Too Levered for Pigou: Carbon Pricing, Financial Constraints, and Leverage Regulation

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Abstract

We analyze jointly optimal carbon pricing and financial policies under financial constraints and endogenous climate-related transition and physical risks. The socially optimal emissions tax may be above or below a Pigouvian benchmark, depending on the impact of physical climate risk on collateral values. We derive necessary conditions for emissions taxes alone to implement a constrained-efficient allocation, and compare the welfare consequences of introducing a cap-and-trade system, green subsidies, or leverage regulation. Our analysis also shows that efficient carbon pricing can be supported by carbon price hedging markets but may be hindered by socially responsible investors in equilibrium.

Keywords: Pigouvian tax, carbon tax, cap and trade, financial constraints, climate risk, financial regulation

JEL classifications: D62, G28, G32, G38, H23

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1 Introduction

Tackling climate change requires large-scale emissions reductions and investments in clean technologies. Absent frictions, such investments can be incentivized through emissions taxes set at a rate equal to the social cost of emissions, also known as Pigouvian taxes in reference to the pioneering work by Pigou (1932). However, during the transition to a low-carbon economy firms and financial institutions may suffer significant losses that can exacerbate the severity of financial frictions. Such frictions may limit the ability of firms to make the necessary investments in green technologies (see Kacperczyk and Peydró, 2022; Martinsson et al., 2023), and constrain regulators in designing environmental policies (see Hoffmann et al., 2017; Oehmke and Opp, 2023; Biais and Landier, 2022). Accordingly, the risks posed by climate change have moved up the agenda of investors and financial policy makers.[1]

Motivated by these considerations, we evaluate jointly optimal climate and financial policies in a tractable model with financial constraints and endogenous climate-related transition and physical risks. The model yields several insights. First, our analysis shows that physical climate risk gives rise to a collateral externality that crucially affects how carbon taxes interact with financial constraints. This collateral externality implies the optimal carbon tax may be above a standard Pigouvian benchmark. Second, we derive necessary conditions under which carbon taxes alone can implement a constrained-efficient allocation in the presence of financial constraints. This enables a clean ranking of several commonly used climate and financial policies such as cap-and-trade systems, green subsidies, and leverage regulation. Third, the model provides novel insights to the theoretical literature on socially responsible investing (e.g., see Heinkel et al., 2001; Chowdhry et al., 2019; Gupta et al., 2022; Hong et al., 2023), by highlighting how socially responsible investors and carbon price hedging markets interact with carbon pricing policies. We show that responsible investors may hinder, while hedging markets can enable, efficient emissions reductions in equilibrium.

In the model there are three dates and two types of agents: borrowers and deep-pocketed, risk-neutral lenders. At the initial date, each borrower finances an investment project using their limited initial endowment and debt raised from investors. The bor-

[1] For example, the European Central Bank and the Bank of England now include climate risks in their stress tests (see Alogoskoufis et al., 2021; Brunnermeier and Landau, 2022), and institutional investors view climate change as an important source of risk that they seek to mitigate (Krueger et al., 2020).

rower's project generates a pecuniary return as well as carbon emissions at the final date. At the interim date a regulator sets an emissions tax in order to incentivize borrowers to reduce their emissions through costly abatement activities. At the same time, borrowers need to roll-over debt raised in the initial period, but debt issuance is limited by a financial constraint because project returns are not fully pledgeable to outside investors. Constrained borrowers can liquidate part of the initial investment at the interim date to generate resources and reduce emissions, yet liquidations are inefficient due to liquidation losses.

To capture the notion of "stranded assets", we assume that agents learn the social cost of emissions – and therefore the optimal level of emissions taxes – only at the interim date, when borrowers' initial investment decisions have already been made. This assumption is also consistent with the uncertainty evident in the wide range of estimates of the social cost of carbon (e.g., see Golosov et al., 2014; Nordhaus, 2019), and implies that borrowers are exposed to *climate transition risk* in the model.[2]

A key feature of our model is that we also incorporate the empirically documented effect of *physical climate risk* on asset values (Giglio et al., 2021; Issler et al., 2020; Ginglinger and Moreau, 2019).[3] In particular, to capture borrowers' exposure to losses in asset values due to (expected) environmental damages caused by a warming climate, we assume the return of the project may decrease in the level of *aggregate* emissions. Both climate-related risks are endogenous in the model: transition risk is a consequence of emissions taxes optimally set by an environmental regulator, and financial losses due to physical climate risks depend on aggregate emissions that are a function of abatement and investment decisions by borrowers. This allows us to explore the differences in how these two types of climate-related risks interact with financial frictions and affect optimal environmental and financial policies in equilibrium.

As a benchmark, we show that a state-contingent emissions tax equal to the social cost of emissions (i.e., a Pigouvian tax) implements the first-best allocation if financial constraints are slack in all states. In the first-best allocation, there are no liquidations

[2]Consistent with transition risks being priced in financial markets, recent evidence documents that firm-level carbon emissions are priced in corporate bonds (see Seltzer et al., 2020), stocks (see Bolton and Kacperczyk, 2021), and options (see Ilhan et al., 2021), and that the risk of stranded fossil fuel assets is priced in bank loans (see Delis et al., 2019).

[3]Giglio et al. (2021) find that the value of real estate in flood zones responds more to changes in climate attention. Issler et al. (2020) document an increase in delinquencies and foreclosures after wildfires in California. Evidence in Ginglinger and Moreau (2019) indicates that physical climate risks affect a firm's capital structure. For a review discussing climate risks, see Giglio et al. (2021).

and the optimal abatement scale trades off the social benefit of lower emissions against abatement costs.

However, in equilibrium the financial constraint may bind (particularly when a high social cost of emissions necessitates high emissions taxes and abatement investments). In this case, Pigouvian taxes cannot implement the first best, and optimal emissions taxes generally differ from the Pigouvian benchmark. The reason is that a constrained borrower has a limited ability to finance abatement and therefore needs to inefficiently liquidate some of the initial investment at the interim date. Consequently, the socially optimal emissions tax needs to trade off the benefit of lower emissions against the cost of triggering inefficient liquidations. This implies an optimal emissions tax below the Pigouvian benchmark as emissions taxes tighten financial constraints.[4]

A key insight from our analysis is that physical climate risks can reverse the relationship between emissions taxes and financial constraints. If physical climate risk has a substantial effect on collateral values, borrowers may benefit from an increase in pledgeable income when the aggregate level of emissions is brought down by a higher emissions tax.[5] Because of this collateral externality the optimal emissions tax may be *above* the Pigouvian benchmark rate if the effects of physical climate risk dominate the effects of transition risk. More broadly, we show that financial constraints call for a generalized Pigouvian tax that takes climate-induced collateral externalities into account.

To evaluate whether it may be welfare-improving to use other policy tools, we analyze under what conditions the allocation implemented with emissions taxes alone is constrained efficient (i.e., equivalent to an allocation chosen by a planner maximizing social welfare subject to the same constraints as private agents). In a first step, we consider a benchmark where emissions taxes are fully rebated to borrowers, and tax rebates are fully pledgeable to outside investors, so that emissions taxes have no *direct* effect on financial constraints. In this case, the competitive equilibrium with optimally set emissions taxes is constrained efficient. This implies that, while financial constraints generally imply optimal emissions taxes different from a Pigouvian benchmark, there is no scope to improve welfare using additional policy instruments when tax rebates are fully pledgeable.

[4]The mechanism behind this result is consistent with recent evidence documenting that financial constraints affect firm abatement activities and emissions, see Xu and Kim (2022) and Bartram et al. (2021).

[5]This effect is similar to collateral externalities in models with pecuniary externalities (for a detailed discussion, see Dávila and Korinek, 2018). In our setting, the collateral externality operates through a reduction in asset values due to (expected) environmental damages.

By contrast, when tax rebates are partially non-pledgeable, the allocation is not constrained efficient, and using other policy tools can improve welfare. We first consider a cap-and-trade system with tradable pollution permits (such as the EU Emissions Trading System, EU ETS). In a frictionless world, emissions taxes are equivalent to a cap-and-trade system, and the initial allocation of pollution permits does not matter for equilibrium emissions (see Montgomery, 1972). We show that in the presence of financial constraints this "Coasean independence" breaks down because the initial allocation of permits affects the tightness of constraints. Additionally, we show that the equivalence between emissions taxes and a cap-and-trade system only holds if the fraction of pledgeable tax rebates is equal to the fraction of freely allocated permits. This implies that freely allocating all pollution permits can eliminate the direct effect of carbon pricing on financial constraints and implement a constrained-efficient allocation. This result highlights the relevance of financial constraints in optimally allocating pollution permits, which is an important policy insight for real-world cap-and-trade systems that typically do not allocate 100% of permits for free.

Given the central role of financial constraints in our framework, we also consider leverage regulation that fixes the initial level of borrowers' equity at a given level. Such a policy can be applied directly to non-financial firms and implemented through taxes and subsidies on initial leverage or leverage ratio requirements (such as loan-to-value limits). Alternatively, we show our setup is equivalent to one in which borrowers represent financial institutions that lend to firms with polluting assets (see Internet Appendix Section IA.3). Under this interpretation, leverage regulation can be implemented within the Basel regulatory framework for financial institutions. Importantly, we show the presence of financial constraints alone does not motivate leverage regulation in the model. This implies any rationale for leverage regulation within our setting is driven by the environmental externality, which allows us to contribute to the debate on whether financial regulatory frameworks should consider climate-related risks beyond the prudential motive behind current regulatory frameworks (such as moral hazard problems due to government guarantees or pecuniary externalities, see, for example, Dewatripont and Tirole, 1994; Hellmann et al., 2000; Lorenzoni, 2008; Martinez-Miera and Repullo, 2010; Bahaj and Malherbe, 2020).

To understand the role of leverage regulation in the model, note that, (i) a borrower's initial leverage affects emissions because it affects financial constraints and therefore liquidations and abatement activities; and (ii) when emissions pricing cannot implement a constrained-efficient allocation, there remains a wedge between the social and the private cost of emissions even if emissions taxes are set optimally. Together, these two points imply that borrowers make socially inefficient leverage choices, and consequently there is a role for leverage regulation to improve welfare – but only if the environmental policy cannot implement the constrained-efficient allocation in the first place.

Overall, in our model there is a pecking order for optimal policy tools under financial constraints. Theoretically, the most effective policy tools combine a stringent emissions tax with transfers of resources from unconstrained investors to constrained borrowers. Such an intervention relaxes polluters' financial constraints and allows the regulator to implement the first-best allocation using a Pigouvian emissions tax. However, such a policy can arguably be politically difficult to implement. If that is the case, a cap-and-trade system with freely allocated permits can still implement the second-best allocation (i.e., constrained efficiency). Only if such policies cannot be implemented optimally, there is a case to complement carbon pricing with leverage regulation.

We also show that carbon price hedging markets can have a positive effect on equilibrium environmental policy, beyond their first-order risk sharing benefits for borrowers. We consider hedging contracts contingent on carbon taxes, which can be implemented through carbon price derivatives or climate-linked bonds that write off part of the principal when carbon taxes are high. Such instruments shift resources from low- to high-carbon price states. If this results in slack constraints in both states, it may enable the regulator to implement the first-best allocation using standard Pigouvian taxes. This highlights an important role the financial sector can play in the transition to a low-carbon economy, distinct from socially responsible investing that aims to reduce emissions by taking environmental and social factors into account in investment decisions.

In another extension, we consider socially responsible investors directly in the model. Socially responsible investors can provide incentives to reduce emissions by demanding a higher financing cost if borrowers fail to reduce emissions. However, our analysis also highlights they can have a perverse negative effect by tightening borrowers' financial constraints, consistent with evidence in Kacperczyk and Peydró (2022). This implies that socially responsible investors are an imperfect substitute for a well-designed carbon pricing policy.

This paper relates to several recent contributions that study environmental externalities and green investment under financial and other economic frictions (Tirole, 2010; Biais and Landier, 2022; Huang and Kopytov, 2023; Allen et al., 2023). Similar to Hoffmann et al. (2017) and Oehmke and Opp (2023), we also show that in the presence of binding financial constraints the optimal tax may be below a Pigouvian benchmark. Our analysis contributes by uncovering that physical climate risk may give rise to a collateral externality that alters the interaction between environmental policy and financial constraints, potentially motivating an emissions tax above a standard Pigouvian benchmark. Moreover, we derive necessary conditions under which emissions taxes can implement a second-best allocation and compare the efficiency under a range of policy tools, including cap-and-trade system and jointly optimal carbon pricing and leverage regulation.

By studying financial policy in this context we also relate to recent contributions by Oehmke and Opp (2022) and Dávila and Walther (2022), who consider risk-weighted capital regulation as a tool for tackling environmental externalities. We follow a different approach in that we take optimal emissions taxes as a starting point and show under what conditions leverage regulation can be valuable as a complementary policy tool.[6]

Our model also provides novel insights on how carbon price hedging markets and socially responsible investors may enable or hinder efficient carbon pricing and emissions reductions in equilibrium, contributing to the literature analyzing the effects of socially responsible investing (Heinkel et al., 2001; Chowdhry et al., 2019; Pástor et al., 2021; Green and Roth, 2022; Broccardo et al., 2022; Gupta et al., 2022; Goldstein et al., 2022; Piccolo et al., 2022; Hong et al., 2023; Oehmke and Opp, 2023).

Section 2 describes the model setup. Section 3 solves the competitive equilibrium. Section 4 analyzes optimal emissions taxation, and compares emissions taxes to a cap-and-trade system and green subsidies. Section 5 introduces leverage regulation, and Section 6 considers carbon price hedging and socially responsible investors. Section 7 concludes.

[6]Another related strand of literature uses DSGE models with financial frictions to simulate the effect and optimal design of macroprudential and monetary policies in the presence of environmental externalities (Carattini et al., 2021; Dafermos et al., 2018; Diluiso et al., 2020; Ferrari and Landi, 2021; Giovanardi and Kaldorf, 2023). We contribute by providing analytical results that allow us to compare different policy tools, pinpoint the friction motivating financial regulation, and study the impact of financing instruments on equilibrium policy.

2 Model Setup

There are three dates, $t = 0, 1, 2$, a unit mass of investors, and a unit mass of borrowers. At $t = 1$ all agents learn whether the economy is in a good state ($s = G$) with a low social cost of emissions, or in a bad state ($s = B$) with a high social cost of emissions. The state of the world is drawn from a binomial distribution with the probability of the bad state given by q_B and that of good state equal to $q_G = 1 - q_B$.

Preferences and Endowments. Investors are risk-neutral and deep-pocketed in that they have a large endowment A_t^i at $t = 0$ and $t = 1$. Borrowers have a limited endowment A_0 only at $t = 0$ and quasi-linear utility over consumption. There is no discounting and all agents suffer disutility from aggregate carbon emissions E_s^a at $t = 2$:

$$U^i = c_0^i + c_{1s}^i + c_{2s}^i - \gamma_s^u E_s^a,$$
$$U^b = u(c_0^b) + c_{1s}^b + c_{2s}^b - \gamma_s^u E_s^a,$$

where γ_s^u is a parameter governing the cost of emissions in agent's utility, which depends on the state of the world $s \in \{G, B\}$. In the bad state γ_s^u takes a high value $\gamma_B^u > \gamma_G^u$. In the good state, we normalize $\gamma_G^u = 0$. Agents are atomistic, so that they do not internalize the effect of their decisions on aggregate emissions E_s^a.

The quasi-linear utility function introduces a meaningful trade-off for borrowers in how much own funds they contribute to the project. To ensure an interior solution we assume that $u(c_0)$ satisfies the Inada conditions, i.e., $u(c_0)$ is strictly increasing and strictly concave, and in the limit $u'(0) = \infty$ and $u'(\infty) = 0$.

Technology. At $t = 0$ borrowers can invest in a productive technology with a fixed scale at an investment cost I_0. At $t = 1$ borrowers can liquidate some of the initial investment and adjust the investment scale to $I_{1s} \leq I_0$. The project generates a return of $R(I_{1s}, E_s^a) = \rho I_{1s} - \gamma_s^p E_s^a$ at $t = 2$, and liquidations generate a payoff $\mu(I_0 - I_{1s})$ at $t = 1$, with $\mu < 1$.

The parameter γ_s^p captures the negative effect of physical climate risk on firms' asset values. As with the utility cost of emissions, $\gamma_B^p \geq 0$ and $\gamma_G^p = 0$. Given that the most severe adverse effects of climate change are likely to occur in the future, we interpret γ_s^p as the overall asset pricing effect of expected environmental damages, rather than the direct impact of extreme weather events in the near future (for a review of evidence on such asset pricing effects, see Giglio et al., 2021). Using a separate parameters γ_s^p allows us to perform key comparative statics on the intensity of climate-related collateral damages compared to other climate-related losses captured by γ_s^u. The total social cost of emissions consists of a direct utility cost as well as losses in asset values from environmental damages, $\gamma_s = 2\gamma_s^u + \gamma_s^p$.

The social cost of emissions is uncertain from an ex-ante perspective, consistent with the wide range of estimates of the social cost of carbon (see Nordhaus, 2019). While uncertainty is not a necessary model ingredient for our baseline results, it allows us to study the role that financial markets can play in facilitating more efficient environmental policy (see Section 6). Moreover, it allows us to frame the analysis in the context of long-run investments that may become stranded due to uncertain climate policies and outcomes.

The project emits carbon emissions $E(X_s, I_{1s})$ at $t = 2$, which can be reduced by non-verifiable abatement investments X_s at a cost $C(X_s, I_{1s})$ paid at $t = 1$. Emissions aggregate to E_s^a and may be subject to emissions taxes $\tau_s \geq 0$. We offer two possible interpretations of this setup. Borrowers may represent non-financial firms that directly invest in a polluting asset, such as manufacturing firms investing in polluting plants. Alternatively, we show in the Internet Appendix (Section IA.3) that, under certain conditions, the setup is equivalent to one in which borrowers are financial institutions that lend to firms with polluting assets. In the latter case, financial institutions pay for emissions taxes and abatement costs indirectly through the profitability of their lending portfolios.

We make the following functional form assumptions.

Assumption 1. *$E(X, I_1)$ and $C(X, I_1)$ satisfy*

1. $\frac{\partial C(X,I_1)}{\partial X} \geq 0,\ \frac{\partial C(X,I_1)}{\partial I_1} \geq 0,\ \frac{\partial E(X,I_1)}{\partial X} \leq 0,\ \frac{\partial E(X,I_1)}{\partial I_1} \geq 0,$

2. $C(0, I_1) = 0,\ C(X, 0) = 0,\ \lim_{X\to\infty} E(X, I_0) = 0,\ E(X, 0) = 0,\ \ E(0, I_0) = \bar{E},$

3. $\frac{\partial^2 E(X,I_1)}{\partial X^2} = 0,\ \frac{\partial^2 C(X,I_1)}{\partial X^2} > 0.$

Assumption 1.1 ensures that abatement investments are costly but reduce emissions, and that a higher final investment scale is associated with higher emissions and abatement costs. Assumption 1.2 defines boundaries such that costs and emissions are non-negative, and there is an upper bound $\bar{E}$ on emissions. Assumption 1.3 implies that emissions are linear in abatement, which simplifies the exposition, and that the cost of abatement is strictly convex, so that the borrower's optimal abatement choice has an interior solution.

Environmental Regulation. An environmental regulator imposes a state-contingent emissions tax τ_s per unit of emissions.[7] Emissions taxes are rebated lump-sum to borrowers, $T_s = \tau_s E_s^a$ (such tax rebates are sometimes referred to as a "carbon dividend" in policy debates). Sections 4.3 and 4.4 consider alternative environmental policies in the form of a cap-and-trade system and green subsidies. In Section 5 we also study whether there is scope for leverage regulation to complement environmental policy.

Financing. Borrowers need to finance the upfront investment I_0 at $t = 0$ and abatement X_s at $t = 1$. At $t = 0$, they can contribute their own funds as inside equity financing $e \leq A_0$. Additionally, borrowers can raise debt financing d_0 and d_{1s} from investors at $t = 0, 1$.[8] In Section 6, we also allow borrowers to write hedging contracts (which could be implemented through state-contingent "climate-linked" bonds), and explore the effect of introducing socially responsible investors. These extensions provide interesting additional insights on how different financial instruments can affect equilibrium environmental policy.

External financing is limited by a moral hazard problem. We assume that borrowers can abscond with any resources except a fraction $\theta \in [0, 1]$ of asset returns, and a fraction $\psi \in [0, 1]$ of tax rebates T_s at $t = 2$. Thus, there is a wedge between the project's return and pledgeable income, with pledgeable project returns given by $\theta R(I_{1s}, E_s^a)$ (as in Rampini and Viswanathan, 2013, among others). The separate pledgeability parameter for tax rebates allows us to perform key comparative statics exercises. For example, when $\psi = 1$ tax rebates are fully pledgeable and emissions taxes have no *direct* effect on financial constraints, while the opposite holds when $\psi < 1$.

At the interim date the liquidation proceeds $\mu(I_0 - I_{1s})$ can be seized by investors who provided $t = 0$ financing (that is, liquidation proceeds are pledgeable). Investors can demand liquidation if they choose not to roll over their debt and are not fully repaid at $t = 1$.

[7] We only consider a linear tax because there is no heterogeneity among borrowers, and therefore a non-linear tax cannot improve upon a linear tax. See Hoffmann et al. (2017) for a model with heterogeneity, in which a non-linear tax can be a superior policy instrument because it transfers less resources from more to less constrained firms.

[8] Internet Appendix Sections IA.1.2 and IA.1.3 discuss the solution when borrowers use external equity or long-term debt financing.

Variable Definitions. For the further analysis it will be useful to introduce the following variable definitions and assumptions:

Definition 1. *The project's private net marginal return on investment* $r(\tau, X, I_1)$ *and pledgeable net marginal return on investment* $\tilde{r}(\tau, X, I_1)$ *are respectively defined as*

$$r(\tau, X, I_1) = \rho - \mu - \frac{\partial C(X, I_1)}{\partial I_1} - \tau \frac{\partial E(X, I_1)}{\partial I_1},$$
$$\tilde{r}(\tau, X, I_1) = \theta\rho - \mu - \frac{\partial C(X, I_1)}{\partial I_1} - \tau \frac{\partial E(X, I_1)}{\partial I_1}.$$

Assumption 2. *Project returns* ρ *are sufficiently large and pledgeability* θ *sufficiently small such that, given a threshold* $\bar{\tau} \geq \gamma_B$*,*

1. $r(\tau, X, I_1) > 0, \ \forall X, I_1, \tau \leq \bar{\tau}$*,*

2. $\tilde{r}(0, X, I_1) < 0, \ \forall X, I_1$*.*

The first condition ensures that continuing the project has a positive NPV even in the bad state with a high social cost of carbon, as long as emissions taxes do not exceed some threshold $\bar{\tau}$. Throughout the paper we focus on the interesting case $\tau_B \leq \bar{\tau}$, so that continuation of the project is privately profitable even when emission taxes are high. The second condition ensures that, while inefficient, liquidations relax financial constraints.

2.1 First-Best Benchmark

Proposition 1. *In the first-best allocation* $I_{1s} = I_0$*, and optimal* $t = 0$ *consumption by borrowers,* c_0^b*, and optimal abatement,* X_s*, are defined by the following conditions:*

$$u'(c_0^b) = 1,$$
$$\gamma_s \frac{\partial E(X_s, I_{1s})}{\partial X_s} = -\frac{\partial C(X_s, I_{1s})}{\partial X_s}.$$

Proof. See Appendix A.1 □

In the first-best allocation, the optimal abatement equates the marginal gain from lower emissions to the marginal cost of abatement. The borrower's consumption is at a level that ensures the marginal utility is equalized across agents and time. Crucially, there are no liquidations because liquidations are inefficient by Assumption 2.[9] The next section shows that this may be different in the competitive equilibrium, where financially constrained borrowers may need to liquidate some of their initial investment.

3 Competitive Equilibrium

This section solves the problem of borrowers and defines a competitive equilibrium given a state-contingent emissions tax τ_s. We analyze optimal emissions taxes and compare the allocation to an equilibrium with financial regulation and other policy tools in later sections.

3.1 Borrower Problem

The borrower's expected utility is given by

$$\mathbb{E}[U^b] = u(c_0^b) + \sum_{s \in \{G,B\}} q_s \left(c_{1s}^b + c_{2s}^b - \gamma_s^u E_s^a \right).$$

Borrowers maximize their expected utility subject to the following constraints:

$$c_0^b = A_0 - e \geq 0, \tag{1}$$

$$c_{1s}^b = (I_0 - I_{1s})\mu + d_{1s} - (I_0 - e) - C(X_s, I_{1s}) \geq 0, \tag{2}$$

$$c_{2s}^b = R(I_{1s}, E_s^a) - \tau_s E(X_s, I_{1s}) - d_{1s} + T_s \geq 0, \tag{3}$$

$$d_{1s} \leq \theta R(I_{1s}, E_s^a) - \tau_s E(X_s, I_{1s}) + \psi T_s, \tag{4}$$

$$I_{1s} \in [0, I_0]. \tag{5}$$

Equations (1), (2) and (3) are non-negativity constraints on consumption at $t = 0, 1$, and 2, respectively. Eq. (4) is a financial constraint that ensures $t = 1$ borrowing does not exceed pledgeable income, which implies borrowers have no incentive to abscond at $t = 2$.[10] Throughout the paper, we focus on the case in which optimally $d_0 < \mu I_0$ (which holds as long as $u'(A_0 - (1-\mu)I_0)$ is not too high), because otherwise borrowers would prefer to forgo the project and consume all of their initial endowment. This also implies that borrowers have no incentive to default on $t = 0$ debt at $t = 1$, as we show in Appendix A.2.2.

[9] Assuming that liquidations are inefficient allows us to cleanly distinguish between efficient abatement spending X_s and inefficient liquidations. In reality, a mix of liquidation and abatement may be optimal.

[10] Eq. (4) is equivalent to an incentive-compatibility condition $c_{2s}^b \geq (1-\theta)R(I_{1s}, E_s^a) + (1-\psi)T_s$.

Additionally, we explore regulatory constraints on $t = 0$ debt in Section 5.

Using the budget constraints to eliminate c_0^b, c_{1s}^b, c_{2s}^b, d_0, and d_{1s}, the borrower's problem can be formulated as a Lagrange function of e, X_s, I_{1s} with Lagrange multipliers λ_s for the $t = 1$ financial constraint in state s, and κ's serving as multipliers for lower and upper bounds on variables. The Lagrangian is formally stated in Eq. (17) in Appendix A.2.1.

3.2 Borrower Decisions at $t = 1$

At $t = 1$ borrowers observe the realization of the aggregate state s and the corresponding tax τ_s, and then choose X_s and I_{1s} according to the following conditions.

$$(1 + \lambda_s)\left(\tau_s \frac{\partial E(X_s, I_{1s})}{\partial X_s} + \frac{\partial C(X_s, I_{1s})}{\partial X_s}\right) = 0, \quad (6)$$

$$r(\tau_s, X_s, I_{1s}) + \lambda_s \tilde{r}(\tau_s, X_s, I_{1s}) - \overline{\kappa}_{Is} + \underline{\kappa}_{Is} = 0, \quad (7)$$

$$\lambda_s[\theta R(I_{1s}, E_s^a) - \tau_s E(X_s, I_{1s}) + \psi T_s + e - I_0 + \mu(I_0 - I_{1s}) - C(X_s, I_{1s})] = 0. \quad (8)$$

The first order condition with respect to X_s in Eq. (6) shows that borrowers choose abatement trading off a reduction in the emissions tax bill against the cost of abatement. Eq. (7) is the first order condition with respect to I_{1s}, and it reflects the trade-off between increasing the private net return and relaxing the financial constraints, captured by $r(\cdot)$ and $\lambda_s \tilde{r}(\cdot)$ respectively. Together with Eq. (8), which combines the complementary slackness conditions of the financial constraint (4) and non-negativity constraint of c_{1s}^b (2), these conditions define the optimal state-contingent $t = 1$ allocations I_{1s}, X_s, and λ_s for a given τ_s and e (the optimality condition for equity is derived below).

Lemma 1. *Borrowers do not liquidate any investment if the financial constraint (4) is slack. That is, if $\lambda_s = 0$, then $I_{1s} = I_0$. In contrast, if $\lambda_s > 0$, then borrowers liquidate some investment so that $I_{1s} < I_0$.*

Proof. In Appendix A.2.3 □

Lemma 1 follows from Assumption 2, which implies that the net marginal return is positive and therefore it is optimal to continue the project without any liquidations, i.e., the optimum is a corner solution with $I_{1s} = I_0$ and $\overline{\kappa}_{Is} > 0$. By contrast, if the financial constraint is binding, $\lambda_s > 0$, the pledgeable income under the full investment scale is insufficient to support the required borrowing. Since liquidations relax financial constraints (by Assumption 2.2), in this case borrowers reduce the investment scale at $t = 1$ by choosing $I_{1s} < I_0$.

3.3 Borrower Decisions at $t = 0$

At $t = 0$ borrowers decide on their capital structure by choosing the optimal inside equity e (debt financing follows as the residual $d_0 = I_0 - e$). The first order condition of the borrower's problem w.r.t. e is given by

$$u'(A_0 - e) = 1 + q_G \lambda_G + q_B \lambda_B. \tag{9}$$

Condition (9) shows that borrowers contribute equity trading off the marginal utility cost of lower $t = 0$ consumption on the left-hand side against the marginal utility of $t = 1$ consumption plus the expected shadow cost of the financial constraint on the right-hand side. The first order conditions and complementary slackness condition together define the competitive equilibrium:

Definition 2. *Given a state-contingent emissions tax τ_s, the competitive equilibrium is the set of allocations $I^*_{1s}(\tau_s), X^*_s(\tau_s), \lambda^*_s(\tau_s), e^*(\tau_G, \tau_B)$, defined by Equations (6), (7), (8), and (9). Aggregate emissions are given by $E^a_s(\tau_s) = E(X^*_s, I^*_{1s})$. The allocations $c^{b*}_0(\tau_G, \tau_B)$, $c^{b*}_{1s}(\tau_s)$, $c^{b*}_{2s}(\tau_s)$, and $d^*_0(\tau_G, \tau_B)$ follow as residuals from Eqs. (1), (2), (3), and $d_0 = I_0 - e$.*

For brevity we sometimes omit the dependence of equilibrium allocations on τ_s. For instance, we refer to $X^*_s(\tau_s)$ as X^*_s, or to $e^*(\tau_G, \tau_B)$ as e^*.

3.4 Pigouvian Benchmark

Proposition 2. *If $\lambda^*_s(\gamma_s) = 0$, $\forall s \in \{G, B\}$, then the competitive equilibrium with $\tau_s = \gamma_s$ is equivalent to the first-best allocation.*

Proof. With $\lambda^*_s(\gamma_s) = 0$, $\forall s \in \{G, B\}$, it follows from Lemma 1 that $I^*_{1s} = I_0$. This investment level, as well as the FOCs of borrowers w.r.t. X_s and e in Eqs. (6) and (9), are then equivalent to those in the first best given in Proposition 1. □

Proposition 2 establishes an important benchmark result. If the financial constraint is slack in all states, then by Lemma 1 borrowers can avoid inefficient liquidations, and the optimal Pigouvian emissions tax can implement the first-best allocation. Accordingly, throughout we refer to a tax $\tau_s = \gamma_s \; \forall s \in \{B, G\}$ as the *Pigouvian benchmark*. In the next section we depart from this benchmark and analyze optimal emissions taxes when the financial constraint binds.

4 Optimal Carbon Pricing

To analyze optimal emissions taxes in the presence of financial constraints, we consider the problem of an environmental regulator who sets a state-contingent emissions tax τ_s^* after observing the social cost of emissions at $t = 1$. We then show under what conditions the resulting equilibrium allocation is constrained efficient, and ask whether there is a case to combine emissions taxes with other policy instruments.

4.1 Socially Optimal Emissions Tax

To derive the optimal τ_s, we solve the problem of a regulator choosing the optimal tax at $t = 1$ so as to maximize social welfare. This problem is formally stated in Appendix A.3.3. The regulator's first order condition with respect to τ_s can be written as:

$$(\tau_s - \gamma_s)\frac{\partial E(X_s^*, I_{1s}^*)}{\partial X_s^*}\frac{\partial X_s^*}{\partial \tau_s} + r(\gamma_s, X_s^*, I_{1s}^*)\frac{\partial I_{1s}^*}{\partial \tau_s} + \kappa_{\tau s} = 0, \tag{10}$$

where $\kappa_{\tau s}$ is the Lagrange multiplier on the non-negativity constraint $\tau_s \geq 0$.

The regulator trades off the effect of the tax on welfare through its impact on emissions, reflected in the first term in Eq. (10), against the welfare implications of the change in the final investment scale induced by the tax, captured in the second term of the equation. In this condition, the final investment scale I_{1s}^* and abatement X_s^* are optimal choices by private agents that respond to changes in emissions taxes.

4.1.1 The Effect of Taxes on Equilibrium Allocations

Higher emissions taxes increase the cost of polluting, which incentivizes borrowers to invest more in abatement. But higher emissions taxes also affect the tightness of financial constraints, which may induce borrowers to abate less. Through this indirect effect, emissions taxes can have a perverse effect and decrease abatement due to tightening financial constraints. To focus on the interesting case in which emissions taxes are a useful tool to incentivize abatement to begin with, we introduce parameter assumptions that ensure the direct effect of emissions taxes on abatement dominates.

Assumption 3. *Model parameters are such that* $\frac{\partial X_s^*}{\partial \tau_s} > 0 \; \forall \tau_s$, *as characterized in Appendix A.3.1.*

The following Lemma additionally clarifies how liquidations and therefore the equilibrium investment scale I_{1s}^* responds to emissions taxes.

Lemma 2. *If the financial constraint is slack,* $\lambda_s^*(\tau_s) = 0$, *then* $\frac{\partial I_{1s}^*}{\partial \tau_s} = 0$ *and* $\frac{\partial X_s^*}{\partial \tau_s} > 0$. *Under Assumption 3, if* $\lambda_s^*(\tau_s) > 0$, *then* $\frac{\partial X_s^*}{\partial \tau_s} > 0$ *and there exists a threshold* $\hat{\gamma}^p(\tau_s)$ *such that*

- $\frac{\partial I_{1s}^*}{\partial \tau_s} < 0$ *if* $\gamma_s^p < \hat{\gamma}^p(\tau_s)$,
- $\frac{\partial I_{1s}^*}{\partial \tau_s} = 0$ *if* $\gamma_s^p = \hat{\gamma}^p(\tau_s)$,
- $\frac{\partial I_{1s}^*}{\partial \tau_s} > 0$ *if* $\gamma_s^p > \hat{\gamma}^p(\tau_s)$.

Proof. See Appendix A.3.2 □

Only if the financial constraint binds, $\lambda_s^*(\tau_s) > 0$, borrowers need to liquidate investments to be able to roll-over their debt. Interestingly, higher emissions taxes can result in more or less liquidations, depending on how strongly asset values are affected by physical climate risk, as captured by γ_s^p. The overall effect of emissions taxes on the final investment scale follows from totally differentiating (8) with respect to τ_s:

$$\frac{\partial I_{1s}^*}{\partial \tau_s} = \frac{\overbrace{(1-\psi)E(X_s^*, I_{1s}^*)}^{\text{Direct effect}} + \overbrace{(\theta\gamma_s^p - \psi\tau_s)\frac{\partial E_s^a}{\partial X_s^*}\frac{\partial X_s^*}{\partial \tau}}^{\text{Collateral externality}}}{\tilde{r}(\tau_s(1-\psi) + \theta\gamma_s^p, X_s^*, I_{1s}^*)} \tag{11}$$

This equation highlights that emissions taxes affect the final investment scale via two channels that operate through financial constraints. First, changes in the tax directly affect the size of the tax bill and the tax rebate. Since only a fraction ψ of the tax rebate is pledgeable this *direct effect* of the emissions tax on the tightness of the financial constraint is proportional to $(1-\psi)E(X_s^*, I_{1s}^*)$.

Second, changes in abatement also affect the aggregate level of emissions, which impact borrowers' pledgeable income via two *collateral externalities.* Physical climate risk represents a negative collateral externality because higher aggregate emissions result in larger physical damages to borrowers' assets, decreasing pledgeable income by $\theta\gamma_s^p$. As a result, higher emissions taxes partly relax financial constraints. At the same time, there is a positive collateral externality because tax rebates are a function of aggregate emissions. Lower aggregate emissions reduce the tax rebate, decreasing pledgeable income by $\psi\tau_s$.

Overall, the effect of emissions taxes on financial constraints and liquidations depends on the relative strength of the direct effect of taxes on pledgeable income, and the indirect effects due to collateral externalities.[11] When borrowers' exposure to physical climate risk is low such that $\gamma_s^p < \hat{\gamma}^p$, the direct effect and tax rebate externality dominate, so that higher emissions taxes imply tighter constraints and more liquidations. If borrowers' exposure to physical climate risk is high such that $\gamma_s^p > \hat{\gamma}^p$, the equilibrium effect of emissions taxes that lowers the physical risk dominates, so that higher emissions taxes relax financial constraints and result in fewer liquidations.

4.1.2 Optimal Emissions Tax

Because emissions taxes interact with financial constraints, the regulator considers not only the direct effect of taxes on emissions, but also their side effect on asset liquidations.

Proposition 3. *The optimal emissions tax τ_s^* solves (10). If $\lambda_s^*(\gamma_s) = 0$ or $\gamma_s = 0$, then $\tau_s^* = \gamma_s$. If $\lambda_s^*(\gamma_s) > 0$ and $\gamma_s > 0$, then the optimal tax depends on the strength of physical risk γ_s^p, and on the pledgeability of tax rebates ψ and cash flows θ. If $\psi \geq \theta$, the optimal emissions tax is below the direct social cost of emissions: $\tau_s^* < \gamma_s$. If $\psi < \theta$, then*

- $\tau_s^* < \gamma_s$ *if* $\gamma_s^p < \hat{\gamma}^p(\tau_s^*)$,
- $\tau_s^* = \gamma_s$ *if* $\gamma_s^p = \hat{\gamma}^p(\tau_s^*)$,
- $\tau_s^* > \gamma_s$ *if* $\gamma_s^p > \hat{\gamma}^p(\tau_s^*)$,

where the threshold $\hat{\gamma}^p(\tau_s^*)$ is defined in Lemma 2 (Appendix A.3.2).

Proof. See Appendix A.3.3 □

With binding financial constraints, $\lambda_s^*(\gamma_s) > 0$, the optimal emissions tax generally differs from the Pigouvian benchmark equal to the direct social cost of emissions γ_s.

[11] Note that, because higher taxes induce an endogenous change in abatement by borrowers, they also affect abatement costs. On one hand, higher abatement increases abatement costs, tightening financial constraints. On the other hand, higher abatement reduces emissions and thereby the tax bill, easing financial constraints. Therefore, an additional term that shows up in the numerator of Eq. (11) is $-\left(\frac{\partial C(X_s^*, I_{1s}^*)}{\partial X_s^*} + \tau \frac{\partial E(X_s^*, I_{1s}^*)}{\partial X_s^*}\right) \frac{\partial X_s^*}{\partial \tau}$. However, by the borrower's optimal abatement choice in Eq. (6), this term is equal to zero, so that this channel has no marginal effect on financial constraints and drops out from Eq. (11).

Put differently, in this case optimal emissions taxes differ from the Pigouvian benchmark because borrowers are "too levered for Pigou".

To disentangle the results in Proposition 3, we discuss three polar cases: (i) tax rebates are not pledgeable and physical climate risk has no effect on collateral values ($\psi = \gamma_s^p = 0$); (ii) tax rebates are not pledgeable but physical climate risk has an effect on collateral values ($\psi = 0, \gamma_s^p > 0$); and (iii) tax rebates are pledgeable and physical climate risk has an effect on collateral values ($\psi > 0, \gamma_s^p > 0$).

(i) No physical risk ($\psi = \gamma_s^p = 0$). With non-pledgeable tax rebates and absent physical climate risk effects, there is no collateral externality and emissions taxes affect financial constraints only through their *direct effect* on pledgeable income. In this case, higher taxes trigger inefficient liquidations (see Lemma 2). Internalizing this undesired side effect, an environmental regulator sets an emissions tax below the direct social cost of emissions, $\tau_s^* < \gamma_s$. Intuitively, regulators set a lower carbon tax because they understand that higher taxes constitute a realization of climate transition risk for financially constrained borrowers.

(ii) Physical risk ($\psi = 0, \gamma_s^p > 0$). Physical climate risk implies that emissions taxes affect borrower's financial constraints not only through their *direct effect*, but also through a *collateral externality*. The relative importance of this effect depends on how strongly collateral values are exposed to physical climate risk, as measured by γ_s^p. If $\gamma_s^p < \hat{\gamma}^p$, the direct effect dominates and the trade-off resembles the one in case (i) above. This case applies when climate transition risks dominate physical climate risk effects, for example in economies with large polluting industries. By contrast, if the effect of physical climate risk on collateral values is sufficiently high such that $\gamma_s^p > \hat{\gamma}^p$, then higher emissions taxes ease financial constraints (see Lemma 2). As a result, the trade-offs faced by an environmental regulator change fundamentally, implying optimal emissions taxes above the direct social cost of emissions, $\tau_s^* > \gamma_s$.[12] Such a case may apply to economies that are heavily exposed to the risk of weather disasters such as droughts or floodings.

[12] Simpson (1995) and Heider and Inderst (2022) highlight other reasons why the optimal emissions tax may be above a Pigouvian benchmark. In Simpson (1995), this is the case if the tax allocates production from less to more efficient firms in a Cournot competition model. In Heider and Inderst (2022) an emissions tax above a Pigouvian benchmark can be beneficial because it improves margins earned by green producers in the product market.

(iii) Plegeability ($\psi > 0, \gamma_s^p > 0$). With (partially) pledgeable tax rebates, the overall collateral externality effect of emissions taxes depends not only on the impact due to physical climate risk, but also due to changes in the size of tax rebates. The latter represents a positive collateral externality of emissions, thereby counteracting the negative collateral externality due to physical risk. Which of the two collateral externalities dominates depends on whether tax rebates or asset returns have a greater pledgeability. If $\psi \geq \theta$, tax rebates are more pledgeable than the firm's asset returns, and the positive collateral externality due to tax rebates dominates. In this case, optimal emissions taxes are unambiguously below the direct social cost of emissions, $\tau_s^* < \gamma_s$, irrespectively of the level of γ_s^p. By contrast, if $\psi < \theta$ the optimal emissions tax may be above the direct social cost of emissions if γ_s^p is sufficiently large, as discussed under case (ii) above.

An interesting implication is that, in economies where firms' assets have a low pledgeability (such as knowledge-based economies with much intangible capital), optimal emissions taxes are lower because the effect of physical risk on collateral values is less relevant (small θ). Similarly, emissions taxes may be optimally lower in economies where tax rebates are more pledgeable (large ψ; for example, due to stronger institutions).

Generalized Pigouvian Tax. Previous literature on collateral externalities focuses primarily on pecuniary externalities, whereby borrowers do not internalize how their choices affect the financial constraint of other agents through their impact on prices (for a detailed discussion, see Dávila and Korinek, 2018). By contrast, in our setting collateral extenalities can emerge because agents do not internalize their impact on aggregate emissions. Consequently, the total social cost of emissions includes not only the direct social cost of emissions γ_s, but also the indirect costs due to collateral externalities driven by physical climate risk, $\lambda_s \theta \gamma_s^p$, and the pledgeability of tax rebates, $-\lambda_s \psi \tau_s$. Therefore, another useful benchmark to compare the optimal emissions tax to is a *generalized* Pigouvian tax, defined as the emissions tax that equalizes the private cost of emissions τ_s to the *total* social cost of emissions $\gamma_s + \lambda_s \theta \gamma_s^p - \lambda_s \psi \tau_s$.

Proposition 4. *Let the generalized Pigouvian tax be defined as*

$$\tau_s^{GP} = \frac{\gamma_s + \lambda_s^* \theta \gamma_s^p}{1 + \psi \lambda_s^*}.$$

With $\lambda_s^* > 0$ and $\gamma_s > 0$, the optimal emissions tax is $\tau_s^* = \tau_s^{GP}$ if $\psi = 1$, and $\tau_s^* < \tau_s^{GP}$

if $\psi < 1$. With $\lambda_s^* = 0$ or $\gamma_s = 0$, the optimal emissions tax is $\tau_s^* = \tau_s^{GP} = \gamma_s$.

Proof. See Appendix A.3.4 □

While the optimal emissions tax may be above a standard Pigouvian benchmark equal to the direct social cost of emissions γ_s (see Proposition 3), Proposition 4 shows that, if tax rebates are not fully pledgeable, the optimal emissions tax is always below a generalized Pigouvian benchmark that accounts for collateral externalities. This highlights that, even with $\tau_s^* > \gamma_s$, the adverse direct effect of emissions taxes on financial constraints can limit the regulator in setting a tax that accounts for all direct and indirect social costs of emissions. The next subsection shows this has implications for the efficiency of the allocation.

4.2 Efficiency

To evaluate efficiency, we compare the allocation that can be implemented with the optimal emissions tax τ_s^* to the constrained-efficient allocation in which a social planner can choose X_s, I_{1s} and e directly, subject to the same resource and financial constraints as private agents. This constrained-efficient allocation is formally defined and characterized in Appendix A.3.5.

Proposition 5. *If $\psi = 1$, then the competitive equilibrium with a socially optimal emissions tax equal to the generalized Pigouvian tax $\tau_s^{GP} = \frac{\gamma_s + \lambda_s^* \theta \gamma_s^p}{1+\lambda_s^*}$ is constrained efficient. If $\psi < 1$ and the financial constraint binds in some state, $\lambda_s^* > 0$, then the competitive equilibrium with a socially optimal emissions tax τ_s^* is not constrained efficient.*

Proof. See Appendix A.3.5 □

We show in Appendix A.3.5 how a constrained planner sets the optimal level of abatement trading off direct and indirect social benefits and costs. In contrast to a social planner, the environmental regulator cannot choose abatement directly, but instead uses emissions taxes as a policy instrument to incentivize abatement. If tax rebates are fully pledgeable, the regulator can implement the constrained-efficient abatement level without introducing additional distortions to the final investment scale by setting the emissions tax equal to the generalized Pigouvian tax τ_s^{GP}. However, if tax rebates are not fully pledgeable, $\psi < 1$, taxes have a direct adverse effect on financial constraints because $\tau_s E(X_s, I_{1s}) - \psi T_s > 0$, and the regulator needs to set an emissions tax below τ_s^{GP} (see

Proposition 4). As a result, emissions taxes can only implement the constrained-efficient allocation if tax rebates are fully pledgeable.

This result implies that, when $\psi < 1$, there may be scope to improve welfare by using policy tools other than carbon taxes. The following subsections discuss two potential alternatives: a cap-and-trade system with tradable pollution permits (Section 4.3) and green subsidies (Section 4.4). Since borrowers' initial leverage directly affects the tightness of the collateral constraint, ex-ante leverage regulation is another natural candidate policy we consider in Section 5.

4.3 Cap and Trade

An alternative policy tool that can curb emissions is a cap-and-trade system with a limited quantity Q_s of tradable pollution permits (similar to the EU ETS). Absent other frictions, such pollution permit markets are equivalent to emissions taxes, and the Coase Theroem implies that the initial allocation of pollution permits does not affect the equilibrium level of emissions (see Coase, 1960; Montgomery, 1972). In what follows we show that this is not necessarily the case in the presence of financial constraints, and explore whether a cap-and-trade system can achieve higher welfare than emissions taxes.

For each unit of emissions the borrower needs to surrender a permit to the regulator at $t = 2$. We assume that a share ϕ of all permits Q_s is freely allocated to borrowers ex-ante, and that the remaining $(1 - \phi)Q_s$ permits need to be purchased by the borrower at the market price p_s.[13] Borrowers can trade permits with each other at the market price p_s. Note that with freely allocated permits borrowers retain the same incentives to invest in abatement because of the opportunity cost of selling unused permits. For now, the regulator takes the freely allocated share ϕ as given. Later we discuss the welfare-maximizing level of ϕ.

4.3.1 Mapping Cap-and-Trade to Emissions Taxes

The budget constraints of the borrower and the first order conditions under the cap-and-trade system are stated in Appendix A.4.1. The FOCs are equivalent to those in the

[13] To simplify the exposition, we assume here that the proceeds from permit sales accrue to investors. Internet Appendix IA.2.1 shows that the insights on the sensitivity of welfare to initial allocation of permits and cap-and-trade being able to implement the constrained efficient allocation hold also when the permit sale proceeds are distributed back to borrowers lump-sum.

baseline problem, with p_s taking the place of τ_s. The borrower's FOC with respect to abatement determines the relationship between the privately optimal level of abatement X_s and the permit price p_s, and mirrors Eq. (6) of the original problem:

$$(1+\lambda_s)\left(p_s\frac{\partial E(X_s, I_{1s})}{\partial X_s}+\frac{\partial C(X_s, I_{1s})}{\partial X_s}\right)=0, \tag{6'}$$

This condition, together with the market clearing for permits, $Q_s = E_s^a$, jointly determine a mapping from p_s to E_s^a. Thus, the regulator can implement a desired market price of permits by altering the total quantity of permits. Consequently, we can express the regulator's problem as maximizing social welfare by choosing p_s in each state $s = \{B, G\}$. Appendix A.4.1 reports the first order condition of the regulator. As in the baseline setting, the regulator internalizes the effect of the policy on borrowers' profits and emissions. Comparing the FOCs under the cap-and-trade system with the one in the original problem yields the following result.

Proposition 6. *The allocation implemented with a pollution permit market in which the quantity of permits is chosen to implement a permit price $p_s = \tau_s$ and a fraction ϕ of permits are allocated freely, is equivalent to the allocation implemented with an emissions tax τ_s if the fraction of freely allocated permits is equal to the fraction of tax rebates that can be pledged, $\phi = \psi$.*

Proof. See Appendix A.4.1 □

In both the baseline setting with carbon taxes and the cap-and-trade system the regulator's policy amounts to choosing the private marginal cost of emissions represented either by the tax rate τ_s or the price of permits p_s. The direct effect of the policies on the financial constraints depend, respectively, on the pledgeability of the tax rebates ψ, and the share of freely allocated permits ϕ. Pollution permits have a direct effect on the financial constraint if the borrower needs to purchase some of them ex-ante (i.e. if $1-\phi > 0$). This corresponds to the direct effect of the tax bill on pledgeable income under emissions taxes. The price of permits also affects the tightness of the financial constraint through the collateral externalities, which mirror those discussed in Section 4.1.2.

Coasean independence. An implication of the Coase Theorem is that absent other frictions the initial allocation of the pollution allowances does not affect the equilibrium level of externality (see Montgomery, 1972). Proposition 6 combined with our previous results show that this "Coasean independence" does not hold under financial frictions.[14] This result is consistent with recent empirical evidence from the EU ETS that indicates that Coasen independence holds for large emitters but not for smaller firms (see Zaklan, 2023). As small firms are more likely to be financially constrained, our framework offers a novel mechanism that may explain these findings.

4.3.2 Free Permits

So far we assumed that the regulator takes the share of freely allocated permits as given. However, the advantage of using a cap-and-trade system instead of emissions taxes is that the regulator can choose ϕ optimally. The equivalence result in Proposition 6 implies that a version of Proposition 5 in which $\tau_s = p_s$ and $\psi = \phi$ holds in the current setting, giving rise to the following corollary.

Corollary 1. *The regulator can implement a constrained-efficient allocation by setting $\phi = 1$ and issuing a quantity of permits that implements a permit price $p_s^* = \frac{\gamma_s + \lambda_s^* \theta \gamma_s^P}{1+\lambda_s^*}$.*

The regulator can avoid the problem of the carbon price's direct effect on borrowers' financial constraints by allocating all permits for free, i.e., setting $\phi = 1$. In this case, the shadow cost of permits induces borrowers to engage in a constrained-efficient level of abatement. As in the baseline with $\psi = 1$, the optimal policy is below the Pigouvian benchmark $p_s^* < \gamma_s$ whenever the financial constraint binds (see Proposition 3).

An important policy implication is that a pollution permit market with free allowances may be a superior policy instrument to carbon taxes in the presence of financial constraints. Yet, in practice cap-and-trade systems often do not allocate permits for free. For example, the EU ETS (the largest emissions permit market in the world), only grants free allowances equal to a fraction of total emissions, and is gradually reducing the amount of free allowances over time.[15]

We acknowledge that there may be considerations outside our model that motivate these real-life policy choices. For example, it may be difficult for regulators to correctly

[14] Previous literature points to transaction costs, market power, uncertainty, allowance allocations being conditioned on past pollution, deviation from cost-minimization by firms, and unequal regulatory treatment of firms as potential sources of break-down of Coasean independence (Hahn and Stavins, 2011).

[15] For example, the manufacturing industry received 80% of its allowances for free in 2013. This proportion had been decreased down to 30% in 2020, see European Commission website.

allocate free permits if polluters were privately informed about heterogeneous abatement costs, potentially triggering undesirable distributional consequences. Similarly, determining the amount of freely allocated permits by past emissions (a policy referred to as "grandfathering"), may weaken incentives to reduce emissions as firms may want to avoid a reduction in the amount of freely allocated permits in the future (see Clò, 2010). While modeling these frictions is beyond the scope of this paper, our results highlight that, when accounting for these additional forces, regulators should also weigh the adverse impact of allowance sales on the tightness of financial constraints.

4.4 Green Subsidies

This subsection considers subsidies. We first analyze a non-redistributive emissions-reductions subsidy financed by lump-sum taxes on borrowers. We then consider subsidies financed by investors, which constitute a net transfer from investors to borrowers.

4.4.1 Emissions-Reduction Subsidy

We assume that abatement is non-verifiable, reflecting the difficulty in assessing the optimal technological choices for a specific polluter.[16] Regulators can nevertheless implicitly subsidize abatement investments through a subsidy σ_s per unit of emissions reductions below a target level $\bar{E}_s$ paid at $t = 2$. For now, suppose the subsidy is financed by lump-sum taxes levied on borrowers equal to $T_s = \sigma_s(\bar{E}_s - E(X_s, I_{1s}))$, so that the subsidy is not redistributive. The first order condition with respect to X_s in Eq. (6) is equivalent to the original first order condition (6) with σ_s taking the place of τ_s. Thus, setting $\sigma_s = \tau_s$ the subsidy can achieve the same incentive effect as an emissions tax.

What about the effect of the subsidy on financial constraints? To map the subsidy to the baseline model, we assume that borrowers can abscond with a fraction $1 - \psi$ of the subsidy payment. As a result, the complementary slackness condition (8) becomes

$$\lambda_s \left[\theta R(I_{1s}, E_s^a) + \psi\sigma_s(\bar{E}_s - E(X_s, I_{1s})) - T_s + e - I_0 + \mu(I_0 - I_{1s}) - C(X_s, I_{1s})\right] = 0.$$

[16]If abatement was verifiable, regulators could implement the constrained-efficient allocation simply through a minimum abatement requirement at $t = 1$ (i.e. using quantity- rather than price-based regulation). Alternatively, the regulator could pay a subsidy on abatement directly to borrowers at $t = 1$ to avoid the negative direct effect of the policy on financial constrains. This would be akin to assuming away the contracting frictions, i.e. setting $\psi = 1$.

This condition maps to Eq. (8) with $\psi T_s - \tau_s E(X_s, I_{1s})$ replaced by $\psi\sigma_s(\bar{E}_s - E(X_s, I_{1s})) - T_s$. Both terms are equal to $-(1-\psi)T_s$ in equilibrium. This implies that the same efficiency properties as in the baseline model apply. Notably, Proposition 5 still holds, so that the allocation is constrained efficient only if the subsidy is fully pledgeable, i.e., if $\psi = 1$.

4.4.2 Redistributive Subsidies

A subsidy may dominate emissions taxes if it is financed through taxes raised from investors. In this case, the subsidy constitutes a net transfer $\mathcal{T}_s = \sigma_s(\bar{E}_s - E(X_s, I_{1s}))$ from unconstrained to constrained agents, and can implement the first-best allocation if the transfer is sufficiently large to ensure financial constraints are slack in all states.

Even if regulators were unable to set an emissions reduction target (for example, due to unobserved heterogeneity), a transfer could nevertheless be implemented in a lump-sum fashion. In this case, a lump-sum transfer $\mathcal{T}_s$ needs to be combined with other carbon pricing policies that incentivize emissions reductions. For example, consider the baseline model with an emissions tax τ_s and a generic transfer $\mathcal{T}_s$ to borrowers paid at $t = 1$, financed by lump-sum taxes from investors. With this transfer the complementary slackness condition (8) becomes

$$\lambda_s \left[\theta R(I_{1s}, E_s^a) - \tau_s E(X_s, I_{1s}) + \psi T_s + \mathcal{T}_s + e - I_0 + \mu(I_0 - I_{1s}) - C(X_s, I_{1s})\right] = 0.$$

Clearly, if $\mathcal{T}_s$ is sufficiently large, then the financial constraint becomes slack. As shown in Proposition 2, this implies that an emissions tax equal to the Pigouvian benchmark can implement the first best. The need to combine a green subsidy with a carbon tax to ensure borrowers have incentives to reduce emissions when emission reduction targets are difficult to establish may be one reason for the simultaneous use of carrots (green subsidies) and sticks (carbon taxes) often observed in practice.[17]

5 Leverage Regulation

Given the central role of financial constraints in the model, and motivated by the recent debate on whether financial regulation should include climate-related goals (for exam-

[17]This result relates to Acemoglu et al. (2016), who find that it is beneficial to combine subsidies for green R&D with carbon taxes, in order to increase the productivity of clean technologies. The difference here is that green subsidies serve the purpose of relaxing financing constraints.

ple, see Brunnermeier and Landau, 2021), this section introduces leverage regulation that could complement emissions taxes. We analyze a leverage mandate that fixes the borrower's equity contribution at a level $\bar{e}$, which can be implemented through a direct mandate, or through taxes and subsidies (see Internet Appendix Section IA.2.2). Such policies could be applied directly to non-financial firms, or introduced into the Basel regulatory framework if borrowers are interpreted as financial institutions (see Internet Appendix Section IA.3). To streamline the discussion, we focus on the case in which the financial constraint binds when $s = B$ and is slack when $s = G$.

5.1 Optimal Leverage Regulation

We consider the problem of a regulator who sets a leverage mandate $\bar{e}$ at $t = 0$ and state-contingent emissions taxes τ_s at $t = 1$, so as to maximize welfare. That is, we re-consider the original optimization problem (23) but allow the regulator to also set $e = \bar{e}$ at $t = 0$.

Each of the policy instruments affects a different decision margin. The emissions tax affects how borrowers trade off abatement and liquidations at $t = 1$, for a *given* tightness of the financial constraint. In contrast, the leverage mandate affects to what extent the borrower uses $t = 0$ resources to consume or to relax the $t = 1$ financial constraints, as can be seen from the regulator's first order condition w.r.t. $\bar{e}$ (see Appendix A.4.2 for the derivation):

$$u'(A_0 - \bar{e}) - 1 = \sum_{s \in \{B,G\}} q_s \left[r(\gamma_s, X_s^*, I_{1s}^*) - (\gamma_s - \tau_s) \frac{\partial E(X_s^*, I_{1s}^*)}{\partial X_s^*} \frac{\partial X_s^*}{\partial I_{1s}^*} \right] \frac{\partial I_{1s}^*}{\partial \bar{e}}. \tag{12}$$

The first order condition trades off the marginal utility of consumption (on the left-hand side) against the marginal social value of more financial slack at $t = 1$ (on the right-hand side). Notice that, by the envelope theorem, the regulator does not consider the effect of the leverage mandate on the emissions tax because the tax is set optimally at $t = 1$.

The marginal social value of financial slack consists of the value due to a higher net return on the project (captured by $r(\gamma_s, X_s, I_{1s})$ in Eq. (12)), and the value due to the change in aggregate emissions (captured by the remaining terms in the square brackets). How the optimal leverage mandate $\bar{e}^*$ is set relative to the privately optimal e^* depends on whether the social value of financial slack diverges from the private value. While both the borrower and regulator account for the effect of equity on the pecuniary return generated by the project, the social and private value of emissions may differ.

Proposition 7. *If in the competitive equilibrium the borrower's financial constraint is slack when $s = G$ and binding when $s = B$, then equity under the optimal leverage mandate coincides with the borrower's choice of equity if and only if*

$$\frac{dE(X_B^*, I_{1B}^*)}{d\bar{e}} \underbrace{\left[\gamma_B - \tau_B^* + \lambda_B \left(\theta \gamma_B^p - \psi \tau_B^*\right)\right]}_{T\text{-}SCC\ wedge} = 0. \tag{13}$$

If $\psi < 1$ the T-SCC wedge is positive and the optimal leverage mandate $\bar{e}^*$ is

- $\bar{e}^* > e^*(\tau_G^*, \tau_B^*)$ *if* $\frac{dE(X_B^*, I_{1B}^*)}{d\bar{e}} < 0$,
- $\bar{e}^* = e^*(\tau_G^*, \tau_B^*)$ *if* $\frac{dE(X_B^*, I_{1B}^*)}{d\bar{e}} = 0$,
- $\bar{e}^* < e^*(\tau_G^*, \tau_B^*)$ *if* $\frac{dE(X_B^*, I_{1B}^*)}{d\bar{e}} > 0$.

Proof. See Appendix A.4.2 □

The left-hand side of Eq. (13) measures the gap in the marginal social and private values of increasing financial slack by contributing more equity. It consists of the marginal effect of equity on emissions, $\frac{dE(X_B^*, I_{1B}^*)}{d\bar{e}}$, and a *total social cost of carbon* (T-SCC) wedge. The T-SCC wedge reflects the difference between the direct social and private cost of emissions, $\gamma_B - \tau_B$, as well as the effect of emissions on pledgeable income due to collateral externalities, $\lambda_B \left(\theta \gamma_B^p - \psi \tau_B\right)$. From Proposition 4, the optimal emissions tax is equal to τ_B^{GP} if $\psi = 1$, which implies a zero T-SCC wedge and no motive for leverage regulation. By contrast, if $\psi < 1$ the optimal emissions tax is below the generalized Pigouvian benchmark, so that the T-SCC wedge is positive and leverage regulation can improve welfare.

Whether the level of equity under the optimal leverage mandate is above or below the level in the competitive equilibrium depends on the effect of borrower equity on emissions. Higher borrower equity loosens financial constraints. This can affect emissions in two ways. On one hand, it implies more emissions due to a higher final investment scale. On the other hand, looser financial constraints affect the optimal abatement choice, which may result in lower emissions. Appendix A.4.2 shows that whether the effect on abatement dominates depends on the cross-derivatives of the emissions and abatement functions (it requires abatement to be more efficient at a higher investment scale).

If the effect of equity on abatement dominates, such that $\frac{dE(X_B^*, I_{1B}^*)}{d\bar{e}} < 0$, then the socially optimal equity is above the privately optimal level, $\bar{e}^* > e^*$. By contrast, if

$\frac{dE(X_B^*, I_{1B}^*)}{d\bar{e}} > 0$, then higher equity implies higher emissions, and the socially optimal equity level is below a borrower's optimal choice of equity in the competitive equilibrium, $\bar{e}^* < e^*$.[18] Note that $\bar{e}^* < e^*$ may be optimal even though liquidations are inefficient. This is because the leverage mandate trades off a high marginal utility of consumption at $t = 0$ against slackening the financial constraint at $t = 1$, while the emissions tax incentivizes the borrower to optimally distribute the available $t = 1$ resources between abatement and avoiding liquidations. If $\frac{dE(X_B^*, I_{1B}^*)}{d\bar{e}} > 0$, then borrowers consume too little because they under-value the benefit of higher leverage reducing emissions.

5.2 Including Climate Externalities in Financial Regulation

The finding in Proposition 7 that leverage regulation can improve welfare may not seem surprising given the large body of literature that shows how financial constraints can motivate financial regulation (for an overview, see Dewatripont and Tirole, 1994). Yet the following corollary shows that the financial constraint in itself does not motivate leverage regulation in our model:

Corollary 2. *If $\gamma_s^u = \gamma_s^p = 0$, then $\bar{e}^* = e^*$ regardless of whether $\lambda_B^* = 0$ or not.*

Proof. Follows from the result in Proposition 3 that $\tau_s = 0$ if $\gamma_s^u = \gamma_s^p = 0$, which implies a zero T-SSC wedge as defined in Proposition 7. □

In our setting, there is no benefit to introducing leverage regulation in the absence of environmental externalities – irrespective of whether the financial constraint binds or not. This is important because it implies that financial constraints alone are not enough to motivate leverage regulation in our model. Instead, the motive for implementing a leverage mandate $\bar{e}$ comes from the interaction between environmental externalities and financial frictions. By showing under what conditions environmental externalities can motivate leverage regulation in such a setting, the results in Proposition 7 contribute to the debate on whether environmental externalities should be included in the mandate of financial regulatory frameworks (also see Dávila and Walther, 2022; Oehmke and Opp, 2022).

[18]This result mirrors insights in Dávila and Walther (2022) that, with constraints on the regulation of some externality-generating activity (here abatement), the optimal second-best regulation of other choices (here leverage) depends on Pigouvian wedges in the constrained regulation and on how the perfectly regulated choices affect the imperfectly regulated activity.

Regulatory Pecking Order. From Proposition 5, a necessary condition for leverage regulation to improve welfare is that environmental regulation alone cannot implement a constrained-efficient allocation. Emissions taxes can implement constrained efficiency if tax rebates are fully pledgeable ($\psi = 1$, see Proposition 5). Alternatively, from Section 4.3, a cap-and-trade system can achieve constrained efficiency if permits are allocated for free. Additionally, Proposition 7 highlights that, even if there is a case for leverage regulation because other policies cannot achieve constrained efficiency, implementing the optimal leverage mandate can be complicated because it may be optimal to either discourage or encourage higher leverage, depending on the specific features of the underlying production and abatement technologies. This suggests a "regulatory pecking order" whereby regulators should first design carbon pricing in a way that minimizes the adverse effect on financial constraints before resorting to targeting climate-related objectives using financial regulation.

6 Financial Instruments

In the baseline model borrowers raise financing using short-term debt. This section considers hedging contracts and socially responsible investors. Long-term debt and external equity financing are covered in the Internet Appendix (Section IA.1).

6.1 Hedging and Climate-Linked Bonds

In this extension we allow fairly-priced hedging contracts that pay h_B in the bad state and h_G in the good state. Such contracts can be implemented through carbon price derivatives, or through state-contingent financing such as "climate linkers" that write off the principal by h_B when carbon taxes (or the social cost of emissions) are high, in return for an interest payment h_G when taxes are low.[19] Fair pricing requires that

$$(1 - q_B)h_G + q_B h_B = 0. \tag{14}$$

[19]Note that the binary risk-structure in the model implies that emissions taxes and the social cost of emissions are perfectly correlated. Therefore, it makes no difference whether the contracts are contingent on the social cost of carbon or the carbon tax.

Using this expression, the problem of borrowers can be expressed in terms of choosing the optimal h_G, while h_B follows as $h_B = -\frac{(1-q_B)h_G}{q_B}$. The borrower's problem is formally stated in the Internet Appendix (Section IA.1.1). The first order conditions are the same as in the baseline model, except for the new first order condition w.r.t. h_G, which states that borrowers equalize the shadow cost of the financial constraints across states:

$$\lambda_G = \lambda_B. \tag{15}$$

This implies that borrowers optimally shift resources from the good, low SCC state to the bad, high SCC state. If this allows borrowers to ensure that financial constraints are slack in both states ($\lambda_G = \lambda_B = 0$), then a Pigouvian emissions tax $\tau_s = \gamma_s, \forall s \in \{B, G\}$ can implement the first-best allocation (see Proposition 2). By allowing firms to hedge climate-related transition risk, the financial sector can enable efficient emissions taxation in equilibrium. This result highlights that hedging of climate-related risks may be an important role the financial sector can play in supporting the transition to a low-carbon economy, distinct from socially responsible investing that aims to direct firm policies by taking into account environmental and social factors in investment decisions (e.g., see Heinkel et al., 2001; Pástor et al., 2021; Oehmke and Opp, 2023). We also contribute to the nascent debate on climate-linked securities. Our analysis shows that supporting such markets can allow more efficient environmental policy in equilibrium, thus pointing to benefits that go beyond the direct risk-sharing and informational gains discussed so far (see Chikhani and Renne, 2022).

If under optimal hedging $\lambda_G = \lambda_B > 0$, then emissions taxes are different from the Pigouvian benchmark, see Proposition 3. We show in the Internet Appendix that in this case the efficiency results in Proposition 5 apply, so that emissions taxes alone can implement a constrained-efficient allocation only if tax rebates are fully pledgeable.

Some degree of hedging climate risks could also be achieved using external equity or long-term debt. However, we show in the Internet Appendix (Sections IA.1.2 and IA.1.3) that the risk-sharing benefits are more limited compared to carbon price hedging.

6.2 Socially Responsible Investing

This subsection introduces socially responsible investors (SRIs). In the spirit of Pástor et al. (2021), we assume that SRIs have a distaste for providing funding to polluting firms, which may incentivize emissions reductions by punishing emitters with a higher cost of funding. For SRIs to have an impact, it must be that borrowers cannot easily substitute away from SRIs to purely financially-motivated investors. For simplicity, we assume here that all investors are socially responsible, so that borrowers cannot substitute SRI capital for cheaper financial capital. This is arguably an extreme case. The main goal of this section is to show that, even in this case, SRIs may have an adverse effect on emissions abatement by tightening financial constraints.[20]

We assume that SRIs derive negative utility proportional to the emissions generated by the firm they provide funding to, weighted by a preference parameter ω (see Internet Appendix Section IA.1.4 for a formal statement of investors' preferences). SRIs' break-even requires that $d_1 = r_1 d_1 - \omega E(X_s, I_{1s})$, where r_1 is the gross interest rate that compensates SRIs for their disutility from investing in a polluting firm. The borrower's problem now yields the following FOC for abatement and complementary slackness condition:

$$(1+\lambda_s)\left[(\tau_s+\omega)\frac{\partial E(X_s, I_{1s})}{\partial X_s} + \frac{\partial C(X_s, I_{1s})}{\partial X_s}\right] = 0, \tag{6"}$$

$$\lambda_s[\theta R(I_{1s}, E_s^a) - (\tau_s+\omega)E(X_s, I_{1s}) + \psi T_s - d_0 + \mu(I_0 - I_{1s}) - C(X_s, I_{1s})] = 0. \tag{8"}$$

These correspond to the original conditions (6) and (8), with $\tau_s + \omega$ taking the place of τ_s. Eq. (6") captures the incentive effect of SRIs on abatement that comes from charging firms a premium on their financing cost proportional to emissions. This incentive effect works in the same way as an emissions tax.

A critical difference between the tax and the SRI premium is the effect on financial constraints, as seen in the complementary slackness condition (8"). The disutility SRIs derive from lending to polluters tightens the constraint by $\omega E(X_s, I_{1s})$. By contrast, the effect

[20] This insight would continue to hold as long as borrowers cannot perfectly substitute away SRI funding. Oehmke and Opp (2023) show that SRIs can achieve impact even when purely financially-motivated capital is abundant. A necessary condition is that investors are consequentialists who care about emissions no matter where they are produced, rather than only about the emissions they are directly responsible for. Considering consequentialist SRIs would require analyzing how they internalize their effect on equilibrium environmental policy, which is beyond the scope of this paper. The SRI preferences here resemble preferences for value-alignment, consistent with experimental evidence in Bonnefon et al. (2019).

of the emissions tax on the financial constraint is (partially) offset by the tax rebate T_s.

Corollary 3. *If socially responsible investors derive a disutility $\omega > 0$ from the emissions of firms they invest in, abatement and liquidations in the laissez-faire allocation (without emissions taxes) are equivalent to those in the baseline model without socially responsible investors but with a carbon tax $\tau_s = \omega$ and non-pledgeable tax rebates, $\psi = 0$.*

This implies that taxes and SRI premiums are imperfect substitutes in incentivizing borrowers to abate. In fact, the presence of SRIs may worsen the trade-offs faced by a regulator setting emissions taxes due to the tightening of borrowers' financial constraints.

7 Conclusion

This paper provides an analytical framework to shed light on how to design and combine carbon pricing with other regulatory tools when firms are subject to financial constraints. We find that emissions taxes alone can only implement a constrained-efficient allocation if tax rebates are fully pledgeable. Otherwise, welfare can be improved by replacing emissions taxes with a cap-and-trade system with ex-ante freely allocated pollution permits, or by complementing carbon taxes with leverage regulation. Fostering financial markets that allow firms to hedge regulatory risk, such as carbon-price derivatives or climate-linked bonds, can improve equilibrium climate policies by enabling firms to shoulder higher carbon taxes.

Another important insight is that physical climate risks give rise to a collateral externality that affects how emissions taxes interact with financial constraints. Higher emissions taxes tighten financial constraints if borrowers have carbon-emitting assets, but emissions taxes can ease financial constraints if they have a positive effect on the collateral value of assets exposed to physical climate risk. Optimal emissions pricing needs to account for climate-induced collateral externalities, and thus may be either above or below a Pigouvian benchmark rate equal to the direct social cost of emissions.

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A Appendix

To simplify the notation in parts of the Appendix we use the following definition

$$N(X_s, I_{1s}, \tau_s) = -\tau_s E(X_s, I_{1s}) - C(X_s, I_{1s}). \tag{16}$$

To make the expressions more legible we sometimes use the following shorthand notation: $F(X_s^*, I_{1s}^*) = F$, $F'_X = \frac{\partial F(X_s, I_{1s})}{\partial X_s}$, $F'_I = \frac{\partial F(X_s, I_{1s})}{\partial I_{1s}}$ for $F = E$ and $F = C$. Similarly, we use $N''_{XI} = \frac{\partial^2 N(X_s, I_{1s}, \tau_s)}{\partial X_s \partial I_s}$.

A.1 First Best (Proof of Proposition 1)

This appendix proofs Proposition 1. The first-best allocation maximizes social welfare defined as $W = U^i + U^b$, subject to aggregate resource constraints at $t = 0$ and $t = 1, 2$ in each state s, respectively:

$$\begin{aligned} c_0^b + c_0^i &= A_0^b + A_0^i - I_0 \\ c_{1s}^b + c_{1s}^i + C(X_s, I_{1s}) &= A_1^i + \mu(I_0 - I_{1s}) \\ c_{2s}^b + c_{2s}^i &= R(I_{1s}, E_s^a) = \rho I_{1s} - \gamma_s E(X_s, I_{1s}) \end{aligned}$$

Using the resources constraints to eliminate $c_0^i, c_{1s}^i, c_{2s}^i, c_{1s}^b, c_{2s}^b$, the problem can be written as the following Lagrangian:

$$\begin{aligned} \max_{c_0^b, I_{1s}, X_s} \mathcal{L} &= u(c_0^b) + A_0^b + A_0^i + A_1^i - I_0 - c_0^b \\ &+ \sum_{s \in \{G,B\}} q_s \left[\mu(I_0 - I_{1s}) - C(X_s, I_{1s}) + \rho I_{1s} - \gamma_s E(X_s, I_{1s}) + \bar{\kappa}_{Is}(I_0 - I_{1s}) \right], \end{aligned}$$

with $\bar{\kappa}_{Is}$ the Lagrange multiplier on the constraint that $I_{1s} \leq I_0$. The FOC's with respect to c_0^b, X_{1s}, and I_{1s} are, respectively:

$$\begin{aligned} u'(c_0^b) &= 1, \\ \gamma_s \frac{\partial E(X_s, I_{1s})}{\partial X_{1s}} + \frac{\partial C(X_s, I_{1s})}{\partial X_{1s}} &= 0, \\ \rho - \mu - \gamma_s \frac{\partial E(X_s, I_{1s})}{\partial I_{1s}} - \frac{\partial C(X_s, I_{1s})}{\partial I_{1s}} - \bar{\kappa}_{I1s} &= 0. \end{aligned}$$

The first two conditions are the ones stated in Proposition 1. The third condition can be used to show that $\bar{\kappa}_{Is} > 0$, which implies that $I_{1s} = I_0$ as stated in Proposition 1. To see this, recall that by Assumption 2.1 liquidations are privately inefficient for any $\tau \leq \bar{\tau}$, with $\bar{\tau} > \gamma_B$. This implies that

$$r(\gamma_B, X, I_1) = \rho - \mu - \frac{\partial C(X, I_1)}{\partial I_1} - \gamma_B \frac{\partial E(X, I_1)}{\partial I_1} > 0$$

Hence, the planner's first order condition w.r.t. I_{1s} can only be satisfied if $\bar{\kappa}_{Is} > 0$, which implies that $I_{1s} = I_0$ in the first-best allocation.

A.2 Competitive Equilibrium

A.2.1 Borrower's Lagrangian

This appendix formally states the Lagrangian for the borrower's problem in Section 3.1, from which the first order conditions in Section 3.2 are derived.

Since $u'(0) = \infty$ it must be that $c_0^b > 0$. Additionally, the financial constraint (4) implies $c_{2s}^b > 0$, so that the non-negativity constraint (3) never binds. Thus, after eliminating c_0^b, c_{1s}^b, and c_{2s}^b using Eqs. (1), (2), and (3), the problem of borrowers can be stated as:

$$\begin{aligned}
&\max_{X_s, I_{1s}, d_{1s}, e} \mathcal{L} = u(A_0 - e) \\
&+ \sum_{s\in\{G,B\}} q_s \left[\mu(I_0 - I_{1s}) + e - I_0 - C(X_s, I_{1s}) + R(I_{1s}, E_s^a) - \tau_s E(X_s, I_{1s}) + T_s\right] \\
&+ \sum_{s\in\{G,B\}} q_s \left\{\lambda_s \left[\theta R(I_{1s}, E_s^a) - \tau_s E(X_s, I_{1s}) + \psi T_s - d_{1s}\right] + \underline{\kappa}_{Is} I_{1s} + \overline{\kappa}_{Is}[I_0 - I_{1s}]\right\} \\
&+ \sum_{s\in\{G,B\}} q_s \kappa_{c_{1s}} \left[d_{1s} + \mu(I_0 - I_{1s}) + e - I_0 - C(X_s, I_{1s})\right],
\end{aligned} \tag{17}$$

The first order condition w.r.t. d_{1s} implies that $\lambda_{1s} = \kappa_{c_{1s}}$. The remaining FOC's of the problem are given in Section 3.2.

A.2.2 Limit on $t = 0$ Borrowing

This appendix shows that borrowers have no incentive to default on $t = 0$ debt at $t = 1$. Throughout the paper we focus on the parameter ranges in which the optimal initial debt is such that $d_0^* < \mu I_0$, because otherwise borrowers would forgo the project.

To see this, in a first step, note that if the borrower defaults at $t = 1$, investors lending at $t = 0$ can force (partial) liquidation of the project and seize the liquidation proceeds $\mu(I_0 - I_{1s})$. This implies that the maximum amount investors can obtain is μI_0, which is what they receive when forcing full liquidation, $I_{1s} = 0$. Could borrowers instead repay d_0 by raising new debt d_1? No, because by Assumption 2.2, pledgeable income at $t = 1$ decreases in I_{1s} and is maximized at μI_0 when $I_{1s} = 0$. Consequently, investors are willing to lend at most μI_0 at $t = 0$, i.e., borrowing is subject to the constraint $d_0 \leq \mu I_0$.

Would borrowers want to borrow to the point where this constraint just binds, $d_0 = \mu I_0$? In this case, investors would force liquidation at $t = 1$ to recoup their initial debt because μI_0 is the highest pledgeable income. Thus, borrower utility is given by $u(A_0 - I_0(1 - \mu))$. But this is dominated by forgoing the project and fully consuming the endowment at $t = 0$, which gives the borrower $u(A_0)$. Therefore, borrowers would always forgo the project if the optimal $d_0^* \geq \mu I_0$, motivating our focus on the case $d_0^* < \mu I_0$.

Finally, note that with $d_0 < \mu I_0$ the borrower has no incentive to default on $t = 0$ debt at $t = 1$. If the borrower defaults, investors force liquidation to the point where they fully recoup their investment, $\mu(I_0 - I_1) = d_0$. The borrower can then decide to continue the project, choose X_s, I_{1s}, and d_{1s} subject to the constraints listed in the baseline problem and the additional constraint $d_0 = \mu(I_0 - I_1)$. The presence of an additional constraint implies that defaulting is weakly dominated by repaying d_0 at $t = 1$.

A.2.3 Proof of Lemma 1

With a slack financial constraint, Equation (7) evaluated at $\lambda_s = 0$ is $r(\tau_s, X_s, I_{1s}) - \overline{\kappa}_{Is} + \underline{\kappa}_{Is} = 0$. By Assumption 2.1 we have that $r(\tau_s, X_s, I_{1s}) > 0$, which implies that the solution requires $\overline{\kappa}_{Is} > 0$ (i.e., $I_0 = I_{1s}^*$).

With a binding financial constraint, the complementary slackness condition (8) can be reformulated as

$$\lambda_s S(\tau_s, X_s, I_{1s}, e) = 0, \tag{8'}$$

where $S(\tau_s, X_s, I_{1s}, e)$ collects the terms in square brackets in Eq. (7). By Assumption 2.2, liquidating investments eases financial constraints, so $\frac{\partial S}{\partial I_{1s}} < 0$. If $S(\tau_s, X_s, I_{1s} = I_0, e) <$

0, the financial constraints binds, $\lambda_s > 0$. In this case, the complementary slackness condition (8') requires that borrowers choose I_{1s}^* s.t. $S(\tau_s, X_s, I_{1s}^*, e) = 0$. Thus, if $\lambda_s > 0$ it must be that $I_{1s}^* < I_0$ and $\overline{\kappa}_{Is} = 0$.

A.3 Optimal Policy

A.3.1 Characterization of Assumption 3

This appendix characterizes parameter conditions under which $\frac{\partial X_s^*}{\partial \tau_s} > 0 \; \forall \tau_s$. Totally differentiating Eq. (6) with respect to τ_s allows us to find $\frac{\partial X_s^*}{\partial \tau_s}$:

$$\frac{\partial X_s^*}{\partial \tau_s} = \frac{\frac{\partial E(X_s^*, I_{1s}^*)}{\partial X_s^*} - \frac{\partial^2 N(X_s^*, I_{1s}^*, \tau_s)}{\partial X_s^* \partial I_{1s}^*} \frac{\partial I_{1s}^*}{\partial \tau_s}}{-\frac{\partial^2 C(X_s^*, I_{1s}^*)}{\partial (X_s^*)^2}} \tag{18}$$

where we use definition of $N(X_s, I_{1s}, \tau_s)$ from Eq. (16) and that $\frac{\partial^2 E(X, I_1)}{(\partial X)^2} = 0$ by Assumption 1.3. Next we evaluate whether this derivative is positive.

If the financial constraint is slack, $\lambda_s^*(\tau_s, \bar{e}) = 0$, then $I_{1s}^* = I_0$, so that $\frac{\partial I_{1s}^*}{\partial \tau_s} = 0$. Together with the fact that $\frac{\partial^2 C(X, I_1)}{\partial X^2} > 0$ by Assumption 1, this implies $\frac{\partial X_s^*}{\partial \tau_s} > 0$ in this case (without further parameter conditions).

If the financial constraint is binding, $\lambda_s^*(\tau_s, \bar{e}) > 0$, the sign of $\frac{\partial X_s^*}{\partial \tau_s}$ is less straightforward. In this case, $\frac{\partial I_{1s}^*}{\partial \tau_s}$ follows from totally differentiating Eq. (8) with respect to τ_s:

$$\frac{\partial I_{1s}^*}{\partial \tau_s} = \frac{(1-\psi) E(X_s^*, I_{1s}^*) - (\psi \tau_s - \theta \gamma_s^p) \frac{\partial E(X_s^*, I_{1s}^*)}{\partial X_s^*} \frac{\partial X_s^*}{\partial \tau}}{\tilde{r}(\tau_s(1-\psi) + \theta \gamma_s^p, X_s^*, I_{1s}^*)} \tag{19}$$

We combine (18) and (19) to isolate the terms $\frac{\partial I_{1s}^*}{\partial \tau_s}$ and $\frac{\partial X_s^*}{\partial \tau_s}$:

$$\frac{\partial I_{1s}^*}{\partial \tau_s} = \frac{(1-\psi) E C''_{X^2} + (\psi \tau_s - \theta \gamma_s^p)(E'_X)^2}{\tilde{r}(\tau_s(1-\psi) + \theta \gamma_s^p, X_s^*, I_{1s}^*)) C''_{X^2} + (\psi \tau_s - \theta \gamma_s^p) E'_X N''_{XI}}, \tag{20}$$

$$\frac{\partial X_s^*}{\partial \tau_s} = \frac{(1-\psi) E N''_{XI} - \tilde{r}(\tau_s(1-\psi) + \theta \gamma_s^p, X_s^*, I_{1s}^*)) E'_X}{\tilde{r}(\tau_s(1-\psi) + \theta \gamma_s^p, X_s^*, I_{1s}^*)) C''_{X^2} + (\psi \tau_s - \theta \gamma_s^p) E'_X N''_{XI}}, \tag{21}$$

where we simplified the expressions using the shorthand notation for E'_X, C'_X, N''_{XI}, etc., introduced at the beginning of the Appendix.

Assumption 3 requires the model parameters to be such that $\frac{\partial X_s^*}{\partial \tau_s} > 0$. This holds if the numerator and denominator of (21) have the same sign. While it is not possible to provide explicit parameter conditions under which this is always the case, it can easily be seen that the denominator of (21) is negative if $\psi = 0$ and $\gamma_s^p = 0$, because $C''_{X^2} > 0$ and $\tilde{r}(\tau_s, X_s^*, I_{1s}^*) < 0$ by Assumptions 1.3 and 2.2 respectively. Therefore, we focus on parameter ranges such that both the denominator and the numerator of (21) are negative. This implicitly defines the parameters required for Assumption 3.

To further characterize the conditions for Assumption 3, note the denominator is negative if and only if $\tilde{r}(\tau_s(1-\psi) + \theta\gamma_s^p, X_s^*, I_{1s}^*)C''_{X^2} < -(\psi\tau_s - \theta\gamma_s^p)N''_{XI}E'_X$. For the numerator, we can use Definition 1 to expand $\tilde{r}(\tau_s(1-\psi)+\theta\gamma_s^p, X_s^*, I_{1s}^*) = \tilde{r}(\theta\gamma_s^p, X_s^*, I_{1s}^*) - (1-\psi)\tau_s E'_I$. Using this in the numerator of (21), we can see that it is negative whenever $\tilde{r}(\theta\gamma_s^p, X_s^*, I_{1s}^*)E'_X > (1-\psi)(\tau_s E'_I E'_X + EN''_{XI})$. This condition is satisfied whenever $\psi = 1$, using Assumption 2.2. Since the RHS of this inequality is monotone in ψ, the numerator of (21) is negative for any ψ if the inequality holds for $\psi = 0$, i.e. if $\tilde{r}(\theta\gamma^p, X_s^*, I_{1s}^*)E'_X > \tau_s E'_I E'_X + EN''_{XI}$.

Thus, Assumption 3 can be restated as $\forall X_s^*(\tau_s), I_{1s}^*(\tau_s), \tau_s < \bar{\tau}$:

- $\tilde{r}(\theta\gamma_s^p, X_s^*, I_{1s}^*)\frac{\partial E(X_s^*, I_{1s}^*)}{\partial X_s^*} > \tau_s \frac{\partial E(X_s^*, I_{1s}^*)}{\partial X_s^*}\frac{\partial E(X_s^*, I_{1s}^*)}{\partial I_{1s}^*} + E(X_s^*, I_{1s}^*)\frac{\partial^2 N(X_s^*, I_{1s}^*, \tau_s)}{\partial X_s^* \partial I_{1s}^*}$
- $\tilde{r}(\tau_s(1-\psi) + \theta\gamma_s^p, X_s^*, I_{1s}^*)\frac{\partial^2 C(X_s^*, I_{1s}^*)}{\partial (X^*)^2} < -(\psi\tau_s - \theta\gamma_s^p)\frac{\partial^2 N(X_s^*, I_{1s}^*, \tau_s)}{\partial X_s^* \partial I_{1s}^*}\frac{\partial E(X_s^*, I_{1s}^*)}{\partial X_s^*}$

A.3.2 Proof of Lemma 2

The derivative $\frac{\partial I_{1s}^*}{\partial \tau_s}$ is defined in Eq. (20). The denominator of (20) is the same as that of $\frac{\partial X^*}{\partial \tau_s}$, i.e. negative under Assumption 3. Lemma 2 follows from observing that the numerator of Eq. (20) is negative if $\gamma_s^p > \hat{\gamma}^p(\tau_s)$ and positive if $\gamma_s^p < \hat{\gamma}^p(\tau_s)$, where

$$\hat{\gamma}^p(\tau_s) \equiv \frac{\psi}{\theta}\tau_s + \frac{(1-\psi)E(X_s^*, I_{1s}^*)\frac{\partial^2 C(X_s^*, I_{1s}^*)}{(\partial X_s)^2}}{\theta\left(\frac{\partial E(X_s, I_{1s}^*)}{\partial X_s}\right)^2} \tag{22}$$

A.3.3 Proof of Proposition 3

The regulator's problem is to set the emissions tax τ_s so as to maximize welfare at $t = 1$ $W_{1s} = c_{1s}^i + c_{2s}^i + c_{1s}^b + c_{2s}^b - 2\gamma_s^u E_s^a$, subject to the non-negativity constraint on τ_s. We eliminate c_{1s}^b, and c_{2s}^b using Eqs. (2) and (3), and substitute $c_{1s}^i = A_1^i + d_{1s} - d_0$, and $c_{2s}^i = d_{1s}$, to write the regulator's problem as the following Lagrangian:

$$\max_{\tau_s} \ \rho I_{1s}^* + \mu(I_0 - I_{1s}^*) - \gamma_s E(X_s^*, I_{1s}^*) - C(X_s^*, I_{1s}^*) + \kappa_{\tau s}\tau_s. \tag{23}$$

The first order condition with respect to τ_s is given by:

$$\left(\gamma_s \frac{\partial E(X_s, I_{1s})}{\partial X_s} + \frac{\partial C(X_s, I_{1s})}{\partial X_s}\right) \frac{\partial X_s^*}{\partial \tau_s} = \left(\rho - \mu - \gamma_s \frac{\partial E(X_s, I_{1s})}{\partial I_{1s}} - \frac{\partial C(X_s, I_{1s})}{\partial I_{1s}}\right) \frac{\partial I_{1s}^*}{\partial \tau_s} + \kappa_{\tau s}$$

Using (6) and the definition of $r(\tau, X, I_1)$ the FOC above simplifies to (10). In Eq. (10), the term $\frac{\partial E(X_s^*, I_{1s}^*)}{\partial X_s^*} \frac{\partial X_s^*}{\partial \tau_s} < 0$ under Assumptions 1 and 3, while $r(\gamma_s, X_s, I_{1s}) > 0$ by Assumption 2. Consequently, for Eq. (10) to hold the optimal tax must be:

- lower than the direct social cost of carbon $\tau_s < \gamma_s$ if $\frac{\partial I_{1s}^*}{\partial \tau_s} < 0$ and $\gamma_s > 0$
- equal to the direct social cost of carbon $\tau_s = \gamma_s$ if $\frac{\partial I_{1s}^*}{\partial \tau_s} = 0$ or if $\frac{\partial I_{1s}^*}{\partial \tau_s} < 0$ and $\gamma_s = 0$
- higher than the direct social cost of carbon $\tau_s > \gamma_s$ if $\frac{\partial I_{1s}^*}{\partial \tau_s} > 0$

The result in Proposition 3 in terms of the threshold $\hat{\gamma}^p(\tau_s)$ follows from using Lemma 2 to determine the sign of $\frac{\partial I_{1s}^*}{\partial \tau_s}$ and noting that if $\gamma_s = 0$ then $\gamma_s^p = 0 \leq \hat{\gamma}^p(\tau_s)$ so $\frac{\partial I_{1s}^*}{\partial \tau_s} < 0$.

A.3.4 Proof of Proposition 4

Using Eq. (11) in Eq. (10) and simplifying yields the following condition characterizing the optimal emissions tax:

$$\begin{aligned} r(\gamma_s, X_s^*, I_{1s}^*)(1-\psi)E(X_s^*, I_{1s}^*) + \kappa_{\tau_s}\tilde{r}(\tau_s(1-\psi) + \theta\gamma_s^p, X_s^*, I_{1s}^*) = \\ [\gamma_s - \tau_s + \lambda_s^*(\theta\gamma_s^p - \psi\tau_s)] \frac{\partial E(X_s^*, I_{1s}^*)}{\partial X_s^*} \frac{\partial X_s^*}{\partial \tau_s} \tilde{r}(\tau_s, X_s^*, I_{1s}^*) \end{aligned} \tag{24}$$

The RHS of Eq. (24) is zero if $\tau_s = \tau_s^{GP} \equiv \frac{\gamma_s + \lambda_s^* \theta \gamma_s^p}{1 + \psi \lambda_s^*}$. The RHS is positive whenever $\tau_s < \tau_s^{GP}$, since $\frac{\partial E(X_s, I_{1,s})}{\partial X_s} < 0$, $\tilde{r}(\tau_s, X_s^*, I_{1s}^*) < 0$ and $\frac{\partial X_s^*}{\partial \tau_s} > 0$ under Assumptions 1.1, 2.2 and 3 respectively.

In the interior solution, $\kappa_{\tau_s} = 0$ and $\tau_s > 0$. If $\psi = 1$, then the LHS of (24) is equal to zero, so the optimal emissions tax must be $\tau_s = \tau_s^{GP}$. If $\psi < 1$, then the LHS is positive, so the optimal emissions tax must satisfy $\tau_s < \tau_s^{GP}$.

In the corner solution, $\kappa_{\tau_s} > 0$ and $\tau_s = 0$. Proposition 3 implies that if $\gamma_s = 0$, then $\tau_s^* = 0$. Evaluating τ_s^{GP} at $\gamma_s = 0$ yields $\tau_s^{GP}(\gamma_s = 0) = 0$. To complete the proof we also need to consider the case when $\tau_s^* = 0$ while $\gamma_s > 0$.

We first show that this can only happen if $\psi < 1$. We do it in two steps: (i) show that if $\psi = 1$ and $\gamma_s > 0$, then $\tau_s^* = 0$ cannot be an equilibrium and (ii) show that if $\psi < 1$ and $\gamma_s > 0$, then $\tau_s^* = 0$ is a feasible equilibrium. Then we show that when $\psi < 1$ and $\gamma_s > 0$, the equilibrium tax satisfies $\tau_s^* < \tau_s^{GP}$ as stated in the Proposition 4 (iii).

(i) Notice that when $\tau_s = 0$ and $\gamma_s > 0$, then the RHS of Eq. (24) is positive. If $\psi = 1$, the LHS of Eq. (24) is weakly negative, since $\tilde{r}(\tau_s(1-\psi) + \theta\gamma_s^p, X_s^*, I_{1s}^*) < 0$ and $\kappa_{\tau_s} \geq 0$. Thus, if $\psi = 1$ and $\gamma_s > 0$, $\tau_s^* = 0$ cannot be an equilibrium.
(ii) If $\psi < 1$, the LHS of Eq. (24) can take any sign, depending on the relative size of κ_{τ_s}. Since the RHS of Eq. (24) is positive whenever $\tau_s^* = 0 < \gamma_s$, such equilibrium is feasible.
(iii) Since $\tau_s^{GP}(\gamma_s) > 0\ \forall\ \gamma_s > 0$, it follows that $\tau_s^* = 0 < \gamma_s$ when $\psi < 1$ is consistent with $\tau_s^* < \tau_s^{GP}$

A.3.5 Proof of Proposition 5

We define the constrained-efficient allocation in which a social planner can choose X_s, I_{1s} and e directly without any policy instruments, but subject to the same constraints as private agents. We eliminate c_0^b, c_{1s}^b, and c_{2s}^b using Eqs. (1), (2), and (3), and use $c_0^i = A_0^i - d_0$, $c_{1s}^i = A_1^i + d_{1s} - d_0$, and $c_{2s}^i = d_{1s}$, to write the planner's problem as the following Lagrangian:

$$\begin{aligned}
\max_{X_s, I_{1s}, e}\ \mathcal{L} &= u(A_0 - e) + e - I_0 + A_0^i + A_1^i \\
&+ \sum_{s\in\{B,G\}} q_s \left\{R(I_{1s}, E_s^a) + \mu(I_0 - I_{1s}) - \gamma_s^u E(X_s, I_{1s}) - C(X_s, I_{1s})\right\} \\
&+ \sum_{s\in\{B,G\}} q_s \left\{\lambda_s^{SP}\left[\theta R(I_{1s}, E_s^a) + \mu(I_0 - I_{1s}) - C(X_s, I_{1s}) + e - I_0\right]\right. \\
&+ \left.\left[\underline{\kappa}_{Is}^{SP} I_{1s} + \overline{\kappa}_{Is}^{SP}(I_0 - I_{1s})\right]\right\}.
\end{aligned} \tag{25}$$

The constrained-efficient levels of $I_{1s}^{SP}, X_s^{SP}, \lambda_s^{SP}, e^{SP}$ are pinned down by the FOCs with respect to X_s, I_{1s}, and e and the complementary slackness condition:

$$-(\gamma_s + \lambda_s^{SP}\theta\gamma_s^p)\frac{\partial E(X_s, I_{1s})}{\partial X_s} - (1 + \lambda_s^{SP})\frac{\partial C(X_s, I_{1s})}{\partial X_s} = 0, \tag{26}$$

$$r(\gamma_s, X_s, I_{1s}) + \lambda_s^{SP}\tilde{r}(\theta\gamma_s^p, X_s, I_{1s}) + \underline{\kappa}_{Is}^{SP} - \overline{\kappa}_{Is}^{SP} = 0, \tag{27}$$

$$-u'(A_0 - e) + 1 + q_G\lambda_G^{SP} + q_B\lambda_B^{SP} = 0, \tag{28}$$

$$\lambda_s^{SP}[\theta R(I_{1s}, E_s^a) + \mu(I_0 - I_{1s}) - C(X_s, I_{1s}) + e - I_0] = 0. \tag{29}$$

To check whether the equilibrium is constrained efficient, we compare the planner's FOCs to the borrowers FOCs (6), (7), (9), and the complementary slackness condition (8). The equilibrium is constrained efficient if and only if $X_s^*(\tau_s^*) = X_s^{SP}$, $I_{1s}^*(\tau_s^*) = I_{1s}^{SP}$,

$e^*(\tau_G^*, \tau_B^*) = e^{SP}$ and $\lambda_s^*(\tau_s^*) = \lambda_s^{SP}$. This is the case if (6) is equivalent to (26), (7) is equivalent to (27), (9) is equivalent to (28), and (8) is equivalent to (29). To check whether this is the case, we postulate that $X_s^*(\tau_s^*) = X_s^{SP}$, $I_{1s}^*(\tau_s^*) = I_{1s}^{SP}$, $e^*(\tau_G^*, \tau_B^*) = e^{SP}$ and $\lambda_s^*(\tau_s^*) = \lambda_s^{SP}$, and verify that each of the borrower-planner FOC pairs are equivalent given τ_s^* $\forall s$ defined in Proposition 4.

Case $\psi < 1$.

- If (29) is satisfied at $X_s^{SP} = X_s^*, I_{1s}^{SP} = I_{1s}^*, e^{SP} = e^*$ for all s, then (8) is satisfied if and only if $\tau_s^* E(X_s^*, I_{1s}^*) - \psi T_s^* = 0$ for all s. This is the case only if $\tau_s^* = 0$ and $T_s^* = 0$ for all s.
- If $I_{1s}^{SP} = I_{1s}^*, e^{SP} = e^*$, then (26) is equivalent to (6) if and only if $\tau_s^* = \frac{\gamma_s + \lambda_s^{SP}\theta\gamma_s^p}{1+\lambda_s^{SP}} \equiv \tau_s^{SP}$.

Thus, for $X_s^{SP} = X_s^*, I_{1s}^{SP} = I_{1s}^*, e^{SP} = e^*$ and (29) to be equivalent to (8) for all s it must be that $\tau_s^{SP} = 0$ for all s, which is the case only if $\gamma_s = 0$ for all s. Since $\gamma_B > 0$, this does not hold, hence if $\psi < 1$, the competitive equilibrium is not constrained efficient.

Case $\psi = 1$. We proceed in four steps:

1. (8) & (29): When $\psi = 1$, then $-\tau_s^* E(X_s^*, I_{1s}^*) + \psi T_s^* = 0$. This implies (8) is equivalent to (29).

2. (6) & (26): The two conditions are equivalent if $\tau_s^* = \frac{\gamma_s + \lambda_s^{SP}\theta\gamma_s^p}{1+\lambda_s^{SP}} \equiv \tau_s^{SP}$. Proposition 4 implies the optimal emissions tax is given by $\tau_s^* = \tau_s^{GP} = \frac{\gamma_s + \lambda_s^*\theta\gamma_s^p}{1+\lambda_s^*}$ when $\psi = 1$. This implies that $\tau_s^* = \tau_s^{SP}$ whenever $\lambda_s^*(\tau_s^*) = \lambda_s^{SP}$. We show that this holds below.

3. (9) & (28): the two conditions are equivalent whenever $\lambda_s^*(\tau_s^*) = \lambda_s^{SP}$. Below we show that this holds at $\tau_s^* = \tau_s^{SP}$.

4. (7) & (27): in the interior solution for I_{1s} the two conditions are equivalent if and only if $\lambda_s^{SP} = -\frac{r(\gamma_s, X_s^{SP}, I_{1s}^{SP})}{\tilde{r}(\theta\gamma_s^p, X_s^{SP}, I_{1s}^{SP})} = -\frac{r(\tau_s^*, X_s^*, I_{1s}^*)}{\tilde{r}(\tau_s^{SP}, X_s^*, I_{1s}^*)} = \lambda_s^*(\tau_s^*)$.

   Verifying that $\lambda_s^{SP} = \lambda_s^*(\tau_s^*)$ at $\tau_s^* = \tau_s^{SP}$ also establishes that (6) is equivalent to (26) (see step 2), and (9) is equivalent to (28) (see step 3).

To verify that $\lambda_s^*(\tau_s^{SP}) = \lambda_s^{SP}$, we first find τ_s^{SP} and then plug it into borrower's FOC (7) to find $\lambda_s^*(\tau_s^{SP})$.

In the interior solution $I_{1s} > 0$, (27) implies that $\lambda_s^{SP} = \frac{r(\gamma_s, X_s^{SP}, I_{1s}^{SP})}{\tilde{r}(\theta\gamma_s^p, X_s^{SP}, I_{1s}^{SP})}$, using this in the expression for τ_s^{SP} yields:

$$\tau_s^{SP} = \frac{\gamma_s \tilde{r}(\theta\gamma_s^p, X_s^{SP}, I_{1s}^{SP})) - \theta\gamma_s^p r(\gamma_s, X_s^{SP}, I_{1s}^{SP}))}{\tilde{r}(\theta\gamma_s^p, X_s^{SP}, I_{1s}^{SP})) - r(\gamma_s, X_s^{SP}, I_{1s}^{SP}))} = \frac{\rho\theta(\gamma_s - \gamma_s^p) - w(\gamma_s - \theta\gamma_s^p)}{\rho(\theta_s - 1) - (\theta\gamma_s^p - \gamma_s)E_I'} \quad (30)$$

where the last step simplifies the expression by substituting $w = \frac{\partial C(X_s^{SP}, I_{1s}^{SP})}{\partial I_{1s}^{SP}} + \mu$ and $E_I' = \frac{\partial E(X_s^{SP}, I_{1s}^{SP})}{\partial I_{1s}^{SP}}$.

Using the expression for τ_s^{SP} in (9) to find $\lambda_s^*(\tau_s^{SP})$:

$$\lambda_s^*(\tau_s^{SP}) = -\frac{r(\tau_s^{SP}, X_s^*, I_{1s}^*)}{\tilde{r}(\tau_s^{SP}, X_s^*, I_{1s}^*)} = -\frac{\rho - w - \tau_s^{SP} E_I'}{\theta\rho - w - \tau_s^{SP} E_I'}$$
$$= -\frac{\rho^2(\theta-1) - \rho(\theta\gamma^p - \gamma)E_I' - w\rho(\theta-1) + w(\theta\gamma^p - \gamma)E_I' - \rho\theta(\gamma - \gamma^p)E_I' + w(\gamma - \theta\gamma^p)E_I'}{\theta\rho^2(\theta-1) - \theta\rho(\theta\gamma^p - \gamma)E_I' - w\rho(\theta-1) + w(\theta\gamma^p - \gamma)E_I' - \rho\theta(\gamma - \gamma^p)E_I' + w(\gamma - \theta\gamma^p)E_I'}$$
$$= -\frac{\rho^2(\theta-1) - w\rho(\theta-1) - \rho\gamma_s(\theta-1)E_I'}{\theta\rho^2(\theta-1) - w\rho(\theta-1) - \rho\theta\gamma_s^p(\theta-1)E_I'} = -\frac{\rho - w - \gamma_s E_I'}{\theta\rho - w - \theta\gamma_s^p E_I'} = -\frac{r(\gamma_s, X_s^{SP}, I_{1s}^{SP})}{\tilde{r}(\theta\gamma_s^p, X_s^{SP}, I_{1s}^{SP})} = \lambda_s^{SP}$$

where we use that $X_s^* = X_s^{SP}$, $I_{1s}^* = I_{1s}^{SP}$ and the last step follows from the definition of λ_s^{SP} for the interior solution of I_{1s} in Eq. (27). This completes the proof that $\lambda_s^*(\tau_s^{SP}) = \lambda_s^{SP}$ for the interior solution $I_{1s} > 0$.

In the corner solution with full liquidations $I_{1s}^* = I_{1s}^{SP} = 0$, so the complementary slackness conditions (8) and (29) pin down $e^* = I_0(1-\mu)$ and $e^{SP} = I_0(1-\mu)$ respectively. Notice that if the investment is in the corner solution in state s, it must be in the interior solution in state $-s$, as otherwise borrowers would be better off not initiating the project at $t = 0$. But we already show above that in the interior solution $\lambda_{-s}^*(\tau_s^{SP}) = \tau_{-s}^{SP}$. Using this together with $e^* = I_0(1-\mu)$ in (9) and $e^{SP} = I_0(1-\mu)$ in (28), implies that $\lambda_s^* = \lambda_s^{SP}$. Thus, if $I_{1s}^* = I_{1s}^{SP} = 0$, then $e^* = e^{SP}$ and $X_s^* = X_s^{SP}$, so the equilibrium is constrained efficient.

A.4 Other Policies

A.4.1 Cap-and-Trade

This appendix derives the borrower's problem under the cap-and-trade system laid out in Section 4.3, and derives the optimal permit price. We assume here that the proceeds from the sale of permits are distributed to investors (Internet Appendix IA.2.1 shows that the insights on implementing the constrained efficient allocation are robust if sale proceeds are distributed to borrowers, instead). The budget constraints of the borrower under the pollution trading scheme are:

$$c_{1s}^b = \mu(I_0 - I_{1s}) + d_{1s} + e - I_0 - C(X_s, I_{1s}) \geq 0, \tag{2'}$$

$$c_{2s}^b = R(I_{1s}, E_s^a) - (1-\phi)Q_s p_s + p_s(Q_s - E(X_s, I_{1s})) - d_{1s} \geq 0, \tag{3'}$$

$$d_{1s} \leq \theta R(I_{1s}, E_s^a). \tag{4'}$$

The borrower's problem is analogous to the one with emissions taxes, but with the pollution permit price p_s taking the place of the tax τ_s, as shown in the budget constraints above. The FOCs and the complementary slackness condition are:

$$(1+\lambda_s)\left(p_s \frac{\partial E(X_s, I_{1s})}{\partial X_s} + \frac{\partial C(X_s, I_{1s})}{\partial X_s}\right) = 0, \tag{6'}$$

$$\rho(1+\lambda_s\theta) - (1+\lambda_s)\left[\mu + \frac{\partial C(X_s, I_{1s})}{\partial I_{1s}} + p_s \frac{\partial E(X_s, I_{1s})}{\partial I_{1s}}\right] - \overline{\kappa}_{Is} + \underline{\kappa}_{Is} = 0, \tag{7'}$$

$$u'(A_0 - e) - 1 - (1-q)\lambda_G - q\lambda_B = 0, \tag{9'}$$

$$\lambda[\theta R(I_{1s}, E_s^a) + I_0 + \mu(I_0 - I_{1s}) + e - C(X_s, I_{1s}) + p_s(\phi Q_s - E(X_s, I_{1s}))] = 0. \tag{8'}$$

Regulator Problem. The regulator sets the amount of emissions Q_s. Condition (6'), together with the market clearing for permits, $Q_s = E_s^a$, jointly determine a mapping from p_s to E_s^a. Thus, the regulator can implement a desired market price of permits by altering the total quantity of permits. Consequently, we can solve the regulator's problem as maximizing social welfare at $t = 1$ by choosing p_s in each state $s = \{B, G\}$, analogous to the regulator problem with emission taxes in Eq. (23). The first order condition of the regulator is:

$$r(\gamma_s, X_s^*, I_{1s}^*)\frac{\partial I_{1s}^*}{\partial p_s} - (\gamma_s - p_s)\frac{\partial E(X_s^*, I_{1s}^*)}{\partial X_s^*}\frac{\partial X_s^*}{\partial p_s} + \kappa_p = 0 \tag{10'}$$

To find $\frac{\partial X_s^*}{\partial p_s}$, we take a total derivative of (6') with respect to p_s. This yields:

$$\frac{\partial X_s^*}{\partial p_s} = \frac{\frac{\partial E(X_s^*, I_{1s}^*)}{\partial X_s^*} - \frac{\partial^2 N(X_s^*, I_{1s}^*, p_s)}{\partial X_s^* \partial I_{1s}^*}\frac{\partial I_{1s}^*}{\partial p_s}}{\frac{\partial^2 C(X_s^*, I_{1s}^*)}{\partial (X_s^*)^2}} \tag{18'}$$

To find $\frac{\partial I_{1s}^*}{\partial p_s}$ take a total derivative of (8') with respect to p_s, keeping in mind that $\phi Q_s = \phi E_s^a$.

$$\frac{\partial I_{1s}^*}{\partial p_s} = \frac{(1-\phi)E(X_s^*, I_{1s}^*) - (\phi p_s - \theta\gamma_s^p)\frac{\partial E(X_s^*, I_{1s}^*)}{\partial X_s^*}\frac{\partial X_s^*}{\partial p_s}}{\tilde{r}(p_s(1-\phi) - \theta\gamma_s^p, X_s^*, I_{1s}^*)} \tag{19'}$$

We define $\frac{\partial X_s^*}{\partial \tau_s} = g_X(\tau_s, \psi)$ and $\frac{\partial I_{1s}^*}{\partial \tau_s} = g_I(\tau_s, \psi)$. Comparing (18) with (18') and (19) with (19'), it is straightforward that $\frac{\partial X_s^*}{\partial p_s} = g_X(p_s, \phi)$ and $\frac{\partial I_{1s}^*}{\partial p_s} = g_I(p_s, \phi)$. Thus, the first order condition of the regulator's problem in the baseline model (10) is equivalent to the first order condition of the problem of choosing Q_s to implement p_s taking as given ϕ, given by (10'). The two problems are exactly the same if $\psi = \phi$. This proofs the statement in Proposition 6.

A.4.2 Leverage Regulation (Proof of Proposition 7)

Consider the problem of a regulator who maximizes social welfare by choosing $\bar{e}$ at $t = 0$ and τ_s at $t = 1$. The first order condition with respect to τ_s is given by Eq. (10). The first order conditions of the regulator with respect to $\bar{e}$ is:

$$u'(A_0 - \bar{e}) - 1 = \sum_{s\in\{G,B\}} q_s \left[\left(\rho - \mu - \gamma_s \frac{\partial E}{\partial I_{1s}} - \frac{\partial C}{\partial I_{1s}}\right)\frac{\partial I_{1s}^*}{\partial \bar{e}} - \left(\gamma_s \frac{\partial E}{\partial X_s} + \frac{\partial C}{\partial X_s}\right)\frac{\partial X_s^*}{\partial \bar{e}}\right] \tag{31}$$

To get Eq. (12) in the main text, combine this FOC with the borrower's FOC w.r.t. X_s, (6), and totally differentiate (6) with respect to $\bar{e}$ to find:

$$\frac{\partial X_s^*}{\partial \bar{e}} = \frac{\partial X_s^*}{\partial I_{1s}^*}\frac{\partial I_{1s}^*}{\partial \bar{e}}. \tag{32}$$

Effect of equity on liquidations and abatement. To derive the result in Proposition 7, we first find $\frac{\partial I_{1s}^*}{\partial \bar{e}}$ and $\frac{\partial X_s^*}{\partial I_{1s}^*}$, which are needed to further expand Eq. (31). Totally differentiating (6) with respect to I_{1s}^* allows us to find:

$$\frac{\partial X_s^*}{\partial I_{1s}^*} = \frac{\frac{\partial^2 N(X_s^*, I_{1s}^*, \tau_s)}{\partial X_s^* \partial I_{1s}^*}}{\frac{\partial^2 C(X_s^*, I_{1s}^*)}{\partial (X_s^*)^2}} \tag{33}$$

where we use $N(X_s, I_{1s}, \tau_s) = -\tau_s E(X_s, I_{1s}) - C(X_s, I_{1s})$.

If $\lambda_s^*(\tau_s) = 0$, then $I_{1s}^* = I_0$, so $\frac{\partial I_{1s}^*}{\partial \bar{e}} = 0$ and $\frac{\partial X_s^*}{\partial \bar{e}} = 0$. If $\lambda_s^*(\tau_s) > 0$, the interior solution of $I^*_{1s}(\tau_s)$ is pinned down by (8). Totally differentiating (8) with respect to $\bar{e}$ yields:

$$\frac{\partial I^*_{1s}}{\partial \bar{e}} = \frac{-1 - (\psi\tau_s - \theta\gamma^p_s)\frac{\partial E(X^*_s, I^*_{1s})}{\partial X^*_s}\frac{\partial X^*_s}{\partial I^*_{1s}}\frac{\partial I^*_{1s}}{\partial \bar{e}}}{\tilde{r}(\tau_s(1-\psi) + \theta\gamma^p_s, X^*_s, I^*_{1s})} \tag{34}$$

Combining (32) and (34) and using the shorthand notation, yields:

$$\frac{\partial I^*_{1s}}{\partial \bar{e}} = \frac{-C''_{X^2}}{\tilde{r}(\tau_s(1-\psi) + \theta\gamma^p_s, X^*_s, I^*_{1s})C''_{X^2} + (\psi\tau_s - \theta\gamma^p_s)E'_X N''_{XI}} \tag{35}$$

$$\frac{\partial X^*_s}{\partial \bar{e}} = \frac{-N''_{XI}}{\tilde{r}(\tau_s(1-\psi) + \theta\gamma^p_s, X^*_s, I^*_{1s})C''_{X^2} + (\psi\tau_s - \theta\gamma^p_s)E'_X N''_{XI}} \tag{36}$$

The denominator of (35) and (36) is negative by Assumption 3. The numerator of (35) is negative because $C''_{X^2} > 0$ by Assumption 1. This implies that $\frac{\partial I^*_{1s}}{\partial \bar{e}} > 0$.

From (35), $\frac{\partial X^*_s}{\partial \bar{e}} > 0$ if and only if the cross-derivative $N''_{XI} = \frac{\partial^2 N(X_s, I_{1s}, \tau_s)}{\partial X_s \partial I_s} > 0$ (defined in Eq. (16)). The economic interpretation of this cross-derivative being positive is that the net benefit of abatement is greater at a higher investment scale, for example, because there are economies of scale in that it is cheaper to reduce emissions on a larger project. However, the effect of higher equity can alternatively be negative if $N''_{XI} < 0$.

Comparing private and socially optimal equity choice (Proposition 7). Focusing on the case where the financial constraint binds only in the bad state and using (7), (35), and (36), we can further rewrite the planner's FOC (12) and the borrower's FOC (9) as, respectively:

$$u'(A_0 - \bar{e}) - 1 = \frac{-r(\tau_B, X^*_B, I^*_{1B})C''_{X^2} + (\gamma_B - \tau_B)[E'_I C''_{X^2} + E'_X N''_{XI}]}{\tilde{r}(\tau_B(1-\psi) + \theta\gamma^p_B, X^*_B, I^*_{1B})C''_{X^2} + (\psi\tau_B - \theta\gamma^p_B)E'_X N''_{XI}}, \tag{37}$$

$$u'(A_0 - e) - 1 = \frac{-r(\tau_B, X^*_B, I^*_{1B})}{\tilde{r}(\tau_B, X^*_B, I^*_{1B})}. \tag{38}$$

Comparing the two, borrowers choose a lower level of equity than the regulator if and only if:

$$\frac{-r(\tau_B, X^*_B, I^*_{1B})C''_{X^2} + (\gamma_B - \tau_B)[E'_I C''_{X^2} + E'_X N''_{XI}]}{\tilde{r}(\tau_B(1-\psi) + \theta\gamma^p_B, X^*_B, I^*_{1B})C''_{X^2} + (\psi\tau_B - \theta\gamma^p_B)E'_X N''_{XI}} > \frac{-r(\tau_B, X^*_B, I^*_{1B})}{\tilde{r}(\tau_B, X^*_B, I^*_{1B})}$$

Note that under Assumption 3:

$$\tilde{r}(\tau_B(1-\psi)+\theta\gamma_B^p, X_B^*, I_{1B}^*, X_B^*, I_{1B}^*)C''_{X^2}+(\psi\tau_B-\theta\gamma_B^p)E'_X N''_{XI}<0,$$

and by Assumption 2: $\tilde{r}(\tau, X_B^*, I_{1B}^*)<0$. Thus with some algebra, the condition can be rewritten as:

$$\left(\frac{\partial E(X_B^*, I_{1B}^*)}{\partial I_{1B}^*}+\frac{\partial E(X_B^*, I_{1B}^*)}{\partial X_B^*}\frac{\frac{\partial^2 N(X_B^*, I_{1B}^*, \tau_B)}{\partial X_B^* \partial I_{1B}^*}}{\frac{\partial^2 C(X_B^*, I_{1B}^*)}{(\partial X_B^*)^2}}\right)\underbrace{\left[(\gamma_B-\tau_B)-\frac{r(\tau_B, X_B^*, I_{1B}^*)}{\tilde{r}(\tau_B, X_B^*, I_{1B}^*)}(\theta\gamma_B^p-\psi\tau_B)\right]}_{T-SCC\ wedge}<0.$$

Borrowers choose a lower level of equity than the regulator if the LHS is smaller than zero. Conversely, borrowers choose a higher level of equity if the LHS is greater than zero, and the same level if it is equal to zero. Notice that, since $\lambda_B=-\frac{r(\tau_B, X_B^*, I_{1B}^*)}{\tilde{r}(\tau_B, X_B^*, I_{1B}^*)}$, the term in the square bracket corresponds to the total social cost of carbon wedge (T-SCC wedge) defined in Proposition 7. The T-SCC wedge is equal to zero when $\tau_B^*=\tau_B^{GP}$ and is positive when $\tau_B^*<\tau_B^{GP}$. Hence, Proposition 4 implies that the T-SCC wedge is zero when $\psi=1$ (in this case there is no motive for leverage regulation) and is positive whenever $\psi<1$.

To arrive at condition (13) in Proposition 7 we use (33) to restate the first term of the condition above as:

$$\begin{aligned}\frac{\partial E(X_B^*, I_{1B}^*)}{\partial I_{1B}^*}+\frac{\partial E(X_B^*, I_{1B}^*)}{\partial X_B^*}\frac{\frac{\partial^2 N(X_B^*, I_{1B}^*, \tau_B)}{\partial X_B^* \partial I_{1B}^*}}{\frac{\partial^2 C(X_B^*, I_{1B}^*)}{(\partial X_B^*)^2}} &= \frac{\partial E(X_B^*, I_{1B}^*)}{\partial I_{1B}^*}+\frac{\partial E(X_B^*, I_{1B}^*)}{\partial X_B^*}\frac{\partial X_B^*}{\partial I_{1B}^*}\\ &\equiv \frac{dE(X_B^*, I_{1B}^*)}{dI_{1B}^*}\end{aligned}$$

Finally, notice that, since $\frac{\partial I_{1B}^*}{\partial \bar{e}}>0$ whenever $\lambda_B^*>0$, we can express the condition in terms of $\frac{dE(X_B^*, I_{1B}^*)}{d\bar{e}}$ as in (13), by using $\frac{dE(X_B^*, I_{1B}^*)}{d\bar{e}}=\frac{dE(X_B^*, I_{1B}^*)}{dI_{1B}^*}\frac{\partial I_{1b}^*}{\partial \bar{e}}$.

The effect of equity on emissions. If the financial constraint is slack in state s, then $\frac{dE(X_s^*, I_{1s}^*)}{d\bar{e}}=0$. To understand how leverage affects emissions when the financial constraint binds in the bad state, $\frac{dE(X_B^*,I_{1B}^*)}{d\bar{e}}$ can be decomposed as follows:

$$\frac{dE(X_B^*, I_{1B}^*)}{d\bar{e}} = \underbrace{\left(\underbrace{\frac{\partial E(X_B^*, I_{1B}^*)}{\partial I_{1B}^*}}_{\text{Direct effect of } I_{1B}^*} + \underbrace{\frac{\partial E(X_B^*, I_{1B}^*)}{\partial X_B^*} \frac{\partial X_B^*}{\partial I_{1B}^*}}_{\text{Indirect effect through } X_B^*} \right)}_{= \frac{dE(X_B^*, I_{1B}^*)}{dI_{1B}^*}} \frac{\partial I_{1B}^*}{\partial \bar{e}}. \tag{39}$$

Higher borrower equity loosens financial constraints, which allows the borrower to liquidate less, and therefore implies a higher final investment scale, $\frac{\partial I_{1B}^*}{\partial \bar{e}} > 0$. The direct effect of a higher investment scale is an increase in emissions, captured by the first term in brackets in Eq. (39). At the same time, looser financial constraints affect the optimal abatement choice. This effect is captured by the second term in brackets in Eq. (39). Note that this is an indirect effect that depends on how the marginal cost and benefit of abatement respond to changes in the final investment scale. As reflected in Eq. (33) the magnitude and direction of this effect depends on the cross-derivatives of $C(X, I_1)$ and $E(X, I_1)$, since $\frac{\partial^2 N(X,I_1,\tau)}{\partial X \partial I_1} = -\tau \frac{\partial^2 E(X,I_1)}{\partial X \partial I_1} + \frac{\partial^2 C(X,I_1)}{\partial X \partial I_1}$:

- The effect of higher borrower equity on abatement is positive if abatement is more efficient at a higher investment scale: i.e. if $\frac{\partial^2 N(X,I_1,\tau)}{\partial X \partial I_1} > 0$
- The effect of higher borrower equity on abatement is negative if abatement is less efficient at a higher investment scale: i.e. if $\frac{\partial^2 N(X,I_1,\tau)}{\partial X \partial I_1} < 0$

Internet Appendix
for
Too Levered for Pigou: Carbon Pricing, Financial Constraints, and Leverage Regulation

Robin Döttling and Magdalena Rola-Janicka

IA Internet Appendix

IA.1 Financial Instruments

IA.1.1 Hedging

With hedging as described in Section 6.1, the borrower's problem can be written as the following Lagrangian:

$$\max_{X_s, I_{1s}, d_1, e, h_s} \mathcal{L} = u(A_0 - e) + \sum_{s \in \{G,B\}} q_s \kappa_{c_{1s}} \left[d_{1s} + \mu(I_0 - I_{1s}) + e + h_s - I_0 - C(X_s, I_{1s}) \right]$$
$$+ \sum_{s \in \{G,B\}} q_s \left[\mu(I_0 - I_{1s}) + e + h_s - I_0 - C(X_s, I_{1s}) + R(I_{1s}, E_s^a) - \tau_s E(X_s, I_{1s}) + T_s \right]$$
$$+ \sum_{s \in \{G,B\}} q_s \left\{ \lambda_s \left[\theta R(I_{1s}, E_s^a) - \tau_s E(X_s, I_{1s}) + \psi T_s + h_s - d_{1s} \right] + \underline{\kappa}_{Is} I_{1s} + \overline{\kappa}_{Is} [I_0 - I_{1s}] \right\} \tag{IA.1}$$

The problem and first order conditions are equivalent to the problem in the main text (17), except that now additionally borrowers choose h_s subject to the fair pricing condition (14). Using (14) to substitute $h_B = -\frac{(1-q_B)h_G}{q_B}$, the first order condition w.r.t. h_G is given by

$$\lambda_G = \lambda_B.$$

Constrained Efficiency With hedging, the problem of a constrained social planner is similar to Eq. (25), but with h_s as an additional choice variable, analogous to the updated borrower problem (IA.1).

$$\begin{aligned} \max_{X_s, I_{1s}, d_{1s}, e, h_s} \mathcal{L} &= A_0^i + A_1^i + u(A_0 - e) + e - I_0 \\ &+ \sum_{s \in \{B,G\}} q_s \left\{ R(I_{1s}, E_s^a) + \mu(I_0 - I_{1s}) + h_s - 2\gamma_s^u E(X_s, I_{1s}) - C(X_s, I_{1s}) \right\} \\ &+ \sum_{s \in \{B,G\}} q_s \lambda_l^{SP} \left\{ \theta R(I_{1s}, E_s^a) + h_s + \mu(I_0 - I_{1s}) - C(X_s, I_{1s}) + e - I_0 \right\} \\ &+ \sum_{s \in \{B,G\}} q_s \left[\underline{\kappa}_{Is} I_{1s} + \overline{\kappa}_{Is} (I_0 - I_{1s}) \right]. \end{aligned} \tag{IA.2}$$

Using (14) to substitute $h_B = -\frac{(1-q)h_G}{q}$, the first order condition w.r.t. h_G is equivalent to the borrower's first order condition:

$$\lambda_G^{SP} = \lambda_B^{SP}.$$

All other first order conditions are the same as in the model without hedging. This implies the efficiency properties of the equilibrium allocation are the same as in the baseline model without hedging, as outlined in Proposition 5.

IA.1.2 External Equity

Some degree of hedging climate risks can also be achieved using external equity or long-term debt. Similar to hedging contracts, these alternative funding sources could enable a more efficient environmental policy if they bring down the shadow cost of the financial constraint in the bad state. However, the capacity to share risks using these contracts is more limited than that of climate-linked securities, as we show formally here.

Intuitively, equity financing provides less flexible risk sharing compared to hedging contacts because equity value is proportional to firm value, rather than flexibly designing the payoffs h_G and h_B to ensure $\lambda_B = \lambda_G$ (see Section 6.1 in the main paper).

To see this formally, suppose at $t = 0$ borrowers raise external equity financing e^{ext} by selling a fraction α of pledgeable firm value. Fair pricing of equity requires that

$$e^{ext} = \alpha \left[\sum_{s \in \{G,B\}} q_s \left(\theta R(I_{1s}, E_s^a) - \tau_s E(X_s, I_{1s}) + \psi T_s \right) \right].$$

This implies that borrower consumption at $t = 2$ is now given by

$$\begin{aligned} c_{2s}^b &= \left[R(I_{1s}, E_s^a) - \tau_s E(X_s, I_{1s}) + T_s \right] - \alpha \left[\theta R(I_{1s}, E_s^a) - \tau_s E(X_s, I_{1s}) + \psi T_s \right] \\ &= R(I_{1s}, E_s^a) - \tau_s E(X_s, I_{1s}) + T_s - e^{ext} \beta_s, \end{aligned}$$

where $\beta_s = \frac{\theta R(I_{1s}, E_s^a) - \tau_s E(X_s, I_{1s}) + \psi T_s}{\sum_{k \in \{G,B\}} q_k \left(\theta R(I_{1k}, E_k^a) - \tau_k E(X_k, I_{1k}) + \psi T_k \right)}$, with $\beta_H \geq 1 \geq \beta_L$. This implies that equity financing results in a transfer of $e^{ext}(\beta_H - \beta_L)$ from the good state to the bad state. Consequently, it is equivalent to an allocation where firms raise $d_0 = e^{ext}$ in debt financing and additionally write a hedging contract with $h_G = e^{ext}(1 - \beta_H) \leq 0$ and

$h_B = e^{ext}(1 - \beta_L) \geq 0$. The benefit of a hedging contract is that borrowers can flexibly design the payoffs h_G and h_B to ensure $\lambda_B = \lambda_G$ (see Section IA.1.1 above).

The efficiency results from our baseline model continue to hold when borrowers fund themselves with outside equity, whenever the resulting risk sharing does not achieve $\lambda_G = \lambda_B = 0$.

IA.1.3 Long-Term Debt

Long-term debt can only provide risk-sharing capacity if borrowers default on debt in the bad state as investors are compensated for that risk with a higher interest rate paid in the good state. As with equity financing, the risk-sharing achieved with long-term debt is less flexible than that with carbon price derivatives or climate-linked securities. Additionally, we show here that defaulting on long-term debt can result in a severe debt overhang problem that hinders abatement investments. This is in contrast to the baseline model with short-term debt, where borrowers optimally do not default (see Appendix A.2.2).

To see this formally, suppose borrowers can raise long-term debt d_{LT} at $t = 0$ due at $t = 2$, with an interest rate r_{LT} between $t = 0$ and $t = 2$. Borrowers can additionally raise short-term debt.

This appendix first shows that the allocation with risk-free long-term debt is equivalent to the one in the baseline model with short-term debt only. We then show that long-term debt may result in default in $s = B$ if the face value is high enough. While default allows for some risk-sharing by shifting repayments from the bad to the good state, we show below that risky long-term debt comes at the expense of exposing borrowers to a debt overhang problem that results in borrowers making no abatement investments.

Risk-free debt. We first consider the case in which the long-term debt is risk-free, so that the promised and realized repayment is $rd_{LT} = d_{LT}$. With long-term debt $I_0 = d_0 + d_{LT} + e$ and the $t = 2$ budget constraint of borrowers reads:

$$c^b_{2s} = R(I_{1s}, E^a_s) - \tau_s E(X_s, I_{1s}) - d_{1s} - r_s d_{LT} + T_s \tag{IA.3}$$

The borrower also faces an updated financial constraint:

$$d_{LT} + d_{1s} \leq \theta R(I_{1s}, E^a_s) - \tau_s E(X_s, I_{1s}) + \psi T_s, \tag{IA.4}$$

Taking these into account gives rise to the following Lagrangian:

$$\max_{X_s, I_{1s}, d_{1s}, d_{LT}, d_0} \mathcal{L} = u(A_0 - I_0 + d_0 + d_{LT})$$
$$+ \sum_{s\in\{G,B\}} q_s \left[\mu(I_0 - I_{1s}) - d_0 - C(X_s, I_{1s}) + R(I_{1s}, E_s^a) - \tau_s E(X_s, I_{1s}) + T_s - d_{LT}\right]$$
$$+ \sum_{s\in\{G,B\}} q_s \left\{\lambda_s \left[\theta R(I_{1s}, E_s^a) - \tau_s E(X_s, I_{1s}) + \psi T_s - d_{1s} - d_{LT}\right] + \underline{\kappa}_{Is} I_{1s} + \overline{\kappa}_{Is}[I_0 - I_{1s}]\right\}$$
$$+ \sum_{s\in\{G,B\}} q_s \kappa_{c_{1s}} \left[d_{1s} + \mu(I_0 - I_{1s}) - d_0 - C(X_s, I_{1s})\right], \tag{IA.5}$$

The FOCs wrt X_s, I_{1s} and d_{1s} are the same as in the original problem. The FOC wrt d_0 and d_{LT} read, respectively,

$$u'(c_0^b) - 1 - q_G\lambda_G - q_B\lambda_B = 0$$
$$u'(c_0^b) - q_G\lambda_G - q_B\lambda_B - 1 = 0$$

Thus, the borrower chooses inside equity e to satisfy $u'(c_0^b) = 1 + q_G\lambda_G + q_B\lambda_B$, and is indifferent between short-term and long-term debt if long-term debt is risk-free. This implies that the allocation with risk-free long-term debt is equivalent to the allocation with short-term debt in the baseline model.

Risky debt. The possibility of default on long-term debt makes the repayment of debt state-contingent. As investors need to break even, they will charge a higher interest rate, thereby allowing borrowers to reallocate resources from $s = G$ to $s = B$.

Anticipating the default, investors are not willing to provide short-term debt at $t = 1$ in $s = B$, so that previous period short-term debt repayment and abatement investments must be funded by liquidations $C(X_B, I_{1B}) + d_0 = \mu(I_0 - I_{1B})$. Using this, the borrower's optimal choice of abatement and liquidation at $t = 1$ in $s = B$, conditional on defaulting at $t = 2$ solves:

$$\max_{X_B, I_{1B}} \mathcal{L} = (1-\theta)R(I_{1B}, E_B^a) + (1-\psi)T_B + \mu(I_0 - I_{1B}) - d_0 - C(X_s, I_{1B})$$
$$+ \underline{\kappa}_{IB} I_{1B} + \overline{\kappa}_{IB}[I_0 - I_{1B}] + \kappa_{c1B}[\mu(I_0 - I_{1B}) - d_0 - C(X_s, I_{1B})],$$

The FOCs wrt X_B, I_{1B} is:

$$(1+\kappa_{c1B})\frac{\partial C(X_B, I_{1B})}{\partial X_B} = 0 \qquad \text{(IA.6)}$$

$$(1-\theta)\rho + \underline{\kappa}_{IB} - \overline{\kappa}_{IB} + (1+\kappa_{c1B})\left(-\mu - \frac{\partial C(X_B, I_{1B})}{\partial I_{1B}}\right) = 0 \qquad \text{(IA.7)}$$

Thus, in $s = B$ the borrower chooses $X_B^{*d} = 0$. The borrower chooses minimum liquidations needed to repay the $t = 0$ short term debt, $\mu(I_0 - I_{1B}^{*d}) = d_0$, whenever $(1-\theta)\rho - \mu > 0$. If $(1-\theta)\rho - \mu < 0$ the borrower liquidates all assets at $t = 1$ and consumes $c_1 = \mu I_0 - d_0$, leaving nothing to the long-term creditors at $t = 2$. In this case, risky long-term debt results in a severe debt overhang problem that induces borrowers to not invest in abatement at all.

Next suppose the borrower does not default at $t = 2$ in a given state s. In this case, the choice of abatement and investment at $t = 1$ follow from

$$\begin{aligned} &\max_{X_s, I_{1s}, d_{1s}} \mathcal{L} = \left[\mu(I_0 - I_{1s}) - d_0 - C(X_s, I_{1s}) + R(I_{1s}, E_s^a) - \tau_s E(X_s, I_{1s}) + T_s - d_{LT}\right] \\ &\lambda_s \left[\theta R(I_{1s}, E_s^a) - \tau_k E(X_s, I_{1s}) + \psi T_s - d_{1s} - r d_{LT}\right] + \underline{\kappa}_{Is} I_{1s} + \overline{\kappa}_{Is}[I_0 - I_{1s}] \\ &\kappa_{c_{1s}} \left[d_{1s} + \mu(I_0 - I_{1s}) - C(X_s, I_{1s}) - d_0\right], \end{aligned} \qquad \text{(IA.8)}$$

The FOCs wrt d_{1s} pins down $\lambda_s = \kappa_{c1s}$, and those wrt X_s, I_{1s} are:

$$(1+\lambda_s)\frac{\partial C(X_s, I_{1s})}{\partial X_s} - (1+\lambda_s)\tau\frac{\partial E(X_s, I_{1s})}{\partial X_s} = 0 \qquad \text{(IA.9)}$$

$$\rho(1+\theta\lambda_s) - (1+\lambda_s)\left[\mu + \frac{\partial C(X_s, I_{1s})}{\partial I_{1s}} + \tau\frac{\partial E(X_s, I_{1s})}{\partial I_{1s}}\right] + \underline{\kappa}_{Is} - \overline{\kappa}_{Is} = 0 \qquad \text{(IA.10)}$$

With the complementary slackness constraint:

$$\theta R(I_{1s}, E_s^a) - \tau_k E(X_s, I_{1s}) + \psi T_s + \mu(I_0 - I_{1s}) - C(X_s, I_{1s}) - r d_{LT} - d_0 = 0 \qquad \text{(IA.11)}$$

Thus, the choice of abatement and liquidations corresponds to the one in the benchmark where d_0 is substituted by $d_0 + r d_{LT}$. Let these choices be denoted by X_s^* and I_{1s}^*

Comparing the payoffs earned in the case of default and non-default in $s = B$, we can find the level of long-term debt at which the debt is indeed risky. If $(1-\theta)\rho > \mu$, this level is given by:

$$d_0 + rd_{LT} >$$
$$\rho I^*_{1B} - (1-\theta)\rho I^{*d}_{1B} - \theta\gamma^p_B E^a_B + \psi T_B + \mu(I_0 - I^*_{1B}) - C(X^*_B, I^*_{1B}) - \tau_B E(X^*_B, I^*_{1B}) = \hat{d}_{LT}$$

If $(1-\theta)\rho < \mu$ this level is given by:

$$rd_{LT} > \rho I^*_{1B} - \gamma^p_B E^a_B + T_B - \mu I^*_{1B} - C(X^*_B, I^*_{1B}) - \tau_B E(X^*_B, I^*_{1B}) = \hat{d}_{LT}$$

Focusing on the case when $(1-\theta)\rho > \mu$, the most that the lender can recover from the borrower in the case of default is $\theta R(I^{*d}_{1B}, E^a_B, \gamma^p_B) + \psi T_B - \tau_B E(X^{*d}_B, I^{*d}_{1B})$. Thus, the lender's participation constraint requires that:

$$d_{LT} \leq q_G r d_{LT} + q_B[\theta R(I^{*d}_{1B}, E^a_B) + \psi T_B - \tau_B E(X^{*d}_B, I^{*d}_{1B})]$$

If $(1-\theta)\rho < \mu$ long-term lender's participation constraint is:

$$d_{LT} \leq q_G r d_{LT}$$

In both cases, the participation constraint of the lender implies that the risk of default and the inefficient abatement and/or liquidation choices at $t = 1$ must be compensated with a sufficiently high interest rate paid to the lender.

Risk sharing vs debt overhang The potential gains from risk-sharing permitted by the risky long-term debt come at the expense of exposing borrowers to a debt overhang problem. As shown above, in the bad state borrowers abscond with resources at $t = 2$, and therefore no longer have incentives to maximize the project's value. As a result, they choose not to engage in any abatement, as the emissions tax bill is paid out of the pledgeable income and thus does not affect borrowers' payoff under default.

In equilibrium investors price in the cost of debt-overhang, demanding a high compensation for holding the long-term debt. Thus, any gains from insurance due to using risky long-term debt come at a premium relative to hedging contracts or external equity.

IA.1.4 Socially Responsible Investing

This appendix section solves the model with socially responsible investors, as described in Section 6.2. We assume that each borrower matches with 1 investor and all investors are socially responsible, with their utility given by:

$$U^i = c_0^i + c_{1s}^i + c_{2s}^i - \gamma_s^u E_s^a - (\omega_0 \mathbb{I}_{d_0^b} + \omega_1 \mathbb{I}_{d_1^b}) E(X_s^b, I_{1s}^b) \tag{IA.12}$$

Where $\mathbb{I}_{d_0^b}$ and $\mathbb{I}_{d_1^b}$ are indicator functions taking the value of 1 if the investor lends to the borrower at $t = 0$ and $t = 1$ respectively. Thus, investors' break-even conditions for lending to borrower b are given by:

$$\begin{aligned} d_0^b =& r_0 d_0^b - \omega_0 \mathbb{E}[E(X^b, I_1^b)] \\ d_1^b =& r_1 d_1^b - \omega_1 E(X_s^b, I_{1s}) \end{aligned}$$

where r_0 and r_1 are the gross interest rates on $t = 0$ and $t = 1$ debt respectively, that compensate SRIs for their disutility from investing in a polluting firm.

In the presence of socially responsible investors, borrower's constraints become:

$$\begin{aligned} c_0^b &= A_0 - (I - d_0) \geq 0, \\ c_{1s}^b &= (I_0 - I_{1s})\mu + d_{1s} - d_0 - \omega \mathbb{E}[E(X, I_1)] - C(X_s, I_{1s}) \geq 0, \\ c_{2s}^b &= R(I_{1s}, E_s^a) - \tau_s E(X_s, I_{1s}) - d_{1s} - \omega E(X_s^b, I_{1s}) + T_s \geq 0, \\ I_{1s} &\in [0, I_0]. \end{aligned}$$

The re-stated financial constraint,

$$d_{1s} + \omega E(X_s^b, I_{1s}) \leq \theta R(I_{1s}, E_s^a) - \tau_s E(X_s, I_{1s}) + \psi T_s,$$

corresponds to the updated complementary slackness condition in Eq. (8") in the main text.

The Lagrangian of the problem is now:

$$\max_{X_s, I_{1s}, d_{1s}, d_0} \mathcal{L} = u(A_0 - I_0 + d_0)$$
$$+ \sum_{s\in\{G,B\}} q_s \left[\mu(I_0 - I_{1s}) + d_{1s} - d_0 - \omega\mathbb{E}[E(X, I_1)] - C(X_s, I_{1s})\right]$$
$$+ \sum_{s\in\{G,B\}} q_s \left[R(I_{1s}, E_s^a) - \tau_s E(X_s, I_{1s}) + T_s - d_{1s} - \omega E(X_s, I_{1s})\right]$$
$$+ \sum_{s\in\{G,B\}} q_s \lambda_s \left[\theta R(I_{1s}, E_s^a)) - \tau_s E(X_s, I_{1s}) + \psi T_s - d_{1s} - \omega E(X_s, I_{1s})\right] + \underline{\kappa}_{Is} I_{1s}$$
$$+ \sum_{s\in\{G,B\}} q_s \left\{\kappa_{c_{1s}} \left[d_{1s} + \mu(I_0 - I_{1s}) - d_0 - \omega\mathbb{E}[E(X, I_1)] - C(X_s, I_{1s})\right] + \overline{\kappa}_{Is}[I_0 - I_{1s}]\right\},$$

The FOCs w.r.t. d_{1s}, X_s, I_{1s}, d_0 are:

$$\begin{aligned}
-\lambda_s + \kappa_{c1s} &= 0 \\
-(1+\lambda_s)\frac{\partial C(X_s, I_{1s})}{\partial X_s} - (1+\lambda_s)(\tau_s + \omega)\frac{\partial E(X_s, I_{1s})}{\partial X_s} &= 0 \\
\rho(1+\theta\lambda_s) - (1+\lambda_s)[\mu + \frac{\partial C(X_s, I_{1s})}{\partial I_{1s}} + (\tau_s + \omega)\frac{\partial E(X_s, I_{1s})}{\partial I_{1s}}] &= 0 \\
u'(c_0^b) - 1 - \sum_{s\in\{G,B\}} q_s \lambda_s &= 0
\end{aligned}$$

The FOC for X_s corresponds to Eq. (6”) in the main text. The other FOCs are similar to those in the baseline model, except for the presence of ω. Comparing the FOCs to the baseline model, they are equivalent if in the baseline model $\tau_s = \omega$ (and $\tau_s = 0$ in the model with socially responsible investors).

IA.2 Alternative policy implementation

IA.2.1 Cap-and-trade: sale proceeds to borrowers

In the baseline analysis of the cap-and-trade system in Section 4.3 in the main text we assume that the proceeds from the sale of permits are redistributed to investors. If the proceeds from sale were distributed to borrowers in the form of a lump-sum rebate $T_s = (1-\phi)Q_s p_s$ to the borrower, the $t = 2$ budget constraint and the financial constraints would be:

$$c_{2s}^b = R(I_{1s}, E_s^a) - (1-\phi)Q_s p_s + p_s(Q_s - E(X_s, I_{1s})) + T_s - d_{1s} \geq 0, \tag{3"}$$

$$d_{1s} \leq \theta R(I_{1s}, E_s^a) + \psi T_s. \tag{4"}$$

The private FOC's are unaffected by the rebate. The regulator's FOC is only altered through the change in $\frac{\partial I_{1s}^*}{\partial p_s}$ which now reads:

$$\frac{\partial I_{1s}^*}{\partial p_s} = \frac{(1-\phi-\psi-\phi\psi)E(X_s^*, I_{1s}^*) - ((\phi+\psi+\phi\psi)p_s - \theta\gamma_s^p)\frac{\partial E(X_s^*, I_{1s}^*)}{\partial X_s^*}\frac{\partial X_s^*}{\partial p_s}}{\tilde{r}(p_s(1-\phi-\psi-\phi\psi) - \theta\gamma_s^p, X_s^*, I_{1s}^*)} \tag{19"}$$

The equivalence between the emissions taxes and the cap-and-trade solution holds now if and only if $\phi + \psi - \phi\psi = \psi$. This implies that, as is the case in the baseline, whem $\phi = 1$ the cap-and-trade solution corresponds to the emissions taxes solution with $\psi = 1$.

IA.2.2 Implementation of the Capital Mandate through Taxes on Leverage

This appendix shows that a capital mandate $\bar{e}$ derived in Section 5 can alternatively be implemented through a tax τ_d on $t = 0$ debt (or a subsidy if $\tau_d < 0$). Given that capital requirements in the Basel Accord apply to financial institutions, leverage taxes and subsidies may be a more likely tool seen in the real world if borrowers in the model are interpreted as non-financial firms (such as manufacturing firms). Tax proceeds are fully rebated to borrowers via a lump-sum rebate T_0^b.

With a leverage tax τ_d, the $t = 0$ budget constraint is given by $I_0 = e + d_0(1-\tau_d) + T_0^b$, which can be re-arranged to $d_0 = \frac{I_0 - e - T_0^b}{(1-\tau_d)}$. With this budget constraint, the borrower's problem (17) is now given by the following Lagrangian:

$$\begin{aligned}
\max_{X_s, I_1, d_1, e} \ & \mathcal{L} = u(A_0 - e) \\
& + \sum_{s\in\{G,B\}} q_s \left[\mu(I_0 - I_{1s}) - \frac{I_0 - e - T_0^b}{1-\tau_d} - C(X_s, I_{1s}) + R(I_{1s}, E_s^a) - \tau_s E(X_s, I_{1s}) + T_s\right] \\
& + \sum_{s\in\{G,B\}} q_s \left\{\lambda_s \left[\theta R(I_{1s}, E_s^a) - \tau_s E(X_s, I_{1s}) + \psi T_s - d_{1s}\right] + \underline{\kappa}_{Is} I_{1s} + \overline{\kappa}_{Is}[I_0 - I_{1s}]\right\} \\
& + \sum_{s\in\{G,B\}} q_s \kappa_{c_{1s}} \left[d_{1s} + \mu(I_0 - I_{1s}) - \frac{I_0 - e - T_0^b}{1-\tau_d} - C(X_s, I_{1s})\right],
\end{aligned} \tag{IA.13}$$

The first order conditions with respect to X_s and I_{1s} are equivalent to those in the main text and given by (6) and (7), respectively. By contrast, the first order condition with respect to equity e is different from the main text Eq. (9), and is now given by

$$u'(A_0 - e) = \frac{1 + (1-q)\lambda_G + q\lambda_B}{1 - \tau_d}.$$

From this equation it is clear that a higher tax on debt induces borrowers to choose a higher level of e, i.e., lower leverage. By fully rebating the taxes, such that $T_0^b = \tau_d d_0$, a regulator can ensure that the tax does not affect any constraints. Consequently, a leverage mandate $\bar{e}^*$ can be implemented by setting a leverage tax τ_d^* such that

$$u'(A_0 - \bar{e}^*) = \frac{1 + (1-q)\lambda_G + q\lambda_B}{1 - \tau_d^*}.$$

IA.3 Interpretation of Borrowers as Financial Institutions

This appendix derives a version of the model in which borrowers are banks that make loans to non-financial firms. A continuum of firms run by risk-neutral owners have access to the same investment project as described in Section 2. Firms have no own funds and must obtain a loan from a bank. Banks have the same preferences and the same limited endowment A_0 as borrowers in the baseline model. Banks can also raise financing from investors as in the baseline model. In contrast, each firm is matched with a bank and can only obtain financing through a loan from its bank, i.e., firms cannot obtain funding from other investors or banks. There is no friction between a firm and its bank, but banks are constrained by the same financial constraint (4) as borrowers in the baseline model. That is, banks can fully seize the firm's assets at $t = 2$ but can only pledge $\theta R(I_{1s}, E_s^a)$ of the seized asset returns to outside investors. In this version of the model, "borrowers" are split into a financial and a real sector, where banks finance loans to bank-dependent firms through bank equity and outside financing, and firms use loans to finance real investment and abatement. We assume that firm owners are risk-neutral and bank owners have the same quasi-linear utility as borrowers in the baseline model. For simplicity, we focus on the case $\psi = 0$.

Firm problem. Banks make a take-it-or-leave-it offer to firms, offering a loan l_t at $t = 0$ and $t = 1$, and repayment D due at $t = 2$. Firms can decide to accept or reject the loan but conditional on accepting take l_t and D as given. When rejecting the loan, the outside option for firms is not to finance the project.

Firms have no own funds, so that $I_0 = l_0$. At $t = 1$ firms can liquidate some initial investment to generate a liquidation value $\mu(I_0 - I_{1s})$, and invest in abatement X_s at a cost $C(X_s, I_{1s})$. Firm owner's consumption is given by

$$
\begin{aligned}
c_0^f &= l_0 - I_0 \\
c_{1s}^f &= \mu(I_0 - I_{1s}) - C(X_s, I_{1s}) + l_{1s} \\
c_{2s}^f &= R(I_{1s}, E_s^a, \gamma_s^p) - \tau E(X_s, I_{1s}) + T_s - D
\end{aligned}
$$

The firm's problem is to choose I_{1s} and X_s so as to maximize $c_0^f + c_1^f + c_2^f$ subject to $I_0 \geq I_{1s} \geq 0$ and non-negativity constraints on consumption. This problem can be written as follows:

$$
\begin{aligned}
&\max_{X_s, I_{1s}, l_{1s}, l_0} \mathcal{L} = l_0 - I_0 \\
&+ \sum_{s \in \{G,B\}} q_s \left[R(I_{1s}, E_s^a) - \tau_s E(X_s, I_{1s}) + T_s - D + l_{1s} + \mu(I_0 - I_{1s}) - C(X_s, I_{1s})\right] \\
&+ \kappa_{c_0^f}(l_0 - I_0) + \sum_{s \in \{G,B\}} q_s \kappa_{c_{1s}^f}\left[\mu(I_0 - I_{1s}) - C(X_s, I_{1s}) + l_{1s}\right] \\
&+ \sum_{s \in \{G,B\}} q_s \left[\kappa_{c_{2s}^f}\left[R(I_{1s}, E_s^a) - \tau_s E(X_s, I_{1s}) + T_s - D\right] + \underline{\kappa}_{Is} I_{1s} + \overline{\kappa}_{Is}(I_0 - I_{1s})\right].
\end{aligned}
\tag{IA.14}
$$

The first order conditions with respect to I_{1s} and X_s are, respectively,

$$
(1 + \kappa_{c_{2s}^f})\left(\rho - \tau \frac{\partial E(X_s, I_{1s})}{\partial I_{1s}}\right) - (1 + \kappa_{c_{1s}^f})\left(\mu + \frac{\partial C(X_s, I_{1s})}{\partial I_{1s}}\right) + \underline{\kappa}_{Is} - \overline{\kappa}_{Is} = 0,
\tag{IA.15}
$$

$$
-\tau_s \frac{\partial E(X_s, I_{1s})}{\partial X_s} - \frac{\partial C(X_s, I_{1s})}{\partial X_s} = 0.
\tag{IA.16}
$$

The first order condition with respect to X_s is the same as in the baseline model, cf. Eq. (6). By Assumption 2 (liquidations are inefficient) and the fact that $\kappa_{c_2^f s} \geq 0$, it also follows that $(1 + \kappa_{c_2^f s})\left(\rho - \tau \frac{\partial E(X_s, I_{1s})}{\partial I_{1s}}\right) - \left(\mu + \frac{\partial C(X_s, I_{1s})}{\partial I_{1s}}\right) > 0$. This implies that either $\overline{\kappa}_{Is} > 0$ or $\kappa_{c_1^f} > 0$, so that I_{1s} is either $I_{1s} = I_0$ or is pinned down by $c_1^f = 0$, which defines $I_{1s}(l_{1s})$.

Bank problem. The bank chooses l_0, l_{1s}, D, d_{1s} and d_0, subject to the financial constraint (4).

$$c_0 = A - e$$
$$c_1 = d_{1s} - d_0 - l_{1s}$$
$$c_2 = D - d_{1s}$$

Firm participation requires that $c_t^f \geq 0$. Banks optimally choose D, l_{1s} and l_0 such that the participation constraints bind, which implies $l_0 = I_0 = e + d_0$, $l_{1s} = -\mu(I_0 - I_{1s}) + C(X_s, I_{1s})$, and $D = R(I_{1s}, E_s^a) - \tau E(X_s, I_{1s}) + T_s$.

If the firm's investment is pinned down by $I_{1s}(l_{1s})$ (defined by $c_1^f = 0$), the bank's problem can be expressed as:

$$\begin{aligned}
&\max_{l_{1s}, d_{1s}, e} \mathcal{L} = u(A-e) - I_0 + e \\
&+ \sum_{s \in \{G,B\}} q_s \left[\mu(I_0 - I_{1s}(l_{1s})) - C(X_s, I_{1s}(l_{1s})) + R(I_{1s}(l_{1s}), E_s^a) - \tau_s E(X_s, I_{1s}(l_{1s})) + T_s\right] \\
&+ \sum_{s \in \{G,B\}} q_s \lambda_s \left(\theta R(I_{1s}(l_{1s}), E_s^a) - \tau_s E(X_s, I_{1s}(l_{1s})) - d_{1s}\right) + \kappa_{c_0}(A - e) \\
&+ \sum_{s \in \{G,B\}} q_s \left[\kappa_{c_{1s}} \left(d_{1s} - I_0 + e + \mu(I_0 - I_{1s}(l_{1s})) - C(X_s, I_{1s}(l_{1s}))\right)\right] \\
&+ \sum_{s \in \{G,B\}} q_s \left[\kappa_{c_{2s}} \left(R(I_{1s}(l_{1s}), E_s^a) - \tau_s E(X_s, I_{1s}(l_{1s})) + T - d_{1s}\right)\right].
\end{aligned} \tag{IA.17}$$

The first order conditions read:

$$u'(A-e) = 1 - \kappa_{c_0} + q_G \kappa_{c_{1G}} + q_B \kappa_{c_{1B}} \tag{IA.18}$$

$$\kappa_{c_{1s}} - \kappa_{c_{2s}} - \lambda_s = 0 \tag{IA.19}$$

$$-(1 + \kappa_{c_{1s}}) \left(\mu + \frac{\partial C}{\partial I_{1s}}\right) + (1 + \kappa_{c_{2s}}) \left(\frac{\partial R}{\partial I_{1s}} - \tau_s \frac{\partial E}{\partial I_{1s}}\right) + \lambda_s \left(\theta \rho - \tau_s \frac{\partial E}{\partial I_{1s}}\right) = 0 \tag{IA.20}$$

Due to the assumptions on $u'(c_0)$, it is never optimal to have $A - e = 0$, so $\kappa_{c_0} = 0$. Because $d_{1s} \leq \theta R(I_{1s}, E_s^a) - \tau_s E(X_s, I_{1s})$, $c_{2s} > 0$ and $\kappa_{c_{2s}} = 0$. It follows that $\lambda_s = \kappa_{c_{1s}} > 0$, so the FOCs simplify to:

$$u'(c_0) = 1 + q_G\lambda_G + q_B\lambda_B \tag{IA.21}$$

$$\lambda_s = -\frac{r(\tau_s, X_s, I_{1s})}{\tilde{r}(\tau_s, X_s, I_{1s})} \tag{IA.22}$$

which are the same as the conditions as (9) and (7) in the baseline model (with $\bar{\kappa}_{Is} = \underline{\kappa}_{Is} = 0$. Since also Eq. (IA.16) is equivalent to Eq. (6), in this case all first order conditions and therefore the equilibrium allocations are the same as in the baseline model.