The Efficiency of Patent Litigation

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Abstract

How efficient is the U.S. patent litigation system? We quantify the extent to which the litigation system shapes innovation using a novel dynamic model, in which heterogeneous firms innovate and face potential patent lawsuits. We show that the impact of a litigation reform depends on how heterogeneous firms endogenously select into lawsuits. Calibrating the model, we find that weakening plaintiff rights through fewer defendant injunctions increases firm innovation and output growth, improving social welfare by 3.32%. Raising plaintiff pleading requirements, which heightens barriers to filing lawsuits, likewise promotes innovation, boosts output growth, and enhances social welfare.

Keywords: patent litigation, innovation, firm value, growth, social welfare.

JEL Classification: G30, E22, O30, O40, K40.

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1. Introduction

Innovation is essential for economic growth. Designed to promote innovation, the U.S. patent system protects an inventor's ability to profit from costly research and development. The patent system relies on the courts to enforce patent-holder rights through patent-infringement litigation. Practitioners, policy makers, and academics disagree about how this litigation should be handled to best promote innovation. As a result, policy proposals for patent-litigation reform abound. We add to this discussion by using a calibrated model to evaluate the efficiency of the patent-litigation system. Our model embeds a realistic patent-litigation system in a dynamic general equilibrium model of corporate innovation and economic growth. These features allow us to evaluate how counterfactual reforms would change the nature of patent litigation. In particular, we show that a reform that makes it twice as costly for a patent-holder plaintiff to file a lawsuit increases social welfare by 2.08%. Similarly, we find weakening plaintiff rights, by granting fewer injunctions against defendants, would increase social welfare by 3.32%.

These types of quantitative results are important given the intense and enduring debate around patent-litigation reform, which centers around whether plaintiff (patent holder) rights are too strong or too weak. For example, a Senate bill introduced in 2023 provides a compromise between tech companies, who believe excessive patent granting has led to frivolous patent lawsuits, and pharmaceutical companies, who believe it is too difficult to protect their innovation with patents.[1] Fourteen patent-reform bills were proposed in the 113th Congress alone, with goals such as increasing plaintiff pleading requirements to strengthen defendant rights (Gugliuzza, 2015).

To flesh out the intuition behind our results, we begin with a simple model, which illustrates how the patent litigation system impacts welfare. In the model, innovation leads to better products, allowing the innovating firm to steal market share from competitors. Competing firms choose their equilibrium level of innovation to maximize profits. Some firms underinvest in innovation, relative to the socially efficient benchmark, because they do not internalize how their innovation creates positive technology spillovers for firms with similar technology. Other firms overinvest in innovation, relative to the socially

[1] See https://www.reuters.com/legal/litigation/tech-pharma-companies-divided-pto-patent-eligibility-comments-2021-10-19/.

efficient benchmark, because they inefficiently internalize the transfer they extract from competitors by stealing customers. In this standard setting (Bloom, Schankerman, and Van Reenen, 2013), we introduce a litigation system. When a firm innovates, there is a chance that an incumbent firm sues to block the innovation. If the court grants an injunction, the innovating firm cannot use its novel technology. We show that an increase in the injunction rate (a plaintiff-friendly reform) leads to less innovation. Importantly, however, this reform has ambiguous effects on welfare. If most litigation occurs in technology classes where firms inefficiently overinvest in innovation, then such a reform improves welfare. Conversely, increasing the injunction rate harms welfare if most litigators are inefficiently underinvesting.

While this simple model provides helpful intuition, it is too stylized to match empirical facts and provide reliable policy counterfactuals. To overcome this limitation, we build a dynamic equilibrium model of innovation and litigation. In the model, heterogeneous firms compete in product markets. Incumbent firms and new potential entrants spend on research and development to innovate a better version of an existing product. After a successful innovation, the owner of the newly improved product enjoys a monopoly on that particular product until a competitor innovates a better version. Firms choose innovation levels and production policies to maximize profits, taking prices as given.

Within this dynamic equilibrium setting, we introduce a patent litigation system. Whenever a firm innovates a better version of a product, there is a chance that the new product infringes on a patent of an existing firm. If infringement occurs, the patent holder observes a random cost of filing a lawsuit. The patent holder sues the infringer if the expected lawsuit payoff exceeds the cost of filing.

In a lawsuit, the defendant (the innovating firm) privately observes its probability of winning a lawsuit. The plaintiff (the patent holder) makes a take-it-or-leave-it offer to settle. The defendant accepts if its continuation value from going to trial is worse than the cost of making the proposed settlement payment. Because of the defendant's private information, both settlements and trials occur in equilibrium. If the defendant declines the settlement offer, the lawsuit goes to trial. In the trial, the defendant has an idiosyncratic random probability of winning. If the defendant loses, there is a chance that the plaintiff obtains an injunction, which prevents the defendant from selling its new product. Otherwise, the defendant patents and sells its newly innovated product.

In our model equilibrium, firms have rational expectations about how the patent system shapes the returns to innovating, which are generated by the following tradeoff. On the one hand, firms recognize that a plaintiff-friendly system makes it likely that their innovation will be blocked by an incumbent's patent. Plaintiff-friendly litigation reforms can thus discourage innovation. On the other hand, firms also recognize that a plaintiff-friendly system increases the returns to successful innovation. Conditional on a successful innovation not getting blocked, the innovating firm enjoys a longer period of monopolist profits because it can sue to block new entrants. Plaintiff-friendly reforms can thus also encourage innovation.

We introduce firm heterogeneity in the model to allow innovation to have differing social values across firms. As in our illustrative model, innovation creates a positive externality, through technology spillovers, for firms using similar technologies. Firms do not internalize this externality, so some firms will underinvest in innovation, relative to a socially efficient benchmark. Other firms innovate products that are barely better than existing products. For these firms, innovation has little social value, but a large private value because these firms inefficiently internalize the value they extract from incumbents by stealing their customers. These firms will thus overinvest in innovation, relative to a socially efficient benchmark. By combining this heterogeneity with an endogenous litigation process, we can model which types of firms select into using the patent litigation system.

We calibrate this realistic model and use it to simulate the effects of patent-litigation reform. For example, in 2006, a Supreme Court ruling made it harder for plaintiffs to obtain injunctions against patent infringers. The analysis reveals several key findings. In response to reduced injunction rates, both incumbents and entrants increase their innovation, with incumbents showing a higher rate of increase. This results in a decline in the contribution of entrants to growth. Despite a decline in the average product line value due to increased creative destruction and reduced litigation protection, the average number of product lines per firm increases, leading to an increase in incumbent firm value. This decline in product line value is outweighed by a decrease in value loss associated with IP risks, driving up entrant innovation and entrepreneur value. Additionally, the increase in creative destruction and the number of product lines increase the average plaintiff probability. Such a reform also has significant aggregation implications. It boosted output growth rate due to increased innovation and resulted in a 3.32% overall increase in social welfare.

Our model allows us to study other potential reforms as well. For example, one recently proposed reform suggested increasing plaintiff pleading requirements (Gugliuzza, 2015). This is analogous to increasing the costs of filing for plaintiffs. Our calibrated model reveals that this would lead to fewer lawsuits, more innovation, and higher welfare and growth.

1.1. Literature Review

This paper contributes to several literatures. First, we contribute to the literature studying how the patent-litigation system interacts with innovation. An empirical literature studies how changes in plaintiff rights impact innovation activity (Sakakibara and Branstetter, 2001; Lerner, 2002; Moser, 2005; Lerner, 2009; Murray and Stern, 2007; Galasso and Schankerman, 2015; Williams, 2013; Cohen, Gurun, and Kominers, 2019; Mezzanotti, 2021; Kempf and Spalt, 2022; Lin, Liu, and Manso, 2021). These types of reduced-form studies are helpful in informing the policy debate around patent-litigation reform. However, a model-based approach provides insights that are impossible to glean from reduced-form settings. For example, our calibrated model can reveal the impact of counterfactual reforms, which is essential given the proliferation of policy proposals. More importantly, by embedding litigation in a fully specified general-equilibrium growth model, we can quantify the impact of reforms on both socially beneficial and socially harmful innovation. Capturing selection into litigation by firms with differing innovation incentives, our model reveals the welfare consequences of litigation reform.

A theoretical literature uses stylized models to qualitatively illustrate how litigation reforms impact ex-post and ex-ante welfare of the litigants (Bessen and Meurer, 2006; Choi and Gerlach, 2017; Antill and Grenadier, 2023). Marco (2006) estimates a quantitative dynamic litigation model but does not estimate the impacts on ex-ante innovation incentives. Relative to this earlier work, we contribute a structural general-equilibrium model of how innovation, litigation, and firm dynamics interact in the presence of heterogeneity across industries and technology classes. A calibrated model like ours is necessary to capture the dynamic equilibrium effects of litigation reforms, as well as to calculate the changes in aggregate growth and welfare under counterfactual policies.

This paper builds on and contributes to the firm dynamics literature featuring endogenous Schumpeterian growth. Our new model extends the seminal Klette and Kortum (2004) framework, which itself is based on the vertical innovation models with creative destruction pioneered by Aghion and Howitt (1992) and Grossman and Helpman (1991). The calibration and the quantification of the model link it to more recent work studying firm dynamics and innovation in general equilibrium, such as Lentz and Mortensen (2008), Akcigit and Kerr (2018), Cavenaile, Celik, and Tian (2019), and Acemoglu, Akcigit, and Celik (2022) among others. Different from most of the existing literature, a successful innovation in our model does not directly translate into success in taking over a product line. Conditional on a successful innovation, different types of patent infringement can occur, leading to a litigation subgame in which the innovating firm is the defendant and the plaintiff is either the incumbent owner of the product line or an unrelated third party whose patent was infringed. The structure of the legal system, litigation costs, and the policy choice regarding whether patent infringements should lead to injunctions all affect the outcome of a successful innovation. The presence of a legal system, in turn, has downstream implications for optimal innovation decisions, firm value, firm dynamics, economic growth, and social welfare. In this respect, the two closest papers to ours are: (i) Abrams, Akcigit, Oz, and Pearce (2020), who study how non-practicing entities ("patent trolls") affect innovation and technological progress; and (ii) Rempel (2023), who studies how heterogeneity in firms' intellectual-property-protection investments affect innovation and firm dynamics. Different from these two papers, we highlight the importance of the heterogeneity in firms' industries and technology classes, the identity and motives of the plaintiff, as well as the implications for inter-firm knowledge spillovers in assessing optimal policy in patent litigation. Our focus on this margin links our work to the literature on misallocation in innovation.[2]

[2]See Acemoglu et al. (2018), Akcigit, Hanley, and Stantcheva (2022), Ayerst (2023), Celik (2023a), Liu and Ma (2021), and Celik (2023b) among others.

2. Institutional Details and Illustrative Model

2.1. Institutional Details

2.1.1. Patents

The US patent system is designed to encourage innovation by giving patent holders the exclusive right to use their patented technology. Following a new discovery, inventors can apply for a patent with the United States Patent and Trademark Office (USPTO). Before granting the patent, the USPTO verifies that the invention is (i) novel; (ii) useful and operable; (iii) a non-obvious improvement relative to prior technology; and (iv) related to a patentable subject matter.[3] To ensure these criteria are met, a patent examiner verifies that the invention is not an obvious extension of an existing patented technology. Once the USPTO grants a patent, the patent expires 20 years after the application date (35 U.S.C §154).[4] During that period, the patent holder has the right to exclude others from making, using, or selling their patented inventions.

2.1.2. Patent litigation and injunctions

Patent holders can enforce their patents through patent-infringement lawsuits. These lawsuits are typically filed in federal district courts. A patent holder can sue anyone who "makes, uses, sells, offers to sell, or imports the patented invention or a product made by a patented process."[5] If the lawsuit proceeds to trial, the plaintiff (the patent holder) and the defendant (the alleged infringer) present evidence to a jury. In a trial, the infringer's product is compared to the plaintiff's patented invention. To establish infringement, the plaintiff must show the infringing product includes every element of the patented product. This is called the "all elements rule."[6] In some instances, the "doctrine of equivalents" allows a plaintiff to show infringement if some element of the patented product is missing in the infringing product but the differences are insubstantial.[7]

[3] See https://www.justia.com/intellectual-property/patents

[4] See https://www.law.cornell.edu/uscode/text/35/154.

[5] See https://content.next.westlaw.com/practical-law/document/I0a46282fd1a011e598dc8b09b4f043e0/Patent-litigation-in-the-United-States-overview?viewType=FullText&transitionType=Default&contextData=(sc.Default).

[6] See https://definitions.uslegal.com/a/all-elements-rule/.

[7] See https://www.law.cornell.edu/wex/doctrine_of_equivalents.

If the plaintiff wins the patent-infringement lawsuit, the judge will typically grant a permanent injunction against the infringing defendant. The permanent injunction is an order forcing the defendant to stop all activity that infringes on the patent. If the injunction covers any step in the production of the defendant's product, the defendant must entirely shut down that product until a noninfringing process is developed. While patent infringement itself is a tort and not a criminal offense, violating a permanent injunction can lead to criminal penalties.[8]

Prior to 2006, permanent injunctions were essentially automatically granted after plaintiff victories. However, in 2006, the US Supreme Court clarified the criteria for granting a permanent injunction. In *eBay Inc v MercExchange L.L.C.* ("eBay"),[9] the Supreme Court ruled that courts must apply a four-factor test to determine whether a permanent injunction is appropriate. In this test, the plaintiff must show (i) it suffered irreparable harm; (ii) other remedies (e.g. monetary damages) are inadequate to compensate the plaintiff; (iii) comparing the resulting hardships for the plaintiff and defendant, equitable relief (e.g., enforcing the patent) is warranted; and (iv) an injunction would not harm the public interest.[10] The rate at which successful plaintiffs obtained injunctions declined from 95% before eBay to 75% after eBay (Seaman, 2015).

Anticipating the trial process described above, many plaintiffs and defendants settle patent infringements out of court. In many instances, a plaintiff files a formal lawsuit that is ultimately settled before trial. In other instances, the plaintiff sends a "demand letter" asking the defendant to pay a license fee for the use of the patented technology. If the defendant agrees, this is a form of out-of-court settlement.

2.2. Illustrative Model

This section provides a simple model to illustrate the role of the litigation system in promoting efficient innovation. We first show that privately optimizing agents can underinvest or overinvest in innovation relative to the socially efficient benchmark. We then show that changing the litigation system to be more plaintiff-friendly can improve

[8]See https://www.mandourlaw.com/patent-injunction/.

[9]See https://www.law.cornell.edu/supct/cert/05-130.

[10]See https://content.next.westlaw.com/practical-law/document/I0a46282fd1a011e598dc8b09b4f043e0/Patent-litigation-in-the-United-States-overview?viewType=FullText&transitionType=Default&contextData=(sc.Default).

or lower welfare, depending on which types of innovating firms use the litigation system.

2.2.1. Illustrative model

There are two technology classes $c = 1, 2$ and two product markets $j = 1, 2$. We refer to a firm by its pair (c, j). In each pair (c, j), there is a continuum of identical firms with measure 1. Every firm starts with one product line. Each firm chooses how much to spend on innovation. Let x_{cj} denote the innovation policy chosen by each firm in pair (c, j). The cost of innovation for each firm is χx_{cj}^2 for a parameter $\chi > 0$.

The probability of successful innovation by a firm in pair (c, j) is x_{cj}. If a firm in (c, j) successfully innovates, it has a chance of stealing the product line from a competitor in market j. We call this competitor the "target firm." All firms in pairs (c, j) and (c', j) are equally likely to be targets when a firm in (c, j) innovates.

If the target firm shares the same technology class as the innovator, the target sues to retain its product line with probability ρ_{cj}. This parameter captures features of the product market and technology class that allow for easier legal action. For example, in some technology classes it might be particularly easy to write broad patents that relate to many potential innovations. If the target firm that previously held the product line sues, it wins the lawsuit and obtains an injunction with probability $\iota \in (0, 1)$. Importantly, we assume that policy makers can vary ι but cannot vary ρ_{cj}. They must take as given the kind of firm that uses the legal system when deciding how to change the legal system. Given this assumption, if a firm in pair (c, j) innovates, its probability of getting a new product line is

$$\underbrace{\frac{1}{2}}_{\text{Target has c}' \neq c} + \underbrace{\frac{1 - \rho_{cj}\iota}{2}}_{\text{Target has c}=c} = \frac{2 - \rho_{cj}\iota}{2}. \tag{1}$$

If a firm keeps its original product line, it gets cashflow π. If a firm with pair (c, j) innovates and steals a product line, it gets cashflow $\pi + \lambda_c + \sigma x_{c,j'}$ from the stolen product line and the target firm losing the product line gets nothing from that line. The parameter λ_c captures the efficacy of innovation in technology class c. The parameter σ captures the technology spillover: if firms in technology class c all innovate more, then their innovation is more impactful across product lines. Note that the parameter π drives the business stealing incentive - the planner does not internalize who gets the cashflow π, but firms do. In contrast, firms do not internalize their technology spillovers. Specifically, fixing the equilibrium strategies of other firms, firm (c, j)'s problem is

$$\sup_{x_{cj}} \underbrace{x_{cj} \frac{2 - \rho_{cj}\iota}{2}(\pi + \lambda_c + \sigma x^*_{cj'})}_{\text{Steal product line}} - \chi(x_{cj})^2. \tag{2}$$

Note that each firm cannot take any action to help retain its original product line - the optimization is thus only a choice over how much effort to exert to steal another product line. Rearranging the first-order condition, the privately optimal innovation policy is:

$$x^*_{cj} = (2 - \rho_{cj}\iota)\frac{\pi + \lambda_c + \sigma x^*_{cj'}}{4\chi}. \tag{3}$$

The total equilibrium value of firms in the pair (c, j), which includes the value of the original product line, is:

$$\text{Value}_{cj}(x^*) = \underbrace{\pi\left(1 - \frac{x^*_{c'j}}{2} - \frac{x^*_{cj}(1 - \rho_{cj}\iota)}{2}\right)}_{\text{Keep original product line}} + \underbrace{x^*_{cj}\frac{2 - \rho_{cj}\iota}{2}(\pi + \lambda_c + \sigma x^*_{cj'})}_{\text{Steal product line}} - \chi(x^*_{cj})^2. \tag{4}$$

We now compare the privately optimal policies (3) to the innovation policies that a social planner would choose to maximize welfare. The social planner chooses a vector innovation x^s of innovation policies for all firms to optimize the sum of all firms' values $\sum_{c,j} Value_{cj}(x^s)$. Cancelling terms, this is:

$$\sup_{x^s} \sum_{c,j} Value_{cj}(x^s) = 4\pi + \sup_{x^s} \sum_{c,j} x^s_{cj} \frac{2 - \rho_{cj}\iota}{2}(\lambda_c + \sigma x^s_{cj'}) - \chi(x^s_{cj})^2. \tag{5}$$

In words, the planner only cares about the resources spent on innovation, the gains λ_c from successful innovation, and the technology spillover. The planner does not care who gets the original cashflow π from each product line, so it does not internalize business stealing effects. Rearranging the first-order condition with respect to x^s_{cj}, we get:

$$x^s_{cj} = \frac{2-\rho_{cj}\iota}{4\chi}(\lambda_c + \sigma x^s_{cj'}) + \sigma x^s_{cj'}\frac{2-\rho_{cj'}\iota}{4\chi}. \tag{6}$$

2.2.2. Illustrative model intuition

The following lemma summarizes useful intuition from the illustrative model.

Lemma 1. *Assume that $\pi > 0$, $\lambda_c > 0$, and $0 < \sigma < \chi$. Then:*

1. *As $\sigma \to 0$, privately optimizing agents overinvest in innovation: $x^*_{cj} > x^s_{cj}$.*
2. *As $\pi \to 0$, privately optimizing agents underinvest in innovation: $x^*_{cj} < x^s_{cj}$. As $\pi \to \infty$, privately optimizing agents overinvest in innovation: $x^*_{cj} > x^s_{cj}$.*
3. *As $\lambda_c \to 0$, privately optimizing agents overinvest in innovation: $x^*_{cj} > x^s_{cj}$.*

Intuitively, private agents inefficiently internalize the transfer π they extract from other agents when innovating to steal their product lines. When this transfer gets large, they spend too much on innovation. Private agents do not internalize the positive externality their innovation creates through technology spillovers. These technology spillovers grow with the parameter σ. As $\sigma \to 0$, there is no positive externality from innovation, so the business stealing again leads to overinvestment in innovation. As $\pi \to 0$, the business stealing incentive disappears, so there is no reason to overinvest, so the private agents underinvest in innovation because they don't internalize technology spillovers.

Finally, we see that λ_c matters for the social value of innovation. As $\lambda_c \to 0$, the main incentive to innovate is to steal business from others, since there is no marginal improvement in technology. This means there is too much innovation relative to the socially efficient benchmark. In summary, the parameters π, σ, λ_c determine whether there is too much or too little innovation relative to the socially efficient benchmark.

2.2.3. The role of litigation

We see from equation (3) that more injunctions (higher ι) tend to discourage innovation. The extent of this effect depends on how likely firms in (c, j) are to use the litigation system (ρ_{cj}). If litigation is primarily used by firms that underinvest in innovation, then further discouraging their innovation through injunctions would harm welfare. If litigation is primarily used by firms that overinvest in innovation, then discouraging their innovation through injunctions would improve welfare. From Lemma 1, we see that firms with low λ_c are more likely to overinvest in innovation. This suggests that increasing the injunction rate helps welfare if low λ_c firms primarily use the litigation system. Conversely, increasing the injunction rate harms welfare if high λ_c firms primarily use the litigation system.

We now formalize this with a simple numerical example. Table 1 shows our parameter value assumptions. We solve the model using equations (6) and (3). We assume one technology class has $\lambda_1 = 3$ while the other technology class has $\lambda_2 = 0.1$. Consistent with lemma 1, we see from Table 1 that firms with $c = 1$ underinvest in innovation, relative to the socially efficient benchmark, while firms with $c = 2$ overinvest in innovation. In our first case (panel 2), we assume that only firms in technology class 1 access the litigation system. In this situation, increasing ι will lead to less innovation for firms in technology class 1 while firms in class 2 are unaffected. This exacerbates the underinvestment in innovation by firms in class 1, harming welfare. In our second case (panel 3), we assume that only firms in technology class 2 access the litigation system. In this situation, increasing ι will lead to less innovation for firms in technology class 2. This ameliorates their overinvestment, improving welfare.

2.2.4. Illustrative model limitations

This illustrative model demonstrates the key intuition behind our exercise. Firms can innovate too much or too little, relative to the socially efficient benchmark, because of the tradeoff between positive technology spillovers and negative business stealing incentives. Changes in the litigation system can encourage or discourage innovation, and the welfare effects of such changes depend on whether litigating firms are over-innovators or under-innovators.

However, the illustrative model has many limitations. First, the litigation system is extremely simple. It doesn't account for how trial outcomes such as injunctions shape the incentives for firms to settle out of court. This is critical since most lawsuits are settled out of court. Second, the model does not capture the potential for incumbent innovation to crowd out new entrants. Since innovation leads to monopolies through patents, it is critical to understand how suppressing incumbents impacts welfare. Third, the model is static. This severely limits its quantitative usefulness. For example, in the illustrative model, potential innovators always expect to be defendants in patent infringement lawsuits. It follows that a defendant-friendly system always encourages innovation in this illustrative model. In practice, however, firms know when they innovate that they might end up using the litigation system to defend their patents in the future. Capturing the potential for an innovator to be a current defendant and a future plaintiff is essential for understanding how litigation shapes innovation.

We now overcome these limitations by introducing a dynamic general equilibrium model. Our model features a realistic litigation system with asymmetric information and endogenous out-of-court settlement. It includes monopolist incentives and new entrants. And importantly, it includes the potential for innovating firms to be both defendants, when they innovate, and future plaintiffs, when they protect their past innovation. These realistic features allow us to more convincingly match the data generated by real-world behavior. Calibrating this model, we can better understand how changing the litigation system shapes innovation.

3. Model

3.1. Environment and preferences

Time is continuous and denoted by $t \geq 0$. There is an infinitely-lived representative household whose preferences are represented by the lifetime utility function

$$\int_0^\infty e^{-\rho t} \ln C_t dt \tag{7}$$

where $\rho > 0$ is the discount rate, and C_t denotes consumption of the final good at time t. The household owns all assets A_t in the economy, which deliver a rate of return equal to r_t. The household supplies labor $L = 1$ inelastically to firms at the real wage rate w_t.

3.2. Technology

3.2.1. Final good production

The final consumption good Y_t is produced by a competitive firm that combines differentiated goods from different industries indexed by $j \in \{1, ..., J\}$. The production function is expressed as

$$\ln Y_t = \sum_{j=1}^{J} \omega_j \ln Y_{jt} \tag{8}$$

where $\omega_j \in (0, 1)$ denotes the Cobb-Douglas weight of industry j's output Y_{jt} in production, with $\sum_{j=1}^{J} \omega_j = 1$. The output of each industry j, in turn, is produced by combining a continuum of differentiated goods in said industry according to the production function:

$$\ln Y_{jt} = \int_0^1 \ln y_{ijt} di \tag{9}$$

where y_{ijt} denotes the quantity of differentiated good $i \in [0, 1]$ in industry j at time t. The price of the final consumption good is set as the numeraire, whereas the price of good i in industry j at time t is denoted as p_{ijt}.

3.2.2. Differentiated good production

Each differentiated good in each industry can be produced by multiple firms who own a blueprint to produce it, using labor as input with productivity q. As is common in the Schumpeterian growth literature, we assume Bertrand competition between firms such that only the productivity leader produces a positive amount of the good in equilibrium. Let q_{ijt} denote the productivity of the leader for good i in industry j at time t. Then the production function for y_{ijt} is written as:

$$y_{ijt} = q_{ijt} l_{ijt} \tag{10}$$

where $l_{ijt} \geq 0$ is the labor hired by the leader for production.

3.2.3. Firms, technology classes, and product markets

As in Klette and Kortum (2004), each firm owns a portfolio of blueprints to produce various differentiated goods. They produce each good for which they are the technology leader, which is called a "product line" of the firm. Ignoring the effects of litigation, a firm can become the leader in a new product line by discovering a better technology than that of the incumbent through innovation. Likewise, a firm can lose its status as the leader if a competitor discovers a technology that is better. In the absence of legal intervention, this creative destruction leads the prior leader to cede its product line to the innovating competitor. A firm with no product lines exits.

Different from the existing Schumpeterian growth literature, we introduce two further dimensions of heterogeneity across firms. Firms fundamentally differ from each other in terms of their innovation process ("technology class") and the industry they operate in ("product market").

Each firm has a technology class $c \in \{1, ..., C\}$ which determines the productivity improvement generated by their successful innovations, as well as the body of knowledge they draw upon to come up with new blueprints. The body of knowledge they rely on affects the knowledge spillovers their innovation activities benefit from, as well as whether their new innovations can infringe upon the intellectual property of other firms. The details are discussed in the following subsections.

The product market of a firm $j \in \{1, ..., J\}$, in turn, determines the industry in which they can produce differentiated goods. This firm characteristic determines the returns to successful innovation from taking over new product lines, as well as the creative destruction risk from competitors in the same product market.

3.2.4. Incumbent innovation

Incumbent firms can engage in risky innovation to discover new blueprints and potentially expand into new product lines. Conditional on successful innovation, the firm improves upon one of the existing technology leaders' blueprints to produce a differentiated good, chosen at random among all possible goods in the innovating firm's product market. The productivity of the new blueprint is given by

$$q_{ijt}^{new} = (1 + \lambda_c) q_{ijt}^{old} \tag{11}$$

where q_{ijt}^{old} is the productivity of the existing leader, and $\lambda_c > 0$ is the step size by which the new innovation improves upon the previous one. The size of λ_c hinges on the technology class c of the innovating firm.

Each owned product line provides the firm with a lab to generate a Poisson arrival rate of successful innovation $x_{ijt} \geq 0$. Coming up with new ideas is costly. To generate this arrival rate, the firm must spend final good on R&D given by the cost function

$$C_c(x_{ijt}) = \frac{\chi_c x_{ijt}^{\psi} Y_t}{1 + \sigma M_{ct}} \tag{12}$$

where $\chi_c > 0$ is a scale parameter, $\psi > 1$ introduces convexity, Y_t ensures the R&D costs scale up with aggregate output along a balanced growth path equilibrium, and σM_{ct} with $\sigma \geq 0$ is the term that captures the knowledge spillovers from other firms in the same technology class c which will be discussed next.[11]

Let I_{cjt} denote the set of goods i in industry j for which the leader has technology class c. Let $\mu_{cjt} \in [0, 1]$ denote the measure of the set I_{cjt}. Then $M_{ct} \in [0, 1]$ is defined as

$$M_{ct} = \sum_{j=1}^{J} \omega_j \mu_{cjt} \tag{13}$$

that is, the fraction of all product lines in the economy currently owned by firms with technology class c, where different industries are weighted in proportion to their Cobb-Douglas share in final good production. The higher the value of M_{ct} is, the cheaper it is for all firms in technology class c to discover new ideas. In other words, past successful innovation by other firms in the same technology class increases a firm's research efficiency. The strength of this technology-class-specific knowledge spillover is governed by the parameter $\sigma \geq 0$, where a higher value of σ corresponds to stronger knowledge spillovers within the same technology class.

3.2.5. Entrant innovation

There is a continuum of identical entrepreneurs of measure one. These entrepreneurs choose the rate at which they found new businesses through successful entrant innovation. To generate a Poisson arrival rate of successful innovation $z_t \geq 0$, they must spend final

[11] Note that this technology-class-specific knowledge spillover is in addition to the inherent Schumpeterian knowledge spillovers from improving the productivity of the existing leader given in Equation (11). That is, our model features within-industry knowledge spillovers that can be both within and across technology classes, as well as within-technology-class spillovers that are both within and across industries.

good on R&D according to the cost function

$$C_e(z_t) = \nu z_t^{\psi} Y_t \tag{14}$$

where $\nu > 0$ is a scale parameter. As was the case for incumbent innovation, $\psi > 1$ is the convexity parameter, and the Y_t term ensures the R&D costs scale up with aggregate output along a balanced growth path equilibrium.

Conditional on successful innovation, the probability that the new firm has technology class c and industry j is denoted as $\eta_{cj} \in [0,1]$. For all c and j, η_{cj} are exogenous parameters that satisfy $\sum_{c=1}^{C} \sum_{j=1}^{J} \eta_{cj} = 1$. The new firm with a single product line is immediately sold off at fair market value by the successful entrepreneur, who remains an entrepreneur and continues to found new businesses.

3.2.6. Patent infringement and litigation

When a firm successfully innovates, its new innovation might potentially infringe upon the intellectual property of incumbents, which creates a risk of litigation. We model two types of potential patent infringement:

1. **Type 1:** If a firm innovates on a product line owned by a firm with the same technology class, then it infringes on the intellectual property of the incumbent with probability $\kappa_1 \in [0,1]$.

2. **Type 2:** If a firm innovates on a product line owned by a firm with a different technology class, then it does not infringe upon the intellectual property of the incumbent due to the differences in technologies. However, with probability $\kappa_2 \in [0,1]$, the firm infringes on the IP of another firm in the same technology class. This reflects the fact that the new innovation is building upon existing patents in the same technology class. The exact infringed patent is chosen at random across all product lines owned by firms with the same technology class as the innovator, which can be in other industries.

In the first scenario, the incumbent has a lot at stake. If the incumbent can successfully litigate, it can avoid losing its product line to the innovating firm. The plaintiff and the defendant are therefore fighting over the ownership of a product line in which patent infringement occured.

In the second scenario, the plaintiff is not the incumbent, and it has no direct stake in whether the innovator or the incumbent ends up as the owner of the product line. However, the plaintiff can extract rents by threatening to deny the innovator the chance to capture the product line through an injunction.

We model the litigation subgame as follows. Conditional on patent infringement, the plaintiff has to make a decision of whether to hire a legal team. A legal team is necessary for making a settlement offer or going to court. The cost of hiring a legal team is γY_t, where $\gamma > 0$ is a random variable drawn from the distribution $\Gamma(\gamma)$, and the Y_t term ensures that litigation costs grow at the same rate as output in a balanced growth path (BGP) equilibrium. If the plaintiff chooses to pay γY_t and hire a legal team, it then makes a take-it-or-leave-it settlement offer to the defendant. The defendant has private information about its chances of winning the trial. Let $\tau \in [0, 1]$ denote the probability that the defendant wins the trial. This probability is drawn from the exogenous distributions $T_1(\tau)$ and $T_2(\tau)$ for type 1 and type 2 infringements, respectively. Given its private information τ, the defendant can accept the settlement or refuse. Refusal leads to a trial. With probability τ, the defendant wins the trial and the product line takeover is realized. With the complementary probability $1 - \tau$, the defendant loses. If the defendant loses, then the court decides on whether to grant an injunction or not. With probability $\iota \in [0, 1]$, an injunction is granted and the product line takeover is blocked. With probability $1 - \iota$, there is no injunction and the defendant can still take over the product line. The parameter ι is a policy parameter that captures the inclination of a court to grant an injunction in the case of a proven patent infringement. Figure 1 illustrates the timeline of the litigation subgame.

3.3. Decision problems

3.3.1. Household's problem

Given initial assets $A_0 > 0$, the utility maximization problem of the representative household is written as

$$\max_{[C_t, A_t]_{t=0}^{\infty}} \left\{ \int_0^{\infty} e^{-\rho t} \ln C_t dt \right\}, \text{ subject to} \tag{15}$$

$$\dot{A}_t = r_t A_t + w_t - C_t, \forall t \geq 0 \tag{16}$$

The Euler equation of this standard problem is

$$\frac{\dot{C}_t}{C_t} = r_t - \rho \tag{17}$$

3.3.2. Final good producer's problem

The static profit maximization problem of the competitive final good producer at time t is written as

$$\max_{\left\{[y_{ijt}]_{i=0}^{1}\right\}_{j=1}^{J}} \left\{ \exp\left(\sum_{j=1}^{J} \omega_j \left(\int_0^1 \ln y_{ijt} di \right) \right) - \sum_{j=1}^{J} \left(\int_0^1 p_{ijt} y_{ijt} di \right) \right\} \tag{18}$$

For any good i in industry j, the first order condition delivers

$$y_{ijt} = \frac{\omega_j Y_t}{p_{ijt}} \tag{19}$$

which pins down the demand function for y_{ijt}.

3.3.3. Differentiated good producer's problem

Due to Bertrand competition, only the productivity leader for any differentiated good produces a positive quantity. The final good production function assures that the leader always chooses to follow a limit pricing strategy since the monopoly price tends to infinity. For a leader with productivity q_{ijt} and technology class c, the productivity of the most productive competitor is $q_{ijt}/(1+\lambda_c)$, and the limit price is therefore $w_t(1+\lambda_c)/q_{ijt}$. Then the static profit flow of the leader for good i in industry j at time t is written as

$$\begin{aligned} \pi_{ijt} &= \left(p_{ijt} - \frac{w_t}{q_{ijt}} \right) y_{ijt} \\ &= \left(\frac{w_t(1+\lambda_c)}{q_{ijt}} - \frac{w_t}{q_{ijt}} \right) \frac{q_{ijt}\omega_j Y_t}{w_t(1+\lambda_c)} \\ &= \frac{\lambda_c}{1+\lambda_c} \omega_j Y_t \end{aligned} \tag{20}$$

which grows at the same rate as aggregate output Y_t, is linearly related to the industry j's share ω_j, and is increasing in the technology class specific productivity step size λ_c, which is also the net markup. Note that this quantity is independent of the productivity q_{ijt}. Therefore, the relevant state variable for an incumbent firm's dynamic problem is not the set of productivities of owned product lines, but simply its cardinality. This dynamic problem will be discussed next.

3.3.4. Incumbent's dynamic problem

The dynamic profit maximization problem of an incumbent firm with technology class c in industry j who is the leader in n product lines at time t is stated as

$$\begin{aligned} r_t V_{cjt}(n) - \dot{V}_{cjt}(n) = \max_{\{x_{mcjt}\}_{m=1}^{n}} \Bigg\{ & \sum_{m=1}^{n} \frac{\lambda_c}{1+\lambda_c}\omega_j Y_t + n \sum_{j'=1}^{J} R_{cj't} - \sum_{m=1}^{n} \frac{(1-s_{cj})\chi_c x_{mcjt}^{\psi} Y_t}{1+\sigma M_{ct}} \\ & + \left(\sum_{m=1}^{n} x_{mcjt}\right)\left(V_{cjt}^{+}(n) - V_{cjt}(n)\right) \\ & + n d_{jt}\left(V_{cjt}^{-}(n) - V_{cjt}(n)\right) + \delta\left(0 - V_{cjt}(n)\right) \Bigg\} \end{aligned} \tag{21}$$

$V_{cjt}(n)$ is the firm's value function. The first term in the maximization is the sum of static profit flows from owned product lines. The second term is the expected rent flow from type 2 patent infringements by other firms on our firm's intellectual property which do not directly contest its ownership of product lines. Such infringements can occur not only due to innovations in the firm's own industry, but also other industries $j' \neq j$, hence the summation. R_{cjt} stands for such rent flows for a single product line from all infringements by firms in industry j, and is thus multiplied by n. The third term is the total R&D bill, where $s_{cj} \in [0,1]$ is the potentially industry- and technology-class specific incumbent R&D subsidy rate. The fourth term is the Poisson arrival rate of a successful innovation by the firm multiplied by the expected increase in firm value. The value function $V_{cjt}^{+}(n)$ stands for the expected value of the firm conditional on successful innovation, but before potential patent infringement and consequent litigation outcomes are realized. The fifth term is the Poisson arrival rate of a successful innovation by another firm on our firm's product lines, nd_{jt}, multiplied by the expected change in firm value. $d_{jt} \geq 0$ is the creative destruction rate in industry j at time t, and is determined by the innovation efforts of all incumbent and entrant firms in the same industry. The value function $V^{-}_{cjt}(n)$ stands for the expected value of the firm conditional on facing this event, once again before patent infringement and litigation outcomes are realized. Finally, the sixth term stands for the risk of exogenous firm exit at rate $\delta \geq 0$.[12]

In order to characterize the value function $V_{cjt}(n)$ in full, the three value functions related to litigation – R_{cjt}, $V^{+}_{cjt}(n)$, and $V^{-}_{cjt}(n)$ – must be calculated. However, we can make a few observations before moving on. First, the static profit flows from owned product lines are the same, and therefore the total flow is linear in n. Second, the first order condition with respect to the innovation rate x_{mcjt} for any m is given by

$$
\begin{aligned}
\frac{(1-s_{cj})\chi_c \psi x_{mcjt}^{\psi-1} Y_t}{1+\sigma M_{ct}} &= \left(V^{+}_{cjt}(n) - V_{cjt}(n)\right) \\
x_{mcjt} &= \left(\frac{\left(V^{+}_{cjt}(n) - V_{cjt}(n)\right)(1+\sigma M_{ct})}{(1-s_{cj})\chi_c \psi Y_t}\right)^{\frac{1}{\psi-1}} \equiv x_{cjt}(n) \qquad (22)
\end{aligned}
$$

showing that $x_{mcjt} = x_{cjt}(n), \forall m$, and the total R&D bill as well as the arrival rate of successful innovation at the firm level are also linear in the number of product lines n. As will be proven later, these properties ensure that the firm value function $V_{cjt}(n)$ itself is linear in n.

3.3.5. Entrepreneur's problem

At any given time t, the static profit maximization problem of an entrepreneur is stated as

$$
\max_{z_t \geq 0} \left\{ -(1-s_e)\nu z_t^{\psi} Y_t + z_t \sum_{c=1}^{C} \sum_{j=1}^{J} \eta_{cj} V^{+}_{cjt}(0) \right\} \qquad (23)
$$

The first term in the maximization is the R&D cost incurred by the entrepreneur, and the second term is the expected return from entrant innovation. $s_e \in [0,1]$ is the entrant

[12] The exogenous exit rate δ captures firm exit events due to reasons other than losing all product lines. When a firm exogenously exits, it is replaced by an identical firm that inherits its product lines.

R&D subsidy rate. The Poisson arrival rate of successful innovation is z_t. Conditional on successful innovation, the new firm has technology class c and industry j with probability η_{cj}.

If there were no litigation, the value of the new firm would simply be $V_{cjt}(1)$. However, due to the risk of litigation, the new firm's value is equal to the expected value of an incumbent firm with zero existing product lines that succeeded in innovation, but before potential patent infringement and consequent litigation outcomes are realized, denoted $V^+_{cjt}(0)$.[13]

The first order condition with respect to entrant innovation z_t pins down its optimal value as

$$\begin{aligned} (1-s_e)\nu\psi z_t^{\psi-1}Y_t &= \sum_{c=1}^{C}\sum_{j=1}^{J}\eta_{cj}V^+_{cjt}(0) \\ z_t &= \left(\frac{\sum_{c=1}^{C}\sum_{j=1}^{J}\eta_{cj}V^+_{cjt}(0)}{(1-s_e)\nu\psi Y_t}\right)^{\frac{1}{\psi-1}} \end{aligned} \tag{24}$$

3.3.6. Litigation and settlements

We solve for the subgame perfect equilibrium of the litigation game by backward induction. The expected payoff from going to court for a defendant firm with technology class c innovating in industry j with n product lines is given by

$$[\tau+(1-\tau)(1-\iota)]\left(V_{cjt}(n+1)-V_{cjt}(n)\right) \tag{25}$$

where the first term is the total probability of taking over the product line (either by winning the trial or the court deciding not to grant an injunction despite recognizing the infringement) and the second term is the increase in firm value from increasing the number of its product lines from n to $n+1$. Facing an out-of-court settlement offer s to be paid to the plaintiff, the defendant's decision problem to accept or refuse the settlement

[13] Note that $V^+_{cjt}(0) = V^+_{cjt}(0) - V_{cjt}(0)$ since $V_{cjt}(0) = 0$.

is written as

$$\max_{\mathbb{I}\in\{0,1\}} \left\{\mathbb{I}\left(V_{cjt}(n+1)-V_{cjt}(n)-s\right)+(1-\mathbb{I})[\tau+(1-\tau)(1-\iota)]\left(V_{cjt}(n+1)-V_{cjt}(n)\right)\right\} \quad (26)$$

where $\mathbb{I}=1$ indicates acceptance and $\mathbb{I}=0$ indicates refusal. Then, the defendant strictly accepts the offer if and only if

$$\begin{aligned} V_{cjt}(n+1)-V_{cjt}(n)-s &> [\tau+(1-\tau)(1-\iota)]\left(V_{cjt}(n+1)-V_{cjt}(n)\right) \\ (1-\tau)\iota\left(V_{cjt}(n+1)-V_{cjt}(n)\right) &> s \end{aligned} \quad (27)$$

where $(1-\tau)\iota$ is the probability of an injunction conditional on going to court. In other words, the defendant strictly accepts the settlement offer only if the value of removing the injunction risk is sufficiently high. Denote the threshold value of s which leaves the defendant indifferent as $\bar{s}_{cjt}(n,\tau)$.

Type 2 patent infringements:

We first consider the decision problem of a plaintiff facing a type 2 patent infringement in which they don't face any risk of losing product lines. They must choose a take-it-or-leave-it settlement offer s without knowing the realization of τ – the defendant's probability of winning at court. Their problem is written as

$$\max_{s\geq 0} \left\{s\mathbb{P}(s\leq \bar{s}_{cjt}(n,\tau))\right\} \quad (28)$$

where the second term is the probability that the offer is accepted. Assume the distribution $T_2(\tau)$ is the continuous uniform distribution $U(\tau_2^l,\tau_2^h)$ with $0\leq\tau_2^l<\tau_2^h\leq 1$. Assume further that $1+\tau_2^l\leq 2\tau_2^h$, which ensures the solution is always interior. Proposition 1 summarizes the equilibrium of this game.

Proposition 1. *When successful innovation by a firm with technology class c in industry j with n product lines leads to a type 2 patent infringement, the following are true in the subgame perfect equilibrium of the litigation game:*

1. *The ex-ante probability that the plaintiff hires a legal team is*

$$p^{LT}_{2,cjt} \equiv \mathbb{P}\left(\gamma \leq \frac{(1-\tau^l_2)^2\iota(V_{cjt}(n+1)-V_{cjt}(n))}{4(\tau^h_2-\tau^l_2)\Upsilon_t}\right) \tag{29}$$

2. *The optimal settlement offer made by the plaintiff is*

$$s^* \equiv \frac{(1-\tau^l_2)\iota(V_{cjt}(n+1)-V_{cjt}(n))}{2} \tag{30}$$

3. *The defendant accepts the settlement if* $\tau \leq \tau^* \equiv \frac{1+\tau^l_2}{2}$*, and rejects otherwise. The ex-ante acceptance probability is* $\mathbb{P}(s^* < \bar{s}_{cjt}(n,\tau)) = \frac{1}{2}\frac{1-\tau^l_2}{\tau^h_2-\tau^l_2}$.

4. *The expected payoff of the plaintiff is*

$$W^{plain}_{2,cjt} \equiv p^{LT}_{2,cjt}\frac{(1-\tau^l_2)^2\iota(V_{cjt}(n+1)-V_{cjt}(n))}{4(\tau^h_2-\tau^l_2)} - \Upsilon_t\int_0^{\frac{(1-\tau^l_2)^2\iota(V_{cjt}(n+1)-V_{cjt}(n))}{4(\tau^h_2-\tau^l_2)\Upsilon_t}} \gamma d\Gamma(\gamma) \tag{31}$$

5. *The expected payoff of the defendant is*

$$W^{def}_{2,cjt} \equiv \Bigg((1-p^{LT}_{2,cjt}) + p^{LT}_{2,cjt}\Bigg[\left(1-\frac{(1-\tau^l_2)\iota}{2}\right)\left(\frac{1}{2}\frac{1-\tau^l_2}{\tau^h_2-\tau^l_2}\right) + (1-\iota)\left(1-\frac{1}{2}\frac{1-\tau^l_2}{\tau^h_2-\tau^l_2}\right) + \frac{\iota}{2(\tau^h_2-\tau^l_2)}\left((\tau^h_2)^2-\frac{(1+\tau^l_2)^2}{4}\right)\Bigg]\Bigg)(V_{cjt}(n+1)-V_{cjt}(n)) \tag{32}$$

Proof. See Section B.2 in the Theory Appendix. □

Type 1 patent infringements:

We now consider the decision problem of a plaintiff facing a type 1 patent infringement which means they are the owner of the product line that is facing the risk of creative destruction. We further know that the plaintiff and the defendant share the same technology class c. Unlike a type 2 infringement, this time the plaintiff cares about more than the potential settlement they can extract from the defendant, since settling out of court also means they lose their product line for sure. Assume the distribution $T_1(\tau)$ is the continuous uniform distribution $U(\tau_1^l, \tau_1^h)$ with $0 \leq \tau_1^l < \tau_1^h \leq 1$. Then the plaintiff's problem is written as

$$\max_{s\geq 0} \left\{ \int_{\tau_1^l}^{1-s/(\iota(V_{cjt}(n^d+1)-V_{cjt}(n^d)))} \left(V_{cjt}(n-1) - V_{cjt}(n) + s\right) dT_1(\tau) \right.$$
$$\left. + \int_{1-s/(\iota(V_{cjt}(n^d+1)-V_{cjt}(n^d)))}^{\tau_1^h} (\tau + (1-\tau)(1-\iota))(V_{cjt}(n-1) - V_{cjt}(n))dT_1(\tau) \right\} \quad (33)$$

where the first integral is the expected payoff from defendants who accept the settlement and the second integral is the expected payoff from those who reject. The term $V_{cjt}(n-1) - V_{cjt}(n)$ is negative, and reflects the cost of losing the product line. In the cases when the defendant accepts, the plaintiff is gives up their $(1-\tau)\iota$ chance of retaining their product line in exchange for a settlement amount s. Proposition 2 summarizes the equilibrium of this game.

Proposition 2. *Suppose $V_{cjt}(n)$ is linear in n. When successful innovation by a firm with technology class c in industry j with n^d product lines leads to a type 1 patent infringement on the IP of an incumbent with n product lines, the following are true in the subgame perfect equilibrium of the litigation game:*

1. *The ex-ante probability that the plaintiff hires a legal team is*

$$p_{1,cjt}^{LT} \equiv \mathbb{P}\left(\gamma \leq \left(-\iota + \iota\frac{\tau_1^h + \tau_1^l}{2}\right) \frac{(V_{cjt}(n-1) - V_{cjt}(n))}{Y_t}\right) \quad (34)$$

2. *Due to adverse selection, the plaintiff never chooses to settle out of court. That is, the plaintiff offers $s^* = 0$, and the defendant always rejects, independent of the realization of τ.*

3. *The expected payoff of the plaintiff is*

$$W_{1,cjt}^{plain} \equiv p_{1,cjt}^{LT}(V_{cjt}(n-1) - V_{cjt}(n))\left(1 - \iota + \iota\frac{\tau_1^h + \tau_1^l}{2}\right) - Y_t \int_0^{\left(-\iota+\iota\frac{\tau_1^h+\tau_1^l}{2}\right)\frac{(V_{cjt}(n-1)-V_{cjt}(n))}{Y_t}} \gamma d\Gamma(\gamma)$$
$$+(1 - p_{1,cjt}^{LT})(V_{cjt}(n-1) - V_{cjt}(n)) \quad (35)$$

4. *The expected payoff of the defendant is*

$$W^{def}_{1,cjt} \equiv p^{LT}_{1,cjt}(V_{cjt}(n^d+1) - V_{cjt}(n^d))\left(1 - \iota + \iota\frac{\tau^h_1 + \tau^l_1}{2}\right) + (1 - p^{LT}_{1,cjt})(V_{cjt}(n^d+1) - V_{cjt}(n^d)) \quad (36)$$

Proof. See Section B.3 in the Theory Appendix. □

3.3.7. Characterizing the value function

Having solved the subgame perfect equilibrium of the litigation game for both types of patent infringements, we are now ready to characterize the value function of an incumbent.

The rent flow for a single product line from type 2 patent infringements by others in industry j, R_{cjt}, is given by

$$R_{cjt} = p^{rent}_{cjt} W^{plain}_{2,cjt} \quad (37)$$

where p^{rent}_{cjt} is the Poisson arrival rate of a type 2 patent infringement from firms in industry j, calculated in general equilibrium. This arrival rate depends on the innovation choices of all other firms with the same technology class c in industry j, as well as the share of product lines that belong to firms with technology classs c across all industries. p^{rent}_{cjt} is increasing in the prior (since more firms innovating means there are more potential infringements) as well as the probability of a type 2 infringement κ_2, and it is decreasing in the latter (since the same amount of infringements is spread over a larger mass of potentially infringed product lines).

The value difference conditional on successful innovation, but before the litigation subgame, denoted as $V^+_{cjt}(n) - V_{cjt}(n)$, is given by

$$\begin{aligned} V^+_{cjt}(n) - V_{cjt}(n) \quad = \quad & p^{def}_{cjt}\kappa_1 W^{def}_{1,cjt} + (1 - p^{def}_{cjt})\kappa_2 W^{def}_{2,cjt} \\ & +[(p^{def}_{cjt}(1-\kappa_1) + (1 - p^{def}_{cjt})(1-\kappa_2)](V_{cjt}(n+1) - V_{cjt}(n)) \quad (38) \end{aligned}$$

where p^{def}_{cjt} is the probability that the firm innovates on the product line of another firm with the same technology class c in its industry, which is again determined in general equilibrium. The first term is the probability of a type 1 patent infringement times the associated defendant payoff, $W^{def}_{1,cjt}$, calculated earlier. Likewise, the second term is the probability of a type 2 patent infringement times the associated defendant payoff, $W^{def}_{2,cjt}$. The last term is the probability that no infringement happens times the value change from adding a new product line for certain.

The value difference conditional on being innovated on (i.e., value loss from creative destruction), but before the litigation subgame, denoted as $V^{-}_{cjt}(n) - V_{cjt}(n)$, is given by

$$\begin{aligned} V^{-}_{cjt}(n) - V_{cjt}(n) &= p^{plain}_{cjt}\kappa_1 W^{plain}_{1,cjt} + (1 - p^{plain}_{cjt})\kappa_2 p^{inj}_{cjt}(V_{cjt}(n) - V_{cjt}(n)) \\ &\quad + (1 - p^{plain}_{cjt}\kappa_1 - (1 - p^{plain}_{cjt})\kappa_2 p^{inj}_{cjt})(V_{cjt}(n-1) - V_{cjt}(n)) \end{aligned} \tag{39}$$

where p^{plain}_{cjt} is the probability that the incoming innovation belongs to a firm with the same technology class c, in which case a type 1 infringement is possible with probability κ_1. The first term is this joint probability times the associated plaintiff payoff, $W^{plain}_{1,cjt}$. The second term is the probability that the incoming innovation belongs to a firm with a different technology class, in which case a type 2 infringement is possible with probability κ_2. In such an event, the innovating firm interacts with a third firm whose patent is infringed, and the probability of an injunction being granted in the litigation subgame is denoted as p^{inj}_{cjt}. In this case, the incumbent retains its product line, and therefore there is no value loss (i.e., the second term equals zero, and it is kept only for clarity). The last term is the remaining probability times the value change from losing a product line for certain.

Given these expressions, we are now ready to solve for the firm value function in a BGP equilibrium in closed form.

Definition 1. *A balanced growth path (BGP) equilibrium of this economy is an equilibrium in which:*

1. *The aggregate variables Y_t, C_t, A_t and the real wage rate w_t grow at the constant rate $g > 0$.*

2. *The real interest rate r, the industry-specific creative destruction rates $\{d_j\}^J_{j=1}$, the fraction of product lines owned by technology class c firms $\{M_c\}^C_{c=1}$ and the probabilities $\{\{p^{rent}_{cj}, p^{LT}_{1,cj}, p^{LT}_{2,cj}, p^{def}_{cj}, p^{plain}_{cj}, p^{inj}_{cj}\}^J_{j=1}\}^C_{c=1}$ are time-invariant.*

Theorem 1. *In a BGP equilibrium, the value function of an incumbent firm with technology class c in industry j who is the leader in n product lines at time t is given by*

$$V_{cj}(n) = v_{cj} n Y_t \tag{40}$$

where $v_{cj} > 0$ is an industry- and technology-class-specific time-invariant scalar given by

$$v_{cj} \quad = \quad \frac{\frac{\lambda_c}{1+\lambda_c}\omega_j + \sum_{j'=1}^{J} \hat{R}_{cj'} - \frac{(1-s_{cj})\chi_c x_{cj}^{\psi}}{1+\sigma M_c}}{\rho + \delta - x_{cj} L_{cj}^{def} + d_j L_{cj}^{plain}} \tag{41}$$

In particular, x_{cj} is the time-invariant per product line incumbent innovation arrival rate given by

$$x_{cj} \quad = \quad \left(\frac{L_{cj}^{def} v_{cj}(1+\sigma M_c)}{(1-s_{cj})\chi_c \psi} \right)^{\frac{1}{\psi-1}} \tag{42}$$

and $\hat{R}_{cj}$, L_{cj}^{def}, and L_{cj}^{plain} are time-invariant terms that summarize the implications of the litigation subgame on firm value, defined in Equations (B.34), (B.37), and (B.40), respectively. Likewise, z is the time-invariant entrant innovation arrival rate given by

$$z \quad = \quad \left(\frac{\sum_{c=1}^{C} \sum_{j=1}^{J} \eta_{cj} L_{cj}^{def} v_{cj}}{(1-s_e)\nu\psi} \right)^{\frac{1}{\psi-1}} \tag{43}$$

Proof. See Section B.4 in the Theory Appendix. □

3.4. General equilibrium

To close the model, we need to derive the equations that pin down the values of endogenous variables in a BGP equilibrium, such as the growth rate g, the stationary product line distribution across industries and technology classes $\{\{\mu_{cj}\}_{c=1}^{C}\}_{j=1}^{J}$, and the associated probabilities of various events discussed earlier. Proposition 3 provides the values of these expressions.

Proposition 3. *In a BGP equilibrium, the following are true:*

1. *The industry-specific creative destruction rate d_j in industry j is*

$$d_j = \sum_{c=1}^{C} (\mu_{cj} x_{cj} + \eta_{cj} z) \tag{44}$$

2. *The probability for plaintiffs of type (c, j) that the incoming innovation belongs to a firm with the same technology class, p_{cj}^{plain}, is*

$$p_{cj}^{plain} = \frac{\mu_{cj} x_{cj} + \eta_{cj} z}{\sum_{c'=1}^{C} (\mu_{c'j} x_{c'j} + \eta_{c'j} z)} \tag{45}$$

3. *The probability for defendants of type (c, j) to innovate on the product line of another firm with the same technology class c in its industry, p_{cj}^{def}, is*

$$p_{cj}^{def} = \mu_{cj} \tag{46}$$

4. *The Poisson arrival rate of a type 2 patent infringement for plaintiffs with technology class c from firms in industry j, p_{cj}^{rent}, is*

$$p_{cj}^{rent} = \frac{(\mu_{cj} x_{cj} + \eta_{cj} z)(1 - \mu_{cj}) \kappa_2}{\sum_{j'=1}^{J} \mu_{cj'}} \tag{47}$$

5. *The probability that an injunction is granted conditional on a type 2 infringement from the perspective of the owner of the product line, p_{cj}^{inj}, is*

$$p_{cj}^{inj} = \frac{\sum_{c' \neq c} (\mu_{c'j} x_{c'j} + \eta_{c'j} z) p_{2,c'j}^{LT}}{\sum_{c' \neq c} (\mu_{c'j} x_{c'j} + \eta_{c'j} z)} \left[\left(\tau_2^h - \frac{1 + \tau_2^l}{2} - \frac{(\tau_2^h)^2}{2} + \frac{(1 + \tau_2^l)^2}{8} \right) \frac{1}{\tau_2^h - \tau_2^l} \right] \iota \tag{48}$$

6. *The time-invariant output growth rate g is given by*

$$g = \sum_{j=1}^{J} \omega_j \sum_{c=1}^{C} \mu_{cj} f_{cj} \tag{49}$$

where

$$f_{cj} = (\mu_{cj}x_{cj} + \eta_{cj}z)\left[1 - \kappa_1 p_{1,cj}^{LT}\left(1 - \frac{\tau_1^h + \tau_1^l}{2}\right)\iota\right]\ln(1+\lambda_c)$$
$$+ \sum_{c' \neq c}(\mu_{c'j}x_{c'j} + \eta_{c'j}z)\left[1 - \kappa_2 p_{c'j}^{inj2}\right]\ln(1+\lambda_c) \tag{50}$$

7. *Define $P(\Theta, \Theta')$ as the transition rate from product lines of type $\Theta = (c, j)$ (origin) to $\Theta' = (c', j')$ (destination). The stationary values of μ_{cj} are pinned down by the following linear system of equations*

$$P^T \mu = \mu \tag{51}$$
$$\sum_{c=1}^{C} \mu_{cj} = 1, \forall j \tag{52}$$

which consists of $CJ + J$ equations.

Proof. See Section B.5 in the Theory Appendix. □

While not needed to compute the balanced growth path equilibrium, we can also compute the stationary firm size distributions $\varphi_{cj}(n)$ for firms of type (c, j). The details of their derivation are relegated to Section B.6 of the Theory Appendix.

3.5. Output and welfare

We would like to compute social welfare in counterfactual economies and compare them against the estimated equilibrium. To calculate welfare, we need to compute the consumption stream of the representative household. From the utility function of the representative household in equation (7), we have:

$$W = \int_0^\infty e^{-\rho t} \ln C_t dt = \int_0^\infty e^{-\rho t} \ln(e^{gt} C_0) dt = \frac{\ln C_0}{\rho} + \frac{g}{\rho^2} \tag{53}$$

which shows how the welfare depends on the initial level of consumption C_0 and the growth rate of the economy g. To compute C_0, we need to calculate the initial output level, Y_0, and the fraction of output spent on R&D by all firms in the economy. The details are relegated to Section B.7 in the Theory Appendix.

For two economies A and B, we can define a consumption equivalent welfare change measure (ϖ) which corresponds to the percentage increase in lifetime consumption that an agent in economy A would need to be indifferent between being in economy A or B:

$$W_B = \frac{\ln(C_0^A(1+\varpi))}{\rho} + \frac{g^A}{\rho^2} \tag{54}$$

Solving for ϖ, we get:

$$\varpi = \exp\left(\left(W_B - \frac{g^A}{\rho^2}\right)\rho - \ln(C_0^A)\right) - 1 \tag{55}$$

4. Calibration and Identification

4.1. Data

We use several datasets to inform our calibration: (i) Compustat North America Fundamentals Annual, which contains annual data from firm financial statements; (ii) the Global Corporate Patent Dataset (GCPD) (Bena, Ferreira, Matos, and Pires, 2017), which contains detailed information on patents linked to the patent-holding firms in Compustat; (iii) the USPTO litigation database, which contains the universe of patent litigation over the period 2003-2016, including identifiers for patents involved in each lawsuit, and (iv) the Federal Judicial Center (FJC) civil-lawsuit database, which includes detailed information on lawsuit outcomes for all patent lawsuits filed in federal courts.

We use this data to construct a firm-year panel over the period 2003-2016. Based on the data, we categorize firms into nine technology classes and four primary industry groups. We provide details regarding the classification procedure in the Online Appendix.

We use these datasets to construct 15 empirical moments. Using our model solution, we can calculate model counterparts for each moment. In the following sections, we describe how we calibrate our model parameters to make the model-implied moments match the empirical counterparts. We provide details on the empirical moment construction in the Online Appendix.

4.2. Parameterizing the Model

To map the model to the data, we need to add a few parametric assumptions. First, we assume the litigation cost γ is drawn from an exponential distribution with parameter ξ: the CDF is $\Gamma(\gamma) = 1 - e^{-\xi\gamma}$. Second, we draw λ_c from a uniform distribution with a mean of μ_λ and a standard deviation of σ_λ. Finally, the R&D cost parameter χ_c is drawn from a uniform distribution with a mean of μ_χ and a standard deviation of σ_χ.

4.3. Parameter Values

4.3.1. Estimates from the Literature

We begin by calibrating a few parameters based on literature conventions. The discount rate ρ is set to 0.04, which implies a real interest rate of 6% when the growth rate is 2%. The R&D cost convexity parameter ψ is set to 2 following Bloom, Griffith, and Van Reenen (2002). The R&D subsidy s_{cj} is set to 8%, the implied tax subsidy rate on R&D expenditures in the US from the OECD database. The exit rate δ is set to 3% following Acemoglu et al. (2018). Additionally, the injunction rate is pinned down by the observed injunction rate - this was 95% before eBay ruling in 2006 and 75% after eBay (Seaman, 2015), leading to an average injunction rate of 81% in our sample.

4.3.2. Directly measured parameters

Next, we use our firm-year panel to calibrate some parameters based on observable proxies. We directly calibrate $\{\omega_j\}$ and $\{\eta_{cj}\}$ to match observed proxies. We provide details in the Online Appendix.

4.3.3. Inferred Parameters

We infer the remaining parameters from the data. Specifically, we choose the following parameters to make model-implied moments match their empirical counterparts: (i) the mean μ_λ and standard deviation σ_λ of λ_c; (ii) the mean μ_χ and standard deviation σ_χ of R&D cost scale χ_c; (iii) the entrant R&D cost scale ν; (iv) the knowledge spillovers σ; (v) the infringement probabilities κ_1, κ_2 for type-1 and type-2 infringement, respectively; (vi) the litigation cost-scale parameter ξ; (vii) the bounds τ_2^l, τ_2^h determining the distribution of defendant win rates in type-2 infringements; (viii) the single parameter $(\tau_1^l + \tau_1^h)/2$

sufficient for summarizing defendant win rates in type-1 infringements. We thus infer the values for 12 parameters by calibrating them jointly to match 15 empirical moments in the data.

We now explain how the 15 empirical moments listed in Table 2 help us infer the values of these 12 parameters.[14] Technically, each model-implied moment depends jointly on all 12 parameters through the model solution. However, certain moments are more sensitive to particular parameters, aiding our identification. We now go through each parameter and explain which moments are particularly helpful in calibrating that parameter. Growth in our model depends on the parameters μ_λ and σ_λ that determine the distribution of λ_c. We calibrate those two parameters using the (a) average GDP growth rate and (b) standard deviation of sales growth across technology classes. Specifically, we calculate these moments in the model and vary $\mu_\lambda, \sigma_\lambda$ until the model-implied moments match the empirical counterparts.

Next, we choose the R&D cost parameters μ_χ, σ_χ to make the model-implied distribution of R&D spending across technology classes match related empirical statistics: the mean and standard deviation of R&D intensity (R&D/sales) across technology classes.

The entrant R&D cost-scale parameter ν is chosen to match the fraction of growth attributable to new entrant's innovation (Garcia-Macia, Hsieh, and Klenow, 2019). The technology-spillover parameter σ is chosen to match the regression coefficient of R&D spending on sales shares of its technology class. If firms in technology classes with a large body of knowledge (a high fraction of sales) spend more on R&D, that suggests the body of knowledge produces spillovers for firms in the same technology class.

Next, we consider the litigation parameters. The observed probability of a firm being a patent-litigation plaintiff in a given year, and the standard deviation of that probability across technology classes and industries, provide valuable information about κ_1 and κ_2. Intuitively, once we have pinned down innovation incentives, the probability of a firm being a plaintiff depends on the probabilities of infringement κ_1, κ_2. If there is a wide standard deviation across technology classes and industries, that suggests a wide gap between κ_1 and κ_2. Moreover, the observed fraction of lawsuits occurring between firms in the same industry provides valuable information about κ_1: a type-1 infringement can only occur between two firms in the same industry. We thus choose κ_1, κ_2 to match these

[14]We provide details on how we construct these moments in the Online Appendix.

three empirical moments related to litigation.

To help pin down the defendant-win-rate parameters $\{\tau_j^l, \tau_j^h\}_{j=1,2}$, we use the observed likelihood of a plaintiff winning a trial for lawsuits in which (i) the plaintiff is in the same industry as the defendant and (ii) the plaintiff is in a different industry than the defendant. We also use the overall fraction of lawsuits that settle, rather than going to trial, since the extent of the asymmetric information determines the likelihood of a settlement. We choose the litigation-cost parameter ξ to match the fact that a typical US firm spends 0.57% of its revenue on litigation.

Finally, as an informal "overidentification test" we choose our parameters to match the distribution of litigation across technology classes. Specifically, for each technology class, we calculate (i) the average probability of a firm being a patent-litigation plaintiff in a given year;[15] (ii) the average sales growth; and (iii) the average R&D intensity. We match the across-technology-class correlation between R&D intensity and the likelihood of being a plaintiff. We also match the corresponding correlation between sales growth and litigation activity. These two correlations help further discipline the interactions between innovation and litigation within the model, ensuring they align well with observed data patterns. The latter correlation is key for our welfare implications. Recall from the illustrative model that a plaintiff-friendly reform (increasing the injunction rate) lowers welfare if high λ_c firms are frequent litigators. We find a positive correlation between sales growth and litigation, suggesting that high λ_c technology classes (high-growth firms) are more frequently involved in litigation. Based on our illustrative model, this implies that increasing the injunction rate should lower innovation activity and harm welfare. The following section shows that our full model delivers this same prediction.

Panel A of Table 2 reports the values of the parameters, whereas Panel B provides an overview of the values of the targeted moments in the data and the estimated model. The model tightly matches the data moments.

[15] Note that we focus on the probability of being a plaintiff because we can more precisely identify plaintiffs in Compustat. However, technology classes with frequent plaintiffs will also be technology classes with frequent defendants since lawsuits tend to occur within technology classes.

5. Quantitative Analysis

5.1. Comparative Statics

Using the calibrated parameters (Table 2), we solve our model. We now examine how our model equilibrium changes as we change parameters from the values listed in Table 2. To begin, we solve and simulate the model 20 times. For each of these 20 simulations, we calculate many model moments of interest: social welfare, the output growth rate, the distribution of firm value, and the innovation and litigation decisions of firms. We then change one parameter at a time, holding every other parameter fixed at the values listed in Table 2, and repeat this moment simulation process.

We first examine the impact of changing the injunction rate parameter ι. The results are shown in Figure 2. Based on our baseline parameter estimates, we find that increasing the injunction rate ι tends to reduce firm innovation, output growth, and consumption-equivalent welfare. Intuitively, a higher injunction rate increases the likelihood that value from successful innovation will be lost due to infringement risks related to other firms' intellectual property. This decrease in potential reward reduces the incentive for firms to innovate, which in turn lowers the output growth rate. While the consumption ratio slightly increases due to saved R&D expenses, this negative impact on growth outweighs its positive effect on consumption, leading to a decrease in social welfare.

Regarding the variables related to litigation, despite the increasing probability of hiring a legal team due to the higher potential gain from litigation, the reduction in disruptive innovation (and the decrease in the number of product lines per firm, as will be shown later) leads to a lower average probability of being a plaintiff.

The influence of the injunction rate on the average product line value is a bit more nuanced. On the one hand, an elevated injunction rate amplifies the value loss risks associated with legal actions when a firm's new innovations potentially infringe upon existing patents (risk channel). The left panel in the first row of Figure 2 illustrates that a rise in the injunction rate results in an expanded proportion of value being forfeited from successful innovations, owing to increased infringement risks. This, in turn, tends to decrease the average value of a product line.

On the other hand, a higher injunction rate can act as a protective shield for a firm's existing product lines (protection channel). As illustrated in the middle panel of the first row of Figure 2, an increased injunction rate enhances the value to a product-line owner by offering the opportunity to fend off new market entrants through patent litigation. This protective mechanism empowers firms to maintain control over their current product lines. Furthermore, the reduction in creative disruption further reinforces these existing productive lines, thereby contributing to an increase in the average product line value. Further taking into account the reduction in R&D expenses and increase in rent incomes from type 2 infringements, the average product line value increases with the injunction rate. As depicted in top left panel of Figure 3, in the estimated equilibrium, the protective effects of higher injunction rates overshadow the associated risks, leading to a modest overall increase in the average product line value.

The decreased innovation, however, diminishes the success rate of taking over product lines, resulting in a decrease in the number of product lines owned by incumbents as shown in top right panel of Figure 3. The decrease in the average number of product lines owned by each firm dominates the increase in average product line value, leading to a decline in the average incumbent value. For new entrants, the adverse effect of the injunction rate (risk channel) dominates, resulting in a negative relationship between the injunction rate and entrant value.

Next, we explore the effects of altering the litigation cost parameter ξ. A higher ξ value signifies a lower average cost of litigation. Figure 4 illustrates the results. When the costs associated with filing lawsuits and hiring legal teams are reduced, firms are more likely to engage in litigation in cases of patent infringement. This heightened litigation risk erodes the expected returns from successful innovations due to the fear of infringing on other firms' intellectual property. Consequently, firms are less motivated to innovate, which negatively affects growth and diminishes social welfare.

We further investigate the impact of lowering litigation costs on firm valuation. Figure 5 illustrates the results. In the estimated equilibrium model, the benefits of reduced litigation expenses, coupled with the decline in creative destruction, outweigh the accompanying risks, resulting in a moderate increase in the value of the average product line. The reduction in innovation leads to a decrease in the average number of product lines. This reduction in the number of product lines dominates the effect of the increase in average product line value when the litigation cost parameter is relatively low. However, as the litigation cost parameter increases, the latter effect dominates the former, leading to a U-shaped average incumbent value. Regarding entrepreneur value, as shown in Figure 5, despite the increase in average product line value, the increased litigation risk reduces the entrants' innovation incentive and consequently the entrepreneur value.

5.2. eBay-Ruling: Automatic Injunction to Case-by-case Injunction

In this subsection, we use the calibrated model to conduct an event study examining the impact of the eBay ruling. This pivotal Supreme Court decision in the *eBay v. MercExchange* case increased judicial flexibility in addressing patent disputes and effectively lowered the injunction rate for future cases. The consequences of the *eBay v. MercExchange* decision are ex ante ambiguous. While removing the automatic injunction might decrease deterrence against violations, thereby lowering incentives for innovation, it could also positively influence innovation by reducing the costs associated with patent litigation for innovative companies. According to Chien and Lemley (2012), the rate at which courts have granted injunctions has decreased from an estimated 95% pre-eBay to about 75% post-eBay. To model the effects of the eBay ruling, we adjust the ι parameter from 0.95 to 0.75, while keeping other parameters unchanged. We then assess its impact on innovation, firm values, litigation, growth, and social welfare.

As shown in Table 3, our analysis reveals that the decrease in injunction rates prompts an increase in innovation among both incumbents and entrants, by 4.06% and 2.57% respectively. This surge in innovation stems from a diminished value loss associated with the risks of infringing on other firms' intellectual property. Notably, incumbent firms exhibit a greater increase in innovation compared to entrants, which in turn results in a slight decline in the contribution of entrants to growth.

The ripple effects of these changes are multifaceted. Both incumbent and entrepreneur values increase. Despite a 1.76% decline in average product line value attributed to factors such as increased creative destruction, R&D expenses, and diminished litigation protection and rent income, the bolstered innovation activity compensates for these losses by driving a 3.04% increase in the average number of product lines per firm, thereby elevating the value of incumbent firms by 1.23%. The reduction in potential value loss due to IP infringement concerns also offsets the decrease in average product line value, leading to enhanced incentives for entrant innovation and an increase in entrepreneur value.

The higher innovation rate leads to a higher success rate of taking over product lines, especially for firms with high research efficiency. Other firms innovate more as well, but they tend to lose to the most efficient firms in the competition. High research efficiency firms have a higher chance of becoming the leaders in product lines and end up holding more product lines, while the number of product lines for other firms declines. This dynamic leads to a wider dispersion in the number of product lines held by different firms across the industry, whereas the average number product lines per firm increases.

Moreover, the eBay ruling affects the potential gain from litigation which in turn influences firms' legal strategies. Despite the decreasing probability of hiring a legal team due to the lower potential gain from litigation, the increase in creative destruction leads to a slight increase in the average plaintiff probability per product line. Together with the increase in the number of product lines, the average probability of being a plaintiff increases by 3.05%.

In terms of aggregate implications, the increase in incumbent and entrant innovation boosts the output growth rate. The higher R&D cost crowds out consumption, leading to a slightly lower consumption level. However, the growth effect dominates the level effect, resulting in an overall increase in social welfare of 3.32%. This welfare improvement is quite substantial. For comparison, recent quantitative evaluations estimate the welfare costs of business cycles to be in the range of 0.1-1.8% (Krusell et al., 2009), the welfare costs of inflation (Lucas, 2000) and managerial short-termism (Terry, 2023) close to 1%, and the static benefits from trade around 2.5% (Melitz and Redding, 2015). These figures are of a similar magnitude to the welfare improvements we have identified from reducing the rate of injunctions in our analysis.

5.3. Increasing Plaintiff Filing Costs

Our model allows us to study other potential reforms as well. For example, one recent proposed reform suggested increasing plaintiff pleading requirements (Gugliuzza, 2015). This is analogous to increasing the costs of filing for plaintiffs. To examine its impact, we vary the litigation cost parameter denoted as ξ. A lower ξ value corresponds to higher average litigation costs for plaintiffs, making it more costly to file lawsuits. We set ξ at half of its baseline value, thereby effectively doubling the average legal costs. The values of other parameters are kept unchanged at their baseline levels. Table 4 reports the impact of increasing the costs of filing for plaintiffs.

In response to the higher litigation cost, the probability of hiring a legal team declines. Both incumbents and entrants carry out more innovation due to the lower value loss associated with the risk of infringing on other firms' IP. Similar to the effect of reducing the injunction rate, we note a larger increase in innovation among incumbents than entrants, diminishing the latter's growth contribution.

The average product line value declines by 1.91% due to the increase in creative destruction. This decline is overshadowed by the benefits stemming from reduced IP infringement risks, culminating in a surge in innovation among entrants and, consequently, an uplift in entrepreneur value. Similar to the effect of reduced injunction rates, the higher litigation cost leads to higher innovation, resulting in an increase in the average and standard deviation of product lines. However, the increment in the average number of product lines per firm is modest at 1.23%, failing to fully offset the 1.91% reduction in average product line value, and resulting in a slight decrease in incumbent firm value by 0.71%.

The higher filing costs significantly influence litigation behavior. The higher filing costs lead to a reduction in the average probability of a plaintiff filing a lawsuit, primarily due to a decline in the per-product-line plaintiff probability. This decline stems from the significantly reduced likelihood of engaging legal teams in the event of infringements, attributed directly to the increased costs of filing.

The increase in filing costs also leads to aggregate implications that are comparable to those observed with the reduction of injunction rates. The rise in innovation from both incumbents and entrants propels the output growth rate. While the increased R&D expenditure tempers consumption levels, the overarching growth effects surpass the adverse level effects. Consequently, the social welfare experiences a sizeable uplift of 2.08%.

5.4. Increasing R&D Subsidies

Lastly, we examine the quantitative effects of increasing R&D subsidies for firms in the calibrated economy. To simulate the influence of R&D subsidies, we conduct three exercises. We double the subsidy parameter s_{cj} from its baseline value for one of three groups: (i) the whole sample, (ii) firms in the lowest tercile of research efficiency (measured by λ_c/χ_c), and (iii) firms in the highest tercile of research efficiency, respectively. The values of other parameters are kept unchanged at their baseline levels. We evaluate the impact of these R&D subsidies on innovation, litigation, growth, and social welfare. Table 5 presents the implications of increasing R&D subsidies.

We find that doubling the R&D subsidy for all firms in the economy can potentially increase innovation and the growth rate, which could lead to an increase in social welfare by 2.84%. This suggests an overall underinvestment in innovation. However, the degree of over-investment and under-investment is highly heterogeneous among firms with different research efficiencies. We find that subsidizing firms in the lowest terciles of research efficiency does not lead to much change in the average innovation and R&D spending. However, the composition of R&D activities shifts, with research activities moving to these relatively less efficient firms, which leads to a decline in the growth rate. As a result, social welfare decreases by 0.087%. In contrast, doubling the R&D subsidy for firms in the highest terciles of research efficiency substantially increases innovation and creative destruction, leading to a significant increase in the growth rate and social welfare. This indicates that firms with low research efficiency tend to overinvest in innovation, while firms with high research efficiency tend to underinvest in the estimated economy. Our calibration suggests that defendant-friendly legal reforms stimulate innovation for high-research-efficiency firms, improving welfare.

6. Conclusion

We develop a novel dynamic general equilibrium model with endogenous growth to quantify the extent to which the litigation system influences innovation, firm value, growth, and social welfare. This model features heterogeneous firms that innovate while facing potential patent lawsuits. In our continuous-time equilibrium model, heterogeneous firms innovate to steal market share from competitors. Firms inefficiently internalize the transfer they extract from competitors by innovating better products. This can lead to an equilibrium level of innovation for some firms that exceeds the level a social planner would choose to maximize welfare. However, innovation also creates positive externalities, through technology spillovers, that firms do not internalize. This can lead some firms to innovate less in equilibrium than the level that a social planner would choose.

We embed a realistic model of patent litigation in this dynamic general equilibrium framework. When firms innovate to steal a competitor's product line, there is a chance that they infringe on an existing patent. The patent holder's decision to file a lawsuit and the joint decision of whether to go to trial are both determined endogenously. In a trial, there is a chance that the court grants an injunction, stopping the innovating firm from taking over the incumbent firm's product line.

By integrating this realistic patent litigation system into a dynamic general equilibrium model with endogenous growth, we are able to assess how changes in the legal landscape affect firm behaviors and social welfare. We calibrate our model to evaluate historical patent-litigation reforms and proposed reforms. In both instances, we find that defendant-friendly reforms increase both innovation and welfare. The 2006 Supreme Court "eBay ruling," which improved defendant rights by lowering injunction rates, improved welfare by 3.32%. A proposed reform to increase plaintiff pleading requirements, making it more difficult to file a lawsuit, would likewise improve welfare. The results of our analysis underscore the significant impact that patent-litigation reforms can have on innovation, firm value, economic growth, and social welfare. Our research adds to the discourse on patent litigation reform, offering policymakers and stakeholders guidance on how to craft reforms that help the patent system fulfill its original purpose of promoting technological progress and economic well-being.

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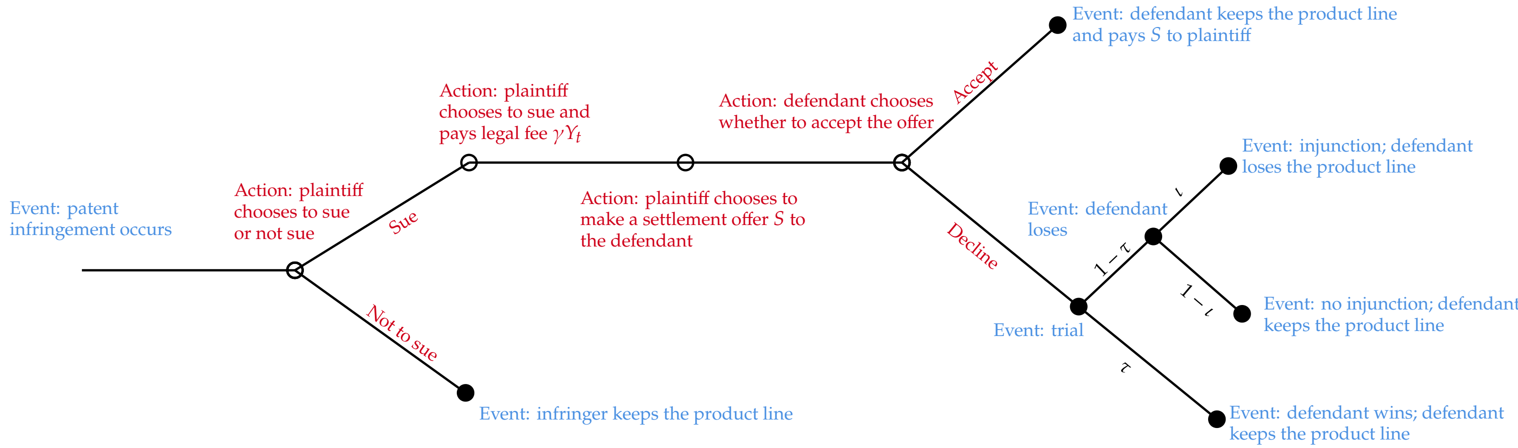

Figure 1: Litigation Subgame Timeline

Notes: This figure illustrates the timeline of the litigation subgame. Conditional on patent infringement, the plaintiff has to make a decision of whether to hire a legal team. A legal team is necessary for making a settlement offer or going to court. The cost of hiring a legal team is γY_t, where $\gamma > 0$ is a random variable drawn from the distribution $\Gamma(\gamma)$, and the Y_t term ensures that litigation costs grow at the same rate as output in a balanced growth path (BGP) equilibrium. If the plaintiff chooses to pay γY_t and hire the legal team, it then makes a take-it-or-leave-it settlement offer to the defendant. The defendant has private information about its chances of winning the trial. Let $\tau \in [0, 1]$ denote the probability that the defendant wins the trial. This probability is drawn from the exogenous distributions $T_1(\tau)$ and $T_2(\tau)$ for type 1 and type 2 infringements, respectively. Given its private information τ, the defendant can accept the settlement or refuse. Refusal leads to a trial. With probability τ, the defendant wins the trial and the product line takeover is realized. With the complementary probability $1 - \tau$, the defendant loses. If the defendant loses, then the court decides on whether to grant an injunction or not. With probability $\iota \in [0, 1]$, an injunction is granted and the product line takeover is blocked. With probability $1 - \iota$, there is no injunction and the defendant can still take over the product line. The parameter ι is a policy parameter that captures the inclination of a court to grant an injunction in the case of a proven patent infringement.

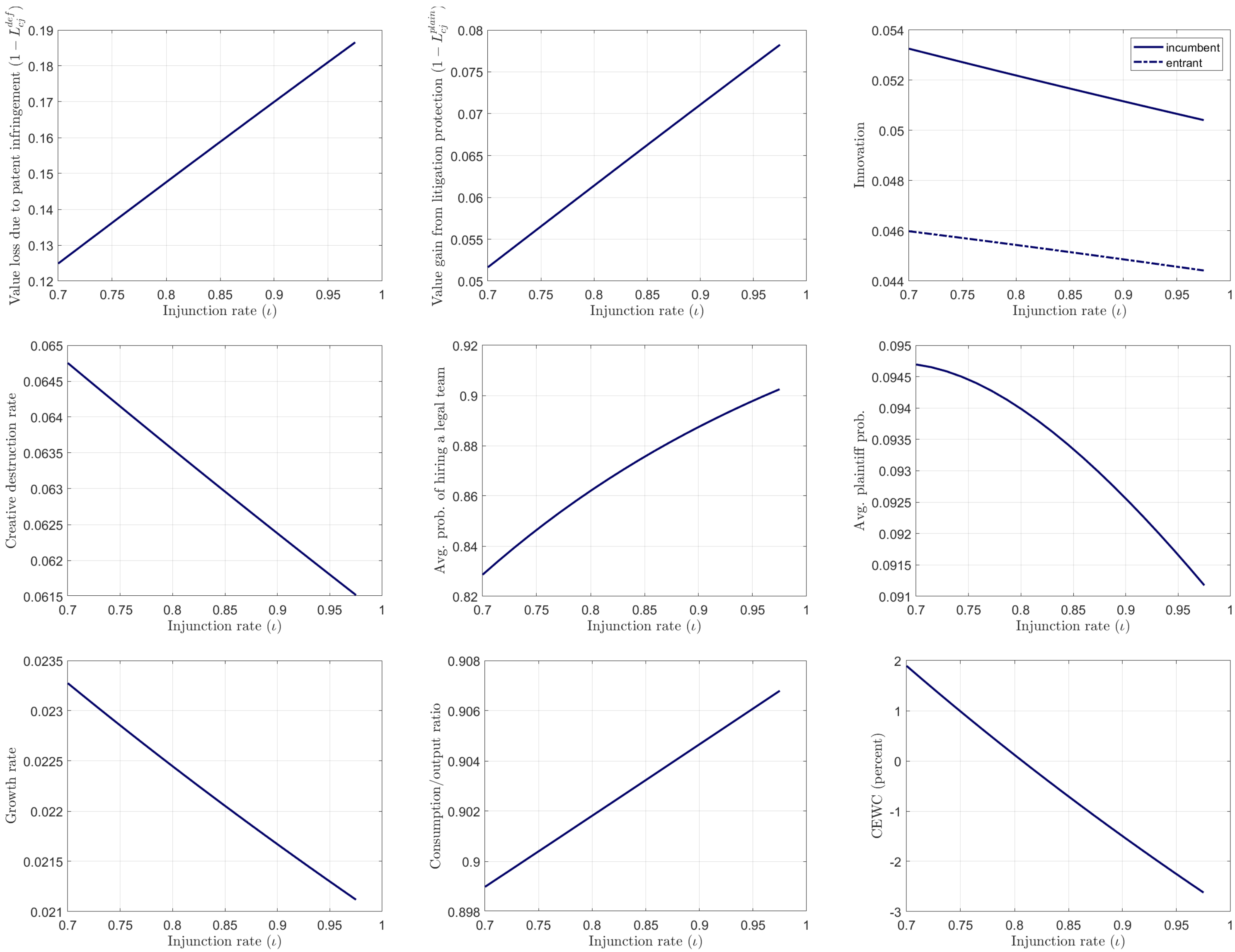

Figure 2: Injunction Rate: Impacts on Firm Innovation, Litigation, and Aggregate Outcomes

Notes: This figure depicts comparative statics under different values of injunction rate ι. For each panel, we solve and simulate the model 20 times, each time corresponding to a different value of injunction rate while keeping other parameters unchanged.

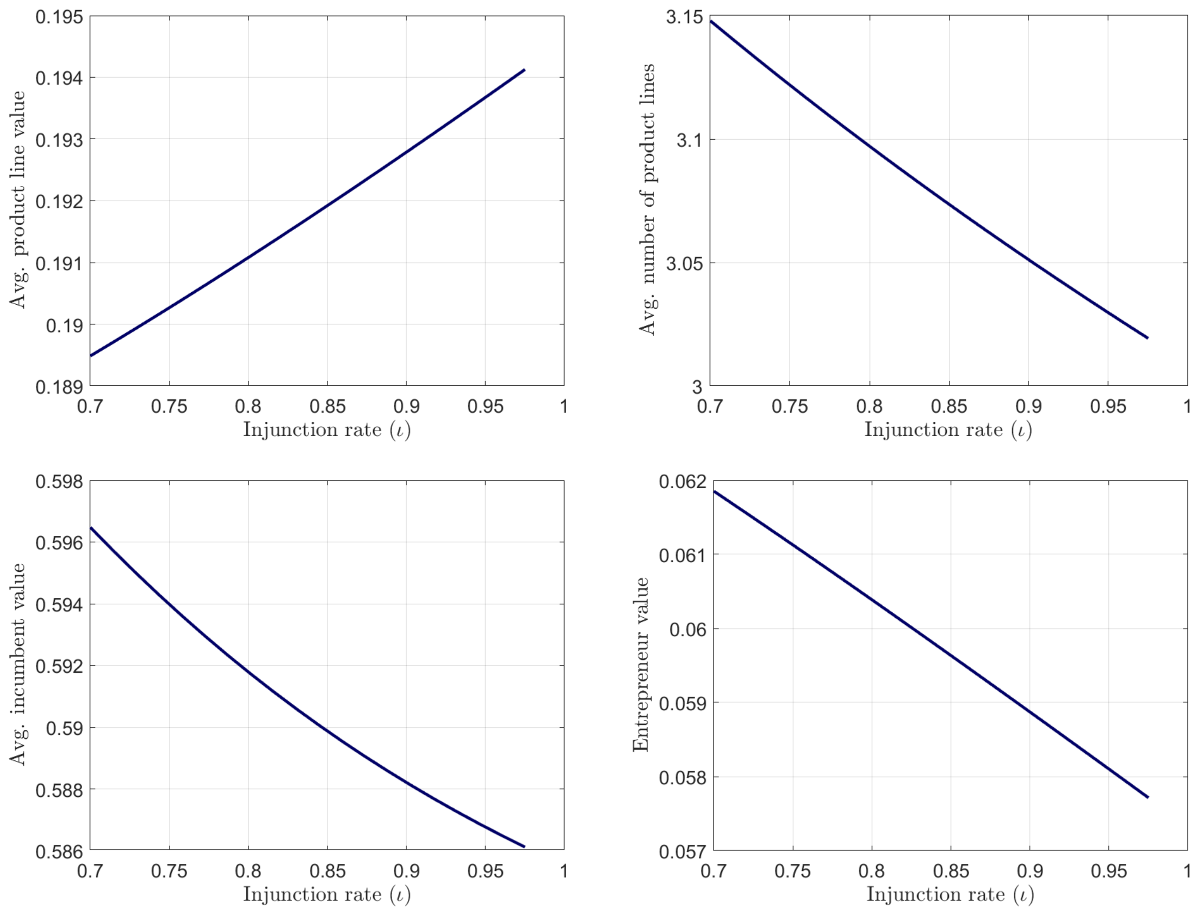

Figure 3: Injunction Rate: Impacts on Firm Values of Incumbents and Entrants

Notes: This figure depicts the impact of the injunction rate on the average value of produce lines, the number of product lines, the incumbent value, and the entrepreneur value. For each panel, we solve and simulate the model 20 times, each time corresponding to a different value of injunction rate while keeping other parameters unchanged.

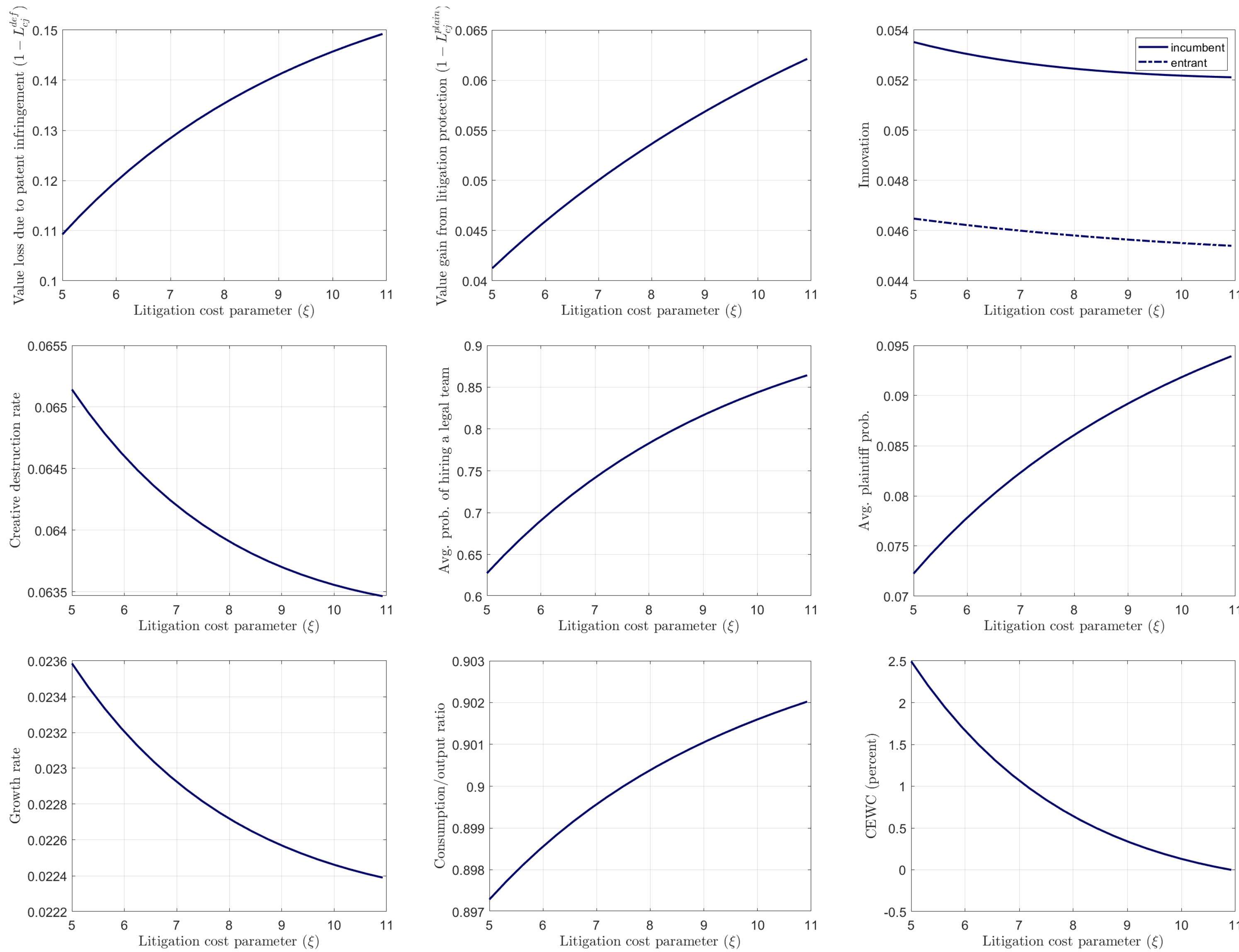

Figure 4: Litigation Cost: Impacts on Firm Innovation, Litigation, and Aggregate Outcomes

Notes: This figure depicts comparative statics under different value of litigation cost parameter ξ. A higher ξ value signifies a lower average cost of litigation. For each panel, we solve and simulate the model 20 times, each time corresponding to a different value of litigation cost parameter ξ while keeping other parameters unchanged.

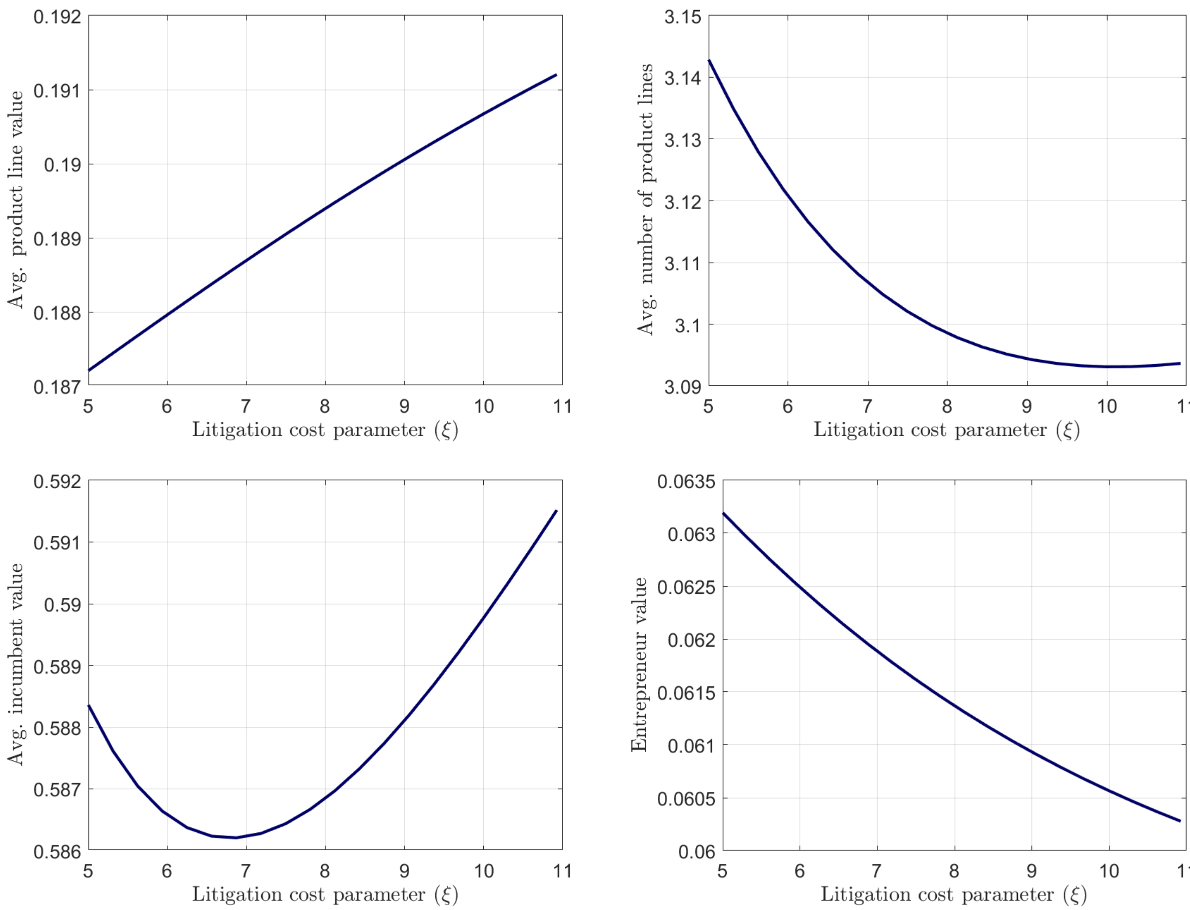

Figure 5: Litigation Cost: Impacts on Firm Values of Incumbents and Entrants

Notes: This figure depicts the impact of litigation cost on the average value of produce lines, the number of product lines, the incumbent value, and the entrepreneur value. A higher ξ value signifies a lower average cost of litigation. For each panel, we solve and simulate the model 20 times, each time corresponding to a different value of litigation cost parameter ξ while keeping other parameters unchanged.

Table 1: Illustrative model

This table shows the results of a simple example from our illustrative model. The first panel shows our assumed parameter values. In the second panel, we consider a case in which only efficient innovators ($c = 1, \lambda_1 = 3$) use the litigation system: $\rho_{1j} > 0 = \rho_{2j}$. We display the gap between the private and socially optimal levels of innovation, and display what happens if ι increases by 0.2 In the third panel, we repeat the exercise assuming only inefficient innovators litigation: $\rho_{2j} > 0 = \rho_{1j}$.

Parameters	
χ	4.5
σ	2
ι	0.75
π	1
λ_1	3
λ_2	0.1

Case 1: efficient innovators litigate	
ρ_{1j}	0.05
ρ_{2j}	0
$x^*_{1j} - x^s_{1j}$	-0.02
Δx^*_{1j} from $\iota \uparrow 0.2$	-0.0036
Δ Welfare from $\iota \uparrow 0.2$	-0.02

Case 2: inefficient innovators litigate	
ρ_{1j}	0
ρ_{2j}	0.05
$x^*_{2j} - x^s_{2j}$	0.13
Δx^*_{2j} from $\iota \uparrow 0.2$	-0.001
Δ Welfare from $\iota \uparrow 0.2$	0.0007

Table 2: Model Calibration

Notes: The table reports calibration results. reports the values of the parameters, whereas Panel B provides an overview of the values of the targeted moments in the data and the estimated model. See Section 4 and Appendix A for the definition and construction of data moments.

Panel A: Parameter Values

	Description	Values
ρ	discount rate	0.04
δ	exogenous exit rate	0.03
ψ	R&D cost convexity	2
ι	avg. injunction rate	81%
μ_λ	mean of innov. step size	0.294
σ_λ	stdv. of innov. step size	0.192
μ_χ	mean of incumbent R&D cost scale	6.504
σ_χ	stdv. of incumbent R&D cost scale	0.361
ν	entrant R&D cost scale	5.061
σ	knowledge spillover strength	0.100
κ_1	type 1 infringement prob.	0.831
κ_2	type 2 infringement prob.	0.397
$(\tau_1^l + \tau_1^h)/2$	type 1 def. win prob. lb and ub	0.677
$[\tau_2^l, \tau_2^h]$	type 2 def. win prob. lb and ub	[0.044, 0.659]
ξ	litigation cost scale	10.915

Panel B: Moments

	Data	Model
output growth rate	2.03%	2.24%
stdv. of sales growth	1.79%	1.80%
mean of R&D intensity	6.43%	5.50%
stdv. of R&D intensity	3.60%	3.34%
entrant innovation's contribution to growth	19.80%	18.45%
β(R&D spending, tech class sales share)	0.028	0.029
mean of the prob. of being a plaintiff	10.53%	9.39%
stdv. of the prob. of being a plaintiff	8.63%	9.15%
fraction of same industry lawsuits	78.22%	76.62%
prob(plaintiff win | same-industry lawsuits)	35.44%	33.58%
prob(plaintiff win | different-industry lawsuits)	40.91%	40.95%
prob(settlement | being a plaintiff)	58.32%	57.65%
mean of litigation costs/revenue	0.57%	0.61%
corr (litigation prob., sales growth)	0.713	0.712
corr (litigation prob., R&D intensity)	0.620	0.813

Table 3: The Impact of 2006 eBay Ruling

Notes: This table reports the impact of 2006 eBay ruling. To model the effects of the eBay ruling, we adjust the ι parameter from 0.95 to 0.75, while keeping other parameters unchanged at their baseline values. We assess its impact on innovation, firm values, litigation, growth, and social welfare.

	Baseline Pre-eBay ruling: $\iota = 0.95$	Counterfactual Post-eBay ruling: $\iota = 0.75$	% changes
incumbent innovation	0.0507	0.0527	4.065%
avg. R&D intensity	5.274%	5.588%	5.943%
entrant innovation	0.0446	0.0457	2.569%
contribution of entrants to growth	18.773%	18.313%	-2.452%
creative destruction rate	6.180%	6.415%	3.795%
avg. incumbent value	0.5868	0.5940	1.230%
avg. entrepreneur value	0.0581	0.0611	5.209%
avg. product line value	0.1937	0.1903	-1.760%
avg. number of product lines	3.0296	3.1218	3.043%
stdv. number of product lines	5.6121	6.1589	9.743%
avg. plaintiff prob.	9.166%	9.445%	3.050%
per product line plaintiff prob.	3.025%	3.026%	0.007%
avg. prob of hiring a legal team	89.782%	84.645%	-5.721%
output growth rate	2.130%	2.285%	7.300%
consumption	0.2277	0.2262	-0.623%
output	0.2513	0.2513	0.005%
CEWC	-	3.316%	-

Table 4: The Impact of Increasing Plaintiff Filing Costs

Notes: This table presents the implications of increasing plaintiff filing costs. To simulate the influence, we halve the ξ parameter from its baseline value, effectively doubling the average expense associated with hiring a legal team for the plaintiff. The values of other parameters are kept unchanged at their baseline levels. We evaluate its effects on innovation, firm values, litigation, growth, and social welfare.

	Baseline $\xi^* = 10.915$	Counterfactual $\xi = 0.5 \times \xi^*$	% changes
incumbent innovation	0.0521	0.0533	2.242%
avg. R&D intensity	5.498%	5.716%	3.971%
entrant innovation	0.0454	0.0464	2.116%
contribution of entrants to growth	18.453%	18.237%	-1.175%
creative destruction rate	6.347%	6.487%	2.219%
avg. incumbent value	0.5915	0.5873	-0.705%
avg. entrepreneur value	0.0603	0.0629	4.276%
avg. product line value	0.1912	0.1876	-1.907%
avg. number of product lines	3.0936	3.1315	1.225%
stdv. number of product lines	5.9896	6.2352	4.101%
avg. plaintiff prob.	9.391%	7.496%	-20.179%
per product line plaintiff prob.	3.036%	2.394%	-21.145%
avg. prob of hiring a legal team	86.408%	65.819%	-23.827%
output growth rate	2.239%	2.340%	4.493%
consumption	0.2266	0.2256	-0.456%
output	0.2513	0.2513	0.0003%
CEWC	-	2.080%	-

Table 5: The Impact of Increasing R&D Subsidies

Notes: This table presents the implications of increasing R&D subsidies. To simulate the influence of R&D subsidies, we conduct three exercises. We double the subsidy parameter s_{cj} from its baseline value for one of three groups: (i) the whole sample, (ii) firms in the lowest tercile of research efficiency (measured by λ_c/χ_c), and (iii) firms in the highest tercile of research efficiency, respectively. The values of other parameters are kept unchanged at their baseline values. We evaluate the impact of these R&D subsidies on innovation, litigation, growth, and social welfare.

	Baseline	Whole Sample		Low λ_c/χ_c Subsample		High λ_c/χ_c Subsample	
		Counterfactual	% Changes	Counterfactual	% Changes	Counterfactual	% Changes
incumbent innovation	0.0521	0.0567	8.891%	0.0521	-0.004%	0.0568	8.939%
avg. R&D intensity	5.498%	6.383%	16.103%	5.508%	0.187%	6.205%	12.857%
entrant innovation	0.0454	0.0444	-2.238%	0.0454	0.069%	0.0441	-2.783%
contribution of entrants to growth	18.453%	17.004%	-7.854%	18.453%	-0.002%	17.005%	-7.849%
creative destruction rate	6.347%	6.784%	6.901%	6.347%	0.009%	6.781%	0.522%
avg. incumbent value	0.5915	0.6350	7.354%	0.5883	-0.541%	0.6548	10.700%
avg. entrepreneur value	0.0603	0.0576	-4.395%	0.0604	0.130%	0.0570	-5.441%
avg. product line value	0.1912	0.1902	-0.510%	0.1908	-0.234%	0.1922	0.522%
avg. number of product lines	3.0936	3.3382	7.904%	3.0841	-0.308%	3.4069	10.125%
stdv. number of product lines	5.9896	6.8654	14.623%	5.9557	-0.565%	7.1630	19.590%
avg. plaintiff prob.	9.391%	11.138%	18.603%	9.335%	-0.595%	11.596%	23.488%
per product line plaintiff prob.	3.036%	3.336%	9.915%	3.027%	-0.288%	3.404%	12.134%
avg. prob of hiring a legal team	86.408%	86.445%	0.044%	86.315%	-0.107%	86.680%	0.522%
output growth rate	2.239%	2.414%	7.809%	2.235%	-0.170%	2.427%	8.413%
consumption	0.2266	0.2231	-1.555%	0.2267	0.008%	0.2230	-1.604%
output	0.2513	0.2513	0.0325%	0.2512	-0.0086%	0.2514	0.0494%
CEWC	-	2.843%	-	-0.087%	-	3.141%	-

Online Appendices:

The Efficiency of Patent Litigation

A. Data and Empirical Moment Construction

A.1. Data

We use several datasets to inform our calibration. First, we download the Compustat North America Fundamentals Annual dataset. We obtain a firm-year dataset with the following variables: (i) total sales (revenue); (ii) R&D spending; (iii) industry (SIC code); (iv) firm name.

Second, we download the Global Corporate Patent Dataset (GCPD) (Bena, Ferreira, Matos, and Pires, 2017).[1] For the period 1980-2017, the GCPD provides a comprehensive link between patents awarded by the U.S. Patent and Trademark Office (USPTO) and the publicly listed Compustat firms that received those patents.

Third, we download the USPTO litigation database, which covers all patent lawsuits filed in Federal courts over the period 2003-2016. For each lawsuit, the dataset includes identifiers for all of the infringed patents.

We merge these three datasets together. Our final sample is a firm-year panel over the period 2003-2016. It includes all Compustat observations for all firms that hold at least one patent in the GCPD.[2] We assign each firm a technology class c and an industry j by the following procedure. We use the first digit of the GCPD technology-class classification to construct nine different potential technology classes. We assign each firm a time-invariant technology class using the GCPD classification.[3] We use SIC codes from Compustat to construct the Fama-French twelve industries. We exclude firms in Finance, Utilities, or Other. We then aggregate the remaining nine industries into four industry groups.[4] We thus have nine technology classes and four industry groups in our data. Since we

[1] See https://patents.darden.virginia.edu/.

[2] We exclude firm years with missing SIC codes, assets, sales or Compustat identifiers. We exclude firm years with under $50 million in sales.

[3] If a firm has multiple patents, we take the modal technology class across its patents. If there is a tie, we consider the firm's technology class to be missing.

[4] Based on the Fama-French industry classification, our analysis focuses on four primary industry groups: (i) Manufacturing: This encompasses Fama-French industry classifications 1 (Consumer NonDurables – Food, Tobacco, Textiles, Apparel, Leather, Toys), 2 (Consumer Durables – Cars, TVs, Furniture, Household Appliances), and 3 (Manufacturing – Machinery, Trucks, Planes, Office Furniture, Paper, Commercial Printing); (ii) Extraction and Chemicals: This group includes Fama-French industry classifications 4 (Oil, Gas, and Coal Extraction and Products) and 5 (Chemicals and Allied Products); (iii) Information and Communication Technology (ICT): Encompasses Fama-French industry classifications 6 (Business Equipment – Computers, Software, and Electronic Equipment) and 7 (Telephone and Television Transmission); (iv) Services: This group includes Fama-French industry classifications 9 (Wholesale, Retail, and Some Services such as Laundries, Repair Shops) and 10 (Healthcare, Medical Equipment, and Drugs). We exclude firms categorized under the 'other' industry classification. Additionally, we exclude firms operating in the financial and utility sectors due to their heavy regulation and distinct business models compared to other industries.

analogously solve our model assuming there are nine technology classes and four industry groups, the reduction in the number of industry groups eases computation.

Our merge links each firm in our sample to all of its patents over the period 1980-2017. It also links each patent to all of the lawsuits filed over that patent. We can thus construct an indicator equal to one for firm i in year t if firm i files a patent lawsuit in year t.

For a separate empirical moment, we download quarterly data on year-over-year US GDP growth from the Federal Reserve Bank of St. Louis.

Finally, we construct some moments using a separate lawsuit-level dataset. To construct this dataset, we begin with the Federal Judicial Center (FJC) database. This contains every civil lawsuit filed in Federal courts. Using the FJC classification, we isolate patent lawsuits. For each patent lawsuit, the FJC contains an indicator equal to one if a lawsuit is settled out of court. It also contains an indicator equal to one if a lawsuit goes to trial. Further, it contains a variable specifying whether the plaintiff or defendant won in trial. We merge the FJC data with the USPTO litigation database to obtain identifiers for litigated patents. Using the patent identifiers, Compustat, and the GCPD, we identify the name and industry j of the plaintiff. Finally, we use defendant names in the FJC data to identify defendants in Compustat.[5] Our final lawsuit-level dataset contains the industry of the plaintiff, the industry of the defendant, the outcome of the lawsuit (settlement versus trial), and the trial outcome (plaintiff or defendant victory) for lawsuits ending in trials.

A.2. Empirical Moments

We use the data described above to calculate 15 empirical moments. We calibrate our model to match these moments, which are listed in Table 2. We first summarize these moments, then describe how we use them to choose parameter values.

First, we calculate the average annual GDP growth rate in our sample period: it is 2.03%.

Second, we use our firm-year panel to calculate several moments. We calculate firm-year level sales growth, take an average for each technology class, then take a standard deviation across technology classes. The corresponding standard deviation, 1.79%, corresponds to the variation in sales growth across technology classes. Next, we calculate R&D intensity as the ratio of R&D spending to sales. We calculate a sample average of 6.43%. We then take the average R&D intensity for each technology class and take a standard deviation across technology classes: we find this standard deviation is 3.6%. Next, we measure whether firms in technologies with a higher share of sales in the economy tend to have higher R&D spending. For each technology class, we calculate the share of all sales attributable to firms in that technology class. Regressing R&D spending on the sales

[5] We verify name matches both algorithmically and manually. We only keep matches for which we are highly confident of the accuracy.

share, controlling for year fixed effects, we get a regression coefficient of 0.028.

Next, we study our indicator for a firm being a plaintiff in a patent lawsuit in a given year. The average for this indicator is 10.53% in our sample. We take an average for each industry j and technology class c. Taking a standard deviation across industries and technologies, we find the standard deviation is 8.63%. We then explore how high-litigation technology classes differ from low-litigation technology classes. We take an average of our litigation indicator for each technology class. We similarly take the average sales growth and R&D intensity at the technology class level. Taking a correlation across technology classes, we find a correlation between litigation and sales growth equal to 0.713. We find a correlation between litigation and R&D intensity equal to 0.62.

We then turn to our lawsuit-level dataset. Conditional on a plaintiff having the same industry as the defendant and a lawsuit going to trial, we find that 35.44% of plaintiffs win in trial. Conditional on a plaintiff having a different industry than the defendant and a lawsuit going to trial, we find that 40.91% of plaintiffs win in trial. Overall , we find that 58.32% of lawsuits end in settlement.

Finally, we use a couple of statistics from external sources. A US courts survey[6] indicates that a typical US company spends roughly 0.57% of revenue on litigation costs annually. Garcia-Macia, Hsieh, and Klenow (2019) show that 19.8% of growth is attributable to innovation by new entrants.

We directly calibrate ω_j to match the share of all sales attributable to industry j.[7] Likewise, we directly calibrate η_{cj} to match the share of all new-entrant sales attributable to industry j.[8] Additionally, the injunction rate is pinned down by the observed injunction rate - this was 95% before eBay ruling in 2006 and 75% after eBay (Seaman, 2015), leading to an average injunction rate of 81% in our sample.

[6]See Figure 6 https://www.uscourts.gov/sites/default/files/litigation_cost_survey_of_major_companies_0.pdf.

[7]In each industry j and year t, we calculate the fraction of all sales attributable to firms in industry j. For each j, we average across years to calculate ω_j.

[8]We call a firm i a new entrant in year t if it is the first year over the period 2003-2017 in which the firm appears in Compustat. In each industry j and each year t, we calculate the fraction of new-entrant sales attributable to industry j. We average across years to construct a share η_j of new-entrant sales. We then divide by nine to construct η_{cj}.

B. Theory Appendix

B.1. Illustrative Model Proof

Define the operator G that maps the vector $[x^s, x^c]$ to a vector Gx defined by

$$Gx^*_{cj} = (2 - \rho_{cj}\iota)\frac{\pi + \lambda_c + \sigma x^*_{cj'}}{4\chi}. \tag{B.1}$$

$$Gx^s_{cj} = \frac{2 - \rho_{cj}\iota}{4\chi}(\lambda_c + \sigma x^s_{cj'}) + \sigma x^s_{cj'}\frac{2 - \rho_{cj'}\iota}{4\chi}. \tag{B.2}$$

The private equilibrium and social optimum are given by a fixed point of G. Moreover, if $\sigma < \chi$, this is clearly a contraction mapping, since each element of $Gx - Gy$ is the product of (i) another element of the vector $x - y$, and (ii) $\sigma/\chi < 1$, and (iii) either $(2 - \rho_{cj}\iota)/4 < 1$ or $1 - \frac{\rho_{cj}\iota + \rho_{cj'}\iota}{4} \leq 1$.

First, note that as $\sigma \longrightarrow 0$,

$$x^*_{cj} = \frac{2 - \rho_{cj}\iota}{4\chi}(\pi + \lambda_c) > \frac{2 - \rho_{cj}\iota}{4\chi}\lambda_c = x^s_{cj}. \tag{B.3}$$

Next, note that as $\pi \longrightarrow 0$,

$$x^*_{cj} = (2 - \rho_{cj}\iota)\frac{\lambda_c + \sigma x^*_{cj'}}{4\chi} > 0. \tag{B.4}$$

while

$$x^s_{cj} = \frac{2 - \rho_{cj}\iota}{4\chi}(\lambda_c + \sigma x^s_{cj'}) + \sigma x^s_{cj'}\frac{2 - \rho_{cj'}\iota}{4\chi} > 0. \tag{B.5}$$

Thus, as $\pi \longrightarrow 0$, G has the property that whenever the vector x has $x^*_{cj} < x^s_{cj}$, the same holds for the corresponding elements of Gx. Since G is a contraction mapping, we can start at any point and iteratively apply G to find the fixed point. It follows that $x^*_{cj} \leq x^s_{cj}$ at the fixed point. From inspection of the above equations, we see that it cannot be that $x^s_{cj} = x^*_{cj}$, so $x^*_{cj} < x^s_{cj}$.

As $\pi \longrightarrow \infty$, we have $x^*_{cj} \longrightarrow \infty$ while x^s_{cj} is fixed, so $x^*_{cj} > x^s_{cj}$.

Next, note that as $\lambda_c \to 0$,

$$x^*_{cj} = (2 - \rho_{cj}\iota)\frac{\pi + \lambda_c + \sigma x^*_{cj'}}{4\chi} \geq \frac{2 - \rho_{cj}\iota}{4\chi}\pi > 0, \tag{B.6}$$

while

$$x^s_{cj} = x^s_{cj'}\left(\frac{2 - \rho_{cj}\iota}{4\chi}\sigma + \sigma\frac{2 - \rho_{cj'}\iota}{4\chi}\right), \tag{B.7}$$

implying that

$$x^s_{cj} = x^s_{cj}\left(\frac{2 - \rho_{cj}\iota}{4\chi}\sigma + \sigma\frac{2 - \rho_{cj'}\iota}{4\chi}\right)\left(\frac{2 - \rho_{cj'}\iota}{4\chi}\sigma + \sigma\frac{2 - \rho_{cj}\iota}{4\chi}\right), \tag{B.8}$$

and thus $x^s_{cj} = 0$ absent a knife edge case. This implies $x^s_{cj} = 0 < x^*_{cj}$ as $\lambda_c \to 0$.

B.2. Proof of Proposition 1

We consider the decision problem of a plaintiff facing a type 2 patent infringement in which they don't face any risk of losing product lines. They must choose a take-it-or-leave-it settlement offer s without knowing the realization of τ – the defendant's probability of winning at court. Their problem is written as

$$\max_{s\geq 0}\left\{s\mathbb{P}(s\leq \bar{s}_{cjt}(n,\tau))\right\} \tag{B.9}$$

where the second term is the probability that the offer is accepted. We can rewrite this probability as

$$\begin{aligned}\mathbb{P}(s\leq \bar{s}_{cjt}(n,\tau)) &= \mathbb{P}\left(s\leq (1-\tau)\iota\left(V_{cjt}(n+1)-V_{cjt}(n)\right)\right)\\ &= \mathbb{P}\left(\tau\leq 1-\frac{s}{\iota\left(V_{cjt}(n+1)-V_{cjt}(n)\right)}\right)\\ &= \int_{-\infty}^{1-s/(\iota(V_{cjt}(n+1)-V_{cjt}(n)))} dT_2(\tau)\end{aligned} \tag{B.10}$$

which, under the distributional assumption, becomes

$$\mathbb{P}(s\leq \bar{s}_{cjt}(n,\tau))=\begin{cases}0 & \text{if } 1-s/(\iota(V_{cjt}(n+1)-V_{cjt}(n)))<\tau_2^l\\ 1 & \text{if } 1-s/(\iota(V_{cjt}(n+1)-V_{cjt}(n)))>\tau_2^h\\ \frac{1-s/(\iota(V_{cjt}(n+1)-V_{cjt}(n))-\tau_2^l}{\tau_2^h-\tau_2^l} & \text{otherwise}\end{cases} \tag{B.11}$$

Note that the optimal s must be such that $\tau_2^l\leq 1-s/(\iota(V_{cjt}(n+1)-V_{cjt}(n)))\leq \tau_2^h$.[9] Then we can rewrite the objective function over this range as

$$\frac{s(1-\tau_2^l)-s^2/(\iota(V_{cjt}(n+1)-V_{cjt}(n))}{\tau_2^h-\tau_2^l} \tag{B.12}$$

[9] Below τ_2^l, the probability of acceptance is zero, and so are the extracted rents. Above τ_2^h, the plaintiff is asking for a smaller payment even though it does not increase the probability of acceptance, thus losing out on rents. Both are suboptimal.

with the first order condition delivering

$$\begin{aligned}1-\tau_2^l &= \frac{2s}{\iota(V_{cjt}(n+1)-V_{cjt}(n))} \\ s &= \frac{(1-\tau_2^l)\iota(V_{cjt}(n+1)-V_{cjt}(n))}{2} \equiv s^* \end{aligned} \tag{B.13}$$

which pins down the optimal s if the solution is interior. Given this expression, the cut-off τ for which the defendant is indifferent is given as

$$\tau^* = \frac{1+\tau_2^l}{2} \tag{B.14}$$

If $\tau^* \leq \tau_2^h$, then the solution is interior, and the optimal s is given by equation (30). If not, then we have a corner solution:

$$s = (1-\tau_2^h)\iota(V_{cjt}(n+1)-V_{cjt}(n)) \tag{B.15}$$

In the case of an interior solution, the identity for the acceptance probability becomes

$$\mathbb{P}(s < \bar{s}_{cjt}(n,\tau)) = \frac{1}{2}\frac{1-\tau_2^l}{\tau_2^h-\tau_2^l} \tag{B.16}$$

which is independent of c, j, t, and n. The optimal expected rent is then

$$s\mathbb{P}(s < \bar{s}_{cjt}(n,\tau)) = \frac{(1-\tau_2^l)^2\iota(V_{cjt}(n+1)-V_{cjt}(n))}{4(\tau_2^h-\tau_2^l)} \tag{B.17}$$

In the case of a corner solution, the probability of acceptance is unity, and the optimal expected rent is simply equal to equation (B.15). Given our assumption that $1+\tau_2^l \leq 2\tau_2^h$, the solution is always interior.

Given the optimal expected rent expression, we can now turn to the plaintiff's decision to hire a legal team or not. The plaintiff will choose to hire a legal team if the expected rent is higher than the cost γY_t where γ is drawn from the distribution $\Gamma(\gamma)$. The probability of hiring a legal team is given by

$$p^{LT}_{2,cjt} \equiv \mathbb{P}\left(\gamma \leq \frac{(1-\tau^l_2)^2\iota(V_{cjt}(n+1)-V_{cjt}(n))}{4(\tau^h_2-\tau^l_2)Y_t}\right) \tag{B.18}$$

and the expected rents conditional on a type 2 patent infringement minus legal team cost is given as

$$W^{plain}_{2,cjt} \equiv p^{LT}_{2,cjt}\frac{(1-\tau^l_2)^2\iota(V_{cjt}(n+1)-V_{cjt}(n))}{4(\tau^h_2-\tau^l_2)} - Y_t\int_0^{\frac{(1-\tau^l_2)^2\iota(V_{cjt}(n+1)-V_{cjt}(n))}{4(\tau^h_2-\tau^l_2)Y_t}} \gamma d\Gamma(\gamma) \tag{B.19}$$

Turning to the defendant's side, conditional on a type 2 patent infringement, they will receive a settlement offer only if the plaintiff chooses to hire a legal team, the probability of which is $p^{LT}_{2,cjt}$. Conditional on receiving a settlement offer, their expected payoff is

$$\begin{aligned}
&\int_{\tau^l_2}^{\tau^*}\left(V_{cjt}(n+1)-V_{cjt}(n)-s^*\right)dT_2(\tau)+\int_{\tau^*}^{\tau^h_2}[\tau+(1-\tau)(1-\iota)]\left(V_{cjt}(n+1)-V_{cjt}(n)\right)dT_2(\tau)\\
= &\left(1-\frac{(1-\tau^l_2)\iota}{2}\right)(V_{cjt}(n+1)-V_{cjt}(n))\left(\frac{1}{2}\frac{1-\tau^l_2}{\tau^h_2-\tau^l_2}\right)\\
&+(1-\iota)(V_{cjt}(n+1)-V_{cjt}(n))\left(1-\frac{1}{2}\frac{1-\tau^l_2}{\tau^h_2-\tau^l_2}\right)\\
&+\iota(V_{cjt}(n+1)-V_{cjt}(n))\int_{\tau^*}^{\tau^h_2}\frac{\tau}{\tau^h_2-\tau^l_2}d\tau\\
= &\left[\left(1-\frac{(1-\tau^l_2)\iota}{2}\right)\left(\frac{1}{2}\frac{1-\tau^l_2}{\tau^h_2-\tau^l_2}\right)+(1-\iota)\left(1-\frac{1}{2}\frac{1-\tau^l_2}{\tau^h_2-\tau^l_2}\right)\right.\\
&\left.+\frac{\iota}{2(\tau^h_2-\tau^l_2)}\left((\tau^h_2)^2-\frac{(1+\tau^l_2)^2}{4}\right)\right](V_{cjt}(n+1)-V_{cjt}(n))
\end{aligned} \tag{B.20}$$

Therefore, the defendant's expected payoff conditional on a type 2 patent infringement and before learning whether they will receive a settlement offer can be written as

$$W_{2,cjt}^{def} \equiv \left((1-p_{2,cjt}^{LT}) + p_{2,cjt}^{LT}\left[\left(1-\frac{(1-\tau_2^l)\iota}{2}\right)\left(\frac{1}{2}\frac{1-\tau_2^l}{\tau_2^h-\tau_2^l}\right) + (1-\iota)\left(1-\frac{1}{2}\frac{1-\tau_2^l}{\tau_2^h-\tau_2^l}\right)\right.\right.$$
$$\left.\left. + \frac{\iota}{2(\tau_2^h-\tau_2^l)}\left((\tau_2^h)^2 - \frac{(1+\tau_2^l)^2}{4}\right)\right]\right)(V_{cjt}(n+1) - V_{cjt}(n)) \tag{B.21}$$

Note that the whole expression is linear in the expected change in firm value if they take over the product line, $V_{cjt}(n+1) - V_{cjt}(n)$, which will be of use in deriving a closed-form expression for the firm value function.

B.3. Proof of Proposition 2

We consider the decision problem of a plaintiff facing a type 1 patent infringement which means they are the owner of the product line that is facing the risk of creative destruction. We further know that the plaintiff and the defendant share the same technology class c. Unlike a type 2 infringement, this time the plaintiff cares about more than the potential settlement they can extract from the defendant, since settling out of court also means they lose their product line for sure. Assume the distribution $T_1(\tau)$ is the continuous uniform distribution $U(\tau_1^l, \tau_1^h)$ with $0 \leq \tau_1^l < \tau_1^h \leq 1$. Then the plaintiff's problem is written as

$$\max_{s\geq 0} \left\{ \int_{\tau_1^l}^{1-s/(\iota(V_{cjt}(n^d+1)-V_{cjt}(n^d)))} \left(V_{cjt}(n-1) - V_{cjt}(n) + s\right) dT_1(\tau) \right.$$
$$\left. + \int_{1-s/(\iota(V_{cjt}(n^d+1)-V_{cjt}(n^d)))}^{\tau_1^h} (\tau + (1-\tau)(1-\iota))(V_{cjt}(n-1) - V_{cjt}(n)) dT_1(\tau) \right\} \tag{B.22}$$

where the first integral is the expected payoff from defendants who accept the settlement and the second integral is the expected payoff from those who reject. The term $V_{cjt}(n-1) - V_{cjt}(n)$ is negative, and reflects the cost of losing the product line. In the cases when the defendant accepts, the plaintiff is gives up their $(1-\tau)\iota$ chance of retaining their product line in exchange for a settlement amount s.

Note that there is an inherent adverse selection problem here: Conditional on a settlement offer s, only firms with the lowest chance of winning the trial τ will accept. From the defendant's problem, we know that a defendant strictly prefers the settlement offer if and only if

$$(1-\tau)\iota(V_{cjt}(n^d+1)-V_{cjt}(n^d)) \quad > \quad s \tag{B.23}$$

where n^d stands for the defendant's number of product lines. On the other hand, the difference in the plaintiff's payoff in the case of acceptance is

$$(1-\tau)\iota(V_{cjt}(n-1)-V_{cjt}(n))+s \tag{B.24}$$

Consider the defendant with the threshold τ^* who is indifferent. Then the abovementioned difference becomes

$$(1-\tau^*)\iota(V_{cjt}(n-1)-V_{cjt}(n))+(1-\tau^*)\iota(V_{cjt}(n^d+1)-V_{cjt}(n^d)) \tag{B.25}$$

which is exactly zero if $V_{cjt}(n)-V_{cjt}(n-1)=V_{cjt}(n^d+1)-V_{cjt}(n^d)$, that is, if the value change from having one more product line in industry j for firms with technology class c is the same regardless of how many product lines the company owns, n. We will later on show that this is exactly the case in a stationary equilibrium, since the value function of the firm will turn out to be linear in n. But this highlights the adverse selection problem: Even in the best case scenario, the plaintiff gains exactly zero from the firm with the highest probability of winning the trial among those who accept. For all other firms who accept that have a probability of winning the trial below the threshold firm with $\tau<\tau^*$, the abovementioned difference becomes

$$(1-\tau)\iota(V_{cjt}(n-1)-V_{cjt}(n))+(1-\tau^*)\iota(V_{cjt}(n^d+1)-V_{cjt}(n^d)) \tag{B.26}$$

which is strictly negative if $V_{cjt}(n)-V_{cjt}(n-1)=V_{cjt}(n^d+1)-V_{cjt}(n^d)$. This means the plaintiff would be making no extra return from the threshold firm that accepts, and would make an extra loss from every other firm that accepts. As a consequence, it is optimal for a plaintiff to always make settlement offers that will be rejected by every defendant – the adverse selection problem completely undermines any chance of out-of-court settlements for type 1 patent infringements.[10]

Having figured out that plaintiffs will always pick a high enough settlement amount s such that every defendant will reject, we can calculate the expected payoffs for the agents.

[10]Note that this result owes to two facts: (1) The defendant and the plaintiff have the same technology class c in type 1 patent infringements, and (2) the firm value function is linear in n.

The payoff of the plaintiff from going to court is:

$$\begin{aligned}
& \int_{\tau_1^l}^{\tau_1^h} (\tau + (1-\tau)(1-\iota))(V_{cjt}(n-1) - V_{cjt}(n)) dT_1(\tau) \\
= & \ (V_{cjt}(n-1) - V_{cjt}(n)) \left(1 - \iota + \iota \int_{\tau_1^l}^{\tau_1^h} \tau dT_1(\tau)\right) \\
= & \ (V_{cjt}(n-1) - V_{cjt}(n)) \left(1 - \iota + \iota \frac{\tau_1^h + \tau_1^l}{2}\right) \qquad \text{(B.27)}
\end{aligned}$$

Given this expression, we can turn to the plaintiff's decision to hire a legal team or not. If the plaintiff does not hire a legal team, then their payoff is simply $V_{cjt}(n-1) - V_{cjt}(n)$ since they will lose their product line for certain. Therefore, they will strictly prefer to hire a legal team if and only if

$$\begin{aligned}
(V_{cjt}(n-1) - V_{cjt}(n)) \left(1 - \iota + \iota \frac{\tau_1^h + \tau_1^l}{2}\right) - \gamma Y_t &> (V_{cjt}(n-1) - V_{cjt}(n)) \\
\left(-\iota + \iota \frac{\tau_1^h + \tau_1^l}{2}\right) (V_{cjt}(n-1) - V_{cjt}(n)) &> \gamma Y_t \qquad \text{(B.28)}
\end{aligned}$$

Then the probability of hiring a legal team is given by

$$p_{1,cjt}^{LT} \equiv \mathbb{P}\left(\gamma \leq \left(-\iota + \iota \frac{\tau_1^h + \tau_1^l}{2}\right) \frac{(V_{cjt}(n-1) - V_{cjt}(n))}{Y_t}\right) \qquad \text{(B.29)}$$

and the expected payoff of the plaintiff conditional on a type 1 patent infringement minus legal team cost is given as

$$\begin{aligned}
W_{1,cjt}^{plain} \equiv & \ p_{1,cjt}^{LT}(V_{cjt}(n-1) - V_{cjt}(n)) \left(1 - \iota + \iota \frac{\tau_1^h + \tau_1^l}{2}\right) - Y_t \int_0^{\left(-\iota + \iota \frac{\tau_1^h + \tau_1^l}{2}\right) \frac{(V_{cjt}(n-1) - V_{cjt}(n))}{Y_t}} \gamma d\Gamma(\gamma) \\
& + (1 - p_{1,cjt}^{LT})(V_{cjt}(n-1) - V_{cjt}(n)) \qquad \text{(B.30)}
\end{aligned}$$

where the first term is the probability to hire a legal team times the expected returns to the plaintiff not including legal team costs, the second term is the expected legal team costs, and the third term is the probability not to hire a legal team times the expected returns, which is simply the value change from losing a product line for certain.

Now, let's turn to the payoff of the defendant. We know the settlement will always be sufficiently high such that every defendant rejects. Then, given τ, the defendant's payoff from going to court is:

$$(\tau + (1-\tau)(1-\iota))(V_{cjt}(n^d+1) - V_{cjt}(n^d)) \tag{B.31}$$

and taking expectation over τ before its realization, we have

$$\begin{aligned}\mathbb{E}\left[(\tau + (1-\tau)(1-\iota))(V_{cjt}(n^d+1) - V_{cjt}(n^d))\right] &= (V_{cjt}(n^d+1) - V_{cjt}(n^d))\left(1-\iota+\iota\int_{\tau_1^l}^{\tau_1^h}\tau dT_1(\tau)\right) \\ &= (V_{cjt}(n^d+1) - V_{cjt}(n^d))\left(1-\iota+\iota\frac{\tau_1^h+\tau_1^l}{2}\right)\end{aligned} \tag{B.32}$$

Then, given the probability of the plaintiff hiring a legal team, the expected payoff of the defendant conditional on a type 1 patent infringement is

$$W_{1,cjt}^{def} \equiv p_{1,cjt}^{LT}(V_{cjt}(n^d+1) - V_{cjt}(n^d))\left(1-\iota+\iota\frac{\tau_1^h+\tau_1^l}{2}\right) + (1-p_{1,cjt}^{LT})(V_{cjt}(n^d+1) - V_{cjt}(n^d)) \tag{B.33}$$

where the first term is the probability to hire a legal team times the expected returns to the defendant, and the second term is the complementary probability times the value change from adding a product line for certain.

B.4. Proof of Theorem 1

The guess-and-verify method will be used. Suppose the value function takes the specified form. Then, we can plug it into the various terms that show up in Equation (21) and recover the equations that pin down the values of the scalars v_{cj} for all technology classes c and all industries j.

First, consider the expected rent flow from type 2 patent infringements by firms in industry j on our firm's IP, nR_{cjt}. Using Equations (31) and (37), we get:

$$\begin{aligned}
R_{cjt} &= p_{cj}^{rent}\left(p_{2,cj}^{LT}\frac{(1-\tau_2^l)^2\iota(V_{cjt}(n^d+1)-V_{cjt}(n^d))}{4(\tau_2^h-\tau_2^l)} - Y_t\int_0^{\frac{(1-\tau_2^l)^2\iota(V_{cjt}(n^d+1)-V_{cjt}(n^d))}{4(\tau_2^h-\tau_2^l)Y_t}}\gamma d\Gamma(\gamma)\right) \\
&= p_{cj}^{rent}\left(p_{2,cj}^{LT}\frac{(1-\tau_2^l)^2\iota(v_{cj}(n^d+1)Y_t-v_{cj}n^dY_t)}{4(\tau_2^h-\tau_2^l)} - Y_t\int_0^{\frac{(1-\tau_2^l)^2\iota(v_{cj}(n^d+1)Y_t-v_{cj}n^dY_t)}{4(\tau_2^h-\tau_2^l)Y_t}}\gamma d\Gamma(\gamma)\right) \\
&= p_{cj}^{rent}\left(p_{2,cj}^{LT}\frac{(1-\tau_2^l)^2\iota v_{cj}}{4(\tau_2^h-\tau_2^l)} - \int_0^{\frac{(1-\tau_2^l)^2\iota v_{cj}}{4(\tau_2^h-\tau_2^l)}}\gamma d\Gamma(\gamma)\right)Y_t \\
nR_{cjt} &= p_{cj}^{rent}\left(p_{2,cj}^{LT}\frac{(1-\tau_2^l)^2\iota v_{cj}}{4(\tau_2^h-\tau_2^l)} - \int_0^{\frac{(1-\tau_2^l)^2\iota v_{cj}}{4(\tau_2^h-\tau_2^l)}}\gamma d\Gamma(\gamma)\right)nY_t \\
&\equiv \hat{R}_{cj}nY_t
\end{aligned} \tag{B.34}$$

where the last line implicitly defines the normalized term $\hat{R}_{cj}$ for convenience.

Second, consider the value difference conditional on successful innovation, but before the litigation subgame, $V_{cjt}^{+}(n) - V_{cjt}(n)$. As gleaned from Equation (38), we must first obtain the expected payoffs of the defendant conditional on type 1 and type 2 patent infringements, denoted as $W_{1,cjt}^{def}$ and $W_{2,cjt}^{def}$, respectively. Plugging the guess in Equation (36) yields:

$$\begin{aligned}
W_{1,cjt}^{def} &= p_{1,cj}^{LT}(V_{cjt}(n^d+1)-V_{cjt}(n^d))\left(1-\iota+\iota\frac{\tau_1^h+\tau_1^l}{2}\right) + (1-p_{1,cj}^{LT})(V_{cjt}(n^d+1)-V_{cjt}(n^d)) \\
&= p_{1,cj}^{LT}(v_{cj}(n^d+1)Y_t-v_{cj}n^dY_t)\left(1-\iota+\iota\frac{\tau_1^h+\tau_1^l}{2}\right) + (1-p_{1,cj}^{LT})(v_{cj}(n^d+1)Y_t-v_{cj}n^dY_t) \\
&= \left(p_{1,cj}^{LT}\left(1-\iota+\iota\frac{\tau_1^h+\tau_1^l}{2}\right) + (1-p_{1,cj}^{LT})\right)v_{cj}Y_t \\
&\equiv \hat{W}_{1,cj}^{def}v_{cj}Y_t
\end{aligned} \tag{B.35}$$

where the last line implicitly defines the normalized term $\hat{W}_{1,cj}^{def}$ which depends on the probability $p^{LT}_{1,cj}$. Likewise, plugging the guess in Equation (32) yields:

$$\begin{aligned} W^{def}_{2,cjt} &= \Bigg((1-p^{LT}_{2,cj}) + p^{LT}_{2,cj}\Bigg[\left(1-\frac{(1-\tau^l_2)\iota}{2}\right)\left(\frac{1}{2}\frac{1-\tau^l_2}{\tau^h_2-\tau^l_2}\right) + (1-\iota)\left(1-\frac{1}{2}\frac{1-\tau^l_2}{\tau^h_2-\tau^l_2}\right) \\ &\quad + \frac{\iota}{2(\tau^h_2-\tau^l_2)}\left((\tau^h_2)^2 - \frac{(1+\tau^l_2)^2}{4}\right)\Bigg]\Bigg) v_{cj}Y_t \\ &\equiv \hat{W}^{def}_{2,cj} v_{cj} Y_t \end{aligned} \tag{B.36}$$

where the last line implicitly defines the normalized term $\hat{W}^{def}_{2,cj}$ which depends on the probability $p^{LT}_{2,cj}$. Using Equations (38), (B.35), and (B.36), we get:

$$\begin{aligned} V^+_{cjt}(n) - V_{cjt}(n) &= p^{def}_{cjt}\kappa_1 W^{def}_{1,cjt} + (1-p^{def}_{cjt})\kappa_2 W^{def}_{2,cjt} \\ &\quad + [(p^{def}_{cj}(1-\kappa_1) + (1-p^{def}_{cj})(1-\kappa_2)](V_{cjt}(n+1) - V_{cjt}(n)) \\ &= \Big(p^{def}_{cj}\kappa_1 \hat{W}^{def}_{1,cj} + (1-p^{def}_{cj})\kappa_2 \hat{W}^{def}_{2,cj} \\ &\quad + [(p^{def}_{cj}(1-\kappa_1) + (1-p^{def}_{cj})(1-\kappa_2)]\Big) v_{cj} Y_t \\ &\equiv L^{def}_{cj} v_{cj} Y_t \end{aligned} \tag{B.37}$$

where the last line implicitly defines L^{def}_{cj}. Notice that, in the absence of any patent infringement – that is, $\kappa_1 = \kappa_2 = 0$ – we have $L^{def}_{cj} = 1$, and the whole expression simplifies to $v_{cj}Y_t$ alone. Therefore, $1 - L^{def}_{cj}$ captures the fraction of the value of a successful innovation that is lost due to the risk of infringing on other firms' IP.

Given Equation (B.37), we can calculate the optimal innovation rate $x_{cjt}(n)$ using Equation (22) as:

$$\begin{aligned} x_{cjt}(n) &= \left(\frac{\left(V^+_{cjt}(n) - V_{cjt}(n)\right)(1+\sigma M_{ct})}{(1-s_{cj})\chi_c \psi Y_t}\right)^{\frac{1}{\psi-1}} \\ &= \left(\frac{L^{def}_{cj} v_{cj}(1+\sigma M_c)}{(1-s_{cj})\chi_c\psi}\right)^{\frac{1}{\psi-1}} \equiv x_{cj} \end{aligned} \tag{B.38}$$

Note that this optimal innovation rate is independent of both the number of product lines owned by the firm, n, and time, t. The latter owes to the fact that the term M_{ct} must be time-invariant in a BGP equilibrium since the firm distribution across industries, technology classes, and number of product lines is stationary.

Third, consider the value difference conditional on being innovated on (i.e., value loss from creative destruction), but before the litigation subgame, denoted as $V^{-}_{cjt}(n) - V_{cjt}(n)$. As gleaned from Equation (39), we must first obtain the expected payoff of the plaintiff conditional on type 1 patent infringement, denoted as $W^{plain}_{1,cjt}$. Plugging the guess in Equation (35) yields:

$$\begin{aligned} W^{plain}_{1,cjt} &= p^{LT}_{1,cj}(V_{cjt}(n-1) - V_{cjt}(n))\left(1 - \iota + \iota\frac{\tau^h_1 + \tau^l_1}{2}\right) - Y_t \int_0^{\left(-\iota+\iota\frac{\tau^h_1+\tau^l_1}{2}\right)\frac{(V_{cjt}(n-1)-V_{cjt}(n))}{Y_t}} \gamma d\Gamma(\gamma) \\ &\quad +(1 - p^{LT}_{1,cj})(V_{cjt}(n-1) - V_{cjt}(n)) \\ &= p^{LT}_{1,cj}(-v_{cj}Y_t)\left(1 - \iota + \iota\frac{\tau^h_1 + \tau^l_1}{2}\right) - Y_t \int_0^{\left(-\iota+\iota\frac{\tau^h_1+\tau^l_1}{2}\right)(-v_{cj})} \gamma d\Gamma(\gamma) \\ &\quad +(1 - p^{LT}_{1,cj})(-v_{cj}Y_t) \\ &= \left(p^{LT}_{1,cj}\left(1 - \iota + \iota\frac{\tau^h_1 + \tau^l_1}{2}\right) + \frac{1}{v_{cj}} \int_0^{\left(-\iota+\iota\frac{\tau^h_1+\tau^l_1}{2}\right)(-v_{cj})} \gamma d\Gamma(\gamma) \right. \\ &\quad \left. +(1 - p^{LT}_{1,cj})\right)(-v_{cj}Y_t) \\ &\equiv \hat{W}^{plain}_{1,cj}(-v_{cj}Y_t) \end{aligned} \tag{B.39}$$

where the last line implicitly defines the normalized term $\hat{W}^{plain}_{1,cj}$ which depends on the probability $p^{LT}_{1,cj}$ and v_{cj}. Using Equations (39) and (B.39), we get:

$$\begin{aligned} V^{-}_{cjt}(n) - V_{cjt}(n) &= p^{plain}_{cj}\kappa_1 W^{plain}_{1,cjt} + (1 - p^{plain}_{cj}\kappa_1 - (1 - p^{plain}_{cj})\kappa_2 p^{inj}_{cj})(V_{cjt}(n-1) - V_{cjt}(n)) \\ &= p^{plain}_{cj}\kappa_1 \hat{W}^{plain}_{1,cj}(-v_{cj}Y_t) + (1 - p^{plain}_{cj}\kappa_1 - (1 - p^{plain}_{cj})\kappa_2 p^{inj}_{cj})(-v_{cj}Y_t) \\ &= \left(p^{plain}_{cj}\kappa_1 \hat{W}^{plain}_{1,cj} + (1 - p^{plain}_{cj}\kappa_1 - (1 - p^{plain}_{cj})\kappa_2 p^{inj}_{cj})\right)(-v_{cj}Y_t) \\ &\equiv L^{plain}_{cj}(-v_{cj}Y_t) \end{aligned} \tag{B.40}$$

where the last line implicitly defines L_{cj}^{plain}. Notice that, in the absence of any patent infringement – that is, $\kappa_1 = \kappa_2 = 0$ – we have $L_{cj}^{plain} = 1$, and the whole expression simplifies to $-v_{cj}Y_t$ alone. Therefore, $1 - L_{cj}^{plain}$ captures the value gain to the owner of a product line from the possibility of using a patent infringement case to fight off an entrant, and by doing so, retain the ownership of their product line.

Before we move on to the HJB equation, there are a few additional expressions that need to be computed. First, notice that the summation of the static profit flows from owned product lines is simply:

$$\sum_{m=1}^{n} \frac{\lambda_c}{1+\lambda_c}\omega_j Y_t = \frac{\lambda_c}{1+\lambda_c}\omega_j n Y_t \tag{B.41}$$

Second, the time derivative of the value function is:

$$\dot{V}_{cjt}(n) = \frac{d}{dt}(v_{cj}nY_t) = v_{cj}n\frac{dY_t}{dt} = gv_{cj}nY_t \tag{B.42}$$

Third, the total R&D bill is given as:

$$\sum_{m=1}^{n} \frac{(1-s_{cj})\chi_c x_{mcjt}^{\psi} Y_t}{1+\sigma M_{ct}} = \frac{(1-s_{cj})\chi_c x_{cj}^{\psi}}{1+\sigma M_c} nY_t \tag{B.43}$$

Given all the previous derivations, we are now ready to plug in all expressions to the

HJB equation given in Equation (21). This yields:

$$
\begin{aligned}
r_t V_{cjt}(n) - \dot{V}_{cjt}(n) = \max_{\{x_{mcjt}\}_{m=1}^{n}} \quad \Bigg\{ & \sum_{m=1}^{n} \frac{\lambda_c}{1+\lambda_c}\omega_j Y_t + n\sum_{j'=1}^{J} R_{cj't} \\
& - \sum_{m=1}^{n} \frac{(1-s_{cj})\chi_c x_{mcjt}^{\psi} Y_t}{1+\sigma M_{ct}} + \left(\sum_{m=1}^{n} x_{mcjt}\right)\left(V_{cjt}^{+}(n) - V_{cjt}(n)\right) \\
& + n d_{jt}\left(V_{cjt}^{-}(n) - V_{cjt}(n)\right) + \delta\left(0 - V_{cjt}(n)\right) \Bigg\}
\end{aligned}
$$

$$
\begin{aligned}
(r-g)v_{cj}nY_t \quad = \quad & \frac{\lambda_c}{1+\lambda_c}\omega_j nY_t + \sum_{j'=1}^{J} \hat{R}_{cj'} nY_t \\
& - \frac{(1-s_{cj})\chi_c x_{cj}^{\psi}}{1+\sigma M_c} nY_t + x_{cj} L_{cj}^{def} v_{cj} nY_t \\
& - d_j L_{cj}^{plain} v_{cj} nY_t - \delta v_{cj} nY_t
\end{aligned}
$$

As can be seen, all the terms are linear in nY_t. Dividing both sides by nY_t and reorganizing, we get:

$$
\begin{aligned}
(r-g)v_{cj} \quad &= \quad \frac{\lambda_c}{1+\lambda_c}\omega_j + \sum_{j'=1}^{J} \hat{R}_{cj'} - \frac{(1-s_{cj})\chi_c x_{cj}^{\psi}}{1+\sigma M_c} + x_{cj} L_{cj}^{def} v_{cj} \\
& \quad\quad - d_j L_{cj}^{plain} v_{cj} - \delta v_{cj} \\
\left(r - g + \delta - x_{cj} L_{cj}^{def} + d_j L_{cj}^{plain}\right) v_{cj} \quad &= \quad \frac{\lambda_c}{1+\lambda_c}\omega_j + \sum_{j'=1}^{J} \hat{R}_{cj'} - \frac{(1-s_{cj})\chi_c x_{cj}^{\psi}}{1+\sigma M_c} \\
v_{cj} \quad &= \quad \frac{\frac{\lambda_c}{1+\lambda_c}\omega_j + \sum_{j'=1}^{J} \hat{R}_{cj'} - \frac{(1-s_{cj})\chi_c x_{cj}^{\psi}}{1+\sigma M_c}}{\rho + \delta - x_{cj} L_{cj}^{def} + d_j L_{cj}^{plain}} \qquad \text{(B.44)}
\end{aligned}
$$

where the last line uses $r - g = \rho$ that must hold in a BGP equilibrium due to the Euler equation of the representative household. Given the probabilities $p_{cj}^{rent}, p_{1,cj}^{LT}, p_{2,cj}^{LT}, p_{cj}^{def}, p_{cj}^{plain}, p_{cj}^{inj}$, the growth rate g, the fraction of product lines owned by technology class c firms M_c, and the creative destruction rate d_j, Equation (B.44) pins down the exact values of the scalars

v_{cj} for all c and j, and thus concludes the proof for the incumbents.

Given the value function of incumbents, the optimal entrant innovation arrival rate z chosen by the entrepreneurs can also be calculated in closed-form. Using Equation (24), we get

$$
\begin{aligned}
z_t &= \left(\frac{\sum_{c=1}^{C} \sum_{j=1}^{J} \eta_{cj} (V_{cjt}^{+}(0) - V_{cjt}(0))}{(1-s_e)\nu\psi Y_t} \right)^{\frac{1}{\psi-1}} \\
&= \left(\frac{\sum_{c=1}^{C} \sum_{j=1}^{J} \eta_{cj} L_{cj}^{def} v_{cj} Y_t}{(1-s_e)\nu\psi Y_t} \right)^{\frac{1}{\psi-1}} \\
&= \left(\frac{\sum_{c=1}^{C} \sum_{j=1}^{J} \eta_{cj} L_{cj}^{def} v_{cj}}{(1-s_e)\nu\psi} \right)^{\frac{1}{\psi-1}} \equiv z \qquad \text{(B.45)}
\end{aligned}
$$

which is time-invariant and the same for all entrepreneurs.

To compute the full BGP equilibrium, the values of these endogenous probabilities must also be calculated. Two of these, the litigation probabilities $p_{1,cj}^{LT}$ and $p_{2,cj}^{LT}$ can be computed without any reference to the stationary distribution of firms. Using Equation (34), we have:

$$
\begin{aligned}
p_{1,cj}^{LT} &= \mathbb{P}\left(\gamma \leq \left(-\iota + \iota \frac{\tau_1^h + \tau_1^l}{2} \right) \frac{(V_{cjt}(n-1) - V_{cjt}(n))}{Y_t} \right) \\
&= \mathbb{P}\left(\gamma \leq \left(-\iota + \iota \frac{\tau_1^h + \tau_1^l}{2} \right) (-v_{cj}) \right) \qquad \text{(B.46)}
\end{aligned}
$$

Likewise, using Equation (29), we have:

$$
\begin{aligned}
p_{2,cj}^{LT} &= \mathbb{P}\left(\gamma \leq \frac{(1-\tau_2^l)^2 \iota (V_{cjt}(n+1) - V_{cjt}(n))}{4(\tau_2^h - \tau_2^l) Y_t} \right) \\
&= \mathbb{P}\left(\gamma \leq \frac{(1-\tau_2^l)^2 \iota v_{cj}}{4(\tau_2^h - \tau_2^l)} \right) \qquad \text{(B.47)}
\end{aligned}
$$

The remaining endogenous variables must be computed numerically, consistent with the stationary firm distribution in the economy.

B.5. Proof of Proposition 3

To close the model, we need to derive the equations that pin down the values of endogenous variables in a BGP equilibrium, such as the growth rate g, the stationary product line distribution across industries and technology classes $\{\{\mu_{cj}\}_{c=1}^{C}\}_{j=1}^{J}$, and the associated probabilities of various events discussed earlier.

Recall that $\mu_{cjt} \in [0,1]$ denotes the measure of all product lines in industry j for which the leader has technology class c at time t, with $\sum_{c=1}^{C} \mu_{cjt} = 1$. In a stationary equilibrium, μ_{cjt} are time-invariant, so time subscripts will be suppressed from here on. Under this definition, total incumbent innovation by firms of technology class c in industry j is $\mu_{cj} x_{cj}$, and the total entrant innovation for the same is $\eta_{cj} z$.

The industry-specific creative destruction rate d_j in industry j depends on total innovation in that industry by both incumbents and entrants with any technology class. This is given by

$$d_j = \sum_{c=1}^{C} (\mu_{cj} x_{cj} + \eta_{cj} z) \tag{B.48}$$

The probability for plaintiffs of type (c, j) that the incoming innovation belongs to a firm with the same technology class, denoted p_{cj}^{plain}, can be calculated as

$$p_{cj}^{plain} = \frac{\mu_{cj} x_{cj} + \eta_{cj} z}{\sum_{c'=1}^{C} (\mu_{c'j} x_{c'j} + \eta_{c'j} z)} \tag{B.49}$$

which is the fraction of total innovation in industry j originating from firms of type (c, j) to that of total innovation in industry j irrespective of technology class.

The probability for defendants of type (c, j) to innovate on the product line of another firm with the same technology class c in its industry, p_{cj}^{def} is simply

$$p_{cj}^{def} = \mu_{cj} \tag{B.50}$$

since $\sum_{c=1}^{C} \mu_{cj} = 1$.

To calculate the the Poisson arrival rate of a type 2 patent infringement for plaintiffs with technology class c from firms in industry j, denoted p_{cj}^{rent}, we need to do an accounting of the measure of type 2 patent infringements that happen in technology class c in industry

j, and the measure of eligible plaintiffs across all industries. The prior is calculated as

$$(\mu_{cj}x_{cj} + \eta_{cj}z)(1 - \mu_{cj})\kappa_2 \tag{B.51}$$

where the first factor is the total innovation in industry j originating from firms of type (c, j), the second factor is the probability that such innovation lands on a product line with technology class $c' \neq c$, and the third factor is the probability of a type 2 patent infringement occuring under this scenario. The latter is simply the sum of all product lines belonging to firms with technology class c across all industries, i.e., $\sum_{j=1}^{J} \mu_{cj}$. Then we can calculate p_{cj}^{rent} as

$$p_{cj}^{rent} = \frac{(\mu_{cj}x_{cj} + \eta_{cj}z)(1 - \mu_{cj})\kappa_2}{\sum_{j'=1}^{J} \mu_{cj'}} \tag{B.52}$$

Recall that the probability that an injunction is granted conditional on a type 2 infringement from the perspective of the owner of the product line was denoted p_{cj}^{inj}. In type 2 infringements, the technology class c' of the innovating firm matters for the injunction probability, since it also influences the rents the third-party plaintiff can extract. Define $p_{c'j}^{inj2}$ as the probability of an injunction conditional on the innovating firm having technology class $c' \neq c$. Then, this probability is calculated as

$$\begin{aligned} p_{c'j}^{inj2} &= p_{2,c'j}^{LT}\left(\int_{\tau_2^l}^{\tau^*} 0 dT_2(\tau) + \int_{\tau^*}^{\tau_2^h} (1-\tau) dT_2(\tau)\right)\iota \\ &= p_{2,c'j}^{LT}\left[\left(\tau_2^h - \tau^* - \frac{(\tau_2^h)^2}{2} + \frac{(\tau^*)^2}{2}\right)\frac{1}{\tau_2^h - \tau_2^l}\right]\iota \\ &= p_{2,c'j}^{LT}\left[\left(\tau_2^h - \frac{1+\tau_2^l}{2} - \frac{(\tau_2^h)^2}{2} + \frac{(1+\tau_2^l)^2}{8}\right)\frac{1}{\tau_2^h - \tau_2^l}\right]\iota \end{aligned} \tag{B.53}$$

where the first factor is the probability that the plaintiff hires a legal team, the second factor is the probability that the defendant rejects the settlement offer and loses at court, and the third factor is the probability that an injunction is granted. Given this, we can calculate p_{cj}^{inj} as

$$
\begin{aligned}
p_{cj}^{inj} &= \frac{\sum_{c'\neq c}(\mu_{c'j}x_{c'j}+\eta_{c'j}z)p_{c'j}^{inj2}}{\sum_{c'\neq c}(\mu_{c'j}x_{c'j}+\eta_{c'j}z)} \\
&= \frac{\sum_{c'\neq c}(\mu_{c'j}x_{c'j}+\eta_{c'j}z)p_{2,c'j}^{LT}}{\sum_{c'\neq c}(\mu_{c'j}x_{c'j}+\eta_{c'j}z)}\left[\left(\tau_2^h-\frac{1+\tau_2^l}{2}-\frac{(\tau_2^h)^2}{2}+\frac{(1+\tau_2^l)^2}{8}\right)\frac{1}{\tau_2^h-\tau_2^l}\right]\iota \qquad \text{(B.54)}
\end{aligned}
$$

To calculate the growth rate of the economy, we must tally not only successful innovations, but also the rate at which successful innovations convert to product line takeovers (i.e., the fraction of successful innovations that are not blocked by an injunction), and the technology classes of both the incumbent and the innovator, since the productivity gains λ_c are heterogeneous, and so are the markups charged over marginal cost.

From the definition of the production technology, we have:

$$
\begin{aligned}
\ln Y_t &= \sum_{j=1}^{J}\omega_j\left(\int_0^1 \ln y_{ijt}di\right) \\
\frac{\ln Y_{t+\Delta t}-\ln Y_t}{\Delta t} &= \sum_{j=1}^{J}\omega_j\left(\int_0^1 \frac{\ln y_{ijt+\Delta t}-\ln y_{ijt}}{\Delta t}di\right) \\
g_t &= \lim_{\Delta t\to 0}\sum_{j=1}^{J}\omega_j\left(\int_0^1 \frac{\ln y_{ijt+\Delta t}-\ln y_{ijt}}{\Delta t}di\right) \qquad \text{(B.55)}
\end{aligned}
$$

Hence, to figure out the output growth rate g_t, we must focus on how log output in each product line $\ln y_{ijt}$ changes over time. From the incumbent firm's static problem, we know

$$
\begin{aligned}
\ln y_{ijt} &= \ln\left(\frac{\omega_j Y_t q_{ijt}}{w_t(1+\lambda_c)}\right) \\
&= \ln\omega_j + \ln\left(\frac{Y_t}{w_t}\right) + \ln q_{ijt} - \ln(1+\lambda_c) \qquad \text{(B.56)}
\end{aligned}
$$

The first term is the function of a parameter, and thus constant. The second term is a function of the relative wage w_t/Y_t, which is time-invariant in a BGP equilibrium. The third term is log productivity, which increases upon successful innovation that is not blocked. The fourth term is the markup distortion, which can change upon successful innovation that is not blocked if the innovator has a different technology class $c' \neq c$.

Now, consider the case of some product line i in industry j owned by a firm with technology class c. The probability that the product line is lost to a firm with the same technology class c over a small time interval Δt is

$$(\mu_{cj} x_{cj} + \eta_{cj} z)\Delta t \left[1 - \kappa_1 p_{1,cj}^{LT} \left(1 - \frac{\tau_1^h + \tau_1^l}{2}\right) \iota\right] \tag{B.57}$$

where the term outside the brackets is the probability of a successful innovation, whereas the term inside the brackets is the probability that an injunction is not granted. An injunction is only granted if there is an infringement (prob. κ_1), the plaintiff pays the legal team cost (prob. $p_{1,cj}^{LT}$), the defendant loses (prob. $1 - (\tau_1^h + \tau_1^l)/2$), and the court grants an injunction (prob. ι). In this scenario, since both firms have the same technology class, the markup distortion is unchanged. However, log productivity increases by $\ln(1 + \lambda_c)$.

For any technology class $c' \neq c$, the probability that the product line is lost to a firm with the technology class c' over a small time interval Δt is

$$(\mu_{c'j} x_{c'j} + \eta_{c'j} z)\Delta t \left[1 - \kappa_2 p_{c'j}^{inj2}\right] \tag{B.58}$$

where the term outside the brackets is the probability of a successful innovation, whereas the term inside the brackets is the probability that an injunction is not granted, which uses the $p_{c'j}^{inj2}$ defined in Equation (B.53). In this scenario, the markup distortion changes from $\ln(1 + \lambda_c)$ to $\ln(1 + \lambda_{c'})$. Log productivity also increases by $\ln(1 + \lambda_{c'})$. The net effect on log output for the product line is therefore $\ln(1 + \lambda_{c'}) + \ln(1 + \lambda_c) - \ln(1 + \lambda_{c'}) = \ln(1 + \lambda_c)$, same as the previous scenario.

Given these observations, for some product line i in industry j owned by a firm with technology class c, we can write:

$$\begin{aligned} \frac{\ln y_{ijt+\Delta t} - \ln y_{ijt}}{\Delta t} &= (\mu_{cj} x_{cj} + \eta_{cj} z)\left[1 - \kappa_1 p_{1,cj}^{LT} \left(1 - \frac{\tau_1^h + \tau_1^l}{2}\right) \iota\right] \ln(1 + \lambda_c) \\ &\quad + \sum_{c' \neq c} (\mu_{c'j} x_{c'j} + \eta_{c'j} z)\left[1 - \kappa_2 p_{c'j}^{inj2}\right] \ln(1 + \lambda_c) \equiv f_{cj} \end{aligned} \tag{B.59}$$

which is called f_{cj} for convenience. Then, we can plug in these expressions in Equation

(B.55) to obtain

$$g = \sum_{j=1}^{J} \omega_j \sum_{c=1}^{C} \mu_{cj} f_{cj} \tag{B.60}$$

which pins down the output growth rate in a BGP equilibrium.

Finally, we need to pin down the equations that determine $\mu_{cj}, \forall c, j$. To this purpose, define a joint product line type as $\Theta = (c, j)$, and define $P(\Theta, \Theta')$ as the transition rate from product lines of type $\Theta = (c, j)$ (origin) to $\Theta' = (c', j')$ (destination). First, note that no event can change the industry of a product line. Therefore, we have

$$P((c, j), (c', j')) = 0, \forall c, \forall j, \forall c', \forall j' \neq j \tag{B.61}$$

Second, if the innovating firm has the same technology class as the incumbent, the type of the product line does not change even if ownership does, so it requires no explicit accounting. So that leaves the third case to consider, with $j = j'$ and $c' \neq c$. In this case, we have:

$$P((c, j), (c', j)) = (\mu_{c'j} x_{c'j} + \eta_{c'j} z) \left[1 - \kappa_2 p_{c'j}^{inj2} \right], \forall c, \forall j, \forall c' \neq c \tag{B.62}$$

in agreement with Equation (B.58). Finally, we have the case $j = j'$ and $c = c'$ which is implicitly defined as

$$P((c, j), (c, j)) = 1 - \sum_{c' \neq c} P((c, j), (c', j)) \tag{B.63}$$

Using the transition matrix defined by $P(\Theta, \Theta')$, we can pin down the stationary values of μ_{cj} by solving the linear system of equations

$$P^T \mu = \mu \tag{B.64}$$

$$\sum_{c=1}^{C} \mu_{cj} = 1, \forall j \tag{B.65}$$

which consists of $CJ + J$ equations.

B.6. Firm size distributions

To compute the firm size distributions, we need to calculate the product line takeover probabilities conditional on successful innovation for every firm type. Define this takeover probability for a firm with technology class c in industry j at time t as $p_{cjt}^{take} \in [0,1]$. This probability is calculated as:

$$\begin{aligned} p_{cjt}^{take} &= p_{cjt}^{def}\kappa_1\left[p_{1,cjt}^{LT}\left(1-\iota+\iota\frac{\tau_1^h+\tau_1^l}{2}\right)+\left(1-p_{1,cjt}^{LT}\right)\right] \\ &\quad +(1-p_{cjt}^{def})\kappa_2\left\{p_{2,cjt}^{LT}\left[\left(\frac{1}{2}\frac{1-\tau_2^l}{\tau_2^h-\tau_2^l}\right)+(1-\iota)\left(1-\frac{1}{2}\frac{1-\tau_2^l}{\tau_2^h-\tau_2^l}\right)\right.\right. \\ &\quad \left.\left.+\frac{\iota}{2(\tau_2^h-\tau_2^l)}\left((\tau_2^h)^2-\frac{(1+\tau_2^l)^2}{4}\right)\right]+\left(1-p_{2,cjt}^{LT}\right)\right\} \\ &\quad +\left(p_{cjt}^{def}(1-\kappa_1)+(1-p_{cjt}^{def})(1-\kappa_2)\right) \end{aligned} \tag{B.66}$$

where the first term is the probability of a type 1 infringement times the conditional takeover probability, the second term is the same for type 2 infringements, and the third term is the complementary event that no infringement occurs, in which case the takeover is assured.

We also need to calculate the flow rate of losing a product line for incumbent firms. Define the per product line product line loss flow rate for a firm with technology class c in industry j at time t as $p_{cjt}^{loss} > 0$. This flow rate is calculated as:

$$\begin{aligned} p_{cjt}^{loss} &= (\mu_{cjt}x_{cjt}+\eta_{cj}z_t)\left[1-\kappa_1 p_{1,cjt}^{LT}\left(1-\frac{\tau_1^h+\tau_1^l}{2}\right)\iota\right] \\ &\quad +\sum_{c'\neq c}(\mu_{c'jt}x_{c'jt}+\eta_{c'j}z_t)\left[1-\kappa_2 p_{c'jt}^{inj2}\right] \end{aligned} \tag{B.67}$$

Define the mass of firms with technology class c in industry j at time t that own n product lines as $\varphi_{cjt}(n) \geq 0$. Using previously-calculated expressions, we can write the ordinary differential equations that govern the evolution of these expressions. Due to new firm entry and endogenous firm exit, $n = 1$ is a special case, which is given by:

$$\dot{\varphi}_{cjt}(1) = z_t\eta_{cj} + 2p_{cjt}^{loss}\varphi_{cjt}(2) - (x_{cjt}p_{cjt}^{take} + p_{cjt}^{loss})\varphi_{cjt}(1) \tag{B.68}$$

where the first term corresponds to new entrants with a single product line, the second term corresponds to firms with two product lines losing one of them, and the third term corresponds to outflows of firms with a single product line due to both successful takeovers, as well as losses.

For all the other cases with $n \geq 2$, we have the general expression:

$$\begin{aligned} \dot{\varphi}_{cjt}(n) &= (n-1)x_{cjt}p_{cjt}^{take}\varphi_{cjt}(n-1) + (n+1)p_{cjt}^{loss}\varphi_{cjt}(n+1) \\ &\quad -n(x_{cjt}p_{cjt}^{take} + p_{cjt}^{loss})\varphi_{cjt}(n) \end{aligned} \tag{B.69}$$

where the first term corresponds to firms with $n-1$ product lines succeeding in taking over a new product line, the second term corresponds to firms with $n+1$ product lines losing one of them, and the third term corresponds to outflows of firms with n product lines due to both successful takeovers, as well as losses.

In a stationary equilibrium, we have $\dot{\varphi}_{cjt}(n) = 0, \forall c, j, t, n$. Therefore, the firm size distributions are time-invariant; that is, $\varphi_{cjt}(n) \equiv \varphi_{cj}(n), \forall c, j, t, n$. Using the previous equations, we can pin down these time-invariant firm size distributions. For any technology class c and industry j, we have the following equations:

$$0 = z\eta_{cj} + 2p_{cj}^{loss}\varphi_{cj}(2) - (x_{cj}p_{cj}^{take} + p_{cj}^{loss})\varphi_{cj}(1) \tag{B.70}$$

$$\begin{aligned} 0 &= (n-1)x_{cj}p_{cj}^{take}\varphi_{cj}(n-1) + (n+1)p_{cj}^{loss}\varphi_{cj}(n+1) \\ &\quad -n(x_{cj}p_{cj}^{take} + p_{cj}^{loss})\varphi_{cj}(n), \forall n \geq 2 \end{aligned} \tag{B.71}$$

In addition, we also know

$$z\eta_{cj} = p_{cj}^{loss}\varphi_{cj}(1) \tag{B.72}$$

$$\sum_{n=1}^{\infty} n\varphi_{cj}(n) = \mu_{cj} \tag{B.73}$$

where the first equation is due to firm entry being equal to firm exit in a stationary equilibrium, and the second equation is an accounting identity that ensures that the total number of product lines owned by firms with technology class c in industry j equals μ_{cj}. Together, equations (B.70), (B.71), and (B.72) pin down $\varphi_{cj}(n), \forall n \geq 1$.

B.7. Computing output and welfare

We would like to compute social welfare in counterfactual economies and compare them against the estimated equilibrium. To calculate welfare, we need to compute the consumption stream of the representative household. In a BGP equilibrium, two components must be known: the growth rate of consumption g, and the initial consumption level C_0. This requires us to compute initial output Y_0 and aggregate spending on R&D. In turn, computing initial output requires computing the (time-invariant) relative wage rate w_t/Y_t. We will compute these in reverse order.

To calculate the relative wage rate, we will use the labor market clearing condition. First, recall that the output y_{ijt} of firm i in industry j at time t is given by:

$$y_{ijt} = \frac{\omega_j Y_t}{p_{ijt}} = \frac{\omega_j Y_t q_{ijt}}{w_t(1+\lambda_c)} \tag{B.74}$$

Then, the labor demand of this firm becomes

$$l_{ijt} = \frac{y_{ijt}}{q_{ijt}} = \frac{\omega_j Y_t}{w_t(1+\lambda_c)} \tag{B.75}$$

which is independent of the firm's productivity q_{ijt}. Since the representative household supplies labor $L = 1$ inelastically, labor market clearing requires:

$$\begin{aligned}
1 &= \sum_{j=1}^{J} \int_0^1 l_{ijt} di \\
1 &= \sum_{j=1}^{J} \int_0^1 \frac{\omega_j Y_t}{w_t(1+\lambda_c)} di \\
\frac{w_t}{Y_t} &= \sum_{j=1}^{J} \omega_j \int_0^1 \frac{1}{(1+\lambda_c)} di \\
\frac{w_t}{Y_t} &= \sum_{j=1}^{J} \omega_j \sum_{c=1}^{C} \frac{\mu_{cj}}{(1+\lambda_c)} \qquad \text{(B.76)}
\end{aligned}$$

which delivers the time-invariant relative wage rate.

The level of output Y_t at time t is given by:

$$\begin{aligned}
\ln Y_t &= \sum_{j=1}^{J} \omega_j \left(\int_0^1 \ln y_{ijt} di \right) \\
&= \sum_{j=1}^{J} \omega_j \left(\int_0^1 \ln \left(\frac{\omega_j Y_t q_{ijt}}{w_t(1+\lambda_c)} \right) di \right) \\
&= -\ln \frac{w_t}{Y_t} + \sum_{j=1}^{J} \omega_j \ln(\omega_j) + \sum_{j=1}^{J} \omega_j \left(\int_0^1 \ln \left(\frac{q_{ijt}}{(1+\lambda_c)} \right) di \right) \\
&= -\ln \frac{w_t}{Y_t} + \sum_{j=1}^{J} \omega_j \ln(\omega_j) - \sum_{j=1}^{J} \omega_j \sum_{c=1}^{C} \mu_{cj} \ln(1+\lambda_c) + \sum_{j=1}^{J} \omega_j \left(\int_0^1 \ln q_{ijt} di \right) \qquad \text{(B.77)}
\end{aligned}$$

where the last term is the log productivity level of the economy at time t, i.e., the weighted sum of the log productivity level in each industry j, where the weights are the Cobb-Douglas shares ω_j. In our counterfactual experiments, we shall hold the initial log productivity level at time $t = 0$ fixed across economies. Without loss of generality, it is normalized to zero.[11]

Let L_{cj} denote the normalized per product line expected litigation cost for product lines owned by firms in industry j with technology class c:

$$L_{cj} = d_j p_{cj}^{plain} \kappa_1 \left(\int_0^{\left(-\iota+\iota\frac{\tau_1^h+\tau_1^l}{2}\right)(-v_{cj})} \gamma d\Gamma(\gamma) \right) + \sum_{j'=1}^{J} \left(p_{cj'}^{rent} \int_0^{\frac{(1-\tau_2^l)^2 \iota v_{cj'}}{4(\tau_2^h-\tau_2^l)}} \gamma d\Gamma(\gamma) \right) \qquad \text{(B.78)}$$

Then the aggregate litigation spending in the whole economy is calculated as

$$\sum_{j=1}^{J} \sum_{c=1}^{C} \mu_{cj} L_{cj} Y_t \qquad \text{(B.79)}$$

From the goods market clearing, we can compute the time-invariant consumption to

[11] This is equivalent to setting all $q_{ij0} = 1$.

output ratio C_t/Y_t as follows:

$$\begin{aligned}
Y_t &= C_t + \sum_{j=1}^{J}\sum_{c=1}^{C}\mu_{cj}\frac{\chi_c x_{cj}^{\psi} Y_t}{1+\sigma M_c} + \nu z^{\psi} Y_t + \sum_{j=1}^{J}\sum_{c=1}^{C}\mu_{cj}L_{cj}Y_t \\
1 &= \frac{C_t}{Y_t} + \sum_{j=1}^{J}\sum_{c=1}^{C}\mu_{cj}\frac{\chi_c x_{cj}^{\psi}}{1+\sigma M_c} + \nu z^{\psi} + \sum_{j=1}^{J}\sum_{c=1}^{C}\mu_{cj}L_{cj} \\
\frac{C_t}{Y_t} &= 1 - \sum_{j=1}^{J}\sum_{c=1}^{C}\mu_{cj}\frac{\chi_c x_{cj}^{\psi}}{1+\sigma M_c} - \nu z^{\psi} - \sum_{j=1}^{J}\sum_{c=1}^{C}\mu_{cj}L_{cj} \qquad \text{(B.80)}
\end{aligned}$$

where the second and third terms are the total incumbent and entrant R&D spending to output ratios, respectively, and the last term is the aggregate litigation spending to output ratio. Then, the initial output level is simply $C_0 = Y_0(C_0/Y_0)$.

We are now ready to compute social welfare in a BGP equilibrium. From the utility function of the representative household in equation (7), we have:

$$W = \int_0^{\infty} e^{-\rho t}\ln C_t dt = \int_0^{\infty} e^{-\rho t}\ln(e^{gt}C_0)dt = \frac{\ln C_0}{\rho} + \frac{g}{\rho^2} \qquad \text{(B.81)}$$

which shows how the welfare depends on the initial level of consumption C_0 and the growth rate of the economy g.

For two economies A and B, we can define a consumption equivalent welfare change measure (ϖ) which corresponds to the percentage increase in lifetime consumption that an agent in economy A would need to be indifferent between being in economy A or B:

$$W_B = \frac{\ln(C_0^A(1+\varpi))}{\rho} + \frac{g^A}{\rho^2} \qquad \text{(B.82)}$$

Solving for ϖ, we get:

$$\varpi = \exp\left(\left(W_B - \frac{g^A}{\rho^2}\right)\rho - \ln(C_0^A)\right) - 1 \qquad \text{(B.83)}$$