Payments, Reserves, and Financial Fragility*

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Abstract

We propose a theory of payments that highlights a conflict between the roles of medium of exchange and store of value. We posit that payments must involve the reciprocal transfer of a scarce reserve good, which holds value for other non-payment purposes. The theory demonstrates that agents make payments only when reserves are abundant enough and when the conflict is low. Otherwise, history-dependent equilibria arise in which an agent's payment decision depends on the payment history of other agents within an equilibrium. The theory explains why payments frequently encounter delays and interruptions. Improving payment technologies may not reduce such fragility when reserves remain scarce and valuable for non-payment functions. The theory helps explain the evolution of money and payment systems, encompassing metallic payments before fiat money, modern bank payments, cross-border payments, and contemporary digital payment systems.

Keywords: payments, reserves, financial fragility, opportunity costs, technology

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1 Introduction

Money and payments play a crucial role in economic transactions, an idea widely acknowledged since Fisher (1911), Baumol (1952), and Tobin (1956). Throughout history, humans have developed various means of payment from wheat to shells to metal coins. Modern payments are based on fiat money and intermediated by banks, and the economic scale is huge.[1] Recently, payment systems have undergone significant digitization with the emergence of fast payment platforms, cryptocurrencies, and central bank digital currencies (CBDCs) (e.g., Duffie, 2019, Brunnermeier, James and Landau, 2019, Brunnermeier and Payne, 2022). Economists have long focused on technology and the ability to maintain stable prices when assessing the efficiency of money and payments. Across generations of evolution, it becomes evident that the payment systems that have thrived are indeed the ones that embrace superior technologies while maintaining stable inflation levels (Friedman and Schwartz, 1963, Hayek, 1976, Steinsson, 2023a,b).

In this paper, we argue that another equally crucial but often overlooked factor shapes the evolution of money and payment systems: financial fragility. Payments are inherently fragile, susceptible to delays and interruptions even without a fundamental shock. Historical records show frequent disruptions in payment systems, particularly in transactions involving small amounts, famously known as the "big problem for small change" (Sargent and Velde, 2001, Steinsson, 2023a). Despite technological advancements, modern bank payments encounter significant voluntary delays and interruptions during both normal and crisis times (McAndrews and Potter, 2002, Afonso and Shin, 2011, Copeland, Duffie and Yang, 2020, Afonso, Duffie, Rigon and Shin, 2022).[2] A notable recent example is the unprecedented spike in Treasury repo rates by over 1,000 basis points in September 2019 (Afonso, Cipriani, Copeland, Kovner, La Spada and Martin, 2020, Correa, Du and Liao, 2020), where dysfunctional bank payments played a key role (Copeland, Duffie and Yang, 2020, Afonso, Duffie, Rigon and Shin, 2022).[3] Understanding the root causes of such fragility is vital for comprehending the development and evolution of payment methods, and the reciprocal nature of payments naturally necessitates a dynamic model.

[1]For example, the Fedwire, the real-time gross settlement funds transfer system for financial institutions operated by the U.S. Federal Reserve Banks, sees a daily volume of more than \$4.2 U.S. trillion in 2022. Payments per se also generate huge revenues for the financial institutions that handle them: global payments revenues totaled \$2.2 U.S. trillion in 2021, roughly 3% of global GDP.

[2]The interbank lending markets, which rely on short-term credits to facilitate interbank payments, also constantly experience disruptions (e.g., Ashcraft and Duffie, 2007, Afonso, Kovner and Schoar, 2011, Ashcraft, McAndrews and Skeie, 2011, Acharya and Merrouche, 2013, Craig and Ma, 2021).

[3]Indeed, when testified before the House Financial Services Committee on June 21, 2023, Federal Reserve Chair Jerome Powell admitted that future interest hikes should avoid a repeat of the 2019 repo market and interbank payment crisis. See "Powell Haunted by Repo Crisis as Fed Aims to Cut Balance Sheet," Bloomberg, July 9, 2023.

This paper builds a new framework to understand the fragility of payments. We posit that settling any payment must involve the transfer of a *reserve* good, which has limited supply and holds value for other non-payment functions (e.g., store of value). We show that these two natural assumptions of reserve scarcity and multi-functionality go a long way: they imply that payments are intrinsically fragile, and advancements in payment technologies cannot eliminate this fragility.

The core insight of our framework lies in uncovering a fundamental conflict between two essential functions of the reserve good. On one hand, it serves as a medium of exchange for payments, while on the other hand, it fulfills non-payment functions such as being a store of value or delivering other service flows. This conflict introduces various opportunity costs when the scarce reserve is transferred as a medium of exchange in the economy. This conflict results in a trade-off in payments: while an agent may benefit from successfully transferring the reserve good to another agent for transactional purposes, she risks losing the reserve for other non-payment functions if reciprocal payments are not made in the future. The trade-off creates a dynamic coordination motive in agents' asynchronous and reciprocal payment decisions. An agent may cease making payments if it expects other agents to do the same in the future, and past payment histories endogenously arise as a coordination device even without a fundamental shock. Our model thus explains the fragility observed in payment systems and the prevalence of delays and history-dependent behaviors regardless of economic fundamentals and payment technologies.

The versatility of the reserve good concept allows us to capture many payment contexts:

- The reserve can be historically interpreted as a durable good, such as gold or silver, which is physically scarce. Apart from being used as a medium of exchange, they are also used as jewelry and conductors, as well as a store of value (Jermann, 2022).

- The reserve can be understood as central bank reserves in a fiat-based monetary system, where their short-term supply is constrained by monetary policy implementations (e.g., Acharya and Rajan, 2022, Lopez-Salido and Vissing-Jorgensen, 2023). Every interbank payment involves an irrevocable transfer of central bank reserves, which are also valuable to banks in fulfilling reserve and regulatory requirements (e.g., Correa, Du and Liao, 2020).

- The reserve can be understood as commercial bank notes, that is, deposits, whose supply is limited by banks' reserve requirements and the money multiplier (Tobin, 1965). Deposits are used by households and firms as means of payment (e.g., Diamond and Rajan, 2006, Gu, Mattesini, Monnet, and Wright, 2013, Donaldson, Piacentino and Thakor, 2018, Parlour, Rajan and Walden, 2020), while they also serve as a store of value (e.g., Stein, 2012, Dang, Gorton, Holmström, and Ordoñez, 2017) and may generate interest income for depositors.

- The reserve can represent a dominant currency like the U.S. dollar due to its extensive use in global trades and payments (e.g., Gopinath and Stein, 2021, Coppola, Krishnamurthy and Xu, 2023). Its supply is limited, and it is also valued as a global safe asset for storing value (e.g., He, Krishnamurthy and Milbradt, 2019, Jiang, Krishnamurthy and Lustig, 2021, Maggiori, Neiman and Schreger, 2021, Brunnermeier, Merkel and Sannikov, 2022) beyond the use in payments.
- The reserve can be also interpreted as a digital currency such as stablecoins or central bank digital currencies (CBDCs), whose supply is constrained by design. Beyond potential payment functions, they deliver other non-payment functions. For example, stablecoins are widely held by investors as collateral for speculating on other crypto-assets (Gorton, Klee, Ross, Ross, and Vardoulakis, 2023).

At the same time, our model incorporates realistic aspects of different reserve goods and the corresponding payment technologies, allowing us to make predictions regarding the relative fragility of different payment systems and their evolution.

Our model shows that, when the conflict between payment and non-payment (e.g., store of value) functions is low (high), the equilibrium is good (bad) in that agents always make (deny) payments to each other, irrespective of past payment histories. These two equilibrium types serve as benchmarks where the payment system functions or freezes. However, when the magnitude of the conflict falls within an intermediate range, payment decisions become history-dependent due to an endogenous asynchronous coordination motive. This can result in fragility even without any fundamental shocks. Agents anticipate reciprocal payments from others based on historical payment patterns within an equilibrium, which emerge as a coordination device. Even well-funded agents may delay or halt payments if they observe delays or halts by others in the past. These history-dependent payment behaviors and potential coordination failures contribute to the fragility widely observed in payment systems.

A methodological contribution of our paper is the development of a dynamic framework of payments in which strategic complementarity endogenously arises and payment fragility happens *within* an equilibrium.[4] Whenever multiple equilibria arise, we can further characterize the wel-

[4]In doing so, we adapt a technique in the literature of repeated games with imperfect public monitoring (e.g., Abreu, Pearce, and Stacchetti, 1990) to stochastic dynamic games. While this technique has been applied in in-

fare outcomes of *all* equilibria in closed form. Thus, our model and the solution method may also inform future studies that focus on asynchronous coordination in dynamic contexts.

Overall, our framework provides a new perspective for evaluating the historical and future evolution of payments, highlighting the fundamental conflict between the reserve good's payment and non-payment functions. Under our framework, "the big problem of small change" and the various payment crises under the metallic system (e.g., Sargent and Velde, 2001, Steinsson, 2023a) arise precisely due to the reserve good's non-payment functions outweighing its payment function. Our model provides one complementary explanation for the decline of the gold standard, and further suggests that money should possess no intrinsic value to mitigate the conflict between its payment and non-payment functions. By the same token, our model suggests that it is optimal to set payment deposits' interest rate to zero, providing an alternative explanation for why commercial banks exert market power on the deposit markets (e.g., Drechsler, Savov and Schnabl, 2017). Regarding interbank payment and repo market disruptions in 2019 and 2020, our framework attributes them to increasing bank balance sheet costs and "quantitative tightening" in recent years, which contribute to reserve scarcity and the conflict between reserves' payment and non-payment functions. This explanation is consistent with recent empirical work such as Afonso, Duffie, Rigon and Shin (2022) and Lopez-Salido and Vissing-Jorgensen (2023) that show a strong relationship between the amount of central bank reserves and payment market efficiency. Furthermore, our framework suggests that advancements in payment technologies through digitalization may not necessarily reduce payment fragility when the underlying reserves remain scarce and useful for other non-payment functions. Instead, we predict that the winning means of payment for the next generation is likely to be those that not only provide superior technologies to improve the payment function but also reduce the demand of holding reserves for other non-payment functions.

Related Literature. Our paper contributes to several branches of the literature on banking, money, and payments. First, our paper is closely related to the theoretical literature on banking, coordination, and financial stability (Diamond and Dybvig, 1983, Allen and Gale, 2000, Diamond and Rajan, 2005, Goldstein and Pauzner, 2005). The most closely related paper is Diamond and Rajan (2006), who analyze the conflict between money's payment function and its "fiscal" function, that is, its use for paying taxes. They demonstrate how this conflict can lead to bank fragility

dustrial organization (e.g., Athey, Bagwell and Sanchirico, 2004), contract economics (Levin, 2003, Halac, 2012), and policy designs (Chang, 1998, Phelan and Stacchetti, 2001, Athey, Atkeson, and Kehoe, 2005), it has not been explored in the contexts of banking, finance, and payments. Notably, our work makes progress by analytically characterizing the set of equilibrium welfare outcomes, in contrast to the existing literature that heavily relies on numerical methods.

when the demand for payments is high, as depositors are incentivized to store money in banks for fiscal purposes but also need to withdraw it for payments. Complementary to their insights, we introduce the notion of reserve scarcity arising from the conflict between reserves' payment and all non-payment functions, and explore the implications on payment fragility. Also relatedly, Freixas, Parigi and Rochet (2000), Donaldson, Piacentino and Thakor (2018), Bolton, Li, Wang, and Yang (2020), Parlour, Rajan and Walden (2020), and Li and Li (2021) show that payment risks lead to inefficient and unstable bank lending through banks' liquidity management.[5] The empirical literature has also widely documented the coordination and history-dependent natures of interbank payments and explored its consequences on financial fragility (e.g., McAndrews and Potter, 2002, Bech and Garratt, 2003, Afonso and Shin, 2011, Copeland, Duffie and Yang, 2020, Afonso, Duffie, Rigon and Shin, 2022). Our key contribution is to uncover a root of the emergence of fragility in the payment context: reserve scarcity. Our message is general: the history-dependence of payments and the resulting fragility may happen regardless of the actual forms (e.g., whether they are intermediated by banks) as long as payments involve transfers of a scarce reserve good that also holds value for non-payment functions. Notably, strategic complementarity endogenously arises in our model even if the stage game does not feature coordination, and fragility may arise within an equilibrium (rather than requiring the switch or selection between multiple equilibria). Therefore, our framework differs not only from static coordination problems (e.g., Diamond and Dybvig, 1983, Morris and Shin, 1998, Goldstein and Pauzner, 2005), but also from Frankel and Pauzner (2000) and He and Xiong (2012) in which fundamental shocks serve as a coordination device to select an equilibrium.

Our paper also contributes to the literature of money and payments (see Kahn and Roberds, 2009, for an early review). Macroeconomic models have increasingly and explicitly incorporated the payment role of money and payment risks, demonstrating their significant impact on macroeconomic outcomes and optimal policy design (e.g., Lagos and Wright, 2005, Lagos and Zhang, 2020, Bianchi and Bigio, 2021, Piazzesi, Rogers and Schneider, 2021, Piazzesi and Schneider, 2021, Bigio, 2022, Bigio and Sannikov, 2023). On the microeconomic side, this literature has experienced a recent revival thanks to the fast development of new payment technologies in the last decade,[6] and a growing literature has explored the potential of next-generation payment systems

[5]Another related literature focuses on bank liquidity management due to uncertainty, asymmetric information, or counterparty risks (e.g., Caballero and Krishnamurthy, 2008, Allen, Carletti and Gale, 2009, Acharya and Skeie, 2011, Gale and Yorulmazer, 2013, Heider, Hoerova and Holthausen, 2015).

[6]Recent empirical literature shows that new payment technologies improve economic efficiencies in consumption, investment, and lending decisions (e.g. Jack and Suri, 2014, Muralidharan, Niehaus, and Sukhtankar, 2016, Higgins, 2020, Ghosh, Vallee, and Zeng, 2022) without examining the financial stability implications.

including stablecoins and CBDCs (see, e.g., Duffie, 2019, Auer, Frost, Gambacorta, Monnet, Rice and Shin, 2022, Brunnermeier and Payne, 2022, for surveys). Complementing this literature, we highlight an intrinsic yet understudied friction in payments: the scarcity of reserves and the conflict between their payment and non-payment functions. We stress that improvements in payment technology may not necessarily reduce payment fragility if reserves remain scarce and valuable for other non-payment functions.

Our paper also joins a literature that studies the implications of monetary policy tightening on financial stability. Since Fisher (1933) and Friedman and Schwartz (1963), it has been recognized that aggregate reserve scarcity may lead to deflation and economic depression, and Brunnermeier and Sannikov (2016) formulate that idea in a model with financial intermediation and different forms of money. Instead of focusing on price levels and money's store of value function, we take a complementary view and show that reserve scarcity has direct financial fragility implications when reserves are used as a medium of exchange. Recently, Acharya and Rajan (2022) argue that the effect of reserves on liquidity provision should be understood in conjunction with banks' endogenous reserve-holding behaviors, which our model highlights. Lopez-Salido and Vissing-Jorgensen (2023) study the effects of quantitative tightening and show that deposit-adjusted reserves play a key role in determining important financial market rates such as the federal funds rate and repo rates. Our framework also complements a fast-growing literature that explores the causes and consequences of the disruptions in interbank payments in September 2019.[7]

Our framework is also inspired by the large literature that highlights the endogenous emergence of money as a means of payment (e.g., Kiyotaki and Wright, 1989, Kocherlakota, 1998, Lagos, Rocheteau and Wright, 2017). We instead analyze the conflict between the payment and non-payment functions of money, rather than to explain why some goods may endogenously emerge as means of payment. Relatedly, we draw inspiration from the literature on credits, market participation, and commitment (e.g., Kehoe and Levine, 1993, Kocherlakota, 1996, Alvarez and Jermann, 2000). To focus on the inalienable role of money and reserves in settling payments, our model deliberately abstracts away from credits. An interesting question is how the co-existence of reserves and credits (as modeled in Townsend, 1989, Gu, Mattesini, and Wright, 2016, etc.) may affect payment fragility, which we leave for future research.

[7]As discussed above, Copeland, Duffie and Yang (2020), Correa, Du and Liao (2020), and Afonso, Duffie, Rigon and Shin (2022) empirically document and explore how the scarcity in reserves leads to delays in interbank payments as well as the disruptions in repo funding markets. Theoretically, d'Avernas and Vandeweyer (2020) and Yang (2021) build dynamic asset pricing models of repos to explain such empirical patterns, highlighting intraday liquidity management and bank regulations.

2 The Model

We build a stochastic, dynamic model of payments. We present two versions of the model which generate identical analytical results but admit different interpretations, helping broaden the applications of the model. Appendix A contains all proofs.

2.1 Baseline model setup

Setup. Consider a discrete-time, infinite-horizon economy, $t = 0, 1, 2, ...$, with two risk-neutral agents, 1 and 2, which we sometimes also call "banks" when interpreting the model. Agents have a common discount factor $\delta \in (0, 1)$, which captures their time preference.

There are two types of goods: a single unit of indivisible *reserve* and potentially many *rewards*. At $t = 0$, only one agent is endowed with the reserve while the other is not. No agents have any rewards at $t = 0$, but rewards can be created when agents make *payments* by transferring the reserve to each other, as we detail below.

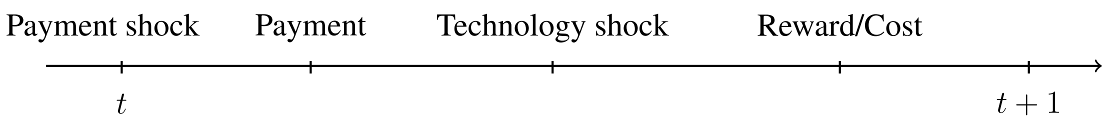

Figure 1: Timeline of the baseline setup

This figure shows the timeline of actions, events, and shocks in the baseline model setup. Both the reward and cost are accrued at the end of time t before the economy continues to time $t + 1$.

The timeline of the economy is illustrated in Figure 1. At any time $t \geq 0$, the agent who holds the reserve, suppose agent i, is subject to a private payment shock: with probability $\lambda \in (0, 1]$ agent i is supposed to transfer the reserve to agent j. Denote by $a \in \{0, 1\}$ agent i's possible private actions: $a = 1$ means she sends the reserve, whereas $a = 0$ means not and she keeps the reserve. When agent i sends the reserve, the potential transfer is subject to another private technology shock: with probability $\mu \in (0, 1]$ the transfer goes through to agent j, whereas with probability $1 - \mu$ the transfer fails and the reserve remains with agent i. Any payment outcome is publicly observable, and we denote by $k = 1$ a successful transfer of the reserve good and $k = 0$ otherwise. If the reserve is successfully transferred to the other agent j, the initially reserve-holding agent i will get $z > 0$ rewards at the end of time t, where z is a parameter. Rewards are perishable so will have to be consumed immediately by agent i at t, and the consumption value of one unit of rewards is normalized to 1. Finally, at the end of any time $t \geq 0$, the agent who does not hold the reserve suffers a per period cost of c, where $c > 0$ is a parameter. In reality, c parsimoniously captures the various opportunity costs incurred from not holding the reserve, which arise from the reserve's non-payment functions. The economy then moves to $t+1$ regardless of whether the payment is made or not.

To highlight the conflict between the reserve's payment and non-payment functions, it is important to note that the reserve good has an effective present value of $\kappa = c/(1-\delta)$ if not used for payments. In other words, permanently losing it would effectively incur a present cost of κ to the current reserve-holding agent. Given that the supply of the reserve is normalized to one, κ thus captures the *scarcity* of the reserve: a higher present value of κ for non-payment functions suggests that the reserve good is more scarce for the payment function. Economically, this observation also implies that our notion of reserve scarcity fundamentally arises from the conflict between the reserve good's payment and non-payment functions. Suppose a hypothetical reserve good is completely useless for other non-payment functions but still accepted as a medium of exchange, it is then not scarce under our framework. Formally, we define:

Definition 1. *The reserve good's scarcity is captured by the present value of the opportunity costs of transferring it:* $\kappa = c/(1-\delta)$*. The reserve good is more (less) scarce if* κ *is higher (lower).*

Equilibrium concept. To focus on to what extent current payment decisions depend on past payment histories, we adopt the equilibrium concept of Perfect Public Equilibrium (PPE), in which agents' optimal strategies are allowed to depend on the public history of past outcomes. Once a PPE exists, we can examine under what conditions agents' optimal payment strategies indeed depend on their past payment outcomes, and if yes, to what extent, and whether there are multiple equilibria.

Formally, denote by $s_t \in \{0,1\}$ the state of the stochastic game at t: $s_t = 1$ means that agent 1 has the reserve and $s_t = 0$ means that agent 2 has it. Since payment outcomes are publicly observable, a generic public history of states is given by $s^t = (s_0, s_1, s_2, \cdots, s_{t-1}) \in \mathcal{S}^t$, where $s^{t+1} = (s^t, s_t)$ and $\mathcal{S}^t \doteq \{0,1\}^t$. The public history at t thus fully summarizes all public signals up to t. Denote by $\mathcal{S}^t =$ all possible public histories at t, and $\mathcal{S} = \bigcup_t \mathcal{S}^t$. A public strategy is then defined by a mapping $\sigma : \mathcal{S} \to \{0,1\}$. Note that it is sufficient to let σ specify the strategy for the reserve-holding agent but not the other agent because only the reserve-holding agent takes action. For the same reason, it is sufficient to use σ to denote both a public strategy and a public strategy profile. If a public strategy σ exhibits $\sigma(s^t) = 1$ for all $s^t \in \mathcal{S}$ (or $\sigma(s^t) = 0$ for all $s^t \in \mathcal{S}$), then we say σ is history-independent. Otherwise, it is history-dependent.

Definition 2. *A public strategy profile is a perfect public equilibrium (PPE) if for all t and all $s^t \in \mathcal{S}^t$, $\sigma|_{s^t}$ is a Nash equilibrium.*

In words, a PPE specifies a sequential equilibrium that only involves public strategies and which also constitutes a sequential equilibrium for the dynamic game from any date and any history. This definition is a direct counterpart to the standard definition of PPE in a repeated game.

We can immediately define whether a PPE is history-independent (or history-dependent) based on the description of history-independent strategies.

Definition 3. *A perfect public equilibrium (PPE) is history-independent if it involves only history-independent public strategies; otherwise, it is history-dependent.*

2.2 Alternative model setup

Our model can be set alternatively to accommodate aggregate shocks and particularly the store of value function of money while keeping all the equilibrium outcomes the same. Consider a discrete-time, infinite-horizon economy, $t = 0, 1, 2, \ldots$, with two risk-neutral agents, 1 and 2, which we also call "banks" when interpreting the model. There is no time discount, but starting at the end of $t = 0$ the economy is subject to an aggregate shock every period: with probability $\delta \in (0, 1)$ the economy continues to $t+1$, whereas with probability $1-\delta$ the economy discontinues. Note that the continuation probably δ maps to the discount factor in the baseline model discussed above. The two types of goods, reserves, and rewards, are specified the same as in the baseline model.

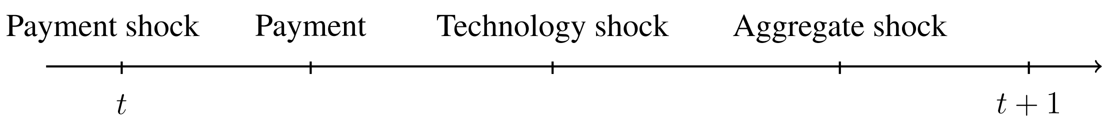

Figure 2: Timeline of the alternative setup

This figure shows the timeline of actions, events, and shocks in the baseline model setup. Both the reward and consumption penalty are accrued at the end of time t before the economy potentially continues to time $t+1$.

The timeline of the economy is illustrated in Figure 2. At any time $t \geq 0$, the agent who holds the reserve, suppose agent i, is subject to a private payment shock: with probability $\lambda \in (0, 1]$ agent i is supposed to transfer the reserve to agent j. Denote by $a \in \{0, 1\}$ agent i's possible private actions: $a = 1$ means she sends the reserve, whereas $a = 0$ means not and she keeps the reserve. When agent i sends the reserve, the potential transfer is subject to another private technology shock: with probability $\mu \in (0, 1]$ the transfer goes through to agent j, whereas with probability $1 - \mu$ the transfer fails and the reserve remains with agent i. Any payment outcome is publicly observable. If the reserve is successfully transferred to the other agent j, that is, a payment is successfully made, agent i will get $z > 0$ rewards, where z is a parameter. Rewards are perishable so will have to be consumed immediately by agent i at t, and the consumption value of one unit of rewards is normalized to 1. If the economy continues at the end of time t, then it moves to $t + 1$ regardless of whether the payment is made or not. Rather, if the economy discontinues at the end of time t, the reserve-holding agent consumes the reserve itself. The reserve-holding agent gets a normalized consumption value of 0, and the other who does not hold the reserve gets $-\kappa < 0$, where $\kappa > 0$. Economically, because κ occurs only in the aggregate bad state, it captures the store of value function of the reserve good. In other words, whoever holds the reserve good in the aggregate bad state is protected by the reserve good from a consumption loss. Note that the consumption penalty is analytically the same as the present value of the reserve good in the baseline model. Therefore, κ similarly captures the value and scarcity of the reserve good as in the baseline model.

An important observation is that the alternative economy is observationally equivalent to the baseline economy in terms of equilibrium profiles. In other words, if a strategy profile is a PPE in the baseline economy, it is then also a PPE in the alternative economy, and vice versa. This important feature allows us to interpret the equilibrium flexibly and map it to many rich economic applications.

2.3 Mapping the model to realistic payment applications

Our model is parsimonious but general enough to accommodate many economically relevant applications of payments. Below, we discuss some of the key elements of the model to illustrate how it covers the essence of various payment applications.

Infinite-horizon economy with patient agents in the baseline setup. We set up an infinite-horizon dynamic economy to capture the general notion that payment activities are dynamic and reciprocal, and they involve long-term interactions among agents in most applications. Payment activities create value by solving the lack of double coincidence of wants, captured by the creation of rewards. However, payment activities are also costly and risky because making an outbound payment means a transfer of the reserve good to the other agent. A successful payment means a drawdown on the paying agent's reserve holdings that are potentially useful for other non-payment functions. In our model, it is captured by that the reserve-holding agent will have not maintained the reserve good after making a payment, incurring a cost. It is also uncertain whether other agents will make reciprocal payments back in the future. This implies that the reserve-holding agent may incur prolonged periods of opportunity costs going forward. The reserve-holding agent thus trades off the current benefit against future costs when making the payment decision, which is in turn affected by her time preference.

Infinite-horizon economy with an aggregate shock in the alternative setup. In the alternative model setup, making an outbound payment particularly means transferring a store of value in the aggregate bad state. The aggregate shock of the game ending, naturally representing a bad aggregate state and all agents having to consume in that bad state, thus helps us parsimoniously capture the idea of reserves being not only a medium of exchange but also a store of value in many applications.

Reserves and rewards. We view the essence of any payment as the transfer of a scarce reserve good that is valuable for other non-payment functions. In the model, we take a fixed unit supply with a flexible present value to capture the notion of scarcity and its magnitude. That is, a reserve good with a higher present value is relatively more scarce given the fixed unit supply. This setting of a fixed unit supply greatly helps uncover the economic mechanism at play while keeping the model tractable. In reality, the net supply of means of payment is unlikely to be completely fixed in the long run. But as long as its supply is relatively inelastic in the short run, it is scarce compared to the demand of transaction needs and the quality of the payment technology. Assuming a fixed unit supply of reserves implies that our model does not directly generate quantitative predictions concerning the amount of reserves. However, the ability to use the reserve good's present value to capture the magnitude of scarcity still allows the model to capture rich relationships between reserve scarcity and payment activities. And in exchange for this simplification, we provide a fairly general characterization of the equilibrium without sacrificing the delivery of the underlying economics.

The scarcity of the reserve good naturally implies an imperfect substitutability between it and other consumption goods, which is the reason we separately model the reserve and reward goods. It is natural that successful payments generate economic value from resolving the lack of double coincidence of wants, which is captured by the creation of rewards. The assumption that rewards are perishable and thus have to be immediately consumed is not crucial. What is crucial in this assumption is that agents cannot generate more reserves by accumulating the rewards in the short run, which, again, fundamentally reflects the scarcity of the reserve.

To be more specific, in what follows we separately describe a number of payment applications and illustrate what are the reserves and rewards, why the reserve has a limited supply and is imperfectly substitutable with the reward, and in what sense the payment and non-payment functions of the reserve good co-exist but can be separated:

- **Metallic payments.** Metallic payments have dominated the payment system for about four millenniums. Silver and gold are the main forms of reserves, whose supply is limited due to physical mint constraints. Around 2000 B.C., the use of metallic coins for payments first appears in ancient Greece and Rome for trading consumption commodities and services – rewards – that are not substitutable with reserves. Greeks employ the Attic silver standard, which becomes the predominant weight standard for coins in the Eastern Mediterranean. The Roman Empire use the Denarius, which has a fixed weight and value, as the basic silver coin. At the same time, high-value trade of consumption goods and services are settled through gold minted in Byzantine or Muslim. The global supply of gold has remained low and stable, only until the 20th century due to vast gold fields being discovered in South Africa and the development of the so-called cyanide process to extract gold from the low-grade ore in these gold fields (Redish, 2000).

  Although silver and gold are used as a medium of exchange, they have high intrinsic value as jewelry and conductors, used on a daily basis. More importantly, they are historically pursued and held by consumers and investors as a store of value in times of low or negative real interest rates and economic crises (Jermann, 2022).

- **Modern interbank payments.** Today, real-time gross settlement (RTGS) interbank payment systems play the leading role in large-size payments. As an example, the Fedwire, the RTGS funds transfer system for financial institutions operated by the U.S. Federal Reserve (Fed) Banks, sees a daily volume of more than $4.2 U.S. trillion in 2022. By construction, Fedwire is open to banks that have accounts at the Fed, and each interbank payment involves the transfer of central bank reserves from one bank to another. Successful payments allow the paying bank to collect fees – rewards – from its clients in the form of lendable cash and deposits, which can be in turn immediately lent out and thus imperfectly substitutable with central bank reserves. Indeed, Diamond, Jiang, and Ma (2022) show that central bank reserves crowd out bank lending, further supporting the notion that lendable funds and reserves are not perfectly substitutable. In addition, the supply of central bank reserves in the U.S. is largely limited and inelastic in the short run, being subject to the

Fed's monetary implementation cycles (e.g., Copeland, Duffie and Yang, 2020, Acharya and Rajan, 2022, Lopez-Salido and Vissing-Jorgensen, 2023).

Beyond the use in interbank payments, central bank reserves provide other non-payment benefits and functions to banks. First, the Fed may use the interest rate on excess reserves (IOER) as an additional monetary policy tool, allowing banks to directly earn interest income by holding reserves. Second, holding excess reserves helps large banks to more easily meet post-crisis regulatory and liquidity requirements (e.g., Duffie, 2019, Correa, Du and Liao, 2020). Finally, holding excess reserves also saves banks from the stigma of tapping the Fed's discount window in volatile market times (e.g. Afonso, Kovner and Schoar, 2011), thus providing a store of value to banks.

- **Payments using bank deposits.** Complementary to interbank payment systems, modern mid- and small-size payments are also settled by commercial bank notes, that is, deposits, within the same bank. The supply of bank deposits – reserve – is limited by banks' reserve requirements and the money multiplier (Tobin, 1965, Steinsson, 2023b) and thus only inelastic in the short run. Households and firms then use bank deposits to settle the purchases of goods and services – rewards. Indeed, various theories and facts are provided (e.g., Diamond and Rajan, 2006, Gu, Mattesini, Monnet, and Wright, 2013, Donaldson, Piacentino and Thakor, 2018, Parlour, Rajan and Walden, 2020) to justify why bank deposits emerge as a medium of exchange. At the same time, they also serve as a store of value (e.g., Stein, 2012, Dang, Gorton, Holmström, and Ordoñez, 2017) and may generate interest income for depositors.

- **Cross-border payments.** U.S. dollar is the dominant currency – reserve – for cross-border payments today. According to the Society for Worldwide Interbank Financial Telecommunication (SWIFT) system, the dollar accounts for 79.5% of payments in international trade between 2010-2020. Additionally, over the period of 1999-2019, the dollar accounted for 96% and 74% of trade invoicing in the Americas and Asia-Pacific regions, respectively (Gopinath and Stein, 2021), and is also predominantly used in settling the payments of global financial contracts (e.g., Coppola, Krishnamurthy and Xu, 2023). The supply of U.S. dollars is limited by the U.S.'s fiscal and monetary capacities, while non-U.S. banks typically collect fees in local currencies after successful payments – rewards – which are not directly substitutable with the U.S. dollar.

  Beyond the use in global payments, the U.S. dollar is widely held by investors, commercial banks, and central banks as a global safe asset for a store of value (e.g., He, Krishnamurthy and Milbradt, 2019, Jiang, Krishnamurthy and Lustig, 2021, Maggiori, Neiman and Schreger, 2021, Brunnermeier, Merkel and Sannikov, 2022). Indeed, long before the U.S. dollar become popular in cross-border payments, it has cemented its role as the global reserve currency since the 1944 Bretton-Woods Agreement.[8] As of 2022, foreign central banks still hold 59% of reserves in U.S. dollars.

- **Digital and crypto payments.** Stablecoins and CBDCs are widely considered as the next generations of payment methods (see, e.g., Duffie, 2019, Auer, Frost, Gambacorta, Monnet, Rice and Shin, 2022, Brunnermeier and Payne, 2022, for surveys). For example, Auer, Frost, Gambacorta, Monnet, Rice and Shin (2022) estimate that the greatest potential for stablecoins is the cross-border remittance markets, on which stablecoins may help reduce the current costs by more than half. In these applications, the transactional gains – rewards – are typically reflected in local currencies, which are not directly substitutable with stablecoins. On the other hand, the supply of stablecoins and CBDCs as reserves is constrained by the various design choices.

  Beyond potential use in payments, stablecoins and CBDCs deliver other non-payment benefits and functions. For example, stablecoins are widely held by investors as collateral for speculating other cryptoassets (Gorton, Klee, Ross, Ross, and Vardoulakis, 2023). CBDCs, regardless of their design choices, are largely perceived to be a store of value accessible to households. Indeed, some recent studies hypothesize that this role of CBDCs may even generate unintended consequences by disintermediating commercial banks, particularly in crisis times (e.g., Auer, Frost, Gambacorta, Monnet, Rice and Shin, 2022).

Separation of the roles of medium of exchange and store of value. Although intrinsic value (e.g., from service flows), if any, can be easily separated from the other functions of the reserve good in the above applications, its payment and non-payment functions may reinforce each other in reality (e.g., Gorton and Pennacchi, 1990). Our separation of them follows from the classic idea of Hayek (1976) who argues the following. The payment function, that is, the role of reserves as a medium of exchange, stems from them resolving the lack of double coincidence of wants across time. In contrast, the role as a store of value, the leading non-payment function

[8] Under the Bretton-Wood System, the U.S. dollar was pegged to gold at $35 an ounce, and other countries peg their currencies to the U.S. dollar. The U.S. abandoned this aspect of the agreement as President Nixon abandoned the gold standard in favor of free-floating exchanges in 1971. However, the end of the gold standard had little impact on the U.S. dollar's role as a store of value.

of reserves, originates from them improving the ability to commit to future risk-sharing across different states of the world. Thus, these two functions reduce fundamentally different economic frictions. This separation has also been studied in the recent literature (e.g. Diamond and Rajan, 2006, Brunnermeier, James and Landau, 2019).

Payment shock. The payment shock in our model is a parsimonious way to capture uncertain transaction needs and the relative scarcity of reserves compared to transaction needs. In reality, at any given date, an individual may receive goods or services, or may not receive anything. A bank may receive clients' payment requests or not as well. An agent only needs to send a payment if there are transaction needs.

We highlight that only agent i (i.e., the reserve-holding agent) is subject to the payment shock. This setting parsimoniously captures the notion that whoever has to make a payment already has enough reserve funds to do so despite reserves being scarce in the aggregate. Thus, we rule out the mechanical and uninteresting case in which an agent does not make a payment or waits for others to make a payment first simply because she does not have enough reserve funds.

How does the model capture delays? Although the reserve-holding agent only chooses between sending (i.e., $a_t = 1$) and not sending (i.e., $a_t = 0$) upon receiving a payment shock, it precisely captures payment delays in the following sense. If the reserve-holding agent does not make a transfer at time t, she does not enjoy any reward (e.g., the delivery of the consumption good, or payment fees) accordingly and keeps the reserve. At time $t + 1$, she is subject to another payment shock with probability $\lambda > 0$. This can be interpreted as with probability λ the household is tempted to purchase the same good again at $t = 1$ she would have purchased at $t = 0$, or the client's payment request still stays on the bank's order book at $t = 1$. If the reserve-holding agent chooses to pay now (i.e., $a_{t+1} = 1$) and the payment goes through, the combined history of $\{s_t = 0, s_{t+1} = 1\}$ suggests that the payment, which could have been made earlier, is delayed for one period. Delays of more periods can be thus captured in the same way.

Technology shock. The technology shock captures the efficiency of the underlying payment technology, highlighting the notion that a payment may fail for reasons that are out of the agents' control. Shell and gold may be stolen in transitions. Even modern, large-scale electronic payment systems are subject to technical errors and failures. For example, the Fedwire system may occasionally break down; it was disrupted twice in 2019 due to undisclosed technical issues, resulting in significant delays in cross-bank settlements.

The modeling of the payment shock and technology shock allows us to cover many realistic frictions in payment activities. We highlight that both the payment shock and the technology shock are only privately known to the potentially payment-sending agent i, but not j. Those two assumptions imply that seeing no payments, the non-reserve-holding agent j is not sure whether it is because agent i chooses not to make a payment, or if it is just because agent i is simply not requested to send any payment at all, or because the technology fails. This signal-jamming problem implies that past payment outcomes are informative but imperfect signals when an agent tries to look at them to anticipate other agents' future payment patterns. Therefore, we are able to characterize to what extent the past dependence of payments is subject to different types and degrees of shocks in various payment applications.

Relationship with repeated games with imperfect public monitoring. As discussed above, the presence of payment and technology shocks introduces a signal-jamming and imperfect monitoring problem. Therefore, the stochastic dynamic economy we set up is similar to a standard repeated game with imperfect public monitoring (e.g., Abreu, Pearce, and Stacchetti, 1990, Fudenberg, Levine and Maskin, 1994). However, some important differences emerge between our model and standard repeated games, which significantly affect the equilibrium analysis.

First, our dynamic economy does not involve the repetition of a stage game. The stage game in our economy more closely resembles a decision problem for the reserve-holding agent, taking into consideration the strategic interaction between the two agents only through the continuation value.

Second, beyond the public history, which agent holds the reserve good also constructs an important state variable. Despite the two agents' preferences being identical, their continuation values in any given period are thus different depending on who owns the reserve good. We view these features being important because they jointly capture the nature of payments being reciprocal and reserves being scarce.

3 Payment equilibria and financial fragility

3.1 The "good" payment equilibrium

We start by characterizing the existence of both a "good" and a "bad" payment equilibrium, in which agents' payment decisions are history-independent. In the good (bad) equilibrium, the agent who holds the reserve good always (never) makes a payment, regardless of the past history of the other agent making payments or not. Those two equilibria thus serve as benchmarks and allow us to later uncover the economic conditions under which payment decisions become history-dependent.

Proposition 1. *There exists a good equilibrium in which the reserve-holding agent always makes a payment, if and only if*

$$\frac{z}{\kappa} \geq \frac{1-\delta}{1-\delta(1-\lambda\mu)}\,. \tag{3.1}$$

The right hand side of (3.1) is strictly smaller than 1, and is decreasing in δ, λ, and μ. The good payment equilibrium is a PPE and is history independent.

The proof is based on Abreu, Pearce, and Stacchetti (1990)'s idea of decomposability, but we give a sketch of the idea here to help build intuition. It is based on the standard one-shot deviation principle to check whether the reserve-holding agent, conditional on being subject to a payment shock and the other agent playing the proposed equilibrium strategy, would find a profitable deviation. Intuitively, $1-\lambda\mu$ is the per period probability of no transfer and the game continues to the next period conditional on the bank with the reserve transferring the reserve upon request. In other words, $1-\lambda\mu$ is the probability that the bank keeps its current status in the next period given that it chooses to transfer the reserve this period (which succeeds with probability $\mu < 1$) given the other agent playing the equilibrium strategy. Then, we have

$$z \geq c\left(1+\delta\left(1-\lambda\mu\right)+\delta^2\left(1-\lambda\mu\right)^2+...\right)\,, \tag{3.2}$$

meaning that the good payment equilibrium exists if and only if z, the gain from making a payment at time t, is no less than the sum of c, the cost at time t if giving up the reserve, $\delta\left(1-\lambda\mu\right)c$, the present value of the expected cost at time $t+1$, $\delta^2\left(1-\lambda\mu\right)^2 c$, the present value of the expected cost at time $t+2$, and so on. By the relationship of $\kappa = c/(1-\delta)$, condition (3.2) immediately yields the equilibrium condition (3.1).

Proposition 1 allows for a number of comparative statics thanks to the explicit characterization of the equilibrium existence region. First, the good payment equilibrium is more likely to happen when the benefit of payment-making is higher (i.e., a larger z). This is intuitive because the benefit from a successful payment can directly compensate for the cost of not having enough reserves in the future, encouraging the reserve-holding agent to make a payment today.

Second, the good payment equilibrium is more likely to happen when the reserve good is less scarce/valuable for non-payment functions (i.e., a smaller κ). Intuitively, when the reserve is more scarce, the reserve-holding agent has a higher incentive to hoard the reserve good because losing it otherwise would incur a larger present loss. By the same token, when the reserve is less scarce, the reserve-holding agent is more encouraged to make a payment. This prediction is directly supported by the recent evidence in, for example, Afonso, Duffie, Rigon and Shin (2022) and Lopez-Salido and Vissing-Jorgensen (2023), that interbank payments are more efficient when reserves are more abundant.

Third, the good payment equilibrium is more likely to happen when the agents are more patient (i.e., when δ is larger in the baseline economy), or when the aggregate negative shock is less likely to happen (i.e., when δ is larger in the alternative economy). This result is consistent with the repeated game literature (e.g. Green and Porter, 1984, Fudenberg and Maskin, 1986) that a cooperation equilibrium is more easily sustained when agents are more patient. This analogy points to the endogenous coordination motive embedded in our dynamic payment game despite that the game is not repeated and each stage game does not feature any coordination motives.

Fourth, the good equilibrium is also more likely to happen when the payment technology is better (i.e., when μ is larger). Intuitively, when the technology is better, the reserve-holding agent is more likely to get a reciprocal payment back from its counterpart in the future. This in turn increases the reserve-holding agent's incentives to make a payment, leading to a more likely good payment equilibrium.

Finally, the good payment equilibrium is more likely to happen when the other non-reserve-holding agent is more likely to receive a payment request next period (i.e., when λ is larger). To understand this, note that the existence of the good payment equilibrium is guaranteed by an unprofitable one-shot deviation by the reserve-holding agent provided a given payment shock. In other words, the magnitude of the private payment shock is already becoming irrelevant for the reserve-holding agent in question. Rather, when the other non-reserve-holding agent is more likely to receive a payment request going forward, the reserve-holding agent is more likely to get a reciprocal payment back. Just like what a better payment technology implies, this increases the reserve-holding agent's incentives to make a payment and makes a good payment equilibrium more likely to happen.

Overall, Proposition 1 provides a useful benchmark to understand what economic conditions contribute to a good and efficient payment system. It also helps provide alternative yet forceful answers to a number of important questions regarding the nature and evolution of money and payment systems:

Bank market power on deposit markets. A growing literature (e.g, Drechsler, Savov and Schnabl, 2017) examine commercial banks' market power on the deposit markets. Particularly, checking deposits often entitle their holders to zero interest rates regardless of the policy rate. It is typically understood from this literature that commercial banks process such market power from their franchise value such as the provision of branches, debit cards, and customer services.

Proposition 1 provides a complementary explanation from a payment efficiency point of view: zero interest rates, by reducing κ, help to minimize the conflict between checking deposits' payment and non-payment functions because these deposits are indeed designed for payments.

Should CBDCs be interest-bearing? An important debate concerning the design of CBDCs is whether they should be interest-bearing (Auer, Frost, Gambacorta, Monnet, Rice and Shin, 2022). Currently, this debate centers how a potentially interest-bearing CBDC would interact with commercial banks in various aspects. Focusing on improving payment efficiency, which is widely perceived as the top reason to introduce a CBDC, Proposition 1 implies that CBDCs should not bear an interest in order to keep κ low and minimize the conflict between CBDCs' payment and non-payment functions. We acknowledge that to analyze the full implications of CBDCs is however beyond the scope of this paper. **Gresham's Law.** In monetary economics, Gresham's Law implies that "bad" money that has a lower intrinsic value will drive "good" money out of circulation. That is, ironically, "bad" money appears to be a more popular means of payment compared to "good" money. Historians and economists have offered various explanations for Gresham's Law and explored its implications (see Sargent and Velde, 2001, for a review). Proposition 1 offers a complementary view: everything else equal, a reserve with lower intrinsic value dominates another with higher intrinsic value for payment efficiency thanks to a higher z/κ. Indeed, Proposition 1 implies that any desirable form of reserves for payments should have minimal or zero intrinsic value to mitigate the conflict from other non-payment functions. This idea is reminiscent of Adam Smith's point in the *Wealth of Nations*, who argues from a different perspective that replacing gold and silver coins with bank notes would free up the gold and silver for other use of higher social value.

3.2 The "bad" payment equilibrium

Next, we turn to study the other benchmark equilibrium in which none of the agents makes payments regardless of the payment history.

Proposition 2. *There exists a bad equilibrium in which the reserve-holding agent never makes a payment if and only if*

$$\frac{z}{\kappa} \leq 1\,. \tag{3.3}$$

The bad payment equilibrium is a PPE and is history independent.

The intuition of Proposition 2 can be also understood by looking at the one-shot deviation of the reserve-holding agent. We have:

$$z \leq c(1 + \delta + \delta^2 + ...) \, , \tag{3.4}$$

In words, the bad payment equilibrium exists if and only if z, the gain from making a payment at time t, is no greater than than the sum of c, the cost at time t if giving up the reserve, δc, the present value of the expected cost at time $t+1$, $\delta^2 c$, the present value of the expected cost at time $t+2$, and so on. By the relationship of $\kappa = c/(1-\delta)$, condition (3.4) immediately yields the equilibrium condition (3.3).

The bad equilibrium is more likely to happen when the benefit of payment-making is smaller (i.e., a smaller z), or when the reserve good is more scarce/valuable for non-payment functions (i.e., a larger κ). This result is similar to its counterpart in Proposition 1.

Similarly, Proposition 2 also provides alternative answers to a number of important questions in the evolution of monetary and payment systems:

"Big problem of small change." In monetary economics, the "big problem of small change" refers to the notorious phenomenon over the many years of metallic systems that low-value coins constantly disappear from circulation, and the economy suffers from the lack of means to complete low-value transactions (Sargent and Velde, 2001, Steinsson, 2023a). So far, the leading explanation for the "big problem of small change" is that these low-value coins are so small that they are easily lost or difficult to pick up and count (Redish, 2000), so they are unlikely to be useful in everyday payments. Proposition 2 provides an alternative view: these coins disappear not because their physical size is small but because their z/κ is too small. Specifically, the size of the economic transactions they aim to support is so small that people tend to instead hoard them to enjoy their other non-payment functions. Indeed, Redish (2000) documents anecdotal evidence that some low-value coins were privately destroyed for building other products such as weapons. Combined with Proposition 1, this view also helps understand why the "big problem of small change" is no longer a problem today because even for small z the corresponding κ is significantly reduced by the use of deposits and digital payments.

End of the Gold Standard. The Gold Standard effectively ends in 1971 when President Nixon abandoned the peg of the U.S. dollar to gold. The literature has offered many explanations for the end of the Gold Standard, the leading one being the fear of deflation and devaluation of the U.S. dollar (see Blinder, 2022, for a review). To synthesize these views is beyond the scope of this paper, but Proposition 2 provides a complementary view from the payment evolution angle.

Indeed, the U.S. dollar has started to rise as a dominant currency for payments during the same period (Gopinath and Stein, 2021), rendering z/κ to be too low for gold compared to the U.S. dollar and therefore gold to be a dominated reserve good for payments.

Finally, different from Proposition 1, an important observation from Proposition 2 is that how likely the bad equilibrium happens does not depend on the agent's time preference (in the baseline economy), the aggregate shock (in the alternative economy), the frequency of payment requests, or the quality of the payment technology. This can be interpreted as a "coordination trap" because the bad equilibrium happens when the reserve-holding agent believes the other agent will never make a returning payment in the future. Intuitively, conditional on the other non-reserve-holding never making a payment in the future regardless of the economic environment, any changes in the various model parameters will not make a one-shot deviation profitable for the reserve-holding agent in question. This result thus points to the importance of trust-building because it suggests that technology improvement, for example, may not necessarily solve the issue of lack of trust.

3.3 Multiple equilibria and payment fragility

Having analyzed the two benchmark equilibria, we immediately have the following result, which directly derives from Propositions 1 and 2:

Proposition 3. *Both the good and bad payment equilibria exist when*

$$\frac{1-\delta}{1-\delta(1-\lambda\mu)} \leq \frac{z}{\kappa} \leq 1\,. \tag{3.5}$$

Proposition 3 implies that the two payment equilibria may co-exist in the same economy for medium values of z/κ. The existence of multiple equilibria resembles the classic notion of coordination such as that in the bank run models (e.g., Diamond and Dybvig, 1983). There, whether a depositor runs the bank depends on whether other depositors run. Here, whether a reverse-holding agent makes payments depend on whether the non-reserve-holding agent will make a reciprocal payment in the future.

Proposition 3 provides a plausible answer to the motivating question of this paper as to why payments are often fragile. When the economy falls into the parameter region where both equilibria exist, the payment patterns cannot be fully pinned down by fundamentals, implying a potential switch between the two equilibria. For example, when the economy sustains the good payment equilibrium at the lower threshold and an arbitrarily small negative shock hits it, the reserve-holding agent will stop making payments, pushing the economy to switch to a bad payment equilibrium. Such a switch between the two equilibria due to relatively small changes in fundamentals may lead to relatively large changes in equilibrium payment behaviors.

Proposition 3 maps to several important episodes of disruptions in various payment contexts. Below we describe these contexts and discuss how they connect to our model.

Repo market crisis in 2019. One of the motivating facts of this paper is the repo market crisis in September 2019. Overnight Treasury repo rates spiked by over 1,000 basis points, a level not seen since the 2008 global financial crisis (Afonso, Cipriani, Copeland, Kovner, La Spada and Martin, 2020, Correa, Du and Liao, 2020). Existing empirical literature has ascribed this crisis to dysfunctional interbank payments (Copeland, Duffie and Yang, 2020, Afonso, Duffie, Rigon and Shin, 2022), which was indeed driven by a number of factors including banks' increased demand for reserves to meet regulatory requirements, a reduction in the supply of reserves due to the Fed's balance sheet normalization process, and corporations withdrawing cash from banks to pay quarterly tax obligations. That said, it is less well understood why the repo spike started to happen exactly on September 16, 2019. Proposition 3 provides a fragility perspective to understand it: the Fed's balance sheet normalization process pushes z/κ lower and falling into the intermediate region where multiple equilibra exists, leading to the potential for interbank payment disruptions due to coordination failure.

German interbank crisis in 1931. Blickle, Brunnermeier, and Luck (2022) provide a comprehensive analysis of the run on the German banking system in 1931, which was one of the largest bank runs in history and a key event of the Great Depression. A notable feature of this run episode is the severe disruption in interbank payments and lending. Importantly, the run happened when the Reichsbank, the German central bank at that time, was constrained by the Gold Standard and was mandated to cover 40% of the circulating currency with gold. Proposition 3 thus provides a complementary view to understand the source of fragility in interbank payments and lending: reserve scarcity. The mandate to follow the Gold Standard could be understood as κ falling in the intermediate region so that multiple equilibria exists, leading to the potential for interbank payments and lending disruptions.

September 11 attacks. In 2001, the September 11 attacks on the World Trade Center and the Pentagon caused significant disruptions in the U.S. financial system, including the interbank payment systems. The Federal Reserve Bank of New York (New York Fed), which serves as the primary operator of Fedwire, was located just blocks away from the World Trade Center. The attack led to halts and significant delays in the settlement of interbank payments despite major banks being sufficiently funded and the Fedwire operation system itself not being attacked

(Afonso and Shin, 2011). In this context, the September 11 Attacks could be interpreted as a sudden decrease in δ: agents suddenly become less patient, or the probability of the bad aggregate shock suddenly increases.

Brexit. The United Kingdom's withdrawal from the European Union (EU) in 2020 created significant challenges for cross-border payments between the UK and the EU. This included disruptions in the settlement of cross-border payments, as well as uncertainty around regulatory compliance and other issues. In this context, similarly, the establishment of Brexit could be interpreted as a sudden decrease in δ.

SWIFT hacks. SWIFT is a global financial messaging network that facilitates cross-border payments between financial institutions. In recent years, there have been several high-profile hacks of the SWIFT network, including the 2016 Bangladesh Bank heist, in which hackers stole \$81 million from the bank's account at the New York Fed (see Hill, 2018, for a comprehensive analysis). Cross-border payment halt after these events, even if the financial institutions in question have not necessarily been subject to these hacks themselves. In this context, the hacks could be interpreted as a sudden quality decrease in the payment technology μ.

Ripple Labs lawsuit. Ripple Labs, the creator of the XRP cryptocurrency, was sued by the U.S. Securities and Exchange Commission (SEC) in 2020, alleging that the company had conducted an unregistered security offering through the sale of XRP. This led to significant disruptions in cross-border payments involving XRP, with many financial institutions halting or limiting their use of the cryptocurrency. In this context, the lawsuit could be interpreted as a sudden decrease in the frequency of payment request λ.

A straightforward but important observation from Proposition 3 is that multiple equilibria and the implied financial fragility would not go away even when there is no uncertainty about the private payment request (i.e., $\lambda = 1$) and when the payment technology is perfect (i.e., $\mu = 1$).

Corollary 1. *When $\lambda \to 1$ and $\mu \to 1$, both the good and bad payment equilibria exist when*

$$1 - \delta \leq \frac{z}{\kappa} \leq 1\,.$$

The importance of Corollary 1 is that it shows what matters most for payment fragility is the scarcity of the reserve good but not the uncertain payment needs or the quality of the payment technology. As long as $\delta < 1$, that is, when the agents are impatient (in the baseline economy) or when the aggregate shock is not zero (in the alternative economy), the reserve good is valuable in the sense that it processes a positive present value $\kappa > 0$. The scarcity of the valuable reserve good thus gives rise to the dynamic coordination motive between the agents in making payments, leading to financial fragility.

4 The history-dependence of payments

Having analyzed the existence of multiple payment equilibria and linked it to observed disruptions in various payment applications, we move forward to analyze how payment fragility may happen *within* an equilibrium. This is important to help further discipline the model and understand the source of fragility without relying on exogenous shocks or sunspots. In other words, we aim to uncover the *endogenous* source that may trigger a payment disruption to better understand the motivating facts, for example, the 2019 Repo market crisis. At the same time, we aim to answer why payments often involve delays and history-dependence in the sense that an agent makes outgoing payments only after receiving incoming payments, even if these agents are well funded during normal times (e.g., Afonso, Duffie, Rigon and Shin, 2022). This question cannot be fully answered by Proposition 3 because both equilibria studied there are history-independent.

4.1 The grim trigger payment equilibrium and history dependence

To illustrate the point of payments being history-dependent and fragile within an equilibrium, we start by focusing on the classic "grim trigger" strategy first introduced in Friedman (1971) and widely studied in repeated games. The grim trigger strategy is a behavioral strategy in game theory that is used to enforce cooperation among players in a repeated game. The basic idea behind this strategy is that a player will initially cooperate with their opponent, but if their opponent ever defects, then the player will retaliate by defecting in all subsequent rounds of the game. In other words, if one player cheats or does not cooperate, the other player will "punish" them by not cooperating in any future rounds, even if it means a worse outcome for both players. The grim trigger strategy is considered as a form of "tit-for-tat" strategy, where players mimic the previous action of their opponent. However, the grim trigger is more extreme in that it involves a permanent switch to defecting if the opponent ever defects. The grim trigger strategy is most effective when the game is played repeatedly over a long period of time and when the players have a long-term perspective. It is also effective when the cost of defecting is higher than the cost of cooperating, as it provides a strong incentive for players to cooperate in order to avoid the long-term consequences of defection.

Formally, we define the grim trigger strategy in our dynamic economy using a two-state automaton, as follows.

Definition 4. *There is a grim trigger payment equilibrium if the reserve-holding agent plays the grim trigger strategy, which is represented by the following automaton. The set of states is* $W = \{w^{(1)}, w^{(0)}\}$*, with the output function* $f\left(w^{(1)}\right) = 1$ *and* $f\left(w^{(0)}\right) = 0$*. The initial state is* $w^{(1)}$*. The transition function is given by*

$$\tau(w,k) = \begin{cases} w^{(1)}, & \textit{if } w = w^{(1)} \textit{ and } k = 1\,, \\ w^{(0)}, & \textit{otherwise.} \end{cases}$$

A grim trigger strategy is described by $(W, w^{(1)}, f, \tau)$*, whereas the continuation strategy profile after any history in which a reserve transfer (i.e.,* $k = 1$*) is not publicly observed in at least one period is described by* $(W, w^{(0)}, f, \tau)$*.*

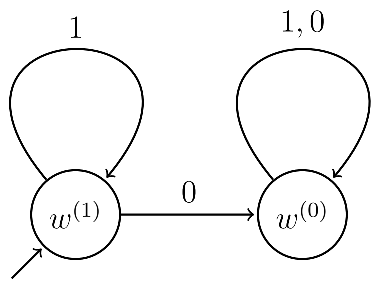

Figure 3: Automaton representation of the grim trigger equilibrium

This figure shows the automaton that represents the grim trigger strategy equilibrium. Circles are states and arrows are transitions, labeled by public outcomes that lead to the transitions.

By Definitions 2, 3, and 4, a grim trigger payment equilibrium is a PPE, and exhibits history dependence. Our next result shows that such an equilibrium exists, and characterizes the conditions under which it exists.

Proposition 4. *There exists a grim trigger payment equilibrium if and only if*

$$1 - \delta\lambda\mu \le \frac{z}{\kappa} \le 1\,. \tag{4.1}$$

Comparing Proposition 4 to Proposition 3 reveals an interesting observation: the region where the grim trigger equilibrium exists is a strict subset of the region where the good and bad payment equilibria co-exist. To see this, note that

$$\frac{1-\delta}{1-\delta(1-\lambda\mu)} < 1 - \delta\lambda\mu\,.$$

The relationship between the good, bad, and grim trigger payment equilibrium can be thus illustrated as in Figure 4.

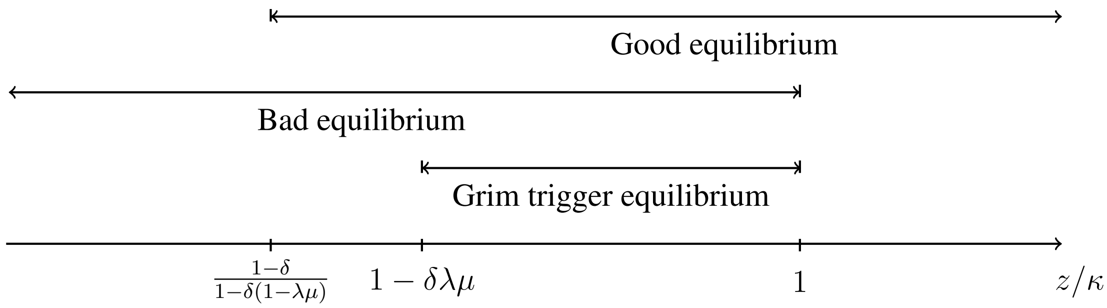

Figure 4: Relationship between the good, bad, and grim trigger equilibria

This figure shows the relationship of the existence regions of the good, bad, and grim trigger equilibria. The good and bad equilibria can co-exist in the intermediate region, and the existence region of the grim trigger equilibrium is a subset of that intermediate region.

The intuition for the relationship between the three equilibria can be understood in two steps. First, notice that the existence of the grim trigger payment equilibrium relies on the existence of the bad payment equilibrium because the bad payment equilibrium constructs one possible sub-game equilibrium of the grim trigger equilibrium. This explains that the region where the grim trigger equilibrium exists is a subset of the region where the bad payment equilibrium exists.

Furthermore, the existence of the grim trigger payment equilibrium also requires stronger economic fundamentals in terms of z/κ than the good payment equilibrium would require. To understand this more subtle result, it is useful to check the one-shot deviation again. Recall that the good payment equilibrium essentially requires the reserve-holding agent to make a payment conditional on the other agent always making a payment in the future. In contrast, the grim trigger payment requirement requires even stronger incentives for the reserve-holding agent, because it implies that the reserve-holding agent is still willing to make a payment initially given the existence of some sub-game equilibrium paths along which the other agent will stop making payments. This finally explains why the region where the grim trigger equilibrium exists is a strict subset of the region where the good and bad payment equilibria co-exist.

The existence of the grim trigger payment equilibrium allows our model to explain why payments often involve delays in the sense that an agent makes outgoing payments only after receiving incoming payments, even if these agents are well funded. As described by Afonso and Shin (2011), Copeland, Duffie and Yang (2020), and Afonso, Duffie, Rigon and Shin (2022), banks typically wait until they have received incoming payments to start sending outgoing payments to other banks, resulting in significant delays in intraday payments during both normal and crisis times. This delay pattern tends to be more pronounced when banks face higher capital costs and when aggregate central bank reserves are more scarce. In our model, this pattern can be explained by the agents playing a grim trigger payment equilibrium when κ, the value of the reserve good is relatively larger, and thus the reserve is more scarce.

Comparative statics of the existence region of the grim trigger payment equilibrium with respect to the economic environment further reveals a number of interesting economic predictions. Notice that the region $[1 - \delta\lambda\mu, 1]$ can only vary with economic parameters at the lower bound, but not the upper bound. Therefore, it suffices to perform comparative statics of the lower bound $1 - \delta\lambda\mu$ with respect to δ, λ, and μ, and it is straightforward to see that the lower bound decreases in all of the three. That is, the grim trigger payment equilibrium is more likely to happen when agents are more patient, when the bad aggregate shock is less likely, when the frequency of payment requests is higher, or when the quality of the payment technology is better.

To understand the intuition, it is useful to be reminded that the grim trigger payment equilibrium requires the bad payment equilibrium as a sub-game equilibrium, but it is itself an improvement of the bad payment equilibrium in terms of payment efficiency in the dynamic economy because the reserve-holding agent is willing to make payment at the beginning. Hence, taking the scarcity of the reserve good as given, an improvement in other economic conditions (including agents being more patient, the bad aggregate shock being less likely, a higher frequency of payment requests, or a better payment technology) gives the reserve-holding agent a higher incentive to pay a payment initially, supporting a grim trigger equilibrium.

However, interestingly, these comparative static results also suggest that an improvement in economic conditions may rather increase rather than decrease the likelihood of payments being history-dependent when the conflict between payment and non-payment is large in the sense that $z/\kappa \leq 1$. And the nature of payments being history-dependent would not go away even if there is no uncertainty about the payment needs and when the payment technology is perfect. Similar to Corollary 1, we have the following result:

Corollary 2. *When $\lambda \to 1$ and $\mu \to 1$, the grim trigger payment equilibrium exists if and only if*

$$1 - \delta \leq \frac{z}{\kappa} \leq 1\,.$$

Like Corollary 1, Corollary 2 shows what matters most for the history dependence of payments is the scarcity of the reserve good but not the uncertain payment needs or the quality of the payment technology. This prediction also relates to Proposition 2, which shows that any improvement in economic conditions other than the scarcity of the reserve good cannot change the region where the bad payment equilibrium exists. Thus, even if improvements in economic conditions increase the chances of the reserve-holding agent making an initial payment upon request, they cannot prevent the economy from eventually switching to the bad payment sub-game equilibrium after some unsuccessful history of payments regardless of the reasons. Proposition 4 and Corollary 1 thus again suggest that economic improvements such as a better payment technology may not necessarily eliminate payment delays that fundamentally arise from reserve scarcity, a point we highlight throughout the paper.

To offer another important perspective to understand the history dependence of payments, we consider the time until when the grim trigger equilibrium "collapses" in that the two agents stop making payments to each other. We can define this time formally and generally:

Definition 5. *For any PPE that admits the bad payment equilibrium as a sub-game equilibrium, the time-to-collapse T is the time when state $w^{(0)}$ is reached, that is, when the bad payment equilibrium is played.*

In our stochastic dynamic game, the time-to-collapse T is itself a random variable in any given equilibrium. Thus, it is useful to consider its distribution and expectation. We have the following result, which immediately follows from Definition 4 and Proposition 4.

Corollary 3. *The time-to-collapse in a grim trigger payment equilibrium follows a geometric distribution with a parameter $1 - \lambda\mu$, and the expected time-to-collapse is*

$$\mathbb{E}[T] = \frac{1}{1 - \lambda\mu}\,.$$

The intuition behind Corollary 3 directly follows from the definition of the grim trigger payment equilibrium and its existence. The two agents start by trusting each other and making payments until no reserve transfer is observed for one period. Looking forward from any given period, we know that the probability of a reserve transfer is $\lambda\mu$, that is, if the reserve-holding agent is requested to make a payment, and her payment goes through. Therefore, the grim trigger payment equilibrium would collapse with probability $1-\lambda\mu$ in any given period.

Together with Proposition 4 and Corollary 2, Corollary 3 offers a complementary view regarding the history dependence of payments in a grim trigger payment equilibrium, and particularly, how payment needs and payment technology affect the history dependence. Although, for example, a better payment technology may not necessarily reduce the chance when history dependence arises in terms of economic fundamentals (as illustrated in Proposition Proposition 4 and Corollary 2), it does reduce the time-to-collapse when history dependence actually happens. In other words, a better payment technology may still reduce the sensitivity of payment decisions on past payment histories, despite they being history-dependent in nature.

4.2 Generalized grim trigger payment equilibria and history dependence

Having studied the classic grim trigger equilibrium, we generalize the analysis to a class of equilibria that allows us to further characterize the magnitude of history dependence in payments. Formally, we define an n-trigger payment equilibrium as follows:

Definition 6. *There is an n-trigger payment equilibrium if the reserve-holding agent plays the n-trigger strategy, which is represented by the following automaton. The set of states is $W = \{w^{(l)} | 0 \leq l \leq n\}$, with the output function $f\left(w^{(l)}\right) = 1$ for $1 \leq l \leq n$ and $f\left(w^{(0)}\right) = 0$. The initial state is $w^{(n)}$. The transition function is given by*

$$\tau(w,k) = \begin{cases} w^{(l)}, & \text{if } w = w^{(l)}, 1 \leq l \leq n, \text{ and } k = 1\,, \\ w^{(l-1)}, & \text{if } w = w^{(l)}, 1 \leq l \leq n, \text{ and } k = 0\,, \\ w^{(0)}, & \text{otherwise.} \end{cases}$$

An n-trigger strategy is then described by $(W, w^{(l)}, f, \tau)$.

By Definitions 2, 3, and 6, an n-trigger payment equilibrium is a PPE, and exhibits history dependence. It is straightforward that the grim trigger payment equilibrium considered in Definition 4 is a special case of the general n-trigger payment equilibrium with $n = 1$. Intuitively, the classic grim trigger strategy represents the most extreme form of punishment in that the reserve-holding agent never makes any future payments after a reserve transfer has been not publicly observed for just one period. The n-trigger payment equilibrium accommodates the same idea of history dependence but is more general to capture its magnitude: the reserve-holding stop making payments after reserve transfers have been unobserved for a total of n accumulated periods. The

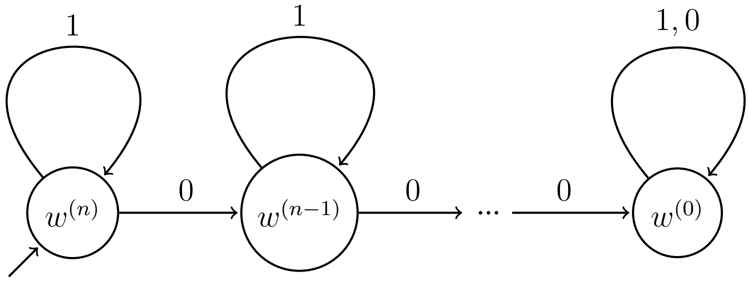

Figure 5: Automaton representation of the n-trigger equilibrium

This figure shows the automaton that represents the n-trigger strategy equilibrium. Circles are states and arrows are transitions, labeled by public outcomes that lead to the transitions.

parameter n thus naturally captures the magnitude of history dependence in payments; a larger n suggests that payment decisions are less sensitive to past payment histories, and thus a lower magnitude of history dependence.

Proposition 5. *There exists an n-trigger payment equilibrium for all $n \geq 1$ if and only if*

$$1 - \delta\lambda\mu \leq \frac{z}{\kappa} \leq 1\,.$$

Comparing Proposition 5 to Proposition 4 reveals that the region where the n-trigger equilibrium exists is exactly the same as that where the grim trigger equilibrium exists. In other words, the region where the n-trigger equilibrium exists is independent of n. This somewhat surprising result can be understood from the following two observations.

On the one hand, by definition, the existence of the n-trigger payment equilibrium relies on the existence of the $n-1$-trigger equilibrium because the latter constructs one possible sub-game equilibrium of the former. This explains that the region where the n-trigger equilibrium exists is a subset of that where the $n-1$-trigger equilibrium exists, and by induction, also a subset of that where the grim trigger equilibrium exists,

On the other hand, the $n+1$-trigger payment equilibrium must exist if the n-trigger payment equilibrium exists, for any $n \geq 1$. This is the key step in the proof of Proposition 5, and its intuition follows from that a larger n represents a lower magnitude of history dependence. To see this, note that the $n+1$-trigger payment equilibrium requires weaker economic fundamentals in terms of z/κ for the reserve-holding agent to make a payment at the initial state compared to the n-trigger payment equilibrium because the former admits one more period of not publicly observing a reserve transfer. In other words, the reserve-holding agent is more encouraged to make a payment in the $n+1$-trigger payment due to a lower threat of experiencing a bad technology shock and the reserve involuntarily failing to be transferred. Hence, the region where the n-trigger equilibrium exists must also be a subset of that where the $n+1$-trigger equilibrium exists. Taken together, these two observations explain why the region where the n-trigger equilibrium exists is independent of n.

Similarly, we can characterize the time-to-collapse for a general n-trigger payment equilibrium, based on Definition 5 and Proposition 5.

Corollary 4. *The time-to-collapse in an n-trigger payment equilibrium follows a negative binomial distribution with parameter n and $1-\lambda\mu$, and the expected time-to-collapse is*

$$\mathbb{E}[T_n] = \frac{n}{1-\lambda\mu}\,.$$

Corollary 4 nests Corollary 3 and provides a complementary view to illustrate the nature of the n-trigger payment equilibrium. The expected time-to-collapse becomes longer when n is larger. This result naturally follows from a lower magnitude of history dependence: a larger n implies that the payment system is more resilient to potential payment failures despite the nature of payments being history-dependent. All these equilibria can be possibly sustained, allowing our model to capture the rich patterns of history dependence in payments in reality.

5 Full equilibrium characterization and welfare outcomes

So far, we have focused on multiple equilibria to understand sudden payment halts for non-fundamental reasons and a specific type of history-dependent equilibrium, the grim trigger payment equilibrium (and its generalized form, the n-trigger payment equilibrium), to understand payments being history-dependent and the resulting payment delays. In this section, we show that the results of payment delays and payments being history-dependent hold much more generally. When the good and bad payment equilibria co-exist, there further exist a set of many history-dependent equilibria, which helps capture the rich pattern of reciprocal payments and their history dependence. As an example, the reserve-holding agent may use a less strict punishment in the sense that it may resume making payments after a number of periods of non-payment. Alternatively, the reserve-holding agent may only stop making payments after having observed multiple periods of non-payment from the other agent.

Given the impossibility to search through a prohibitively immense set of possible equilibria, the repeated game literature has developed an alternative methodology to study the equilibrium outcomes by focusing on the payoffs of agents, that is, the welfare outcomes, rather than imposing any restrictions on the space of the strategy profiles per se. To that end, we adapt the methodology for computing subgame-perfect equilibrium payoffs in repeated games comes from Abreu, Pearce, and Stacchetti (1990), hereafter APS, to our dynamic economy. APS show that the set of equilibrium payoffs satisfies a recursive relationship that is analogous to the Bellman equation from dynamic programming. In particular, any equilibrium payoff can be decomposed into a flow payoff from the first period of play plus the expected discounted payoff from the next period onward, which, by subgame perfection, is also an equilibrium payoff. APS call this idea "decomposability." Just as the value function is the fixed point of the Bellman operator, so too is the equilibrium payoff set is the largest fixed point of an operator that produces the set of payoffs that can be generated using continuation values chosen from a given set. Moreover, APS show that iterating this operator on any set that contains all equilibrium payoffs yields a sequence of sets that asymptotically converges to the set of equilibrium payoffs, which they call "self-generation." Although APS focus on repeated games with imperfect monitoring and without a state variable, we show that their methodology can be extended to the class of games studied here, where there is not a repeated stage game and payoffs are generated in each state using continuation payoffs drawn from a received payoff correspondence.

Our main methodological contribution is an adaption of the APS methodology to a class of non-repeated stochastic dynamic games. In the analogy with dynamic programming, the APS algorithm is identified with value function iteration. We combine this approach with a new form of policy iteration, which is used to solve for equilibrium payoffs when incentive constraints are slack at times. We perform an analysis that yields a recursive characterization of contractual equilibrium payoffs, along the lines of APS, where one relates continuation values that can be achieved from a given period to the continuation values in the next period. The key complication we face here is that the sets of continuation values are different for the two agents depending on their reserve-holding status, and generally differ across periods. Thus, instead of looking for a fixed point set of continuation values, as is the case in the large literature of repeated games, we are looking for a fixed point in the space of indexed collections of sets of continuation values. The approach also leads to new structural insights about equilibria that generate extreme equilibrium payoffs, namely, that play must be stationary until the first period in which an incentive constraint binds. Thanks to the design of our stochastic dynamic game, we are further able to characterize the full set of equilibrium payoffs in closed form, which is typically hard to achieve in the repeated game literature even after applying modern methods (Abreu and Sannikov, 2014, Abreu, Brooks, and Sannikov, 2020).

We first provide some primitive analysis to illustrate the structure of a generic PPE under our framework. In our two-agent stochastic dynamic game, a PPE can be fully characterized by a pair of both agents' continuation values, where the two components in the pair separately capture the continuation values depending on whether a payment is successfully made, that is, whether the reserve good is transferred from one agent to the other. This is because, different from a classic repeated game, the stochastic dynamic game we consider requires an additional state variable of who owns the reserve good, which is scarce. We note that this is different from APS and the subsequent work on repeated games in which the stage game is repeated and does not change over time. This also implies that our methodology involves significant differences from the original APS framework. This methodological contribution provides a useful tool to study long-term interactions in non-repeated stochastic dynamic games and can inform future work in related areas.

Formally, consider any time $t \geq 0$. Suppose agent 1 holds the reserve good at the beginning of date t while agent 2 does not. Let w_i be agent i's equilibrium payoff, that is, per-period continuation value at the beginning of date t, $i \in \{1, 2\}$. Let w_i^k, where $k \in \{0, 1\}$, be agent i's per period continuation value at the beginning of date $t+1$, with $k = 1$ meaning that a transfer of the reserve good successfully occurs at time t and $k = 0$ meaning not.

In what follows, we extend the APS framework of equilibrium payoff construction to our stochastic dynamic game in several steps.

Decomposability. First, following the idea of decomposability in APS, we can decompose the two agents' equilibrium payoff into the current period's payoff and the expected continuation payoff. Note that, any meaningful payment equilibrium profile other than the bad payment equilibrium (described in Proposition 2) must involve the reverse-holding agent choosing to make a payment (upon receiving the private payment shock) at least at some history; otherwise it is the bad payment equilibrium. Thus, without loss of generality, we start from the initial state that agent 1 chooses to make a payment, and we can write:

$$\begin{aligned} w_1 &= (1-\delta)\,\lambda\mu\,(z-c) + \delta\left[\lambda\mu w_1^1 + (1-\lambda\mu)\,w_1^0\right] \\ &= \lambda\mu\left[\delta w_1^1 + (1-\delta)\,(z-c)\right] + (1-\lambda\mu)\,\delta w_1^0 \end{aligned} \tag{5.1}$$

and

$$\begin{aligned} w_2 &= (1-\delta)(1-\lambda\mu)(-c) + \delta\left[\lambda\mu w_2^1 + (1-\lambda\mu) w_2^0\right] \\ &= \lambda\mu\delta w_2^1 + (1-\lambda\mu)\left[\delta w_2^0 - (1-\delta)c\right] \ , \end{aligned} \tag{5.2}$$

which can be summarized by the following vector operation:

$$\mathbf{w} \triangleq \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} = \left[\delta \cdot \mathbf{w}^1 + \begin{pmatrix} (1-\delta)(z-c) \\ 0 \end{pmatrix}, \delta \cdot \mathbf{w}^0 - \begin{pmatrix} 0 \\ (1-\delta)c \end{pmatrix}\right] \begin{bmatrix} \lambda\mu \\ 1-\lambda\mu \end{bmatrix}, \tag{5.3}$$

where the bold letters denote vectors, and $\mathbf{w}^1$ ($\mathbf{w}^0$) is the pair of the two agents' date-$t+1$ per period continuation values if a transfer occurs (does not occur) at date t.

We provide some intuition to help understand conditions (5.1) and (5.2). In (5.1), the reserve-holding agent's current period payoff is $\lambda\mu(z-c)$ because she gets a net payoff of $z-c$ if and only if a successful payment is made, which happens with probability $\lambda\mu$. Otherwise, she does not enjoy any rewards from making a payment but does not suffer from any cost either, resulting in a net payoff of 0. Next period, there are two possible states. If the reserve good is successfully transferred (with probability $\lambda\mu$), her continuation value is w_1^1, while it is w_1^0 if not. Taking expectations of these two continuation values and combining them with the current period payoff gives condition (5.1).

Similarly, (5.2) decomposes the non-reserve-holding agent's equilibrium payoff. Her current period payoff is $(1-\lambda\mu)(-c)$ because she suffers from a cost of lacking the reserve good if and only if she has not received any payment from the other reserve-holding agent, which happens with probability $1-\lambda\mu$. Otherwise, she gets the reserve good and avoids the cost, getting a net payoff of 0. Next period, there are still two possible states. If the reserve good is successfully transferred (with probability $\lambda\mu$), her continuation value is w_2^1, while it is w_2^0 if not. Taking expectations of these two continuation values and combining them with the current period payoff gives condition (5.2).

Based on the construction above, we can define the following operator for any $\mu \in (0,1)$, $\mathbf{w}^1, \mathbf{w}^0 \in \mathbb{R}^2$,

$$\phi\left(\mu, \mathbf{w}^1, \mathbf{w}^0\right) = \left[\delta \cdot \mathbf{w}^1 + \begin{pmatrix} (1-\delta)(z-c) \\ 0 \end{pmatrix}, \delta \cdot \mathbf{w}^0 - \begin{pmatrix} 0 \\ (1-\delta)c \end{pmatrix}\right] \begin{bmatrix} \lambda\mu \\ 1-\lambda\mu \end{bmatrix}. \tag{5.4}$$

Then, (5.3) can be written as

$$\mathbf{w} = \phi\left(\mu, \mathbf{w}^1, \mathbf{w}^0\right) \ .$$

Enforceability (incentive compatibility). The second step is to incorporate the two agents' incentive compatibility conditions, which APS call "enforceability." Recall that, due to the imperfect payment technology, a payment will go through with probability μ when the reserve-holding agent chooses to send it. Facing future continuation values $\mathbf{w}^1$ and $\mathbf{w}^0$, therefore, the reserve-holding agent effectively chooses the probability at which a payment is successfully made:

$$\mu = \mu\left(\mathbf{w}^1, \mathbf{w}^0\right) \triangleq \begin{cases} \mu & \text{if } (1-\delta)(z-c) + \delta w_1^1 \geq \delta w_1^0 \\ 0 & \text{if } (1-\delta)(z-c) + \delta w_1^1 < \delta w_1^0 \ , \end{cases} \tag{5.5}$$

where we break the tie by assuming that any agent makes a payment when she is indifferent.

The intuition of the enforceability condition (5.5) is straightforward: conditional on receiving the private payment shock, the reserve-holding agent chooses to make a payment if and only if this action gives her a higher total payoff (including current period's payoff and next period's expected payoff) compared to not making a payment.

Generating function and evolution of the state variable. Third, we construct the generating function as in APS. Notably, we develop an approach to accommodate the evolution of the state variable (i.e., the ownership of the scarce reserve good), which is not present in APS. The change of reserve ownership differentiates our stochastic dynamic game from standard repeated games in which the stage game is repeated and the space of actions in a stage game does not change over time.

For any payoff set $W \subset \mathbb{R}^2$, define its transpose as

$$\mathbb{T}(W) = \{(w_1, w_2) : (w_2, w_1) \in W\} \ .$$

With a little abuse of notation but no confusion, for any $\mathbf{w} = (w_1, w_2) \in \mathbb{R}^2$, we also write $\mathbb{T}(\mathbf{w}) = (w_2, w_1) \in \mathbb{R}^2$.

Let $V_s \subset \mathbb{R}^2$ denote the set of per-period continuation value pairs that can be supported in a PPE when the state is $s \in \{0, 1\}$, that is, the equilibrium payoff set for the two agents, where s is the same as defined in Section 2. Specifically, V_1 is set of the equilibrium value pairs when agent 1 has the reserve good, while V_0 is that when agent 1 does not have the reserve good.

Definition 7. *Let $B(\cdot)$ be a set operator such that for any $W \subset \mathbb{R}^2$,*

$$B(W) = \left\{\mathbf{w} \in \mathbb{R}^2 : \exists \mathbf{w}^0 \in W \textit{ and } \mathbf{w}^1 \in \mathbb{T}(W), \textit{ such that } \mathbf{w} = \phi\left(\mu\left(\mathbf{w}^1, \mathbf{w}^0\right), \mathbf{w}^1, \mathbf{w}^0\right)\right\}\ .$$

Following APS, we call the operator $B(\cdot)$ the generating function. As discussed before, however, the structure of this generating function is significantly different from its original version in APS and the large literature of repeated games following APS. Specifically, $B(W)$ in our non-repeated dynamic stochastic game consists of pairs of continuation values that can be supported by future continuation value pairs $\mathbf{w}^1$ and $\mathbf{w}^0$ chosen from $\mathbb{T}(W)$ and W, respectively, contingent on whether there is a successful transfer of the reserve good or not. Note that $\mathbf{w}^1$, the pair of future continuation values after a transfer, is chosen from $\mathbb{T}(W)$ instead of W, because after a transfer of the reserve good the state of the game switches from s to $1 - s$, and $V_{1-s} = \mathbb{T}(V_s)$. Fortunately, due to the symmetry between V_0 and V_1, we need to characterize one of them only. We will focus on V_1 below, a generic element of which is (w_1, w_2), with w_1 being the continuation value of the reserve-holding agent, and w_2 being the continuation value of the non-reserve-holding agent.

We provide another perspective to understand why we can handle the evolution of the reserve good ownership as the state variable in the generating function without actually tracking it. In our economy, this stems from the fact that the two agents jointly own one indivisible reserve good. Whenever a successful transfer of the reserve good takes place, the initially reserve-holding (non-reserve-holding) agent becomes the non-reserve-holding (reserve-holding) agent. The resulting new stage game, albeit different from the initial stage game due to the change of ownership, mirrors the initial game by switching the roles of the two agents. Mathematically, this can be thus handled by the transpose of the initial vector that consists of the two agents' payoffs. This feature plays an important role in the analysis of our economy. As discussed above, this feature requires a setup that prevents us from directly making quantitative predictions regarding the amount of the reserve goods. However, it does sufficiently capture the scarcity of the reserve good, which is the key. Furthermore, we also gain the analytical tractability to make significant progress in characterizing the full equilibrium payoff set.

Self-generation and equilibrium payoff set. Having developed an approach to handle the evolution of the state variable, we extend the notion of self-generation in APS to our framework and present an analytical procedure to characterize the equilibrium payoff set V_1, as the last step.

Lemma 1. *The equilibrium payoff set is self-generating in the sense that $V_1 \subset B(V_1)$.*

Lemma 1 holds by definition of PPE: any sub-game equilibrium of a PPE is itself a PPE. Economically, this means that the set of equilibrium payoffs V_1 should be self-generating in the sense that it is possible to sustain average payoffs in V_1 by promising different continuation values in V_1.

Lemma 2. *If $W \subset \mathbb{R}^2$ is bounded and $W \subset B(W)$, then $B(W) \subset V_1$.*

Lemma 2 then gives us a criterion for identifying subsets of the equilibrium payoff set V_1, because any self-generating set is such a subset. Interestingly, other than boundedness, Lemma 2 does not impose any restrictions on the payoff set W. One might expect that applying the generating function $B(\cdot)$ on W would generate payoffs that are not attainable in our dynamic economy. Indeed, the real requirement for Lemma 2 is that W must be able to generate a superset of itself. Intuitively, the generating function given by Definition 7 implies that any $B(W)$ must be itself enforceable in our dynamic economy. Because any enforceable payoff constructs an equilibrium payoff by the definition of PPE, we have the desired result.

Proposition 6. *The equilibrium payoff set satisfies $V_1 = B(V_1)$.*

Proposition 6 is important and directly follows from Lemmas 1 and 2. It states that the equilibrium payoff set V_1 is a fixed point of the generating function $B(\cdot)$. Following APS, we thus call that the equilibrium payoff set V_1 can be factorized. Economically, this implies that V_1 can be found by characterizing the largest fixed point of $B(\cdot)$.

So far, we have extended the APS methodology to our non-repeated stochastic dynamic game and show that the equilibrium payoff set can be similarly characterized by factorization despite the change of reserve ownership as the state variable. However, we note that it is generally hard to analytically characterize the equilibrium payoff set in repeated games even with the powerful tool developed by APS. Rather, the literature has largely focused on describing some potential features of the set, for example, whether the set is compact or closed as APS initially focuses on, or imposing restrictions on the discount factor (e.g., Fudenberg, Levine and Maskin, 1994), or imposing restrictions on the strategy space with the notable example of focusing on strongly symmetric PPE in that all players use the same strategy after every history (e.g., Athey, Bagwell and Sanchirico, 2004). Alternatively, a literature has focused on developing methods to solve for the equilibrium payoff set numerically (e.g., Abreu and Sannikov, 2014, Abreu, Brooks, and Sannikov, 2020). Thanks to the structure of our dynamic economy, we are able to further make significant progress by analytically solving for the full equilibrium payoff set without imposing any restrictions on the discount factor or the strategy space.

To proceed, we first identify a boundary of the equilibrium payoff set V_1 by highlighting some intuitive features of our dynamic payment game:

Lemma 3. *Let $X \subset \mathbb{R}^2$ be a set of pairs of per-period continuation values satisfying the above four constraints, i.e.,*

$$X = \left\{(w_1, w_2) \in \mathbb{R}^2 : -c \leq w_1 + w_2 \leq \lambda\mu z - c,\, w_1 \geq 0 \text{ and } w_2 \geq -c\right\} .$$

Then $V_1 \subset X$.

The proof of Lemma 3 is constructive; the intuition can be understood in the following four steps. Let $\mathbf{w} = (w_1, w_2) \in V_1$. First, $w_1 + w_2 \leq \lambda\mu z - c$. This is because at any given time t, the maximum expected gain from making a payment is $\lambda\mu z$ (i.e., when the good payment equilibrium is played), and one and only one agent suffers from a (per period) cost for falling short of the reserve good, which is c. This captures the upper bound of the two agents' equilibrium payoff sums.

Second, $w_1 + w_2 \geq -c$. This is because in the worst case, that is, when the bad payment equilibrium is played, the reserve-holding agent does not send it upon the payment shock, and the two agents as a whole enjoy no gain but suffer from a (per period) cost of c. This thus captures the lower bound of the two agents' equilibrium payoff sums.

Third, $w_1 \geq 0$. This is because the reserve-holding agent can guarantee a zero payoff for all future periods by not sending the transfer. This captures the lower bound for the reserve-holding agent's equilibrium payoff.

Fourth, $w_2 \geq -c$. This is because the non-reserve-holding agent will at most suffer from a per period cost of $-c$ in the bad payment equilibrium. This captures the lower bound for the non-reserve-holding agent's equilibrium payoff.

Lemma 4. *If $W \subset \widetilde{W}$, then $B(W) \subset B\left(\widetilde{W}\right)$.*

Lemma 4 further states that the generating function $B(\cdot)$ is monotone with respect to the partial order induced by set inclusion "$\subset$". In addition, since all subsets of X form a complete lattice with respect to "$\cap$" and "$\cup$" and the partial order, according to Tarski's fixed point theorem, the fixed points of $B(\cdot)$ form a complete lattice and thus there is a maximal one, which is V_1. In particular, by definition, $B^n(X)$ forms a non-increasing set sequence and $V_1 = \lim_{n\to\infty} B^n(X)$.

Using Proposition 6 and Lemmas 3 and 4, we are finally able to present the following main result of this section, which shows that all the equilibrium-sustainable payoff outcomes for the two agents can be characterized by a triangle. Further, we are able to solve the three extreme points of this triangle in closed form.

Proposition 7. *The equilibrium payoff set V_1 can be characterized by a triangle with the following three extreme points:*

$$\begin{cases} X_1 = (0, -c)', \\ X_2 = \left(\dfrac{\lambda\mu((1-\delta(1-\lambda\mu))z - c)}{1+\delta(2\lambda\mu - 1)}, \dfrac{\delta(\lambda\mu)^2 z - (\delta\lambda\mu + (1-\delta)(1-\lambda\mu))c}{1+\delta(2\lambda\mu-1)} \right)', \\ X_3 = \left(0, -c + \dfrac{\lambda\mu z(2c - (1-\delta)z)}{c + \delta\lambda\mu z} \right)'. \end{cases}$$

The two triangles in Figure 6 illustrate the sets of equilibrium payoffs V_1 for two different sets of model parameters. Specifically, the full set of PPEs can be characterized by a triangle in the space of (w_1, w_2), in which every pair of continuation values are attainable. The equilibrium payoff set V_1 fully captures all the possible welfare outcomes for the two agents in our economy.

It is also worth noting that the two extreme points X_1 and X_2 in V_1, according to Propositions 1 and 2, denote the equilibrium payoffs in the bad and good payment equilibria, respectively, in which payment decisions are history independent. Excluding these two extreme points, the other equilibrium payoffs in V_1 can be achieved by potentially history-dependent payment equilibria, the grim trigger payment equilibrium being a notable example.

The closed-form solution provided in Proposition 7 allows for easy and explicit comparative statics of the full equilibrium payoff set with respect to important model parameters such as the payment technology and agents' time preferences. Figure 6 shows that the equilibrium payoff set expands in that more equilibrium outcomes can be supported in which both agents enjoy higher payoffs when the quality of the payment technology μ increases. In other words, the equilibrium payoff set for a smaller μ is a subset of that for a larger μ. Intuitively, a better payment technology benefits both agents despite payment decisions potentially being history-dependent. This result is reminiscent of Kandori (1992), which shows that the set of PPE payoffs in repeated games is increasing as the public monitoring technology becomes more precise. We note again, however, an improvement in the payment technology may not necessarily reduce payment fragility, as shown in Corollaries 1 and 2.

Similarly, Figure 7 shows the evolution of the equilibrium payoff set when agents become more patient or when the aggregate bad shock is less likely, that is, when δ increases. Interestingly, an increase in δ leads to more equilibrium outcomes that disproportionately benefit the

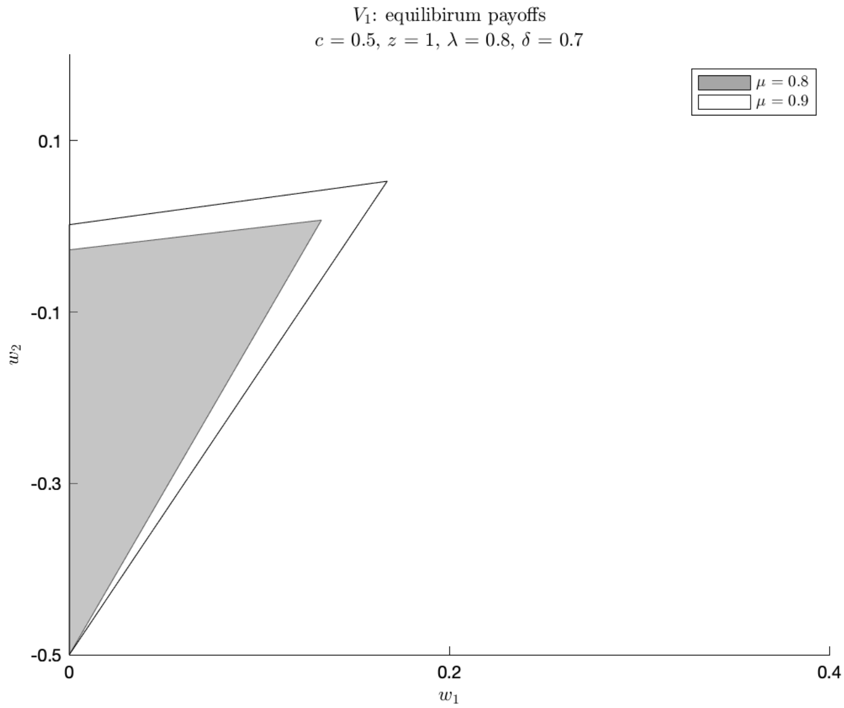

Figure 6: Equilibrium payoffs and comparative statics w.r.t. payment technology

This graph plots the equilibrium set V_1 for two sets of parameters. The shaded area denotes the equilibrium payoffs of equilibria in which payment decisions are history-dependent. Parameters: $c = 0.5$, $z = 1$, $\delta = 0.7$, $\lambda = 0.8$, and μ increases from 0.8 to 0.9.

non-reserve-holding agent (i.e., a higher w_2) while hurting the reserve-holding agent (i.e., a lower w_1). This result significantly differs from the standard APS result for a generic repeated game in that under mild conditions the equilibrium PPE set for a larger discount factor is a superset of that for a smaller discount factor. To understand the intuition of Figure 7 and the contrast to existing results in the literature, notice that the two agents' roles are not symmetric in our (non-repeated) dynamic game that captures asynchronous but reciprocal payments. Intuitively, when δ increases, the reserve-holding agent cares more about the future. This implies a higher incentive for her to send the requested payment to the non-reserve-holding agent, everything else equal. The higher incentive thus implies an expected welfare transfer from the reserve-holding agent to the non-reserve-holding agent, which fundamentally arises from the scarcity of the reserve good. Put differently, a higher δ encourages the reserve-holding agent to give up the scarce reserve good now in exchange for potentially future coordination due to the history-dependence of payments, which disproportionally benefits the currently non-reserve-holding agent.

Furthermore, it is useful to compare Proposition 7 to the results in Abreu, Pearce, and Stacchetti (1986) and Athey, Bagwell and Sanchirico (2004) in which the equilibrium payoff set

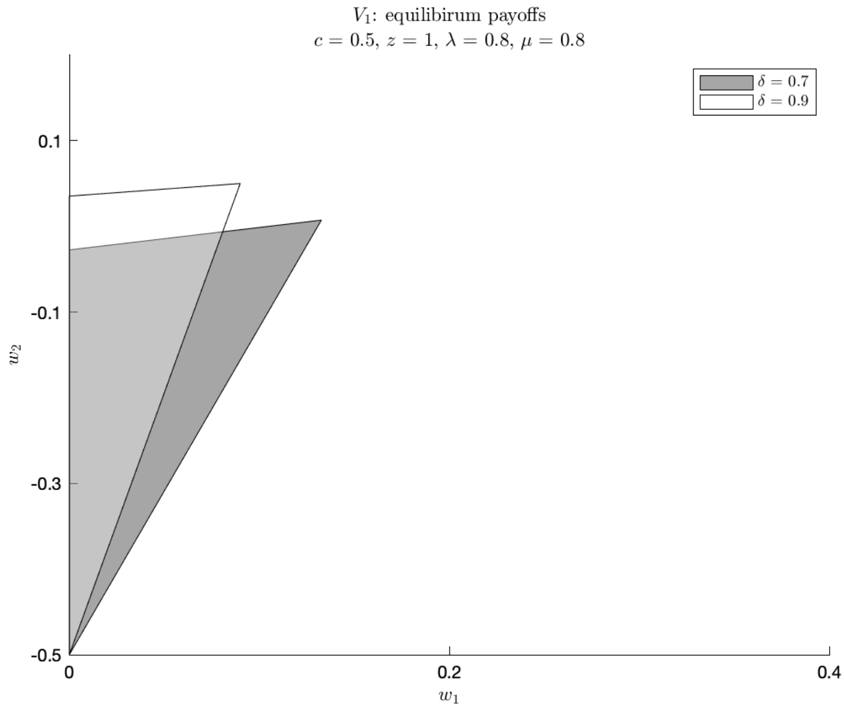

Figure 7: Equilibrium payoffs and comparative statics w.r.t. time preferences

This graph plots the equilibrium set V_1 for two sets of parameters. The shaded area denotes the equilibrium payoffs of equilibria in which payment decisions are history-dependent. Parameters: $c = 0.5$, $z = 1$, $\lambda = 0.8$, $\mu = 0.8$, and δ increases from 0.7 to 0.9.

can be also analytically characterized when only strongly symmetric equilibria are considered. Abreu, Pearce, and Stacchetti (1986) first introduce the concept of symmetric games to focus on PPEs that are strongly symmetric in the sense that each player uses the same strategy after each history. This is a useful simplification because the equilibrium payoff set of strongly symmetric PPE can be analytically characterized by a compact interval $[\overline{\mathbf{w}}, \underline{\mathbf{w}}]$, where $\overline{\mathbf{w}}$ and $\underline{\mathbf{w}}$ are the lowest and highest strongly symmetric PPE payoffs. Specifically, Abreu, Pearce, and Stacchetti (1986) analyze strongly symmetric equilibria in Green and Porter (1984)'s oligopoly game where players choose quantities and the price is a noisy function of the aggregate quantity. Athey, Bagwell and Sanchirico (2004) study strongly symmetric equilibria in a repeated Bertrand pricing game where firms have private cost information. In contrast, we do not impose any restrictions on the strategy space and show that the full set of equilibrium payoffs in our dynamic economy is captured by a compact area rather than an interval.

Indeed, as Proposition 7 shows, the equilibrium payoff set for strongly symmetric PPEs is captured by the interval $[X_1, X_2]$, where X_1 and X_2 are the equilibrium payoffs for the bad and good payment equilibria that are themselves strongly symmetric PPEs. However, an important observation from Proposition 7 is that the dynamic economy we consider admits many other non-symmetric PPEs. We are able to fully characterize them, which represents significant progress compared to the existing literature.

Finally, it is also useful to compare our methodology to other commonly used equilibrium selection mechanisms in coordination games. In classic static coordination games (e.g., Diamond and Dybvig, 1983), the strategic complementarity lies in agents' simultaneous decisions. The global games technique has been widely adopted to select a unique equilibrium and link it to economic fundamentals (e.g. Morris and Shin, 1998, Goldstein and Pauzner, 2005). However, it is well known that it is difficult to apply the global games technique to general stochastic dynamic games (Angeletos, Hellwig and Pavan, 2007). A related but independent literature considers dynamic games in which random fundamental shocks (rather than past histories of decisions) serve as a coordination device in the presence of dynamic strategic complementarity (e.g., Frankel and Pauzner, 2000, He and Xiong, 2012). This literature can explain financial fragility in dynamic interactions but is not designed to explain endogenous patterns of history-dependence in agents' actions and strategies, which we view as a prevalent feature of payments. Given these challenges, we take an alternative approach that is commonly used in the repeated games literature. Specifically, we choose not to select any specific equilibrium but rather directly characterize the equilibrium outcomes of all the equilibria. This approach helps us capture the rich dynamics of history-dependence in the various applications of payments. Our framework and the solution method may also inform future studies that focus on asynchronous coordination in dynamic contexts.

6 Conclusion

We present a dynamic theory of payments that establishes a crucial link to the scarcity of reserves. Our proposition posits that all payments entail the transfer of a reserve good, which, apart from serving payment functions, holds value for non-payment purposes and exhibits inelastic supply. The model's insights illuminate the contrasting behaviors of agents in different reserve abundance scenarios: agents make payments when reserves abound, while they cease payments when reserves become scarce relative to the payment technology. When reserve scarcity falls within an intermediate range, the model unfolds multiple equilibria, with agents' payment decisions becoming linked to the payment history of their counterparts within an equilibrium. The model explains why payments frequently encounter delays and halts, even when agents possess adequate funding. The model indicates that advancements in payment technologies might not always be the panacea for fragility reduction. We develop a new methodology to analyze the welfare outcomes of all feasible equilibria, capturing the intricate dynamics governing the history-dependent nature of payments. The model applies to various payment contexts, encompassing metallic payments preceding fiat money, modern bank payments, cross-border transactions, and modern digital payment systems.

Our work extends to illuminating a conflict between price stability and financial stability. In an era grappling with surging inflationary pressures, contemporary economic debate centers around the optimal intensity of monetary policy tightening, reminiscent of the spirited contest between Arthur Burns and Paul Volcker (Blinder, 2022). Yet, we argue that this debate should be understood at a deeper level. As we commemorate the 150th anniversary of Walter Bagehot's seminal publication, *Lombard Street*, a treatise that reshaped the central banking history, our work sheds light on the role of modern central banks. Bagehot's work laid the foundation for central banks' role as lenders of last resort for the money market and the payment function of money. In that spirit, our work highlights that any monetary policies that involve an increase in interest rates or a reduction in the supply of the monetary base may inevitably result in increased reserve scarcity for the payment function, potentially leading to payment fragility. As such, our work advocates for a judicious calibration of monetary policies, navigating the trade-off between price stability and the financial stability.

References

ACHARYA, VIRAL and RAGHU RAJAN (2022). "Liquidity, Liquidity Everywhere, Not A Drop to Use Why Flooding Banks with Central Bank Reserves May Not Expand Liquidity." Working Paper.

ABREU, DILIP, BENJAMIN BROOKS, and YULIY SANNIKOV (2020). "Algorithms for Stochastic Games with Perfect Monitoring." *Econometrica*, 88: 1661-1695.

ABREU, DILIP, DAVID PEARCE, and ENNIO STACCHETTI (1986). "Optimal Cartel Equilibria with Imperfect Monitoring." *Journal of Economic Theory*, 39: 251-269.

ABREU, DILIP, DAVID PEARCE, and ENNIO STACCHETTI (1990). "Toward a Theory of Discounted Repeated Games with Imperfect Monitoring." *Econometrica*, 58: 1041-1063.

ABREU, DILIP and YULIY SANNIKOV (2014). "An Algorithm for Two-Player Repeated Games with Perfect Monitoring." *Theoretical Economics*, 9: 313-338.

ACHARYA, VIRAL and OUARDA MERROUCHE (2013). "Precautionary Hoarding of Liquidity and Interbank Markets: Evidence from the Subprime Crisis." *Review of Finance*, 17: 107-160.

ACHARYA, VIRAL and DAVID SKEIE (2011). "A Model of Liquidity Hoarding and Term Premia in Inter-bank Markets." *Journal of Monetary Economics*, 58: 436-447.

AFONSO, GARA, MARCO CIPRIANI, ADAM COPELAND, ANNA KOVNER, GABRIELE LA SPADA, and ANTOINE MARTIN (2020). "The Market Events of Mid-September 2019." Working Paper.

AFONSO, GARA, ANNA KOVNER, and ANTOINETTE SCHOAR (2011). "Stressed, not Frozen: The Federal Funds Market in the Financial Crisis." *Journal of Finance*, 66: 1109-1139.

AFONSO, GARA and HYUN SONG SHIN (2011). "Precautionary Demand and Liquidity in Payment Systems." *Journal of Money, Credit and Banking*, 43: 589-619.

AFONSO, GARA, DARRELL DUFFIE, LORENZO RIGON, and HYUN SONG SHIN (2022). "Strategic Complementarities in the Payment System in the Era of Ample Reserves." Working Paper.

ALLEN, FRANKLIN, ELENA CARLETTI, and DOUGLAS GALE (2009). "Interbank Market Liquidity and Central Bank Intervention." *Journal of Monetary Economics*, 56: 639-652.

ALLEN, FRANKLIN and DOUGLAS GALE (2000). "Financial Contagion." *Journal of Political Economy*, 108: 1-33.

ALVAREZ, FERNANDO and URBAN JERMANN (2000). "Efficiency, Equilibrium, and Asset Pricing with Risk of Default." *Econometrica*, 68: 775-797.

ANGELETOS, GEORGE-MARIOS, CHRISTIAN HELLWIG, and ALESSANDRO PAVAN (2007). "Dynamic Global Games of Regime Change: Learning, Multiplicity, and the Timing of Attacks.." *Econometrica*, 75: 711-756.

ASHCRAFT, ADAM and DARRELL DUFFIE (2007). "Systemic Illiquidity in the Federal Funds Market." *American Economic Review*, 97: 221-225.

ASHCRAFT, ADAM, JAMES MCANDREWS, and DAVID SKEIE (2011). "Precautionary Reserves and the Interbank Market." *Journal of Money, Credit and Banking*, 43: 311-348.

ATHEY, SUSAN, KYLE BAGWELL, and CHRIS SANCHIRICO (2004). "Collusion and Price Rigidity." *Review of Economic Studies*, 71: 317-349.

ATHEY, SUSAN, ANDREW ATKESON, and PATRICK KEHOE (2005). "The Optimal Degree of Discretion in Monetary Policy." *Econometrica*, 73: 1431-1475.

AUER, RAPHAEL, JON FROST, LEONARDO GAMBACORTA, CYRIL MONNET, TARA RICE, and HYUN SONG SHIN (2022). "Central Bank Digital Currencies: Motives, Economic Implications, and the Research Frontier." *Annual Review of Economics*, 14: 697-721.

BAUMOL, WILLIAM (1952). "The Transactions Demand for Cash: An Inventory Theoretic Approach." *Quarterly Journal of Economics*, 66: 545-556.

BECH, MORTEN and ROD GARRATT (2003). "The Intraday Liquidity Management Game." *Journal of Economic Theory*, 109: 198-219.

BIANCHI, JAVIER and SAKI BIGIO (2021). "Banks, Liquidity Management, and Monetary Policy." *Econometrica*, forthcoming.

BIGIO, SAKI (2022). "A Theory of Payments-Chain Crises." Working Paper.

BIGIO, SAKI and YULIY SANNIKOV (2023). "A Model of Credit, Money, Interest, and Prices." Working Paper.

BLICKLE, KRISTIAN, MARKUS BRUNNERMEIER and STEPHAN LUCK (2022). "Who Can Tell Which Banks Will Fail?" Working Paper.

BLINDER, ALAN (2022). *A Monetary and Fiscal History of the United States, 1961-2021.* Princeton University Press.

BRUNNERMEIER, MARKUS, HAROLD JAMES and JEAN-PIERRE LANDAU (2019). "The Digitalization of Money." Working Paper.

BRUNNERMEIER, MARKUS, SEBASTIAN MERKEL and YULIY SANNIKOV (2022). "Safe Assets: A Dynamic Retrading Perspective." Working Paper.

BRUNNERMEIER, MARKUS and JONATHAN PAYNES (2022). "Central Bank Digital Currency, FinTech and Private Money Creation." in Dirk Niepelt (eds.), *Central Bank Digital Currenty: Considerations, Projects, and Outlook*, Center for Economic and Policy Research.

BRUNNERMEIER, MARKUS and YULIY SANNIKOV (2016). "The I Theory of Money." Working Paper.

BOLTON, PATRICK, YE LI, NENG WANG, and JINQIANG YANG (2020). "Dynamic Banking and the Value of Deposits." Working Paper.

CABALLERO, RICARDO and ARVIND KRISHNAMURTHY (2008). "Collective Risk Management in a Flight to Quality Episode." *Journal of Finance*, 63: 2195-2230.

CHANG, ROBERTO (1998). "Credible Monetary Policy in an Infinite Horizon Model: Recursive Approaches." *Journal of Economic Theory*, 81: 431-461.

COPELAND, ADAM, DARRELL DUFFIE, and DAVID YANG (2020). "Reserves Were Not So Ample After All: Evidence from the U.S. Treasury Repo Market." Working Paper.

COPPOLA, ANTONIO, ARVIND KRISHNAMURTHY, and CHENZI XU (2023). "Liquidity, Debt Denomination, and Currency Dominance." Working Paper.

CORREA, RICARDO, WENXIN DU, and GORDON LIAO (2019). "U.S. Banks and Global Liquidity." Working Paper.

CRAIG, BEN and YIMING MA (2021). "Intermediation in the Interbank Lending Market." *Journal of Financial Economics*, forthcoming.

D'AVERNAS, ADRIEN and QUENTIN VANDEWEYER (2020). "Intraday Liquidity and Money Market Dislocations." Working Paper.

DANG, TRI VI, GARY GORTON, BENGT HOLMSTRÖM and GUILLERMO ORDOÑEZ (2017). "Banks as Secret Keepers." *American Economic Review*, 107: 1005-1029.

DIAMOND, DOUGLAS and PHILIP DYBVIG (1983). "Bank Runs, Deposit Insurance, and Liquidity." *Journal of Political Economy*, 91: 401-419.

DIAMOND, DOUGLAS and RAGHURAM RAJAN (2005). "Liquidity Shortages and Banking Crises." *Journal of Finance*, 60: 615-647.

DIAMOND, DOUGLAS and RAGHURAM RAJAN (2006). "Money in a Theory of Banking." *American Economic Review*, 96: 30-53.

DIAMOND, WILLIAM, ZHENGYANG JIANG, and YIMING MA (2022). "The Reserve Supply Channel of Unconventional Monetary Policy." Working Paper.

DONALDSON, JASON, GIORGIA PIACENTINO, and ANJAN THAKOR (2018). "Warehouse Banking." *Journal of Financial Economics*, 129: 250-267.

DRECHSLER, ITAMAR, ALEXI SAVOV and PHILIPP SCHNABL (2017). "The Deposits Channel of Monetary Policy." *Quarterly Journal of Economics*, 132: 1819-1876.

DUFFIE, DARRELL (2019). "Digital Currencies and Fast Payment Systems: Disruption is Coming." Asian Monetary Policy Forum.

FISHER, IRVING (1911). *The Purchasing Power of Money: Its Determination and Relation to Credit, Interest, and Crisis.* Macmillan, New York.

FISHER, IRVING (1933). "The Debt-Deflation Theory of Great Depression." *Econometrica*, 1: 337-357.

FRANKEL, DAVID and ADY PAUZNER (2000). "Resolving Indeterminacy in Dynamic Settings: the Role of Shocks." *Quarterly Journal of Economics*, 115: 285-304.

FREIXAS, XAVIER, BRUNO PARIGI, and JEAN-CHARLES ROCHET (2000). "Systemic Risk, Interbank Relations, and Liquidity Provision by the Central Bank." *Journal of Money, Credit and Banking*, 32: 611-638.

FRIEDMAN, JAMES (1971). "A Non-Cooperative Equilibrium for Supergames." *Review of Economic Studies*, 38: 1-12.

FRIEDMAN, MILTON and ANNA SCHWARTZ (1963). *A Monetary History of the United States, 1867-1960.* Princeton University Press.

FUDENBERG, DREW, DAVID LEVINE, and ERIC MASKIN (1994). "The Folk Theorem with Imperfect Public Information." *Econometrica*, 62: 997-1039.

FUDENBERG, DREW and ERIC MASKIN (1986). "The Folk Theorem in Repeated Games with Discounting or with Incomplete Information." *Econometrica*, 53: 533-554.

GALE, DOUGLAS and TANJU YORULMAZER (2013). "Liquidity Hoarding." *Theoretical Economics*, 8: 291-324.

GHOSH, PULAK, BORIS VALLEE, and YAO ZENG (2022). "FinTech Lending and Cashless Payment." Working Paper.

GOLDSTEIN, ITAY and ADY PAUZNER (2005). "Demand Deposit Contracts and the Probability of Bank Runs." *Journal of Finance*, 60: 1293-1328.

GONZALEZ, RODRIGO, LARS NORDEN, BERNARDUS VAN DOORNIK, and YAO ZENG (2023). "The Macroeconomics of Instant Payments and CBDC: Evidence from Brazil's Pix." Working Paper.

GOPINATH, GITA and JEREMY STEIN (2021). "Banking, Trade, and the Making of a Dominant Currency." *Quarterly Journal of Economics*, 136: 783-830.

GORTON, GARY, ELIZABETH C. KLEE, CHASE ROSS, SHARON ROSS, and ALEXANDROS P. VARDOULAKIS (2023). "Leverage and Stablecoin Pegs." Working Paper.

GORTON, GARY and GEORGE PENNACCHI (1990). "Financial Intermediaries and Liquidity Creation." *Journal of Finance*, 45: 49-71.

GREEN, EDWARD and ROBERT PORTER (1984). "Noncooperative Collusion under Imperfect Price Information." *Econometrica*, 52: 87-100.

GU, CHAO, FABRIZIO MATTESINI, and RANDALL WRIGHT (2016). "Money and Credit Redux." *Econometrica*, 84: 1-32.

GU, CHAO, FABRIZIO MATTESINI, CYRIL MONNET, and RANDALL WRIGHT (2013). "Banking: A New Monetarist Approach." *Review of Economic Studies*, 80: 636-662.

HALAC, MARINA (2012). "Relational Contracts and the Value of Relationships." *American Economic Review*, 102: 750-779.

HAYEK, FRIEDREICH A. (1976). *Choice in Currency: A Way to Stop Inflation.* Ludwig von Mises Institute.

HE, ZHIGUO, ARVIND KRISHNAMURTHY, and KONSTANTIN MILBRADT (2019). "A Model of Safe Asset Determination." *American Economic Review*, 109: 1230-1262.

HE, ZHIGUO and WEI XIONG (2012). "Dynamic Debt Runs." *Review of Financial Studies*, 25: 1799-1843.

HEIDER, FLORIAN, MARIE HOEROVA, and CORNELIA HOLTHAUSEN (2015). "Liquidity Hoarding and Interbank Market Rates: The Role of Counterparty Risk." *Journal of Financial Economics*, 118: 336-354.

HIGGINS, SEAN (2020). "Financial Technology Adoption." *American Economic Review*, forthcoming.

HILL, JULIE (2018). "SWIFT Bank Heists and Article 4A." *Journal of Consumer and Commercial Law*, 22: 25-30.

JACK, WILLIAM and TAVNEET SURI (2014). "Risk Sharing and Transactions Costs: Evidence from Kenya's Mobile Money Revolution." *American Economic Review*, 104: 183-223.

JERMANN, URBAN (2022). "Gold's Value as An Investment." Working Paper.

JIANG, ZHENGYANG, ARVIND KRISHNAMURTHY, and HANNO LUSTIG (2021). "Dollar Safety and the Global Financial Cycle." Working Paper.

KAHN, CHARLES and WILLIAM ROBERDS (2009). "Why Pay? An Introduction to Payments Economics." *Journal of Financial Intermediation*, 18: 1-23.

KANDORI, MICHIHIRO (1992). "The Use of Information in Repeated Games with Imperfect Monitoring." *Review of Economic Studies*, 59: 581-593.

KEHOE, TIMOTHY and DAVID LEVINE (1993). "Debt-Constrained Asset Markets." *Review of Economic Studies*, 60: 865-888.

KIYOTAKI, NOBUHIRO and RANDALL WRIGHT (1989). "On Money as A Medium of Exchange." *Journal of Political Economy*, 97: 927-954.

KOCHERLAKOTA, NARAYANA (1996). "Implications of Efficient Risk Sharing without Commitment." *Review of Economic Studies*, 63: 595-609.

KOCHERLAKOTA, NARAYANA (1998). "Money Is Memory." *Journal of Economic Theory*, 81: 232-251.

LAGOS, RICARDO, GUILLAUME ROCHETEAU, and RANDALL WRIGHT (2017). "Liquidity: A New Monetarist Perspective." *Journal of Economic Literature*, 55: 371-440.

LAGOS, RICARDO and RANDALL WRIGHT (2005). "Unified Framework for Monetary Theory and Policy Analysis." *Journal of Political Economy*, 113: 463-484.

LAGOS, RICARDO and SHENGXING ZHANG (2020). "Turnover Liquidity and the Transmission of Monetary Policy." *American Economic Review*, 110: 1635-1672.

LEVIN, JONATHAN (2003). "Relational Incentive Contracts." *American Economic Review*, 93: 835-857.

LI, YE and YI LI (2021). "Payment Risk and Bank Lending." Working Paper.

LOPEZ-SALIDO, DAVID and ANNETTE VISSING-JORGENSEN (2023). "Reserve Demand, Interest Rate Control, and Quantitative Tightening." Working Paper.

MAGGIORI, MATTEO, BRENT NEIMAN, and JESSE SCHREGER (2021). "International Currencies and Capital Allocation." *Journal of Political Economy*, 128: 2019-2066.

MCANDREWS, JAMES and SIMON POTTER (2002). "Liquidity Effects of the Events of September 11, 2001." *Federal Reserve Bank of New York Economic Policy Review*, 8: 59-79.

MORRIS, STEPHEN and HYUN SONG SHIN (1998). "Unique Equilibrium in A Model of Self-Fulfilling Currency Attacks" *American Economic Review*, 88: 587-597.

MURALIDHARAN, KARTHIK, PAUL NIEHAUS, and SANDIP SUKHTANKAR (2016). "Building State Capacity: Evidence from Biometric Smartcards in India." *American Economic Review* 106: 2895-2929.

PARLOUR, CHRISTINE, UDAY RAJAN, and JOHAN WALDEN (2020). "Payment System Externalities and the Role of Central Bank Digital Currency." *Journal of Finance*, forthcoming.

PHELAN, CHRISTOPHER and ENNIO STACCHETTI (2001). "Sequential Equilibria in a Ramsey Tax Model." *Econometrica*, 69: 1491-1518.

PIAZZESI, MONIKA, CIARAN ROGERS and MARTIN SCHNEIDER (2021). "Money and Banking in a New Keynesian Model." Working Paper.

PIAZZESI, MONIKA and MARTIN SCHNEIDER (2021). "Payments, Credit and Asset Prices." Working Paper.

REDISH, ANGELA (2000). *Bimetallism: An Economic and Historical Analysis.* Cambridge University Press.

SARGENT, THOMAS and FRANCOIS VELDE (2001). *The Big Problem of Small Change.* Princeton University Press.

TOBIN, JAMES (1956). "The Interest Elasticity of the Transactions Demand for Cash." *Review of Economics and Statistics*, 38: 241-247.

TOBIN, JAMES (1965). "The Monetary Interpretation of History." *American Economic Review*, 55: 464-485.

TOWNSEND, ROBERT (1989). "Currency and Credit in a Private Information Economy." *Journal of Political Economy*, 97: 1323-1344.

STEIN, JEREMY (2012). "Monetary Policy as Financial Stability Regulation." *Quarterly Journal of Economics*, 127: 57-95.

STEINSSON, JÓN (2023a). "Money, Inflation, and Output: A Quantity-Theoretic Introduction." Chapter 13, *Lectures in Macroeconomics*.

STEINSSON, JÓN (2023b). "Money and Banking." Chapter 14, *Lectures in Macroeconomics*.

YANG, DAVID (2021). "Why Repo Spikes?" Working Paper.

Appendix for

Payments, Reserves, and Financial Fragility

Itay Goldstein Ming Yang Yao Zeng

A Proofs omitted from the main text

Proof of Proposition 1. The proof follows the idea of APS's decomposability and is based on the formal setup we lay out in Section 5. Let $\mathbf{w} = (w_1, w_2)$ be the pair of continuation values, that is, the equilibrium payoffs of the two agents in the good payment equilibrium. Note that if a good payment equilibrium exists, (w_1, w_2) must be unique since the agents' actions are history independent. By the nature of the good payment equilibrium, the currently reserve-holding agent always has continuation value w_1 and the non-reserve-holding agent always has continuation value w_2 regardless of the public history. Plugging $\mathbf{w}^1 = \mathbb{T}(\mathbf{w})$ and $\mathbf{w}^0 = \mathbf{w}$ into equation (5.3), we obtain

$$w_1 = (1-\delta)\,\lambda\mu\,(z-c) + \delta\,[\lambda\mu w_2 + (1-\lambda\mu)\,w_1] \tag{A.1}$$

and

$$w_2 = (1-\delta)\,(1-\lambda\mu)\,(-c) + \delta\,[\lambda\mu w_1 + (1-\lambda\mu)\,w_2]\ . \tag{A.2}$$

Taking the difference of (A.1) and (A.2) yields

$$w_1 - w_2 = \frac{(1-\delta)\,[\lambda\mu z - (2\lambda\mu - 1)\,c]}{1+\delta\,(2\lambda\mu-1)}\ . \tag{A.3}$$

Taking the sum of (A.1) and (A.2) yields

$$w_1 + w_2 = \lambda\mu z - c\,, \tag{A.4}$$

which is intuitive. It is straightforward to see that the good payment equilibrium, if exists, is the only equilibrium that attains the boundary $\{(w_1, w_2) : w_1 + w_2 = \lambda\mu z - c\}$. According to (5.5), this equilibrium exists if and only if

$$\delta w_2 + (1-\delta)\,(z-c) \geq \delta w_1\,,$$

that is,

$$\delta\left(w_1 - w_2\right) \leq (1-\delta)\,(z-c)\ .$$

By (A.3), this is equivalent to

$$z/c \geq \left[1-\delta\left(1-\lambda\mu\right)\right]^{-1}\ . \tag{A.5}$$

By the relationship of $\kappa = c/(1-\delta)$, this immediately yields the equilibrium condition (3.1).

We can further solve the two continuation values from (A.3) and (A.4), which are given by

$$w_1 = \frac{\lambda\mu}{1+\delta\left(2\lambda\mu-1\right)}\left[\left(1-\delta\left(1-\lambda\mu\right)\right)z - c\right] \tag{A.6}$$

and

$$w_2 = \frac{\delta\left(\lambda\mu\right)^2 z - \left[\delta\lambda\mu + (1-\delta)\left(1-\lambda\mu\right)\right]c}{1+\delta\left(2\lambda\mu-1\right)}\ . \tag{A.7}$$

Note that (A.5) implies that w_1, the continuation value of the agent with the reserve in this good equilibrium as given by (A.6), is non-negative. This concludes the proof. □

Proof of Proposition 2. The bad payment equilibrium consists of the strategy that the agent with the reserve never transfer the reserve upon request. Without loss of generality, let agent 1 have the reserve at the beginning of the period. Then, if both agents follow the bad strategy, Agent 1's per period continuation value is 0 and agent 2's is $-c$. This strategy is an equilibrium if and only if one-shot deviation is not profitable. No profitable one-shot deviation is equivalent to

$$\mu\left(z - (1-\delta)\cdot\frac{c}{1-\delta} + \delta\cdot\frac{-c}{1-\delta}\right) \leq 0\,,$$

i.e.,

$$z \leq \frac{c}{1-\delta}\ .$$

By the relationship of $\kappa = c/(1-\delta)$, this immediately yields the equilibrium condition (3.3).

Similarly, we can easily calculate the continuation values for the two agents in the bad payment equilibrium as $w_1^{(0)} = 0$ and $w_2^{(0)} = -c$. This concludes the proof. □

Proof of Proposition 4. The proof proceeds in two steps. These two steps give the upper and lower bounds of the region where the grim trigger payment equilibrium exists.

STEP 1. Note that the grim trigger payment equilibrium admits the bad payment equilibrium as a sub-game equilibrium. By Definition 2, the existence of the grim trigger payment equilibrium thus requires the existence of the bad payment equilibrium. That is, condition (3.3) must hold.

STEP 2. Following Definition 4 and APS's decomposability conditions, we can write:

$$w_1^{(1)} = \lambda\mu \left(\delta w_2^{(1)} + (1-\delta)(z-c) \right) + (1-\lambda\mu)\, \delta w_1^{(0)} \tag{A.8}$$

and

$$w_2^{(1)} = \lambda\mu\delta w_1^{(1)} + (1-\lambda\mu) \left(\delta w_2^{(0)} - (1-\delta)c \right) . \tag{A.9}$$

Note that, different from the counterparts in the proofs of Propositions 1 and 2, the pair of continuation values as well as agents' payment decisions are no longer history independent in the grim trigger equilibrium. Rather, they follow the automaton as the state of the game evolves.

By Proposition 2, we easily have $w_1^{(0)} = 0$ and $w_2^{(0)} = -c$. Plugging them into (A.8) and (A.9) yields:

$$w_1^{(1)} = \lambda\mu \left(\delta w_2^{(1)} + (1-\delta)(z-c) \right) \tag{A.10}$$

and

$$w_2^{(1)} = \lambda\mu\delta w_1^{(1)} - (1-\lambda\mu)c\,. \tag{A.11}$$

Further plugging (A.11) into (A.10) yields:

$$w_1^{(1)} = \frac{\lambda\mu\,(\delta(1-\lambda\mu)(-c) + (1-\delta)(z-c))}{1-\delta^2\lambda^2\mu^2} \tag{A.12}$$

On the other hand, the existence of the grim trigger payment equilibrium requires incentive compatibility for the reserve-holding agent in state $w^{(1)}$:

$$\delta w_2^{(1)} + (1-\delta)(z-c) \geq \delta w_1^{(0)}\,. \tag{A.13}$$

Plugging (A.11) and $w_1^{(0)} = 0$ into (A.13) simplies the incentive compatability condition to:

$$w_1^{(1)} \geq 0\,. \tag{A.14}$$

Combining (A.12) and (A.14) finally yields:

$$\frac{z}{c} \geq \frac{1-\delta\lambda\mu}{1-\delta}\,.$$

We can now close the proof by combining results from the two steps above:

$$\frac{1-\delta\lambda\mu}{1-\delta} \leq \frac{z}{c} \leq \frac{1}{1-\delta}\,.$$

By the relationship of $\kappa = c/(1-\delta)$, this immediately yields the equilibrium condition (4.1). This concludes the proof. □

Proof of Proposition 5. The proof uses mathematical induction and builds upon the proof of Proposition 4. It proceeds in three steps.

STEP 1. Based on the proof of Proposition 4, it is known that the grim trigger strategy, that is, the 1-trigger strategy constitutes a PPE when

$$\frac{1-\delta\lambda\mu}{1-\delta} \leq \frac{z}{c} \leq \frac{1}{1-\delta}\,.$$

STEP 2. We show that the $n+1$-trigger strategy must constitute a PPE if the n-trigger strategy constitutes a PPE, for all $n \geq 1$. Following Definition 4 and APS's decomposability conditions, we start from:

$$w_1^{(n+1)} = \lambda\mu\left(\delta w_2^{(n+1)} + (1-\delta)(z-c)\right) + (1-\lambda\mu)\,\delta w_1^{(n)}$$

and

$$w_2^{(n+1)} = \lambda\mu\delta w_1^{(n+1)} + (1-\lambda\mu)\left(\delta w_2^{(n)} - (1-\delta)c\right)\,.$$

that is,

$$\mathbf{w}^{(n+1)} \triangleq \begin{bmatrix} w_1^{(n+1)} \\ w_2^{(n+1)} \end{bmatrix} = \left[\delta\cdot\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\mathbf{w}^{(n+1)} + \begin{pmatrix} (1-\delta)\,(z-c) \\ 0 \end{pmatrix}, \delta\cdot\mathbf{w}^{(n)} - \begin{pmatrix} 0 \\ (1-\delta)\,c \end{pmatrix}\right]\begin{bmatrix} \lambda\mu \\ 1-\lambda\mu \end{bmatrix},$$

where we have $w_1^0 = 0$ and $w_2^0 = -c$ by Proposition 2. Recast the equation above to get:

$$\left[\mathbf{I} - \lambda\mu\delta\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\right]\mathbf{w}^{(n+1)} = (1-\lambda\mu)\delta\mathbf{w}^{(n)} + (1-\delta)\begin{pmatrix} \lambda\mu(z-c) \\ -(1-\lambda\mu)c \end{pmatrix},$$

which implies

$$\mathbf{w}^{(n+1)} = (1-\lambda\mu)\delta\mathbf{A}^{-1}\mathbf{w}^{(n)} + (1-\delta)\mathbf{A}^{-1}\begin{pmatrix} \lambda\mu(z-c) \\ -(1-\lambda\mu)c \end{pmatrix},$$

where $\mathbf{A} \triangleq \mathbf{I} - \lambda\mu\delta \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$.

On the other hand, incentive compatibility requires that

$$(0, \delta)\mathbf{w}^{(n+1)} \geq (\delta, 0)\mathbf{w}^{(n)} - (1-\delta)(z-c)\,. \tag{A.15}$$

In order to further characterize $\mathbf{w}^{(n)}$ and the incentive compatibility condition, let $\mathbf{y}^{(n)} = \mathbf{w}^{(n)} + \mathbf{x}$ for some $\mathbf{x}$, such that $\mathbf{y}^{(n)}$ takes the form

$$\mathbf{y}^{(n+1)} = (1-\lambda\mu)\delta\mathbf{A}^{-1}\mathbf{y}^{(n)}\,, \tag{A.16}$$

that is,

$$\begin{aligned} \mathbf{w}^{(n+1)} + \mathbf{x} &= (1-\lambda\mu)\delta\mathbf{A}^{-1}\left(\mathbf{w}^{(n)} + \mathbf{x}\right)\,, \\ \Rightarrow \mathbf{x} &= (1-\delta)\left[(1-\lambda\mu)\delta\mathbf{I} - \mathbf{A}\right]^{-1} \begin{pmatrix} \lambda\mu(z-c) \\ -(1-\lambda\mu)c \end{pmatrix}. \end{aligned}$$

Now we can characterize $\mathbf{y}^{(n)}$ and equilibirum payoff $\mathbf{w}^{(n)}$ with boundary conditions. Note that (A.16) implies contraction, hence $\mathbf{y}^{(\infty)} = 0$ is the unique fixed point and $\mathbf{w}^{(\infty)} = -\mathbf{x}$. The incentive compatibility condition (A.15) now becomes

$$(\delta, 0)\left[\mathbf{y}^{(n)} - \mathbf{x}\right] - (0, \delta)\left[\mathbf{y}^{(n+1)} - \mathbf{x}\right] \leq (1-\delta)(z-c)\,.$$

Plugging in (A.16) and that $\mathbf{w}^{(\infty)} = -\mathbf{x}$ yields

$$\begin{aligned} \left[(\delta, 0) - (0, \delta)\mathbf{A}^{-1}(1-\lambda\mu)\delta\right]\mathbf{y}^{(n+1)} &\leq \delta(1,-1)\mathbf{x} + (1-\delta)(z-c)\,, \\ \Rightarrow \frac{1}{1-\lambda\mu}(1, -\delta)\mathbf{y}^{(n+1)} &\leq \delta(1,-1)\mathbf{w}^{(\infty)} + (1-\delta)(z-c)\,. \end{aligned} \tag{A.17}$$

To show that the $n+1$-trigger strategy must constitute a PPE if the n-trigger strategy constitutes a PPE, it now suffices to show that (A.17) must hold if $\mathbf{y}^{(n)}$ satisfies

$$\frac{1}{1-\lambda\mu}(1, -\delta)\mathbf{y}^{(n)} \leq \delta(1,-1)\mathbf{w}^{(\infty)} + (1-\delta)(z-c)\,. \tag{A.18}$$

By definition, $\mathbf{w}^{(\infty)}$ is the payoff of the good payment equilibrium, thus the RHS of (A.17) must be positive. This is because the RHS is exactly the expected payoff from the good payment equilibrium minus the expected payoff from one-shot deviation, hence the RHS is non-negative if and only if the good payment equilibrium exists.

It is straightforward to see that the eigenvalues and the corresponding normalized eigenvectors of $(1-\lambda\mu)\delta\mathbf{A}^{-1}$ are

$$m_1 = \frac{(1-\lambda\mu)\delta}{1-\lambda\mu\delta}, \quad \mathbf{e_1} = 2^{-\frac{1}{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix},$$
$$m_2 = \frac{(1-\lambda\mu)\delta}{1+\lambda\mu\delta}, \quad \mathbf{e_2} = 2^{-\frac{1}{2}}\begin{pmatrix} 1 \\ -1 \end{pmatrix},$$

where $\mathbf{y}^{(0)}$ is given by

$$\mathbf{y}^{(0)} = \mathbf{w}^{(0)} - \mathbf{w}^{(\infty)} = \begin{pmatrix} -w_1^{(\infty)} \\ -c-w_2^{(\infty)} \end{pmatrix}.$$

Note that $w_1^{(\infty)} \geq 0$, $w_2^{(\infty)} \leq -c$ implies $y_1^{(0)} \leq 0$, $y_2^{(0)} \leq 0$. Let $\mathbf{y}^{(0)} = (\mathbf{e_1}, \mathbf{e_2})(\beta_1, \beta_2)^T$, where β_1, β_2 are given by

$$\begin{pmatrix} \beta_1 \\ \beta_2 \end{pmatrix} = 2^{-\frac{1}{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} y_1^{(0)} \\ y_2^{(0)} \end{pmatrix},$$

and hence $\beta_1 \leq 0$. By (A.16), $\mathbf{y}^{(n)} = [(1-\lambda\mu)\delta\mathbf{A}^{-1}]^n\,\mathbf{y}^{(0)} = (m_1^n\mathbf{e_1}, m_2^n\mathbf{e_2})(\beta_1,\beta_2)^T$. Since the n-trigger strategy constitutes a PPE, plugging $\mathbf{y}^{(0)}$ into (A.18) to get

$$\begin{aligned}
\frac{1}{1-\lambda\mu}(1,-\delta)(m_1^n\mathbf{e_1}, m_2^n\mathbf{e_2})\begin{pmatrix} \beta_1 \\ \beta_2 \end{pmatrix} &\leq \delta(1,-1)\mathbf{w}^{(\infty)} + (1-\delta)(z-c),\\
\Rightarrow \frac{2^{-\frac{1}{2}}}{1-\lambda\mu}(1-\delta, 1+\delta)\begin{pmatrix} \mathbf{e_1}^T \\ \mathbf{e_2}^T \end{pmatrix}(m_1^n\mathbf{e_1}, m_2^n\mathbf{e_2})\begin{pmatrix} \beta_1 \\ \beta_2 \end{pmatrix} &\leq \delta(1,-1)\mathbf{w}^{(\infty)} + (1-\delta)(z-c),\\
\Rightarrow \frac{2^{-\frac{1}{2}}}{1-\lambda\mu}\left[m_1^n(1-\delta)\beta_1 + m_2^n(1+\delta)\beta_2\right] &\leq \delta(1,-1)\mathbf{w}^{(\infty)} + (1-\delta)(z-c).
\end{aligned} \tag{A.19}$$

To show (A.17), it suffices to show (A.19) holds for $n+1$. To see this, note that

$$m_1^{n+1}(1-\delta)\beta_1 + m_2^{n+1}(1+\delta)\beta_2 = m_2\left[m_1^n(1-\delta)\beta_1 + m_2^n(1+\delta)\beta_2\right] + (m_1-m_2)m_1^n(1-\delta)\beta_1,$$

where $0 < m_2 < m_1 < 1$. As shown above, $\beta_1 \leq 0$, hence $(m_1 - m_2)m_1^n(1-\delta)\beta_1 \leq 0$. Recall that $\delta(1,-1)\mathbf{w}^{(\infty)} + (1-\delta)(z-c) \geq 0$. If $m_1^n(1-\delta)\beta_1 + m_2^n(1+\delta)\beta_2 \leq 0$, we have

$$\begin{aligned}
& \frac{2^{-\frac{1}{2}}}{1-\lambda\mu}\left[m_1^{n+1}(1-\delta)\beta_1 + m_2^{n+1}(1+\delta)\beta_2\right] \\
= & \frac{2^{-\frac{1}{2}}}{1-\lambda\mu}\left[m_2\left[m_1^n(1-\delta)\beta_1 + m_2^n(1+\delta)\beta_2\right] + (m_1-m_2)m_1^n(1-\delta)\beta_1\right] \\
\leq & \ 0 \\
\leq & \ \delta(1,-1)\mathbf{w}^{(\infty)} + (1-\delta)(z-c)\,,
\end{aligned}$$

otherwise,

$$\begin{aligned}
& \frac{2^{-\frac{1}{2}}}{1-\lambda\mu}\left[m_1^{n+1}(1-\delta)\beta_1 + m_2^{n+1}(1+\delta)\beta_2\right] \\
= & \frac{2^{-\frac{1}{2}}}{1-\lambda\mu}\left[m_2\left[m_1^n(1-\delta)\beta_1 + m_2^n(1+\delta)\beta_2\right] + (m_1-m_2)m_1^n(1-\delta)\beta_1\right] \\
\leq & \frac{2^{-\frac{1}{2}}}{1-\lambda\mu}\left[m_1^n(1-\delta)\beta_1 + m_2^n(1+\delta)\beta_2 + (m_1-m_2)m_1^n(1-\delta)\beta_1\right] \\
\leq & \frac{2^{-\frac{1}{2}}}{1-\lambda\mu}\left[m_1^n(1-\delta)\beta_1 + m_2^n(1+\delta)\beta_2\right] \\
\leq & \ \delta(1,-1)\mathbf{w}^{(\infty)} + (1-\delta)(z-c)\,,
\end{aligned}$$

where the last inequality follows from (A.19) for n. This thus concludes the second step of induction. It implies that the region where the n-trigger payment equilibrium must be a superset of that where the 1-trigger payment equilibrium exists.

STEP 3. Note that, by definition, the n-trigger payment equilibrium admits the 1-trigger payment equilibrium as a possible sub-game equilibrium. Hence, the region where the n-trigger payment equilibrium must also be a subset of that where the 1-trigger payment equilibrium exists. This concludes the proof. □

Proof of Lemma 2. For any $\mathbf{w} \in B(W)$, by the definition of $B(\cdot)$, there exist $\mathbf{w}^0 \in W$ and $\mathbf{w}^1 \in \mathbb{T}(W)$ such that $\mathbf{w} = \phi(\mu(\mathbf{w}^1, \mathbf{w}^0), \mathbf{w}^1, \mathbf{w}^0)$. Since $W \subset B(W)$, we can find for $\mathbf{w}^0$ two pairs of continuation values $\mathbf{w}^{00} \in W$ and $\mathbf{w}^{01} \in \mathbb{T}(W)$ such that $\mathbf{w}^0 = \phi(\mu(\mathbf{w}^{01}, \mathbf{w}^{00}), \mathbf{w}^{01}, \mathbf{w}^{00})$, where the right superscript of each pair of continuation values denote the associated history of public signals (e.g., $\mathbf{w}^{01} = (w_1^{01}, w_2^{01})$ is the pair of continuation values for the two agents after "no transfer" in period 1 and "transfer" in period 2). Similarly, since $\mathbb{T}(\mathbf{w}^1) \in W \subset B(W)$, we can find for $\mathbf{w}^1$ two pairs of continuation values $\mathbf{w}^{10} \in W$ and $\mathbf{w}^{01} \in \mathbb{T}(W)$ such that

$\mathbb{T}(\mathbf{w}^1)=\phi(\mu(\mathbb{T}(\mathbf{w}^{11}),\mathbb{T}(\mathbf{w}^{10})),\mathbb{T}(\mathbf{w}^{11}),\mathbb{T}(\mathbf{w}^{10}))$. In this way, we can find for every public history $h^t=(h_1,h_2,\cdots,h_{t-1})\in\{0,1\}^{t-1}$ (where h^0 denotes the null history right before period 1 with the initial pair of continuation values $\mathbf{w}$, and write $h^{t+1}=(h^t,h_t)$), a pair of continuation values

$$\mathbf{w}^{\left(h^t,0\right)}\in\begin{cases} W & \text{if } s_t\oplus 0=1\\ \mathbb{T}(W) & \text{if } s_t\oplus 0=0\,,\end{cases}$$

and a pair of continuation values

$$\mathbf{w}^{\left(h^t,1\right)}\in\begin{cases} W & \text{if } s_t\oplus 1=1\\ \mathbb{T}(W) & \text{if } s_t\oplus 1=0\,,\end{cases}$$

where $s_t\in\{0,1\}$ is the state of the game (specifying which agent has the reserve) after history h^t and $s_{t+1}=(s_t+h_t)\bmod 2\triangleq s_t\oplus h_t$, such that i) if $s_t=1$, then $\mathbf{w}^{h^t}=\phi\left(\mu\left(\mathbf{w}^{\left(h^t,1\right)},\mathbf{w}^{\left(h^t,0\right)}\right),\mathbf{w}^{\left(h^t,1\right)},\mathbf{w}^{\left(h^t,0\right)}\right)$ and Agent 1's action after h^t is $\mu\left(\mathbf{w}^{\left(h^t,1\right)},\mathbf{w}^{\left(h^t,0\right)}\right)$; ii) if $s_t=0$, then $\mathbb{T}\left(\mathbf{w}^{h^t}\right)=\phi\left(\mu\left(\mathbb{T}\left(\mathbf{w}^{\left(h^t,1\right)}\right),\mathbb{T}\left(\mathbf{w}^{\left(h^t,0\right)}\right)\right),\mathbb{T}\left(\mathbf{w}^{\left(h^t,1\right)}\right),\mathbb{T}\left(\mathbf{w}^{\left(h^t,0\right)}\right)\right)$ and agent 2's action after h^t is $\mu\left(\mathbb{T}\left(\mathbf{w}^{\left(h^t,1\right)}\right),\mathbb{T}\left(\mathbf{w}^{\left(h^t,0\right)}\right)\right)$.

Define a public strategy profile σ as

$$\sigma\left(h^t\right)=\begin{cases} \text{agent 1 chooses } \mu\left(\mathbf{w}^{\left(h^t,1\right)},\mathbf{w}^{\left(h^t,0\right)}\right)\text{, agent 2 no action} & \text{if } s_t=1\\ \text{agent 2 chooses } \mu\left(\mathbb{T}\left(\mathbf{w}^{\left(h^t,1\right)}\right),\mathbb{T}\left(\mathbf{w}^{\left(h^t,0\right)}\right)\right)\text{, agent 1 no action} & \text{if } s_t=0\,.\end{cases}$$

Then the original $\mathbf{w}\in B(W)$ is attained, and $\mathbf{w}^{h^t}$ is also attained after every public history h^t because W is bounded. Moreover, by construction, there is no profitable one-shot deviation and thus σ is a PPE. Therefore, $B(W)\subset V_1$. This concludes the proof. □

Proof of Proposition 6. The proof takes several steps to explicitly construct the equilibrium payoff set V_1. These steps are developed and presented as lemmas below.

For any closed convex set $V\in\mathbb{R}^2$, define

$$\mathbf{Ext}(V)=\left\{\begin{array}{c}\mathbf{w}\in V:\ \text{there do not exist } \mathbf{w}' \text{ and } \mathbf{w}'' \text{ in } V \text{ such that}\\ \mathbf{w}=\alpha\cdot\mathbf{w}'+(1-\alpha)\cdot\mathbf{w}'' \text{ for some } \alpha\in(0,1)\end{array}\right\}$$

as the set of extreme points of V. We will allow public randomization so that V_1 is convex. It is also straightforward to see that V_1 is closed. Since a closed and convex set can be characterized by its extreme points, we next study $\mathbf{Ext}(V_1)$.

Note that the good equilibrium is the only equilibrium that attains the boundary $\{(w_1, w_2) \in \mathbb{R}^2 : w_1 + w_2 = \lambda\mu z - c\}$. Hence, $\mathbf{w} = (w_1, w_2)$ is an extreme point of V_1. Let $\underline{\mathbf{w}} = (0, -c)$. By Lemma 2, $\underline{\mathbf{w}}$ is the pair of continuation values of the equilibrium in which agents never transfer the reserve. Since $\underline{\mathbf{w}} \in V_1 \subset X$ and $\underline{\mathbf{w}}$ is an extreme point of X, it is also an extreme point of V_1.

Lemma 5. *If $(0, -c) \in V_1$, then $\forall \mathbf{w} \in \mathbf{Ext}(V_1) \backslash \{(0, -c)\}$, the associated current-period equilibrium decision for the reserve-holding agent is $\sigma = 1$.*

Proof. We prove by contradiction. First, if $\sigma = 0$, (5.3) implies that $\mathbf{w} = \delta \mathbf{w}^0 + (1 - \delta)(0, -c)^T$ for some $\mathbf{w}^0 \in V_1$. Since $\mathbf{w} \neq (0, -c)^T$, we also have $\mathbf{w}^0 \neq (0, -c)^T$. Then $\mathbf{w}$ is a strict convex combination of $\mathbf{w}^0$ and $(0, -c)^T$, which are two different points in V_1, contradicting to the assumption that $\mathbf{w} \in \mathbf{Ext}(V_1)$.

Second, if $\sigma \in (0, 1)$, that is, the reserve-holding agent is indifferent between making a payment or not, there must exist $\mathbf{w}^0 \in V_1$ and $\mathbf{w}^1 \in \mathbb{T}(V_1)$ such that $\mathbf{w} = \phi(\mu, \mathbf{w}^1, \mathbf{w}^0)$. Since the reserve-holding agent is indifferent, both $\phi(0, \mathbf{w}^1, \mathbf{w}^0)$ and $\phi(\mu, \mathbf{w}^1, \mathbf{w}^0)$ belong to V_1 as they should be both supported in a PPE. However, by Lemma 2, for $\mu \in (0, \mu_1)$, $\phi(\mu, \mathbf{w}^1, \mathbf{w}^0)$ is a strict convex combination of $\phi(0, \mathbf{w}^1, \mathbf{w}^0)$ and $\phi(\mu, \mathbf{w}^1, \mathbf{w}^0)$, contradicting to the assumption that $\mathbf{w} \in \mathbf{Ext}(V_1)$.

Therefore, it must be that $\sigma = 1$. This concludes the proof. $\square$

Lemma 5 states the current-period equilibrium decision associated with any extreme point of V_1 other than that of the bad payment equilibrium is to transfer the reserve good.

For any $W \subset \mathbb{R}^2$, let ∂W denote the set of boundary points of W.

Lemma 6. *For any $\mathbf{w} \in \mathbf{Ext}(V_1)$ and $\mathbf{w} \neq (0, -c)$, let $\mathbf{w} = \phi(\mu, \mathbf{w}^1, \mathbf{w}^0)$ for some $\mathbf{w}^0 \in V_1$ and $\mathbf{w}^1 \in \mathbb{T}(V_1)$. Then $\mathbf{w}^0 \in \partial V_1$ and $\mathbf{w}^1 \in \partial \mathbb{T}(V_1)$. Moreover, at least one of $\mathbf{w}^0$ and $\mathbf{w}^1$ is an extreme point of V_1.*

Proof. Denote the line segment connecting $\underline{\mathbf{w}}$ and $\overline{\mathbf{w}}$ by $L(\underline{\mathbf{w}}, \mathbf{w})$, i.e.,

$$L(\underline{\mathbf{w}}, \mathbf{w}) = \left\{ \mathbf{w} \in \mathbb{R}^2 : \mathbf{w} = \alpha \cdot \mathbf{w} + (1 - \alpha) \cdot \underline{\mathbf{w}} \text{ for some } \alpha \in [0, 1] \right\} .$$

Then $L(\underline{\mathbf{w}}, \mathbf{w}) \subset V_1$. Thus, $V_1 = \lim_{n \to \infty} B^n (L(\underline{\mathbf{w}}, \mathbf{w}))$. This concludes the proof. $\square$

Lemma 6 states that there exist non-extreme equilibria in which agents stop making transfers after observing histories of non-transfers, capturing the idea that anticipating that the other agent may not transfer the reserve back in the future, the reserve-holding agent is reluctant to transfer the reserve today.

Recall from Propositions 1 and 2 that the good payment equilibrium exists if and only if $\frac{z}{c} \geq \frac{1}{1-\delta(1-\lambda\mu_1)}$ and the bad payment equilibrium exists if and only if $\frac{z}{c} \leq \frac{1}{1-\delta}$. Also note that $\frac{1}{1-\delta(1-\lambda\mu_1)} < \frac{1}{1-\delta}$. For any $\mathbf{x} \in \mathbb{R}^2, \alpha \in \mathbb{R}, W \subset \mathbb{R}^2$, define

$$\alpha \cdot W \triangleq \{\alpha \cdot \mathbf{w} : \mathbf{w} \in W\}$$
$$\mathbf{x} + W \triangleq \{\mathbf{x} + \mathbf{w} : \mathbf{w} \in W\}$$

Denote the stage game payoffs as

$$g(0) = \begin{pmatrix} 0 \\ -c \end{pmatrix}, \quad g(1) = \begin{pmatrix} \lambda\mu_1(z-c) \\ -(1-\lambda\mu_1)c \end{pmatrix}$$

For any $\mathbf{w}^0 \in \mathbb{R}^2$ and $W \subset \mathbb{R}^2$, define

$$Q^0_a(\mathbf{w}^0, W) = \begin{cases} \{\mathbf{w}^1 \in \mathbb{T}(W) : \delta w^1_1 + (1-\delta)(z-c) \leq \delta w^0_1\} & \text{if } a = 0, \\ \{\mathbf{w}^1 \in \mathbb{T}(W) : \delta w^1_1 + (1-\delta)(z-c) \geq \delta w^0_1\} & \text{if } a = 1. \end{cases}$$

Given $\mathbf{w}^0$, the continuation value when no transfer occurs, $Q^0_a(\mathbf{w}^0, W)$ is the set of $\mathbf{w}^1$ in W such that action $a \in 0, 1$ is incentive compatible.

Analogously, define

$$Q^1_a(\mathbf{w}^1, W) = \begin{cases} \{\mathbf{w}^0 \in \mathbb{T}(W) : \delta w^1_1 + (1-\delta)(z-c) \leq \delta w^0_1\} & \text{if } a = 0, \\ \{\mathbf{w}^0 \in \mathbb{T}(W) : \delta w^1_1 + (1-\delta)(z-c) \geq \delta w^0_1\} & \text{if } a = 1. \end{cases}$$

For any $W \subset \mathbb{R}^2$, let $Co(W)$ denotes the convex hull of W, i.e.,

$$Co(W) = \{\mathbf{w} : \exists \mathbf{x}, \mathbf{y} \in W \text{and } \alpha \in [0,1], \text{s.t. } \mathbf{w} = \alpha\mathbf{x} + (1-\alpha)\mathbf{y}\}$$

Modify the definition of $\mu(\mathbf{w}^1, \mathbf{w}^0)$ to allow mixed actions when the reserve-holding agent is indifferent, i.e.,

$$\mu(\mathbf{w}^1, \mathbf{w}^0) = \begin{cases} \{\mu_1\} & \text{if } \delta w_1^1 + (1-\delta)(z-c) > \delta w_1^0, \\ [0, \mu_1] & \text{if } \delta w_1^1 + (1-\delta)(z-c) = \delta w_1^0, \\ \{0\} & \text{if } \delta w_1^1 + (1-\delta)(z-c) < \delta w_1^0. \end{cases}$$

Accordingly, $\phi(\mu(\mathbf{w}^1, \mathbf{w}^0), \mathbf{w}^1, \mathbf{w}^0)$ should be understood as a set, especially when $\mu(\mathbf{w}^1, \mathbf{w}^0)$ is a set. We can now generalize the definition of $B(.)$:

Definition 4. *Let B(.) be a set operator such that for any $W \subset \mathbb{R}^2$,*

$$B(W) = \left\{\mathbf{w} \in \mathbb{R}^2 : \exists \mathbf{w}^0 \in W \textit{ and } \mathbf{w}^1 \in \mathbb{T}(W), \textit{ s.t. } \mathbf{w} \in \phi(\mu(\mathbf{w}^1, \mathbf{w}^0), \mathbf{w}^1, \mathbf{w}^0)\right\}$$

Since public randomization is allowed, following the approach in Abreu and Sannikov (2014), we can write $B(W)$ as

$$B(W) = Co(B_0(W) \cup B_1(W)), \tag{A.20}$$

where

$$\begin{aligned} B_a(W) = & (1-\delta)g(a) \\ & + \delta \left[\left(\bigcup_{\mathbf{w}^0 \in W} ((1-\lambda\mu_a)\mathbf{w}^0 + \lambda\mu_a Q_a^0(\mathbf{w}^0, W)) \right) \right. \\ & \left. \bigcup \left(\bigcup_{\mathbf{w}^1 \in \mathbb{T}(W)} (\lambda\mu_a \mathbf{w}^1 + (1-\lambda\mu_a) Q_a^1(\mathbf{w}^1, W)) \right) \right] \end{aligned} \tag{A.21}$$

Note that $a \in \{0, 1\}, \mu_0 = 0, \mu_1 \in [0, 1]$. Also, note that any closed convex set can be identified as the convex hull of its extreme points. We will also characterize V_1, by its extreme points. First, by (A.20), for any $W \subset \mathbb{R}^2$, e can characterize $B(W)$ by the extreme points of $B_a(W)$. Since public randomization is allowed, without loss of generality, we focus on closed convex set.

Lemma 7. *Suppose $W \subset \mathbb{R}^2$ is closed and convex. Let $\mathbf{w} \in \mathbf{Ext}(B_a(W))$ and $\mathbf{w} = \psi(\mu_a, \mathbf{w}^1, \mathbf{w}^0)$ for some $\mathbf{w}^1 \in \mathbb{T}(W)$ and $\mathbf{w}^0 \in W$. Then,*

1. *$\mathbf{w}^0 \in \mathbf{Ext}(Q_a^1(\mathbf{w}^1, W))$, $\mathbf{w}^1 \in \mathbf{Ext}(Q_a^0(\mathbf{w}^0, W))$;*
2. *At least one of the following two statements is true: (a) $\mathbf{w}^0 \in \mathbf{Ext}(W)$, (b) $\mathbf{w}^1 \in \mathbf{Ext}(\mathbb{T}(W))$.*

Proof. Since $\mathbf{w}^1$ and $\mathbf{w}^0$ induce action a, $\mathbf{w}^1 \in Q_a^0(\mathbf{w}^0, W)$ and $\mathbf{w}^0 \in Q_a^1(\mathbf{w}^1, W)$.

1. Suppose $\mathbf{w}^1 \notin \mathbf{Ext}(Q_a^0(\mathbf{w}^0, w))$, then there exist $\mathbf{w}^{1x}, \mathbf{w}^{1y} \in Q_a^0(\mathbf{w}^0, w)$ s.t. $\mathbf{w}^1$ is a strict convex combination of $\mathbf{w}^{1x}$ and $\mathbf{w}^{1y}$. Then $\mathbf{w} = \phi(\mu_a, \mathbf{w}^1, \mathbf{w}^0)$ is also a strict convex combination of $\phi(\mu_a, \mathbf{w}^{1x}, \mathbf{w}^0)$ and $\phi(\mu_a, \mathbf{w}^{1y}, \mathbf{w}^0)$, which are both in $B_a(W)$, contradicting to $\mathbf{w} \in \mathbf{Ext}(B_a(W))$. Thus $\mathbf{w}^1 \in \mathbf{Ext}(Q_a^0(\mathbf{w}^0, W))$. A similar argument can be shown for $\mathbf{w}^0$.

2. Suppose $\mathbf{w}^0 \notin \mathbf{Ext}(W)$ and $\mathbf{w}^1 \notin \mathbf{Ext}\mathbb{T}(W)$.
   Case (1): At least one of $\mathbf{w}^0$ and $\mathbf{w}^1$ is on a vertical boundary of W and $\mathbb{T}(W)$, respectively.
   If $\mathbf{w}^0$ is on a vertical boundary of W, then since $\mathbf{w}^0 \notin \mathbf{Ext}(W)$, we can find $\mathbf{w}^{0x}$ and $\mathbf{w}^{0y}$ on that vertical boundary of W, s.t. $\mathbf{w}^{0x} \neq \mathbf{w}^{0y}$ and $\mathbf{w}^0$ is a strict convex combination of $\mathbf{w}^{0x}$ and $\mathbf{w}^{0y}$. Since $\mathbf{w}_1^{0x} = \mathbf{w}_1^{0y} = \mathbf{w}_1^0$ and $\mathbf{w}^1 \in Q_a^0(\mathbf{w}^0, W)$, we have $\mathbf{w}^1 \in Q_a^0(\mathbf{w}^{0x}, W) \cap Q_a^0(\mathbf{w}^{0y}, W)$. Then $\phi(\lambda\mu_a, \mathbf{w}^1, \mathbf{w}^{0x})$ and $\phi(\lambda\mu_a, \mathbf{w}^1, \mathbf{w}^{0y})$ are both in $B_a(W)$, and $\mathbf{w} = \phi(\lambda\mu_a, \mathbf{w}^1, \mathbf{w}^0)$ is a strict convex combination of them, contradicting to $\mathbf{w} \in \mathbf{Ext}(B_a(W))$.
   If $\mathbf{w}^1$ is on a vertical boundary of $\mathbb{T}(W)$, then since $\mathbf{w}^0 \notin \mathbf{Ext}(\mathbb{T}(W))$, we can find $\mathbf{w}^{1x}$ and $\mathbf{w}^{1y}$ on that vertical boundary of $\mathbb{T}(W)$, s.t. $\mathbf{w}^{1x} \neq \mathbf{w}^{1y}$ and $\mathbf{w}^1$ is a strict convex combination of $\mathbf{w}^{1x}$ and $\mathbf{w}^{1y}$. Since $\mathbf{w}_1^{1x} = \mathbf{w}_1^{1y} = \mathbf{w}_1^1$ and $\mathbf{w}^1 \in Q_a^0(\mathbf{w}^0, W)$, we have $\{\mathbf{w}^{1x}, \mathbf{w}^{1y}\} \subset Q_a^0(\mathbf{w}^{0y}, W)$. Then $\phi(\lambda\mu_a, \mathbf{w}^{1x}, \mathbf{w}^0)$ and $\phi(\lambda\mu_a, \mathbf{w}^{1y}, \mathbf{w}^0)$ are both in $B_a(W)$, and $\mathbf{w} = \phi(\lambda\mu_a, \mathbf{w}^1, \mathbf{w}^0)$ is a strict convex combination of them, contradicting to $\mathbf{w} \in \mathbf{Ext}(B_a(W))$.
   Case (2): Neither $\mathbf{w}^0$ nor $\mathbf{w}^1$ is on a vertical boundary of W and $\mathbb{T}$ respectively. Then, there exist $\mathbf{w}^{0x} \neq \mathbf{w}^{0y}$ in W and $\mathbf{w}^{1x} \neq \mathbf{w}^{1y}$ in $\mathbb{T}(W)$, s.t. $\mathbf{w}^0$ is a strict convex combination of $\mathbf{w}^{0x}$ and $\mathbf{w}^{0y}$ and $\mathbf{w}^1$ is a strict convex combination of $\mathbf{w}^{1x}$ and $\mathbf{w}^{1y}$, and $w_1^{0x} = w_1^0 = w_1^{1x} - w_1^1$, $w_1^{0y} - w_1^0 = w_1^{1y} - w_1^1$. Note that these two equations ensure that (a)$\exists \alpha \in [0,1]$, s.t. $\mathbf{w}^0 = \alpha\mathbf{w}^{0x} + (1-\alpha)\mathbf{w}^{0y}$, $\mathbf{w}^1 = \alpha\mathbf{w}^{1x} + (1-\alpha)\mathbf{w}^{1y}$. Note that this α is common for the combination of both $\mathbf{w}^0$ and $\mathbf{w}^1$; (b) $\mathbf{w}^{1x} \in Q_a^0(\mathbf{w}^{0x}, W)$, $\mathbf{w}^{1y} \in Q_a^0(\mathbf{w}^{0y}, W)$, because $\mathbf{w}^1 \in Q_a^0(\mathbf{w}^0, W)$. Then both $\phi(\lambda\mu_a, \mathbf{w}^{1x}, \mathbf{w}^{0x})$ and $\phi(\lambda\mu_a, \mathbf{w}^{1x}, \mathbf{w}^{0y})$ belong to $B_a(W)$. Then,

$$
\begin{aligned}
\mathbf{w} &= \phi(\lambda\mu_a, \mathbf{w}^1, \mathbf{w}^0) \\
&= \alpha\phi(\lambda\mu_a, \mathbf{w}^{1x}, \mathbf{w}^{0x}) + (1-\alpha)\phi(\lambda\mu_a, \mathbf{w}^{1y}, \mathbf{w}^{0y})
\end{aligned}
$$

contradicting to $\mathbf{w} \in \mathbf{Ext}(B_a(W))$.

This completes the proof. $\square$

Lemma 7 allow us to rewrite (A.21) as

$$
\begin{aligned}
B_a(W) &= (1-\delta)g(a) \\
&\quad + \delta Co\left(\bigcup_{\mathbf{w}^0 \in \mathbf{Ext}(W)} \left((1-\lambda\mu_a)\mathbf{w}^0 + \lambda\mu_a \mathbf{Ext}(Q_a^0(\mathbf{w}^0, W))\right)\right. \\
&\quad \left.\cup \left(\bigcup_{\mathbf{w}^1 \in \mathbf{Ext}(\mathbb{T}(W))} \left(\lambda\mu_a \mathbf{w}^1 + (1-\lambda\mu_a)\mathbf{Ext}(Q_a^1(\mathbf{w}^1, W))\right)\right)\right)
\end{aligned}
$$

which simplifies the characterization below. Define

$$
\begin{cases}
\mathbf{X}^1 = (0, -c)', \\
\mathbf{X}^2 = \left(\dfrac{\lambda\mu((1-\delta(1-\lambda\mu))z - c)}{1+\delta(2\lambda\mu - 1)}, \dfrac{\delta(\lambda\mu)^2 z - (\delta\lambda\mu + (1-\delta)(1-\lambda\mu))c}{1+\delta(2\lambda\mu-1)}\right)', \\
\mathbf{X}^3 = \left(0, -c + \dfrac{\lambda\mu z(2c - (1-\delta)z)}{c + \delta\lambda\mu z}\right)'.
\end{cases}
$$

Note that $\mathbf{X}^2$ is the average continuation payoff vector in the good payment equilibrium. If such a equilibrium does not exist, $\mathbf{X}^2$ can be understood as the average continuation payoff vector resulted from the history-independent strategy that the two agents always make transfers upon receive a request. Define $W^* = Co(\mathbf{X}^1, \mathbf{X}^2, \mathbf{X}^3)$.

Lemma 8. *When* $\frac{1}{1-\delta(1-\lambda\mu_1)} < \frac{z}{c} < \frac{1}{1-\delta}$, $W^* = B(W^*)$.

Proof. We need to show $W^* \subset B(W^*)$ and $B(W^*) \subset W^*$. To show $W^* \subset B(W^*)$, it is sufficient to show $\{\mathbf{X}^1, \mathbf{X}^2, \mathbf{X}^3\} \subset B(W^*)$. Since $\frac{1}{1-\delta(1-\lambda\mu_1)} < \frac{z}{c} < \frac{1}{1-\delta}$, both $\mathbf{X}^1$ and $\mathbf{X}^2$ are equilibrium payoff vectors. In particular, $\mathbf{X}^1 = \phi(0, \mathbb{T}(\mathbf{X}^1), \mathbf{X}^1)$, $\mathbf{X}^2 = \phi(\mu_1, \mathbb{T}(\mathbf{X}^2), \mathbf{X}^2)$. Thus $\{\mathbf{X}^1, \mathbf{X}^2\} \subset B(W^*)$.

For $\mathbf{X}^3$, it is straightforward to verify that

$$
\begin{pmatrix} -(\delta^{-1}-1)(z-c) \\ \frac{\mathbf{X}_2^3 - \mathbf{X}_2^2}{\mathbf{X}_1^3 - \mathbf{X}_1^2}(c - (\delta^{-1}-1)(z-c)) \end{pmatrix} \in Q_1^0(\mathbf{X}^3, W^*)
$$

and

$$
\mathbf{X}^3 = \phi\left(\mu_1, \begin{pmatrix} -(\delta^{-1}-1)(z-c) \\ \frac{\mathbf{X}_2^3 - \mathbf{X}_2^2}{\mathbf{X}_1^3 - \mathbf{X}_1^2}(c - (\delta^{-1}-1)(z-c)) \end{pmatrix}, \mathbf{X}^3\right).
$$

Thus, $\mathbf{X}^3 \in B(W^*)$.

To show $B(W^*) \subset W^*$, it suffices to show $\mathbf{Ext}(B_1(W^*)) \cup \mathbf{Ext}(B_0(W^*)) \subset W^*$. By Lemma 7, we know that any extreme point of $B(W^)$ can be generated by some $\mathbf{w}^0 \in \mathbf{Ext}(W^*)$ and $\mathbf{w}^1 \in \mathbf{Ext}(Q_a^0(\mathbf{w}^0, W^*))$, or some $\mathbf{w}^1 \in \mathbf{Ext}(\mathbb{T}(W^*))$ and $\mathbf{w}^0 \in \mathbf{Ext}(Q_a^1(\mathbf{w}^1, W^*))$, for some $a \in \{0, 1\}$.

Therefore, we can check all points generated in this way to show that they are all in W^*. We give an example below: let $\mathbf{w}^1 = \mathbb{T}(\mathbf{X}^2)$, then $\mathbf{Ext}(Q_1^1(\mathbf{w}^1, W^*)) = \{\mathbf{X}^1, \mathbf{X}^2, \mathbf{X}^3\}$, $Q_0^1(\mathbf{w}^1, W^*) = \varnothing$. Then we have three potential extreme points:

1. $\phi(\mu_1, \mathbb{T}(\mathbf{X}^2), \mathbf{X}^1)$, which equals to $\delta(1-\lambda\mu_1)\mathbf{X}^1 + [1-\delta(1-\lambda\mu_1)]\mathbf{X}^2 \in W^*$.

2. $\phi(\mu_1, \mathbb{T}(\mathbf{X}^2), \mathbf{X}^2) = \mathbf{X}^2 \in W^*$, because this corresponds to the good payment equilibrium.

3. $\phi(\mu_1, \mathbb{T}(\mathbf{X}^2), \mathbf{X}^3)$, which equals to $[1-\delta(1-\lambda\mu_1)]\mathbf{X}^2 + \delta(1-\lambda\mu_1)\mathbf{X}^3 \in W^*$.

Other potential extreme points of $B(W^*)$ can be checked accordingly. This concludes the proof.

□

This concludes the proof of Proposition 6. □