

# ROBUST AND ACCELERATED SUBZONE-BASED NONLINEAR INVERSION IN ELASTOGRAPHY

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## Introduction

*In-vivo* mechanical characterization of soft tissues has emerged as a valuable diagnostic and therapeutic tool [1]. Techniques such as Magnetic Resonance Elastography (MRE) provide a platform for encoding motion-sensitive information with a phase-contrast pulse sequence, thereby providing an estimate of full-field displacement data. Using the measured data, an iterative inversion algorithm can then be used to extract the mechanical properties of soft tissue. Gradient-based optimization techniques are often employed to solve inverse problems. However, the inversion using the whole domain is computationally intensive and problematic. An alternative parallelized subzone-based approach [2] is implemented to expedite the inversion, in which the inverse problem is solved on each subzone independently. In addition, the superior convergence properties of the Levenberg-Marquardt algorithm have been utilized to accelerate the inversion algorithm.

## Methods

The inverse problem is solved using synthetic data generated by modelling the physical system of MRE as a Boundary Value Problem (BVP) of elastodynamics. A linear viscoelastic and nearly incompressible material behaviour is assumed. The boundary conditions, along with the assumed distribution of storage ( $G'$ ) and loss ( $G''$ ) moduli, are given in Fig. 1. A FEM-based mixed formulation is used for solving the forward problem to

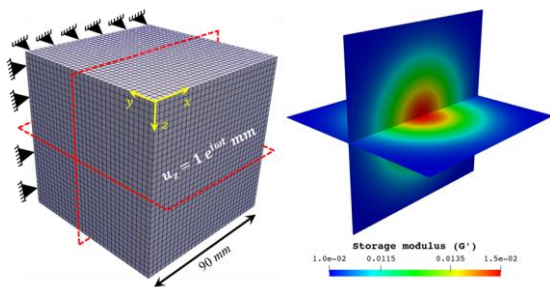


Figure 1: BVP using a cubic domain with assumed storage and loss moduli (MPa) distribution. The loss modulus follows the same distribution as the storage modulus but with values one order less.

obtain 3D harmonic displacement and pressure field solutions ( $f = 60$  Hz). Next, the generated displacement field is used as measured data ( $\mathbf{u}_m$ ) for the objective function evaluation and enforcing boundary conditions. In iterative approaches, a forward problem is solved at every iteration, followed by sensitivity analysis utilizing the latest estimate of the unknown material parameters

( $\mu$ ). In elastography, the forward problem is solved on the domain  $\Omega \subset \mathbb{R}^d$  bounded by  $\Gamma$  utilizing the following PDEs and boundary conditions:

$$\begin{aligned} \nabla \cdot (\mu(\nabla \mathbf{u} + \nabla \mathbf{u}^T) - 2/3 \mu(\nabla \cdot \mathbf{u})\mathbf{I} + p\mathbf{I}) &= -\rho \omega^2 \mathbf{u} \\ K(\nabla \cdot \mathbf{u}) - p &= 0 && \text{on } \Omega \\ \boldsymbol{\sigma} &= \lambda \text{tr}(\boldsymbol{\epsilon})\mathbf{I} + 2\mu\boldsymbol{\epsilon} && \text{on } \Omega \\ \mathbf{u} &= \mathbf{u}_m && \text{on } \Gamma \end{aligned}$$

where the scientific symbols have their usual meanings. In the subzone-based approach, rather than using the whole domain, the domain is divided into smaller overlapping spherical or cubic subzones. The inversion is then performed on each subzone in a parallel fashion.

## Results

The subzone-based approach has been found to accelerate the inversion process. For a mesh grid of  $32 \times 32 \times 32$ , the parallelized algorithm converged in 50 minutes on an 8 Core Intel Xeon(R) W-2245 processor (see Fig. 2). In addition, it has been observed that the subzone size, overlap, and number of iterations per subzone influence the accuracy of the solution.

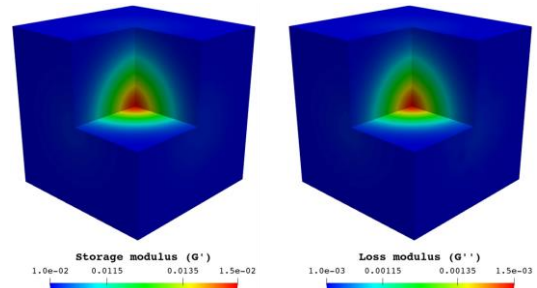


Figure 2: Reconstructed storage and loss moduli using the subzone-based approach.

## Discussion

A sensitivity analysis is being conducted to quantify the influence of each variable on reconstruction. Further, the developed algorithm is being tested on the publicly available *in-vivo* MRE dataset for the human brain [3].

## References

1. Manduca et al, Magn Reson Med, 85:2377-2390, 2021.
2. Van Houten et al, Magn Reson Med, 45:827-837, 2001.
3. <https://www.nitrc.org/projects/bbir/> (accessed 29/01/2023)

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