AN ITERATIVE REGULARISATION ALGORITHM TO ESTIMATE PERMEABILITY IN MYOCARDIAL PERFUSION

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Introduction

Myocardial perfusion is the blood supply to heart muscle provided by the coronary arteries. The blood flow can be described by Darcy's law as fluid flows through a porous media at the macro-scale. Additionally, contrast enhanced myocardial perfusion imaging uses an injected contrast agent (CA) as a proxy for blood flow in cardiac tissue. This imaging modality has previously been modelled as the advection-diffusion of the CA through a porous medium [1], which provided a computational framework enabling the direct mapping from permeability to local perfusion defect. In this work we propose a method to estimate permeability from observed contrast agent concentration.

Methods

An iterative algorithm was developed for inversely estimating the permeability by the observed CA concentration field, as a route to more accurate and detailed quantitative analysis. Several test cases of varying porous parameters, Peclet number (Pe) and boundary conditions were used to generate 'observed' data, on which to test the inverse estimation. The forward models of Darcy flow and CA advection-diffusion solved using the finite element method, implemented in FEniCS. The inverse problem was solved by Tikhonov regularisation approach: minimising an objective function containing two penalty terms: (1) the error between estimated concentration and observed concentration, and (2) the gradient of the permeability tensor. However, this inverse problem is ill-posed as the solution is not unique. Therefore, an iterative regularisation algorithm was developed, which iteratively adjusts the weighting of the penalty terms and updates the initial guess. This allows the optimiser to converge to a low overall error while preserving local inhomgeneities in permeability.

Results

The algorithm was tested for four types of permeability distribution: (a) homogeneous permeability, (b) homogeneous permeability with defect, (c) linearly inhomogeneous permeability, (d) linearly inhomogeneous permeability with defect. For (a), only one iteration of regularisation was required. For (b), the location, shape, defect and background permeability were successfully estimated within few iterations. For (c), the estimation was still acceptable with a L2 error norm about 0.12, although the permeability at the region where has no CA filled in are slightly mis-estimated. And for the most challenging case (d), the location and extent of the defect region was estimated with a same precision of error norm of (c), see Figure 1 for the contour plot of permeability estimation of (d). In addition, a parametric analysis was studied to find out the sensitivity to the parameter Peclet number (Pe) and observation frequency (T_m, the time scale of advection and observation). For test case (a), the error was very small and negligible. For (b)(c)(d), generally, both the error norm of estimated permeability (Error K) and concentration (Error c) increase when Pe increases, and the error remain stable when T_m is greater than 5 (See Figure 2).

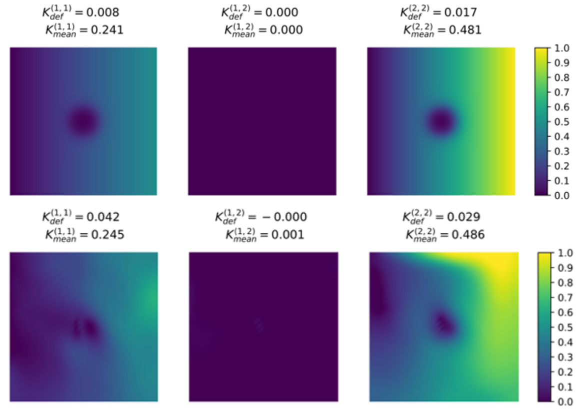

Figure 1: Contour plot of exact permeability (top) and estimated permeability (bottom) in test case (d). Superscript indicates the index of tensor element, and subscript indicates the defective and mean permeability.

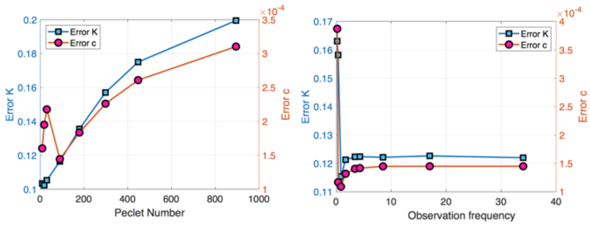

Figure 2: The error K and Error c against Peclet number (left) and observation frequency (right) of test case (d)

Discussion

Generally, when Pe increases, diffusion takes less effect on CA's movement, the estimation becomes harder due to not having enough concentration information. And T_m is noncritical for purely homogeneous permeability estimation. For inhomogeneous or defective permeability cases, when T_m is greater than a threshold, the performance of the estimation cannot be improved.

References

1. Cookson, A. N., et al., Med Image Anal 18(7), 1200 (2014).