EFFICIENT SENSITIVITY ANALYSIS FOR BIOMECHANICAL MODELS WITH CORRELATED INPUTS

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Introduction

Patient-specific biomechanical models are highly suited for outcome prediction of clinical interventions. Variance-based sensitivity analysis (SA) can be used for parameter-prioritization of these models based on their contribution to the total output uncertainty. Most variance-based SA approaches assume statistical input independence for the sake of computability while in many cases the inputs are correlated which may affect input importance ranking. Li et al. [1] proposed a new method to compute these correlated sensitivity indices. However, this method is computationally costly, especially for biomechanical models with large sets of inputs. Therefore, this study aimed to introduce an efficient SA methodology for biomechanical models while considering correlation.

Methods

The SA methodology is optimized concerning computational costs by using surrogate models. This methodology will be referred to as surrogate-based sensitivity analysis (SSA). A *vectorial kernel orthogonal greedy algorithm* [2] is used to create surrogate models of a 1D pulse wave propagation model (PWPM). This approach allows for the creation of a kernel-based surrogate model trained on a set of inputs and a specific output generated with the PWPM. The kernel-based surrogate model relates new input variables to an output of interest, thereby drastically speeding up the evaluation time. Afterward, a variance-based correlated SA, based on the work of Li et al. [1], was performed on the surrogate models. The trained surrogate models were verified using benchmark problems and an independent test set. To verify the two-step methodology whilst assuming input independence, SSA was compared to an established SA method for uncorrelated parameters based on adaptive generalized polynomial chaos expansion (*agPCE)* [3]. Later, the effects of considering correlations on the sensitivity indices (SIs) were investigated by changing the correlation coefficient ρ between the aortic length and diameter from $\rho \in (-1,1)$.

Results

Verification with the benchmark problems and the independent test set showed that the surrogates were able to correctly mimic the PWPM. The largest normalized root mean square test error was 0.031 [-]. The *agPCE* and SSA methods were both used to compute the uncorrelated SIs of the systolic and diastolic aortic pressure. The biggest observed difference between the computed SIs of both methods was only $6.7 \cdot 10^{-4}$ [-].

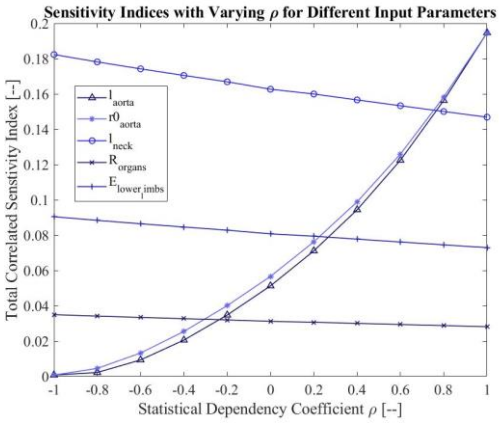

Figure 1: Computed SI for varying ρ[-].

Figure 1 shows the computed SIs for a subset of input parameters when considering correlations. Using a different value for the ρ considerably affects the importance ranking of the correlated input parameters. Moreover, the SIs of non-correlated input parameters were also affected, but to a smaller degree.

Discussion

The low errors and small differences found during verification contribute to the robustness of our newly proposed methodology. The SSA allowed for a correlated SA at a low computational cost for a relatively complex cardiovascular model. By applying SSA on the pulse wave propagation model, a speed-up of a factor 27000 [-] was achieved. In the future, this method could be applied to more complex models. Taking into account the correlations had a large impact on computed SIs. Therefore, when performing SA, it is necessary to take correlations into account and to learn the dependency structure of the input space beforehand.

References

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