CODE VERIFICATION OF THE MICRO FINITE ELEMENT SOLVER PAROSOL USING THE METHOD OF MANUFACTURED SOLUTIONS

Frederik Trommer (1,2), Pinaki Bhattacharya (1,2)

1. Insigneo Institute for in silico Medicine, University of Sheffield, UK
2. Department of Mechanical Engineering, University of Sheffield, UK

Introduction

Despite the complexity and widespread use of micro-FE solvers in bone mechanics research, little attention has been paid to their code verification [1]. The present study uses the method of manufactured solutions (MMS) to verify the open-source micro-FE solver ParOSol [2] which is widely used in bone mechanics research.

Methods

MMS was originally developed for CFD codes [3], [4]. It has only recently been used to verify commercial FE solvers [1]. Here, one "manufactures" an analytical displacement vector field $\boldsymbol{u}(\boldsymbol{x})$, which need not be physically meaningful. This is used to derive strain and stress tensor ($\boldsymbol{\sigma}$) fields, the latter by applying a set of chosen constitutive laws. The stress divergence $\nabla \cdot \boldsymbol{\sigma}$ is usually non-zero, and has to be balanced by "fictitious body forces" ($\boldsymbol{b}$) [1], the analytical expression for which can be determined easily using $\boldsymbol{b} = -\nabla \cdot \boldsymbol{\sigma}$, as the right hand side is already known. The computational problem is set up by defining the mesh, posing the manufactured displacement at all boundary nodes and the manufactured body forces at all nodes, and setting material properties for all elements.

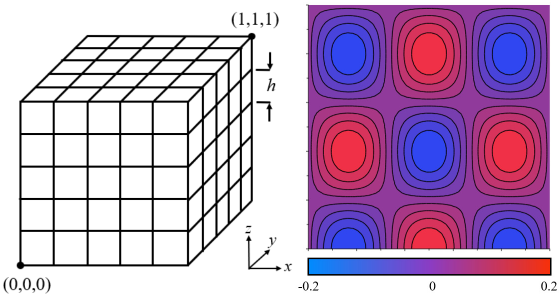

Fig. 1: Unit cube domain and grid spacing h (left), contour plot of the chosen displacement field in y direction at a cross section of x= 0.5(right)

The manufactured solution used in the present study is adapted from [1]. The domain is a unit cube (homogenous, isotropic linear elastic) with a grid spacing of h (Fig. 1). The displacement field is infinitely differentiable and sufficiently complex to exercise all terms in the governing equations. The analytical forms of strains, stresses and body forces based on this displacement field are obtained using the symbolic computing software Maple. HDF5 input files with this problem set-up were created corresponding to 5 different grid spacing values (h = 0.2, 0.1, 0.05, 0.025, 0.0125). The original version of ParOSol was modified to be able to apply the distributed body force. The error at each node is defined as the magnitude of the difference between the numerical and analytical (i.e. manufactured) displacement vectors $|u_{\text{num}} - u_{\text{MMS}}|$, normalised by the maximum value of the error across all nodes. The l_2 and l_∞ norms of these errors were analysed in dependence of grid spacing. At each refinement step, the observed order of convergence of the l_2 and l_∞ error norms were calculated as $OOC_{obs} = \ln(l_c/l_f)/\ln(r)$. Here, l_c and l_f correspond respectively to the error norms (l_2 or l_∞) at the immediately coarser and finer meshes, and r is the ratio of grid spacings between the two meshes.

Results and Discussion

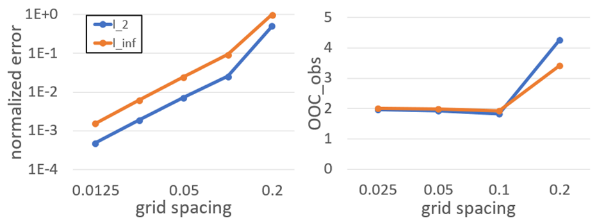

Fig. 2: Normalized error norms (left) and observed orders of convergence OOC_{obs} (right)

The observed convergence rates of l_2 and l_∞ error norms asymptotically approach the expected theoretical convergence rate (Fig. 2), evidencing a low likelihood of coding errors in the tested ParOSol version that can negatively influence the simulation results for linear elastostatic problems.

Conclusion

Expanding the suite of manufactured solutions will reduce this likelihood further. The methodology can be applied to newer versions of ParOSol that involve different constitutive models or require additional governing equations such as contact interactions.

References

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