CONTINUUM MECHANICS OF OPTIMAL TRABECULAR BONE

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Background

Trabecular bone contributes importantly to the various functions of the human skeleton and its heterogeneous architecture can be described by numerous parameters. Interestingly, bone volume fraction (BV/TV) and fabric (axes and extent of structural anisotropy) alone explain up to 98% of the variation in anisotropic elastic and yield properties independently of anatomical location. Accordingly, BV/TV and fabric are now widely used to estimate the apparent mechanical properties of trabecular bone in homogenized finite element models based on quantitative computed tomography images.
In fact, Cowin and others integrated BV/TV and fabric in their continuum level remodeling algorithms. The principal fabric axes align with the principal stresses when free energy density is minimized for a given stress state [1], but the extent of the fabric orientation remains unknown. Moreover, no explicit results are available for optimality criteria other than free energy density.

Recent Advances

In the past year, the above optimization problem was formulated, and resolved analytically with three criteria for trabecular bone in the light of fabric-elasticity relationships, and their solutions were compared.

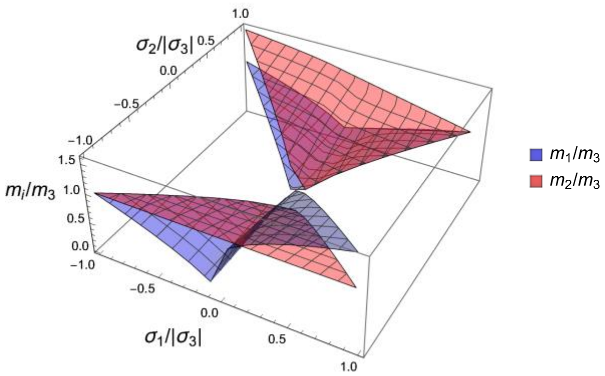

Figure 1: Fabric eigenvalue ratios as a function of the principal stress ratios for the isotropic, asymmetric maximum principal strain criterion.

For a given stress tensor, an optimal fabric was calculated that minimize volume fraction (BV/TV) under the constraint of a maximal biomechanical criterion. Three such criteria were selected: 1) the classical complementary free energy density, 2) an anisotropic measure of damage proposed in [2] and 3) an isotropic, but asymmetric measure of principal tensile and compressive strains. Fabric eigenvalue ratios were computed as a function of the principal stress ratios, while volume fraction was derived from the universal fabric-based material constants.
Analytical solutions were obtained for all criteria. For all criteria the minimum BV/TV was achieved for a fabric tensor that was aligned with the principal stresses. The extent of fabric anisotropy with respect to the stress anisotropy was derived for the first time and differed among the three criteria. The outcome of the principal strain-based criterion is shown in Fig. 1.

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Future Directions

This recent work confirms the alignment of the fabric tensor with the principal stresses for the free energy criterion obtained in [1]. It also extends this result for two additional criteria and delivers the optimal volume fraction and fabric for a given stress tensor. The most realistic criterion needs to be confirmed on experimental grounds, but the principal strain hypothesis matches the poro-elastic mechano-transduction principle at the extracellular matrix level and accounts for a different threshold in tension and compression.
The obtained results will allow to run and compare bone adaptation simulations at the continuum level for the three criteria. Most interestingly, the analytical resolution of the forward problem also provides the solution of the broader inverse problem that consists in finding the optimal stress tensor for an existing morphology. This last observation will contribute to the evaluation of musculoskeletal loading.

References

1. Luo Z. and An K., J Math Biology 36(6):557-568, 1998.
2. Schwiedrzik J. et al., Biomech Model Mechanobiol, 12(6):1155-1168, 2013.

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