A NEW HYPERELASTIC MODEL FOR HUMAN MYOCARDIUM

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Introduction

Human myocardium is formed by locally parallel muscle fibres which are arranged in layers, sometimes called sheets [1]. This structure determines the orthotropic mechanical response of myocardium, as evidenced by the results of biaxial extension tests and simple shear tests [2]. A suitable hyperelastic model for myocardium should reflect its structure and produce an orthotropic response with highest stiffness in the fibre direction, **f**, intermediate in the sheet direction, **s**, and lowest in the sheet-normal direction, **n** (**f**, **s** and **n** are mutually orthogonal). The most widely used model which satisfies these requirements is that proposed by Holzapfel and Ogden [3]. It introduces two orthogonal families of fibres, one in the **f** direction and the other in the **s** direction. The families are represented by two exponential terms in the strain-energy density function, formulated in terms of invariants $I_{4\mathrm{f}} = \mathbf{f} \cdot \mathbf{Cf}$ and $I_{4\mathrm{s}} = \mathbf{s} \cdot \mathbf{Cs}$ which equal the square of stretch in **f** and **s** directions, respectively (**C** is the right Cauchy-Green tensor). However, the family in the **s** direction is somewhat artificial since in myocardium there is no distinct family of fibres (collagen of others) arranged predominantly perpendicular to the muscle fibres; instead, the chains of myocytes are bundled by endomysial connective tissue with membraneous appearance [4]. For this reason, we present a modification of the model which reflects more accurately the laminar structure of myocardium and turns out to have better capability to reproduce experimental responses.

Methods

In our modification of the strain-energy density function the exponential term with the invariant I_{4s} was replaced by the term

$$\Psi_{\mathrm{fs}} = \frac{a_{\mathrm{fs}}}{2b_{\mathrm{fs}}}\left(\exp(b_{\mathrm{fs}}(K_1 - 1)^2 - 1)\right) \quad (1)$$

which employs an uncommon invariant K_1 defined in terms of the cofactor of $\mathbf{C}$, $\mathrm{cof}(\mathbf{C})$, and the sheet-normal unit vector **n** (perpendicular to the sheets) as $K_1 = \mathbf{n} \cdot \mathrm{cof}(\mathbf{C})\mathbf{n}$ [5]. This invariant is essentially a 2-dimensional analogue of the invariant I_4 because it expresses the square of stretch of an infinitesimal area initially perpendicular to **n**; more precisely it is the ratio of the area of the deformed infinitesimal sheet to its referential (initial) area. Thus, unlike the original model [3], our model explicitly includes a mathematical representation of the layered blocks of endomysial connective tissue (apparent in electron microscope [4]) that bind together the muscle fibres. The fibres and the isotropic matrix are modelled in the same way as in the original model [3] (i.e. by means of invariants $I_{4\mathrm{f}}$ and $I_1 = \mathrm{tr}(\mathbf{C})$).

Results

The modified model was fitted to the results of 5 different biaxial tests and 6 different simple shear tests published in [2]. Although the proposed modification is quite subtle, it significantly improved the ability of the model to reproduce all the above-mentioned experimental data (R^2 increased from 0.90 to 0.98). As an example, Figure 1 shows the final fit to the data from equibiaxial extension test.

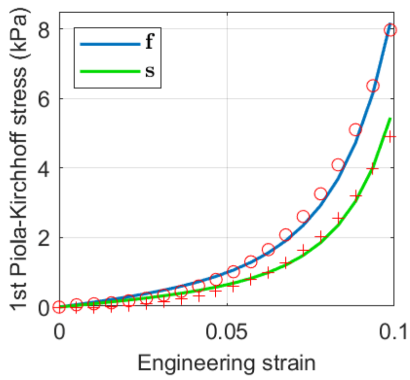

Figure 1: Fit of the proposed hyperelastic model to the equibiaxial experimental data from [2].

Discussion

The proposed model reflects more accurately the microstructure of ventricular myocardium and it has great capability to describe the mechanical response of myocardium in different biaxial and simple shear loading modes. However, derivation of the spatial elasticity tensor (necessary for implementation into commercial finite-element packages) is rather complicated due to the new term (1) of the strain-energy function.

References

1. Agger et al, J Cardiovasc Magn Reson, 19(93):1-16, 2017.
2. Sommer et al, Acta Biomater, 24:172-192, 2015.
3. Holzapfel & Ogden, Philos Trans A Math Phys Eng Sci, 367(1902):3445-3475, 2009.
4. Lunkenheimer & Niederer, Technol Health Care, 20:423-434, 2012.
5. Schröder & Neff, Int J Solids Struct, 40(2):401-445, 2003.

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