical illustrations for optimal design***

The positive implication of Theorem 1 is that raising liquidity with asset sales should prevail in environments where the issuer can commit to learn noisy information about high cash flow realizations and more granular information about lower cash flow realizations (corresponding to one of optimal signal distributions in Proposition 1). This prediction is in line with the reality that, in many situations, corporations simply sell assets to raise liquidity despite potential concerns for a high degree of adverse selection. In Section 6, we discuss several specific applications where this is the case, and how in these applications, issuers can commit to an optimal signal distribution.

On the normative side, Theorem 2 provides a counter-point to the established view in the literature that security design serves as a remedy for adverse selection. With sufficient flexibility in information design, it is strictly suboptimal to resort to security design (within the class of double-monotone securities). Thus, security design is relevant only when the issuer cannot design an optimal signal about the asset. (Proposition 1 suggests that this is the case when the issuer cannot commit not to learn granular information about high cash flows.) Taking Theorem 2 as a normative benchmark, a justification of a particular security observed in reality must start from identifying the realistic restrictions on the information/security design that make the security design relevant in the first place. As an illustration of this approach, we study in the next section how our insights change with the introduction of realistic external liquidity requirements on securities that the issuer can offer.

Remark 1. *We can think of the classical literature as imposing the extreme assumption that information is completely exogenous. In particular, Biais and Mariotti (2005), which is the closest to our framework, assume that the issuer perfectly observes X at $t = 1$, which in our setup corresponds to the issuer restricted to choose H^{φ} in program*

(2). They obtain debt as the optimal security for any prior distribution of cash flows H. Note that there is no contradiction between this result and Theorem 2. First, since $\mathcal{X}$ is bounded, selling the asset is a special case of debt with face value $\overline{x}$. Second, Biais and Mariotti (2005) assume a very specific form of exogenous private information – learning the cash flows X perfectly. As Proposition 1 shows, perfectly learning the security payoff is generally suboptimal. An optimal signal distribution instead produces a noisy signal about high valuations.[8] *Relatedly, DeMarzo and Duffie (1999) show optimality of debt under certain conditions on the issuer's private information in the model where the issuer signals to the competitive liquidity suppliers the security value by retaining a fraction of it. We show in Section 5 that selling the asset is also optimal in our model with competitive liquidity suppliers.*

Optimality of Live-or-Die Security. While we view double-monotonicity as a natural restriction of securities that captures relevant agency frictions that are not explicitly modeled, relaxing it provides interesting theoretical insights that we present next. We solve (2) for monotone securities ($\Phi = \Phi_1$). Let us call securities of the form $\varphi(X) = \mathbf{1}\{X \geq L\}X$ *live-or-die securities* – they pay all cash flows above L, but pay nothing ("die") if cash flows fall short of L.

Theorem 3. *Suppose H admits a density on $\mathcal{X}$. A live-or-die security $\varphi^*(X) = \mathbf{1}\{X \geq L^*\}X$ is optimal within the class of monotone securities Φ_1. Further, φ^* strictly dominates selling the asset (in particular, $L^* > \underline{x}$) and gives a payoff of 0 to the liquidity supplier.*

Theorem 3 implies that, for continuous distributions of cash flows, relaxing the double monotonicity assumption in Theorem 2 strictly increases the issuer's payoff: a live-or-die security φ^* that retains cash flows below L^* is optimal among monotone securities. This result reveals interesting theoretical insights into the optimal joint use of information and security design. In conjunction with Theorem 2, Theorem 3 shows how the set of equilibrium payoffs expands compared to the attainable sets in the two benchmarks where the issuer either (i) sells the asset and only uses information design (Kartik and Zhong 2023), or (ii) only uses security design (Biais and Mariotti 2005). By Theorem 2, the expansion of equilibrium payoffs compared to only security design is obtained already within the class of double-monotone securities.

[8] To the best of our knowledge, it is an open question whether in Biais and Mariotti (2005), debt is still optimal for more general distributions of exogenous signals about X.

Further, Kartik and Zhong (2023) show that by selling the asset in combination with an optimal information design, the issuer attains a payoff of $\delta\left(u^X - \mu^X\right)$ (where we denote by X the security $\varphi(X) = X$). This outcome also minimizes the liquidity supplier's payoff across all signal distributions, and equals $\mu^X - \delta u^X$. In this outcome, the asset is always sold, and so, there is no tension between maximization of information rents and efficiency. While Theorem 2 shows that, within the class of double monotone securities, no further increase of the issuer's payoff is possible despite the added flexibility to design the security φ, Theorem 3 reveals that this result is an artifact of the double monotonicity assumption. The optimal live-or-die security allows the issuer to attain a strictly higher payoff of $\delta\left(u^{\varphi^*} - \mu^{\varphi^*}\right)$. This outcome also lowers the liquidity supplier's minimal equilibrium payoff, who is up against her individual rationality constraint. Thus, somewhat surprisingly, despite having all the bargaining power at the trading stage, the liquidity supplier gets 0 in expectation when the issuer offers the optimal live-or-die security.[9] In contrast to Kartik and Zhong (2023), in order to attain this outcome, the issuer needs to sacrifice efficiency – he retains asset cash flows in bad states of the world when $X < L^*$.

Interestingly, the optimal type of retention with information design is the opposite to the one with exogenous private information. As we discussed above, in the latter case, debt is generally optimal, which makes the issuer retain cash flows in high states when X exceeds the debt face value. With optimal information design, retention of low cash flows increases the information sensitivity of the security, which according to Theorem 1 tends to increase the issuer's freedom in information design. In fact, holding the average payoff fixed, live-or-die securities are the most informationally sensitive monotone securities as illustrated in Figure 1b. This implies that there is a live-or-die security that is optimal among all monotone securities.

Theorem 3 shows a stronger result that retention of low cash flows (up to L^*) *strictly dominates* selling the asset (which is a special case of a live-or-die security with $L = \underline{x}$). This result is subtle, because while the retention at the bottom increases the issuer's flexibility in information design, a reduction in the average payoff of the security reduces the gains from trade, and this way, may restrict information design. The following lemma establishes that the former effect dominates whenever $\mu^X - \delta u^X > 0$, i.e., the liquidity supplier's profit is positive when trading the whole

[9] In other words, combining information and security design expands the Pareto frontier of attainable equilibrium payoffs compared to those in Kartik and Zhong (2023).

asset. In fact, this result holds more generally for any security.

Lemma 3. *Suppose H admits density on $\mathcal{X}$. For any $\varphi \in \Phi_1$ satisfying $\mu^{\varphi} - \delta u^{\varphi} > 0$, there is $\tilde{\varphi}(X) = \varphi(X) \mathbf{1}\left\{X \geq \tilde{L}\right\}$ for some $\tilde{L} > \underline{x}$ such that $\boldsymbol{V}(\tilde{\varphi}) > \boldsymbol{V}(\varphi)$.*

Lemma 1 follows from inspection of the constraint $\mathcal{L}(y|\varphi, u^{\varphi}) \geq 0, y \in [0, u^{\varphi}]$, of the information design program explicitly stated in (5). Observe that $\mathcal{L}(y|\varphi, u^{\varphi})$ is affected by the right tail of H^{φ}. Since $\tilde{\varphi}(x) = \varphi(x) \mathbf{1}\left\{x \geq \tilde{L}\right\}$ coincides with $\varphi(x)$ above $\tilde{L}$, it is immediate that $\mathcal{L}(y|\tilde{\varphi}, u^{\varphi}) \geq 0$ for $y \geq \varphi\left(\tilde{L}\right)$. Since $\tilde{\varphi}(x) = 0$ for $x < \tilde{L}$, $H^{\tilde{\varphi}}(f) = H\left(\tilde{L}\right)$ for $f \in \left[0, \varphi\left(\tilde{L}\right)\right)$, and so,

$$\mathcal{L}(y|\tilde{\varphi}, u^{\varphi}) = \varphi(\overline{x}) - \delta u - (1-\delta) y^{\frac{1}{1-\delta}} u^{-\frac{\delta}{1-\delta}} - \int_{\varphi(\tilde{L})}^{\varphi(\overline{x})} H^{\varphi}(f) \, \mathrm{d}f - H\left(\tilde{L}\right)\left(\varphi\left(\tilde{L}\right) - y\right)$$

is a strictly concave function on $\left[0, \varphi\left(\tilde{L}\right)\right]$. Thus, $\mathcal{L}(y|\tilde{\varphi}, u^{\varphi})$ attains its minimum either at $y = \varphi\left(\tilde{L}\right)$ (which is non-negative by the argument above for $y \geq \varphi\left(\tilde{L}\right)$) or at $y = 0$. In turn, $\mathcal{L}(0|\tilde{\varphi}, u^{\varphi}) \geq 0$ is equivalent to $\mu^{\tilde{\varphi}} - \delta u^{\varphi} \geq 0$, which by $\mu^{\varphi} - \delta u^{\varphi} > 0$, indeed holds for some $\tilde{L} > \underline{x}$. Hence, $\mathcal{L}(y|\tilde{\varphi}, u^{\varphi}) \geq 0$ for all $y \in [0, u^{\varphi}]$ which, by Lemma 1, implies that $u^{\tilde{\varphi}} \geq u^{\varphi}$. The issuer thus strictly benefits, because he obtains a weakly higher price $\delta u^{\tilde{\varphi}}$ but retains more cash flows ($\mu^{\tilde{\varphi}} < \mu^{\varphi}$).

Intuitively, Lemma 3 shows that for any security φ leaving a strictly positive payoff to the liquidity supplier, the issuer can always retain cash flows at the bottom and preserve the optimal price δu^{φ} offered by the liquidity supplier. This is profitable for the issuer who receives the same price but retains a larger fraction of the security's cash flows. At the optimum, the issuer retains as much cash flows at the bottom as possible while respecting the liquidity supplier's individual rationality constraint.

The existence of density of H is important in Theorem 3, as it ensures continuity of μ^{φ} in L for live-or-die securities $\varphi(X) = X \mathbf{1}\{X \geq L\}$, which in turn guarantees that focusing on them is without loss of optimality. When this assumption fails, live-or-die securities need not be optimal. For instance, one can verify that selling the asset is strictly optimal in the example in Section 2. However, the intuition that retaining cash flows in low states might be optimal is robust even for discrete distributions. To see this, consider the model with two cash flow realizations: $X = \overline{x}$ with probability $\gamma \in (0,1)$ and $X = \underline{x}$ with probability $1 - \gamma$. Let u^X be the solution to (4) for $\varphi(X) = X$ and $l^X = \left(\frac{\mu^X / u^X - \delta}{1-\delta}\right)^{1-\delta} u^X$. The following proposition (proved in the Online Appendix) provides a sufficient condition for the optimality of cash flow retention in state $\underline{x}$.[10]

Proposition 2. *In the model with two cash flow realizations, if $l^X > \underline{x}$, then there is a uniquely optimal φ within Φ_1 such that $\varphi(\overline{x}) = \overline{x}$ and $\varphi(\underline{x}) < \underline{x}$.*

4 External Liquidity Requirements

Theorem 2 offers a new perspective on the analysis of optimal securities. With sufficient flexibility in information design, the issuer does not need to resort to security design (at least within the class of double monotone securities commonly assumed in the literature). Thus, a theoretical justification of a particular security should begin with a question: which are the restrictions on the information design in the specific environment under consideration that make the security design relevant in the first place. In this section, we take this approach to provide a novel microfoundation for the prevalence of debt in highly regulated environments.

We impose the following restrictions that we call *external liquidity requirements* (or simply, *liquidity requirements*) on the securities that the issuer can offer: (i) the whole security is always sold at $t = 1$; (ii) for a fixed $\rho \in [0, 1]$, the security price satisfies $p \geq \rho\varphi(\overline{x})$. Since $\delta\varphi(\overline{x})$ is always accepted by the issuer, $p \geq \rho\varphi(\overline{x})$ is not sustainable when $\rho > \delta$, and we suppose $\rho \in [0, \delta]$.

Liquidity requirements of this form are often encountered in practice. For example, they naturally arise in the design of mortgage-backed securities (MBSs) or collateralized loan obligations (CLOs). To better fit this application, we slightly modify the model and make the issuer and the seller of the security separate entities. At $t = 0$, the issuer designs the security and the information that will be privately revealed to the security holder at $t = 1$. The issuer then sells the security to one of many competitive institutional investors (the seller) that we call for concreteness banks (alternatively, they can represent pension funds, insurance companies, or other institutional investors subject to strict regulation on the liquidity of their assets). At $t = 1$, the bank that bought the security observes signal Z about its value and, if hit by a liquidity shock, sells it to the monopolistic liquidity supplier. Since banks are

[10] The condition $l^X > \underline{x}$ in Proposition 2 is sufficient but not necessary. For example, for parameter values $\delta = 3/4$, $\underline{x} = 1$, $\overline{x} = 3/2$, and $\gamma = 0.3$, the uniquely optimal security is given by $\varphi(\underline{x}) \approx 0.85 < \underline{x}$ and $\varphi(\overline{x}) = \overline{x}$, which retains cash flows in the low state, yet, $l^X = \underline{x}$.

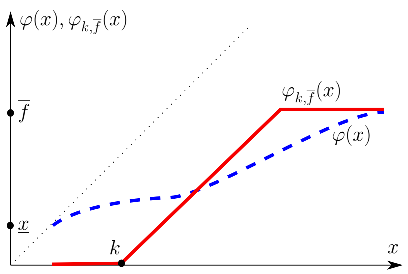
competitive and there is no information asymmetry at $t = 0$, the issuer extracts all information rents, $\boldsymbol{V}(\varphi)$, from the bank buying the security.[11]

In the context of MBSs and CLOs, the issuer is the underwriter who securitizes mortgages/loans after the origination and sells these securities to competitive institutional investors. Information design is relevant in this situation, as investors in MBSs and CLOs receive proprietary information about the asset pool and its performance from the asset-pool manager and the underwriter.[12] The underwriter specifies what information is contained in these private disclosures when he designs securities, which justifies flexibility of information design at the ex-ante stage. Institutional investors normally have a strong preference for securities satisfying regulatory liquidity requirements, because they allow them to comply more easily with regulatory liquidity ratios (e.g., liquidity coverage and net stable funding ratios). These liquidity requirements often take the form specified above. For example, Basel III qualifies securities as "high-quality liquid" if they can be liquidated within a short period of time with no significant loss of value. Say, banks should be able to liquidate level-2 assets over a 30-day period with a maximal decline in price of 10%. In the context of our model, this translates into the ability to always sell the security (irrespective of the realization of Z) and $\rho = 90\%$. Requiring $p \geq \rho\varphi(\overline{x})$ is an informationally robust way to ensure compliance, which guarantees that the maximal haircut on the security value is at most $1 - \rho$ without the regulator knowing the bank's private information or having to trust the bank's reporting.

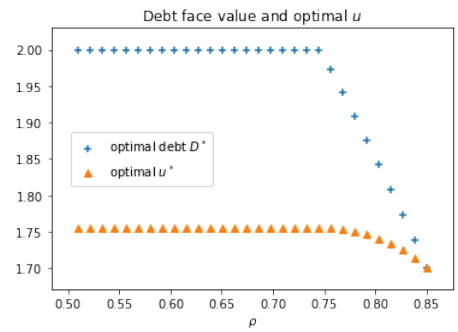
Although we focus on the regulatory nature of liquidity requirements, similar requirements on securities can also arise for corporations due to the shareholders' oversight. Corporations' shareholders (or boards of directors representing them) can be concerned that insiders sell securities at a large discount. If they believe that the security price is much lower than the true value, say below $\rho\varphi(x)$, they might block the sale. If shareholders do not have the insiders' private information, they can impose the floor on the price $\rho\varphi(\overline{x})$, which guarantees that the security is never sold below a fraction ρ of its true value.

[11] Note that competitive banks are not necessary for this argument. If instead the issuer gets only a $\gamma \in (0, 1)$ share of information rents, he maximizes $\gamma\boldsymbol{V}(\varphi)$, and so, his objective is still to maximize information rents.

[12] Under Regulation AB, the SEC imposes disclosure requirements for asset-backed securities offerings (e.g., see https://www.sec.gov/corpfin/divisionscorpfinguidanceregulation-ab-interpshtm).



(a) Illustration for Theorem 4



(b) Optimal debt face value D^* and u^*

Note: $H(x) = x - 1, x \in [1, 2]$ and $\delta = 0.85$.

Figure 2: ***Graphical illustrations for optimal design under liquidity requirements***

Optimality of Debt. We solve the security design problem under the liquidity requirements. In this section, we focus on theoretical results and postpone the discussion of implications to Section 6.

Let us first show that the liquidity requirements impose non-trivial joint restrictions on the security and information design.

Lemma 4. *Security φ satisfies liquidity requirements if and only if $u^\varphi \geq (\rho/\delta)\varphi(\overline{x})$.*

Thus, the security design program under liquidity requirements becomes

$$\max_{\varphi \in \Phi} \mathbf{V}(\varphi) \text{ s.t. } u^\varphi \geq (\rho/\delta)\varphi(\overline{x}). \tag{7}$$

Theorem 4. *A debt security $\hat{\varphi}(X) \equiv \min\{X, D^*\}$, $D^* \geq 0$, solves the program (7) for $\Phi = \Phi_2$. If in addition $\rho < \delta$, then $\hat{\varphi}$ is uniquely optimal.*

The optimality of debt is restored under liquidity requirements. The intuition for this result goes as follows. By Proposition 1, optimal signals for security φ restrict the issuer not to learn too positive information about the security value. In particular, the highest signal u^φ of an optimal signal distribution is generally below the highest security payoff $\varphi(\overline{x})$. By Lemma 4, the liquidity requirements act in the opposite direction and "force" the issuer to learn granular information about high values of the security. Intuitively, they put a floor on the security price, and in order to make this price optimal for the liquidity supplier, it is necessary that the issuer learns information about sufficiently high values of the security with positive probability. Because of this tension between the optimal information design and the liquidity requirements, certain securities might be disqualified, particularly selling the asset might not be possible. Theorem 4 establishes that, when facing such restrictions on the information design, the issuer finds it optimal to take advantage of the informational insensitivity of debt.

Remark 2. *Biais and Mariotti (2005) show optimality of debt, when the issuer perfectly learns X at $t = 1$. In this case, the issuer learns high values of the security (in particular, $\varphi(\overline{x})$) whenever they occur. Similarly, in the example in Section 2, the signal technology S^{II}, which makes debt optimal and which is generally suboptimal, perfectly reveals high cash flows $\overline{x}$ with positive probability. This in conjunction with Theorem 4 suggests a general insight that informational insensitivity of debt is valuable when the private information is more granular about high cash flow realizations.*

More formally, the proof sketch proceeds as follows. Observe that if $\varphi(X)$ satisfies the liquidity requirements so does $\varphi(X)+\Delta, \Delta > 0$. Hence, by Lemma 2, it is without loss of optimality in (7) to focus on securities satisfying $\varphi(\underline{x}) = \underline{x}$. Fix any such security $\varphi(X)$ satisfying the liquidity requirements. Consider security $\varphi_{k,\overline{f}}(X) = \min\left\{\overline{f}, \max\{0, X-k\}\right\}$, where $\overline{f} = \varphi(\overline{x})$ and k is such that $\mu^{\varphi_{k,\bar{f}}} = \mu^{\varphi}$ (which exists by continuity of $\mu^{\varphi_{k,\bar{f}}}$ in k). Security $\varphi_{k,\overline{f}}$ modifies the call option with strike price k by capping its payout at $\overline{f}$ (see Figure 2a). Since security $\varphi_{k,\bar{f}}$ is more informationally sensitive than φ, $\boldsymbol{V}\left(\varphi_{k,\overline{f}}\right) \geq \boldsymbol{V}(\varphi)$ (by Theorem 1), and so, $u^{\varphi_{k,\overline{f}}} \geq u^{\varphi}$. At the same time, the cap $\overline{f}$ on the payout ensures that $\varphi_{k,\overline{f}}$ also satisfies the liquidity requirements. Hence, it is without loss of optimality in (7) to restrict attention to $\varphi_{k,\overline{f}}$ and vary parameters k and $\overline{f}$. Observing that $\varphi_{0,\overline{f}}$ is a debt security and also the only such security with $\varphi(\underline{x}) = \underline{x}$, we get that debt is optimal.

Note that debt with face value $\overline{x}$ is equivalent to selling the asset, and so, there is no contradiction between Theorems 2 and 4. Debt is optimal when the liquidity requirements are sufficiently stringent (ρ is high), and selling the asset is optimal when they do not bind. As an illustration, Figure 2b depicts the optimal security for different ρ's in the uniform example. For high ρ's, the constraint $u^{\varphi} \geq (\rho/\delta)\varphi(\overline{x})$ is binding and the optimal security is debt with face value $D^*(\rho)$ that is weakly decreasing in ρ. For low ρ's, the constraint is not binding, and it is optimal to sell the asset (that is, $D^*(\rho) = \overline{x}$).

Remark 3. *The optimal debt in Theorem 4 is generally larger than the optimal debt in Biais and Mariotti (2005)'s model where the issuer learns X at $t = 1$. Indeed, Biais* *and Mariotti (2005) show that the optimal debt with face value D^{BM} in their model is also perfectly liquid, that is, it is traded at price δD^{BM}. Hence, debt D^{BM} satisfies the liquidity requirements in (7), and so, $D^* \geq D^{BM}$. Thus, despite the restrictions imposed by liquidity requirements, the issuer still gains from the possibility of optimally choosing the signal distribution, which is generally more complex than simply learning cash flows.*

Additionally, our predictions about investors' private information about debt securities differ from Biais and Mariotti (2005). In their paper, the issuer perfectly learn cash flows X. In this case, he believes that debt is risk-free in most scenarios (when $X \geq D^{BM}$). In contrast, optimal signals described in Proposition 1 reveal to the issuer an expected debt value consistently lower than its face value, resulting in a generally positive credit spread recorded by investors. This prediction aligns with the industry's standard practice of marking securities to market value, rather than valuing them at face value on the balance sheet.

Optimality of AT1 Debt. We next relax the double-monotonicity assumption. Interestingly and in contrast to Theorem 3, this leads to the optimality of a new class of securities often encountered in practice.

Let us call securities of the form $\varphi(x) = \mathbf{1}\{X \geq L\} \min\{X, D\}$ *additional tier-1 (AT1) debt securities.* AT1 debt is a fairly new type of security that has become popular in recent years among European banks. In normal times, it is a debt security that promises a fixed payment. Yet, unlike debt, in distress times, it is junior to other debt but also to equity under certain circumstances (in particular, when the bank is taken over by regulators due to inadequate capital or liquidity). Securities of the form $\varphi(x) = \mathbf{1}\{X \geq L\} \min\{X, D\}$ capture this by specifying a threshold L above which φ coincides with debt D and below L the security is junior to all other claims and pays 0.

Theorem 5. *Suppose H admits a density on $\mathcal{X}$. An AT1 debt security $\varphi_{D^*,L^*}(X) \equiv \min\{X, D^*\} \mathbf{1}\{X \geq L^*\}$ for some D^* and L^* solves the program (7) for $\Phi = \Phi_1$. Further, φ_{D^*,L^*} strictly dominates standard debt (i.e., $L^* > \underline{x}$) and gives a payoff of 0 to the liquidity supplier.*

The proof of Theorem 5 combines the insights developed in Theorems 3 and 4. As we argued in Section 3, live-or-die securities are optimal among monotone securities due to their high informational sensitivity, while as shown in this section, a cap on the security payout allows the issuer to comply with the external liquidity requirements. Taken together, these insights result in optimality of AT1 debt, which is a combination of debt (at high cash flows) and live-or-die security (at low cash flows).

5 Robustness

In this section, we discuss the robustness of our results.

Scope of Information Design. Throughout the paper, we consider full flexibility in information design. In doing so, we reveal which features of the information design are most valuable to the issuer and how this allows for an improvement upon the case of exogenous information considered in the literature. We stress that our results do not require a commitment to a specific information design. Any information design that satisfies two economic properties stated in Proposition 1 is optimal.[13] In this respect, our results do not require full flexibility in information design and obtain as long as the issuer has access to at least one such optimal signal distribution.

The literature considers the opposite extreme of assuming a completely exogenous signal. In the setting closest to ours, Biais and Mariotti (2005) show that debt is optimal, when the issuer perfectly learns future cash flows X before trading. The reality is somewhere on the spectrum. Covering the extremes allows us to clearly show the economic forces that push towards one or the other security design. Specifically, commitment to not learn about high cash flow realizations benefits the issuer and allows him to sell the asset completely, while the absence of such commitment favors the optimality of debt. It is an open question what happens in between these two extremes, which is beyond the scope of the present study.

In our analysis, we abstract from public disclosures of information, which has been shown to be optimal in settings with exogenous private information (Glode et al. 2018). This restriction, however, is without loss of generality. Indeed, leveraging Theorem 2 in Kartik and Zhong (2023), one can show that under a natural refinement, signal distributions in Proposition 1 are optimal in a richer class of information structures, where the liquidity supplier also gets a private signal that is less

[13] Note that we use ICD distributions in Lemma 1 as a tool to obtain more explicit constraints on u^{φ}. However, this does not mean that the issuer needs to choose the optimal ICD distribution.

informative than the issuer's signal (namely, the issuer's signal is a sufficient statistic for the liquidity supplier's signal with respect to the value of security). Thus, the issuer does not gain from publicly disclosing information about the security value.

Competitive Liquidity Suppliers. In this section, we relax the assumption of a monopolistic liquidity supplier. Suppose there is a high-liquidity state ω_H and a low-liquidity state ω_L. The issuer chooses at $t = 0$ state-contingent securities and signals, $(\varphi_H, G^{\varphi_H})$ and $(\varphi_L, G^{\varphi_L})$. In state ω_L, the liquidity supply is scarce and there is a single monopolistic liquidity supplier as in the baseline model. In state ω_H, there are competitive liquidity suppliers and the issuer chooses the security price to maximize his payoff subject to liquidity suppliers breaking even. Formally, he offers price p in state ω_H solving

$$\max_p \int_{\varphi_H(\underline{x})}^{p/\delta} (p - \delta z)\, \mathrm{d}G^{\varphi_H}(z) \ \text{ s.t. } \ \pi\left(p|G^{\varphi_H}\right) \geq 0. \tag{8}$$

The analysis is unchanged in state ω_L. In state ω_H, the maximal surplus from trading security φ^H is $(1 - \delta)\, \mu^{\varphi_H}$. This outcome is attained when the issuer is uninformed about X and offers price $p = \mu^{\varphi_H}$, which gives the liquidity supplier a payoff of 0. Thus, in state ω_H, it is optimal for the issuer to sell the asset and trade it at price μ^X, thereby fully extracting all the trade surplus.

Note that the same outcome in state ω_H is obtained if, instead of no information, the issuer receives the optimal signal described in Proposition 1. In this case, the highest valuation of the issuer δu^X is weakly below μ^X (by the fact that the liquidity supplier gets non-negative profit $\mu^X - \delta u^X$ in state ω_L), and so, all issuer types find it optimal to offer price μ^X in state ω_H. To summarize,

Proposition 3. *Within the class Φ_2, it is optimal to sell the asset in both states. Then,*

1. *In state ω_L, it is optimal for the issuer to choose an optimal signal distribution described in Proposition 1.*

2. *In state ω_H, it is optimal for the issuer to receive no information, or alternatively, to choose an optimal signal distribution described in Proposition 1.*

The security price is δu^X in state ω_L and μ^X in state ω_H.

The asset sale is optimal irrespective of the state. Further, part 2 of Proposition 3 states that both no information and the same information as in state ω_L are optimal in state ω_H. We can select one of two outcomes by introducing small costs of committing to certain information features.

First, in the presence of vanishingly small, but positive costs of non-trivial signals, no information would be uniquely optimal in the high-liquidity state. While the issuer sells the asset in both states, the signal distribution differs across states, establishing a counter-cyclical pattern of private information among financial institutions. When liquidity is abundant and liquidity suppliers are competitive, issuers have no incentive to possess private information, as it might be costly to produce it and may potentially hinder liquidity without inducing further benefits. In periods of scarce liquidity when liquidity suppliers hold significant market power, issuers acquire private information about the downside of securities, which allows them to capture some information rents while preserving the liquidity of their securities. This result is also in contrast to Biais and Mariotti (2005) showing that, when the issuer learns X at $t = 1$, debt is optimal in both competitive and monopolistic setting and the face value of debt is sensitive to the degree of competition.

Instead, if there are vanishingly small, but positive costs of introducing state-contingent information design, it is uniquely optimal for the issuer to keep the information the same in both states. In other words, the private information learned in the low-liquidity state ω_L does not prevent the issuer from attaining the optimal outcome in the high-liquidity state ω_H.

6 Empirical Implications

In this section, we highlight positive implications of our theory and discuss the relevance of our assumptions.

Debt vs Asset Sale. Our theory predicts some common ways of raising liquidity in practice – selling assets as an unconstrained optimum, and debt and AT1 debt as a constrained optimum under the external liquidity requirements. Our microfoundation for these securities differs from that in the classical literature in which retaining assets' cash flows typically serves as a credible signal of quality in markets with significant information asymmetry. In models with exogenous information, debt arises as the optimal security, and selling the entire asset occurs only in cases when information asymmetry is relatively mild.

In contrast, with optimal information design, retention is generally unnecessary, and selling the entire asset is strictly optimal (Theorem 2).[14] As a result, our novel empirical prediction is that, even in environments where information asymmetry is a major concern, investors can raise liquidity by selling assets rather than issuing more complex securities. This requires the seller to commit to having noisy private information about high security valuations and more detailed information about low valuations. Below, we discuss several applications where such a commitment is realistic and indeed asset sales are prevalent.

In contrast, exogenous positive news about the asset can interfere with such a commitment making debt optimal as in classical models. Hence, in environments where exogenous private news about the upside potential of the company are anticipated by the issuer and outside investors, we expect to see more debt issuances. In the paper, we provide a particular case of such an interference. We show that the optimal information design can conflict with external liquidity requirements imposed by regulators or shareholders. As these liquidity requirements become stricter, the optimal security switches from the asset sale to debt (Theorem 4 and Figure 2b).

We next discuss several contexts in which our model is applicable.

Multidivisional Firms. Multidivisional firms generally consist of core and periphery divisions. Under this organization structure, periphery divisions receive a great deal of autonomy in both daily operations and short and medium-term strategic planning. The firm's general management maintains a hands-off approach and only launches thorough investigations when a crisis occurs. This organization design serves as a commitment device for the management not to learn granular information about periphery divisions and be more aware of negative news.

Consistent with our theory, despite potentially a high degree of asymmetric information versus outsiders, liquidity-constrained multidivisional firms often divest entire divisions to raise funds (Lang, Poulsen and Stulz 1995, Officer 2007) rather than issue securities backed by division cash flows. Further, Kaplan and Weisbach (1992)

[14]In reality, companies often sell fractions of assets, e.g., equity stakes in the multidivisional firms or parts of asset portfolios for investment funds. In the ongoing research, we obtain this fractional sale as an optimal security design in response to several liquidity shocks (details are available upon request).

and Maksimovic and Phillips (2001) find that parent units usually divest periphery, non-core divisions. A key insight of our analysis is that by not monitoring too closely periphery divisions, firms can maintain the liquidity of these assets. Consistent with this prediction, Schlingemann et al. (2002) find that multidivisional firms divest their divisions in highly-liquid markets, and that, perhaps surprisingly, firms are less likely to divest their worst-performing units but rather tend to divest their most liquid divisions.[15]

Investment Funds. Our model provides insights into the common practice of investment funds liquidating their assets, even though they may be susceptible to significant information asymmetry, instead of utilizing asset-backed securities as a means to raise liquidity.

In private equity funds, general partners (GPs) oversee investments and secure capital from limited partners (LPs). Despite LPs having access to internal performance reports, their ability to evaluate investment strategies is limited, leading them to delegate decisions to GPs. Our theory suggests that the passive role of LPs enables them to raise liquidity by selling their stakes, whereas GPs face more constraints in this regard. This aligns with the existence of an active secondary-market for LP stakes, where buyers, often funds of funds, provide liquidity to selling LPs impacted by unexpected liquidity needs (Nadauld et al. 2019). Interestingly, there is a segment of collateralized fund obligations issuing highly-rated bonds backed by pools of stakes in private equity funds, but its size remains relatively small. This indicates that the secondary markets for LP stakes are adequately liquid.[16]

Mutual and hedge funds face the possibility of meeting large redemptions, which can lead to the liquidation of less liquid assets like private equity or large blocks of public shares in decentralized markets. While these funds usually hold liquid securi-

[15] Robot maker Boston Dynamics provides an interesting case study. It was bought by Google in 2013 who sold it to Softbank in 2017. In turn, Softbank sold it to Hyundai in 2021 partially in response to a liquidity shock caused by losses in its investment portfolio, such as Uber, WeWork, OneWeb. Throughout the years, Boston Dynamics maintained a high degree of autonomy by keeping the headquarters in Boston and maintaining its own research team. Our theory predicts that, because of this autonomy, the head companies were able to easily raise liquidity by selling it. Importantly, despite the complex nature of the business, the sale did not involve designing complex securities backed by Boston Dynamics' cash flows. *Heater, Brian. 2021. "Hyundai completes deal for controlling interest in Boston Dynamics." Tech Crunch, June 21. https://techcrunch.com/2021/06/21/hyundai-completes-deal-for-controlling-interest-in-boston-dynamics/.*

[16] *Wiggins, Kaye. 2022. "Collateralised fund obligations: how private equity securitised itself." Financial Times, November 22. https://www.ft.com/content/e4c4fd61-341e-4f5b-9a46-796fc3bdcb03*

ties as a safeguard, severe shocks during crises can disrupt this buffer. In such times, buyers in decentralized markets wield considerable market power due to limited liquidity and heightened demand. Except for activist funds with concentrated positions, fund managers oversee numerous firms and have limited knowledge and capabilities to provide effective governance for each company in its portfolio. Consequently, majority of investment funds tend to be passive, prioritizing liquidity needs in the face of shocks. Consistent with our theory, these funds do not issue securities backed by their holdings, opting instead to raise liquidity through portfolio liquidation.

VCs specialize in early-stage financing of startups with a finite life-span of around 12 years after which the fund must return money to the investors. Due to the growth of private equity markets and a recent cooling down of the IPO market, VC-backed startups currently prefer to stay private for longer time. This shift makes conventional exit strategies of IPOs or mergers and acquisitions more challenging, leading VCs to liquidate their stakes in startups in an illiquid market for early-stage private equity (Nigro and Stahl 2021, Bian et al. 2022).[17] Because of the significant information asymmetry between VCs and external investors, the practice of VCs divesting their entire holdings without implementing more sophisticated securities arrangements contradicts the predictions of the classical theory.

Nevertheless, this aligns with our theory, which stresses the role of information design. VCs often restrict themselves either contractually or through reputational mechanisms to take a hands-off approach to their investment, wherein they provide financial and operational support of the startup but refrain from interference unless the startup fails to meet predetermined milestones. This approach allows VCs to gain more detailed information when the firm performs poorly, prompting them to investigate the underlying causes. Conversely, as long as the startup remains on track, VCs have limited insight into its potential and day-to-day progress, ensuring that they do not set unrealistically high valuation expectations. This hands-off approach, which contrasts with the governance approach involving intensive monitoring of startups, has become prominent in recent years, with many leading VCs maintaining a founder-friendly reputation (Ewens et al. 2018, Lerner and Nanda 2020). In a recent study, Fu

[17]Recently, the VC market has seen a rise in alternative strategies, such as the use of continuation funds or strip sales, that allow for an extension of the fund's lifespan. Fundamentally, these strategies are variations of the sale of stakes in the fund by its investors. *Kinder, T., Hammond, G., and Louch, W. 2023. "Tech funds adopt private equity strategies in race to return cash to investors." Financial Times, December 3. https://www.ft.com/content/6e70fed4-eecf-46f1-83cb-9d20da9c1d7f*

(2024) uses cell phone tracking data to document that VCs do not increase monitoring of successful startups (with high exit multiples), which is broadly consistent with the commitment not to learn too detailed positive information about assets.

Commitment to Information Design. In the above examples, companies employ several commitment mechanisms that restrict their ex-post information acquisition about the upside. This can be done through organizational design (in the case of multi-divisional firms or LPs), portfolio structure for investment funds, or contractual and reputational mechanisms for VCs.

Further, the qualitative properties of optimal information designs in Proposition 1 – specifically, the focus on downside risks rather than the upside potential – are in line with accounting principles and risk-management practices. Accounting standard-setters, such as GAAP, recommend the conservatism principle that is widely adopted by investment funds. According to the dictum, financial institutions should record losses as soon as they learn about them, whereas gains are not supposed to be recorded until they are realized (see Ruch and Taylor 2015). Similarly, standard risk management involves keeping track of market and credit risk exposures of the investment portfolio and the likelihood of potential losses.

Corporations allocate substantial resources into compliance with accounting standards and a proper risk management to avert catastrophic outcomes or costly lawsuits. In a world where resources for information acquisition and processing are limited, this means fewer resources being directed toward refining the upside projections that are by their nature more challenging to gauge with precision. Thus, corporations tend to produce more refined information about risks than the upside potential.

We stress that we do not posit that these commitment mechanisms are employed for the sole purpose of ensuring adequate asset liquidity in crisis times. On the contrary, they are most likely implemented for orthogonal reasons, such as regulatory compliance, adequate risk management, optimal delegation of decisions within the firm. However, this fact in itself strengthens the commitment power of these mechanisms. Namely, they are already in place for other reasons, and additionally, they do not create a tension with the firm's ability to raise liquidity. This way, the firm does not need to take further (potentially costly) actions to modify its private information beyond what is already in place.

Signaling with Retention. A distinct prediction of our theory is that retention of cash flows is suboptimal if the issuer can properly curb his informational advantage. This prediction squares with recent evidence on the market for syndicated loans. Blickle et al. (2020) report that lead arrangers for syndicated loans, who are arguably the most informed investors in loans due to their prominent role in the underwriting process, often sell their entire loan stake to other investors, e.g., collateralized loan obligations, loan mutual funds, insurance companies, pension funds. They also show that reputational concerns seem to be important: lead arrangers of loans that turned sour tend to subsequently lose the market share. While this evidence contradicts the standard theory that highlights retention by the underwriter as a credible signaling device (e.g., Leland and Pyle 1977), it is consistent with our model. Maintaining reputation for focusing on the downside risk in their due diligence rather than the upside potential enables lead arrangers to offload completely their loan stake to institutional investors.

Regulation-Induced Debt. Many securities, such as MBSs and CLOs, are structured as debt securities. The traditional theory posits that debt is optimal under exogenous information. An alternative viewpoint is that debt is an optimal response to prudential regulation: institutional investors demand debt because their regulators view it as adequately safe and liquid. These two explanations are often presented as contradictory to each other.

Our theory of debt in Section 4 reconciles these view points. Similar to the traditional theory, debt is an optimal security, but only under additional external liquidity requirements. These requirements arise from regulatory or shareholder oversight over the securities holders and are similar in nature to the prudential regulation of banks, pension funds, and insurance companies. Importantly, similar to how they are formulated in practice, our liquidity requirements do not restrict the class of securities, but rather require that adequately liquid securities are sold in a short period of time without a significant loss of value. That the optimal security is debt comes as a result not an assumption. This result formalizes the idea of regulation-induced debt: debt allows financial institutions to optimally address their liquidity needs while complying with regulatory requirements. This theory aligns with the widespread presence of MBSs and CLOs marketed and held by heavily regulated entities such as banks, insurance companies, and pension funds. On the other hand, less regulated institutions (e.g., investment funds) prefer to sell assets to generate liquidity.

Additionally, we diverge from the traditional theory regarding investors' private information about debt securities. As discussed in Remark 3, in existing models, it is often assumed that investors perfectly learn cash flows before trading, leading to investors believing that debt is risk-free in most scenarios. In contrast, our optimal information design reveals to investors an expected debt value consistently lower than its face value, resulting in a generally positive credit spread. This prediction aligns better with the industry's standard practice of marking securities to market value, rather than valuing them at face value on the balance sheet.

AT1 Debt After the financial crisis of 2007-2009, AT1 debt has become popular among banks as a quick built-in way to reduce leverage when banks suffer losses. This is done through write-downs of the principal at regulators' discretion (see Avdjiev et al. 2015 for an overview of the market). Theorem 5 provides a novel and complementary liquidity-based microfoundation for AT1 debt. AT1 debt is optimal in environments where buyers of securities have demand for securities that satisfy external liquidity requirements (e.g., banks) and the issuer can offer any monotone security. The monotonicity assumption is particularly realistic for banks issuing AT1 debt. Due to regulators' scrutiny, banks are less likely to engage in cash flow destruction, and the sabotage argument justifying double-monotone securities is less applicable to them. Our results demonstrate the apart from attaining financial stability goals, AT1 debt might also have special liquidity properties that make it appealing for banks.

Counter-Cyclicality of Private Information. Our findings in Section 5 establish a connection between the competitive environment and the presence of private information. When liquidity is abundant and liquidity suppliers are competitive, issuers have no incentive to possess private information, as it would hinder liquidity without benefiting them. However, in periods of scarce liquidity when liquidity suppliers hold significant market power, issuers acquire private information about the downside of securities. This allows them to capture some information rents while preserving the liquidity of their securities. In essence, it is optimal to remain "ignorant" about asset quality during booms but gain sufficient private information during downturns to maintain market liquidity. As a result, our theory predicts a counter-cyclical pattern of private information among financial institutions.

Our prediction aligns with previous theoretical studies that highlight a negative relationship between economic activity and the extent of asymmetric information (e.g., Gorton and Ordonez 2014, Fishman and Parker 2015). Nonetheless, our findings diverge by attributing the correlation to shifts in investors' bargaining power prompted by fluctuations in economic activity, rather than external shocks impacting asset quality.

7 Conclusion

In this study, we take a broader approach to the optimal design of securities, encompassing not only payout design but also information design. Optimal information design involves the issuer refraining from obtaining overly positive information about the security value while ensuring perfect liquidity of the security. Contrary to common intuition, when the issuer optimally selects his signal, it is optimal to simply sell the asset rather than design more complex securities. Additionally, we propose a theory linking regulatory liquidity requirements to the prevalence of debt securities.

One broad takeaway from our analysis is that the security design is shaped by external institutional or technological restrictions on the joint information and security design. The present study focuses on the classical question of determining the optimal shape of the security and therefore its information sensitivity. In reality, there are many other features of securities that involve design of both payoffs and private information. For example, pooling and tranching commonly used in mortgage-backed securities and collateralized loan obligations; convertibility features often introduced in debt securities; downside protection common in startup equity contracts. We believe that our approach can be instrumental and fruitful in shedding light on these and other contractual features.

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Appendix: Omitted Proofs

Proofs of Propositions 1 and 2, and lemmas are relegated to Online Appendix.

Proof of Theorem 1. 1) Since $\tilde{\varphi}$ is more informationally sensitive than φ, there is $x^* \in [\underline{x}, \overline{x}]$ such that $\tilde{\varphi}(x) \leq \varphi(x)$ for $x < x^*$ and $\tilde{\varphi}(x) \geq \varphi(x)$ for $x > x^*$. Let $f^* \equiv \varphi(x^*)$ and $f_* \equiv \lim_{x \uparrow x^*} \varphi(x)$. By monotonicity, choose x^* such that $f^* = \varphi(x^*) \leq \tilde{\varphi}(x^*)$. Since $\mu^{\tilde{\varphi}} = \mu^{\varphi}$ and $[\varphi(\underline{x}), \varphi(\overline{x})] \subseteq [\tilde{\varphi}(\underline{x}), \tilde{\varphi}(\overline{x})]$, $\int_{-\infty}^{y} \left(H^{\tilde{\varphi}}(f) - H^{\varphi}(f)\right) \mathrm{d}f = 0$ for $y \leq \tilde{\varphi}(\underline{x})$ and $y \geq \tilde{\varphi}(\overline{x})$. Since $H^{\varphi} = H \circ \varphi^{-1}$, $H^{\varphi}(f) = H(\sup\{x : \varphi(x) \leq f\})$ for any f. Thus, $H^{\tilde{\varphi}}(f) \geq H^{\varphi}(f), f \in (\tilde{\varphi}(\underline{x}), f_*)$; $H^{\tilde{\varphi}}(f) = H^{\varphi}(f), f \in [f_*, f^*)$ when $f_* < f^*$; and $H^{\tilde{\varphi}}(f) \leq H^{\varphi}(f), f \in [f^*, \tilde{\varphi}(\overline{x}))$. Hence, $\int_{-\infty}^{y} \left(H^{\tilde{\varphi}}(f) - H^{\varphi}(f)\right) \mathrm{d}f \geq 0$ for $y \in (\tilde{\varphi}(\underline{x}), f^*]$. For $y \in (f^*, \tilde{\varphi}(\overline{x}))$,

$$\begin{aligned}\int_{-\infty}^{y} H^{\tilde{\varphi}}(f)\,\mathrm{d}f =& \tilde{\varphi}(\overline{x}) - \underbrace{\mu^{\tilde{\varphi}}}_{=\mu^{\varphi}} - \int_{y}^{\tilde{\varphi}(\overline{x})} \underbrace{H^{\tilde{\varphi}}(f)}_{\leq H^{\varphi}(f)} \mathrm{d}f \geq \tilde{\varphi}(\overline{x}) - \mu^{\varphi} - \int_{y}^{\tilde{\varphi}(\overline{x})} H^{\varphi}(f)\,\mathrm{d}f \\ =& \varphi(\overline{x}) - \mu^{\varphi} - \int_{y}^{\varphi(\overline{x})} H^{\varphi}(f)\,\mathrm{d}f = \int_{-\infty}^{y} H^{\varphi}(f)\,\mathrm{d}f.\end{aligned} \tag{9}$$

Thus, $\int_{-\infty}^{y} \left(H^{\tilde{\varphi}}(f) - H^{\varphi}(f)\right) \mathrm{d}f \geq 0$ for all y, and so, $\mathcal{G}^{\varphi} \subseteq \mathcal{G}^{\tilde{\varphi}}$ and $\boldsymbol{V}(\tilde{\varphi}) \geq \boldsymbol{V}(\varphi)$.

2) Suppose there is $\varepsilon > 0$ such that $H^{\tilde{\varphi}}(f) > H^{\varphi}(f)$ for all $f \in (\tilde{\varphi}(\underline{x}), \tilde{\varphi}(\underline{x}) + \varepsilon)$ and $H^{\tilde{\varphi}}(f) < H^{\varphi}(f)$ for all $f \in (\tilde{\varphi}(\overline{x}) - \varepsilon, \tilde{\varphi}(\overline{x}))$. Hence, for $y \in (\tilde{\varphi}(\underline{x}), f^*]$, $\int_{-\infty}^{y} \left(H^{\tilde{\varphi}}(f) - H^{\varphi}(f)\right) \mathrm{d}f \geq \int_{\tilde{\varphi}(\underline{x})}^{\min\{y, \tilde{\varphi}(\underline{x}) + \varepsilon\}} \left(H^{\tilde{\varphi}}(f) - H^{\varphi}(f)\right) \mathrm{d}f > 0$, and for $y \in (f^*, \tilde{\varphi}(\overline{x}))$,

$$\int_{-\infty}^{y} H^{\tilde{\varphi}}(f)\,\mathrm{d}f = \tilde{\varphi}(\overline{x}) - \mu^{\tilde{\varphi}} - \int_{y}^{\tilde{\varphi}(\overline{x})} H^{\tilde{\varphi}}(f)\,\mathrm{d}f > \tilde{\varphi}(\overline{x}) - \mu^{\varphi} - \int_{y}^{\tilde{\varphi}(\overline{x})} H^{\varphi}(f)\,\mathrm{d}f = \int_{-\infty}^{y} H^{\varphi}(f)\,\mathrm{d}f,$$

which proves (6). This together with $\mu^{\tilde{\varphi}} = \mu^{\varphi}$ and $G_{u^{\varphi}, \mu^{\varphi}} \in \mathcal{G}^{\varphi}$ implies $\int_{-\infty}^{y} G_{u^{\varphi}, \mu^{\tilde{\varphi}}}(z)\,\mathrm{d}z < \int_{-\infty}^{y} H^{\tilde{\varphi}}(f)\,\mathrm{d}f, y \in (\tilde{\varphi}(\underline{x}), \tilde{\varphi}(\overline{x}))$. If in addition $[l^{\varphi}, u^{\varphi}] \subset (\tilde{\varphi}(\underline{x}), \tilde{\varphi}(\overline{x}))$, then there is $\epsilon > 0$ such that, for $u^{\epsilon} \equiv u^{\varphi} + \epsilon$ and $l^{\epsilon} = \left(\frac{\mu^{\tilde{\varphi}}/u^{\epsilon} - \delta}{1-\delta}\right)^{1-\delta} u^{\epsilon}$, $\int_{-\infty}^{y} G_{u^{\epsilon}, \mu^{\tilde{\varphi}}}(z)\,\mathrm{d}z < \int_{-\infty}^{y} H^{\tilde{\varphi}}(f)\,\mathrm{d}f, y \in (\tilde{\varphi}(\underline{x}), \tilde{\varphi}(\overline{x}))$ and $[l^{\epsilon}, u^{\epsilon}] \subset (\tilde{\varphi}(\underline{x}), \tilde{\varphi}(\overline{x}))$. Thus, $G_{u^{\epsilon}, \mu^{\tilde{\varphi}}} \in \mathcal{G}^{\tilde{\varphi}}$, and by Proposition 1, $\boldsymbol{V}(\tilde{\varphi}) \geq \delta\left(u^{\epsilon} - \mu^{\tilde{\varphi}}\right) > \delta\left(u^{\varphi} - \mu^{\varphi}\right) = \boldsymbol{V}(\varphi)$. □

Proof of Theorem 2. Suppose φ solves (2) for $\Phi = \Phi_2$. Suppose to contradiction that $\mu^{\varphi} < \mu^{X} \equiv \mathbb{E}_H[X]$. For any $\varphi \in \Phi_2$, if $\Delta \equiv \underline{x} - \varphi(\underline{x}) > 0$, then $\tilde{\varphi}(X) \equiv$

$\varphi(X)+\Delta \in [0, X]$, and so, $\tilde{\varphi} \in \Phi_2$. By Lemma 2, $\boldsymbol{V}(\tilde{\varphi}) \geq \boldsymbol{V}(\varphi)$. Thus, it is without loss of optimality within Φ_2 to focus on securities that include safe debt $\underline{x}$: $\varphi(\underline{x}) = \underline{x}$. For any such φ, there is a security $\varphi_k(X) = \underline{x}/2 + \max\{0, X-k\}$ such that $\mu^{\varphi} = \mu^{\varphi_k}$ and $\varphi_k \in \Phi_2$ (see Figure 1a). This follows from the continuity of $\mathbb{E}_H[\varphi_k(X)]$ in k (indeed, $\mathbb{E}_H[\varphi_k(X) - \varphi_{k+\varepsilon}(X)] \in [0, \varepsilon]$ for any $\varepsilon > 0$) and $\varphi_{\overline{x}}(x) \leq \varphi(x) \leq \varphi_{\underline{x}/2}(x)$ for all $x \in \mathcal{X}$. Since $\mu^{\varphi} < \mu^X$, $k > \underline{x}/2$, and so, $H^{\varphi_k}(f) > H^{\varphi}(f) = 0$ for $f \in (\underline{x}/2, \underline{x})$. This, in conjunction with $\mu^{\varphi} = \mu^{\varphi_k}$, implies that $\varphi_k(\overline{x}) > \varphi(\overline{x})$, and so, $H^{\varphi_k}(f) < H^{\varphi}(f) = 1$ for $f \in (\varphi(\overline{x}), \varphi_k(\overline{x}))$. By construction, φ_k it is more informationally sensitive than φ. Further, $[l^{\varphi}, u^{\varphi}] \subseteq [\varphi(\underline{x}), \varphi(\overline{x})] \subset (\varphi_k(\underline{x}), \varphi_k(\overline{x}))$. By Theorem 1, $\boldsymbol{V}(\varphi_k) > \boldsymbol{V}(\varphi)$, which is a contradiction to optimality of φ. Therefore, $\mu^{\varphi} = \mu^X$. $\square$

Proof of Theorem 3. Since H admits a density h, $\mathbb{E}_H[X\mathbf{1}\{X \geq L\}] = \int_L^{\overline{x}} xh(x)\,\mathrm{d}x$ is continuous in L and ranges from 0 (for $L = \overline{x}$) to μ^X (for $L = \underline{x}$). Hence, for any $\mu \in [0, \mu^X]$, there is L such that $\mathbb{E}_H[\mathbf{1}\{X \geq L\}X] = \mu$. Further, given the same average payoff, a live-or-die security is more informationally sensitive than any security in Φ_1. By Theorem 1, there is a live-or-die security that solves (2) for $\Phi = \Phi_1$.

Since H admits a density on $\mathcal{X}$, $\mathcal{L}_y(\underline{x}|X, u^X) = -(\underline{x}/u^X)^{1/(1-\delta)} < 0$, and so, $\mathcal{L}(\underline{x}|X, u^X) > 0$, which implies $\mu^X - \delta u^X > 0$. By Lemma 3, $\varphi(X) = X (= X\mathbf{1}\{X \geq \underline{x}\})$ cannot be optimal, and so, in the optimum, $L^* > \underline{x}$. By Lemma 3, the optimal φ^* must satisfy $\mu^{\varphi^*} - \delta u^{\varphi^*} = 0$. $\square$

Proof of Theorem 4. *Part 1.* Consider the securities $\varphi_{k,\overline{f}}(X) = \min\{\overline{f}, \max\{0, X - k\}\}$ parameterized by k and $\overline{f}$. For any $k, \varepsilon > 0$, $0 \leq \mathbb{E}_H[\varphi_{k,\overline{f}}(X) - \varphi_{k+\varepsilon,\overline{f}}(X)] \leq \varepsilon$. Hence, $\mathbb{E}_H[\varphi_{k,\overline{f}}(X)]$ is continuous in k and takes all values between 0 (for $k = \overline{x}$) and $\mathbb{E}_H[\min(X, \overline{f})]$ (for $k = 0$). Thus, for any $\varphi \in \Phi_2$, there is k such that $\mathbb{E}_H[\varphi_{k,\overline{f}}(X)] = \mu^{\varphi}$, where $\overline{f} = \varphi(\overline{x})$. We argue that $\varphi_{k,\overline{f}}$ dominates φ. By Lemma 2, it is without loss of generality to assume that $\varphi(\underline{x}) = \underline{x}$.[18] Since $\varphi(\underline{x}) = \underline{x}$ and $\varphi \in \Phi_2$, $\overline{x} - k > \varphi(\overline{x})$, and so, $\varphi_{k,\overline{f}}(x) = \overline{f}$ on $x \in [\overline{f} + k, \overline{x}]$. Since $\varphi_{k,\overline{f}}$ is more informationally sensitive than φ, $\mathcal{G}^{\varphi} \subseteq \mathcal{G}^{\varphi_{k,\overline{f}}}$ and $\boldsymbol{V}(\varphi_{k,\overline{f}}) \geq \boldsymbol{V}(\varphi)$ (by Theorem 1). Note that if $u^{\varphi} \geq (\rho/\delta)\overline{f}$, then (by Proposition 1 and $\varphi_{k,\overline{f}}(\overline{x}) = \overline{f}$), $u^{\varphi_{k,\overline{f}}} \geq u^{\varphi} \geq (\rho/\delta)\varphi_{k,\overline{f}}(\overline{x})$.

To summarize, for any φ satisfying the constraint in (7), there is $\varphi_{k,\overline{f}}$ that dominates φ and also satisfies it. By Lemma 2, $\tilde{\varphi}_0(X) = \varphi_{k,\overline{f}}(X) + \min\{k, \underline{x}\}$ weakly dominates $\varphi_{k,\overline{f}}$. By the argument in footnote 18, $\tilde{\varphi}_0$ satisfies the constraint in (7). If

[18] Indeed, our argument applies to $\tilde{\varphi}(X) = \varphi(X) + \underline{x} - \varphi(\underline{x})$, which by Lemma 2 weakly dominates φ. Further, $u^{\varphi} \geq (\rho/\delta)\varphi(\overline{x})$ implies $u^{\tilde{\varphi}} \geq u^{\varphi} + \underline{x} - \varphi(\underline{x}) \geq (\rho/\delta)\varphi(\overline{x}) + \underline{x} - \varphi(\underline{x}) \geq (\rho/\delta)\tilde{\varphi}(\overline{x})$.

$k \leq \underline{x}$, then $\tilde{\varphi}_0$ is debt. Otherwise, $\mu^{\tilde{\varphi}_0} = \mu^{\varphi} + \underline{x}$. In this case, we repeatedly apply the same argument to construct $(\tilde{\varphi}_i)_{i=1}^{I}$ such that for all i, $\tilde{\varphi}_i$ weakly dominates $\tilde{\varphi}_{i-1}$ and $\tilde{\varphi}_i$ satisfies the constraint in (7). Since $\mu^{\tilde{\varphi}_i} = \mu^{\tilde{\varphi}_{i-1}} + \underline{x}$ and $\mu^{\tilde{\varphi}_i} \leq \mu^X$, I is finite, and so, $\tilde{\varphi}_I$ must be debt.

Part 2. Let us show that debt uniquely solves (7) when $\rho < \delta$. Consider a non-debt security φ satisfying the constraint in (7) and such that $\varphi(\underline{x}) = \underline{x}$, which is without loss of optimality by Lemma 2. We prove that there is a security $\hat{\varphi}$ that strictly dominates φ and satisfies the constraint in (7). Thus, the only solution to (7) is a debt security. By the argument above, $\varphi_{k,\overline{f}}$ weakly dominates φ, where $\overline{f} \equiv \varphi(\overline{x})$. Since φ is not debt, $k > 0$. Fix $\Delta \in (0, \min\{k, \underline{x}\})$. By Lemma 2, $\tilde{\varphi}(X) \equiv \varphi_{k,\overline{f}}(X) + \Delta$, weakly dominates $\varphi_{k,\overline{f}}$ and $u^{\tilde{\varphi}} \geq u^{\varphi} + \Delta$. Further, $u^{\tilde{\varphi}} \geq u^{\varphi} + \Delta \geq (\rho/\delta)\varphi(\overline{x}) + \Delta = (\rho/\delta)\tilde{\varphi}(\overline{x}) + \Delta(1-\rho/\delta)$, where $\Delta(\delta/\rho - 1) > 0$ by $\rho < \delta$. For any $\varepsilon \in (0, \Delta)$, consider security $\hat{\varphi}_{\varepsilon,K}(X) = \max\{\tilde{\varphi}(X) - \varepsilon, X - K\}$. Since $0 \leq \mathbb{E}_H[\hat{\varphi}_{\varepsilon,K}(X) - \hat{\varphi}_{\varepsilon,K+\epsilon}(X)] \leq \epsilon$ for $\epsilon > 0$, $\mathbb{E}_H[\hat{\varphi}_{\varepsilon,K}(X)]$ is continuous in K and it takes values between $\mu^{\tilde{\varphi}} - \varepsilon$ (for $K = \overline{x}$) and μ^X (for $K = 0$). Hence, there is $K(\varepsilon)$ such that $\mathbb{E}_H\left[\hat{\varphi}_{\varepsilon,K(\varepsilon)}(X)\right] = \mu^{\tilde{\varphi}}$. Further, $\hat{\varphi}_{\varepsilon,K(\varepsilon)}$ converges point-wise to $\tilde{\varphi}$ as $\varepsilon \to 0$. Hence, for sufficiently small $\varepsilon > 0$, $\tilde{\varphi}(\overline{x}) + \Delta(\delta/\rho - 1) \geq \hat{\varphi}_{\varepsilon,K(\varepsilon)}(\overline{x})$, which combined with $u^{\tilde{\varphi}} \geq (\rho/\delta)\tilde{\varphi}(\overline{x}) + \Delta(1-\rho/\delta)$ implies $u^{\tilde{\varphi}} \geq (\rho/\delta)\tilde{\varphi}(\overline{x}) + \Delta(1-\rho/\delta) \geq (\rho/\delta)\hat{\varphi}_{\varepsilon,K(\varepsilon)}(\overline{x})$. Choose one such $\varepsilon > 0$ and let $\hat{\varphi} = \hat{\varphi}_{\varepsilon,K(\varepsilon)}$.

By construction, $\hat{\varphi}$ is more informationally sensitive than $\tilde{\varphi}$, and $\left[l^{\tilde{\varphi}}, u^{\tilde{\varphi}}\right] \subseteq [\tilde{\varphi}(\underline{x}), \tilde{\varphi}(\overline{x})]$. Further, for $x \in \left[\overline{f} + k, \overline{x}\right]$, $\varphi_{k,\overline{f}}(x) = \overline{f}$, and so, $\tilde{\varphi}(x) = \overline{f} + \Delta$. Hence, $\tilde{\varphi}(x) = \overline{f} + \Delta < x - K(\varepsilon) = \hat{\varphi}(x)$ for x in some left neighborhood of $\overline{x}$, and $\tilde{\varphi}(x) = \hat{\varphi}(x) + \varepsilon$ for x in some right neighborhood of $\underline{x}$. Hence, there is $\epsilon > 0$ such that $H^{\hat{\varphi}}(f) < 1 = H^{\tilde{\varphi}}(f)$ for $f \in (\hat{\varphi}(\overline{x}) - \epsilon, \hat{\varphi}(\overline{x}))$, $H^{\hat{\varphi}}(f) > 0 = H^{\tilde{\varphi}}(f)$ for $f \in (\hat{\varphi}(\underline{x}), \hat{\varphi}(\underline{x}) + \epsilon)$, and $\left[l^{\tilde{\varphi}}, u^{\tilde{\varphi}}\right] \subseteq [\tilde{\varphi}(\underline{x}), \tilde{\varphi}(\overline{x})] \subset (\hat{\varphi}(\underline{x}), \hat{\varphi}(\overline{x}))$. By Theorem 1, $\boldsymbol{V}(\hat{\varphi}) > \boldsymbol{V}(\tilde{\varphi})$, which given that $\mu^{\tilde{\varphi}} = \mu^{\hat{\varphi}}$ means that $u^{\hat{\varphi}} > u^{\tilde{\varphi}}$. As we argued above, $u^{\tilde{\varphi}} \geq (\rho/\delta)\hat{\varphi}(\overline{x})$, and so, $\hat{\varphi}$ satisfies the constraint in (7) and strictly dominates $\tilde{\varphi}$.

To summarize, any non-debt security φ satisfying the constraint in (7) is dominated by $\hat{\varphi}$ that also satisfies the constraint. This combined with Part 1 that establishes that there is a debt security solving (7), gives the desired uniqueness of debt as solution to (7). □

Proof of Theorem 5. Consider AT1 debt $\varphi_{D,L}$ parametrized by L and D. For a fixed $D \in [0, \overline{x}]$, since H admits a density h on $\mathcal{X}$, $\mathbb{E}_H[\varphi_{D,L}(X)] = \int_L^{\overline{x}} \min\{x, D\}\, h(x)\, \mathrm{d}x$ is continuous in L and ranges from 0 (for $L = \overline{x}$) to $\mu^D = \mathbb{E}_H[\min\{X, D\}]$ (for $L = \underline{x}$).

Hence, for any $\mu \in \left[0, \mu^D\right]$, there is L such that $\mathbb{E}_H\left[\varphi_{D,L}(X)\right] = \mu$. Further, given the same average payoff, an AT1 debt is more informationally sensitive than any other security $\varphi \in \Phi_1$ satisfying $\varphi(\overline{x}) = D$. By Theorem 1, there is a live-or-die security that solves (7) for $\Phi = \Phi_1$. The proof of $L^* > \underline{x}$ and $\pi(\varphi_{D^*,L^*}) = 0$ follows from the same argument as in Theorem 3. □

Online Appendix (Not for Publication)

Auxiliary Results

In the paper, we use the following integration by parts formula for Lebesgue-Stieltjes integrals.

Lemma 5 (**Integration by parts**). *Suppose random variable F is distributed according to the c.d.f. H with support $\left[\underline{f},\overline{f}\right]$. Then, for any $y \in \left[\underline{f},\overline{f}\right]$, $\int_{\underline{f}}^{y} f \mathrm{d}H(f) = yH(y) - \int_{\underline{f}}^{y} H(f)\,\mathrm{d}f$.*

Proof. Let $H^{-}(f)$ be the left-continuous regularization of H with the convention that $H^{-}\left(\underline{f}\right) = 0$. Then, $\int_{\underline{f}}^{y} f\mathrm{d}H(f) = yH(y) - \int_{\underline{f}}^{y} H^{-}(f)\,\mathrm{d}f = yH(y) - \int_{\underline{f}}^{y} H(f)\,\mathrm{d}f$, where the first equality is by Theorem VI.90 in Dellacherie and Meyer (1982, Probabilities and Potential B, Theory of Martingales, Elsevier), and the second equality is by the fact that a monotone function can have at most a countable number of discontinuities, and so, $H^{-}(f) = H(f)$ almost everywhere. □

Lemma 6. *$G^{\varphi} \in \mathcal{G}^{\varphi}$ if and only if $\mathbb{E}_{G^{\varphi}}[Z] = \mathbb{E}_{H^{\varphi}}[F]$, and $\int_{-\infty}^{y} H^{\varphi}(f)\,\mathrm{d}f \geq \int_{-\infty}^{y} G^{\varphi}(z)\,\mathrm{d}z$ for all y.*

Proof of Lemma 6. Consider a signal Z about $F = \varphi(X)$ described by the probability space $(\mathcal{F} \times \mathcal{Z}, \mathscr{F} \times \mathscr{Z}, \nu_{F,Z})$. Here, $\mathcal{F} \equiv \varphi(\mathcal{X})$ is the set of payoffs of security F, $\mathcal{Z}$ is a sufficiently rich Polish space of signal realizations (in particular, $\mathcal{F} \subseteq \mathcal{Z}$), and (Z, F) is distributed according to the probability measure $\nu_{F,Z}$ on the product of Borel σ-algebras, $\mathscr{F} \times \mathscr{Z}$, with $\mathrm{marg}_{\mathcal{F}}\nu_{F,Z} = H^{\varphi}$. If $\mathcal{J}^{\varphi}$ is the set of all possible CDFs of unbiased signals Z about F (for which $Z = \mathbb{E}[\varphi(X)|Z]$ almost surely), then by the Strassen theorem (Theorem 7.A.1 in Shaked and Shanthikumar 2007), $G^{\varphi} \in \mathcal{J}^{\varphi}$ if and only if $\mathbb{E}_{G^{\varphi}}[Z] = \mathbb{E}_{H^{\varphi}}[F]$, and G^{φ} second-order stochastically dominates H^{φ}:$\int_{-\infty}^{y} H^{\varphi}(f)\,\mathrm{d}f \geq \int_{-\infty}^{y} G^{\varphi}(z)\,\mathrm{d}z$ for all y. To prove the lemma, we show that $\mathcal{J}^{\varphi} = \mathcal{G}^{\varphi}$.

To show $\mathcal{G}^{\varphi} \subseteq \mathcal{J}^{\varphi}$, consider any $G^{\varphi} \in \mathcal{G}^{\varphi}$ and a corresponding signal S about X described by the probability space $(\mathcal{X} \times \mathcal{S}, \mathscr{X} \times \mathscr{S}, \nu_{X,S})$ such that G^{φ} is the CDF of $Z = \mathbb{E}[\varphi(X)|S]$. Then, (X, Z) is distributed according to the probability measure $\nu_{X,Z}$ on the probability space $(\mathcal{X} \times \mathcal{Z}, \mathscr{X} \times \mathscr{Z}, \nu_{X,Z})$ with $\mathcal{Z} \equiv \{\mathbb{E}[\varphi(X)|S = s], s \in \mathcal{S}\}$ and $\nu_{X,Z}(b_X \times b_Z) \equiv \mathbb{P}_{\nu_{X,S}}[X \in b_X, \mathbb{E}[\varphi(X)|S] \in b_Z]$, $b_X \times b_Z \subseteq \mathscr{X} \times \mathcal{Z}$. By the law of iterated expectations, $\mathbb{E}[\varphi(X)|Z] = \mathbb{E}[\mathbb{E}[\varphi(X)|S]|Z] =$

Z almost surely. Hence, Z is an unbiased signal about F described by the probability space $(\mathcal{F} \times \mathcal{Z}, \mathscr{F} \times \mathscr{Z}, \nu_{F,Z})$ with $\text{marg}_{\mathcal{Z}}\nu_{F,Z} = G^{\varphi}$, where $\nu_{F,Z}(b_F \times b_Z) \equiv \nu_{X,Z}(\varphi^{-1}(b_F) \times b_Z)$ for any $(b_F, b_Z) \in \mathscr{F} \times \mathscr{Z}$. Thus, $G^{\varphi} \in \mathcal{J}^{\varphi}$.

To show $\mathcal{J}^{\varphi} \subseteq \mathcal{G}^{\varphi}$, consider any $G^{\varphi} \in \mathcal{J}^{\varphi}$ and a corresponding signal Z about F described by the probability space $(\mathcal{F} \times \mathcal{Z}, \mathscr{F} \times \mathscr{Z}, \nu_{F,Z})$ with $\text{marg}_{\mathcal{Z}}\nu_{F,Z} = G^{\varphi}$. Let $\mathcal{S} = \mathcal{Z}$ and $\mathscr{S} = \mathscr{Z}$. For any x such that $[\underline{x}, x] \in \varphi^{-1}(\mathscr{F})$ and any $b_Z \in \mathscr{Z}$, define $\nu_{X,Z}([\underline{x}, x] \times b_Z) \equiv \mathbb{P}_{\nu_{F,Z}}[F \leq \varphi(x), Z \in b_Z]$. Next, consider any x such that $[\underline{x}, x] \notin \varphi^{-1}(\mathscr{F})$, which is the case when φ is flat in some neighborhood of x. Let $[\check{x}, \hat{x})$ be the largest interval on which φ is constant and equals $\varphi(x)$. Then, $[\underline{x}, \check{x}) \in \varphi^{-1}(\mathscr{F})$ and $[\check{x}, \hat{x}) \in \varphi^{-1}(\mathscr{F})$. For any $b_Z \in \mathscr{Z}$, define $\nu_{X,Z}([\underline{x}, x] \times b_Z) \equiv \mathbb{P}_{\nu_{F,Z}}[F < \varphi(x), Z \in b_Z] + \mathbb{P}_{\nu_{F,Z}}[F = \varphi(x), Z \in b_Z]\,\mathbb{P}_H[X \in [\underline{x}, x] | X \in [\check{x}, \hat{x}]]$ .By construction, $\text{marg}_{\mathcal{X}}\nu_{X,Z} = H$. Hence, we specified a signal Z about X described by the probability space $(\mathcal{X} \times \mathcal{Z}, \mathscr{X} \times \mathscr{Z}, \nu_{X,Z})$ with $\text{marg}_{\mathcal{X}}\nu_{X,Z} = H$ and $\text{marg}_{\mathcal{Z}}\nu_{X,Z} = G^{\varphi}$. Thus, $G^{\varphi} \in G^{\varphi}$, which completes the proof of $\mathcal{J}^{\varphi} = G^{\varphi}$. $\square$

Omitted Proofs

Proof of Proposition 1. By Proposition 2 in Kartik and Zhong (2023), the minimal liquidity supplier's profit over all $G^{\varphi} \in \mathcal{G}^{\varphi}$ equals $\underline{\Pi} \equiv \mu^{\varphi} - \delta u^{\varphi}$ and is attained at $G_{u^{\varphi}, \mu^{\varphi}}$. By Theorem 2 in Kartik and Zhong (2023), for any $G^{\varphi} \in \mathcal{G}^{\varphi}$, $V(G^{\varphi}) \leq (1 - \delta)\mu^{\varphi} - \underline{\Pi} = \delta(u^{\varphi} - \mu^{\varphi})$. Since $G_{u^{\varphi}, \mu^{\varphi}}$ attains this upper bound, $\boldsymbol{V}(\varphi) = \max_{G^{\varphi} \in \mathcal{G}^{\varphi}} V(G^{\varphi}) = \delta(u^{\varphi} - \mu^{\varphi})$. For any G^{φ} satisfying (i) and (ii) in the statement of the proposition, $V(G^{\varphi}) = \boldsymbol{V}(\varphi)$, and so, G^{φ} is optimal for φ. Conversely, if G^{φ} is optimal, then $V(G^{\varphi}) = \delta(u^{\varphi} - \mu^{\varphi})$, and so, inequality $V(G^{\varphi}) \leq (1 - \delta)\mu^{\varphi} - \underline{\Pi}$ cannot be strict, which implies that trade always happens under G^{φ}. This in turn implies that $u(G^{\varphi}) = u^{\varphi}$, otherwise, we get a contradiction to optimality of either G^{φ} or $G_{u^{\varphi}, \mu^{\varphi}}$. $\square$

Proof of Lemma 1. The constraint in (4) is $\int_{-\infty}^{y}(H^{\varphi}(z) - G_{u,\mu^{\varphi}}(z))\,\mathrm{d}z \geq 0$, which given equation (3) for $G_{u,\mu^{\varphi}}$ and $l = \left(\frac{\mu^{\varphi} - \delta u}{1-\delta}\right)^{1-\delta} u^{\delta}$, is equivalent to $\mathcal{L}(y|\varphi, u) \geq 0, y \in [l, u]$, $u \leq \varphi(\overline{x})$, and $l \geq \varphi(\underline{x})$ (the latter is equivalent to $\mathcal{L}(\varphi(\underline{x})|\varphi, u) \geq 0$).

Inequalities $\mathcal{L}(y|\varphi, u) \geq 0, y \in \{\varphi(\underline{x})\} \cup [l, u]$ imply that, for $y \in [0, l)$,

$$\begin{aligned}\mathcal{L}(y|\varphi, u^\varphi) &= \mu^\varphi - \delta u^\varphi - (1-\delta) y^{1/(1-\delta)} (u^\varphi)^{-\delta/(1-\delta)} + \int_{-\infty}^{y} H^\varphi(x)\, \mathrm{d}x \\ &> \mu^\varphi - \delta u^\varphi - (1-\delta) l^{1/(1-\delta)} (u^\varphi)^{-\delta/(1-\delta)} + \int_{-\infty}^{y} H^\varphi(x)\, \mathrm{d}x = \int_{-\infty}^{y} H^\varphi(x)\, \mathrm{d}x \geq 0,\end{aligned}$$

where the first inequality is by $y < l$; the first equality is by integration by parts; the second equality is by $l = \left(\frac{\mu^\varphi/u^\varphi - \delta}{1-\delta}\right)^{1-\delta} u^\varphi$. Thus, the constraint in (4) is also equivalent to (5) and $u \leq \varphi(\overline{x})$, which is the desired conclusion. □

Proof of Lemma 2. Since $\varphi \in \Phi_1$ and $\tilde{\varphi}$ satisfies limited liability, $\tilde{\varphi} \in \Phi_1$. By Proposition 1, there is an optimal signal distribution for security φ, G^φ, and under any such G^φ, the liquidity supplier always trades at price δu^φ. Let $G^{\tilde{\varphi}}(z) \equiv G^\varphi(z - \Delta)$ for all z. Since $G^\varphi \in \mathcal{G}^\varphi$, $G^{\tilde{\varphi}} \in \mathcal{G}^{\tilde{\varphi}}$. By Lemma 4 in Biais and Mariotti (2005), under $G^{\tilde{\varphi}}$, the liquidity supplier optimally chooses a screening cutoff type that is weakly greater than $u^\varphi + \Delta = u\left(G^{\tilde{\varphi}}\right)$. Hence, the liquidity supplier finds it optimal under $G^{\tilde{\varphi}}$ to offer $\delta u\left(G^{\tilde{\varphi}}\right) = \delta(u^\varphi + \Delta)$ and buys from all types. Thus, $\boldsymbol{V}(\tilde{\varphi}) \geq V\left(G^{\tilde{\varphi}}\right) = \delta\left(u^\varphi + \Delta - \mu^{\tilde{\varphi}}\right) = \delta(u^\varphi - \mu^\varphi) = V(G^\varphi) = \boldsymbol{V}(\varphi)$. By Proposition 1, $\boldsymbol{V}(\tilde{\varphi}) = \delta\left(u^{\tilde{\varphi}} - \mu^{\tilde{\varphi}}\right)$, and so, $u^{\tilde{\varphi}} \geq u^\varphi + \Delta$. □

Proof of Lemma 3. Let L be the maximal $L \geq \underline{x}$ such that $\varphi(X)\mathbf{1}\{X \leq L\} = 0$ with probability 1. Since $\mu^\varphi > \delta u^\varphi \geq 0$, $L < \overline{x}$. By Lemma 1, for $y \in [0, u^\varphi]$, $\mathcal{L}(y|\varphi, u^\varphi) = \varphi(\overline{x}) - \delta u^\varphi - (1-\delta) y^{1/(1-\delta)} (u^\varphi)^{-\delta/(1-\delta)} - \int_y^{\varphi(\overline{x})} H^\varphi(f)\, \mathrm{d}f \geq 0$. Consider $\tilde{\varphi}(X) = \varphi(X)\mathbf{1}\left\{X \geq \tilde{L}\right\}, \tilde{L} > L$. By $\tilde{L} > L$, $H^{\tilde{\varphi}}(f) = H^\varphi(f)$ for $f \geq \varphi\left(\tilde{L}\right)$, and so, $\mathcal{L}(y|\tilde{\varphi}, u^\varphi) = \mathcal{L}(y|\varphi, u^\varphi) \geq 0$ for $y \in [0, u^\varphi] \cap \left[\varphi\left(\tilde{L}\right), \infty\right)$. For $y \in \left[0, \varphi\left(\tilde{L}\right)\right)$, $\mathcal{L}_y(y|\tilde{\varphi}, u^\varphi) = -(y/u^\varphi)^{\delta/(1-\delta)} + H\left(\tilde{L}\right)$ is strictly decreasing in y. Moreover, $\mathcal{L}(y|\tilde{\varphi}, u^\varphi)$ is continuous at $y = \varphi\left(\tilde{L}\right)$. Hence, $\mathcal{L}(y|\tilde{\varphi}, u^\varphi)$ is strictly concave in y on $\left[0, \varphi\left(\tilde{L}\right)\right]$, and it attains its minima on $\left[0, \tilde{L}\right]$ at $y = 0$ or $y = \varphi\left(\tilde{L}\right)$. We showed that $\mathcal{L}\left(\varphi\left(\tilde{L}\right)\middle|\tilde{\varphi}, u^\varphi\right) = \mathcal{L}\left(\varphi\left(\tilde{L}\right)\middle|\varphi, u^\varphi\right) \geq 0$. Further, $\mathcal{L}(0|\tilde{\varphi}, u^\varphi) \geq 0$ is equivalent to $\mu^{\tilde{\varphi}} - \delta u^\varphi \geq 0$. Since H admits a density h on $\mathcal{X}$, $\mathbb{E}_H[\tilde{\varphi}(X)] = \int_{\tilde{L}}^{\varphi(\overline{x})} \varphi(x) h(x)\, \mathrm{d}x$ is continuous in $\tilde{L}$. This together with $\mu^\varphi - \delta u^\varphi > 0$ implies that for $\tilde{L}$ sufficiently close to L, $\mu^{\tilde{\varphi}} - \delta u^\varphi \geq 0$, and so, $\mathcal{L}(0|\tilde{\varphi}, u^\varphi) \geq 0$. Thus, for such $\tilde{L}$, $\mathcal{L}(y|\tilde{\varphi}, u^\varphi) \geq 0$ on $y \in [0, u^\varphi]$. By Lemma 1, $u^{\tilde{\varphi}} \geq u^\varphi$, and so, $\boldsymbol{V}(\tilde{\varphi}) \geq \delta\left(u^\varphi - \mu^{\tilde{\varphi}}\right) > \delta(u^\varphi - \mu^\varphi) = \boldsymbol{V}(\varphi)$, which is the desired conclusion. □

Proof of Proposition 2. We first argue that for any $\varphi \in \Phi_1$ with $\varphi(\overline{x}) < \overline{x}$, there is $\tilde{\varphi} \in \Phi_1$ with $\tilde{\varphi}(\overline{x}) = \overline{x}$ such that $\boldsymbol{V}(\tilde{\varphi}) > \boldsymbol{V}(\varphi)$. Consider a $\varphi \in \Phi_1$ with $\varphi(\overline{x}) < \overline{x}$. By Lemma 2, if it were that $\varphi(\underline{x}) < \underline{x}$, then there is $\varepsilon > 0$ such that $\varphi(X) + \varepsilon \in \Phi_1$ and $\boldsymbol{V}(\varphi + \varepsilon) \geq \boldsymbol{V}(\varphi)$. Hence, without loss of generality, suppose that $\varphi(\underline{x}) = \underline{x}$. Consider $\tilde{\varphi}$ such that $\tilde{\varphi}(\overline{x}) = \varphi(\overline{x}) + \overline{\varepsilon}$ and $\tilde{\varphi}(\underline{x}) = \varphi(\underline{x}) - \underline{\varepsilon}$ for some $\underline{\varepsilon}, \overline{\varepsilon} > 0$ such that $\tilde{\varphi} \in \Phi_1$ and $\gamma\overline{\varepsilon} = (1-\gamma)\underline{\varepsilon}$. By construction, $\mu^{\tilde{\varphi}} = \mu^{\varphi}$, $\tilde{\varphi}$ is more informationally sensitive than φ, $H^{\tilde{\varphi}}(f) = 1 - \gamma > 0 = H^{\varphi}(f)$ for $y \in (\tilde{\varphi}(\underline{x}), \tilde{\varphi}(\underline{x}) + \underline{\varepsilon})$ and $H^{\tilde{\varphi}}(f) = 1 - \gamma < 1 = H^{\varphi}(f)$ for all $y \in (\tilde{\varphi}(\overline{x}) - \overline{\varepsilon}, \tilde{\varphi}(\overline{x}))$, and $[l^{\varphi}, u^{\varphi}] \subseteq [\varphi(\underline{x}), \varphi(\overline{x})] \subset (\tilde{\varphi}(\underline{x}), \tilde{\varphi}(\overline{x}))$. By Theorem 1, $\boldsymbol{V}(\tilde{\varphi}) > \boldsymbol{V}(\varphi)$, and so, any optimal security satisfies $\varphi(\overline{x}) = \overline{x}$.

By Lemma 1 and X taking only two values, for any $\varphi \in \Phi_1$, u^{φ} is the largest $u \leq \varphi(\overline{x})$ satisfying

$$\mathcal{L}(y|\varphi, u) = \mu^{\varphi} - \delta u - (1-\delta) y^{1/(1-\delta)} u^{-\delta/(1-\delta)} + \max\{0, y - \varphi(\underline{x})\}(1-\gamma) \geq 0, y \in [0, u].$$

Note that it is sufficient to check that this inequality holds for $y \in [\varphi(\underline{x}), u]$. Since $\mathcal{L}_y(y|\varphi, u) = 1 - \gamma - (y/u)^{\delta/(1-\delta)}$, $\mathcal{L}(y|\varphi, u)$ is strictly concave in y, hence, it attains its minimum at $y = u$ or $y = \varphi(\underline{x})$. That $\mathcal{L}(u|\varphi, u) \geq 0$ follows from $u \leq \varphi(\overline{x})$. Thus,

$$u^{\varphi} = \max\left\{u \in [\varphi(\underline{x}), \varphi(\overline{x})] : \mathcal{L}(\varphi(\underline{x})|\varphi, u) = \mu^{\varphi} - \delta u - (1-\delta)\varphi(\underline{x})^{\frac{1}{1-\delta}} u^{-\frac{\delta}{1-\delta}} \geq 0\right\}. \tag{10}$$

For φ that is not a safe debt (with $\mu^{\varphi} > \varphi(\underline{x})$), $u^{\varphi} > \varphi(\underline{x})$. Since $\frac{\partial}{\partial u}\mathcal{L}(\varphi(\underline{x})|\varphi, u) = -\delta\left(1 - (\varphi(\underline{x})/u)^{1/(1-\delta)}\right) < 0$ for $u > \varphi(\underline{x})$, if $l^{\varphi} > \varphi(\underline{x})$ (equivalently, $\mathcal{L}(\varphi(\underline{x})|\varphi, u^{\varphi}) > 0$), then $u^{\varphi} = \varphi(\overline{x})$.

Suppose $l^X > \underline{x}$. We show that $\varphi(X) = X$ is suboptimal. By the argument above, because $l^X > \varphi(\underline{x}) = \underline{x}$, $\mathcal{L}\left(\underline{x}\middle|X, u^X\right) > 0$ and $u^X = \varphi(\overline{x}) = \overline{x}$. Consider $\tilde{\varphi}$ with $\tilde{\varphi}(\overline{x}) = \overline{x}$ and $\tilde{\varphi}(\underline{x}) = \underline{x} - \varepsilon$ for some $\varepsilon > 0$ such that $\mathcal{L}\left(\tilde{\varphi}(\underline{x})\middle|\tilde{\varphi}, u^X\right) > 0$, which exists by $\mathcal{L}\left(\underline{x}\middle|X, u^X\right) > 0$. This implies that $u^{\tilde{\varphi}} = u^X$. By construction, $\mu^{\tilde{\varphi}} < \mu^X$. Therefore, $\boldsymbol{V}(\tilde{\varphi}) = \delta\left(u^X - \mu^{\tilde{\varphi}}\right) > \delta\left(u^X - \mu^X\right) = \boldsymbol{V}(\varphi)$, which proves that $\varphi(X) = X$ cannot be optimal. □

Proof of Lemma 4. The "if" direction is trivial. To prove the "only if" part, suppose to contradiction $u^{\varphi} < (\rho/\delta)\varphi(\overline{x})$ but security φ is always sold at price $p \geq \rho\varphi(\overline{x})$. The latter implies that the issuer's payoff equals $p - \delta\mu^{\varphi} \geq \rho\varphi(\overline{x}) - \delta\mu^{\varphi} > \delta(u^{\varphi} - \mu^{\varphi})$,

	$X=\underline{x}$	$X=\overline{x}$
B	$\tau^I(1-\gamma)$	0
G	$\left(1-\tau^I\right)(1-\gamma)$	γ

(a) Signal S^I

	$X=\underline{x}$	$X=\overline{x}$
B	$1-\gamma$	$(1-\tau^{II})\gamma$
G	0	$\tau^{II}\gamma$

(b) Signal S^{II}

Table 2: ***Signal distributions***

Tables describe joint distributions of signals S^I, S^{II} and cash flows X. Parameter τ controls the precision of signals with $\tau = 0$ corresponding to uninformative signals and $\tau = 1$ corresponding to perfectly revealing signals.

which contradicts Proposition 1. □

Model with Two States and Two Signals

Consider the model with two states: $X = \underline{x}$ with probability $1-\gamma$ and $X = \overline{x}$ with probability $\gamma \in (0,1)$. We assume that

$$(1-\delta)\underline{x}(1-\gamma) > \gamma\overline{x} + (1-\gamma)\underline{x} - \delta\overline{x} \tag{11}$$

so that if the issuer perfectly learns X, the liquidity supplier prefers to screen and offers $\delta\underline{x}$ rather than making the pooling offer $\delta\overline{x}$.

Consider the two binary signals S^I and S^{II} in Table 2. Signal distribution S^I in Table 2a perfectly reveals "bad news" that $X = \underline{x}$ when $S^I = B$, while signal $S^I = G$ leads to the posterior probability of $\overline{x}$ equal to $\frac{\gamma}{1-(1-\gamma)\tau}$. Signal S^{II} in Table 1b perfectly reveals "good news" that $X = \overline{x}$ when $S^{II} = G$, whereas signal $S^{II} = B$ leads to the posterior probability of $\overline{x}$ equal to $\frac{(1-\tau)\gamma}{(1-\tau)\gamma+1-\gamma}$. The precision of the signal distributions is parameterized by $\tau^i \in [0,1]$, $i \in \{I, II\}$. We solve for the optimal security $\varphi^i \in \Phi_2$ and precision $\tau^i \in [0,1]$ for each type of signal, $i \in \{I, II\}$. By Lemma 2, it is without loss to assume that $\varphi^i(\underline{x}) = \underline{x}$, $i \in \{I, II\}$. Thus, the issuer's problem in each case boils down to finding $(\varphi^i(\overline{x}), \tau^i)$ to maximize his expected payoff.

Proposition 4. *Suppose that inequality (11) holds. Then,*

1. $\varphi^I(\overline{x}) = \overline{x}$, $\tau^I = \frac{1}{1-\gamma}\left(1 - 2\delta\left(1+\sqrt{1+\frac{4\delta(1-\delta)\underline{x}}{\gamma(\overline{x}-\underline{x})}}\right)^{-1}\right)$;
2. $\varphi^{II}(\overline{x}) = \delta\underline{x}(1-\gamma)/(\delta-\gamma)$, $\tau^{II} = 1$.

Proof. We solve each case separately.

Perfectly revealing bad news: We first fix the value of $\varphi^I(\overline{x})$ and solve for τ^I. The issuer gets positive information rents only if the liquidity supplier prefers the pooling offer $p = \delta\mathbb{E}[\varphi(X)|\text{G}]$ to the screening offer $p = \delta\underline{x}$ that is only accepted by B-type (with probability $\tau^I(1-\gamma)$). Thus, it is necessary that

$$(1-\delta)\,\underline{x}\tau^I(1-\gamma) \leq \mu^{\varphi^I} - \delta\mathbb{E}\left[\varphi^I(X)|\text{G}\right]. \tag{12}$$

In this case, the issuer's expected payoff equals $\delta\left(\mathbb{E}\left[\varphi^I(X)|\text{G}\right] - \mu^{\varphi^I}\right)$. Since $\mathbb{E}\left[\varphi^I(X)|\text{G}\right] = \underline{x} + \left(\varphi^I(\overline{x}) - \underline{x}\right)\frac{\gamma}{1-(1-\gamma)\tau^I}$, making signal S^I more precise (by increasing τ^I) increases the issuer's expected payoff but tightens the constraint (12). Thus, the optimal value τ^I is the highest value that makes (12) bind (unless $\tau^I = 1$). Given $\mathbb{E}\left[\varphi^I(X)|\text{G}\right] = \underline{x} + \frac{\gamma(\varphi^I(\overline{x})-\underline{x})}{1-(1-\gamma)\tau^I}$ and $\mu^{\varphi^I} = \varphi^I(\overline{x})\gamma + \underline{x}(1-\gamma)$, we can re-write (12) as

$$0 \leq (\varphi^I(\overline{x}) - \underline{x})\left(\gamma - \frac{\gamma\delta}{1-(1-\gamma)\tau^I}\right) + \underline{x}(1-\delta)\left(1-(1-\gamma)\tau^I\right),$$

or equivalently,

$$\left(\varphi^I(\overline{x}) - \underline{x}\right)\left(\frac{\gamma\delta}{(1-(1-\gamma)\tau^I)^2} - \frac{\gamma}{1-(1-\gamma)\tau^I}\right) - \underline{x}(1-\delta) \leq 0.$$

Let us denote $a = \frac{\gamma\left(\varphi^I(\overline{x})-\underline{x}\right)}{1-(1-\gamma)\tau^I}$. Then,

$$\frac{\delta}{\gamma(\varphi^I(\overline{x}) - \underline{x})}a^2 - a - \underline{x}(1-\delta) \leq 0.$$

The monotonicity of φ^I implies that the last inequality holds for all $a \in \left[0, a^*\left(\varphi^I(\overline{x})\right)\right]$, where

$$a^*\left(\varphi^I(\overline{x})\right) \equiv \gamma\left(\varphi^I(\overline{x}) - \underline{x}\right)\frac{1+\sqrt{1+\frac{4\delta(1-\delta)\underline{x}}{\gamma(\varphi^I(\overline{x})-\underline{x})}}}{2\delta}.$$

Thus, whenever $\tau^I < 1$,

$$\tau^I = \frac{1}{1-\gamma}\left(1 - 2\delta\left(1+\sqrt{1+\frac{4\delta(1-\delta)\underline{x}}{\gamma\left(\varphi^I(\overline{x}) - \underline{x}\right)}}\right)^{-1}\right). \tag{13}$$

Then, the requirement $\tau^I < 1$ is equivalent to

$$1+\sqrt{1+\frac{4\delta(1-\delta)\underline{x}}{\gamma\left(\varphi^I\left(\overline{x}\right)-\underline{x}\right)}}<\frac{2\delta}{\gamma}$$

or given that $\delta > \gamma$,

$$\frac{\varphi^I\left(\overline{x}\right)}{\underline{x}}>\frac{\delta\left(1-\gamma\right)}{\delta-\gamma}.$$

Note that this condition holds for $\varphi^I\left(\overline{x}\right)=\overline{x}$ by (11).

Given the optimal signal precision in (13), the issuer's expected payoff equals

$$\begin{aligned}
\delta\left(\mathbb{E}\left[\varphi^I\left(X\right)|\mathrm{G}\right]-\mu^\varphi\right) =&\delta\left(\underline{x}+\frac{\gamma(\varphi^I\left(\overline{x}\right)-\underline{x})}{1-(1-\gamma)\tau^I}-(1-\gamma)\underline{x}-\gamma\varphi^I\left(\overline{x}\right)\right)\\
=&\delta\gamma(\varphi^I\left(\overline{x}\right)-\underline{x})\frac{(1-\gamma)\tau^I}{1-(1-\gamma)\tau^I}\\
=&\delta\gamma\left(1-\frac{2\delta}{1+\sqrt{1+\frac{4\delta(1-\delta)\underline{x}}{\gamma(\varphi^I(\overline{x})-\underline{x})}}}\right)\left(\frac{1+\sqrt{1+\frac{4\delta(1-\delta)\underline{x}}{\gamma(\varphi^I(\overline{x})-\underline{x})}}}{2\delta}\right)\left(\varphi^I\left(\overline{x}\right)-\underline{x}\right)\\
=&\gamma\left(1-2\delta+\sqrt{1+\frac{4\delta(1-\delta)\underline{x}}{\gamma\left(\varphi^I\left(\overline{x}\right)-\underline{x}\right)}}\right)\left(\frac{\varphi^I\left(\overline{x}\right)-\underline{x}}{2}\right).
\end{aligned}$$

The derivative of this function with respect to $\varphi^I(\overline{x}) - \underline{x}$ equals:

$$\begin{aligned}
&\frac{\gamma}{2}\left(1-2\delta+\sqrt{1+\frac{4\delta(1-\delta)\underline{x}}{\gamma\left(\varphi^I\left(\overline{x}\right)-\underline{x}\right)}}-\frac{\frac{4\delta(1-\delta)\underline{x}}{\gamma(\varphi^I(\overline{x})-\underline{x})}}{2\sqrt{1+\frac{4\delta(1-\delta)\underline{x}}{\gamma(\varphi^I(\overline{x})-\underline{x})}}}\right)\\
=&\frac{\gamma}{2}\left(1-2\delta+\frac{2+2\frac{4\delta(1-\delta)\underline{x}}{\gamma(\varphi^I(\overline{x})-\underline{x})}-\frac{4\delta(1-\delta)\underline{x}}{\gamma(\varphi^I(\overline{x})-\underline{x})}}{2\sqrt{1+\frac{4\delta(1-\delta)\underline{x}}{\gamma(\varphi^I(\overline{x})-\underline{x})}}}\right)\\
=&\frac{\gamma}{2}\left(1-2\delta+\frac{2+\frac{4\delta(1-\delta)\underline{x}}{\gamma(\varphi^I(\overline{x})-\underline{x})}}{2\sqrt{1+\frac{4\delta(1-\delta)\underline{x}}{\gamma(\varphi^I(\overline{x})-\underline{x})}}}\right)\\
=&\frac{\gamma}{4}\left(2\left(1-2\delta\right)+\underbrace{\sqrt{1+\frac{4\delta(1-\delta)\underline{x}}{\gamma\left(\varphi^I\left(\overline{x}\right)-\underline{x}\right)}}+\frac{1}{\sqrt{1+\frac{4\delta(1-\delta)\underline{x}}{\gamma(\varphi^I(\overline{x})-\underline{x})}}}}_{\geq 2}\right)\\
\geq&\gamma\left(1-\delta\right).
\end{aligned}$$

Thus, optimal $\varphi^I(\overline{x}) = \overline{x}$.

Perfectly revealing good news: We first fix the value of $\varphi^{II}(\overline{x})$ and solve for τ^{II}. The issuer gets positive information rents only if the liquidity supplier makes a pooling offer $p = \delta\varphi^{II}(\overline{x})$ equal to the security value for type G. The liquidity supplier prefers to do so rather than make screening offer $\mathbb{E}\left[\varphi^{II}(X)|\mathrm{B}\right]$ accepted with probability $1-\gamma+\left(1-\tau^{II}\right)\gamma$ if and only if

$$(1-\delta)\,\mathbb{E}\left[\varphi^{II}(X)|\mathrm{B}\right]\left(1-\gamma+\left(1-\tau^{II}\right)\gamma\right) \leq \mu^{\varphi^{II}} - \delta\varphi^{II}(\overline{x}). \tag{14}$$

Then, the issuer's expected payoff is $\delta\left(\varphi^{II}(\overline{x}) - \mu^{\varphi^{II}}\right) = \delta\left(\varphi^{II}(\overline{x}) - \underline{x}\right)(1-\gamma)$ and is independent of the signal precision τ^{II}. Increasing informativeness of signal τ^{II} decreases the payoff from making a screening offer $\mathbb{E}\left[\varphi^{II}(X)|\mathrm{B}\right]$, as it lowers both $\mathbb{E}\left[\varphi^{II}(X)|\mathrm{B}\right]$ and the probability of acceptance. Hence, $\tau^{II} = 1$ is optimal for any φ^{II}, that is, the issuer perfectly learns the value of security. Plugging it into (14), we get $\varphi^{II}(\overline{x}) \leq \delta\underline{x}(1-\gamma)/(\delta-\gamma)$. Thus, the issuer optimally sets $\varphi^{II}(\overline{x}) = \delta\underline{x}(1-\gamma)/(\delta-\gamma)$ and $\tau^{II} = 1$. Further, inequality (11) implies that $\varphi^{II}(\overline{x}) < \overline{x}$, i.e., the issuer strictly benefits from retaining cash flows and issuing debt. □