SINE-BASED ACTIVATION FUNCTION IS SUPERIOR IN PHYSICS-INFORMED NEURAL NETWORK FOR CARDIOVASCULAR FLOWS

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Introduction

Physics-Informed Neural Networks (PINNs) has emerged as a powerful approach to encode governing partial differential equations (PDEs) and training data to solve complex engineering problems. In cardiovascular applications, PINNs can be designed to encode Navier-Stokes equations and clinically-acquired hemodynamic data (e.g., from 4D Flow MRI) into the loss function to improve predictions. However, minimizing the loss function during the training process is challenging.

One strategy to improve the training operation is to use functions that maintain differentiability, non-linearity and minimize vanishing gradients; however, most popular activation functions lack these properties.

We propose to use a recently-introduced Fourier-based activation function that utilizes a periodic *sine* function [1]. The derivative of a *sine* function is a cosine, a phase-shifted *sine*, and thus, inherit the properties of the original function.

Methods

Test Case – 2D Stenosis: A 2D eccentric stenosis was created based on Varghese et al [2]. CFD simulations were performed using a higher-order finite-element solver, *Oasis [3],* at Re=5000 on a 240k triangle mesh.

PINNs Model: The solution $u(\boldsymbol{x},t)$ was approximated with a deep learning network $f(\boldsymbol{x},t;\theta)$, where θ represents the trainable parameters of the neural network. The loss function is defined as:

$$\mathcal{L}_{\text{total}}(\theta) = \mathcal{L}_{PDE} + \lambda_{BC}\mathcal{L}_{BC} + \lambda_{data}\mathcal{L}_{data} \qquad (1)$$

where $\mathcal{L}_{PDE}$, $\mathcal{L}_{BC}$ and $\mathcal{L}_{data}$ correspond to the loss term for the Navier-Stokes equation, boundary conditions, and known sensor data, respectively. The parameters λ_{BC} and λ_{data} aim to balance the interplay of the different terms in the loss function. The network consisted of 4 layers with 128 neurons/layer. The boundary conditions and velocity field at the mesh nodes were treated as unknown quantities. Sensitivity to sensor data (obtained from CFD) were tested by increasing sensor points from 25 to 400.

Activation Functions: We compared two standard activation functions, *swish* and *tanh* against a Fourier-based, periodic *sine* function (i.e., SIREN[1]). The formulation of the three activations functions are

$$swish: f(x) = x.sigmoid(x) \qquad (4)$$

$$tanh: f(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}} \qquad (5)$$

$$Sinus: f(x) = \sin(\omega_o x) \qquad (6)$$

Sinus requires special initialization for the first layer and normalization of the input parameters from -1 to 1. We applied a strategy proposed by Pan et al. in our work [3].

Results

While PINNs model with *sinus* and *swish* activation functions converged, the *tanh* solution diverged due to diminishing gradients issues. Figure 2A shows presence of unsteady vortical structures due to the geometric perturbation (i.e., non-axisymmetric stenosis). Figure 2B shows a monotonic decrease in L_2 errors as the number of sensor data increases from 25 to 400. There is a rapid decrease in error norm from 25 to 100 points for *sine* function compared to *swish* function. Figure 2C shows the velocity field estimated by PINNs for 25 and 400 sensor points, marked with P1 and P4 respectively. Even with 25 sensor points, the sine-based velocity field shows the spatial dynamics that are not captured by *swish* activation function. At 400 sensor points *swish* and *sine* show similar L_2 norms; however, the qualitative velocity maps demonstrate that *sine*-based predictions better match the CFD results.

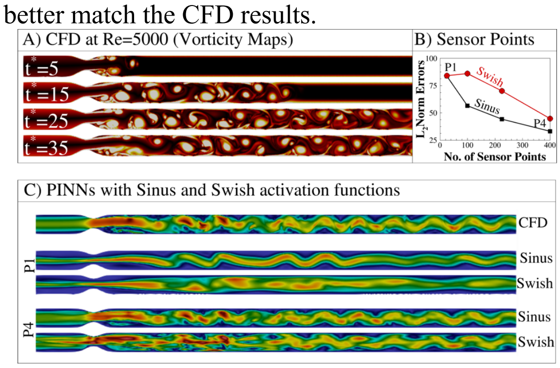

Figure 1: A) Temporal evolution of vortical structures over non-dimensional time, $t^* = tu_i/D$. B) Decrease in error with increasing sensor data, and C) PINNs-predicted velocity for sine and swish activation functions at $t^* = 35$.

Discussion

We have demonstrated that *sine* activation function can substantially improve velocity field predictions compared to conventional activation functions. Second, *sine* activation function was able to reconstruct the gross velocity field even with 25 sensor points. The reduced requirements on sensor data are beneficial since clinically-acquired hemodynamic data is often scarce and sparse, for example, in 4D Flow MRI or dynamic perfusion CT imaging.

References

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