The redistributive effects of bank capital regulation

Elena Carletti[*] Robert Marquez Silvio Petriconi

May 29, 2017

Abstract

We build a general equilibrium model of banks' optimal capital structure, where investors are reluctant to invest in financial products other than deposits, and where bankruptcy is costly. We first show that banks raise both deposits and equity, and that investors are willing to provide equity only if adequately compensated. We then introduce (binding) capital requirements and show that: (i) it distorts investment away from productive projects toward storage; or (ii) it increases the cost of raising capital for banks, with the bulk of this cost accruing to depositors. These results hold also when we extend the model to incorporate various rationales justifying capital regulation.

Keywords: limited market participation, bank capital structure, capital regulation

JEL classifications: G18, G2, G21

[*]We thank Paolo Fulghieri for helpful comments and suggestions. We also thank seminar participants at the Bank of Canada, as well as at the 2016 Baffi CAREFIN Annual International Banking Conference.
Address for correspondence: Carletti: Boconi University and CEPR, Via Röntgen 1, 20136 Milan, Italy, elena.carletti@unibocconi.it. Marquez: University of California, Davis. 3410 Gallagher Hall. One Shields Avenue. Davis, CA 95616, USA, rsmarquez@ucdavis.edu. Petriconi: Bocconi University, IGIER and BIDSA, Università Bocconi, Via Röntgen 1, 20136 Milan, Italy, silvio.petriconi@unibocconi.it.

1 Introduction

The regulation of financial institutions, and banks in particular, has been at the forefront of the policy debate for a number of years. Much of the concern over financial institutions relates to the perceived negative consequences associated with a bank's failure, and with how losses may be distributed across various sectors, such as borrowers (either corporate or individual) and creditors, including depositors and the government, with the ultimate bearers of the losses being households and shareholders.

A primary tool for bank regulation is the imposition of minimum capital standards, which amount to requirements that banks limit their leverage and issue at least a minimal amount of equity. Capital requirements thus act as a portfolio restriction on the liability side of banks' balance sheets,[1] and have two primary roles. First, by creating a junior security held by the ultimate shareholders of the bank, capital (i.e., equity) stands as a first line of defense against losses, with shareholders absorbing losses before those accrue to other bank claimants, thus triggering distress and ultimately bankruptcy. Second, by forcing shareholders to have "skin in the game", capital is seen as a way to control moral hazard or asset substitution problems that may otherwise arise as a result of investment decisions by levered banks. In fact, recent calls among regulators, policy makers, and even academics (see, e.g., Admati, DeMarzo, Hellwig, and Pfleiderer (2013) for a discussion of this issue) have been for banks to dramatically increase the amount of capital they issue as way of reducing risk and ultimately increasing social welfare.

What is less understood in the discussion related to bank capital requirements is who bears the costs, to the extent that there are any, associated with requiring banks to increase their capitalization beyond their preferred levels. If banks', or bankers', primary goal is to maximize profits, and capital structure is chosen taking this objective into account, then the imposition of a leverage constraint, or any other restriction on banking activities, should lead

[1] Of course, the imposition of *risk weighted* capital requirements serve to place portfolio restrictions on the asset side as well, in addition to constraining how banks must finance themselves.

to a reduction in bank profitability and consequently in the return available to bank claimants. The regulatory perspective, of course, is that the possible loss in profitability associated with such constraints is more than compensated by increases in social welfare through other channels, such as a reduced social burden associated with government guarantees or the internalization of externalities that might arise when a bank fails. Nevertheless, at the bank level, the reduction in bank profitability should fall on the shoulders of its various claimants, such as depositors and shareholders, and there has been little study of which party bears the brunt of this burden. Do shareholders, as is commonly argued, primarily bear the costs associated with regulation, or are depositors and other creditors the parties more affected by tighter regulatory requirements? This is an important consideration for understanding the incidence of regulation. Moreover, if the aim of (capital) regulation is to protect specific agents, such as retail depositors and by extension the deposit insurance fund, an important question is whether capital regulation is likely to achieve this goal.

To tackle these questions, we present a general equilibrium model of banks' optimal capital structures where all parties are risk neutral, but investors are reluctant to participate in financial markets, preferring to invest only in safe assets, such as deposits. This key friction, which is well documented in the literature on household finance (see, e.g., Guiso and Sodini (2013)), implies that investors must be induced through additional compensation to hold anything other than a bank deposit. In particular, investors (or households, more generally) will be unwilling to become equity holders unless they receive compensation sufficient to overcome their reluctance.

Specifically, in our model banks exist to channel funds from investors, who have limited outside options for storing their savings, into productive but risky investments. Investors are naturally disinclined to invest in anything other than in storage or in "safe" bank deposits, but can be induced to hold equity if they view the equilibrium return to equity investment as being sufficiently high relative to that of holding a deposit. As investors are otherwise risk neutral, however, there is no premium for holding risk and thus effects related purely to leverage changes (as in Modigliani and Miller (1958)) are shut down.

Banks can finance themselves with either debt or equity, but using too much debt exposes them to default and, hence, to bankruptcy risk. Given that bankruptcy is costly, and that banks ultimately are owned by shareholders and thus try to maximize shareholder value, they endogenously limit the amount of debt financing to reduce expected bankruptcy costs. However, raising equity capital is difficult because investors face costs in becoming equity holders and thus need to be compensated, with greater compensation demanded the larger amount of equity capital the bank wants to raise. In other words, investors' participation in financial markets as suppliers of bank capital (i.e., equity) is endogenous, and will depend on the difference in the equilibrium returns of deposits versus equity. The trade-off between these two forces leads to an optimal capital structure, with banks always finding it optimal to raise some amount of debt financing (i.e., deposits), with the exact amount depending on how profitable their investment projects are, and how variable are their returns.

Importantly, we characterize the equilibrium return to equity holders as well as to depositors. We show that, while the marginal capital supplier is indifferent between being an equity holder or a depositor, inframarginal equity holders earn a strictly positive rent as a result of investing in a bank's equity. The bank therefore creates value for investors by channeling funds from storage into real investment projects, allowing investors to earn a return that more than compensates them for the disutility they associate with being an equity market investor (i.e., their unwillingness to participate in financial markets). When the expected return to investment projects is sufficiently high, all available funds are put to productive use, with no funds going into storage. In this scenario, even depositors earn a premium in that their expected return is strictly higher than what they would earn in storage.

Despite the friction of (endogenous) limited market participation, the individually optimal capital structure decisions for banks are socially efficient, and the market solution yields no distortions that can be directly improved with capital regulation. This occurs because banks are competitive in our model, and all profits ultimately accrue to the banks' shareholders, who are the residual claimants. The model thus exhibits an efficient benchmark solution that provides us with an ideal laboratory in which to study how the distortion (on bank profits and, thus, social output) affects the claimants of the bank, namely depositors and shareholders who are, ultimately, represented by the households who are making portfolio decisions on how to allocate their savings.[2]

We identify two main sources of inefficiency associated with (binding) minimum capital requirements. One is that, when project returns are relatively low and not all funds are being invested in productive projects but are rather being held as storage, requiring banks to hold even greater amounts of capital reduces further the number of projects that are funded. This, in turn, reduces aggregate surplus since investment projects yield a higher surplus than investing in storage, so that the capital requirement introduces a distortion away from productive investments toward storage.

The second inefficiency arises at the other extreme, once all investment funds are being allocated to productive projects. Here, an increased requirement to hold capital raises the overall cost borne by investors. The reason is that satisfying the capital requirement necessitates that a larger number of investors be induced to become capital suppliers, and thus bear the disutility associated with holding equity. Much of this increased cost is ultimately borne by those investors who remain as depositors, since their equilibrium return goes down and they earn a lower return on their deposits relative to what they could get in the absence of a capital requirement. In other words, much of the incidence of the increased costs associated with satisfying the capital requirement falls on those investors who are least willing to be financial market participants, rather than on investors who, by holding equity, are the

[2] As described below, we later introduce three frictions that can be at least partly resolved through capital requirements, so that capital regulation plays a role in solving a social problem. We show that even in that context, where capital regulation can increase social welfare, the incidence of the regulation falls differentially on different classes of investors. In particular, it is not the households that are most inclined to be equity investors that bear the brunt of the regulatory burden.

residual claimants of the banks.

We then extend our results to consider various market failures that may justify a need to introduce minimum capital requirements. Specifically, we study three cases that represent three different sources of inefficiency. We start by considering a standard limited-liability induced risk shifting story where capital reduces the moral hazard problem for banks (see, e.g., Holmstrom and Tirole, 1997, for a classic treatment). In such a context, we first show that capital adequacy standards can increase social welfare. We then show that the primary beneficiaries are not depositors but rather those investors who would have been inclined to hold equity anyway.

We then examine externalities that arise as a result of "fire sales" when a large number of banks fail, thus depressing recovery values on bank assets. In that setting, recovery values are lower the more banks are unable to meet their repayment obligations, so that a bank's failure has a negative externality on other banks, which ultimately is reflected in decreased overall value. By reducing the overall number of banks, capital regulation helps solve the inefficiency as it reduces the externality. Nevertheless, the impact of regulation is felt most strongly by households that would normally choose to be depositors rather than shareholders, as above. Finally, we study the implications of having deposit insurance. We first show that deposit insurance entails a trade-off. On the one hand, by reducing the interest rate that needs to be promised to depositors, deposit insurance can reduce the threshold for bankruptcy and thus increase social welfare, all things equal. On the other hand, it introduces a distortion in banks' capital structure decisions since it leads banks to leverage as much as possible, using no equity and overly exposing themselves to bankruptcy risk. We then show that capital regulation can again help rectify the distortion and achieve greater surplus, but the primary benefit of the regulation accrues to shareholders rather than to households that hold deposits.

Our work is related to various strands of literature. The first relates to the role of capital at financial institutions and, more specifically, to the incentives institutions have to use leverage but also finance themselves with equity capital. Allen, Carletti, and Marquez (2015) show that bank capital earns a higher expected return in equilibrium than deposits when banks' funding markets are exogenously segmented. In contrast, we study the endogenous degree of participation of investors in bank equity markets, and outline the consequences for the imposition of binding capital requirements. Bertsch and Mariathasan (2015) develop a model of endogenous bank equity prices during large-scale recapitalizations. A sizable literature has also studied the role of bank capital as a buffer that protects from bankruptcy, or confines moral hazard (e.g. Holmström and Tirole (1997), Dell'Ariccia and Marquez (2006), Allen, Carletti, and Marquez (2011), Hellmann, Murdock, and Stiglitz (2000)). Our basic model incorporates the first effect – buffer stock view of bank capital – and we explore the second in an extension, with the focus on understanding who primarily bears the costs associated with capital regulation.

The second strand of literature relates to the well-documented limited degree of participation in financial markets by households. Taking households' unwillingness to participate as given, an important body of literature has focused on the implications of limited participation on the equilibrium pricing of assets. For instance, Allen and Gale (1994) study how limited market participation by potential investors can lead to amplified volatility of asset prices relative to the case where there is complete participation. Vissing-Jørgensen (2002) uses limited market participation by households to help explain part of the equity premium puzzle in financial economics, which argues that the observed differences between average returns in equity markets and debt markets are difficult to explain through risk aversion alone. In our approach, we assume that investors are heterogeneous in their willingness to participate, with some being very reluctant and others more willing. The equilibrium level of participation is then endogenous, and is pinned down at the same time as are equilibrium returns to all investors. To the best of our knowledge, we are the first paper to focus on the capital structure implications for either financial or non-financial firms of investors' reluctance to hold risky assets.

The paper proceeds as follows. The next section lays out the model. Section 3 contains the main analysis of the model, and the characterization of equilibrium. Section 4 looks at social welfare, whereas Section 5 studies the effects of a binding capital requirement. Section 6 extends the baseline model to include various settings, where there is a social efficiency which capital regulation can help address. Finally, Section 7 concludes.

2 The Model

We develop a simple one period ($T = 0, 1$) model of financial intermediation with banks and investors that can provide funds in the form of equity capital or deposits. There exist two investment options: one is a safe *storage* technology which yields in $t = 1$ a return of one on every unit of funds invested at $t = 0$; the other is a *risky* investment which, for every unit of funds invested at $t = 0$, yields in $t = 1$ a risky return of r uniformly distributed in $[R - S, R]$, with $0 < S \leq R$ and $R > 2$. These conditions ensure that the expected return of the risky technology is at least as high as investing in the storage technology, that is $E[r] = \mu \equiv \frac{2R-S}{2} \geq 1$.

There is a continuum of mass M of risk-neutral investors, endowed with one unit of wealth each. Investors may either invest directly in the storage technology, or they can place their wealth in a bank, either as depositors or as equity holders.[3] However, investors are reluctant to hold anything other than a bank deposit, so that they have to be induced to hold risky, more junior securities such as bank equity. Specifically, each investor i incurs a non-pecuniary cost c_i for becoming *sophisticated* and being willing to hold bank equity. This cost may stand for the cost of acquiring the necessary skills to trade in equity markets or for heterogeneous taste for safe assets in the investor population, and a consequent disutility

[3] We obtain similar results if we also allow investors to invest directly in the risky technology, earning r per dollar they invest. Therefore, for ease of exposition we abstract from this possibility.

associated with holding junior, leveraged claims. The cost c_i, which is i.i.d. across investors and known to each investor i, is drawn from a uniform distribution on the support $[0, C]$. As a result, investors will only be willing to hold bank equity capital if their expected return from doing so exceeds their return from investing in either storage or a bank deposit by at least the cost c_i.

Banks are primarily vehicles that provide investors with access to the risky technology. Each bank finances itself with an amount of capital k and an amount of (uninsured) deposits $1-k$, and invests in the risky technology.[4] This implies that, by becoming shareholders in a bank, investors take de facto a position in the risky technology. We denote the promised per unit deposit rate as r_D, and the equilibrium expected return to bank deposits and to bank capital as u and ρ, respectively.

Banks are be subject to bankruptcy if they are unable to repay their debt obligations. This occurs when $r < (1-k)r_D$, that is when the realized return from the risky technology is lower than the total promised repayment to depositors. Bankruptcy is assumed to be costly. For simplicity, we make the extreme assumption that in the event of bankruptcy, all the project's return is dissipated; that is, that the entire return r is dissipated whenever $r < (1-k)r_D$. Finally, we assume that the banking sector in our model is perfectly competitive. Free entry reduces profits to zero, and banks behave as price takers with respect to the equilibrium expected return on bank capital ρ and bank deposits u.

3 Optimal capital structure

The equilibrium of the model is pinned down by the following conditions:

1. Investors optimally decide whether to become sophisticated, and invest to maximize their expected wealth;

[4] Given there are constant returns to scale, normalizing the size of every bank to 1 is without loss of generality.

2. Banks choose capital k and a promised deposit rate r_D to maximize expected profits;

3. The market for bank deposits clears;

4. The market for bank equity clears;

5. Free entry reduces bank profits to zero in equilibrium.

We start by analyzing investors' optimal investment strategy. At time $t = 0$, each investor i decides whether to become sophisticated based on how the difference in returns between equity and deposits, $\rho - u$, compares to the relevant cost c_i. Whenever $\rho - u < C$, there exists a marginal investor that is indifferent between earning the return u on deposits and earning the return ρ on bank capital minus his cost c_i. The marginal investor's cost must therefore satisfy $\hat{c} = \rho - u$. For a given spread $\rho - u$, there is then a mass of investors K_{supply} that choose to become sophisticated and provide equity to the bank, where

$$K_{supply} = \begin{cases} M\frac{\rho - u}{C} & \text{if } 0 \leq \rho - u < C \\ M & \text{if } \rho - u \geq C. \end{cases}$$

The remaining mass of investors, $D_{supply} = M - K_{supply}$, find it optimal to either deposit their funds at a bank, or invest in storage.

Turning to the banks' problem, each bank chooses its capital k and its promised deposit rate r_D to solve the following problem:

$$\max_{k, r_D} E\left[\Pi_B\right] = \frac{1}{S} \int_{\max\{r_D(1-k), R-S\}}^{R} (r - (r_D(1-k))dr - \rho k \tag{1}$$

subject to

$$E\left[U_D\right] = \frac{1}{S} \int_{\max\{r_D(1-k), R-S\}}^{R} r_D \, dr \geq u \tag{2}$$

$$E\left[\Pi_B\right] \geq 0 \tag{3}$$

$$0 \leq k \leq 1. \tag{4}$$

Expression (1) represents bank expected profit. The first term captures the bank's return from investing in the risky technology, net of the payment $r_D(1-k)$ to depositors. Such a return is positive only when the bank does not go bankrupt, that is for $r \geq r_D(1-k)$ if $r_D(1-k) > R-S$ and for any r if $r_D(1-k) < R-S$. The second term ρk reflects the return to shareholders for providing bank capital. Constraint (2) captures depositors' participation constraint. It requires that the payoff depositors receive when the bank remains solvent, that is for $r \geq \max\{r_D(1-k), R-S\}$, is at least equal to their opportunity cost u. Constraints (3) and (4) ensure that the bank is active and that the chosen capital structure lies within the feasible range. Given that there is free entry into the banking market, each bank's expected profit must be zero. This implies that the return to shareholders ρ adjusts so that $E[\Pi_B] = 0$.

We can now characterize the equilibrium.

Proposition 1. *The unique equilibrium is as follows, where* $\hat{u} \equiv \frac{R^2}{4S}\left(1+\sqrt{\frac{R^2}{R^2+8CS}}\right)$, u^* *is the unique positive solution of* $\frac{2R-S}{2} - u^* - C\left(1-\frac{R-S}{u^*}\right)^2 = 0$, *and* N *is the number of active banks:*

i) For $\hat{u} \leq 1 < R/2$, *banks choose a risky capital structure with* $k = \frac{4S}{R^2} - 1$, $r_D = \frac{R}{2}$, $\rho = \frac{2S}{4S-R^2}$, $u = 1$, $E[\Pi_B] = 0$, $E[U_D] = 1$, $N = MR^2\frac{R^2-2S}{C(4S-R^2)^2}$, $K = Nk$, *and* $D = N(1-k) < M-K$.

ii) For $1 < \hat{u} < R/2$, *banks choose a risky capital structure with* $k = \frac{R}{\sqrt{R^2+8CS}}$, $r_D = \frac{R}{2}$, $\rho = \frac{R}{4S}\left(R + \frac{R^2+4CS}{\sqrt{R^2+8CS}}\right)$, $u = \hat{u} > 1$, $E[\Pi_B] = 0$, $E[U_D] = \hat{u}, N = M, K = Nk$, *and* $D = N(1-k) = M-K$.

iii) For $\hat{u} \geq R/2$, *banks choose a safe capital structure with* $k = 1 - \frac{R-S}{u^*}$, $r_D = u^*$, $\rho = \frac{Su^*}{2(u^*-R+S)}$, $u = u^*$, $E[\Pi_B] = 0$, $E[U_D] = u^*, N = M$, $K = Nk, D = N(1-k) = M-K$.

In all three cases, $\rho > E[r] > u$ *holds.*

Proof. See appendix. □

The proposition shows, first, that banks always find it optimal to raise some deposit financing, with the exact amount depending on the profitability of the risky technology and the variance of returns as captured by the variable $\hat{u}$, which represents a measure of the trade-offs between maximal project return, R, dispersion of returns, S, and investor disutility associated with being an equity holder, C.

Second, although banks are always leveraged, they do not always choose a capital structure that exposes them to bankruptcy risk. Depending again on the tradeoff between the project's expected returns and the dispersion in those returns, they may choose a safe capital structure characterized by very low debt and no bankruptcy risk. In regions i) and ii) where $\hat{u} < R/2$, banks choose to raise enough deposits that they risk bankruptcy with some strictly positive probability. The reason is that in these regions the project is highly risky and not very profitable. Given this, using leverage increases the return to shareholders since deposits earn a lower equilibrium return, allowing a greater premium to be paid to investors willing to become equityholders. As project returns increase, using more capital becomes progressively costly since it requires the bank to induce more unwilling investors to become equity holders. The bank then chooses to reduce the amount of capital and instead increase leverage, even if this means incurring some bankruptcy risk as a result. As project return increases further, bankruptcy becomes increasingly more costly because it destroys higher project returns. At some point when $\hat{u}$ reaches $R/2$, losses under bankruptcy become larger than the participation cost c_i, and banks find it optimal to employ relatively little leverage so to avoid bankruptcy risk. In other words, when project returns are high enough and return dispersion is low (i.e., with either a high R or a low S), banks optimally choose a safe capital structure that has no risk of bankruptcy, and hence no expected bankruptcy costs. This choice is mantained in the whole region where $\hat{u} \geq R/2$. As project returns increase further, banks find it optimal to keep increasing their level of capital so to avoid bankruptcy.

Third, the proposition shows that, irrespective of the specific capital structure banks choose, the equilibrium return for capital suppliers is always higher than the project expected return and the return for depositors, that is $\rho > E[r] > u$. The reason is that deposits earns a lower return given the alternative investment for investors to deposit is storage. This in turn allows for greater surplus to be distributed to bank equity holders.

Fourth, the proposition establishes a boundary, represented by the variable $\hat{u}$, characterizing the circumstances when all funds from the mass M of investors are invested in the banking sector (i.e., $N = M$) and when instead some of them are invested in storage (i.e., $N < M$). We refer to the former case as "full inclusion" and to the latter case as "partial inclusion" in financial markets. Storage will be used when the return for depositors is the same as the return from storage, i.e., $u = 1$. By contrast, when $u > 1$, all funds are invested in the banking sector. This latter case emerges when the project's expected return is sufficiently high, so that enough compensation for participation can be offered to both capital suppliers ($\rho > E[r]$) and depositors ($u > 1$).

Given the main features of the equilibrium, we can now pin down the expressions of the equilibrium bankruptcy probability, denoted p_B, as follows:

Corollary 1.1. *The equilibrium probability of bankruptcy is given by*

$$p_B = \begin{cases} 1 - \frac{2}{R} & \text{if } \hat{u} \leq 1 \\ 1 - \frac{R}{2S} - \frac{R^2}{2S\sqrt{8CS+R^2}} & \text{if } R/2 > \hat{u} > 1 \\ 0 & \text{if } \hat{u} \geq R/2 \end{cases}$$

In the regions where $\hat{u} < R/2$ and risky capital structure, the probability of bankruptcy increases in R if $\hat{u} \leq 1$ while decrease in R otherwise.

Proof. Because bankruptcy cost are 100%, we must have $u = (1 - p_B)r_D$ from which the statement follows immediately. □

The corollary establishes that when banks choose a risky capital structure, bankruptcy risk changes with the model parameters and in particular it decreases with the project;s return when $N = M$ and $\hat{u} > 1$. This result, which arises from the endogenous supply of funds by investors, contrasts with Allen, Carletti and Marquez (2015), where both capital and the bankruptcy probability become fixed once all funds are invested in the banking sector since they assume markets are segmented and households cannot move from one investment into the other at any cost.

Although the result that banks may choose a safe capital structure is interesting per se, in what follows we will concentrate the analysis on the regions with positive bankruptcy risk. This case is indeed more empirically relevant and plausible. Moreover, it is a more sensible scenario for studying the implications of the model in terms of social efficiency and the effects of capital regulation.

Given this, we next conduct some comparative statics exercise to study how the equilibrium values of k, ρ and u described in parts i) and ii) of Proposition 1 change as a function of the various underlying parameters, in particular project expected return and variance.

Corollary 1.2. *The following comparative statics results hold:*

i) For given variance of the project returns $var(r)$, an increase in the expected project return $E[r]$ decreases k and increases ρ for $\hat{u} \leq 1$, while it increases k, ρ and u for $\hat{u} > 1$.

ii) For given $E[r]$, an increase in $var(r)$ increases k and decreases ρ for $\hat{u} \leq 1$, while it reduces k, ρ and u for $\hat{u} > 1$.

iii) An increase in the average participation cost of becoming equity holder $E[c_i]$ leaves k and ρ unchanged and reduces the number of banks N for $\hat{u} \leq 1$, while it decreases k and u, increases ρ and leaves N unchanged for $\hat{u} > 1$.

Part i) of the corollary shows that capital is not monotonic in the expected project return, being counter-cyclical in recessions (as defined by $\hat{u} \leq 1$) and pro-cyclical in boom phases (i.e., for $\hat{u} \leq 1$). The corollary also shows that as $E[r]$ shareholders always benefit through an increased compensation ρ, while depositors do so through an increased u only in the case of full inclusion. When $\hat{u} \leq 1$ so that $N < M$ and storage is in use, an increase in $E[r]$ creates a greater wedge between the return of the risky projects and that of storage. This leads to greater demand for funds by banks, inducing more investors to place funds in bank deposits. In aggregate, there is an increase in the number of banks N and in the aggregate amount of capital K employed in the banking sector, thus pushing up the equity return ρ. The amount of capital employed by each individual bank, k, decreases in this region because at the margin, the only source of funding for new banks to form (i.e., for N to increase) is for investors to move away from storage to bank deposits. This switch is costless, whereas inducing more investors to become equity suppliers is increasingly costly as it requires investors with ever increasing costs of sophistication, c_i, to participate.

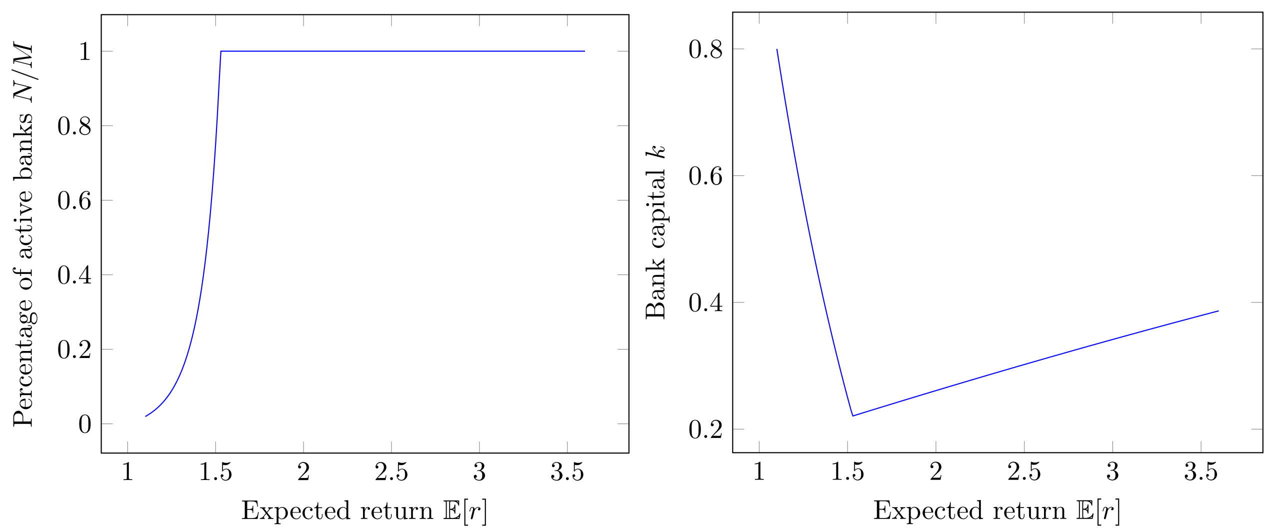

Figure 1: Equilibrium number of banks N and level of bank capital k as a function of the expected return $\mathbb{E}[r]$ of the risky technology. The lowest capitalization is attained exactly at the moment of full inclusion. Parametrization: $C = 8, S = 1.8$

Once $\hat{u} > 1$ and $N = M$ (equivalently, when $u > 1$), all funds are in use in the banking sector and it is thus no longer possible to allocate more funds to productive projects. For a given bankruptcy probability, a higher $E[r]$ increases the expected bankruptcy losses since project returns are greater. As a result, banks find it optimal to reduce their leverage and protect themselves through higher equity capital. This reduces bankruptcy risk and lead to an increase in the equilibrium return to capital suppliers, ρ, as well as in the aggregate amount of capital being used by the banking sector, K.

Part ii) of the corollary states that k, ρ and u react differently from part i) when a mean preserving spread (MPS) increases the variance of the project returns without affecting the average $E[r]$. Capital increases now in the region where $\hat{u} \leq 1$ and decreases when $\hat{u} > 1$, while ρ decreases throughout as well as u when $\hat{u} \leq 1$. With partial inclusion (i.e., $N < M$) and $u = 1$, a MPS increases the probability of bankruptcy, which, ceteris paribus, reduces the expected payoff to bank depositors. As a result, banks increase capital to reduce bankruptcy risk and retain depositors. This leads to fewer banks and less aggregate capital. A similar logic applies to the case with full inclusion (i.e., $N = M$) and $u > 1$ but now banks can offset part of the increased bankruptcy risk by reducing the equilibrium return to depositors, without risking that depositors move back into storage. Banks find it optimal to reduce capital for the following reason: an incremental unit of capital reduces bankruptcy risk and increases bank return by a factor of $\frac{1}{S}$, which corresponds to the density of the distribution of project returns around the default boundary (with a uniform distribution, this density is the same everywhere).[5] As MPS increases S, it reduces the marginal value of holding capital for the bank, thus inducing them to reduce its use. Thus, k falls with a MPS, as do ρ and u, the equilibrium returns to capital suppliers and depositors, respectively.

Finally, part iii) of the corollary describes the effects of an increase in the average participation cost for investors.

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[5] This can be seen directly from the maximization problem for the bank: $\max_{r_D,k} \frac{1}{S} \int_{r_D(1-k)}^{R} (r - r_D (1-k))\, dr - pk$, subject to depositors' participation constraint, $\frac{1}{S} \int_{r_D(1-k)}^{R} r_D dr = u$. Substituting the participation constraint into the bank's maximization problem yields

$$\max_k \frac{1}{S} \int_{r_D(1-k)}^{R} r dr - (1-k)\, u - pk.$$

The first order condition is $\frac{1}{S} r_D^2 (1-k) - (p - u) = 0$, or $k = 1 - \frac{S(p-u)}{r_D^2}$, which is decreasing in S for given ρ, u, and r_D, meaning that the marginal incentive to hold capital is lower.

4 Social efficiency

We now turn to the question how the banks' capital structure, which is determined as part of the market equilibrium, compares to a socially efficient benchmark. To this end, we consider the case where a social planner chooses capital k_P to maximize social welfare (i.e., investors' aggregate returns net of aggregate participation costs to become capital supplier), while deposit rates are still set by the banks in order to maximize their expected profits. This means that the planner solves

$$\max_{k^{FB}} SW = \rho K + uD - \int_0^{\rho-u} M \frac{c}{C}\, dc \tag{5}$$

subject to

$$r_D = \arg\max_{r_D} \frac{1}{S} \int_{r_D(1-k^{FB})}^{R} (r - (r_D(1-k^{FB}))dr - \rho k^{FB} \tag{6}$$

$$E\left[U_D\right] = \frac{1}{S} \int_{r_D(1-k^{FB})}^{R} r_D dr \geq u \tag{7}$$

$$E\left[\Pi_B\right] \geq 0 \tag{8}$$

$$0 \leq k^{FB} \leq 1 \tag{9}$$

The constraints are as in the market solution except constraint (6), which indicates that the deposit rate r_D is chosen by shareholders to maximize bank profits. We can now state the following result:

Proposition 2. *The socially optimal allocation of bank capital coincides with the competitive capital structure:* $k^{FB} = k^*$.

Proof. See appendix. □

Although banks behave as price takers in the competitive equilibrium and do not individually consider the impact of their capital structure choices on the equilibrium rate of return on capital, they end up holding a socially optimal level of bank capital. The reason is that there is no pecuniary externality in the bank equity market in our model. Banks maximize profits to the benefit of bank shareholders and, given the market for capital is competitive, they ultimately internalize investors' costs to become sophisticated and willing to hold bank equity. The first welfare theorem thus applies, with the market equilibrium being Pareto optimal and in fact equivalent to the socially optimal solution.

5 The incidence of binding capital requirements

Since, as demonstrated above, the competitive level of capital in our baseline model is efficient, any requirement to hold capital $k^{reg} > k^*$, where k^* denote the market solution found previously, must necessarily reduce aggregate surplus. This is simply because there are no externalities in the model, and the competitive banking sector fully internalizes the (social) cost borne by investors in overcoming their aversion to holding bank equity. The model therefore provides an efficient benchmark where binding regulation is distortionary and represents a cost that reduces total output. More importantly, the model can be used to understand who primarily bears the burden of regulation, without any offsetting benefit that can be used to generate transfers to the affected parties. In the next section, we will study cases where social inefficiencies create a rationale for regulation, and highlight the need to perhaps incorporate transfers into any regulatory rules that might be adopted.

To study the issue of the incidence of regulation, and specifically of binding capital requirements, note first that any capital requirement imposing $k^{reg} \leq k^*$ will not be binding since banks will prefer to choose k^* over the regulatory minimum. Therefore, we restrict our analysis to cases where $k^{reg} > k^*$ and we consider separately the case with partial inclusion, where the equilibrium return to depositors is the same as that in storage ($u = 1$), and that with full inclusion, where depositors earn a premium relative to storage ($u > 1$).

Proposition 3. *For both cases below, suppose that $k^{reg} > k^*$, and define K^* as the aggregate amount of capital in the market solution: $K^* = N^* k^*$.*

1. *Assume* $N^{reg} < M$*, with* $u^{reg} = 1$*. Then* $\frac{d\rho^{reg}}{dk^{reg}} < 0$ *and* $\frac{dN^{reg}}{dk^{reg}} < 0$*. This also implies that* $K^{reg} < K^*$*, where* K^{reg} *is the aggregate level of capital used in the regulatory solution:* $K^{reg} = N^{reg}k^{reg}$*.*

2. *Assume* $N^{reg} = M$*, with* $u^{reg} > 1$*. Then,* $\frac{du^{reg}}{dk^{reg}} < \frac{d\rho^{reg}}{dk^{reg}} < 0$*. This then implies that* $K^{reg} > K^*$*.*

Proof: See appendix.

The proposition establishes that binding capital requirements either lead to fewer banks operating, $N^{reg} < N^*$, or to the burden being borne disproportionately by depositors. The first result, which arises when $N^{reg} < M$, so that some investors use storage and not all funds are allocated to the banking sector, gives rise to an inefficiency since productive projects are funded through banks. Therefore, reducing the total funds that flow to the banking sector, and thus reducing the number of banks in equilibrium, leads to less output being produced. In other words, the deadweight loss associated with capital requirements here is reflected in lower output being produced. Moreover, since $N^{reg} < N^*$, this implies that there is a region of parameter values R, S, and C such that for $N^{reg} < N^* = M$, depositors are made strictly worse off by binding capital requirements since in the market solution they would earn an expected return of $u^* > 1$, whereas they earn only a return equal to 1 when banks are subject to capital regulation.

The second result highlights how binding capital requirements affect the different classes of investors. Since the market solution also maximizes aggregate output, a binding capital requirement leads to less total output being produced. Since the number of projects that are financed remains constant with regulation (for local changes in the amount of capital around the market solution k^*), the deadweight loss arises from the increased participation costs borne by the additional investors that need to be induced to hold bank capital. While the equilibrium return to equity holders, ρ, decreases, the return to depositors, u, decreases *even* *more* because the difference between them, $\rho - u$, must increase in order for more investors to be willing to hold bank capital. Thus, while shareholders earn a lower return, reflecting the greater deadweight losses and lower aggregate output, the bulk of the losses are borne by depositors, who bear much of the burden of the increased capital requirement. It is therefore the investors who in principle should be protected most by larger capital requirements (as indeed they are since bankruptcy risk is reduced) who ultimately pay for this protection through more than commensurate reductions in the return they earn in equilibrium.

Finally, it is worth noting that while we focus on marginal changes in the capital requirement, larger changes may compound both effects. For instance, starting from a market solution with $N^* = M$, a large increase in a capital requirement could cause depositors' return u to hit its lower bound of $u = 1$. At that point, investors are indifferent between holding deposits and putting their savings into storage, so that further increases in the capital requirement would lead to less market participation, fewer projects being financed, and further reductions in total output.

6 Extensions

The baseline model we have so far developed has no inefficiencies, so that banks' capital structures decisions are socially efficient and there is thus no scope for capital regulation. Any binding capital requirement, therefore, leads to a social loss whose incidence must ultimately fall on households, either through lower equilibrium returns to investors, or through lower aggregate output since fewer productive investments get funded. Here, we extend the model to incorporate various market inefficiencies which can be, at least partly, resolved through capital regulation, thus providing a rationale for why capital requirements may exist and help increase social welfare.

Specifically, we study three canonical market failures associated with financial intermediaries: (1) moral hazard (or risk shifting) induced by limited liability, which leads banks to take excessive risk; (2) externalities in the recovery value of assets that may arise when many banks fail at once ("fire sales") and which may depress asset values; and (3) deposit insurance, which provides an implicit or explicit subsidy for raising deposits rather than equity, and tilts banks' capital structures toward being excessively levered. For all of these instances, capital regulation can improve social welfare. However, we show that the primary beneficiaries of such regulation are not depositors, but rather those households that would choose to be providers of capital (i.e., equity holders) anyway. Thus, much like before, capital regulation redistributes returns in favor of equity holders, and in fact allows these households to capture much of the welfare gain associated with capital regulation.

For simplicity, in what follows we assume that $S = R$, so that project returns are distributed uniformly in $[0, R]$. This simplifies calculations substantially, without affecting the main implications of the analysis related to the incidence of bank capital requirements.

6.1 Moral hazard induced by limited liability

Limited liability is often argued to induce moral hazard or risk shifting, and thus may create an inefficiency. To study such a setting, we modify the model slightly to allow the bank or, equivalently, bank shareholders to take a costly action a aimed at increasing project returns, but which is privately costly. Specifically, assume that by putting in effort a at time $t = \frac{1}{2}$, the bank can increase the project's return by $a > 0$, but bears a cost of $\frac{\eta}{2}a^2$. This action is taken after the bank has chosen its capital structure and financed its project (which occurs at $t = 0$), so that all variables of interest – u, ρ, r_D, and k – are taken as fixed when choosing a.

With this, we can write the bank's capital structure problem at $t = 0$ as

$$\max_{k,r_D} E\left[\Pi_B\right] = \frac{1}{R}\int_{\max\{r_D(1-k),a\}}^{R+a} (r - (r_D(1-k))dr - \rho k - \frac{\eta}{2}a^2, \tag{10}$$

subject to the same constraints as before, except that the depositor's participation constraint must be modified as follows:

$$E\left[U_D\right] = \frac{1}{R}\int_{\max\{r_D(1-k),a\}}^{R+a} r_D\, dr \geq u.$$

At $t=\frac{1}{2}$, the bank maximizes

$$\max_e E\left[\Pi_B\right] = \frac{1}{R}\int_{\max\{r_D(1-k),a\}}^{R+a} \left(r - r_D\left(1-k\right)\right) dr - \rho k - \frac{\eta}{2}a^2, \tag{11}$$

taking k, ρ, and r_D as given.

Assuming that $r_D(1-k) > a$,[6] the FOC for (11) is

$$\frac{1}{R}\left(R + a - r_D + k r_D - Ra\right) = 0,$$

which solving for a yields

$$a^* = \frac{R - r_D\left(1-k\right)}{R-1}.$$

Note that the optimal action a^* is increasing in k, the amount of capital at the bank. This is a result of the limited liability effect, that when the bank chooses its action a it takes the deposit rate r_D and the capital structure k as given. A central planner, by contrast, would want to maximize project returns, which would yield $a^{FB} = \frac{R}{R-1} > a^*$.

At time 0, the bank's maximization of (10) can be expressed as the solution to the first order condition $\frac{dE[\Pi_B]}{dk} = \frac{\partial E[\Pi_B]}{\partial k} + \frac{\partial E[\Pi_B]}{\partial r_D}\frac{dr_D}{dk} + \frac{\partial E[\Pi_B]}{\partial a}\frac{da}{dk} = \frac{\partial E[\Pi_B]}{\partial k} + \frac{\partial E[\Pi_B]}{\partial r_D}\frac{dr_D}{dk}$ since $\frac{\partial E[\Pi_B]}{\partial a} = 0$ from the envelope theorem.

Consider now the social planner's problem, which as usual is to maximize total surplus given by

$$\begin{aligned}\max_{k_P} SW &= N\frac{1}{R}\int_{\max\{r_D(1-k),a\}}^{R+a^*} r dr - \int_0^{\widehat{c}} M\frac{c}{C}\, dc + M - N - \frac{1}{2}\left(a^*\right)^2 \\ &= N\frac{1}{R}\int_{\max\{r_D(1-k),a\}}^{R+a^*} r dr - \frac{1}{2C}M\widehat{c}^2 + M - N - \frac{1}{2}\left(a^*\right)^2,\end{aligned}$$

[6] It is straightforward to see that, for any given parameters, there is always a value of η large enough such that the action a that will be chosen will be small relative to the amount of deposits $1-k$. For smaller values of η, the bank may choose an action high enough that default never occurs, i.e., so that $r_D(1-k) < a$. The result also extends to this case, but the derivation is slightly different.

where the social planner recognizes that the bank will choose $a^* = \frac{R - r_D(1-k_p)}{R-1}$ given the planner's choice of k_p. As for the bank, the first order condition to this problem yields $\frac{dSW}{dk} = \frac{\partial SW}{\partial k} + \frac{\partial SW}{\partial r_D}\frac{dr_D}{dk} + \frac{\partial SW}{\partial a}\frac{da}{dk}$. This expression will only be the same as for the banks if $\frac{\partial SW}{\partial a} = \frac{\partial E[\Pi_B]}{\partial a} = 0$, which does not hold since $\frac{\partial SW}{\partial a} > 0$ and the social planner desires a higher level of effort. This implies that the social planner prefers a higher level of capital k at each bank than in the market solution as a way of inducing greater effort. Capital regulation is thus socially valuable, and leads to binding capital requirements which increase SW.

To see who ultimately benefits from capital regulation, consider that, unlike in the case we have studied so far in Section 5 where binding capital requirements lead to a reduction of SW, here capital regulation leads to an increase in SW through a reduction in moral hazard. This increase must be allocated to either households who become depositors, or households who become shareholders. In other words, capital regulation is socially valuable. Compounding this, since the capital requirement that pushes the level of capital above the market solution leads to less leverage at each bank, bankruptcy costs at each bank will be lower, which creates a second reason why capital requirements increase aggregate output. Given this, a decrease in ρ, which would be equivalent to a decrease in K, the total capital used in the banking sector, would not be consistent with an increase in SW. Depositors cannot be getting the surplus since $1-k$ goes down at each bank and, for $N < M$, we have that $u = 1$. Therefore, shareholders must be capturing the increase in surplus, implying that the return to shareholders and, consequently, the total amount of capital must increase. In other words, it must be that $\rho^{reg} > \rho$ and $K^{reg} > K^*$ in order to be consistent with using regulation to maximize social welfare, and social welfare being higher with (optimally chosen) binding capital requirements.

6.2 Fire sale externalities

Suppose that under bankruptcy, triggered by the bank inability to repay its depositors, liquidation of the bank's asset yields a recovery value equal to a fraction $h < 1$ of the realized cash flow r. In other words, unlike in previous sections, losses under bankruptcy are equal for $(1-h)\,r$. To model the notion that the failure of many banks at once depresses asset prices for all banks that are being liquidated – a "fire sale" externality – we assume that h is decreasing in N, the number of active banks, or, equivalently, that the proportional losses $1-h$ are increasing in N.

Consider now a bank's problem of maximizing profits, which is still given by (1). The only change to the bank's problem stems from depositors' participation constraint, which now incorporates that depositors receive something in the event of bankruptcy, and is given by

$$E\left[U_D\right] = \frac{1}{R}\int_0^{r_D(1-k)} \frac{hr}{1-k}\,dr + \frac{1}{R}\int_{r_D(1-k)}^{R} r_D\,dr \geq u. \tag{12}$$

We can now state the following result.

Proposition 4. *When there are externalities due to possible fire sales, $N^{reg} \leq N^*$, with the inequality strict whenever $N^{reg} < M$; in other words, a social planner would prefer fewer banks to operate ($N^{reg} < N^*$), and for each bank to hold more capital ($k^{reg} > k^*$).*

The result above establishes that there is a social value to requiring that banks hold more capital than what they are inclined to do as a way of controlling the number of banks, and thus reducing the externalities associated with having a large number of banks all failing at once. Since $k^{reg} > k^*$ but $N^{reg} < N^*$, the comparison of K^{reg} relative to K^* is at first glance ambiguous. However, an argument similar to that in the previous section shows that it is shareholders that primarily benefit from the binding capital requirements: since the increased capital requirement leads to less leverage at each bank, bankruptcy costs at each bank will be lower. Compounding this, the reduction in the number of banks to N^{reg} further reduces bankruptcy costs through the greater recovery: $h' < 0$. With this, a decrease in K

and ρ would again not be consistent with an increase in SW since all the surplus goes to either depositors or shareholders. As above, depositors cannot be getting the surplus since $1-k$ goes down at each bank and, for $N < M$, we have that $u = 1$. Therefore, shareholders must be capturing the increase in surplus, implying that the return to shareholders and, consequently, the total amount of capital must increase. In other words, exactly as above, it must be that $\rho^{reg} > \rho$ and $K^{reg} > K^*$ in order to be consistent with using regulation to maximize social welfare.

This section therefore shows that, once we have a rationale for capital regulation related to "fire sale" externalities, it is primarily shareholders who benefit through higher returns and greater surplus. Households that would normally choose to be depositors, while not made worse off, also do not benefit at all from the capital requirements that increase aggregate surplus.

6.3 Deposit Insurance

Another motive for minimum bank capital regulations can be the existence of government-provided deposit insurance which tends to encourage excessive risk taking by bank shareholders. This form of moral hazard can be mitigated by sufficiently high capital requirements which ensure that losses are borne to greater extent by shareholders.

Let us now assume that deposit insurance is in place, and that it is fully financed from the proceeds of non-distortionary lump sum taxes. Banks then choose capital k as to maximize

$$\max_{k,r_D} \frac{1}{R} \int_{r_D(1-k)}^{R} r - r_D(1-k)\, dr - \rho k \tag{13}$$

s.t.

$$E\left[U\right] = \frac{1}{R} \int_{0}^{R} r_D\, dr \geq u. \tag{14}$$

and deposit insurance covers the cost of compensating depositors for their losses,

$$DI = N \cdot \frac{1}{R} \int_{0}^{r_D(1-k)} r_D(1-k)\, dr = N \cdot \frac{\left(r_D(1-k)\right)^2}{R} \tag{15}$$

As deposit insurance makes bank deposits riskless, the promised return r_D becomes equal to u and does not depend on the risk that the bank assumes. This induces banks to hold too little capital:

Proposition 5. *In the presence of deposit insurance, the unique equilibrium is characterized by* $k = 0$, $N = M$ *and* $r_D = u = R$.

Proof. see appendix. □

Since in equilibrium banks hold zero capital and promise depositors always the highest possible return, they go bankrupt with a probability of one and deposit insurance always pays a total of $DI = M \cdot R$ to depositors. This puts a considerable strain on all investors: they incur a lump-sum tax of R per capita to fund deposit insurance.

Can minimum capital standards reduce the deadweight losses due to bankruptcy and increase welfare? Suppose that a regulator imposes a minimum bank capital standard k^{reg} which is chosen as to maximize welfare,

$$\max_{k^{reg}} SW = -DI + \rho K + uD - \int_0^{\rho-u} M \frac{c}{C}\, dc \tag{16}$$

subject to

$$r_D = \arg\max_{r_D} \frac{1}{R} \int_{r_D(1-k^{reg})}^{R} r - r_D(1-k^{reg})\, dr - \rho k^{reg} \tag{17}$$

$$E\left[U_D^{reg}\right] = r_D \geq u \tag{18}$$

$$E\left[\Pi_B^{reg}\right] \geq 0 \text{ and } 0 \leq k^{reg} \leq 1 \tag{19}$$

Constraints $(17) - (19)$ ensure that the solution satisfies both bank profit maximization with respect to r_D and the individual rationality constraint of depositors. In fact, the regulatory optimal solution is characterized by a positive amount of capital:

Corollary 5.1. *In the presence of deposit insurance, the regulator will optimally require banks to hold a strictly positive amount of capital* k^{reg}. *Social welfare is strictly higher under this allocation than in the unregulated case.*

The redistributive consequences of this regulatory intervention are very similar to those that we have observed in the uninsured baseline model. Whilst higher capital standards alleviate the tax burden on shareholders and depositors alike by making banks more stable and reducing deposit insurance expenditures, shareholders again will benefit disproportionally from the regulatory intervention because the regulatory allocation calls for a higher level K^{reg} of aggregate bank capital than the market solution. In order to endogenously generate sufficient supply of such capital, shareholder returns ρ must increase disproportionally over depositor returns u, and depositor returns must fall below their unregulated market equilibrium value of R. We therefore see that similarly to the previously discussed cases, it will be depositors who bear the primary burden of this regulation.

7 Conclusion

This paper presents an analysis of banks' optimal capital structures in a setting where investors may be reluctant to participate in financial markets by holding anything other than safe assets, such as bank deposits, and have to be induced to do so through the promise of higher returns. The equilibrium amount of stock market participation in the banking sector is thus endogenous, and depends on the distribution of returns associated with the investment opportunity set available to banks. We use this framework to study the incidence of capital regulation, and shed light on whether requirements geared toward reducing bank failure and absorbing losses that would otherwise accrue to depositors and by extension the deposit insurance fund affect various classes of investors differently.

In our analysis, we have explicitly sidestepped issues related to the interaction of risk and leverage that are present when systematic risk is priced by assuming risk neutrality. Therefore, the standard results stemming from the work by Modigliani and Miller (1958) are not present, allowing us to isolate the effects stemming from limited market participation and capital regulation. An interesting issue, however, would be to consider how risk aversion, coupled with the existence of systematic risk, interact with the results we obtain here. At present, investors' reluctance to invest in equities implies in our framework that greater demand for capital (i.e., equity) by banks requires that investors earn a higher return in order to induce them to participate. By contrast, the usual logic of risk aversion and systematic risk implies that greater leverage makes equity riskier on a systematic basis, and increases its required return. The study of this issue introduces additional complexities to understand the exact source of households' unwillingness to participate in financial markets, and is left for future research.

A Appendix

Proof of Proposition 1:

We consider banks' optimal choice of capital k. Define $\kappa \equiv 1 - \frac{R-S}{r_D}$. There are two fundamentally distinct scenarios regarding the level of bank capital: if banks choose any capital stock $k \geq \kappa$, they are fully protected against bankruptcy because banks owe less to depositors than the lowest possible state, $r_D(1-\kappa) = R - S$. We will turn to this case later. If banks choose $k \in (0, \kappa)$, bankruptcy will occur with positive probability. Solving $EU = u$ for k we see that the bank chooses a capital stock of

$$k = 1 - \frac{R - S(u/r_D)}{r_D} \tag{20}$$

which is smaller than κ if and only if $u < r_D$; hence, the equilibrium which we derive now can hold only as long as $u < r_D$. Back-substituting (20) into the expression for bank profits we find

$$E\left[\Pi_B\right] = \frac{Su^2 - 2Su\rho + 2\rho R r_D - 2r_D^2\rho}{2r_D^2} \tag{21}$$

Maximization of profits for r_D results in

$$r_D^* = \frac{Su(2\rho - u)}{\rho R} \tag{22}$$

which gives a total maximum profit of

$$E\left[\Pi_B\right] = \frac{\rho\left(\rho R^2 - 2Su(2\rho - u)\right)}{2Su(2\rho - u)} \tag{23}$$

There is free entry, so in equilibrium expected profits in eq. (23) are zero. Solving for ρ yields

$$\rho = \frac{2Su^2}{4Su - R^2} \tag{24}$$

Substituting this expression into eq. (22) and collecting terms yields the remarkable result that the promised depositor rate is always half of the maximal upside of the risky technology, independently of the dispersion S:

$$r_D = \frac{R}{2} \tag{25}$$

Substituting r_D in (20), we obtain

$$k = \frac{4Su}{R^2} - 1 \tag{26}$$

Market clearing in the bank capital market commands that

$$M \frac{\hat{c}}{C} = Nk \tag{27}$$

which can be rewritten as

$$\rho - u = \frac{NCk}{M} \tag{28}$$

Let us for now assume that there is full inclusion, i.e. no storage is used and all investors invest in either equity or deposits: $N = M$. Substituting (26) and (24) in eq. (28) and solving for u, we find two possible roots, $u = \frac{R^2}{4S}\left(1 \pm \sqrt{\frac{R^2}{R^2+8CS}}\right)$. However, back-substituted into (26), one can see that the only economically sensible solution is

$$u = \frac{R^2}{4S}\left(1 + \sqrt{\frac{R^2}{R^2 + 8CS}}\right) \tag{29}$$

because it implies a nonnegative capital stock of

$$k = \sqrt{\frac{R^2}{R^2 + 8CS}} \tag{30}$$

Combining these results with eq. (28), we finally find

$$\rho = \frac{R^2}{4S} + \frac{R\left(4CS + R^2\right)}{4S\sqrt{8CS + R^2}}$$

If $\hat{u} \equiv u = \frac{R^2}{4S}\left(1 + \sqrt{\frac{R^2}{R^2+8CS}}\right) \geq 1$, this full participation equilibrium also satisfies the depositor participation constraint and is the unique equilibrium of the game. However, in case $\hat{u} < 1$, the full participation equilibrium is not feasible because violates the depositor participation constraint: at full participation returns, depositors would strictly prefer storage over deposits. The unique equilibrium must in this case exhibit incomplete (partial) participation, with some funds going to storage, and the return of deposits matching this storage outside option, thus $u = 1$. We obtain the remaining equilibrium variables

$$k = \frac{4S}{R^2} - 1 \tag{31}$$

$$\rho = \frac{2S}{4S - R^2} \tag{32}$$

$$K = Nk = M\frac{2S - R^2}{C(R^2 - 4S)} \tag{33}$$

directly from equations (24), (26) and (27). This concludes our analysis of the risky bank equilibrium, with the important qualification that $u < r_D$. This means that this risky equilibrium only applies when $\max\left\{\frac{R^2}{4S}\left(1 + \sqrt{\frac{R^2}{R^2+8CS}}\right), 1\right\} < R/2$.

We now turn to the safe equilibrium that is obtained whenever banks choose a safe level of capital, such that

$$r_D(1-k) \geq R - S \Leftrightarrow k \geq 1 - \frac{R-S}{r_D}$$

In this case, the probability of bankruptcy is zero. The expected utility of depositors becomes $EU = r_D = u$, whereas the expected return of shareholders is pinned down by the equation

$$E\Pi = E[r] - (1-k)u - k\rho = E[r] - k(\rho - u) - u = 0$$

which implies that

$$\rho = \frac{E[r] - (1-k)u}{k} = \frac{R - S/2 - (1-k)u}{k}$$

Assume $\rho > u$ (which it must be in equilibrium to attract a nonzero capital stock). Then maximization of profits dictates that banks choose the smallest possible k which still attains safety, $k = 1 - \frac{R-S}{u}$. The return to capital becomes then

$$\rho = \frac{R - S/2 - (R-S)}{1 - \frac{R-S}{u}} = \frac{Su/2}{u - R + S}$$

The equilibrium value of u is determined by market clearing,

$$M\frac{\hat{c}}{C} = Nk$$

with $M = N$ and $\hat{c} = \rho - u$.

Substituting previous results, we find

$$\frac{Su/2}{u-R+S} - u = C\left(1 - \frac{R-S}{u}\right)$$

which can be transformed to read

$$\frac{Su^2 - 2u^2(u-R+S) - 2C(u-R+S)^2}{2(u-R+S)^2} = 0$$

$$\Rightarrow (2R - S - 2u)u^2 - 2C(u-R+S)^2 = 0$$

The latter is the defining equation for the equilibrium $u = u^*$ in the safe regime, and the remaining allocations are obtained by inserting u^* into the previously derived equations. Note that a safe equilibrium can never be optimal when an "interior" risky equilibrium with $u > r_D$ exists: by choosing k arbitrarily close to, and just marginally below, κ, banks can attain any safe equilibrium return in a risky setting. Whenever the risky equilibrium choice does not feature such corner solution, it is thus clear that the benefit from the risky equilibrium must be strictly higher.

In order to show that $\rho > E[r] > u$, we first show that $\rho > E[r]$ implies $u < E[r]$. Consider eq. (1) and eq. (2). The former, combined with the free entry condition, can rewritten as

$$E[\Pi_B] = \frac{1}{S}\int_{-\infty}^{\infty} \max(r - r_D(1-k), 0)\, dr - \rho k = 0$$

whereas the latter, rewritten as strict equality (which it must be in equilibrium) and multiplied on both sides with $(1-k)$, can be bounded as follows: Consider that $\frac{1}{S}\int_{-\infty}^{r_D(1-k)} r\, dr + \frac{1}{S}\int_{r_D(1-k)}^{\infty} r_D(1-k)\, dr = \frac{1}{S}\int_{-\infty}^{\infty} \min(r, r_D(1-k))\, dr$. The first term must be nonnegative since the lower bound of the support of the distribution of r is nonnegative. Hence,

$$\frac{1}{S}\int_{r_D(1-k)}^{\infty} r_D(1-k)\, dr = u(1-k) \leq \frac{1}{S}\int_{-\infty}^{\infty} \min(r, r_D(1-k))\, dr$$

Summing both expressions yields

$$(1-k)u + k\rho \leq \frac{1}{S}\int_{-\infty}^{\infty} r\, dr = E[r]$$

which shows that $E[r]$ is greater or equal to a convex combination of ρ and u. Thus, for any $k \in (0, 1)$ having $\rho > E[r]$ implies necessarily that $u < E[r]$.

We conclude the proof by showing that $\rho > E[r]$ for any value of $\hat{u}$.

For $\hat{u} < 1$, we rewrite $\rho - E[r] = \frac{2S}{4S-R^2} - \frac{2R-S}{2} = \frac{(S-2R)\left(4S-R^2\right)+4S}{2(4S-R^2)}$. Since know that $\hat{u} = \frac{R^2}{4S}\Delta < 1$ with $\Delta \equiv 1 + \sqrt{\frac{R^2}{R^2+8CS}} \geq 1$, it must be that $R^2 < 4S$. The denominator of the previous expression is thus always positive and the sign of the expression depends only on the numerator. It is straightforward to show that the numerator has (within the relevant domain of all R, S for which $\hat{u} < 1$ and $0 < S \leq R$) a minimum of zero at $R = 2$, $S = 2$ and is strictly increasing and convex in R for all $R \in (2, 2\sqrt{S})$. Therefore, $\rho > E[r]$ holds as long as $R > 2$, which was part of our initial assumptions.

For $R/2 > \hat{u} \geq 1$, we can calculate

$$\rho - E[r] = \frac{2Su^2}{4Su - R^2} - \frac{2R - S}{2} \tag{34}$$

$$= \frac{2S^2\sqrt{8CS + R^2} + R^2\sqrt{8CS + R^2} - 4RS\sqrt{8CS + R^2} + 4CRS + R^3}{4S\sqrt{8CS + R^2}} \tag{35}$$

which is positive if the numerator is positive. Rewriting the numerator,

$$R(R^2 + 4CS) + \left[R^2 - 4RS + 2S^2\right]\sqrt{R^2 + 8CS} \tag{36}$$

$$\ldots = R(R^2 + 4CS) + \left[2(R - S)^2 - R^2\right]\sqrt{R^2 + 8CS} \tag{37}$$

$$\geq R^2(R^2 + 4CS) - R^2\sqrt{R^2 + 8CS} \tag{38}$$

A lengthy but straightforward analysis yields that for any $x > 0$ the function $R(R^2 + x) - R^2\sqrt{R^2 + 2x}$ is zero at $R = 0$, is strictly increasing in R for $R \in (0, \sqrt{\frac{2}{3}x})$, reaches a maximum value of $\frac{1}{3}\sqrt{\frac{2}{3}x}$ at $R_m = \sqrt{\frac{2}{3}x}$ and is strictly decreasing for $R > R_m$, with $\lim_{R\to\infty} R(R^2 + x) - R^2\sqrt{R^2 + 2x} = 0^+$. Hence, its range for $R \in (0, \infty)$ is strictly positive, and thus the expression in eq. (38) is positive, which establishes that $\rho > E[r]$.

Finally, for $\hat{u} > R/2$ it is immediate from market clearing that $\rho > u$. Since there is no bankruptcy, there are also no bankruptcy cost, and $E[r] = \rho k + u(1 - k)$ holds with equality for a $k \in (0, 1)$, which proves $\rho > E[r] > u$. □

Proof of Proposition 2:

To establish the result, it is useful to start with the problem of bank profit maximization, which can be expressed as

$$\max_{k} E\left[\Pi_B\right] = \frac{1}{S}\int_{r_D(1-k)}^{R} \left(r - r_D(1-k)\right) dr - \rho k$$

subject to the same constraints (6), (7), and (8) as above, for the social planner's problem. Note that the first order condition implied by (6), for any level of capital k, is always negative, meaning that the bank (and the social planner) always finds it optimal to choose the lowest deposit rate r_D that is consistent with satisfying depositors' participation constraint. Therefore, (7) will always be satisfied with equality, allowing us to substitute the constraint into the bank's maximization problem to obtain

$$\max_{k} E\left[\Pi_B\right] = \frac{1}{S}\int_{r_D(1-k)}^{R} rdr - u(1-k) - \rho k.$$

The necessary first order condition that must now be satisfied is

$$\frac{1}{S}\left(r_D^2\left(1-k\right) - \frac{\partial r_D}{\partial k}\left(1-k\right)^2 r_D\right) - \left(\rho - u\right) = 0,$$

where $\frac{\partial r_D}{\partial k}$ is obtained directly from (7). This first order condition must be satisfied in equilibrium whatever the values for ρ and u, which are obtained from market clearing as in Proposition 1.

Consider now the maximization problem for the social planner, which can alternatively be written as

$$\begin{aligned}\max_{k_P} SW &= N\frac{1}{S}\int_{r_D(1-k)}^{R} rdr - \int_0^{\widehat{c}} M\frac{c}{C}\,dc + M - N \\ &= N\frac{1}{S}\int_{r_D(1-k)}^{R} rdr - \frac{1}{2C}M\widehat{c}^2 + M - N,\end{aligned}$$

reflecting the fact that maximizing the return to all stakeholders is equivalent to maximizing aggregate output, $N\frac{1}{S}\int_{r_D(1-k)}^{R} rdr$, since all output is allocated to either depositors or capital suppliers. The last term, $M - N$, represents the funds that are not invested in the banking sector but rather held as storage, to the extent that M may be strictly greater than N. Recall now the market clearing condition, $M\frac{\widehat{c}}{C} = kN$, which implies that $\widehat{c} = \frac{C}{M}kN$, or that $\widehat{c}^2 = \left(\frac{C}{M}kN\right)^2$, and which is taken into account by the social planner. We can thus write the maximization problem above as

$$\begin{aligned}\max_{k_P} SW &= N\frac{1}{S}\int_{r_D(1-k)}^{R} rdr - \frac{1}{2C}M\left(\frac{C}{M}kN\right)^2 + M - N \\ &= N\frac{1}{S}\int_{r_D(1-k)}^{R} rdr - \frac{1}{2}\frac{C}{M}k^2N^2 + M - N.\end{aligned}$$

For an interior solution, the necessary first order condition to this problem is

$$N\frac{1}{S}\left(r_D^2(1-k) - \frac{\partial r_D}{\partial k}(1-k)^2 r_D\right) - \frac{C}{M}kN^2 + \frac{\partial N}{\partial k}\frac{1}{S}\int_{r_D(1-k)}^{R} rdr - \frac{C}{M}k^2N\frac{\partial N}{\partial k} - \frac{\partial N}{\partial k} = 0.$$

Grouping terms obtains

$$N\left(\frac{1}{S}\left(r_D^2(1-k) - \frac{\partial r_D}{\partial k}(1-k)^2 r_D\right) - \frac{C}{M}kN\right) + \frac{\partial N}{\partial k}\left(\frac{1}{S}\int_{r_D(1-k)}^{R} rdr - \frac{C}{M}k^2N - 1\right) = 0.$$

Since at equilibrium $\rho - u = \widehat{c} = \frac{C}{M}kN$, we can further rewrite as

$$N\left(\frac{1}{S}\left(r_D^2(1-k) - \frac{\partial r_D}{\partial k}(1-k)^2 r_D\right) - (\rho - u)\right) + \frac{\partial N}{\partial k}\left(\frac{1}{S}\int_{r_D(1-k)}^{R} rdr - k(\rho - u) - 1\right) = 0$$

Now observe that $\frac{\partial N}{\partial k} \neq 0 \Rightarrow u = 1$ since, if $u > 1$, all funds are being used in the banking sector, so a marginal increase in k cannot change $N = M$.

Consider first the case that $u > 1$, so that $\frac{\partial N}{\partial k} = 0$. We are then left with only

$$N\left(\frac{1}{S}\left(r_D^2(1-k) - \frac{\partial r_D}{\partial k}(1-k)^2 r_D\right) - (\rho - u)\right) = 0,$$

which is the same condition as must be satisfied for profit maximization problem.

Alternatively, suppose that $u = 1$, which allows us to express the first order condition for the social planner as

$$N\left(\frac{1}{S}\left(r_D^2(1-k) - \frac{\partial r_D}{\partial k}(1-k)^2 r_D\right) - (\rho - 1)\right) + \frac{\partial N}{\partial k}\left(\frac{1}{S}\int_{r_D(1-k)}^{R} rdr - k\rho - (1-k)\right) = 0.$$

Note now that term in the parentheses of the second line, $\frac{1}{S}\int_{r_D(1-k)}^{R} rdr - k\rho - (1-k)$, is simply $E[\Pi_B]$ for the case where $u = 1$, which in equilibrium is equal to zero, with all rents going to shareholders through ρ. This leaves exactly the term below, after eliminating the N:

$$\frac{1}{S}\left(r_D^2\,(1-k)-\frac{\partial r_D}{\partial k}\,(1-k)^2\,r_D\right)-(\rho-1)=0,$$

again exactly as in the problem of profit maximization for the case $u=1$. Therefore, for all cases the necessary condition to be satisfied is identical to that which maximizes profits. Given that the market clearing condition that pins down the equilibrium returns u and ρ is the same across both maximization problems, we can conclude that both problems must have the same solution. □

Proof of Proposition 3:

We first establish a preliminary result, which is that the per-unit return to shareholders ρ is maximized at the market solution k^*. To see this, recall that bank profits are given by

$$E\left[\Pi_B\right]=\frac{1}{S}\int_{\widehat{r}_D(1-k)}^{R}\left(r-\widehat{r}_D(1-k)\right)dr-\rho k,$$

where we use $\widehat{r}_D$ to represent the deposit rate that satisfies depositors' participation constraint, (2) and is a function of the level of capital k. For ease of notation, define $F(k)=\frac{1}{S}\int_{\widehat{r}_D(1-k)}^{R}(r-\widehat{r}_D(1-k))\,dr$, and note that F is increasing and strictly concave in k. This allows us to write bank profits as

$$E\left[\Pi_B\right]=F\left(k\right)-\rho k,$$

where, as always, ρ is chosen such that $E\left[\Pi_B\right]=0$ in equilibrium. We now want to determine how ρ changes with a change in k: The implicit function theorem tells us that

$$\frac{d\rho}{dk}=-\frac{\frac{\partial E[\Pi_B]}{\partial k}}{\frac{\partial E[\Pi_B]}{\partial \rho}}=-\frac{F'(k)-\rho}{-\rho}=\frac{F'(k)}{\rho}-1. \tag{39}$$

Consider now the bank's choice of k which, from the first-order condition defining the optimal k^*, satisfies

$$F'(k^*)=\rho. \tag{40}$$

We can now show that returns to equity are maximal at k^*: substituting (40) into (39) to obtain

$$\frac{d\rho}{dk} = \frac{F'(k^*)}{\rho} - 1 = 0.$$

Given concavity of F, which implies that $F'(k) < F'(k^*)$, this means that ρ achieves its maximum at $k = k^*$, and is decreasing in k beyond that. This establishes that $\frac{d\rho}{dk} < 0$ for both cases (1) and (2) in the proposition.

To establish the rest of the result, consider first the case where $N < M$. Market clearing implies

$$\rho - u = Ck\frac{N}{M}.$$

Given the result above that ρ is lower for larger k, and the fact that $u = 1$, the RHS must decrease to satisfy market clearing. Given that we are considering an increase in k, N must fall, and it must fall enough that even kN must be lower. Since $K = kN$, it follows that $K^{reg} < K^*$.

Consider next the case where $N = M$ and therefore $u > 1$. For a marginal increase in k, market clearing simplifies to

$$\rho - u = kC.$$

The RHS must be larger than in the market solution because $k > k^*$. Given that ρ is lower, per the argument above, we must have that u decreases more than proportionally. That is, $0 > \frac{d\rho}{dk} > \frac{du}{dk}$, as desired. Finally, $K^{reg} = Nk > Nk^* = K^*$, which concludes the proof. □

Proof of Proposition 4:

The proof follows a similar approach as that of Proposition 2. Since at equilibrium (12) will be satisfied with equality, we can rewrite (1) as

$$\max_k E\left[\Pi_B\right] = \frac{1}{R}\int_{r_D(1-k)}^{R} r dr + \frac{1}{R}\int_0^{r_D(1-k)} hr\, dr - u(1-k) - \rho k.$$

The necessary first order condition that must now be satisfied is

$$\frac{1}{R}\left(r_D^2\left(1-k\right)-\frac{\partial r_D}{\partial k}\left(1-k\right)^2 r_D\right)-\left(\rho-u\right)+h\frac{1}{R}\left(\frac{\partial r_D}{\partial k}\left(1-k\right)^2 r_D-r_D^2\left(1-k\right)\right)=0,$$

or

$$\frac{1}{R}\left(r_D^2\left(1-k\right)-\frac{\partial r_D}{\partial k}\left(1-k\right)^2 r_D\right)\left(1-h\right)-\left(\rho-u\right)=0,$$

where $\frac{\partial r_D}{\partial k}$ is obtained directly from (12). This first order condition must be satisfied in equilibrium whatever the values for ρ and u, which are obtained from market clearing as in Proposition 1.

Consider now the maximization problem for the social planner, which can alternatively be written as

$$\begin{aligned}\max_{k^{FB}} SW &= N\frac{1}{R}\int_{r_D(1-k)}^{R} rdr-\int_0^{\widehat{c}} M\frac{c}{C}\,dc+M-N+N\frac{1}{R}\int_0^{r_D(1-k)} hr\,dr\\ &= N\frac{1}{R}\int_{r_D(1-k)}^{R} rdr-\frac{1}{2C}M\widehat{c}^2+M-N+N\frac{1}{R}\int_0^{r_D(1-k)} hr\,dr,\end{aligned}$$

reflecting the fact that maximizing the return to all stakeholders is equivalent to maximizing aggregate output, $N\frac{1}{S}\int_{r_D(1-k)}^{R} rdr$. Recall now the market clearing condition, $M\frac{\widehat{c}}{C}=kN$, which implies that $\widehat{c}=\frac{C}{M}kN$, or that $\widehat{c}^2=\left(\frac{C}{M}kN\right)^2$. We can thus write the maximization problem above as

$$\begin{aligned}\max_{k_{FB}} SW &= N\frac{1}{R}\int_{r_D(1-k)}^{R} rdr-\frac{1}{2C}M\left(\frac{C}{M}kN\right)^2+M-N+N\frac{1}{R}\int_0^{r_D(1-k)} hr\,dr\\ &= N\frac{1}{R}\int_{r_D(1-k)}^{R} rdr-\frac{1}{2}\frac{C}{M}k^2N^2+M-N+N\frac{1}{R}\int_0^{r_D(1-k)} hr\,dr.\end{aligned}$$

For an interior solution, the necessary first order condition to this problem is

$$\begin{aligned}&N\frac{1}{R}\left(r_D^2\left(1-k\right)-\frac{\partial r_D}{\partial k}\left(1-k\right)^2 r_D\right)-\frac{C}{M}kN^2+\frac{\partial N}{\partial k}\frac{1}{R}\int_{r_D(1-k)}^{R} rdr-\frac{C}{M}k^2N\frac{\partial N}{\partial k}-\frac{\partial N}{\partial k}\\ &+Nh\frac{1}{R}\left(\frac{\partial r_D}{\partial k}\left(1-k\right)^2 r_D-r_D^2\left(1-k\right)\right)+\frac{\partial N}{\partial k}\frac{1}{R}\int_0^{r_D(1-k)} hr\,dr+N\frac{1}{R}\int_0^{r_D(1-k)}\frac{\partial h}{\partial k}r\,dr=0.\end{aligned}$$

Grouping terms obtains

$$N\left(\frac{1}{R}\left(r_D^2\left(1-k\right)-\frac{\partial r_D}{\partial k}\left(1-k\right)^2 r_D\right)\left(1-h\right)-\frac{C}{M}kN\right)$$
$$+\quad \frac{\partial N}{\partial k}\left(\frac{1}{R}\int_{r_D(1-k)}^{R} rdr+\frac{1}{R}\int_0^{r_D(1-k)} hr\,dr-\frac{C}{M}k^2N-1\right)+N\frac{1}{R}\int_0^{r_D(1-k)}\frac{\partial h}{\partial k}r\,dr=0.$$

Since at equilibrium $\rho-u=\widehat{c}=\frac{C}{M}kN$, we can further rewrite as

$$N\left(\frac{1}{R}\left(r_D^2\left(1-k\right)-\frac{\partial r_D}{\partial k}\left(1-k\right)^2 r_D\right)\left(1-h\right)-\left(\rho-u\right)\right)$$
$$+\quad \frac{\partial N}{\partial k}\left(\frac{1}{R}\int_{r_D(1-k)}^{R} rdr+\frac{1}{R}\int_0^{r_D(1-k)} hr\,dr-k\left(\rho-u\right)-1\right)+N\frac{1}{R}\int_0^{r_D(1-k)}\frac{\partial h}{\partial k}r\,dr=0$$

Now observe that $\frac{\partial N}{\partial k}\neq 0\Rightarrow u=1$ since, if $u>1$, all funds are being used in the banking sector, so a marginal increase in k cannot change $N=M$.

Consider now the market solution for the case that $u>1$, so that $\frac{\partial N}{\partial k}=0$. In that case, $\frac{\partial h}{\partial k}=0$ as well, and there is thus no scope for capital regulation.

Alternatively, suppose that under the market solution $u=1$, and again substitute these values into the social planner's first order condition. This would give us

$$N\left(\frac{1}{R}\left(r_D^2\left(1-k\right)+\frac{\partial r_D}{\partial k}\left(1-k\right)^2 r_D\right)\left(1-h\right)-\left(\rho-1\right)\right)$$
$$+\quad \frac{\partial N}{\partial k}\left(\frac{1}{R}\int_{r_D(1-k)}^{R} r\,dr+\frac{1}{R}\int_0^{r_D(1-k)} hr\,dr-k\rho-\left(1-k\right)\right)+N\frac{1}{R}\int_0^{r_D(1-k)}\frac{\partial h}{\partial k}r\,dr.$$

Under the k, r_D, u, and ρ implied by the market solution, the term in the parentheses, $\frac{1}{R}\int_{r_D(1-k)}^{R} rdr+\frac{1}{R}\int_0^{r_D(1-k)} hr\,dr-k\rho-(1-k)$, is simply $E\left[\Pi_B\right]$ for the case where $u=1$, which in equilibrium is equal to zero, with all rents going to shareholders through ρ. This leaves the term below:

$$N\left(\frac{1}{R}\left(r_D^2\left(1-k\right)+\frac{\partial r_D}{\partial k}\left(1-k\right)^2 r_D\right)\left(1-h\right)-\left(\rho-1\right)\right)+N\frac{1}{R}\int_0^{r_D(1-k)}\frac{\partial h}{\partial k}r\,dr.$$

Again, at the market solution, the first term is equal to zero. This means that, evaluated at the market solution, the derivative with respect to k of the social planner's objective function is $N\frac{1}{R}\int_0^{r_D(1-k)}\frac{\partial h}{\partial k}r\,dr>0$. Therefore, the market solution involves banks holding too little capital relative to what a social planner would like and, consequently, too many banks. □

Proof of Proposition 5:

Note that the optimization problem in (13) can be rewritten as

$$E\left[\Pi_B\right] = \frac{1}{2R}\left(R - r_D(1-k)\right)^2 - \rho k, \tag{41}$$

which is a convex function of k (as verified by $\frac{d^2}{dk^2}E\left[\Pi_B\right] = \frac{r_D^2}{R} > 0$) with a minimum at $k = 1 - R\frac{1-(\rho/r_D)}{r_D}$ which is no less than 1 for $\rho \geq r_D = u$. Since $\rho \geq u$ holds for any $k \geq 0$, the unique maximum of $E\left[\Pi_B\right]$ is attained by choosing the lowest capital level in the $[0,1]$ interval. Therefore, $k^* = 0$, and all M investors become depositors: $N = M$. Since bank profit (41) in equilibrium is zero, we finally obtain $u = r_D = R$.

To see that the regulator will optimally require a positive amount of capital to be held, note that the social welfare function can be written as

$$\begin{aligned} SW &= -N(1-k)\frac{1}{R}\int_0^{r_D(1-k)} r_D\,dr + \rho K + uD - M\cdot\int_0^{\rho-u}\frac{c}{C}\,dc \\ &= -\underbrace{N\cdot r_D(1-k)}_{=Dr_D}\cdot\underbrace{\frac{r_D(1-k)}{R}}_{=\text{Prob(bankr.)}} + \rho K + uD - M\frac{(\rho-u)^2}{2C} \end{aligned}$$

Applying the market clearing condition for capital, $K = M\frac{\rho-u}{C}$, to the last term and substituting $r_D = u$ from the depositor participation constraint we obtain

$$SW = uD\cdot(1-\text{Prob(bankr.)}) + \frac{K}{2}\cdot(\rho+u) \tag{42}$$

Taking derivatives wrt. k at the point of the unregulated equilibrium $k = 0$, we get

$$\begin{aligned} \frac{\mathrm{d}}{\mathrm{d}k}SW &= \frac{\mathrm{d}D}{\mathrm{d}k}\underbrace{u(1-\text{Prob(bankr.)})}_{=0} + D\cdot\frac{\mathrm{d}}{\mathrm{d}k}\left[u\cdot(1-\text{Prob(bankr.)})\right] \\ &\quad + \frac{\mathrm{d}K}{\mathrm{d}k}\underbrace{\left(\frac{\rho+u}{2}\right)}_{\geq 0} + \underbrace{K}_{=0}\cdot\frac{\mathrm{d}}{\mathrm{d}k}\left(\frac{\rho+u}{2}\right) \end{aligned}$$

The term $D\cdot\frac{\mathrm{d}}{\mathrm{d}k}\left[u\cdot(1-\text{Prob(bankr.)})\right]$ must be positive because

$$\frac{\mathrm{d}}{\mathrm{d}k}\left[u\cdot(1-\text{Prob(bankr.)})\right] = \frac{\mathrm{d}u}{\mathrm{d}k}\cdot\underbrace{(1-\text{Prob(bankr.)})}_{=0} + u\cdot\underbrace{\left(-\frac{\mathrm{d}}{\mathrm{d}k}\text{Prob(bankr.)}\right)}_{>0} > 0$$

and hence social welfare must increase strictly in k at $k = 0$. The planner therefore optimally chooses a strictly positive amount of k to maximize welfare. $\square$

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