# Common Risk Factors in the Returns on Stocks, Bonds (and Options), Redux ${ }^{\dagger}$ 

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#### Abstract

Are there risk factors that are pervasive across all major classes of corporate securities, including stocks, bonds, and options? We employ a novel procedure that builds on the ability of asset characteristics to capture the dynamics of asset returns to estimate a conditional latent factor model. A common risk factor structure prominently emerges across asset classes. The first factor that corresponds to the dominant principal component of the joint cross section significantly explains a substantial component of time-series variation of individual asset returns across all three asset classes, has a Sharpe ratio over twice that of the stock market. Other common factors that are less pervasive, i.e. describe a smaller portion of common variation in returns over time. Some of the common factors highly correlated with some of asset-class-specific factors as well as several macroeconomic and financial variables. However, we also document that the factor structure does not fully capture the cross-section of average returns. Portfolios that have zero loadings on the top latent risk factors can earn substantial Sharpe ratios, with different asset classes hedging each other's exposures to the common factors.


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## 1 Introduction

Finance theory predicts a tight connection between the main types of corporate securities, such as equity, equity options, and bonds (Black and Scholes, 1973; Merton, 1974). Close relationship of expected returns to assets' loadings on pervasive common factors (Ross, 1976) forms another pillar of financial economics. And yet factors common to all of these asset classes that also explain a substantial amount of variation in expected returns have been elusive. Consequently, much of the asset pricing literature has pursued factors that are specific to a particular asset class. ${ }^{1}$

In this paper, we employ a novel econometric approach to extracting latent factors directly from individual asset returns by employing the predictive power of well-known asset characteristics. The key advantage of this approach as it allows working with short time series (especially important for securities with finite maturities, such as bonds, and, especially, options) and large cross-sections. The Regressed-PCA (RPCA) approach proposed by Chen, Roussanov, and Wang (2022) employs frontier econometric tools yet can be implemented and interpreted using familiar cross-sectional regressions of Fama and MacBeth (1973) as well as principal component analysis. This semi-parametric method allows us to extract latent factors directly from the large panel of individual assets from different asset classes. The time-varying factor exposures as well as the pricing errors are modeled as a function of observable asset characteristics. The regressed-PCA translates the large-dimensional (and unbalanced) panel individual assets into a much smaller set of characteristic-managed portfolios constructed via period-by-period Fama-MacBeth regressions (Fama and MacBeth, 1973). It then applies standard principal component analysis to these characteristic-managed portfolios. Chen, Roussanov, and Wang (2022) show that the regressed-PCA displays attractive large- $N$ (cross-section) asymptotic properties even when the time series $T$ dimension is relatively small, making it particularly useful for studying large cross-sections of asset returns.

We are able to extract the common factors directly from individual assets across different classes jointly, or consider asset-class-specific factors. We focus on the time period during which data for all three asset classes are easily available. Therefore, the sample period of the monthly data analyzed in this paper is from June 2004 to December 2021. We include

[^1]35 characteristics for stocks, 26 characteristics for corporate bonds, and 19 characteristics for options in our empirical analysis. The characteristics are both firm- or security-specific. We start from asset-class-specific characteristic-managed portfolios of individual assets and then extract the joint latent factors from these portfolios.

Our key result is the strong commonality across the different asset classes. In particular, the "first" regressed-PCA latent factor (i.e. the one corresponding to the principal component that explains the largest share of common variability in returns across the three asset classes jointly) behaves as as a truly "common" risk factor. First of all, we find that the first regressed-PCA factor is highly correlated with the the first principal component of a large set of "observable" pricing factors proposed by the extant literature, most of them specific to a particular asset class. Moreover, the first latent factor is significantly correlated with fifteen out of eighteen asset-class-specific observable factors we study, for example, the market factors for the three asset classes, stock momentum factor, the corporate bond credit risk factor, and the idiosyncratic volatility factor in options. The observable factors together can explain more than half of the variation in the first joint factor. Second, the first joint latent factor correlates highly with the leading latent factors that are extracted from each asset class in isolation. Third, the first joint factor is related to several macroeconomic variables and, most notably, has a correlation of 0.48 with the intermediary capital factor of He, Kelly, and Manela (2017). And lastly, the first joint factor provides both a good in-sample fit and strong out-of-sample predictability of returns in each of the asset classes. These results all indicate an apparent existence of a common factor across different asset classes.

Although different asset classes exhibit a common factor structure, the risk factors alone do not fully capture the cross-sectional returns across these assets. The "arbitrage portfolio" that is based on the estimated conditional "alpha" function, i.e. exploits the ability of characteristics to predict returns but has zero loadings on the joint common factors generates a substantial Sharpe ratio. We also find that the out-of-sample Sharpe ratio does not decline when more factors are included, suggesting that the high average return on the pure-alpha strategy is not simply attributed to model misspecification. Indeed, a model specification test shows that the pricing errors are significantly non-zero, even with ten risk factors included. By considering the characteristics' weights of the pure-alpha portfolio we show that the portfolio loads heavily on the characteristics emanating from options, such as option implied volatility and option gamma. This evidence suggests that pricing in option markets is less "efficient" than in equity markets, where most of the "abnormal" return predictability with characteristics has been driven out by arbitrageurs over time.

Our paper speaks to a voluminous empirical asset pricing literature that studies the joint cross-section of multiple asset classes. Gebhardt, Hvidkjaer, and Swaminathan (2005b) also suggests that equity momentum spills over to the corporate bond market. Other papers such as Chordia, Goyal, Nozawa, Subrahmanyam, and Tong (2017) and Choi and Kim (2018) find evidence suggesting segmentation between equity and bond markets. Bali, Goyal, Huang, Jiang, and Wen (2022) shows that the predictability of bond returns using equity characteristics significantly improves when the Merton model is explicitly imposed, suggesting that bond-equity integration is not captured by the reduced-form approach without restrictions. Cao and Han (2013), Bali, Beckmeyer, Moerke, and Weigert (2021) and Christoffersen, Goyenko, Jacobs, and Karoui (2018) show that the stock-level characteristics such as idiosyncratic volatility with respect to the Fama-French three-factor, momentum, stock illiquidity among others could predict stock option returns. At the same time, Bali and Hovakimian (2009), Johnson and So (2012) and Xing, Zhang, and Zhao (2010) present evidence that the volatility spread, volatility smirk and option to stock volume ratio help predict future stock returns. Furthermore, Goyenko and Zhang (2021) illustrate that the option characteristics are dominant predictor of stock returns and indicate that the options markets lead the stock market. Beyond their advantage in predicting the stock market, some option characteristics also provide information for predicting the corporate bond returns. Cao, Goyal, Xiao, and Zhan (2022) demonstrate that importance of the information related to the default risk which is embedded in the option volatility in predicting corporate bond returns. Across multiple asset classes, studies such as He, Kelly, and Manela (2017) and Lettau, Maggiori, and Weber (2014) suggest that the intermediary and downside risk factors are significant, although they are questioned by Gospodinov and Robotti (2021). Lin, Wang, and Wu (2011) show that liquidity risk is priced in the corporate bonds. Bali, Subrahmanyam, and Wen (2021) find that long-term reversal factor carries a sizable premium in corporate bond markets, which can be related to investors' ex-ante risk assessment and institutional constraints. Elkamhi, Jo, and Nozawa (2022) propose a one-factor model related to long-run consumption risk. Kelly, Palhares, and Pruitt (2022) propose a 5 -factor IPCA model to explain the corporate bond returns. Recent studies in the option pricing literature building on the noarbitrage parametric option pricing models (Duffie, Pan, and Singleton, 2000, etc.) study what characteristics or factors could explain or predict the cross-section of option returns directly. Some focus on the specific characteristics (Goyal and Saretto, 2009; Zhan, Han, Cao, and Tong, 2022; Frazzini and Pedersen, 2021). While some others propose the factor structure to understand the cross-section of option returns. For example, Karakaya (2013)
suggests a three-factor model with a level, slope and value factor to explain the cross-section of delta-hedged individual equity option returns. Christoffersen, Fournier, and Jacobs (2018) apply the principal components of equity volatility, skews and term structures to explain the cross-section of the option prices (Black-Scholes implied volatility). Horenstein, Vasquez, and Xiao (2020) exploit the asymptotic principal component analysis to study the common factor structure of around 100 equity option portfolios. Büchner and Kelly (2022) use the instrumented principal components analysis to explore the latent factor structure of index options. Our paper specifically contributes to this strand of option literature by extracting the latent factors that could capture the common variation in individual asset returns on equity options.

The rest of the paper is organized as follows. Section 2 summarizes the factor model, the model estimation procedure - the regressed-PCA, and the model evaluation metrics used in the analysis. Section 3 introduces the data. Section 4 presents the main results which analyze the extracted latent joint factors. Section 5 discusses the role of the common risk factors in assets' returns. Section 6 concludes the paper.

## 2 Methodology

In this section, we present the general factor models for individual assets' excess returns and adapt it to extract joint latent factors from the returns on stocks, corporate bonds and options. The models can be used to perform on any single asset class as well. To estimate the conditional factor model, we employ the methodology proposed in Chen, Roussanov, and Wang (2022), which is also known as regressed-PCA. We introduce various measures of fit and predictability to assess the empirical performance of the factor models. Based on the estimation, we further construct the arbitrage (pure-alpha) and mean-variance efficiency portfolio (MVE) or beta trading strategies.

### 2.1 Model

Following Chen, Roussanov, and Wang (2022) and Kelly, Pruitt, and Su (2019), we consider the following factor model, for $i=1, \ldots, N$ and $t=1, \ldots, T$,

$$
\begin{equation*}
r_{i, t}=\alpha\left(z_{i, t-1}\right)+\beta\left(z_{i, t-1}\right)^{\prime} f_{t}+\epsilon_{i, t}, \tag{2.1}
\end{equation*}
$$

where $r_{i, t}$ is the monthly excess return of asset $i$ at time $t$, and $f_{t}$ is the $K \times 1$ vector of latent factors. As in Chen, Roussanov, and Wang (2022) and Kelly, Pruitt, and Su (2019), we also specify both the factor loadings $\beta(\cdot)$ and the mispricing errors $\alpha(\cdot)$ are the functions of $z_{i, t-1}$, which is a $J \times 1$ vector summarizing the observable time-varying characteristics of asset $i$ at time $t-1$, and $\epsilon_{i, t}$ is the idiosyncratic error term. Here, we do not necessarily require a balanced panel, that is to say, the sample size of each period is allowed to be time-varying (the number of assets $N$ is time-varying). This is particularly important for options as they have varying and mostly short life spans. The model in (2.1) allows us to disentangle the mispricing errors (alpha) and common variations (beta). In contrast, a restricted version of this model which sets $\alpha(\cdot)=0$ only allows characteristics to explain the risk exposures.

We next introduce the specifications of the mispricing errors $\alpha(\cdot)$ and the dynamic factor loadings $\beta(\cdot)$. In this paper, we mainly focus on the linear approximation of unknown functional forms of $\alpha\left(z_{i, t-1}\right)$ and $\beta\left(z_{i, t-1}\right)$ in (2.1). Specifically, we assume that $\alpha\left(z_{i, t}\right)$ and $\beta\left(z_{i, t}\right)$ are approximated by

$$
\begin{align*}
& \alpha_{i, t}=a^{\prime} z_{i, t}+\eta_{\alpha, i, t}, \\
& \beta_{i, t}=B^{\prime} z_{i, t}+\eta_{\beta, i, t}, \tag{2.2}
\end{align*}
$$

where $z_{i, t}=\left(1, z_{i, t, 1}, \ldots, z_{i, t, J}\right)^{\prime}$, and $a_{(J+1) \times 1}$ and $B_{(J+1) \times K}$ are the corresponding loading coefficients on characteristics, $\eta_{\alpha}\left(z_{i, t}\right)$ and $\eta_{\beta}\left(z_{i, t}\right)$ are the approximation errors.

Letting $R_{t} \equiv\left(r_{1, t}, \ldots, r_{N, t}\right)^{\prime}, Z_{t-1} \equiv\left(z_{1, t-1}, \ldots, z_{N, t-1}\right)^{\prime}, \varepsilon_{t} \equiv\left(\epsilon_{1, t}, \ldots, \epsilon_{N, t}\right)^{\prime}, H_{\alpha, t-1} \equiv$ $\left(\eta_{\alpha, 1, t-1}, \ldots, \eta_{\alpha, N, t-1}\right)^{\prime}$ and $H_{\beta, t-1} \equiv\left(\eta_{\beta, 1, t-1}, \ldots, \eta_{\beta, N, t-1}\right)^{\prime}$, we rewrite (2.1) in a matrix form:

$$
\begin{equation*}
R_{t}=Z_{t-1} a+Z_{t-1} B f_{t}+H_{\alpha, t-1}+H_{\beta, t-1} f_{t}+\varepsilon_{t} . \tag{2.3}
\end{equation*}
$$

We further define $\xi_{t}=H_{\alpha, t-1}+H_{\beta, t-1} f_{t}+\varepsilon_{t}$ and rewrite (2.3) into the following matrix form:

$$
\begin{equation*}
R_{t}=Z_{t-1} a+Z_{t-1} B f_{t}+\xi_{t} . \tag{2.4}
\end{equation*}
$$

In order to extract the joint latent factors, we model that the returns on all three asset classes load on some joint latent factors $F^{J}$ and returns on each asset class follow the factor
model (2.4), which is given by ${ }^{2}$

$$
\begin{aligned}
& R_{t}^{s}=Z_{t-1}^{s} a^{s}+Z_{t-1}^{s} B^{s} F_{t}^{J}+\xi_{t}^{s} \\
& R_{t}^{c}=Z_{t-1}^{c} a^{c}+Z_{t-1}^{c} B^{c} F_{t}^{J}+\xi_{t}^{c} \\
& R_{t}^{o}=Z_{t-1}^{o} a^{o}+Z_{t-1}^{o} B^{o} F_{t}^{J}+\xi_{t}^{o}
\end{aligned}
$$

Then we stack them into the following matrix form:

$$
\underbrace{\left[\begin{array}{c}
R_{t}^{s}  \tag{2.5}\\
R_{t}^{c} \\
R_{t}^{o}
\end{array}\right]}_{\mathbf{R}_{t}}=\underbrace{\left[\begin{array}{ccc}
Z_{t-1}^{s} & \mathbf{0} & \mathbf{0} \\
\mathbf{0} & Z_{t-1}^{c} & \mathbf{0} \\
\mathbf{0} & \mathbf{0} & Z_{t-1}^{o}
\end{array}\right]}_{\mathbf{Z}_{t-1}} \underbrace{\left[\begin{array}{c}
a^{s} \\
a^{c} \\
a^{o}
\end{array}\right]}_{\mathbf{a}}+\underbrace{\left[\begin{array}{ccc}
Z_{t-1}^{s} & \mathbf{0} & \mathbf{0} \\
\mathbf{0} & Z_{t-1}^{c} & \mathbf{0} \\
\mathbf{0} & \mathbf{0} & Z_{t-1}^{o}
\end{array}\right]}_{\mathbf{Z}_{t-1}} \underbrace{\left[\begin{array}{c}
B^{s} \\
B^{c} \\
B^{o}
\end{array}\right]}_{\mathbf{B}} F_{t}^{J}+\underbrace{\left[\begin{array}{c}
\xi_{t}^{s} \\
\xi_{t}^{c} \\
\xi_{t}^{o}
\end{array}\right]}_{\boldsymbol{\xi}_{t}} .
$$

Consequently, the factor model across different asset classes can also be expressed in a more compact way as in (2.4).

$$
\begin{equation*}
\mathbf{R}_{t}=\mathbf{Z}_{t-1} \mathbf{a}+\mathbf{Z}_{t-1} \mathbf{B} F_{t}^{J}+\boldsymbol{\xi}_{t} . \tag{2.6}
\end{equation*}
$$

More generally, our framework allows any asset specific factors (non-joint factors) in Model (2.5). Here, to simplify notations, we combine them with the idiosyncratic error term together in each asset class.

### 2.2 Regressed-PCA

We use the model in (2.4) to demonstrate the estimation method. Our target is to identify $a, B$ and $f_{t}$ ( $B$ and $f_{t}$ up to rotation matrix) in (2.4) and obtain the consistent estimators. Following Chen, Roussanov, and Wang (2022), the regressed-PCA procedure simply takes two steps. First, we run the cross-sectional regression of $R_{t}$ on $Z_{t-1}$ period-by-period and get:

$$
\begin{equation*}
\tilde{R}_{t}=a+B f_{t}+\left(Z_{t-1}^{\prime} Z_{t-1}\right)^{-1} Z_{t-1}^{\prime} \xi_{t} \tag{2.7}
\end{equation*}
$$

where $\tilde{R}_{t}=\left(Z_{t-1}^{\prime} Z_{t-1}\right)^{-1} Z_{t-1}^{\prime} R_{t}$. This first step involves the period-by-period cross-sectional regression, which is known as Fama-MacBeth regression (Fama and MacBeth, 1973). $\tilde{R}_{t}$ is interpreted as the vector of returns on $J+1$ characteristic-managed portfolios or a set of

[^2]cross-sectional factors as in Fama and French (2020). More asset pricing interpretations can be found in Chen, Roussanov, and Wang (2022).

Then, we perform the standard PCA procedure on managed portfolios $\left(\left\{\tilde{R}_{t}\right\}_{1: T}\right)$ in model (2.7). Let $M_{T} \equiv I_{T}-1_{T} 1_{T}^{\prime} / T$ where $1_{T}$ denotes a $T \times 1$ vector of ones and $\tilde{R} \equiv\left(\tilde{R}_{1}, \ldots, \tilde{R}_{T}\right)$. Then in the second step, by imposing the following identification and normalization: $a^{\prime} B=0$, $B^{\prime} B=I_{K}$ and $F^{\prime} M_{T} F / T$ being diagonal with diagonal entries in descending order, we can get $\hat{B}$, the estimator of $B$, as the eigenvectors corresponding to the first $K$ largest eigenvalues of the $(J+1) \times(J+1)$ matrix $\tilde{R} M_{T} \tilde{R}^{\prime} / T$ and $\hat{a}=\left(I_{J+1}-\hat{B} \hat{B}^{\prime}\right) \sum_{t=1}^{T} \tilde{R}_{t} / T$. Furthermore, we can get the estimators of $\alpha(\cdot), \beta(\cdot)$ and $F=\left(f_{1}, \ldots, f_{T}\right)^{\prime}$ as

$$
\hat{\alpha}(z)=\hat{a}^{\prime} z, \hat{\beta}(z)=\hat{B}^{\prime} z \text { and } \hat{F}=\left(\hat{f}_{1}, \ldots, \hat{f}_{T}\right)^{\prime}=\tilde{R}^{\prime} \hat{B}
$$

The model in (2.6) with joint factors could be estimated in a similar way. The first step is running the cross-sectional regression of $\mathbf{R}_{t+1}$ on $\mathbf{Z}_{t}$, which is exactly equivalent to running cross-sectional regression within all asset class separately. Then we get characteristicmanaged portfolios across different asset classes. The second step is the same as beforeperforming PCA on the pooled characteristic-managed portfolios from all three asset classes.

The above estimation procedure is conditional on the known number of factors $K$. In empirical analysis, we pin down the number of factors $K$ consistently by maximizeing the eigenvalues ratios (see Ahn and Horenstein (2013) and Chen, Roussanov, and Wang (2022)).

Compared to other popular methods, in addition to the easy computation via the two steps, one of the desirable properties for regressed-PCA is that Chen, Roussanov, and Wang (2022) establish the large- $N$-asymptotic properties for the regressed-PCA without requiring large $T$. In asset pricing, this property is particularly crucial and appealing for the empirical analysis on individual assets, since the sizes of the cross-section of individual assets are larger than the length of their monthly time series, i.e., $N \gg T$. As shown in Section 3, the data we analyze in this paper contains only 210 periods but has more than 1,000 individual assets for each asset class at each time period. Another desirable property of regressed-PCA is that the method can be applied to unbalanced panels. Intuitively, the regressed-PCA translates the unbalanced panels of individual asset returns into a balanced $J+1$ characteristic-managed portfolios via cross-sectional Fama-MacBeth regressions in each period. Chen, Roussanov, and Wang (2022) show that as long as the sample size in each period $N_{t}$ satisfies $\min _{t \leqslant T} N_{t} \rightarrow \infty$, all the asymptotic properties are well established. In particular, this makes the factor analysis on options realistic since individual option's time-to-maturity is short and the long-maturity options are mostly illiquid, therefore it is
impractical to obtain a balanced panel dataset with large enough number of options or long enough time period. More desirable properties and comparisons among different methods can be found in Chen, Roussanov, and Wang (2022).

Finally, we employ the weighted bootstrap developed in Chen, Roussanov, and Wang (2022) to conduct the testing for $\alpha(\cdot)=0$, and the detailed procedure is shown in their paper.

### 2.3 Evaluation metrics

We first consider several types of $R^{2}$ statistics to check the in-sample goodness-of-fit of our models on assets' returns. The first one, $R_{K}^{2}$, measures the model's ability of explaining the characteristic-managed portfolios from the Fama-MacBeth cross-sectional regression with different number of factors $K$, this is directly from the second step - PCA. The second one is $R_{\tilde{R}}^{2}$, which measures how much variation in individual assets is explained by the characteristic-managed portfolios, this is the $R^{2}$ of the Fama-MacBeth cross-sectional regression in the first step of the estimation procedure. In addition, the total $R^{2}$ evaluates the performance of our models in explaining individual assets' returns directly,

$$
\begin{equation*}
R^{2}=1-\frac{\sum_{i, t}\left[r_{i, t}-\hat{\alpha}\left(z_{i, t-1}\right)-\hat{\beta}\left(z_{i, t-1}\right)^{\prime} \hat{f}_{t}\right]^{2}}{\sum_{i, t} r_{i, t}^{2}} . \tag{2.8}
\end{equation*}
$$

Second, we evaluate the out-of-sample performance. In the empirical analysis, we apply the expanding-window scheme. Specifically, we let the initial window size equal to 60 months, then for $t \geqslant 60$, we obtain the model estimates $\hat{a}_{t-1}, \hat{B}_{t-1}, \hat{F}^{(t-1)} \equiv\left(\hat{f}_{1}^{(t-1)}, \ldots, \hat{f}_{t-1}^{(t-1)}\right)$ and $\hat{\alpha}_{t-1}\left(z_{i, t-1}\right)=\hat{a}_{t-1}^{\prime} \phi\left(z_{i, t-1}\right), \hat{\beta}_{t-1}\left(z_{i, t-1}\right)=\hat{B}_{t-1}^{\prime} \phi\left(z_{i, t-1}\right)$ correspondingly, by estimating the model using data through $t-1$. Next we approximate the time $t$ factors $f_{t}$ by the time-series average of all previous factor estimators $\hat{\lambda}_{t}=\sum_{s \leqslant t-1} \hat{f}_{s}^{(t-1)} /(t-1)$. Then, we evaluate the three out-of-sample predictive $R^{2}$ statistics for both the unrestricted and restricted models, the first one is the total out-of-sample $R_{O}^{2}$, the second one measures the cross-sectional average of model's predictability on time series of each asset $R_{T, N, O}^{2}$, and the third one examines the time-series average of model's predictability on the cross-section $R_{N, T, O}^{2}$ which shows how well the model explain the cross-section of average returns, as it relates to the
$R^{2}$ of the Fama-Macbeth cross-sectional regression. The formulas are shown below.

$$
\begin{align*}
R_{O}^{2} & =1-\frac{\sum_{i, t \geqslant 60}\left[r_{i, t}-\hat{\alpha}_{t-1}\left(z_{i, t-1}\right)-\hat{\beta}_{t-1}\left(z_{i, t-1}\right)^{\prime} \hat{\lambda}_{t}\right]^{2}}{\sum_{i, t \geqslant 60} r_{i, t}^{2}},  \tag{2.9}\\
R_{T, N, O}^{2} & =1-\frac{1}{N} \sum_{i} \frac{\sum_{t \geqslant 60}\left[r_{i, t}-\hat{\alpha}_{t-1}\left(z_{i, t-1}\right)-\hat{\beta}_{t-1}\left(z_{i, t-1}\right)^{\prime} \hat{\lambda}_{t}\right]^{2}}{\sum_{t \geqslant 60} r_{i, t}^{2}},  \tag{2.10}\\
R_{N, T, O}^{2} & =1-\frac{1}{T-60} \sum_{t \geqslant 60} \frac{\sum_{i}\left[r_{i, t}-\hat{\alpha}_{t-1}\left(z_{i, t-1}\right)-\hat{\beta}_{t-1}\left(z_{i, t-1}\right)^{\prime} \hat{\lambda}_{t}\right]^{2}}{\sum_{i} r_{i, t}^{2}} . \tag{2.11}
\end{align*}
$$

Third, we assess the out-of-sample fitness by computing the out-of-sample realized factor return at $t: \quad \hat{f}_{t-1, t}=\left[\sum_{i=1}^{N} \hat{\beta}_{t-1}\left(z_{i, t-1}\right) \hat{\beta}_{t-1}\left(z_{i, t-1}\right)^{\prime}\right]^{-1} \sum_{i=1}^{N} \hat{\beta}_{t-1}\left(z_{i, t-1}\right)\left[r_{i t}-\hat{\alpha}\left(z_{i, t-1}\right)\right]$. By doing that, we are able to evaluate how much the cross-sectional variation of individual assets can be explained by the pre-estimated $\hat{\beta}_{t-1}(\cdot)$. Then the associated $R^{2}$ 's are defined as:

$$
\begin{align*}
R_{f, O}^{2} & =1-\frac{\sum_{i, t \geqslant 60}\left[r_{i, t}-\hat{\beta}_{t-1}\left(z_{i, t-1}\right)^{\prime} \hat{f}_{t-1, t}\right]^{2}}{\sum_{i, t \geqslant 60} r_{i, t}^{2}},  \tag{2.12}\\
R_{f, T, N, O}^{2} & =1-\frac{1}{N} \sum_{i} \frac{\sum_{t \geqslant 60}\left[r_{i, t}-\hat{\beta}_{t-1}\left(z_{i, t-1}\right)^{\prime} \hat{f}_{t-1, t}\right]^{2}}{\sum_{t \geqslant 60} r_{i, t}^{2}},  \tag{2.13}\\
R_{f, N, T, O}^{2} & =1-\frac{1}{T-60} \sum_{t \geqslant 60} \frac{\sum_{i}\left[r_{i, t}-\hat{\beta}_{t-1}\left(z_{i, t-1}\right)^{\prime} \hat{f}_{t-1, t}\right]^{2}}{\sum_{i} r_{i, t}^{2}} . \tag{2.14}
\end{align*}
$$

Fourth, we further examine the out-of-sample performance by an arbitrage portfolio from a pure-alpha trading strategy based on our model estimation of the anomaly terms $\hat{\alpha}(\cdot)$. In the unrestricted model, the characteristics may help capture the mispricing as reflected in the anomaly intercepts that should be independent of the varying risk-based compensation, we then could expect a high Sharpe ratio on this trading strategy if the model accurately captures the risk factor parts of the returns. We assign the portfolio weights as $w_{t}=Z_{t-1}\left(Z_{t-1}^{\prime} Z_{t-1}\right)^{-1} \hat{a}_{t-1}$, where $\hat{a}_{t-1}$ is estimated from the data through $t-1$. Chen, Roussanov, and Wang (2022) show that the return on this portfolio should converge to $\|a\|^{2}$.

## 3 Data

In this section, we introduce the data for stocks, corporate bonds and options. Important filters for corporate bonds and options, as well as summary statistics for the returns are
presented. We also briefly show the characteristics as the compositions of the latent factor loadings and mispricing errors in our model of returns on the three asset classes. Appendix A1 provides a detailed description of the characteristics used in our analyses of corporate bonds and options.

In order to extract the joint factors, we focus on the sample period during which data for all three asset classes are available. Therefore, the sample period of the monthly stocks, corporate bonds and options data analyzed in this paper for all the in-sample analysis is from July 2004 to December 2021.

For the out-of-sample analysis in the later sections, our sample period is from July 2004 to December 2019. We omit two years that covers unprecedented events including the COVID and the GameStop episode, because these events dramatically affect the out-ofsample predictability, especially for equity options. ${ }^{3}$

For all three asset classes, we study the excess returns while the risk-free rates are from Kenneth French's data library. ${ }^{4}$ In addition, following Kelly, Pruitt, and Su (2019), we rescale all characteristics cross-sectionally into the range $[-0.5,+0.5]$ to restrict the impact of outliers.

### 3.1 Stock

The stock returns and characteristics data are originally from Freyberger, Neuhierl, and Weber (2020) and Kim, Korajczyk, and Neuhierl (2021). To model stock returns, we pick 35 characteristics that are available from Freyberger, Höppner, Neuhierl, and Weber (2022), out of 36 characteristics which are used in Kelly, Pruitt, and Su (2019) and Chen, Roussanov, and Wang (2022). ${ }^{5}$ The 35 characteristics are market beta (beta), market capitalization (mktcap), book-to-market ratio (bm), gross profitability (prof), investment (invest), idiosyncratic volatility (idiovol), book leverage (lev), operating leverage (ol), momentum (mom), intermediate momentum (intmom), short-term reversal (strev), long-term reversal

[^3](ltrev), average daily bid-ask spread (bidask), standard unexplained volume (suv), price to 52 -week high price ( $w 52 h$ ), total assets (asset), total-assets-to-size (a2me), sales-to-lagged-net-operating-assets (ato), sales-to-price ( $s 2 p$ ), cash-to-short-term-investment (c), capital turnover (cto), ratio of change in property, plants and equipment to the change in total assets (dpi2a), earnings-to-price ( $e 2 p$ ), return on net operating assets (rna), return on assets (roa), return on equity (roe), price-to-cost margin ( $p c m$ ), profit margin ( $p m$ ), Tobin's Q $(q)$, cash flow-to-book (freecf), last month's volume to shares outstanding (turn), capital intensity ( $d 2 a$ ), operating accruals (oa), ratio of sales and general administrative costs to sales (sga2s), and net operating assets (noa). For detailed construction and summary statistics of these characteristics, one can refer to Freyberger, Neuhierl, and Weber (2020) and Freyberger, Höppner, Neuhierl, and Weber (2022).

### 3.2 Corporate Bond

For corporate bonds, we use the dataset constructed by Dickerson, Robotti, and Rossetti (2023). ${ }^{6}$ This corporate dataset sources from the WRDS bond database and Mergent's FISD. A highlight in this dataset is that the corporate bond prices are properly adjusted for market microstructure noises (MMN) in the trades by following the procedure proposed by Andreani, Palhares, and Richardson (2023), so that the asset pricing implications can be closely aligned with the industry-grade quote data such as ICE. In particular, the authors impose two prominent filtering criteria with respect to the issue size: (1) remove investment grade bonds of less than $\$ 150$ ( $\$ 250$ ) million outstanding prior to (after) November 2004, and (2) remove high-yield bonds that have less than $\$ 100$ ( $\$ 250$ ) million outstanding prior to (after) September 2016. Also, different from the WRDS bond database which the returns are truncated at $100 \%$, the authors adjust the returns that are over $100 \%$ with returns computed from ICE quote database. Besides, the authors follow the standard data preparation procedure to clean the corporate bond data. ${ }^{7}$ They collapse the transaction-level prices into daily prices by taking the par volume-weighted average of intraday prices (Bessembinder, Kahle, Maxwell, and Xu, 2008). They remove transaction records in TRACE Enhanced that are canceled and adjust records that are subsequently corrected or reversed. They eliminate bonds with non-standard transactions which are labeled as when-issued (WIS_FL), lockedin $\left(L C K D_{-} I N_{-} I N D\right)$, have special sales conditions $\left(S P C L_{-} T R D_{-} F L\right)$, or have trading-

[^4]volume of less than $\$ 100,000 .{ }^{8}$ They also exclude bonds with non-standard issuance, i.e., bonds that are issued through private placement (private_placement) or under the 144A rule (rule_144a) and bonds that do not trade in US dollars. They further drop bonds that are structured notes, mortgage backed or asset backed, agency backed, or equity linked, as well as convertible bonds, bonds that trade under $\$ 5$ or above $\$ 1000$, bonds that have a floating or zero coupon rate, and bonds that have less than one year to maturity. They restrict the bond's interest payment frequency between monthly and annual.

The dataset from Dickerson, Robotti, and Rossetti (2023) include monthly variables of corporate bond returns, as well as bond-level characteristics. We compliment their dataset with Mergent's FISD to construct additional bond characteristics. The Mergent's FISD dataset has basic issue information such as bond interest rates, convertible terms, bondholder protections, and unit offerings. It also provides issuer information as well as corresponding agencies. We merge the dataset from Dickerson, Robotti, and Rossetti (2023) with Mergent's FISD based on bond security's CUSIP. The bond returns and characteristics are then merged with firm-level characteristics using the WRDS Bond CRSP Link table.

The monthly corporate bond returns are computed using representative price $(P)$ for each end-of-month date and each bond, accrued interests ( $A I$ ), and coupons (cpn). First, since corporate bond markets are illiquid, and trades may or may not occur frequently within the month, the end-of-month prices should balance the trade-offs between keeping a large enough sample size and extrapolating from the last available prices. Specifically, for each corporate bond on each month-end date, we select the price if it is available within 5 calendar days before the month-end; otherwise, we mark the price as missing. ${ }^{9}$ Second, we compute the accrued interest over the fractional period between the last coupon payment date and the month-end date. We can compute the monthly return as:

$$
R_{t+1}^{\text {corpbond }}=\frac{P_{t+1}+A I_{t+1}+c p n_{t+1}}{P_{t}+A I_{t}}-1
$$

We employ 26 characteristics that are widely studied by the literature on corporate bond returns (e.g., Kelly, Palhares, and Pruitt, 2022; Bao, Pan, and Wang, 2011) in our model. There are 12 bond contract level characteristics, including bond age (age), coupon (cpn), rating (rating), issue amount (issue_size) duration (duration), spread (CS), bond momentum (bond_mom), spread momentum (spread_mom), value-at-risk (VaR), bond short-term

[^5]reversal (bond ${ }_{r}$ ev), bond long-term reversal (bond ${ }_{l}$ trev), and illiquidity (illiq). For bond characteristics that use bond prices of the most recent month, we use the values from Dickerson, Robotti, Rossetti (2023) which adjust for microstructure noises using ICE quote data. We also include 16 stock-level variables, which are idiosyncratic volatility (idiovol), momentum (mom), book leverage (lev), Fama-French five-factor related characteristics (beta, prof, mktcap, invest, bm), operating leverage (ol), earnings-to-price ratio ( $e 2 p$ ), tangibility (tan), total debt (debt), debt-to-EBITDA (d2ebitda), and distance-to-default ( $D D$ ). Appendix A1 presents the sources and detailed description of these characteristics.

### 3.3 Option

The individual equity options data is from OptionMetrics, and underlying stock information such as stock returns, prices, share code, and trading volume is from CRSP. ${ }^{10}$

In order to avoid recording errors, and extremely illiquid options, we follow the literature and retain option contracts after the following filtering process. ${ }^{11}$ All the filters only utilize the information available on the portfolio formation date $t$ to avoid the look-ahead bias. The option price is defined as the mid-point of the bid and ask prices. First, being consistent with the stocks chosen, we study the options of common stocks. Second, to avoid microstructure noise, we keep only options in which bid price is positive, the bid price is smaller than the ask price, the mid-point of the bid and ask is at least $\$ 0.125$, and the bid-ask spread is between the minimum tick size ( $\$ 0.05$ for options trading below $\$ 3$ and $\$ 0.1$ otherwise) and $\$ 5$. Third, we retain only at-the-money options which expire in 1 to 12 months and have positive trading volume at time $t$ to have a focus on the most liquid options. Fourth, we keep standard options which expire on the third Friday of certain months, have non-missing and positive implied volatility, and have non-missing option's Greek delta which is between -1 and 1. Fifth, since individual stock options are of the American type, we control for the early expiration by dropping options with time value in percentage of option value $\frac{F-V}{F}$ is too small (below 5\%), where $F$ is the option price and $V$ is the option's intrinsic value which is defined as $\max (S-K, 0)$ for calls and $\max (K-S, 0)$ for puts with $K$ as the strike price and $S$ as the underlying price (Frazzini and Pedersen, 2021). We focus on the at-the-money

[^6]options, thus we remain only options with the absolute delta between 0.375 to 0.625 . Lastly, we impose the obvious no-arbitrage conditions, see for example Zhan, Han, Cao, and Tong (2022).

We also notice that the outliers in the options data dramatically affect the estimation of the factors (e.g., during the GME episode, the options market exhibited significant volatility and unpredictability.). Consequently, we address this issue by systematically trimming the data, excluding data points below the 1st percentile and above the 99th percentile of the return distribution in each period. The trimmed data is then employed exclusively for insample analysis and estimation.

It is important to acknowledge the potential look-ahead bias introduced by this trimming process when applied to out-of-sample studies. Consequently, in our out-of-sample analysis, we refrain from utilizing the trimmed data. To make our portfolio as realistic as possible, we use the prevailing market quotes to unwind our positions at the end of the holding period (the last trading day of next month) unless we notice recording errors, e.g., the bid price is 998 or 999. ${ }^{12}$

Then we compute the delta-hedged holding returns on call options since calls are more actively traded (Zhan, Han, Cao, and Tong, 2022; Christoffersen, Goyenko, Jacobs, and Karoui, 2018). Specifically, at the portfolio formation date, we buy one contract of the call option and sell delta shares of the underlying stock, where delta is from OptionMetrics and calculated under the Black-Scholes model. We hold the position for one month without daily hedging to reduce the transaction cost and make portfolio more practical (Zhan, Han, Cao, and Tong, 2022). Thus the delta-hedged return is defined as

$$
R_{t+1}^{\text {option }}=\frac{C_{t+1}-\Delta_{t} S_{t+1}}{C_{t}-\Delta_{t} S_{t}}-1
$$

We apply 19 characteristics in our model of option returns. The 19 characteristics are well-documented by the literature that are useful for describing and predicting option returns (e.g., Büchner and Kelly, 2022; Zhan, Han, Cao, and Tong, 2022; Bali, Beckmeyer, Moerke, and Weigert, 2021). We have 7 characteristics on the contract level, including implied volatility (impl_vol), option's Greeks (delta, gamma, theta,volga), embedded leverage (embed_lev), and option illiquidity measure (optspread). The 12 stock level characteristics

[^7]consists of stock illiquidity measure (bidask), idiosyncratic volatility (idiovol), volatility deviation (vol_dev), momentum (strev, intmom, mom), book leverage (lev), and Fama-French five-factor related characteristics (beta, prof, mktcap, invest, bm). Appendix A1 provides a thorough explanation of these characteristics and lists references to papers that examine these characteristics in empirical studies.

Table 1 provides an overview of the descriptive statistics for the returns on the three asset classes ${ }^{13}$. Despite the aforementioned filtering process, our study maintains its focus on a comprehensive panel consisting of the three asset classes. The final sample comprises a substantial number of observations, including 738,518 stock-month observations, 208,652 corporate bond-month observations, and 760,836 option-month observations. Importantly, each period contains a minimum of 386 observations for each asset class. The inclusion of such a large panel enables us to conduct a thorough analysis and gain a deeper understanding of the common factor structure among all individual assets from the three asset classes.

Table 1: Summary statistics of monthly returns on stocks, corporate bonds, and options ${ }^{\dagger}$

|  | No. Obs. | Unique firms | Min No. Obs. | Mean | Std | P10 | P25 | P50 | P75 | P90 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Stock | 738,518 | 8,082 | 2,987 | $1.02 \%$ | $17.42 \%$ | $-14.29 \%$ | $-5.98 \%$ | $0.43 \%$ | $6.75 \%$ | $15.32 \%$ |
| Corp. Bond | 208,652 | 927 | 386 | $0.50 \%$ | $3.56 \%$ | $-1.66 \%$ | $-0.34 \%$ | $0.34 \%$ | $1.30 \%$ | $2.94 \%$ |
| Option | 760,836 | 5,052 | 1,723 | $-0.61 \%$ | $6.96 \%$ | $-6.41 \%$ | $-3.39 \%$ | $-1.20 \%$ | $1.32 \%$ | $5.83 \%$ |

$\dagger$ This tables reports the summary statistics of monthly returns on stocks, corporate bonds, and options used throughout the paper. The sample period is from July 2004 to December 2021. The columns represent the number of monthly observation of individual assets, number of unique firms covered through the sample period, the minimum number of observations in each period, the mean of the return, the standard deviation, and 10th percentile, lower quartile, median, upper quartile and $90 t h$ percentile of the return distribution, respectively.

From Table 1, we observe a notable discrepancy in the standard deviations of returns among stocks, corporate bonds, and options. This discrepancy indicates a significant difference of idiosyncratic volatility and signal-to-noise ratios among different asset classes, which would distort PCA estimation due to finite sample errors. Given that standardization is a fundamental step in the standard protocol for conducting PCA, which is also the second stage of our estimation procedure, the regressed PCA, we standardize the returns of each asset class by dividing them by their respective standard deviations and use the standardized returns throughout the paper. ${ }^{14}$ In this way, we mitigate the distortion of idiosyncratic volatility when we perform PCA on the managed portfolios across different asset classes.

[^8]
## 4 Common Latent Factors

In this section, we extract the latent factors directly and jointly from the individual assets from three asset classes using the joint factor model described by Equation (2.5). We evaluate the factors along several dimensions: (1) commonality among asset classes, (2) relations between latent factors and observable factors as well as macroeconomic variables, (3) insample and out-of-sample performance of the latent factors in returns of different asset classes, and (4) the role of characteristics in capturing the beta loadings of assets on the latent factors - as well as the alpha relative to those factors.

For the following discussion, unless explicitly stated otherwise, our primary focus shall be on the unrestricted model $(\alpha(\cdot) \neq 0)$. We also show in the next section that a model specification test indicates that the pricing errors $\alpha(\cdot)$ are significantly non-zero.

### 4.1 Commonality among asset classes

We start by comparing the regressed-PCA latent factors with the principal components derived from pricing factors as illustrated in Section Appendix A3. If the latent factor(s) are common across asset classes, then they should be correlated with the dominant principal component among observable factors from different classes. This is because the dominant principal component would capture a large proportion of common variations among asset-class-specific factors.

The top panel of Figure 1 illustrates the correlations between regressed-PCA latent factors and principal component of observable pricing factors. Notably, the first latent factor from the regressed-PCA has a correlation of over 0.6 with the first principal component of observable factors, the one which we recognize as common in Section Appendix A3.

The bottom panel of Figure 1 plots the time series of the first regressed-PCA factor with the first PC of observable factors, which shows a strong co-movement between the two. Moreover, the first latent factor is also highly correlated with several other principal components of observable pricing factors. The result suggests that the first latent factor is a candidate of "common" factor across asset classes.

Figure 2 plots the cumulative sum of the first five regressed-PCA latent factors. The first two factors earn significantly higher returns compared to the other factors. Most of the premium for the first factor is earned following sharp market downturns/periods of high
uncertainty. The first factor have two spikes, one after the global financial crisis in 2009, and the other after the first COVID-19 lockdown in the US in 2021. The pattern suggests that the first factor is significantly associated with the systemic market movement.

Next, we compare the latent factors extracted jointly from the three asset classes with the factors extracted from each single asset class using the same regressed-PCA method (asset-class-specific latent factors). Table 2 shows the correlations between the joint latent factors versus class-specific factors. The first regressed-PCA factor (Joint 1) is significantly correlated with latent factors from all three asset classes. Specifically, it has a correlation of 0.85 with the first stock-specific latent factor (Stock 1 ), $0.62,0.33$, and 0.24 respectively with the first three corporate-bond specific factors (Corpbond 1, 2, and 3), and 0.44 and 0.13 with the first two option factors respectively (Option 1 and 2). The findings strongly support the idea that the first regressed-PCA joint latent factor serves as a common factor for all three asset classes.

Beyond the first regressed-PCA factor, we also discover other factors that exhibit common variations across asset classes. The second joint latent factor (Joint 2) has a correlation of 0.80 with the first option-class latent factor (Option 1), and -0.53 with the first corporate bond factor (Corpbond 1), while it has a low correlation with the stock-specific factors, suggesting that it is primarily related to the joint pricing of options and corporate bonds. The third factor (Joint 3) significantly correlates with the dominant principal components of observable pricing factors, with a value around 0.5 . The factor also significantly correlates with a number of class-specific latent factors for stocks, corporate bonds, and options.

To summarize, we find a significant common factor from the individual assets of different asset classes. Notably, when we combine our findings of the PCA results from Section Appendix A3 and the correlations with asset-class-specific latent factors, it becomes evident that the first regressed-PCA joint factor is prominently a common pricing factor. In the next steps, we zoom into this factor and examine its relation with the fundamentals.

### 4.2 Relations with observable factors and macro variables

A natural question follows: which fundamental risk is related to this common factor? We examine the question from two perspectives: (1) its relation to observable factors established in the literature, and (2) its relation to macroeconomic variables associated with business cycles and financial conditions.

Table 3 presents the regression results of each regressed-PCA factor on observable pricing factors. The first regressed-PCA factor (Joint 1) loads positively on the SMB and RMW factors, and negatively on the equity MOM factor, which is consistent with the findings by Asness, Moskowitz, and Pedersen (2013) and Fama and French (2015). Asness, Moskowitz, and Pedersen (2013) find that value and momentum are ubiquitous across different financial markets and several asset classes including equities, commodities, currencies, and government bonds. They also find that value and momentum are negatively correlated with each other within and across asset classes. While Fama and French (2015) show that the value factor can be subsumed by profitability and investment factors. Relating to corporate bond, the first latent factor loads significantly on the credit risk factor and liquidity risk factor from Dickerson, Mueller, and Robotti (2023).,The first latent factor is substantially explained by observable pricing factors from all three asset classes, with adjusted R-squared of $57 \%$, reaffirming its role as a common factor. Notably, we find that almost all regressed-PCA factors earn positive and significant returns beyond the tradable factors. For example, the first factor earns a $0.24 \%$ standardized monthly alpha after controlling for the observable factors from the three asset classes. The second factor also has a monthly alpha of $0.35 \%$. The result suggests that some additional premium is unspanned by observable pricing factors in the existing literature for specific asset classes.

We also present the correlations between the regressed-PCA factors and observable factors in Table 4 in addressing the possible issue of collinearity of observable factors. ${ }^{15}$ The first joint factor is highly correlated with fifteen out of eighteen observable factors. In particular, market factors constructed from equities, corporate bonds, and options all show positive and high correlations with the first latent factor, consistent with the idea that this factor extract common variations across different types of assets.

Next, we examine the relation between latent factors and macroeconomic and financial variables. We consider the following series of three categories: (1) indicators of economic activities including core inflation and growth in industrial production, (2) indicators of uncertainty including economic policy uncertainty (EPU) from Baker, Bloom, and Davis (2016), and financial uncertainty (FINU) and macro uncertainty (MACU) from Jurado, Ludvigson, and Ng (2015), (3) indicators of financial conditions including federal funds rate, term spread, credit spread, VIX index, and He, Kelly, and Manela (2017) intermediary capital factor (HKM). Some series are first-differenced (denoted in $\Delta(\cdot))$ for stationarity. Table 5

[^9]shows the pairwise correlations of joint latent factors and macroeconomic variables of interest. The first latent factor is evidently correlated with a number of macroeconomic and financial series. It shows negative and statistically significant correlations with economic policy uncertainty ( -0.16 ), financial uncertainty ( -0.41 ), industrial production growth ( -0.22 ), and macro uncertainty ( -0.28 ), while modestly negative correlations with inflation ( -0.05 ) and consumption growth (-0.06). This suggests that the common factor is associated with the uncertainty about future economic activities instead of concurrent ones. The factor is also significantly correlated with the HKM series (0.44), changes in the term spread (0.22), and innovations in the VIX index (-0.29), which may correspond to the funding liquidity risk of financial intermediaries. Studies including Brunnermeier and Pedersen (2009), He and Krishnamurthy (2013), and Brunnermeier and Sannikov (2014) show that intermediary liquidity constraints significantly contribute to the risk premia as well as commonality across securities. Adrian, Etula, and Muir (2014) and He and Krishnamurthy (2013) find that the intermediary funding risk is significantly priced in across different asset classes. Our result is consistent with their findings. Lastly, the first latent factor has positive correlations with term spreads, suggesting that duration risk also plays a role in explaining the common risk factor.

To investigate the macro-financial determinants of the joint latent factors, we regress them on the macroeconomic variables. Table 6 summarizes the results. The first latent factor loads heavily on HKM. The Shapley-Owen $R^{2}$ (Huettner and Sunder, 2012; Fournier, Jacobs, and Orłowski, 2023), which describes the marginal importance of the individual explanatory variable, is the highest for HKM, with a value of $28 \%$. The financial uncertainty index has a high $R^{2}$ value of $18 \%$, and the change in term spread has $11 \%$. The results from Tables 5 and 6 reconcile with the earlier evidence that the first latent factor exhibits characteristics of a common factor.

### 4.3 In-sample fit and out-of-sample predictability

We analyze the in-sample and out-of-sample performance of the joint factor model (2.5) following the evaluation metrics introduced in Section 2.3. The four sub-tables of Table 7 present the performance of the unrestricted joint factor model for returns on all three asset classes, stocks, corporate bonds, and options, respectively.

We first document the significant explanatory and predictive power of the first latent joint factor across all three asset classes. The in-sample total $R^{2}$ is $6.11 \%$ for all three asset
classes, $6.27 \%$ for stock returns, $11.21 \%$ for corporate bond returns, and $5.40 \%$ for option returns. In terms of the out-of-sample fitness $R_{f, O}^{2}$, the first joint latent factor can explain $25.96 \%$ for all three asset classes. Specifically, the R-squareds are $9.49 \%$ for stocks, $2.40 \%$ for corporate bonds, and $29.30 \%$ for option returns. These $R^{2} \mathrm{~s}$ explained by the first factor of the joint model (2.5) are comparable to the those from the model (2.4) for each specific asset class (the corresponding results are shown in Table A4) with even more than one factor. ${ }^{16}$ The results indicate that estimating common factors jointly from three asset classes are more efficient in capturing variations in different assets, especially in the explanatory and predictive power of the first latent joint factor.

Beyond the significant common variations among all asset classes captured by the first joint factor, the joint factor model is also able to capture asset-class-specific variations. For instance, the second latent factor predominately contributes to option and corporate bond pricing. This is evident when examining the $R_{K}^{2}$, which assesses the model's capacity to account for the variation in the characteristic-managed portfolios. Notably, adding the second joint latent factor increases the $R_{K}^{2}$ value from $13.45 \%$ to $23.70 \%$ for corporate bond returns and from $8.82 \%$ to $29.73 \%$ for option returns, but only increases by less than $1 \%$ for stock. This observation implies that the second joint factor explain the variations in characteristics-managed portfolios associated with options and corporate bonds.

### 4.4 Constituents of common factors

A convenient feature of regressed-PCA is that we can examine the relative weight of each characteristic on the beta loading of a latent factor. This feature helps us to gain an understanding of the nature of the factor. Figure 3 plots the relative weight ( $B$ coefficients) in the first joint latent factor. Evidently, beta loadings on the factor are exposed to characteristics from all three asset classes. In Appendix A2, we discuss in details about the relative weights of characteristics on the beta loadings of the factor for each of the three asset-class segments.

### 4.5 MVE portfolios

In this section, we construct the in-sample and out-of-sample mean-variance efficient (MVE) and tangency portfolios based on the set of managed portfolios that arise from the Fama-

[^10]Macbeth regression procedure across (and within) different asset classes. For in-sample analysis, we approximate the conditional SDF with in-sample MVE portfolios. We then study the hedging relationships among different asset classes by examining the hedging relationship among the MVE portfolios in different asset classes. We also construct the out-of-sample MVE and tangency portfolios within and across different classes.

### 4.5.1 SDF and in-sample MVE portfolios

In this section, we derive the conditional SDF as a function of the MVE portfolio following Kozak and Nagel (2023). Here, we assume that all the related assumptions as in Kozak and Nagel (2023) hold in our framework. Hence, we can span the conditional SDF with the managed portfolios from Fama-Macbeth regression within/across different asset classes.

In Figure 3, it is hard to draw the conclusions on the hedging relationship among different asset classes due to the large number of managed portfolios. However, based on the results in Kozak and Nagel (2023), under suitable conditions, we can approximate the SDF with the MVE portfolio of managed portfolios. Specifically, as in Kozak and Nagel (2023), we can express the conditional SDF into the following form under suitable conditions:

$$
\begin{equation*}
M_{t+1}=1-\boldsymbol{b}_{t}^{\prime}\left(\tilde{\boldsymbol{R}}_{t+1}-\boldsymbol{\mu}_{\tilde{\boldsymbol{R}}, t}\right) \tag{4.1}
\end{equation*}
$$

where $\tilde{\boldsymbol{R}}_{t+1}=\left(\mathbf{Z}_{t}^{\prime} \mathbf{Z}_{t}\right)^{-1} \mathbf{Z}_{t}^{\prime} \mathbf{R}_{t+1}$ with the related variables defined in (2.6). And, $\boldsymbol{b}_{t}=\left[\boldsymbol{\Sigma}_{\tilde{\boldsymbol{R}}, t}+\right.$ $\left.\sigma_{s}^{2}\left(\mathbf{Z}_{t}^{\prime} \mathbf{Z}_{t}\right)^{-1}\right]^{-1} \boldsymbol{\mu}_{\tilde{\boldsymbol{R}}, t}$ with the associated population version of conditional covariance matrix $\boldsymbol{\Sigma}_{\tilde{\boldsymbol{R}}, t}$ and conditional mean vector $\boldsymbol{\mu}_{\tilde{\boldsymbol{R}}, t}$ of $\tilde{\boldsymbol{R}}$ at time $t$, which is the weight of conditional MVE portfolio on managed portfolios from Fama-MacBeth regression. ${ }^{17}$

We further decompose the conditional SDF in (4.1) into three components as follows, which is corresponding to three asset classes.

$$
\begin{equation*}
M_{t+1}=1-\boldsymbol{b}_{t}^{s \prime}\left(\tilde{\boldsymbol{R}}_{t+1}^{s}-\boldsymbol{\mu}_{\tilde{\boldsymbol{R}}^{s}, t}\right)-\boldsymbol{b}_{t}^{c l}\left(\tilde{\boldsymbol{R}}_{t+1}^{c}-\boldsymbol{\mu}_{\tilde{\boldsymbol{R}}^{c}, t}\right)-\boldsymbol{b}_{t}^{o \prime}\left(\tilde{\boldsymbol{R}}_{t+1}^{o}-\boldsymbol{\mu}_{\tilde{\boldsymbol{R}}^{o}, t}\right) \tag{4.2}
\end{equation*}
$$

with $\boldsymbol{b}_{t}=\left[\boldsymbol{b}_{t}^{s \prime}, \boldsymbol{b}_{t}^{c \prime}, \boldsymbol{b}_{t}^{o \prime}\right]^{\prime}, \tilde{\boldsymbol{R}}_{t+1}=\left[\tilde{\boldsymbol{R}}_{t+1}^{s \prime}, \tilde{\boldsymbol{R}}_{t+1}^{c \prime}, \tilde{\boldsymbol{R}}_{t+1}^{o \prime}\right]^{\prime}$ and $\boldsymbol{\mu}_{\tilde{\boldsymbol{R}}, t}=\left[\boldsymbol{\mu}_{\tilde{\boldsymbol{R}}^{s}, t}^{\prime}, \boldsymbol{\mu}_{\tilde{\boldsymbol{R}}^{c}, t}^{\prime}, \boldsymbol{\mu}_{\tilde{\boldsymbol{R}}^{o}, t}^{\prime}\right]^{\prime}$.

[^11]Importantly, based on (4.2), we can examine the hedging relationship among different asset classes by studying the covariation between the different components of MVE portfolios. This translation makes the study of hedging relationship possible.

We report the annualized Sharpe ratios of MVE and tangency portfolio across/within different asset classes in the first panel of Table 8. First of all, we find the strong hedging relationship among different asset classes. The Sharpe ratio of MVE portfolios among all asset classes (7.43) is much larger than the Sharpe ratio of MVE portfolios from any single asset class, where the largest Sharpe ratio of MVE portfolio based on single asset class is from stock (4.40). Second, for the MVE portfolio from all asset classes, the component from stocks has the largest Sharpe ratio, then option and corporate bond.

To have a better understanding about the hedging relationship, we further report the correlation of different components of MVE portfolios across different asset classes in Table 8. Very importantly, we find the negative correlation between stock and corporate bond ( -0.45 ) or option ( -0.53 ), and the much smaller negative correlation between the components from corporate bond and option ( -0.06 ). This shows that stock can hedge corporate bonds and options, while the hedging relationship between corporate bonds and options is much weaker. This conclusion is further verified by the finding that the stock component's loadings on the first joint common factors are negative, while the loadings of corporate bond and option components are positive. For the second joint factor, all the signs are flipped. We also find that the extracted single common factors (with restriction $a=0$ ) can explain the variations of common MVE portfolios significantly. This finding further supports the existence of a common factor across different asset classes. Moreover, the regression results in the third panel are also consistent with the correlation/hedging relationships in the second panel. Finally, we provide the extension of correlation between MVE components and joint factors in Table 9 . The results are exactly consistent with the hedging relationship until $K=4$. More importantly, we find that the loadings of common MVE portfolio on the joint factors are positive but rather small, which can also reflect that stocks hedge out corporate bond and options.

### 4.5.2 The out-of-sample MVE/tangency portfolios

Next, we consider how to construct the implementable trading on large individual assets across/within different asset classes. We consider the following conditional mean-variance
efficient portfolio choice problem at time $t$ :

$$
\begin{equation*}
\max _{\mathbf{w}_{t}}\left\{\mathbf{w}_{t}^{\prime} E_{t}\left[\mathbf{R}_{t+1}\right]-\frac{1}{2} \mathbf{w}_{t}^{\prime} \operatorname{Var}_{t}\left[\mathbf{R}_{t+1}\right] \mathbf{w}_{t}\right\}, \tag{4.3}
\end{equation*}
$$

with excess return $\mathbf{R}_{t+1}$ of individual assets across/within different asset classes and the associated weight $\mathbf{w}_{t}$. However, it is challenging to estimate the conditional covariance matrix of individual asset returns due to large cross-sectional observations and relatively short time series. Meanwhile, if we model the excess return $\mathbf{R}_{t+1}$ with the factor structure in (2.4) or (2.6), the portfolio choice problem above can be rewritten as:

$$
\begin{equation*}
\max _{\mathbf{w}_{t}}\left\{\mathbf{w}_{t}^{\prime} \mathbf{Z}_{t} E_{t}\left[\check{\mathbf{R}}_{t+1}\right]-\frac{1}{2} \mathbf{w}_{t}^{\prime} \mathbf{Z}_{t} \operatorname{Var}_{t}\left[\check{\mathbf{R}}_{t+1}\right] \mathbf{Z}_{t}^{\prime} \mathbf{w}_{t}-\frac{1}{2} \mathbf{w}_{t}^{\prime} \operatorname{Var}_{t}\left[\boldsymbol{\xi}_{t+1}\right] \mathbf{w}_{t}\right\} . \tag{4.4}
\end{equation*}
$$

with $\check{\mathbf{R}}_{t+1}=a+B F_{t+1}$ by noticing $E_{t}\left[\mathbf{R}_{t+1}\right]=\mathbf{Z}_{t} E_{t}\left[\check{\mathbf{R}}_{t+1}\right]$. Under the assumption of the homoskedastic idiosyncratic errors, following Kozak and Nagel (2023), the solution of (4.4) is given by:

$$
\begin{equation*}
\mathbf{w}_{t}=\mathbf{Z}_{t}\left(\mathbf{Z}_{t}^{\prime} \mathbf{Z}_{t}\right)^{-1}\left[\operatorname{Var}_{t}\left[\check{\mathbf{R}}_{t+1}\right]+\sigma_{s}^{2}\left(\mathbf{Z}_{t}^{\prime} \mathbf{Z}_{t}\right)^{-1}\right]^{-1} E_{t}\left[\check{\mathbf{R}}_{t+1}\right] \tag{4.5}
\end{equation*}
$$

The optimal weight on individual asset is further given by:

$$
\begin{equation*}
\mathbf{w}_{t}=\mathbf{Z}_{t}\left(\mathbf{Z}_{t}^{\prime} \mathbf{Z}_{t}\right)^{-1}\left[B \operatorname{Var}\left[F_{t+1}\right] B^{\prime}+\sigma_{s}^{2}\left(\mathbf{Z}_{t}^{\prime} \mathbf{Z}_{t}\right)^{-1}\right]^{-1}\left(a+B E_{t}\left[F_{t+1}\right]\right) \tag{4.6}
\end{equation*}
$$

If we also assume that the risk premium of managed portfolios or latent factors is constant, then it implies

$$
E_{t}\left[\check{\mathbf{R}}_{t+1}\right]=a+B E_{t}\left[F_{t+1}\right] \text { and } \operatorname{Var}_{t}\left[\check{\mathbf{R}}_{t+1}\right]=B \operatorname{Var}_{t}\left[F_{t+1}\right] B^{\prime}
$$

with the ample analogue for managed portfolios: $\widehat{E_{t}}\left[\check{\mathbf{R}}_{t+1}\right]=\frac{\sum_{j=1}^{t} \check{\mathbf{R}}_{j}}{t}$ and $\widehat{\operatorname{Var}}_{t}\left[\check{\mathbf{R}}_{t+1}\right]=$ $\frac{1}{t-1} \sum_{j=1}^{t}\left(\check{\mathbf{R}}_{j}-\widehat{E}_{t}\left[\check{\mathbf{R}}_{t+1}\right]\right)\left(\check{\mathbf{R}}_{j}-\widehat{E}_{t}\left[\check{\mathbf{R}}_{t+1}\right]\right)^{\prime}$, which are based on the estimated results from Fama-MacBeth regression. For factors, $\widehat{E_{t}}\left[F_{t+1}\right]=\frac{\sum_{j=1}^{t} F_{t}}{t}$ and $\widehat{\operatorname{Var}}_{t}\left[F_{t+1}\right]=\frac{1}{t-1} \sum_{j=1}^{t}\left(F_{j}-\right.$ $\left.\widehat{E_{t}}\left[F_{t+1}\right]\right)\left(F_{j}-\widehat{E_{t}}\left[F_{t+1}\right]\right)^{\prime}$, which are based on the estimated results from regressed-PCA. If the sample size until $t$ is large enough, the mean vector and covariance matrix are estimated consistently.

Alternatively, if we assume that investors simply ignore the idiosyncratic errors, then the problem in (4.4) can be approximated by:

$$
\begin{equation*}
\max _{\mathbf{w}_{t}}\left\{\mathbf{w}_{t}^{\prime} \mathbf{Z}_{t} E_{t}\left[\check{\mathbf{R}}_{t+1}\right]-\frac{1}{2} \mathbf{w}_{t}^{\prime} \mathbf{Z}_{t} \operatorname{Var}_{t}\left[\check{\mathbf{R}}_{t+1}\right] \mathbf{Z}_{t}^{\prime} \mathbf{w}_{t}\right\} \tag{4.7}
\end{equation*}
$$

Define $\mathbf{w}_{t}^{\prime} \mathbf{Z}_{t}=\mathbf{w}_{t}^{p \prime}$, then the objective function in (4.4) can be expressed as:

$$
\begin{equation*}
\max _{\mathbf{w}_{t}^{p}}\left\{\mathbf{w}_{t}^{p \prime} E_{t}\left[\check{\mathbf{R}}_{t+1}\right]-\frac{1}{2} \mathbf{w}_{t}^{p \prime} \operatorname{Var}_{t}\left[\check{\mathbf{R}}_{t+1}\right] \mathbf{w}_{t}^{p}\right\} . \tag{4.8}
\end{equation*}
$$

Given the optimal solution for $\mathbf{w}_{t}^{p}=\operatorname{Var}_{t}\left[\check{\mathbf{R}}_{t+1}\right]^{-1} E_{t}\left[\check{\mathbf{R}}_{t+1}\right]$, the optimal solution for $\mathbf{w}_{t}$ is then given by:

$$
\begin{equation*}
\mathbf{w}_{t}=\mathbf{Z}_{t}\left(\mathbf{Z}_{t}^{\prime} \mathbf{Z}_{t}\right)^{-1} \mathbf{w}_{t}^{p} \tag{4.9}
\end{equation*}
$$

Furthermore, if we impose the restriction: $a=0$, then we can further write the problem in (4.8) as:

$$
\begin{equation*}
\max _{\mathbf{w}_{p t}^{p}}\left\{\mathbf{w}_{t}^{p \prime} B E\left[F_{t+1}\right]-\frac{1}{2} \mathbf{w}_{t}^{p \prime} B \operatorname{Var}\left[F_{t+1}\right] B^{\prime} \mathbf{w}_{t}^{p}\right\} \tag{4.10}
\end{equation*}
$$

Define $\mathbf{w}_{t}^{p \prime} B=\mathbf{w}_{t}^{f \prime}$. Then, we have a set of solutions given by:

$$
\mathbf{w}_{t}^{p}=B\left(B^{\prime} B\right)^{-1} \mathbf{w}_{t}^{f}
$$

Consequently, the solution is approximately given by

$$
\begin{equation*}
\mathbf{w}_{t}=\mathbf{Z}_{t}\left(\mathbf{Z}_{t}^{\prime} \mathbf{Z}_{t}\right)^{-1} \mathbf{w}_{t}^{p}=\mathbf{Z}_{t}\left(\mathbf{Z}_{t}^{\prime} \mathbf{Z}_{t}\right)^{-1} B\left(B^{\prime} B\right)^{-1} \mathbf{w}_{t}^{f} \tag{4.11}
\end{equation*}
$$

With the consistently estimated conditional mean vector and covariance matrix for factors, the feasible optimal solution of $\widehat{\mathbf{w}}_{t}^{p}$ is given by

$$
\widehat{\mathbf{w}}_{t}^{p}=\widehat{\operatorname{Var}}_{t}\left[\check{\mathbf{R}}_{t+1}\right]^{-1} \widehat{E}_{t}\left[\check{\mathbf{R}}_{t+1}\right] \text { or } \widehat{\mathbf{w}}_{t}^{f}=\widehat{\operatorname{Var}}_{t}\left[F_{t+1}\right]^{-1} \widehat{E}_{t}\left[F_{t+1}\right] .
$$

Intuitively, with the parametric factor model for individual asset excess return specified in (2.4), we can translate the optimal portfolio choice problem on large individual assets within/across different asset classes into the portfolio choice on the characteristics managed portfolio from Fama-MacBeth cross-sectional regression within/across different asset classes. As emphasized by the traditional literature, it is rather challenging to construct the optimal portfolio over the individual assets directly due to the challenge of estimation of high dimensional covariance matrix of individual assets. We overcome this issue based on the dimension reduction from Fama-MacBeth cross-sectional regression so that a complicated conditional optimal problem can be translated into a much simpler one. More importantly, we can examine the hedging property of individual equity, corporate bond and option by studying the hedging property of the associated managed portfolios across different asset classes.

To obtain the tangency portfolio, we impose the restriction that $\mathbf{w}_{t}^{\prime} 1=1$ in (4.4). If the first column of $Z_{t}$ is all-one vector, then we have $\sum_{i} w_{i, t}=w_{1, t}^{p}=1$, which implies that the weight on level return or "diversified portfolio" (see Chen, Roussanov, and Wang (2022)) is
unit. Formally, it is easy to obtain this by noting

$$
\left[w_{1, t}, \ldots, w_{N, t}\right] \cdot\left[\begin{array}{cccc}
1 & z_{1, t, 1} & \ldots & z_{1, t, J}  \tag{4.12}\\
1 & z_{2, t, 1} & \ldots & z_{2, t, J} \\
\vdots & \vdots & \ldots & \vdots \\
1 & z_{N, t, 1} & \ldots & z_{N, t, J}
\end{array}\right]=\left[w_{1, t}^{p}, \ldots, w_{J, t}^{p}\right]
$$

If we consider the portfolio election among three asset classes, then we have

$$
\begin{aligned}
& {\left[w_{1, t}^{s}, \ldots, w_{N_{s}, t}^{s}, w_{1, t}^{c}, \ldots, w_{N_{c}, t}^{e}, w_{1, t}^{o}, \ldots, w_{N_{o}, t}^{o}\right]\left[\begin{array}{ccc}
Z_{t}^{s} & \mathbf{0} & \mathbf{0} \\
\mathbf{0} & Z_{t}^{c} & \mathbf{0} \\
\mathbf{0} & \mathbf{0} & Z_{t}^{o}
\end{array}\right] } \\
= & {\left[w_{1, t}^{p, s}, \ldots, w_{J_{s}, t}^{p, s}, w_{1, t}^{p, c}, \ldots, w_{J_{c}, t}^{p, c}, w_{1, t}^{p, o}, \ldots, w_{J_{o}, t}^{p, o}\right] }
\end{aligned}
$$

The restriction $\mathbf{w}_{t}^{\prime} 1=1$ reduces to $w_{1, t}^{p, e}+w_{1, t}^{p, c}+w_{1, t}^{p, o}=1$.
Finally, we solve the optimal portfolio problem within/across different asset classes with the following Lagrangian:

$$
\begin{equation*}
\max _{\mathbf{w}_{t}^{p}}\left\{\mathbf{w}_{t}^{p \prime} \widehat{E}_{t}\left[\check{\mathbf{R}}_{t+1}\right]-\frac{1}{2} \mathbf{w}_{t}^{p \prime} \widehat{\operatorname{Var}}_{t}\left[\check{\mathbf{R}}_{t+1}\right] \mathbf{w}_{t}^{p}\right\} \tag{4.13}
\end{equation*}
$$

s.t.

$$
w_{1, t}^{p}=1 \text { or } w_{1, t}^{p, s}+w_{1, t}^{p, c}+w_{1, t}^{p, o}=1 .
$$

or in a matrix form:

$$
A^{\prime} w_{t}^{p}=1
$$

where $A$ is a section matrix such that the constraints hold. Then the feasible optimal weight is given by

$$
\begin{equation*}
\widehat{\mathbf{w}}_{t}=\widehat{\operatorname{Var}}_{t}\left[\check{\mathbf{R}}_{t+1}\right]^{-1}\left\{\widehat{E}_{t}\left[\check{\mathbf{R}}_{t+1}\right]-A\left(A^{\prime} \widehat{\operatorname{Var}}_{t}\left[\check{\mathbf{R}}_{t+1}\right]^{-1} A\right)^{-1}\left(A^{\prime} \widehat{\operatorname{Var}}_{t}\left[\check{\mathbf{R}}_{t+1}\right]^{-1} \widehat{E}_{t}\left[\check{\mathbf{R}}_{t+1}\right]-1\right)\right\} \tag{4.14}
\end{equation*}
$$

We report the out-of-sample Sharpe ratios of the tangency portfolios in Table 10, which is based on the expanding window regressions starting with the sample of first ten years. We report the results for both MVE and tangency portfolios, where the optimal weights are given by (4.9) and (4.14), separately. First of all, since the calculation is out-of-sample, the Sharpe ratios of MVE portfolios are not necessarily higher than tanegency portfolios, but are rather close. Due the much larger dimensions of covariance matrix estimation for joint estimation, the Sharpe ratios of MVE/tangency portfolios across all asset classes are not
necessarily higher than Sharpe ratios of MVE/tangency portfolios from any single asset class. Second, for the joint estimation, the Sharpe ratios of MVE/tangency portfolios across all asset classes are higher than any single asset class, and Sharpe ratios of tangency portfolios for stocks are the highest among all asset classes. Third, the Sharpe ratios of tangency portfolios across different asset classes are close to the Sharpe ratios of tangency portfolios for corporate bonds. Finally, comparing the Sharpe ratios of single asset class, the Sharpe ratios of tangency portfolios for stocks and options are higher than the tangency portfolio of corporate bonds.

Finally, we implement the out-of-sample MVE based on the extracted joint common factors, where the weight is given by (4.11). First, we find the increasing of Sharpe ratios with the increasing number of factors. Second, the Sharpe ratios of portfolios constructed from factors are smaller than these based om managed portfolios.

## 5 Risk Premium

Do the regressed-PCA factors carry a sizeable risk premium? We answer this question by computing several measures, starting from the in-sample and out-of-sample Sharpe ratios of the regressed PCA factors. We find that the regressed-PCA factors carry economically large in-sample risk premia. Table 12 shows the reports the in-sample and out-of-sample Sharpe ratio for each regressed-PCA factor. The first regressed-PCA latent factor, which we deem as the common factor for all three asset classes, has an annualized in-sample Sharpe ratio of 0.83 , which is economically high. In the out-of-sample, the Sharpe ratio for the first latent factor can also achieve a value of 0.45 .

Albeit large in values, the risk premium from the regressed-PCA factors does not fully capture the cross-sectional returns among the asset classes. In a pure-alpha or zero-beta strategy, the portfolio has zero beta loading on the proposed risk factors. Table 13 reports the annualized means, standard deviations, and Sharpe ratios of the pure-alpha strategy, $S R_{\alpha}$. Interestingly, we find that the portfolio that has zero-beta loading on the first regressedPCA factor can generate a strikingly high out-of-sample Sharpe ratio of 2.14. However, the Sharpe ratios of zero-beta portfolio do not diminish when more factors are included: A pure-alpha strategy with respect to the ten-regressed-PCA-factor model still earns a high Sharpe ratio of 1.96. In the table, we observe that the mean return of the zero-beta portfolio $\mu_{\alpha}$ decline monotonically in the number of included factors; however, the volatility of the
portfolios $\sigma_{\alpha}$ decreases even further with the number of risk factors, leading an increase in the Sharpe ratios. The reduction in volatility indicates that the regressed-PCA factors capture substantial common variation among returns from three asset classes: when the portfolio has zero-beta loading on the factors, it hedges against a sizeable proportion of the common variation. Though the common variation is large, the high Sharpe ratio of the pure-alpha strategy suggests that a substantial part of the cross-sectional risk premium is not explained by the common risk factors ${ }^{18}$.

Furthermore, the formal model specification test demonstrates presence of significantly non-zero pricing errors. To be specific, we conduct the $\alpha(\cdot)=0$ test using the weighted bootstrap method established in Chen, Roussanov, and Wang (2022), presenting estimates and their corresponding $95 \%$ confidence intervals for coefficients in $\alpha(\cdot)$, with 10 factors incorporated, as illustrated in Figure 4. Our findings indicate that despite the inclusion of a substantial number of risk factors, a number of characteristics remarkably contribute to these pricing errors. ${ }^{19}$ The test further empowers us to examine the contributors to the remarkably good performance of the pure-alpha portfolio. First, we find that stock characteristics have modest contribution to the high alphas. This outcome aligns with the mature and extensively traded nature of the equity market, in which most anomalies tend to be rectified through arbitrage activities. Similarly, contributions from corporate bonds are also modest, consistent with the recent literature such as Dickerson, Robotti, and Rossetti (2023) that return anomalies in the corporate bond market are significantly reduced after micro-structure noises are properly adjusted. However, it is not the case in option markets. We find particularly large contributions from option gamma. It suggests that considerable anomalies persist in these two markets, which result in a highly profitable risk-return tradeoffs in the pure-alpha strategy.

[^12]
## 6 Conclusion

In this paper, we find the strong evidence of commonality across different asset classes. Using the regressed-PCA approach, we extract the joint latent factors directly from individual assets from different asset classes. In particular, some latent factors exhibit strong features that resemble a common systematic risk factor. Although a common factor structure exists across different asset classes, it does not capture the entirety of cross-sectional returns. Indeed, a portfolio based on zero beta loadings on the joint factors has a much higher Sharpe ratio, for both in- and out-of-sample.

This leaves the question for future research: what contributes to the risk-return tradeoff in the asset market? On one hand, the CAPM will suggest that assets that carry a high loading of systematic risk should earn a higher risk premium. On the other hand, anomalies that has zero loading on the systematic risk seem to receive a much larger premium than the compensation for risk, as our findings suggest. This opens the door for further examination on the risk-return relation, in which the systematic risk may not be the only risk that requires a positive reward from the market participants.

## Figures and Tables

Figure 1: Regressed-PCA latent factors vs. principal components of observable factors


Note: Joint refers to the regressed-PCA latent factor. $P C$ is the principal component of observable pricing factors as illustrated in Section Appendix A3. The top panel shows the pair wise correlations. A darker square represents a higher correlation coefficient in absolute value. The bottom panel plots the cumulative sum of the 1st regressed-PCA joint factor (in red) and 1st PC of observable factors (in blue). Both time series are standardized to mean zero and variance one.

Figure 2: Cumulative return of the first five Regressed-PCA joint latent factors


Figure 3: Estimation of $B$ coefficients in the first regressed-PCA joint factor


Figure 4: Estimates and $95 \%$ confidence intervals of $a$ coefficients in $\alpha(\cdot)$ with number of common factors $K=10$ for three asset classes

Stock


## Option



Table 2: Correlations between the extracted regressed-PCA joint factors and factors from each asset class ${ }^{\dagger}$

| Factors | Stock 1 | Stock 2 | Stock 3 | Stock 4 | Stock 5 | Stock 6 | Stock 7 | Stock 8 | Stock 9 | Stock 10 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Joint 1 | 0.8549*** | 0.0833 | 0.1166* | -0.1430** | $0.2143^{* * *}$ | 0.1875*** | 0.0107 | 0.0276 | 0.0136 | 0.0264 |
| Joint 2 | -0.0858 | 0.0404 | -0.0216 | -0.0701 | 0.1117 | -0.1315* | -0.0984 | -0.0676 | -0.0412 | -0.0410 |
| Joint 3 | $-0.4596{ }^{* * *}$ | $0.3705^{* * *}$ | $0.3782^{* * *}$ | $-0.1400^{* *}$ | $0.2443^{* * *}$ | $0.2737^{* * *}$ | -0.0066 | 0.0511 | -0.0016 | 0.0323 |
| Joint 4 | -0.0235 | $0.4180^{* * *}$ | $-0.4300^{* * *}$ | $-0.2085^{* * *}$ | $0.1468^{* *}$ | $0.1831^{* * *}$ | 0.0939 | 0.0536 | -0.0007 | 0.1289* |
| Joint 5 | $0.1956^{* * *}$ | $0.6905^{* * *}$ | 0.1341* | $0.2727^{* * *}$ | $-0.3160^{* * *}$ | $-0.3442^{* * *}$ | -0.0663 | 0.0052 | -0.1013 | -0.0384 |
| Joint 6 | 0.0506 | $-0.3514^{* * *}$ | $0.2402^{* * *}$ | 0.0373 | -0.2716*** | 0.0451 | $0.2191^{* * *}$ | 0.0450 | -0.0835 | 0.0919 |
| Joint 7 | -0.0134 | $0.2236^{* * *}$ | $0.1547^{* *}$ | $-0.1361^{* *}$ | $-0.3645^{* * *}$ | 0.1500** | $0.2783^{* * *}$ | -0.0774 | 0.1096 | 0.0211 |
| Joint 8 | 0.0324 | -0.0565 | $0.6348^{* * *}$ | 0.0083 | $0.2426^{* * *}$ | $-0.1711^{* *}$ | -0.0422 | 0.0435 | -0.0473 | 0.0808 |
| Joint 9 | 0.0041 | 0.0032 | $-0.1734^{* *}$ | $0.7846^{* * *}$ | $0.1626^{* *}$ | $0.3024^{* * *}$ | 0.0510 | 0.0491 | -0.0147 | -0.0377 |
| Joint 10 | 0.0100 | -0.1075 | -0.2580*** | $-0.2835^{* * *}$ | -0.1598** | -0.1697** | -0.0926 | 0.2291*** | -0.1260* | 0.0084 |
| Factors | Corpbond 1 | Corpbond 2 | Corpbond 3 | Corpbond 4 | Corpbond 5 | Corpbond 6 | Corpbond 7 | Corpbond 8 | Corpbond 9 | Corpbond 10 |
| Joint 1 | 0.6230*** | 0.3250 *** | $0.2442^{* * *}$ | 0.0036 | 0.0573 | 0.1410** | 0.0447 | -0.1094 | 0.0678 | 0.1010 |
| Joint 2 | $-0.5321^{* * *}$ | $0.3010^{* * *}$ | 0.0234 | -0.0439 | 0.1121 | -0.0003 | 0.1164* | -0.1147* | 0.0113 | 0.0894 |
| Joint 3 | $0.5062^{* * *}$ | 0.0879 | $-0.2815^{* * *}$ | 0.0088 | 0.0929 | $-0.1877^{* * *}$ | $0.1426^{* *}$ | 0.0519 | $-0.1785^{* * *}$ | -0.0005 |
| Joint 4 | $-0.1523^{* *}$ | $0.6083^{* * *}$ | $0.3116^{* * *}$ | $0.2002^{* * *}$ | -0.0117 | 0.0501 | 0.0554 | 0.0071 | -0.1084 | -0.0885 |
| Joint 5 | -0.1150* | -0.0693 | $-0.4720^{* * *}$ | -0.0044 | -0.0595 | 0.0166 | $0.2411^{* * *}$ | 0.1350* | 0.0665 | -0.0999 |
| Joint 6 | -0.0499 | $0.3867^{* * *}$ | -0.3820*** | $-0.2489^{* * *}$ | 0.0731 | $-0.2620^{* * *}$ | 0.0203 | -0.0657 | $0.2618^{* * *}$ | 0.2569*** |
| Joint 7 | -0.0440 | $-0.3662^{* * *}$ | $0.5487^{* * *}$ | $-0.2192^{* * *}$ | 0.1175* | $-0.2777^{* * *}$ | 0.2779*** | 0.0262 | 0.1479** | 0.1540** |
| Joint 8 | -0.0876 | -0.0285 | 0.1347* | $0.2431^{* * *}$ | 0.0314 | -0.0477 | $-0.1977^{* * *}$ | -0.0235 | -0.1025 | 0.1193* |
| Joint 9 | 0.0141 | -0.1620** | -0.0308 | 0.1095 | $-0.1563^{* *}$ | -0.0926 | $-0.2574 * * *$ | $-0.1861^{* * *}$ | 0.0063 | 0.0240 |
| Joint 10 | 0.0552 | $-0.2682^{* * *}$ | -0.1362** | $0.5124^{* * *}$ | $0.1366^{* *}$ | 0.0615 | 0.1264* | $-0.1991 * * *$ | $0.2041^{* * *}$ | 0.1325* |
| Factors | Option 1 | Option 2 | Option 3 | Option 4 | Option 5 | Option 6 | Option 7 | Option 8 | Option 9 | Option 10 |
| Joint 1 | $0.4434^{* * *}$ | $0.1357^{* *}$ | 0.0737 | $-0.2313^{* * *}$ | 0.0472 | $0.1874^{* * *}$ | -0.0763 | $-0.1443 * *$ | -0.0865 | 0.0573 |
| Joint 2 | $0.7977^{* * *}$ | 0.0512 | 0.0496 | 0.1649** | 0.0106 | -0.0613 | 0.0202 | 0.0200 | 0.0307 | -0.0505 |
| Joint 3 | $0.2398^{* * *}$ | $0.2476{ }^{* * *}$ | 0.0819 | -0.1209* | $-0.1904^{* * *}$ | -0.0703 | -0.0626 | 0.1295* | -0.0077 | 0.0974 |
| Joint 4 | $-0.2736^{* * *}$ | $0.4915^{* * *}$ | $0.4589^{* * *}$ | $-0.2170^{* * *}$ | -0.0681 | 0.0523 | -0.0444 | -0.0648 | 0.0448 | 0.0447 |
| Joint 5 | -0.0641 | -0.0319 | -0.0830 | $0.4948^{* * *}$ | $-0.1654^{* *}$ | 0.0480 | 0.1197* | -0.0284 | -0.0281 | 0.0242 |
| Joint 6 | -0.1007 | $0.6086^{* * *}$ | -0.4786 ${ }^{* * *}$ | $0.1382^{* *}$ | -0.0221 | 0.0508 | -0.0523 | 0.0956 | -0.0503 | -0.0858 |
| Joint 7 | 0.0360 | $0.3065^{* * *}$ | $-0.3341 * * *$ | -0.0197 | 0.0960 | -0.0058 | 0.0520 | -0.0861 | 0.1037 | -0.0525 |
| Joint 8 | -0.0830 | 0.1168* | $0.4217^{* * *}$ | $0.3472^{* * *}$ | $0.3923^{* * *}$ | 0.0314 | -0.1145* | -0.0301 | -0.0780 | 0.0042 |
| Joint 9 | 0.0649 | $0.2522^{* * *}$ | 0.0826 | -0.0790 | $0.2747^{* * *}$ | $-0.1990 * * *$ | 0.0571 | -0.0008 | 0.0478 | -0.0393 |
| Joint 10 | 0.0629 | $0.2875^{* * *}$ | $0.1658^{* *}$ | $0.2521^{* * *}$ | -0.1185* | 0.0468 | $0.1465^{* *}$ | 0.0163 | 0.1194* | -0.0804 |

$\dagger * * *: p$-value $<1 \% ;{ }^{* *}: p$-value $<5 \%$; ${ }^{*}: p$-value $<10 \%$.

Table 3: Observable factors and the extracted regressed-PCA joint factors ${ }^{\dagger}$

| Factors | Joint1 | Joint2 | Joint3 | Joint4 | Joint5 | Joint6 | Joint7 | Joint8 | Joint9 | Joint10 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| (Intercept) | $\begin{aligned} & 0.24^{* * *} \\ & (4.64) \end{aligned}$ | $\begin{aligned} & 0.35^{* * *} \\ & (4.82) \end{aligned}$ | $\begin{aligned} & 0.12^{* *} \\ & (2.45) \end{aligned}$ | $\begin{aligned} & 0.12^{* *} \\ & (2.21) \end{aligned}$ | $\begin{gathered} 0.03 \\ (0.58) \end{gathered}$ | $\begin{aligned} & 0.30^{* * *} \\ & (6.76) \end{aligned}$ | $\begin{aligned} & 0.23^{* * *} \\ & (2.93) \end{aligned}$ | $\begin{aligned} & 0.27^{* * *} \\ & (4.88) \end{aligned}$ | $\begin{aligned} & 0.35^{* * *} \\ & (6.61) \end{aligned}$ | $\begin{aligned} & 0.37^{* * *} \\ & (6.12) \end{aligned}$ |
| MKTstock | $\begin{gathered} 0.01 \\ (0.06) \end{gathered}$ | $\begin{gathered} 0.08 \\ (0.73) \end{gathered}$ | $\begin{gathered} 0.11 \\ (0.73) \end{gathered}$ | $\begin{aligned} & -0.18^{* *} \\ & (-1.99) \end{aligned}$ | $\begin{gathered} -0.30^{* *} \\ (-2.54) \end{gathered}$ | $\begin{aligned} & -0.01 \\ & (-0.19) \end{aligned}$ | $\begin{gathered} -0.26^{*} \\ (-1.86) \end{gathered}$ | $\begin{gathered} -0.02 \\ (-0.16) \end{gathered}$ | $\begin{gathered} -0.06 \\ (-0.55) \end{gathered}$ | $\begin{aligned} & -0.09 \\ & (-1.08) \end{aligned}$ |
| SMB | $\begin{gathered} 0.13^{*} \\ (1.92) \end{gathered}$ | $\begin{gathered} 0.13 \\ (1.32) \end{gathered}$ | $\begin{gathered} 0.10 \\ (1.49) \end{gathered}$ | $\begin{gathered} -0.07 \\ (-0.89) \end{gathered}$ | $\begin{gathered} -0.06 \\ (-0.70) \end{gathered}$ | $\begin{gathered} 0.07 \\ (0.96) \end{gathered}$ | $\begin{gathered} -0.03 \\ (-0.37) \end{gathered}$ | $\begin{aligned} & 0.18^{* *} \\ & (2.03) \end{aligned}$ | $\begin{gathered} 0.05 \\ (0.54) \end{gathered}$ | $\begin{gathered} -0.06 \\ (-0.72) \end{gathered}$ |
| HML | $\begin{aligned} & -0.00 \\ & (-0.08) \end{aligned}$ | $\begin{gathered} -0.16^{*} \\ (-1.78) \end{gathered}$ | $\begin{aligned} & 0.18^{* * *} \\ & (2.84) \end{aligned}$ | $\begin{aligned} & 0.21^{* * *} \\ & (2.62) \end{aligned}$ | $\begin{aligned} & 0.25^{* *} \\ & (2.47) \end{aligned}$ | $\begin{aligned} & -0.21^{* * *} \\ & (-2.64) \end{aligned}$ | $\begin{gathered} -0.19 \\ (-1.65) \end{gathered}$ | $\begin{gathered} 0.06 \\ (0.77) \end{gathered}$ | $\begin{gathered} 0.11 \\ (1.31) \end{gathered}$ | $\begin{gathered} -0.18 \\ (-1.53) \end{gathered}$ |
| RMW | $\begin{gathered} 0.11^{* *} \\ (2.02) \end{gathered}$ | $\begin{gathered} -0.06 \\ (-0.86) \end{gathered}$ | $\begin{gathered} -0.06 \\ (-0.92) \end{gathered}$ | $\begin{gathered} 0.01 \\ (0.11) \end{gathered}$ | $\begin{gathered} -0.03 \\ (-0.40) \end{gathered}$ | $\begin{gathered} 0.06 \\ (1.05) \end{gathered}$ | $\begin{gathered} 0.04 \\ (0.42) \end{gathered}$ | $\begin{gathered} -0.17^{* *} \\ (-2.18) \end{gathered}$ | $\begin{gathered} 0.10 \\ (1.29) \end{gathered}$ | $\begin{gathered} 0.08 \\ (1.25) \end{gathered}$ |
| CMA | $\begin{gathered} 0.02 \\ (0.41) \end{gathered}$ | $\begin{gathered} 0.08 \\ (1.15) \end{gathered}$ | $\begin{aligned} & -0.22^{* * *} \\ & (-3.74) \end{aligned}$ | $\begin{gathered} -0.01 \\ (-0.11) \end{gathered}$ | $\begin{gathered} 0.06 \\ (1.03) \end{gathered}$ | $\begin{gathered} 0.10^{*} \\ (1.90) \end{gathered}$ | $\begin{gathered} -0.06 \\ (-0.76) \end{gathered}$ | $\begin{gathered} -0.02 \\ (-0.33) \end{gathered}$ | $\begin{gathered} -0.08 \\ (-0.95) \end{gathered}$ | $\begin{gathered} -0.10 \\ (-1.07) \end{gathered}$ |
| MOM | $\begin{aligned} & -0.35^{* * *} \\ & (-5.39) \end{aligned}$ | $\begin{gathered} -0.08 \\ (-0.82) \end{gathered}$ | $\begin{gathered} -0.02 \\ (-0.19) \end{gathered}$ | $\begin{aligned} & 0.42^{* * *} \\ & (4.64) \end{aligned}$ | $\begin{gathered} 0.13 \\ (1.25) \end{gathered}$ | $\begin{aligned} & -0.21^{* * *} \\ & (-2.85) \end{aligned}$ | $\begin{gathered} -0.32 \\ (-1.52) \end{gathered}$ | $\begin{gathered} 0.09 \\ (0.93) \end{gathered}$ | $\begin{gathered} 0.06 \\ (0.74) \end{gathered}$ | $\begin{gathered} -0.03 \\ (-0.36) \end{gathered}$ |
| MKTB | $\begin{gathered} -0.06 \\ (-0.60) \end{gathered}$ | $\begin{gathered} -0.02 \\ (-0.12) \end{gathered}$ | $\begin{aligned} & -0.45^{* * *} \\ & (-3.84) \end{aligned}$ | $\begin{aligned} & 0.51^{* * *} \\ & (5.32) \end{aligned}$ | $\begin{aligned} & -0.38^{* * *} \\ & (-3.75) \end{aligned}$ | $\begin{aligned} & -0.28^{* * *} \\ & (-3.01) \end{aligned}$ | $\begin{aligned} & 0.31^{* * *} \\ & (3.29) \end{aligned}$ | $\begin{aligned} & 0.24^{* * *} \\ & (2.71) \end{aligned}$ | $\begin{gathered} -0.13 \\ (-0.92) \end{gathered}$ | $\begin{gathered} -0.18 \\ (-1.36) \end{gathered}$ |
| CRF | $\begin{aligned} & 0.27^{* * *} \\ & (3.98) \end{aligned}$ | $\begin{aligned} & -0.52^{* * *} \\ & (-4.13) \end{aligned}$ | $\begin{gathered} -0.11 \\ (-0.88) \end{gathered}$ | $\begin{gathered} 0.00 \\ (0.06) \end{gathered}$ | $\begin{gathered} -0.16 \\ (-1.10) \end{gathered}$ | $\begin{aligned} & -0.44^{* * *} \\ & (-3.46) \end{aligned}$ | $\begin{gathered} -0.00 \\ (-0.02) \end{gathered}$ | $\begin{gathered} -0.01 \\ (-0.08) \end{gathered}$ | $\begin{gathered} -0.07 \\ (-0.52) \end{gathered}$ | $\begin{gathered} -0.02 \\ (-0.23) \end{gathered}$ |
| LRF | $\begin{aligned} & 0.20^{* * *} \\ & (2.86) \end{aligned}$ | $\begin{gathered} 0.03 \\ (0.27) \end{gathered}$ | $\begin{gathered} 0.15 \\ (1.58) \end{gathered}$ | $\begin{gathered} 0.11 \\ (1.05) \end{gathered}$ | $\begin{aligned} & 0.33^{* * *} \\ & (2.78) \end{aligned}$ | $\begin{gathered} -0.05 \\ (-0.45) \end{gathered}$ | $\begin{gathered} -0.01 \\ (-0.12) \end{gathered}$ | $\begin{aligned} & -0.21^{* *} \\ & (-2.23) \end{aligned}$ | $\begin{gathered} -0.22 \\ (-1.51) \end{gathered}$ | $\begin{gathered} 0.17 \\ (1.63) \end{gathered}$ |
| LTR | $\begin{gathered} 0.08 \\ (0.89) \end{gathered}$ | $\begin{gathered} -0.15 \\ (-1.47) \end{gathered}$ | $\begin{gathered} 0.14^{* *} \\ (1.97) \end{gathered}$ | $\begin{gathered} -0.00 \\ (-0.02) \end{gathered}$ | $\begin{gathered} -0.05 \\ (-0.60) \end{gathered}$ | $\begin{aligned} & 0.44^{* * *} \\ & (5.56) \end{aligned}$ | $\begin{aligned} & 0.23^{* *} \\ & (2.15) \end{aligned}$ | $\begin{gathered} -0.03 \\ (-0.28) \end{gathered}$ | $\begin{gathered} 0.14 \\ (1.26) \end{gathered}$ | $\begin{gathered} -0.19^{*} \\ (-1.73) \end{gathered}$ |
| MOMB | $\begin{gathered} -0.09 \\ (-1.30) \end{gathered}$ | $\begin{gathered} -0.09 \\ (-0.76) \end{gathered}$ | $\begin{aligned} & -0.24^{* *} \\ & (-2.48) \end{aligned}$ | $\begin{aligned} & -0.24^{* *} \\ & (-2.31) \end{aligned}$ | $\begin{aligned} & -0.22^{* *} \\ & (-2.27) \end{aligned}$ | $\begin{aligned} & -0.31^{* *} \\ & (-2.35) \end{aligned}$ | $\begin{aligned} & 0.28^{* *} \\ & (2.06) \end{aligned}$ | $\begin{gathered} 0.09 \\ (0.77) \end{gathered}$ | $\begin{gathered} 0.01 \\ (0.12) \end{gathered}$ | $\begin{gathered} 0.03 \\ (0.39) \end{gathered}$ |
| REVstar | $\begin{gathered} 0.07 \\ (0.84) \end{gathered}$ | $\begin{gathered} 0.13 \\ (1.31) \end{gathered}$ | $\begin{gathered} -0.20 \\ (-1.18) \end{gathered}$ | $\begin{gathered} 0.05 \\ (0.77) \end{gathered}$ | $\begin{gathered} -0.01 \\ (-0.15) \end{gathered}$ | $\begin{gathered} 0.17^{*} \\ (1.67) \end{gathered}$ | $\begin{gathered} -0.00 \\ (-0.06) \end{gathered}$ | $\begin{aligned} & 0.21^{* * *} \\ & (2.74) \end{aligned}$ | $\begin{aligned} & 0.18^{* *} \\ & (2.06) \end{aligned}$ | $\begin{gathered} -0.04 \\ (-0.70) \end{gathered}$ |
| MKToption | $\begin{gathered} 0.16 \\ (1.64) \end{gathered}$ | $\begin{gathered} 0.21 \\ (1.17) \end{gathered}$ | $\begin{aligned} & 0.27^{* *} \\ & (2.25) \end{aligned}$ | $\begin{gathered} -0.18 \\ (-1.19) \end{gathered}$ | $\begin{gathered} 0.24^{*} \\ (1.66) \end{gathered}$ | $\begin{gathered} -0.23 \\ (-1.61) \end{gathered}$ | $\begin{gathered} 0.21 \\ (1.22) \end{gathered}$ | $\begin{gathered} 0.26 \\ (1.38) \end{gathered}$ | $\begin{aligned} & 0.40^{* *} \\ & (2.36) \end{aligned}$ | $\begin{gathered} -0.06 \\ (-0.48) \end{gathered}$ |
| LEVEL | $\begin{aligned} & -0.08 \\ & (-0.62) \end{aligned}$ | $\begin{gathered} 0.41^{*} \\ (1.75) \end{gathered}$ | $\begin{gathered} 0.10 \\ (0.77) \end{gathered}$ | $\begin{aligned} & 0.42^{* * *} \\ & (3.19) \end{aligned}$ | $\begin{gathered} -0.16 \\ (-1.40) \end{gathered}$ | $\begin{aligned} & -0.07 \\ & (-0.51) \end{aligned}$ | $\begin{gathered} -0.18 \\ (-0.98) \end{gathered}$ | $\begin{aligned} & 0.41^{* * *} \\ & (2.88) \end{aligned}$ | $\begin{gathered} -0.17 \\ (-1.05) \end{gathered}$ | $\begin{aligned} & 0.47^{* * *} \\ & (3.87) \end{aligned}$ |
| SKEW | $\begin{gathered} 0.01 \\ (0.16) \end{gathered}$ | $\begin{gathered} -0.17 \\ (-1.18) \end{gathered}$ | $\begin{gathered} -0.01 \\ (-0.07) \end{gathered}$ | $\begin{gathered} 0.00 \\ (0.03) \end{gathered}$ | $\begin{gathered} 0.01 \\ (0.13) \end{gathered}$ | $\begin{aligned} & 0.25^{* *} \\ & (2.47) \end{aligned}$ | $\begin{gathered} 0.02 \\ (0.14) \end{gathered}$ | $\begin{aligned} & -0.53^{* * *} \\ & (-4.14) \end{aligned}$ | $\begin{gathered} 0.13 \\ (1.16) \end{gathered}$ | $\begin{gathered} 0.13 \\ (1.53) \end{gathered}$ |
| IVOL | $\begin{gathered} -0.04 \\ (-0.60) \end{gathered}$ | $\begin{gathered} 0.11 \\ (0.91) \end{gathered}$ | $\begin{gathered} 0.01 \\ (0.13) \end{gathered}$ | $\begin{gathered} -0.11 \\ (-1.41) \end{gathered}$ | $\begin{gathered} -0.17^{*} \\ (-1.89) \end{gathered}$ | $\begin{gathered} -0.08 \\ (-0.63) \end{gathered}$ | $\begin{gathered} -0.17 \\ (-1.50) \end{gathered}$ | $\begin{aligned} & -0.21^{*} \\ & (-1.71) \end{aligned}$ | $\begin{gathered} 0.05 \\ (0.43) \end{gathered}$ | $\begin{gathered} 0.15 \\ (1.01) \end{gathered}$ |
| ILQ | $\begin{gathered} 0.05 \\ (0.68) \end{gathered}$ | $\begin{aligned} & -0.25^{*} \\ & (-1.81) \end{aligned}$ | $\begin{gathered} -0.06 \\ (-0.46) \end{gathered}$ | $\begin{gathered} 0.04 \\ (0.64) \end{gathered}$ | $\begin{gathered} 0.10 \\ (1.35) \end{gathered}$ | $\begin{gathered} 0.15 \\ (1.53) \end{gathered}$ | $\begin{gathered} -0.10 \\ (-0.80) \end{gathered}$ | $\begin{gathered} 0.08 \\ (0.83) \end{gathered}$ | $\begin{gathered} -0.14 \\ (-1.44) \end{gathered}$ | $\begin{aligned} & -0.23^{* *} \\ & (-2.42) \end{aligned}$ |
| VOLDEV | $\begin{gathered} -0.04 \\ (-0.96) \end{gathered}$ | $\begin{gathered} 0.11 \\ (1.51) \end{gathered}$ | $\begin{gathered} -0.10^{*} \\ (-1.71) \end{gathered}$ | $\begin{gathered} 0.06 \\ (0.85) \end{gathered}$ | $\begin{gathered} -0.09 \\ (-0.88) \end{gathered}$ | $\begin{gathered} -0.04 \\ (-0.53) \end{gathered}$ | $\begin{gathered} -0.05 \\ (-0.52) \end{gathered}$ | $\begin{aligned} & -0.31^{* * *} \\ & (-3.00) \end{aligned}$ | $\begin{aligned} & 0.19 * * \\ & (2.43) \end{aligned}$ | $\begin{aligned} & -0.38^{* * *} \\ & (-4.03) \end{aligned}$ |
| $R_{\text {adj }}^{2}$ | 57.40\% | 29.26\% | 31.20\% | 51.55\% | 35.89\% | $45.21 \%$ | 18.55\% | 33.28\% | 19.18\% | 24.13\% |
| No. Obs. | 210 | 210 | 210 | 210 | 210 | 210 | 210 | 210 | 210 | 210 |

$\dagger * * *: p$-value $<1 \% ;{ }^{* *}: p$-value $<5 \% ; *: p$-value $<10 \%$. $t$-statistics are reported in parentheses. The regressed PCA factors and the observable factors are standardised using the time series standard deviation. We report the t-statistics using Newey-West standard errors with four lags.

Table 4: Correlations between the regressed-PCA joint latent factors and observable factors ${ }^{\dagger}$

|  | Joint1 | Joint2 | Joint3 | Joint4 | Joint5 | Joint6 | Joint7 | Joint8 | Joint9 | Joint10 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| MKTstock | $0.48^{* * *}$ | 0.00 | $0.22^{* * *}$ | $0.17^{* *}$ | $-0.45^{* * *}$ | $-0.19^{* * *}$ | -0.11 | $0.20^{* * *}$ | 0.04 | -0.10 |
| SMB | $0.35^{* * *}$ | -0.04 | $0.21^{* * *}$ | 0.02 | $-0.20^{* * *}$ | 0.01 | $-0.12^{*}$ | $0.25^{* * *}$ | 0.05 | $-0.19^{* * *}$ |
| HML | $0.32^{* * *}$ | $-0.19^{* * *}$ | $0.21^{* * *}$ | 0.02 | 0.10 | -0.09 | $-0.11^{*}$ | 0.04 | 0.08 | $-0.20^{* * *}$ |
| RMW | -0.07 | 0.02 | $-0.18^{* * *}$ | 0.01 | 0.10 | 0.03 | 0.05 | $-0.20^{* * *}$ | 0.09 | $0.13^{*}$ |
| CMA | 0.02 | -0.06 | $-0.13^{*}$ | -0.09 | $0.24^{* * *}$ | $0.15^{* *}$ | -0.05 | 0.02 | -0.06 | $-0.13^{*}$ |
| MOM | $-0.64^{* * *}$ | 0.08 | $-0.16^{* *}$ | 0.00 | $0.17^{* *}$ | $-0.14^{* *}$ | $-0.16^{* *}$ | 0.09 | 0.05 | $0.14^{* *}$ |
| MKTB | $0.41^{* * *}$ | $0.13^{*}$ | -0.01 | $0.58^{* * *}$ | $-0.39^{* * *}$ | -0.09 | $0.20^{* * *}$ | $0.13^{*}$ | -0.11 | 0.05 |
| CRF | $0.60^{* * *}$ | $-0.25^{* * *}$ | $0.26^{* * *}$ | $0.14^{* *}$ | $-0.35^{* * *}$ | $-0.21^{* * *}$ | -0.09 | $0.14^{* *}$ | 0.02 | -0.10 |
| LRF | $0.42^{* * *}$ | 0.08 | $0.13^{*}$ | $0.37^{* * *}$ | -0.04 | 0.06 | $0.20^{* * *}$ | -0.10 | $-0.18^{* * *}$ | 0.07 |
| LTR | $0.46^{* * *}$ | -0.06 | $0.33^{* * *}$ | $0.24^{* * *}$ | -0.09 | $0.31^{* * *}$ | $0.16^{* *}$ | -0.00 | 0.04 | $-0.13^{*}$ |
| MOMB | $-0.49^{* * *}$ | -0.05 | $-0.27^{* * *}$ | $-0.36^{* * *}$ | 0.06 | $-0.32^{* * *}$ | 0.02 | 0.01 | -0.01 | 0.04 |
| REVstar | $0.21^{* * *}$ | $0.19^{* * *}$ | $-0.16^{* *}$ | $0.17^{* *}$ | $-0.13^{*}$ | $0.22^{* * *}$ | -0.02 | $0.19^{* * *}$ | $0.15^{* *}$ | -0.06 |
| MKToption | $0.35^{* * *}$ | $0.23^{* * *}$ | $0.34^{* * *}$ | $0.28^{* * *}$ | $-0.27^{* * *}$ | $-0.23^{* * *}$ | -0.11 | $0.15^{* *}$ | $0.26^{* * *}$ | 0.07 |
| LEVEL | $0.36^{* * *}$ | $0.25^{* * *}$ | $0.27^{* * *}$ | $0.48^{* * *}$ | $-0.35^{* * *}$ | $-0.18^{* * *}$ | -0.08 | $0.14^{* *}$ | $0.17^{* *}$ | $0.17^{* *}$ |
| SKEW | $0.33^{* * *}$ | $0.16^{* *}$ | $0.21^{* * *}$ | $0.39^{* * *}$ | $-0.29^{* * *}$ | -0.06 | -0.01 | -0.03 | $0.18^{* *}$ | $0.16^{* *}$ |
| IVOL | $0.19^{* * *}$ | $0.17^{* *}$ | $0.21^{* * *}$ | 0.07 | $-0.18^{* * *}$ | $-0.17^{* *}$ | $-0.16^{* *}$ | -0.02 | $0.17^{* *}$ | -0.02 |
| ILQ | $0.21^{* * *}$ | -0.01 | $0.15^{* *}$ | 0.02 | $-0.18^{* * *}$ | $-0.18^{* *}$ | $-0.21^{* * *}$ | $0.19^{* * *}$ | 0.05 | $-0.14^{* *}$ |
| VOLDEV | 0.09 | $0.24^{* * *}$ | 0.07 | 0.10 | $-0.21^{* * *}$ | -0.08 | -0.08 | $-0.26_{* * *}^{0}$ | $0.28^{* * *}$ | $-0.19^{* * *}$ |

$\dagger$ ***: $p$-value $<1 \% ;{ }^{* *}: p$-value $<5 \% ; *: p$-value $<10 \%$.

Table 5: Correlations between the regressed-PCA joint latent factors and macroeconomic variables ${ }^{\dagger}$

|  | Joint1 | Joint2 | Joint3 | Joint4 | Joint5 | Joint6 | Joint7 | Joint8 | Joint9 | Joint10 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Core inflation | -0.05 | -0.05 | -0.06 | -0.04 | 0.03 | 0.02 | -0.03 | -0.10 | $0.14^{* *}$ | 0.05 |
| $\Delta c$ | -0.06 | -0.10 | $0.48^{* * *}$ | 0.06 | -0.03 | $-0.08^{2}$ | $-0.23^{* * *}$ | 0.10 | $-0.16^{* *}$ | -0.02 |
| $\Delta I N D P R O$ | $-0.22^{* * *}$ | $-0.17^{* *}$ | $0.40^{* * *}$ | 0.03 | 0.06 | -0.03 | $-0.21^{* * *}$ | 0.05 | $-0.15^{* *}$ | -0.01 |
| $\Delta(E P U)$ | $-0.16^{* *}$ | 0.08 | $-0.36^{* * *}$ | $-0.17^{* *}$ | $0.13^{*}$ | 0.05 | $0.17^{* *}$ | $-0.14^{* *}$ | 0.03 | $0.12^{*}$ |
| $\Delta(F F R)$ | 0.01 | $-0.25^{* * *}$ | $0.14^{* *}$ | 0.09 | $-0.22^{* * *}$ | $-0.38^{* * *}$ | -0.11 | 0.03 | $-0.14^{* *}$ | -0.04 |
| $\Delta(T E R M)$ | $0.22^{* * *}$ | -0.07 | $0.22^{* * *}$ | $-0.19^{* * *}$ | $0.23^{* * *}$ | 0.10 | $-0.13^{*}$ | -0.10 | 0.10 | $-0.15^{* *}$ |
| $\Delta(O A S)$ | -0.08 | $-0.17^{* *}$ | 0.00 | $-0.21^{* * *}$ | -0.05 | $-0.19^{* * *}$ | $0.12^{*}$ | -0.05 | 0.11 | $0.17^{* *}$ |
| $\Delta(V I X)$ | $-0.40^{* * *}$ | $-0.17^{* *}$ | -0.07 | $-0.29^{* * *}$ | $0.46^{* * *}$ | 0.09 | 0.06 | -0.00 | -0.03 | -0.05 |
| $\Delta(F I N U)$ | $-0.41^{* * *}$ | $-0.17^{* *}$ | $-0.18^{* *}$ | $-0.30^{* * *}$ | $0.15^{* *}$ | $0.17^{* *}$ | 0.04 | $-0.21^{* * *}$ | 0.07 | $-0.15^{* *}$ |
| $\Delta(M A C U)$ | $-0.28^{* * *}$ | 0.04 | $-0.14^{* *}$ | $-0.21^{* * *}$ | $0.17^{* *}$ | $0.25^{* * *}$ | $0.12^{*}$ | -0.11 | $0.13^{*}$ | 0.02 |
| HKM | $0.48^{* * *}$ | $-0.15^{* *}$ | $0.29^{* * *}$ | 0.04 | $-0.33^{* * *}$ | $-0.17^{* *}$ | -0.10 | $0.12^{*}$ | 0.00 | $-0.16^{* *}$ |

$\dagger$ ***: $p$-value $<1 \% ;{ }^{* *}: p$-value $<5 \% ; *: p$-value $<10 \%$.

Table 6: Regression of regressed-PCA joint latent factors on macroeconomic variables ${ }^{\dagger}$

|  | Joint1 | Joint2 | Joint3 | Joint4 | Joint5 | Joint6 | Joint7 | Joint8 | Joint9 | Joint10 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Core inflation | 0.06 | -0.02 | -0.00 | -0.00 | 0.06 | -0.01 | -0.06 | -0.06 | 0.14*** | 0.06 |
|  | (1.38) | (-0.25) | (-0.17) | (-0.05) | (1.27) | (-0.13) | (-1.38) | (-1.36) | (2.92) | (1.21) |
|  | [0.00] | [0.01] | [0.00] | [0.00] | [0.00] | [0.00] | [0.03] | [0.05] | [0.22] | [0.02] |
| $\Delta c$ | 0.08 | 0.19 | 0.24** | -0.01 | -0.09 | -0.02 | -0.12 | 0.11 | -0.18 | 0.01 |
|  | (0.96) | (1.62) | (2.55) | (-0.08) | (-0.89) | (-0.14) | (-0.75) | (0.95) | (-0.87) | (0.11) |
|  | [0.02] | [0.03] | [0.29] | [0.01] | [0.01] | [0.01] | [0.23] | [0.05] | [0.15] | [0.01] |
| $\triangle I N D P R O$ | $-0.33^{* * *}$ | -0.19 | 0.25* | -0.01 | 0.12 | 0.14 | -0.09 | -0.06 | 0.00 | 0.08 |
|  | (-3.59) | (-1.59) | (1.69) | (-0.04) | (1.12) | (0.9) | (-0.47) | (-0.43) | (0.02) | (0.72) |
|  | [0.12] | [0.08] | [0.21] | [0.01] | [0.02] | [0.02] | [0.19] | [0.01] | [0.10] | [0.01] |
| $\Delta(E P U)$ | -0.03 | 0.15 | -0.16** | -0.06 | 0.02 | -0.03 | 0.03 | -0.02 | -0.17 | 0.12 |
|  | (-0.46) | (1.64) | (-2.22) | (-0.7) | (0.35) | (-0.36) | (0.35) | (-0.18) | (-1.52) | (1.19) |
|  | [0.03] | [0.04] | [0.14] | [0.05] | [0.02] | [0.01] | [0.09] | [0.06] | [0.08] | [0.10] |
| $\Delta(F F R)$ | -0.10 | -0.20* | -0.09 | 0.06 | -0.10 | -0.33** | -0.07 | -0.15** | -0.06 | -0.04 |
|  | (-1.29) | (-1.66) | (-1.14) | (0.63) | (-1.2) | (-2.25) | (-0.7) | (-1.96) | (-1.06) | (-0.5) |
|  | [0.02] | [0.22] | [0.02] | [0.02] | [0.07] | [0.46] | [0.05] | [0.04] | [0.10] | [0.02] |
| $\Delta($ TERM $)$ | 0.19*** | -0.08 | 0.18** | -0.10 | 0.22*** | 0.00 | -0.17*** | -0.20** | 0.12 | -0.10 |
|  | (2.91) | (-1.04) | (2.47) | (-1.44) | (3.84) | (0.02) | (-2.62) | (-2.15) | (1.55) | (-1.1) |
|  | [0.11] | [0.03] | [0.12] | [0.12] | [0.15] | [0.02] | [0.21] | [0.15] | [0.10] | [0.12] |
| $\Delta(O A S)$ | 0.01 | -0.21 | 0.13* | -0.16** | -0.11 | -0.21* | 0.07 | -0.02 | 0.13 | 0.15 |
|  | (0.15) | (-1.3) | (1.69) | (-2.18) | (-1.5) | (-1.83) | (0.87) | (-0.29) | (1.64) | (1.44) |
|  | [0.00] | [0.16] | [0.03] | [0.16] | [0.03] | [0.19] | [0.08] | [0.01] | [0.13] | [0.18] |
| $\Delta(V I X)$ | -0.04 | -0.17* | 0.01 | -0.24* | 0.43*** | -0.02 | 0.11 | 0.27* | -0.07 | -0.07 |
|  | (-0.39) | (-1.82) | (0.1) | (-1.71) | (4.97) | (-0.16) | (1.07) | (1.79) | (-0.55) | (-0.78) |
|  | [0.13] | [0.13] | [0.01] | [0.28] | [0.45] | [0.01] | [0.04] | [0.14] | [0.03] | [0.04] |
| $\Delta(F I N U)$ | $-0.20^{* * *}$ | -0.19* | -0.11 | -0.17 | -0.23** | 0.04 | -0.05 | -0.24** | 0.02 | $-0.27 * * *$ |
|  | (-2.7) | (-1.94) | (-1.34) | (-1.2) | (-2.32) | (0.67) | (-0.62) | (-2.08) | (0.25) | (-2.83) |
|  | [0.18] | [0.14] | [0.03] | [0.23] | [0.05] | [0.05] | [0.01] | [0.31] | [0.02] | [0.29] |
| $\Delta(M A C U)$ | -0.22** | 0.05 | 0.11 | -0.01 | 0.08 | 0.21* | 0.03 | 0.01 | 0.06 | 0.06 |
|  | (-2.33) | (0.4) | (1.11) | (-0.07) | (0.8) | (1.79) | (0.22) | (0.1) | (0.72) | (0.81) |
|  | [0.11] | [0.02] | [0.02] | [0.07] | [0.03] | [0.17] | [0.04] | [0.03] | [0.07] | [0.02] |
| HKM | 0.31** | -0.20** | 0.21 | -0.18 | -0.17** | -0.03 | 0.03 | 0.24*** | -0.02 | -0.18* |
|  | (2.26) | (-2.12) | (1.59) | (-1.3) | (-2.26) | (-0.17) | (0.18) | (2.72) | (-0.26) | (-1.8) |
|  | [0.28] | [0.14] | [0.13] | [0.06] | [0.18] | [0.05] | [0.03] | [0.15] | [0.01] | [0.19] |
| $R_{\text {adj }}^{2}$ | 39.07\% | 16.59\% | 32.49\% | 13.85\% | 28.43\% | 18.19\% | 4.70\% | 6.07\% | 3.84\% | 8.67\% |
| No.Obs | 210 | 210 | 210 | 210 | 210 | 210 | 210 | 210 | 210 | 210 |

$\dagger * * *: p$-value $<1 \% ;{ }^{* *}: p$-value $<5 \% ; *: p$-value $<10 \%$. $t$-statistics are reported in parentheses. The ShapleyOwen $R^{2}$ 's are in square brackets. The regressed PCA factors and the macroeconomic variables are standardised using the time series standard deviation. We report the t-statistics using Newey-West standard errors with four lags.

Table 7: In-sample and out-of-sample performance of the joint factor model when $\alpha(\cdot) \neq 0$

| (i) All the returns on three asset classes |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| K | $R_{K}^{2}$ | $R^{2}$ | $R_{f, O}^{2}$ | $R_{f, T, N, O}^{2}$ | $R_{f, N, T, O}^{2}$ |
| 1* | 15.21 | 6.11 | 25.96 | 5.18 | 12.56 |
| 2 | 25.15 | 7.27 | 26.55 | 8.95 | 13.65 |
| 3 | 32.91 | 8.88 | 26.85 | 12.46 | 14.29 |
| 4 | 40.16 | 11.46 | 26.98 | 11.80 | 14.50 |
| 5 | 46.02 | 13.34 | 27.19 | 11.45 | 14.79 |
| 6 | 52.12 | 14.26 | 27.42 | 11.15 | 15.14 |
| 7 | 56.59 | 14.76 | 27.52 | 10.01 | 15.31 |
| 8 | 60.40 | 15.51 | 27.69 | 12.45 | 15.63 |
| 9 | 63.87 | 16.31 | 27.84 | 13.25 | 15.91 |
| 10 | 67.01 | 16.76 | 27.95 | 13.11 | 16.09 |
| K | $R_{O}^{2}$ | $R_{T, N, O}^{2}$ | $R_{N, T, O}^{2}$ | $R_{\tilde{R}}^{2}$ |  |
| 1-10 | 23.97 | 8.90 | 8.86 | 24.83 |  |
| (ii) Stock Returns |  |  |  |  |  |
| K | $R_{K}^{2}$ | $R^{2}$ | $R_{f, O}^{2}$ | $R_{f, T, N, O}^{2}$ | $R_{f, N, T, O}^{2}$ |
| 1* | 21.85 | 6.27 | 9.49 | 12.06 | 8.13 |
| 2 | 22.57 | 6.29 | 10.06 | 12.52 | 8.59 |
| 3 | 32.97 | 7.04 | 10.95 | 14.09 | 9.48 |
| 4 | 38.11 | 7.41 | 11.18 | 14.30 | 9.65 |
| 5 | 48.35 | 9.27 | 11.25 | 13.93 | 9.70 |
| 6 | 52.09 | 9.56 | 11.55 | 14.39 | 10.00 |
| 7 | 54.47 | 9.77 | 11.78 | 14.50 | 10.26 |
| 8 | 59.07 | 10.74 | 11.79 | 14.33 | 10.25 |
| 9 | 65.15 | 11.12 | 11.96 | 14.30 | 10.43 |
| 10 | 67.73 | 11.51 | 12.00 | 14.25 | 10.45 |
| K | $R_{O}^{2}$ | $R_{T, N, O}^{2}$ | $R_{N, T, O}^{2}$ | $R_{\tilde{R}}^{2}$ |  |
| 1-10 | 0.22 | 0.71 | 0.04 | 18.64 |  |
| (iii) Corporate Bond Returns |  |  |  |  |  |
| K | $R_{K}^{2}$ | $R^{2}$ | $R_{f, O}^{2}$ | $R_{f, T, N, O}^{2}$ | $R_{f, N, T, O}^{2}$ |
| 1 | 13.45 | 11.21 | 2.40 | -27.78 | $-6.20$ |
| 2 | 23.70 | 11.58 | 0.52 | -31.78 | -12.08 |
| 3 | 32.58 | 13.74 | 2.14 | -27.16 | -11.54 |
| 4 | 40.98 | 23.30 | 2.85 | -27.23 | -11.93 |
| 5 | 45.08 | 27.79 | 4.46 | -27.98 | -9.35 |
| 6 | 49.94 | 29.35 | 11.29 | -19.45 | -1.63 |
| $7 *$ | 58.40 | 32.20 | 13.07 | -17.79 | 0.46 |
| 8 | 59.73 | 32.99 | 14.03 | -18.28 | 1.09 |
| 9 | 60.92 | 33.74 | 16.60 | -15.09 | 4.65 |
| 10 | 65.12 | 33.99 | 17.43 | $-14.70$ | 5.37 |
| K | $R_{O}^{2}$ | $R_{T, N, O}^{2}$ | $R_{N, T, O}^{2}$ | $R_{\tilde{R}}^{2}$ |  |
| 1-10 | 2.22 | 5.48 | 1.67 | $\stackrel{R}{R}$ |  |
| (iv) Option Returns |  |  |  |  |  |
| K | $R_{K}^{2}$ | $R^{2}$ | $R_{f, O}^{2}$ | $R_{f, T, N, O}^{2}$ | $R_{f, N, T, O}^{2}$ |
| 1 | 8.82 | 5.40 | 29.30 | 5.47 | 12.97 |
| $2^{*}$ | 29.73 | 7.66 | 29.93 | 9.38 | 14.59 |
| 3 | 33.15 | 9.97 | 30.10 | 12.94 | 15.12 |
| 4 | 41.88 | 13.76 | 30.20 | 12.26 | 15.30 |
| 5 | 44.07 | 15.35 | 30.41 | 11.92 | 15.48 |
| 6 | 54.30 | 16.76 | 30.50 | 11.49 | 15.54 |
| 7 | 57.40 | 17.26 | 30.56 | 10.28 | 15.56 |
| 8 | 62.69 | 17.81 | 30.74 | 12.82 | 15.99 |
| 9 | 65.20 | 19.00 | 30.83 | 13.61 | 16.20 |
| 10 | 67.97 | 19.53 | 30.94 | 13.46 | 16.41 |
| K | $R_{O}^{2}$ | $R_{T, N, O}^{2}$ | $R_{N, T, O}^{2}$ | $R_{\tilde{R}}^{2}$ |  |
| 1-10 | 28.55 | 9.12 | 11.24 | 27.80 |  |

Note: $K$ : the number of factors specified, * denotes the estimator of $K$ which maximizes the ratio of two adjacent eigenvalues; $R_{\tilde{R}}^{2}$ : Fama-MacBeth cross-sectional regression $R^{2} ; R_{K}^{2}$ measures the variations in the characteristic-managed portfolios captured by different numbers of factors from PCA; $R^{2}$ : total in-sample $R^{2}(\%)$, see (2.8); $R_{O}^{2}, R_{T, N, O}^{2}, R_{N, T, O}^{2}, R_{f, O}^{2}, R_{f, T, N, O}^{2}, R_{f, N, T, O}^{2}$ : out-of-sample fits $R^{2}$,s (\%), see (2.9)-(2.14);

Table 8: MVE portfolios and joint factors ${ }^{\dagger}$

| Sharpe ratios | Common | Stock | Corpbond | Option |
| :---: | :---: | :---: | :---: | :---: |
| MVE | 7.43 | 2.65 | 1.09 | 2.75 |
|  | - | 4.40 | 2.22 | 4.31 |
| $\rho$ | Common | Stock | Corpbond | Option |
| Common | - | 0.36 | 0.15 | 0.37 |
| Stock | - | - | -0.45 | -0.53 |
| Corpbond | - | - | - | -0.06 |
| Joint factor $1(a \neq 0)$ | 0.11 | -0.41 | 0.37 | 0.31 |
| Joint factor $2(a \neq 0)$ | 0.08 | 0.26 | -0.08 | -0.17 |
| Joint factor $1(a=0)$ | 0.15 | -0.29 | 0.32 | 0.25 |
| Joint factor $2(a=0)$ | 0.04 | 0.39 | -0.22 | -0.27 |
| $\beta$ | Common | Stock | Corpbond | Option |
| Joint factor $1(a \neq 0)$ | 1.24 | -5.75 | 3.25 | 3.74 |
| $t$-value | [1.66] | [-6.40] | [5.82] | [4.70] |
| $R^{2}$ | 1.32\% | 16.44\% | 14.00\% | 9.59\% |
| Joint factor $2(a \neq 0)$ | 0.94 | 3.76 | -0.74 | -2.08 |
| $t$-value | [1.21] | [3.82] | [-1.19] | [-2.43] |
| $R^{2}$ | 0.70\% | 6.54\% | 0.67\% | 2.75\% |
| Joint factor $1(a=0)$ | 1.69 | -4.18 | 2.84 | 3.03 |
| $t$-value | [2.26] | [-4.40] | [4.93] | [3.70] |
| $R^{2}$ | 2.39\% | 8.51\% | 10.47\% | 6.16\% |
| Joint factor $2(a=0)$ | 0.45 | 5.70 | -1.94 | -3.31 |
| $t$-value | [0.57] | [6.09] | [-3.20] | [-3.96] |
| $R^{2}$ | 0.16\% | 15.13\% | 4.68\% | 7.02\% |

[^13]Table 9: MVE portfolios and joint factors correlation ${ }^{\dagger}$

| Correlation ( $\rho$ ) | Common | Stock | Corpbond | Option |
| :---: | :---: | :---: | :---: | :---: |
| Joint factor $1(a \neq 0)$ | 0.11 | -0.41 | 0.37 | 0.31 |
| Joint factor $2(a \neq 0)$ | 0.08 | 0.26 | -0.08 | -0.17 |
| Joint factor $3(a \neq 0)$ | 0.06 | -0.09 | 0.07 | 0.11 |
| Joint factor $4(a \neq 0)$ | 0.00 | -0.17 | 0.10 | 0.13 |
| Joint factor $5(a \neq 0)$ | 0.03 | 0.17 | -0.06 | -0.13 |
| Joint factor $6(a \neq 0)$ | 0.07 | -0.22 | 0.21 | 0.18 |
| Joint factor $7(a \neq 0)$ | 0.05 | 0.03 | -0.28 | 0.22 |
| Joint factor $8(a \neq 0)$ | 0.05 | 0.09 | -0.14 | 0.04 |
| Joint factor $9(a \neq 0)$ | 0.06 | -0.03 | -0.04 | 0.11 |
| Joint factor $10(a \neq 0)$ | 0.00 | 0.28 | -0.22 | -0.17 |
| Joint factor $1(a=0)$ | 0.15 | -0.29 | 0.32 | 0.25 |
| Joint factor $2(a=0)$ | 0.04 | 0.39 | -0.22 | -0.27 |
| Joint factor $3(a=0)$ | 0.06 | -0.08 | 0.05 | 0.11 |
| Joint factor $4(a=0)$ | 0.00 | -0.17 | 0.10 | 0.13 |
| Joint factor $5(a=0)$ | 0.06 | 0.05 | 0.02 | -0.01 |
| Joint factor 6 ( $a=0$ ) | 0.04 | -0.26 | 0.17 | 0.22 |
| Joint factor 7 ( $a=0$ ) | 0.06 | 0.08 | -0.33 | 0.21 |
| Joint factor $8(a=0)$ | 0.05 | 0.11 | -0.12 | 0.00 |
| Joint factor $9(a=0)$ | 0.05 | -0.02 | -0.01 | 0.07 |
| Joint factor $10(a=0)$ | 0.01 | 0.28 | -0.22 | -0.17 |

${ }^{\dagger}$ The first and second panel report the correlation ( $\rho$ ) of MVE portfolios across all asset classes with the components from individual asset classes, and the first and second joint factors with and without restriction $a=0$.

Table 10: Out-of-sample Sharpe ratios of MVE/tangency portfolios ${ }^{\dagger}$

| MVE | Common | Stock | Corpbond | Option |
| :---: | :---: | :---: | :---: | :---: |
| Sharpe ratio | 2.30 | 1.55 | 0.12 | 0.93 |
|  | - | 2.24 | 0.40 | 3.10 |
| $\rho$ | Common | Stock | Corpbond | Option |
| Common | - | 0.60 | 0.40 | 0.29 |
| Stock | - | - | -0.12 | -0.48 |
| Corpbond | - | - | - | 0.09 |
| Tangency | Common | Stock | Corpbond | Option |
| Sharpe ratio | 2.32 | 1.51 | 0.14 | 1.00 |
|  | - | 1.68 | 0.37 | 3.18 |
| $\rho$ | Common | Stock | Corpbond | Option |
| Common | - | 0.54 | 0.40 | 0.36 |
| Stock | - | - | -0.19 | -0.48 |
| Corpbond | - | - | - | 0.16 |

${ }^{\dagger}$ The reported Sharpe ratios are annualized. "MVE" is the optimal portfolio without restriction that sum of the weights is 1; "Tangency" is the optimal portfolio with this restriction. Within each mode, the first column reports the Sharpe ratios for the joint estimation across different assets classes; the second column reports the Sharpe ratios for the estimation within each asset class. The calculation is based on expanding window estimation starting with the sample of the first ten years.

Table 11: Out-of-sample Sharpe ratios of approximated MVE portfolios with different number of factors ${ }^{\dagger}$

| Sharpe ratio | Common | Stock | Corpbond | Option |
| :---: | :---: | :---: | :---: | :---: |
| $K=5$ | 1.24 | -0.47 | 0.24 | 1.83 |
|  | - | 0.14 | 0.36 | 2.18 |
| $K=6$ | 1.32 | -0.34 | 0.33 | 1.85 |
|  | - | -0.12 | 0.40 | 2.20 |
| $K=7$ | 1.38 | 0.01 | 0.15 | 2.19 |
|  | - | -0.20 | 0.42 | 2.11 |
| $K=8$ | 1.41 | 0.40 | 0.10 | 2.30 |
|  | - | -0.05 | 0.52 | 2.29 |
| $K=9$ | 1.64 | 0.61 | 0.60 | 2.31 |
|  | - | 0.11 | 0.22 | 2.18 |
| $K=10$ | 1.69 | 0.54 | 0.43 | 2.22 |
|  | - | 0.18 | 0.66 | 2.18 |

[^14]Table 12: Sharpe ratios of the regressed-PCA factors ${ }^{\dagger}$

| Factors | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Joint | $\mathbf{0 . 8 3}$ | $\mathbf{1 . 2 3}$ | 0.41 | 0.43 | 0.10 | $\mathbf{1 . 0 3}$ | $\mathbf{0 . 7 8}$ | $\mathbf{0 . 9 5}$ | $\mathbf{1 . 2 1}$ | $\mathbf{1 . 2 8}$ |
|  | 0.45 | $\mathbf{2 . 4 7}$ | 0.10 | $\mathbf{0 . 8 1}$ | 0.00 | -0.02 | $\mathbf{- 0 . 5 9}$ | 0.19 | -0.03 | -0.33 |
| Stock | 0.47 | 0.09 | 0.24 | 0.30 | 0.16 | 0.11 | 0.12 | $\mathbf{0 . 5 4}$ | 0.26 | 0.39 |
|  | 0.07 | -0.32 | -0.15 | -0.31 | 0.24 | 0.26 | -0.01 | -0.06 | -0.18 | -0.21 |
| Corpbond | 0.01 | $\mathbf{0 . 5 1}$ | 0.25 | 0.24 | 0.47 | 0.23 | $\mathbf{0 . 6 3}$ | 0.05 | 0.01 | $\mathbf{0 . 7 5}$ |
|  | $\mathbf{- 0 . 5 8}$ | $\mathbf{0 . 7 0}$ | -0.46 | -0.15 | $\mathbf{0 . 8 3}$ | $\mathbf{0 . 6 2}$ | $\mathbf{- 0 . 7 9}$ | 0.30 | -0.28 | $\mathbf{0 . 6 6}$ |
| Option | $\mathbf{1 . 3 7}$ | $\mathbf{2 . 3 0}$ | 0.20 | $\mathbf{1 . 2 9}$ | $\mathbf{1 . 3 5}$ | 0.41 | 0.35 | $\mathbf{1 . 5 5}$ | $\mathbf{0 . 7 4}$ | $\mathbf{1 . 7 3}$ |
|  | $\mathbf{2 . 6 3}$ | $\mathbf{- 0 . 6 5}$ | $\mathbf{- 0 . 6 2}$ | $\mathbf{0 . 8 3}$ | 0.12 | $\mathbf{0 . 5 1}$ | $\mathbf{0 . 9 3}$ | $\mathbf{1 . 0 1}$ | -0.05 | $\mathbf{1 . 2 6}$ |

$\dagger$ The reported Sharpe ratios are annualized. The first row for each category is the in-sample Sharpe ratios, the second row is the out-of-sample Sharpe ratios. Sharpe ratios with $t$-statistics greater than 2.0 are highlighted in bold print.

Table 13: Out-of-sample pure-alpha strategy and tangency portfolio performance of the joint factor model

| (i) All the returns on three asset classes |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| K | $\mu_{\alpha}$ | $\sigma_{\alpha}$ | $S R_{\alpha}$ | $S R_{\beta, U}$ |
| 1 | 7.50 | 3.50 | 2.14 | 0.45 |
| 2 | 3.33 | 3.25 | 1.02 | 2.09 |
| 3 | 3.29 | 3.18 | 1.03 | 2.07 |
| 4 | 3.05 | 3.20 | 0.95 | 2.14 |
| 5 | 3.07 | 3.20 | 0.96 | 2.12 |
| 6 | 3.06 | 3.03 | 1.01 | 1.86 |
| 7 | 3.61 | 2.94 | 1.23 | 0.89 |
| 8 | 3.27 | 2.36 | 1.39 | 0.66 |
| 9 | 3.26 | 2.16 | 1.51 | 0.60 |
| 10 | 3.52 | 1.79 | 1.96 | 0.33 |


| $($ ii Stock Returns |  |  |  |  |
| :--- | :--- | :--- | :---: | :---: |
|  | $K$ | $\mu_{\alpha}$ | $\sigma_{\alpha}$ | $S R_{\alpha}$ |
| 1 | 0.27 | 0.72 | 0.37 |  |
| 2 | 0.18 | 0.66 | 0.28 |  |
| 3 | 0.24 | 0.60 | 0.39 |  |
| 4 | 0.29 | 0.63 | 0.45 |  |
| 5 | 0.32 | 0.60 | 0.52 |  |
| 6 | 0.25 | 0.62 | 0.41 |  |
| 7 | 0.26 | 0.53 | 0.49 |  |
| 8 | 0.36 | 0.62 | 0.58 |  |
| 9 | 0.54 | 0.64 | 0.85 |  |
| 10 | 0.51 | 0.62 | 0.82 |  |


| (iii) Corporate Bond Returns |  |  |  |
| :---: | :---: | :---: | :---: |
|  | $K$ | $\mu_{\alpha}$ | $\sigma_{\alpha}$ |
| 1 | 0.71 | 0.67 | $S R_{\alpha}$ |
| 2 | -0.51 | 0.97 | -0.52 |
| 3 | -0.50 | 0.99 | -0.51 |
| 4 | -0.42 | 1.01 | -0.42 |
| 5 | -0.39 | 1.00 | -0.39 |
| 6 | -0.49 | 1.06 | -0.46 |
| 7 | -0.31 | 1.13 | -0.28 |
| 8 | -0.24 | 0.94 | -0.26 |
| 9 | -0.21 | 0.75 | -0.28 |
| 10 | -0.12 | 0.70 | -0.17 |
|  |  |  |  |
|  | $(\mathbf{i v})$ Option Returns |  |  |
| $K$ | $\mu_{\alpha}$ | $\sigma_{\alpha}$ | $S R_{\alpha}$ |
| 1 | 6.52 | 3.42 | 1.91 |
| 2 | 3.65 | 2.97 | 1.23 |
| 3 | 3.55 | 2.92 | 1.22 |
| 4 | 3.18 | 2.94 | 1.08 |
| 5 | 3.14 | 2.93 | 1.07 |
| 6 | 3.29 | 2.71 | 1.21 |
| 7 | 3.66 | 2.62 | 1.40 |
| 8 | 3.15 | 2.05 | 1.53 |
| 9 | 2.94 | 1.89 | 1.55 |
| 10 | 3.14 | 1.61 | 1.95 |

Note: $K$ : the number of factors specified; $\mu_{\alpha}, \sigma_{\alpha}$ and $S R_{\alpha}$ : out-of-sample annualized means, standard deviations, and Sharpe ratios of the pure-alpha arbitrage strategy (\%). $S R_{\beta, U}$ : out-of-sample annualized Sharpe ratios of the tangency portfolio of the first $K$ factors under the unrestricted case.

Table 14: Out-of-sample pure-alpha strategy and tangency portfolio performance of the joint factor model with nonlinear $\alpha(\cdot)$ and $\beta(\cdot)$

|  | (i) All the returns on three asset classes |  |  |  |  |
| :---: | :--- | :---: | :---: | :---: | :---: |
|  | $K$ | $\mu_{\alpha}$ | $\sigma_{\alpha}$ | $S R_{\alpha}$ | $S R_{\beta, U}$ |
| 1 | 12.36 | 8.99 | 1.38 | 0.00 |  |
| 2 | 11.05 | 9.26 | 1.19 | 0.49 |  |
| 3 | 6.02 | 6.48 | 0.93 | 1.25 |  |
| 4 | 5.94 | 6.37 | 0.93 | 1.22 |  |
| 5 | 6.22 | 5.41 | 1.15 | 0.88 |  |
| 6 | 5.81 | 4.26 | 1.37 | 0.70 |  |
| 7 | 5.45 | 4.16 | 1.31 | 0.74 |  |
| 8 | 5.14 | 3.17 | 1.62 | 0.71 |  |
| 9 | 5.37 | 3.10 | 1.73 | 0.65 |  |
| 10 | 5.50 | 2.86 | 1.92 | 0.56 |  |

(ii) Stock Returns

| (ii) Stock Returns |  |  |  |
| :--- | :--- | :--- | :--- |
|  | $K$ | $\mu_{\alpha}$ | $\sigma_{\alpha}$ |
| 1 | 0.87 | 1.95 | 0.44 |
| 2 | 0.85 | 1.67 | 0.51 |
| 3 | 0.51 | 1.78 | 0.28 |
| 4 | 0.41 | 1.63 | 0.25 |
| 5 | 0.52 | 1.66 | 0.31 |
| 6 | 0.49 | 1.55 | 0.32 |
| 7 | 0.50 | 1.51 | 0.33 |
| 8 | 0.30 | 1.10 | 0.27 |
| 9 | 0.53 | 1.05 | 0.50 |
| 10 | 0.45 | 0.96 | 0.46 |


|  | (iii) Corporate Bond Returns |  |  |
| :---: | :---: | :---: | :---: |
|  | $\mu_{\alpha}$ | $\sigma_{\alpha}$ | $S R_{\alpha}$ |
| 1 | 0.55 | 1.32 | 0.41 |
| 2 | 0.81 | 0.84 | 0.96 |
| 3 | -0.27 | 1.13 | -0.24 |
| 4 | -0.20 | 1.13 | -0.18 |
| 5 | 0.01 | 1.37 | 0.01 |
| 6 | 0.08 | 1.04 | 0.08 |
| 7 | -0.10 | 0.91 | -0.11 |
| 8 | 0.00 | 0.81 | 0.00 |
| 9 | -0.03 | 0.82 | -0.03 |
| 10 | -0.03 | 0.88 | -0.04 |
| (iv) Option Returns |  |  |  |
| K | $\mu_{\alpha}$ | $\sigma_{\alpha}$ | SR ${ }_{\alpha}$ |
| 1 | 10.95 | 8.69 | 1.26 |
| 2 | 9.39 | 9.15 | 1.03 |
| 3 | 5.79 | 6.37 | 0.91 |
| 4 | 5.73 | 6.28 | 0.91 |
| 5 | 5.70 | 5.30 | 1.07 |
| 6 | 5.24 | 3.90 | 1.34 |
| 7 | 5.05 | 3.68 | 1.37 |
| 8 | 4.84 | 2.93 | 1.65 |
| 9 | 4.87 | 2.89 | 1.69 |
| 10 | 5.08 | 2.64 | 1.92 |

Note: $K$ : the number of factors specified; $\mu_{\alpha}, \sigma_{\alpha}$ and $S R_{\alpha}$ : out-of-sample annualized means, standard deviations, and Sharpe ratios of the pure-alpha arbitrage strategy (\%). $S R_{\beta, U}$ : out-of-sample annualized Sharpe ratios of the tangency portfolio of the first $K$ factors under the unrestricted case.

## Appendix A1 Characteristics

In this section, we describes the characteristics that are applied to model returns on corporate bonds and options in detail. We also cite several papers that study these characteristics in empirical applications or show how to construct the characteristics.

We first provide detailed information on the 30 characteristics for corporate bonds. The first 12 characteristics are on the contract level, and the next 18 characteristics are on the stock level.

1. Bond age (age): Following Israel, Palhares, and Richardson (2017). Years since the date the bond was issued.
2. Coupon (cpn): Following Chung, Wang, and Wu (2019). Coupon payment adjusted for payment frequency.
3. Rating (rating): Numerical credit rating from 1 to 22, based on S\&P rating and Moody's rating.
4. Issue size (issue_size): The offering amount outstanding of the bond at issuance.
5. Duration (duration): Following Israel, Palhares, and Richardson (2017) and van Binsbergen and Schwert (2021). The sensitivity of bond value to credit spread.
6. Spread (spread): The yield spread, defined as the yield-to-maturity in excess of the one-month treasury yield.
7. Mom 6m (bond_mom): Following Gebhardt, Hvidkjaer, and Swaminathan (2005a). The most recent 6-2 cumulative bond returns, with a minimum period of 3 months.
8. Mom 6m Spread (spread_mom): Following Kelly, Palhares, and Pruitt (2022). The credit spread 6 months earlier minus current log spread.
9. Value-at-risk (VaR): Following Bai, Bali, and Wen (2019). The 2nd lowest credit excess return (in excess of one-mo Treasury bill rate) over the past 24 months, with a minimum of 12 months.
10. Short-term reversal (bond_strev): bond return reversal from Dickerson, Robotti, Rossetti (2023)
11. Long-term reversal (bond_ltrev): 48-minus-12-month reversal from Dickerson, Robotti, Rossetti (2023)
12. Bond illiquidity (illiq): MMN-adjusted bond illiquidity as per Bao, Pan, and Wang (2011) and Dickerson, Robotti, Rossetti (2023)
13. Tangibility (tan): Following Hahn and Lee (2009), defined as ( $0.715 \times$ total receivables $($ RECT $)+0.547 \times$ inventories $($ INVT $)+0.535 \times$ property, plant and equipment $($ PPENT $)+$ cash and short-term investments $(\mathrm{CHE})) /$ total assets (AT).
14. Total debt (debt): Defined as the sum of long-term debt and debt in current liabilities.
15. Debt-to-EBITDA ( $d 2 e b i t d a$ ): Total debt divided by EBITDA.
16. Distance-to-default ( $D D$ ): Merton model implied firm-specific distance to default, following Gilchrist and Zakrajšek (2012).

17-26. Book leverage (lev), Market beta (beta), Market capitalization (mktcap), Book-to-market ratio (bm), Gross profitability (prof), Investment (invest), Idiosyncratic volatility (idiovol), Stock momentum (mom), Operating leverage (ol), and Earnings-to-price ratio ( $e 2 p$ ): The data on these stock-level characteristics are from Freyberger, Höppner, Neuhierl, and Weber (2022). The above stock-level characteristics are also included in a number of studies such as Gebhardt, Hvidkjaer, and Swaminathan (2005b), Choi and Kim (2018), and Kelly, Palhares, and Pruitt (2022) to examine the effect of stock on corporate bond pricing.

For options, we present details on the 19 characteristics, the first 7 of them are on the contract level and the remaining 12 are on the stock level.

1. Implied volatility (impl_vol): Following Büchner and Kelly (2022), the American option implied volatility is computed by the Ivy DB database of OptionMetrics using the binomal tree model (Cox, Ross, and Rubinstein (1979)).
2. Delta (delta): Following Büchner and Kelly (2022), the delta of the option contract computed by OptionMetrics.
3. Gamma (gamma): Following Büchner and Kelly (2022), the gamma of the option contract computed by OptionMetrics.
4. Theta (theta): Following Büchner and Kelly (2022), the theta of the option contract computed by OptionMetrics.
5. Volga (volga): Following Büchner and Kelly (2022), the volga of the option contract, the sensitivity of vega to changes in volatility, i.e.

$$
\text { volga }=\frac{\partial V e g a}{\partial \sigma} .
$$

This is not provided by OptionMetrics, and hence we compute it by using standard Black-Scholes pricing formula with zero dividend rate.
6. Embedded leverage (embed_lev): Following Büchner and Kelly (2022) and Frazzini and Pedersen (2021), the embedded leverage of the option contract is the amount of market exposure per unit of committed capital, defined as

$$
\Omega=\left|\frac{\Delta \cdot S}{F}\right|
$$

where $\Delta$ is the option delta, $S$ is the underlying price and $F$ is the option price.
7. Option illiquidity (optspread): Following Christoffersen, Goyenko, Jacobs, and Karoui (2018), Bali, Beckmeyer, Moerke, and Weigert (2021) and Goyenko and Zhang (2021), the option illiquidity is the ratio of the bid-ask spread to the mid-point of bid and ask for each option contract.
8. Volatility deviation (vol_dev): Following Zhan, Han, Cao, and Tong (2022), Cao and Han (2013) and Goyenko and Zhang (2021), we use the definition in Goyal and Saretto (2009). The volatility deviation is defined as the difference between historical realized volatility and the ATM option implied volatility. The historical realized volatility is the standard deviation of of daily realized returns over the past 360 days (this is extracted from the Historical_Volatility File in OptionMetrics), and the ATM option implied volatility is the average of the implied volatility of one at-the-money call (with delta equal to 0.5 ) and one at-the-money put (with delta equal to -0.5 ) which have 30 days to maturity (these are extracted from the Volatility_Surface File in OptionMetrics).

9-19. Market beta (beta), Market capitalization (mktcap), Book-to-market ratio ( $b m$ ), Gross profitability ( $p r o f$ ), Investment (invest), Idiosyncratic volatility (idiovol), Book leverage (lev), Average daily bid-ask spread (bidask), Momentum (mom), Intermediate momentum (intmom), Short-term reversal (strev), and Book leverage (lev): The data on these stock-level characteristics
are from Freyberger, Höppner, Neuhierl, and Weber (2022). These characteristics have been demonstrated to have a significant impact on option returns, for example, idiosyncratic volatility (idiovol) by Cao and Han (2013) and Zhan, Han, Cao, and Tong (2022), average daily bid-ask spread (bidask) and momentum (mom) by Bali, Beckmeyer, Moerke, and Weigert (2021) and Goyenko and Zhang (2021).

## Appendix A2 Characteristics on Beta Loadings of the RegressedPCA Latent Factors

In this Appendix, we first explain the beta loadings of the first regressed-PCA latent factors presented in Section 4.4. Then we present the beta loadings of the rest of the first ten regressed-PCA factors to gain insights of what those factors are.

## Appendix A2.1 Beta loadings on the First Regressed-PCA Latent Factor

On the equity segment, book assets and market capitalization dominate. These two characteristics have weights with similar magnitude yet opposite signs, which can be interpreted as a "value" or "leverage" factor involved in the beta loadings. Interestingly, book-to-market ratio has a negligible weight, which implies that the weights of assets and market capitalization estimated via regressed-PCA subsume the book-to-market values. The result from the equity side coincides with Kelly, Pruitt, and Su (2019), who also finds that the beta of the first IPCA factor is driven by high book assets and low market equity.

On the bond segment, ratings, duration, bond momentum, spread momentum and Value-at-Risk contribute to the beta loadings of the first latent factor. The exposure to ratings and duration is consistent with the literature that credit and duration risks contribute significantly to the bond risk premium. Interestingly, duration has a negative weight on the beta loadings, which suggests that securities with higher duration earn lower average returns. This result may be related to the negative term structure of risk premia, as discussed on equity in van Binsbergen, Brandt, and Koijen (2012) and van Binsbergen and Koijen (2017). The exposure to bond momentum is negative and marginally significant. Jostova, Nikolova, Philipov, and Stahel (2013) show that bond momentum is large yet positive, and is primarily concentrated in non-investment grade bonds. Consistent with their findings, our result suggests that the positive bond momentum is associated with return anomalies rather than loading on risk. The beta loading of VaR is positive and significant. Dickerson, Mueller, and Robotti (2023) find that the downside risk factor significantly correlates with the bond market returns. We also discover that the VaR measure is highly and positively correlated with bond return volatility (with value over 0.9 ), which suggests that VaR can be an alternative proxy for the bond's return volatility.

On the option segment, embedded leverage, option's Greek such as gamma and theta, have salient weights on the beta loadings, and some characteristics of the underlying equity
such as book-to-market and momentum are also significant. Specifically, for example, embedded leverage which measures option's return magnification relative to the return of the underlying asset, has a significant positive loading on the risk factor, as buying options with higher embedded leverage increases investors' risk exposure, thus investors require higher risky return. Frazzini and Pedersen (2021) further argue that as investors buy options with high embedded leverage, they earn higher risky return, in compensation, they are giving up the risk-free part of the returns, hence the risk-adjusted return (alpha) should be lower. Our results are also consistent with their argument, as Figure 4 shows that embedded leverage contributes negatively to the alpha of the option returns.

## Appendix A2.2 Beta loadings on Other Regressed-PCA Joint Factors

Figures 5 to 7 present the beta loadings on the second to the tenth regressed-PCA joint factors.

## Appendix A3 Observable factors

Is there a common factor structure among stocks, corporate bonds, and options? To explore this, we start with a straightforward econometric approach known as Principal Component Analysis (PCA) applied to observable pricing factors. These pricing factors have been wellestablished in the asset pricing literature for different asset classes. The rationale behind this exercise is that if the various asset classes are integrated, then the observable factors that explain their returns should exhibit a shared component structure. We opt for PCA as it is specifically designed to extract common components from multiple time series data.

To conduct PCA, we first construct the matrix of observable portfolio factors (standardized to zero mean and unit variance) by sorting them according to their corresponding asset class:

$$
P=\left[\begin{array}{lll}
P_{\text {stock }} & P_{\text {corpbond }} & P_{\text {option }}
\end{array}\right]
$$

The PCA transforms the matrix of observed portfolio factors $P$ into a multiplication of sorted eigenvector weights $B$ and principal components $F$ :

$$
P_{T \times L}=F_{T \times K} \cdot B_{K \times L}+\epsilon_{T \times L}
$$

In constructing the portfolio factor matrix $P$, we consider $L=19$ observable portfolio factors that are used to price stocks, corporate bonds, and options respectively: the six equity-

Figure 5: Estimation of $B$ coefficients in the second to the fourth regressed-PCA joint factor




Figure 6: Estimation of $B$ coefficients in the fifth to the seventh regressed-PCA joint factor




Figure 7: Estimation of $B$ coefficients in the eighth to the tenth regressed-PCA joint factor



related factors are Fama and French (2015) five factors (MKTstock, SMB, HML, RMW, $C M A$ ) and momentum factor ( $M O M$ ); the six corporate bond factors are proposed by Bai, Bali, and Wen (2019) and Dickerson, Robotti, and Rossetti (2023) (MKTbond, CRF, $\left.L R F, B O N D M O M, R E V^{*}, L T R\right) ;{ }^{20}$ lastly, the six option-related factors are volatility level ( $L E V E L$ ), and moneyness skewness (SKEW) factors from Büchner and Kelly (2022), option idiosyncratic volatility (IVOL) and illiquidity ( $I L Q$ ) factors from Zhan, Han, Cao, and Tong (2022), option-market factor (MKToption) and volatility deviation factor (VOLDEV) from Goyal and Saretto (2009). ${ }^{21}$ The date range is from July 2004 to December 2021, in which the corporate bond factors begin. From these observable factors, we extract and examine the first ten principal components $(K=10)$, which jointly explain $88 \%$ of variations.

Figure 8: First principal component of observable factors


Note: This graph plots the series of cumulative sum of the first principal component from the PCA of observable pricing factors from three different asset classes from Aug 2004 to Dec 2021. The series of observable pricing factors include 6 equity factors, 6 corporate bond factors, and 7 equity option factors.

Figure 8 plots the cumulative sum of the first principal component from the PCA. The component manifests a systematic pattern associated with the market downturns: in the time series, it presents the 2008 financial crisis, 2015-2016 global stock market selloff, December 2018 market crash, and 2020 COVID crisis. The pattern indicates that the first component is potentially a common factor that explains the comovements of returns across asset classes.

To measure the degree of commonality across asset classes, we compute the explained variance ratio for the ten principal components, which is shown in Table A1. The first PC

[^15]is able to explain around $30.51 \%$ of variations among the pricing factors of interest. The next two components also explain $12.52 \%$ and $9.57 \%$ of variations, respectively. The results suggest that common components significantly present among pricing factors across asset classes.

Table A1: Explained variance ratios of principal components

|  | PC1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| Variance Ratio (\%) | 30.51 | 12.52 | 9.57 | 7.0 | 6.56 | 5.82 | 5.17 | 4.21 | 3.89 | 2.79 |

Note: The explained variance ratio is the percentage of variations among pricing factors that is attributed to the selected principal component.

A following question to consider is whether these principal components can effectively explain the observable factors specific to each of the three asset classes. A sufficient condition for a principal component to be deemed "common" is that it demonstrates comparable explanatory power for the factors from all asset classes. Conversely, if the variations explained by a particular component predominantly stem from equity-related factors, for example, then that component would be characterized as an equity-specific component rather than a common one. It is crucial to distinguish between common components, which capture shared variations across asset classes, and asset-specific components, which predominantly capture variations specific to a particular asset class.

To examine whether the principal components explain the pricing factors from each asset class, we can compute, for each principal component $k$, its marginal R-squared on the factors from their associative class. Specifically, we can fit portfolio factors $P$ by each principal component $F^{k}$ and its corresponding weight $B^{k}$ :

$$
\hat{P}_{t}^{k}=B^{k \prime} F_{t}^{k}
$$

where $\hat{F}^{k}$ is the predicted factors using $k$ th principal component. Because $F$ is sorted on asset classes, so does $\hat{F}^{k}$ :

$$
\hat{P}^{k}=\left[\begin{array}{lll}
\hat{P}_{\text {stock }}^{k} & \hat{P}_{\text {corpbond }}^{k} & \hat{P}_{\text {option }}^{k}
\end{array}\right] .
$$

We can subsequently compute marginal R-squareds for principal component $k$ to asset class $g \in\{$ stock, corpbond, option $\}$ as:

$$
R_{k, g}^{2}=\frac{1-\sum_{t}\left(P_{g}^{k}-\hat{P}_{g}^{k}\right)^{2}}{\sum_{g, t}\left(P_{g}^{k}\right)^{2}}
$$

Table A2: Marginal R-squareds of principal components on asset-class specific factors

| Marginal $R^{2}(\%)$ | PC 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| Stock | 20.47 | 16.99 | 10.18 | 17.26 | 2.78 | 1.62 | 3.22 | 6.66 | 1.63 | 7.54 |
| Corporate Bond | 31.72 | 3.82 | 15.15 | 1.18 | 4.71 | 13.32 | 2.85 | 4.65 | 7.97 | 0.76 |
| Option | 44.91 | 16.27 | 4.04 | 2.67 | 14.64 | 2.71 | 1.80 | 1.10 | 0.72 | 0.07 |

Table A2 reports the results. The first principal component is able to explain the pricing factors respectively from three different asset classes, which implies that the component is a common factor. It explains over $26.24 \%$ of variations in stock-related factors, $30.33 \%$ in corporate bonds, and $38.65 \%$ in options. The explanatory power of the second principal component is predominantly a corporate bond factor, with the marginal R-squareds of $22 \%$. The third principal component explains primarily stock and option factors.

The PCA analyses conducted on observable factors convey a significant finding: the presence of a common factor that accounts for variations across different asset classes. Furthermore, there is suggestive evidence indicating that this common factor exhibits a systematic relationship with economic cycles. However, it is important to acknowledge that the implications of these exercises may be limited since they are primarily applied to asset classes at the level of observable factors.

Given this limitation, we are motivated to pursue a search for a common factor structure directly from individual assets, which will be the focus of the subsequent sections. By examining individual assets, we aim to uncover a more comprehensive and nuanced understanding of the underlying common factor structure across the three asset classes.

## Appendix A4 Performance of the Restricted Joint Factor Model and Factor Models for Single Asset Class

In this section, we first report the in-sample and out-of-sample performance of the restricted joint factor model $(\alpha(\cdot)=0)$ in Table A3.

We then present the in-sample and out-of-sample performance of the factor model (2.4) for each individual asset class separately. The evaluation metrics are introduced in Section 2.3. Table A4 and A5 report the results for the unrestricted and restricted cases, respectively. Table A6 demonstrates the pure-alpha and beta strategy performance.

Table A3: In-sample and out-of-sample performance of the joint factor model when $\alpha(\cdot)=0$

| (i) All the returns on three asset classes |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| K | $R_{K}^{2}$ | $R^{2}$ | $R_{O}^{2}$ | $R_{T, N, O}^{2}$ | $R_{N, T, O}^{2}$ | $R_{f, O}^{2}$ | $R_{f, T, N, O}^{2}$ | $R_{f, N, T, O}^{2}$ |
| 1* | 15.80 | 6.20 | 24.05 | 5.34 | 8.96 | 26.08 | 8.85 | 12.81 |
| 2 | 26.16 | 7.54 | 24.03 | 8.30 | 8.91 | 26.62 | 10.88 | 13.85 |
| 3 | 33.90 | 8.86 | 24.00 | 8.59 | 8.84 | 26.88 | 13.92 | 14.42 |
| 4 | 41.13 | 10.21 | 24.02 | 7.14 | 8.88 | 27.05 | 14.92 | 14.70 |
| 5 | 47.25 | 12.75 | 24.00 | 6.92 | 8.87 | 27.29 | 14.84 | 15.01 |
| 6 | 53.09 | 14.18 | 23.94 | 6.92 | 8.81 | 27.43 | 15.18 | 15.22 |
| 7 | 57.62 | 14.41 | 23.92 | 7.30 | 8.77 | 27.60 | 15.64 | 15.57 |
| 8 | 61.47 | 15.26 | 23.95 | 8.85 | 8.82 | 27.75 | 15.77 | 15.88 |
| 9 | 64.78 | 16.01 | 23.93 | 9.12 | 8.79 | 27.87 | 15.89 | 16.10 |
| 10 | 67.67 | 16.32 | 23.94 | 9.08 | 8.80 | 27.98 | 16.07 | 16.27 |
| (ii) Stock Returns |  |  |  |  |  |  |  |  |
| K | $R_{K}^{2}$ | $R^{2}$ | $R_{O}^{2}$ | $R_{T, N, O}^{2}$ | $R_{N, T, O}^{2}$ | $R_{f, O}^{2}$ | $R_{f, T, N, O}^{2}$ | $R_{f, N, T, O}^{2}$ |
| 1 | 18.95 | 5.71 | 0.42 | 1.19 | -0.35 | 8.62 | 11.57 | 7.11 |
| 2 | 23.15 | 6.11 | 0.38 | 1.14 | -0.30 | 10.28 | 12.89 | 8.86 |
| 3 | 32.80 | 6.76 | 0.33 | 1.11 | -0.38 | 11.12 | 14.44 | 9.71 |
| 4 | 36.74 | 6.90 | 0.35 | 1.06 | -0.31 | 11.32 | 14.59 | 9.85 |
| 5 | 41.54 | 7.91 | 0.19 | 0.57 | -0.31 | 11.35 | 14.30 | 9.85 |
| 6* | 52.50 | 9.45 | -0.03 | 0.04 | -0.14 | 11.66 | 14.73 | 10.17 |
| 7 | 53.78 | 9.52 | -0.03 | 0.11 | -0.13 | 11.72 | 14.79 | 10.24 |
| 8 | 57.07 | 10.00 | 0.16 | 0.53 | -0.07 | 11.94 | 14.95 | 10.44 |
| 9 | 63.50 | 10.97 | 0.07 | 0.37 | -0.13 | 12.08 | 15.00 | 10.59 |
| 10 | 68.18 | 11.41 | 0.06 | 0.37 | -0.15 | 12.12 | 14.98 | 10.63 |

(iii) Corporate Bond Returns

| $K$ | $R_{K}^{2}$ | $R^{2}$ | $R_{O}^{2}$ | $R_{T, N, O}^{2}$ | $R_{N, T, O}^{2}$ | $R_{f, O}^{2}$ | $R_{f, T, N, O}^{2}$ | $R_{f, N, T, O}^{2}$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 10.89 | 10.40 | 2.18 | 4.09 | 1.96 | 3.91 | -24.47 | -5.28 |
| 2 | 22.71 | 11.20 | 2.53 | 4.68 | 0.72 | 1.89 | -29.37 | -10.60 |
| 3 | 33.05 | 13.36 | 2.36 | 4.30 | 0.29 | 3.66 | -24.13 | -9.85 |
| 4 | 40.24 | 21.21 | 2.41 | 4.28 | 0.96 | 3.73 | -25.29 | -10.66 |
| 5 | 43.25 | 25.13 | 2.29 | 3.84 | 1.04 | 6.02 | -24.60 | -7.29 |
| 6 | 46.62 | 28.66 | 2.11 | 4.15 | 1.22 | 6.62 | -23.93 | -5.79 |
| $7^{*}$ | 57.96 | 31.43 | 2.09 | 4.21 | 1.31 | 13.81 | -15.62 | 1.93 |
| 8 | 60.52 | 32.06 | 2.18 | 4.77 | 1.39 | 17.06 | -12.51 | 6.70 |
| 9 | 62.29 | 33.43 | 2.10 | 4.58 | 1.12 | 19.00 | -10.90 | 8.87 |
| 10 | 64.66 | 33.60 | 2.17 | 5.64 | 1.37 | 18.07 | -12.71 | 7.81 |

(iv) Option Returns

| $R_{K}^{2}$ | $R^{2}$ | $R_{O}^{2}$ | $R_{T, N, O}^{2}$ | $R_{N, T, O}^{2}$ | $R_{f, O}^{2}$ | $R_{f, T, N, O}^{2}$ | $R_{f, N, T, O}^{2}$ |  |
| :--- | ---: | ---: | ---: | ---: | :---: | :---: | :---: | :---: |
| 1 | 16.76 | 6.17 | 28.61 | 5.45 | 11.38 | 29.56 | 9.25 | 13.73 |
| $2^{*}$ | 33.23 | 8.40 | 28.59 | 8.50 | 11.35 | 29.95 | 11.35 | 14.80 |
| 3 | 36.08 | 10.24 | 28.56 | 8.81 | 11.29 | 30.09 | 14.41 | 15.21 |
| 4 | 47.37 | 11.92 | 28.58 | 7.31 | 11.32 | 30.25 | 15.46 | 15.51 |
| 5 | 58.15 | 15.68 | 28.58 | 7.10 | 11.34 | 30.49 | 15.38 | 15.73 |
| 6 | 60.15 | 16.78 | 28.55 | 7.11 | 11.26 | 30.59 | 15.72 | 15.81 |
| 7 | 61.98 | 16.86 | 28.53 | 7.50 | 11.19 | 30.65 | 16.09 | 15.99 |
| 8 | 67.76 | 18.07 | 28.53 | 9.09 | 11.21 | 30.73 | 16.17 | 16.14 |
| 9 | 68.79 | 18.56 | 28.53 | 9.37 | 11.21 | 30.80 | 16.27 | 16.33 |
| 10 | 69.98 | 18.77 | 28.53 | 9.31 | 11.23 | 30.95 | 16.49 | 16.56 |

Note: $K$ : the number of factors specified, * denotes the estimator of $K$ which maximizes the ratio of two adjacent eigenvalues; $R_{\overparen{R}}^{2}$ : Fama-MacBeth cross-sectional regression $R^{2} ; R_{K}^{2}$ measures the variations in the characteristic-managed portfolios captured by different numbers of factors from PCA; $R^{2}$ : total in-sample $R^{2}(\%)$, see (2.8); $R_{O}^{2}, R_{T, N, O}^{2}, R_{N, T, O}^{2}, R_{f, O}^{2}, R_{f, T, N, O}^{2}, R_{f, N, T, O}^{2}$ : out-of-sample fits $R^{2}$ 's (\%), see (2.9)-(2.14);

Table A4: In-sample and out-of-sample performance of the regressed-PCA factor models for each asset class when $\alpha(\cdot) \neq 0$

| (i) Stock Returns |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| K | $R_{K}^{2}$ | $R^{2}$ | $R_{f, O}^{2}$ | $R_{f, T, N, O}^{2}$ | $R_{f, N, T, O}^{2}$ |
| 1* | 28.63 | 3.10 | 8.57 | 7.86 | 7.46 |
| 2 | 43.55 | 3.27 | 9.32 | 9.11 | 8.16 |
| 3 | 53.93 | 4.22 | 12.17 | 15.04 | 10.79 |
| 4 | 61.52 | 5.85 | 12.54 | 15.60 | 11.10 |
| 5 | 67.93 | 11.46 | 13.04 | 16.18 | 11.61 |
| 6 | 73.19 | 14.38 | 13.48 | 16.58 | 12.03 |
| 7 | 77.04 | 14.68 | 13.70 | 16.85 | 12.24 |
| 8 | 80.63 | 14.95 | 13.84 | 16.83 | 12.38 |
| 9 | 83.84 | 15.20 | 14.12 | 17.09 | 12.68 |
| 10 | 86.29 | 15.49 | 14.22 | 17.00 | 12.78 |
| K | $R_{O}^{2}$ | $R_{T, N, O}^{2}$ | $R_{N, T, O}^{2}$ | $R_{\tilde{R}}^{2}$ |  |
| 1-10 | 0.22 | 0.71 | 0.04 | 18.64 |  |

(ii) Corporate Bond Returns

| $R_{K}^{2}$ | $R^{2}$ | $R_{f, O}^{2}$ | $R_{f, T, N, O}^{2}$ | $R_{f, N, T, O}^{2}$ |  |
| :--- | :---: | :---: | :---: | :---: | :---: |
| 1 | 27.85 | 8.09 | 8.68 | -9.67 | 4.91 |
| 2 | 44.54 | 16.37 | 25.28 | 5.03 | 19.75 |
| $3^{*}$ | 59.16 | 30.76 | 29.40 | 11.77 | 23.08 |
| 4 | 66.93 | 34.84 | 30.50 | 11.05 | 23.95 |
| 5 | 73.39 | 37.24 | 33.54 | 17.13 | 27.35 |
| 6 | 78.23 | 39.47 | 34.26 | 17.70 | 28.15 |
| 7 | 81.61 | 40.12 | 35.18 | 18.59 | 29.21 |
| 8 | 84.54 | 40.90 | 37.40 | 19.36 | 31.92 |
| 9 | 87.21 | 42.25 | 37.76 | 20.02 | 32.29 |
| 10 | 89.29 | 42.97 | 38.16 | 20.69 | 32.79 |
| $K$ | $R_{O}^{2}$ | $R_{T, N, O}^{2}$ | $R_{N, T, O}^{2}$ | $R_{\tilde{R}}^{2}$ |  |
| $1-10$ | 2.22 | 5.48 | 1.67 | 47.68 |  |

(iii) Option Returns

| $K$ | $R_{K}^{2}$ | $R^{2}$ | $R_{f, O}^{2}$ | $R_{f, T, N, O}^{2}$ | $R_{f, N, T, O}^{2}$ |
| :--- | :---: | :---: | :---: | :---: | :---: |
| $1^{*}$ | 32.16 | 7.04 | -443.77 | 45.56 | -448.34 |
| 2 | 52.18 | 9.33 | -440.09 | 47.81 | -445.63 |
| 3 | 62.41 | 18.11 | -435.84 | 47.87 | -441.94 |
| 4 | 70.07 | 22.22 | -434.60 | 47.11 | -440.66 |
| 5 | 77.06 | 22.75 | -432.62 | 46.67 | -438.92 |
| 6 | 81.53 | 23.28 | -431.36 | 46.55 | -437.83 |
| 7 | 84.66 | 23.58 | -430.62 | 46.45 | -437.09 |
| 8 | 87.61 | 23.89 | -429.77 | 46.26 | -436.21 |
| 9 | 89.79 | 24.15 | -428.69 | 46.06 | -435.04 |
| 10 | 91.91 | 24.46 | -426.85 | 46.05 | -433.16 |
| $K$ | $R_{O}^{2}$ | $R_{T, N, O}^{2}$ | $R_{N, T, O}^{2}$ | $R_{\tilde{R}}^{2}$ |  |
| $1-10$ | -453.05 | 46.21 | -454.85 | 27.80 |  |

Note: $K$ : the number of factors specified, * denotes the estimator of $K$ which maximizes the ratio of two adjacent eigenvalues; $R_{\tilde{R}}^{2}$ : Fama-MacBeth cross-sectional regression $R^{2} ; R_{K}^{2}$ measures the variations in the characteristic-managed portfolios captured by different numbers of factors from PCA; $R^{2}$ : total in-sample $R^{2}(\%)$, see (2.8); $R_{O}^{2}, R_{T, N, O}^{2}, R_{N, T, O}^{2}, R_{f, O}^{2}, R_{f, T, N, O}^{2}, R_{f, N, T, O}^{2}$ : out-of-sample fits $R^{2}$ 's (\%), see (2.9)-(2.14);

Table A5: In-sample and out-of-sample performance of the regressed-PCA factor models for each asset class when $\alpha(\cdot)=0$

| (i) Stock Returns |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| K | $R_{K}^{2}$ | $R^{2}$ | $R_{O}^{2}$ | $R_{T, N, O}^{2}$ | $R_{N, T, O}^{2}$ | $R_{f, O}^{2}$ | $R_{f, T, N, O}^{2}$ | $R_{f, N, T, O}^{2}$ |
| 1* | 28.65 | 2.91 | 0.35 | 0.71 | 0.04 | 8.68 | 8.41 | 7.56 |
| 2 | 43.57 | 3.08 | 0.34 | 0.69 | 0.03 | 9.40 | 9.49 | 8.23 |
| 3 | 53.95 | 4.02 | 0.29 | 0.73 | -0.07 | 12.13 | 15.30 | 10.76 |
| 4 | 61.54 | 5.52 | 0.27 | 0.77 | -0.06 | 12.56 | 15.88 | 11.13 |
| 5 | 67.96 | 11.32 | 0.11 | 0.22 | -0.05 | 13.07 | 16.40 | 11.66 |
| 6 | 73.21 | 14.22 | 0.18 | 0.66 | -0.05 | 13.51 | 16.83 | 12.05 |
| 7 | 77.07 | 14.52 | 0.18 | 0.59 | -0.04 | 13.73 | 17.04 | 12.26 |
| 8 | 80.67 | 14.74 | 0.18 | 0.53 | -0.04 | 13.86 | 17.18 | 12.41 |
| 9 | 83.88 | 15.04 | 0.15 | 0.43 | -0.06 | 14.16 | 17.32 | 12.72 |
| 10 | 86.33 | 15.34 | 0.10 | 0.30 | -0.11 | 14.26 | 17.38 | 12.82 |

(ii) Corporate Bond Returns

| $R_{K}^{2}$ | $R^{2}$ | $R_{O}^{2}$ | $R_{T, N, O}^{2}$ | $R_{N, T, O}^{2}$ | $R_{f, O}^{2}$ | $R_{f, T, N, O}^{2}$ | $R_{f, N, T, O}^{2}$ |  |
| :--- | ---: | ---: | :--- | ---: | ---: | ---: | ---: | ---: |
| 1 | 27.85 | 6.40 | 0.29 | 1.66 | 0.43 | 9.47 | -8.20 | 6.29 |
| 2 | 44.57 | 17.20 | 2.19 | 6.38 | 2.87 | 26.48 | 8.44 | 21.60 |
| $3^{*}$ | 59.18 | 30.76 | 1.91 | 6.07 | 2.39 | 30.56 | 14.15 | 25.21 |
| 4 | 66.96 | 34.97 | 1.76 | 5.72 | 2.17 | 31.54 | 14.85 | 25.99 |
| 5 | 73.41 | 37.22 | 2.17 | 7.12 | 2.49 | 33.99 | 19.07 | 28.25 |
| 6 | 78.26 | 39.33 | 2.20 | 6.25 | 2.26 | 34.58 | 19.25 | 28.90 |
| 7 | 81.65 | 40.06 | 2.04 | 5.88 | 2.13 | 35.76 | 20.40 | 30.08 |
| 8 | 84.57 | 40.82 | 2.10 | 4.85 | 1.78 | 37.69 | 21.15 | 32.44 |
| 9 | 87.24 | 42.18 | 2.04 | 4.96 | 1.71 | 38.01 | 20.99 | 32.75 |
| 10 | 89.32 | 42.90 | 1.98 | 4.79 | 1.49 | 38.36 | 21.35 | 33.19 |

(iii) Option Returns

| $K$ | $R_{K}^{2}$ | $R^{2}$ | $R_{O}^{2}$ | $R_{T, N, O}^{2}$ | $R_{N, T, O}^{2}$ | $R_{f, O}^{2}$ | $R_{f, T, N, O}^{2}$ | $R_{f, N, T, O}^{2}$ |
| :--- | :---: | ---: | ---: | :---: | ---: | ---: | ---: | :---: |
| $1^{*}$ | 35.08 | 7.61 | -453.35 | 46.37 | -455.06 | -442.25 | 46.44 | -446.96 |
| 2 | 53.90 | 8.21 | -454.20 | 47.31 | -455.85 | -439.93 | 46.95 | -445.04 |
| 3 | 64.11 | 16.51 | -453.87 | 47.05 | -455.40 | -435.79 | 46.71 | -441.50 |
| 4 | 71.58 | 21.44 | -453.31 | 46.34 | -455.12 | -434.31 | 46.51 | -440.22 |
| 5 | 77.89 | 22.59 | -453.21 | 46.24 | -455.06 | -432.62 | 46.62 | -438.78 |
| 6 | 82.32 | 23.13 | -453.21 | 46.27 | -455.01 | -431.52 | 46.50 | -437.69 |
| 7 | 85.45 | 23.38 | -453.14 | 46.18 | -454.98 | -430.79 | 46.52 | -437.05 |
| 8 | 88.22 | 23.86 | -453.09 | 46.10 | -454.94 | -430.01 | 46.40 | -436.27 |
| 9 | 90.35 | 24.06 | -453.05 | 46.14 | -454.90 | -428.59 | 46.33 | -434.68 |
| 10 | 92.20 | 24.52 | -453.09 | 46.26 | -454.89 | -427.00 | 46.21 | -433.15 |

Note: $K$ : the number of factors specified, * denotes the estimator of $K$ which maximizes the ratio of two adjacent eigenvalues; $R_{\tilde{R}}^{2}$ : Fama-MacBeth cross-sectional regression $R^{2} ; R_{K}^{2}$ measures the variations in the characteristic-managed portfolios captured by different numbers of factors from PCA; $R^{2}$ : total in-sample $R^{2}(\%)$, see (2.8); $R_{O}^{2}, R_{T, N, O}^{2}, R_{N, T, O}^{2}, R_{f, O}^{2}, R_{f, T, N, O}^{2}, R_{f, N, T, O}^{2}$ : out-of-sample fits $R^{2}$ 's (\%), see (2.9)-(2.14);

Table A6: Out-of-sample pure-alpha strategy and tangency portfolio performance of the regressed-PCA factor models for each asset class

| (i) Stock Returns |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| K | $\mu_{\alpha}$ | $\sigma_{\alpha}$ | $S R_{\alpha}$ | $S R_{\beta, U}$ | $S R_{\beta, R}$ |
| 1 | 0.26 | 0.51 | 0.51 | 0.07 | 0.08 |
| 2 | 0.33 | 0.44 | 0.73 | -0.02 | -0.01 |
| 3 | 0.34 | 0.45 | 0.75 | -0.05 | -0.05 |
| 4 | 0.39 | 0.45 | 0.87 | -0.15 | -0.16 |
| 5 | 0.36 | 0.44 | 0.82 | -0.05 | 0.07 |
| 6 | 0.29 | 0.32 | 0.91 | 0.16 | 0.23 |
| 7 | 0.29 | 0.29 | 0.99 | 0.13 | 0.16 |
| 8 | 0.29 | 0.27 | 1.07 | 0.11 | 0.14 |
| 9 | 0.32 | 0.27 | 1.19 | 0.03 | 0.07 |
| 10 | 0.34 | 0.26 | 1.34 | -0.06 | 0.02 |

(ii) Corporate Bond Returns

| $\mu_{\alpha}$ |  |  |  |  |  |
| :--- | :--- | :--- | ---: | ---: | ---: |
|  | $\mu_{\alpha}$ | $\sigma_{\alpha}$ | $S R_{\alpha}$ | $S R_{\beta, U}$ | $S R_{\beta, R}$ |
| 1 | 0.06 | 0.05 | 1.09 | -0.58 | -0.57 |
| 2 | 0.02 | 0.04 | 0.56 | 0.60 | 0.63 |
| 3 | 0.03 | 0.04 | 0.76 | 0.33 | 0.36 |
| 4 | 0.03 | 0.03 | 1.06 | 0.13 | 0.19 |
| 5 | 0.01 | 0.02 | 0.50 | 0.70 | 0.75 |
| 6 | 0.01 | 0.02 | 0.39 | 0.90 | 0.95 |
| 7 | 0.01 | 0.02 | 0.62 | 0.57 | 0.61 |
| 8 | 0.01 | 0.02 | 0.60 | 0.80 | 0.75 |
| 9 | 0.01 | 0.02 | 0.89 | 0.40 | 0.41 |
| 10 | 0.01 | 0.01 | 0.75 | 0.66 | 0.72 |

(iii) Option Returns

| $K$ | $\mu_{\alpha}$ | $\sigma_{\alpha}$ | $S R_{\alpha}$ | $S R_{\beta, U}$ | $S R_{\beta, R}$ |
| :--- | :--- | :--- | :---: | :---: | :---: |
| 1 | 0.08 | 0.42 | 0.19 | 2.63 | 2.44 |
| 2 | 0.21 | 0.33 | 0.65 | 1.45 | 1.07 |
| 3 | 0.37 | 0.18 | 2.06 | 0.47 | 0.67 |
| 4 | 0.28 | 0.15 | 1.83 | 0.74 | 1.04 |
| 5 | 0.27 | 0.14 | 1.94 | 0.75 | 1.00 |
| 6 | 0.22 | 0.11 | 2.04 | 0.89 | 1.43 |
| 7 | 0.17 | 0.10 | 1.73 | 1.11 | 1.64 |
| 8 | 0.14 | 0.10 | 1.47 | 1.25 | 1.66 |
| 9 | 0.15 | 0.08 | 1.77 | 1.24 | 1.88 |
| 10 | 0.08 | 0.06 | 1.36 | 1.65 | 1.99 |

Note: $K$ : the number of factors specified; $\mu_{\alpha}, \sigma_{\alpha}$ and $S R_{\alpha}$ : out-of-sample annualized means, standard deviations, and Sharpe ratios of the pure-alpha arbitrage strategy (\%). $S R_{\beta, U}$ and $S R_{\beta, R}$ : out-of-sample annualized Sharpe ratios of the tangency portfolio (beta strategy) of $K$ factors under the unrestricted and restricted case.

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[^1]:    ${ }^{1}$ Fama and French (1993) explore common variation in stock and bond returns and find that stock returns are linked to bond returns through shared variation in the bond-market factors, but conclude that the key factors responsible for the risk premia are largely asset class-specific, which is the approach pursued by Coval and Shumway (2001) for option returns and Lustig, Roussanov, and Verdelhan (2011) for currency returns.

[^2]:    ${ }^{2}$ We could also study the asset-class-specific factors by analyzing the returns on each asset class under the factor model (2.4), separately. Here, "s", "c" and "o" briefly stand for "stock", "corporate bond" and "option", respectively.

[^3]:    ${ }^{3}$ As a robustness check, we also conduct the analysis until December 2021. The main results still hold, except that out-of-sample fitness for returns of stocks and corporate bonds become worse off. This is because the joint factor model accommodates to match some very extreme returns observed from the equity options.
    ${ }^{4}$ http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html
    ${ }^{5}$ Kelly, Pruitt, and Su (2019) and Chen, Roussanov, and Wang (2022) choose 36 stock characteristics from Freyberger, Neuhierl, and Weber (2020), but the sample ends in May 2014. Freyberger, Höppner, Neuhierl, and Weber (2022) extend the characteristics data in Freyberger, Neuhierl, and Weber (2020) to December 2021 and impute the missing values in a GMM framework. The extended dataset is generously provided by the authors. The only characteristic that is absent from this new dataset is fixed costs-to-sales ( $f c 2 y$ ).

[^4]:    ${ }^{6}$ We are sincerely grateful that the authors kindly provide the dataset to us.
    ${ }^{7}$ See Dick-Nielsen (2009), Dick-Nielsen (2014), Nozawa (2017), and van Binsbergen and Schwert (2021)

[^5]:    ${ }^{8}$ Dickerson, Robotti, and Rossetti (2023) mention that the volume filter of $\$ 100,000$ can significantly reduce the noises from potential retail trades.
    ${ }^{9}$ In the WRDS bond database, the variable name is $R E T \_L 5 M$.

[^6]:    ${ }^{10}$ The two datasets are merged using the linking table provided by WRDS: https://wrds-www. wharton. upenn.edu/pages/get-data/linking-suite-wrds/option-metrics-crsp-link/
    ${ }^{11}$ See Büchner and Kelly (2022), Frazzini and Pedersen (2021), Zhan, Han, Cao, and Tong (2022), Bali, Beckmeyer, Moerke, and Weigert (2021), Goyenko and Zhang (2021), Goyal and Saretto (2009), and Boyer and Vorkink (2014).

[^7]:    ${ }^{12}$ We follow Duarte, Jones, Mo, and Khorram (2023) and Duarte, Jones, and Wang (Forthcoming) which also mention that look-ahead bias significantly affects the performance of the out-of-sample trading strategies studied in the empirical options literature.

[^8]:    ${ }^{13}$ We report the summary statistics for trimmed options data, i.e., trimming options data below (above) 1st (99th) percentile of the return distribution.
    ${ }^{14}$ To mitigate any potential look-ahead bias, we rely on the standard deviations calculated over the initial 60 periods aligning with the initial window size used in the out-of-sample analysis.

[^9]:    ${ }^{15}$ The observable factors are correlated, hence the regression analysis shown in Table 3 may be subject to the multicollinearity issue.

[^10]:    ${ }^{16}$ For example, we see from Table A4 that the out-of-sample fitness $R_{f, O}^{2}$ are overwhelmingly negative for options, due to the several outliers in the option returns. But the joint factor model shows superiority of the out-of-sample fitness as indicated in Table 7.

[^11]:    ${ }^{17}$ Kozak and Nagel (2023) provide a set of necessary and sufficient conditions when the MVE portfolios on OLS factors can expand the same conditional SDF based on MVE portfolios on individual assets. Overall, the model specified in (2.6) with restriction $\mathbf{a}=0$ satisfies the related conditions for OLS factors in Kozak and Nagel (2023). Fist of all, their restriction that the latent factor with loadings are orthogonal to characteristics holds trivially here, since we ignore these factors directly. This restriction can also be relaxed, and holds approximately in our analysis due to large number of characteristics we incorporated in the estimation. Lastly, since we normalize the excess return of individual assets with the standard deviations of the respective asset classes and then scale the individual asset returns across all asset classes with unconditional variance of stocks $\left(\sigma_{s}^{2}\right)$, therefore, the idiosyncratic errors are approximately homoskedastic. More details on the conditions can be found in Example 3 and Equation (19) in Kozak and Nagel (2023).

[^12]:    ${ }^{18}$ In Table 14, we present the performance of the pure-alpha strategy using non-linear $\alpha(\cdot)$ and $\beta(\cdot)$ by expanding $\alpha(\cdot)$ and $\beta(\cdot)$ with splines (for details, one can refer to Chen, Roussanov, and Wang (2022)). Incorporating non-linearity does not significantly perform better in capturing common variations then eliminating the pricing errors, although we observe moderate decreases in the Sharpe ratios of the pure-alpha strategies using the joint factors.
    ${ }^{19}$ Many of these characteristics are documented as return anomalies in the asset pricing literature. For example, idiosyncratic volatility for stocks, credit spread for corporate bonds, and gamma for options all have significant loadings in the alpha portfolio.

[^13]:    ${ }^{\dagger}$ In the first panel (Sharpe ratios), the first row reports the Sharpe ratios for the joint estimation across different assets classes; the second rows reports the Sharpe ratios for the estimation within each asset class. The second panel reports the correlation ( $\rho$ ) of MVE portfolios across all asset classes with the components from individual asset classes, and the first and second joint factors with and without restriction $a=0$. The third panel reports the regression results ( $\beta$ ) of MVE portfolios onto the first and second joint factors.

[^14]:    ${ }^{\dagger}$ The reported Sharpe ratios are annualized. The calculation is based on expanding window estimation starting with the sample of the first ten years.

[^15]:    ${ }^{20}$ We remove $D R F$ since it is highly correlated with MKTbond after lead-lag correction. See Dickerson, Mueller, and Robotti (2023) for more details on the lead-lag correction for the factors.
    ${ }^{21}$ We remove $S L O P E$ since it is highly correlated with $L E V E L$.

