SPATIALLY VARYING MULTI-COMPARTMENT MODEL OF BLOOD FLOW AND OXYGEN TRANSPORT IN THE HUMAN BRAIN

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Introduction

The brain relies on a continuous supply of oxygen and other metabolic supplies since its storage capacity if very limited. Brain tissue is thus very highly perfused with every brain cell within a few tens of micrometers of a blood vessel. However, obtaining information about flow and metabolism is highly challenging and mathematical models play an important role in interpreting clinical data. Most models, however, are based on highly simplified compartmental models or highly detailed network models. A new multiple compartment model of blood flow and oxygen transport is thus proposed obtained via a porous medium model.

Theory

Three blood compartments (arterial, capillary, and venous) are assumed for simplicity, although additional compartments can be added as required. Conservation of mass in each compartment gives:

$$\frac{\partial \phi_a}{\partial t} + \nabla.(\phi_a \mathbf{u}_a) = -\phi_a \beta_{ac}(p_a - p_c)$$

$$\frac{\partial \phi_c}{\partial t} + \nabla.(\phi_c \mathbf{u}_c) = \phi_a \beta_{ac}(p_a - p_c) - \phi_c \beta_{cv}(p_c - p_v)$$

$$\frac{\partial \phi_v}{\partial t} + \nabla.(\phi_v \mathbf{u}_v) = \phi_c \beta_{cv}(p_c - p_v)$$

where each compartment has volume fraction ϕ_i, pressure p_i and velocity field $\mathbf{u}_i$ respectively. It is assumed that perfusion coupling between compartments is linearly proportional to the volume fraction and the driving pressure difference, with coefficient β_{ij}. Darcy flow is assumed, following the homogenization procedure derived in [1]. Displacement of the solid phase is neglected, and a linear pressure-volume relationship assumed.

Continuity for oxygen transport is then applied, based on the same assumptions. These are re-written using the flow equations and conservation of volume. Further analysis of the relative magnitudes of the terms to simplify the equations then gives:

$$\frac{\partial S_A}{\partial t} = \nabla(S_A).(\mathbf{K}_a \nabla p_a) - \frac{k_a s_a}{c_{Hb}}(p_A - p_T)$$

$$\frac{\partial S_C}{\partial t} = \beta_{ac}(p_a - p_c)(S_A - S_C) - \frac{k_c s_c}{c_{Hb}}(p_C - p_T)$$

$$\frac{\partial p_T}{\partial t} = \frac{p_T}{\phi_t}[\nabla.(\phi_a \mathbf{K}_a \nabla p_a) + \nabla.(\phi_v \mathbf{K}_v \nabla p_v)] + \frac{k_a s_a}{\alpha_T}\frac{\phi_a}{\phi_t}(p_A - p_T) + \frac{k_c s_c}{\alpha_T}\frac{\phi_c}{\phi_t}(p_C - p_T) - \frac{M_o p_T}{\alpha_T(p_o + p_T)}$$

These are written in terms of the blood oxygen saturation and the tissue partial pressure of oxygen for convenience and a non-linear relationship for metabolism is assumed. Note that the venous compartment reduces to a constant oxygen concentration sink under the relevant assumptions, simplifying the governing equations significantly.

Values and boundary conditions are taken from [2]. The equations are then solved in a spherically symmetric annular shell, with properties scaled between grey and white matter as in [2].

Results

The radial variations in arterial and capillary blood oxygen saturation are shown in Figure 1. There is significant spatial variability with saturation values dropping towards the ventricles, although the tissue partial pressure remains high enough to ensure a sufficient metabolic rate throughout the tissue. Such variability is thus likely to be masked in imaging data.

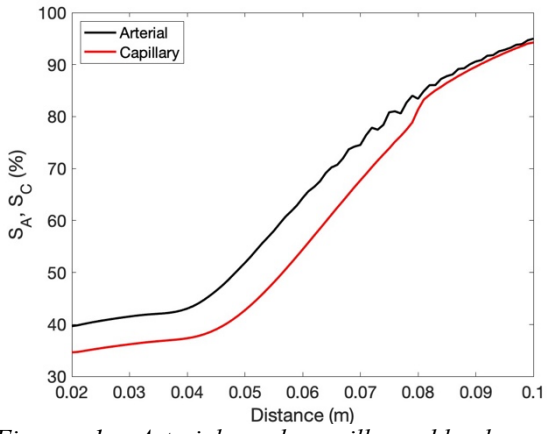

Figure 1: Arterial and capillary blood oxygen saturation against radial distance (both grey and white matter).

Conclusions

A new multi-compartmental framework for spatially varying cerebral blood flow and oxygen concentration is presented. Future work will focus on the validation of these results against available experimental data and testing the model against medical imaging models of perfusion and oxygen extraction factor (OEF).

References

1. El-Bouri WK, Payne SJ. J Theor Biol. 2015;380:40-7.
2. Jozsa IT et al. Ann Biomed Eng. 2021;49(12)3647-3665.

Acknowledgements

SJP is supported by a Yushan Fellowship from the Ministry of Education, Taiwan (# 111V1004-2).