MODELLING BLOOD FLOW IN A MICRO-VESSEL BIFURCATION

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Introduction

Microvasculature accounts for most vessel length in the human body and plays a key role in oxygen transport and nutrients delivery. Currently, a widely used way to simulate microcirculation is the flow network model, where blood vessels are treated as rigid straight pipes and Poiseuille flow is assumed in each vessel. However, the network model oversimplifies the details of blood flow characteristics, especially near the vessel junctions, limiting its wider applications. In this work, we study the blood flow in 2D micro-vessel junctions of various geometries and boundary conditions using a recently developed lightning Stokes solver [1].

Methods

Blood flow in microvasculature is governed by Stokes equations (Reynolds number is much smaller than 1):

$$\nabla p = \mu \nabla^2 \mathbf{u},$$
$$\nabla \cdot \mathbf{u} = 0.$$

Fully developed parabolic profile was imposed at both inlet and outlets of the junction and a no slip condition was assumed on the vessel walls.

Stokes flow was solved using a complex variable method, where the stream function was represented by two analytic complex functions (known as Goursat functions). Each analytic function was approximated using a rational function with poles exponentially clustered near each sharp corner of the flow domain [2]. Coefficients of the rational functions were found by imposing boundary conditions and solving a linear system.

A single vessel bifurcation consists of a parent branch and two child branches. We examined blood flow in bifurcations with different branching angle, diameter ratio and flow ratio between two child branches. The lightning Stokes solver allows for simulation of each case within one second on a standard laptop using Matlab.

Results

Figure 1 presents the solution of blood flow in a symmetric Y-junction with even flow partition and an angle of 90 degrees between two child branches. In this case, the maximum velocity was set as 1 and the streamlines were also plotted on top of the flow field. The smooth streamlines around the corners highlight the capability of the lightning Stokes solver to capture flow characteristics accurately around sharp edges. Note that this method is also capable of simulating blood flow in asymmetric geometries with asymmetric boundary conditions.

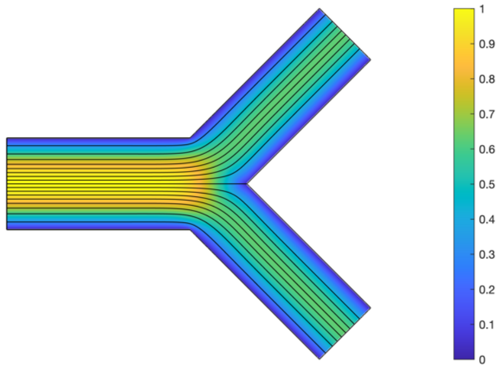

Figure 1: Blood flow in a symmetric Y-junction with even flow partition and an angle of 90 degrees between two child branches.

Discussion

One advantage of this new method for solving microcirculation is its great speed, accuracy and adaptability to different geometries and flow conditions. This mesh-free approach also makes it suitable for solving physiological fluid dynamics problems involving moving objects or boundaries.

A limitation of this work is that we neglect the effects of blood cells, especially red blood cells, on blood flow. In addition, our model geometry is 2D and purely passive. In future work, we will apply this method to simulate microthrombi transport in cerebral microcirculation. This will allow us to investigate the effects of cerebral microvascular geometry on microthrombi distribution, and thus their impact on blood flow and oxygen transport [3,4]. This will complement current *in silico* models of ischaemic stroke and its treatment [5].

References

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