A SEMI-ANALYTICAL MODEL FOR STRESSES IN THE PERIODONTAL LIGAMENT FOR A TOOTH UNDER LOADING

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Introduction

The periodontal ligament (PDL) binds the tooth to the alveolar crest. The orthodontic process involves exploiting the properties of this ligament to achieve the desired tooth displacement and orientation. Controlling the stress patterns can help optimize treatment in terms of speed and success of correction [1]. Previous studies ([1],[2]) have presented an analytical method for calculating the stiffness of the PDL in a single-rooted tooth. These models approximate the tooth as a paraboloid, where the strain distribution throughout the PDL for any given displacement is presented. The following research extends on the work by [1] and [2] by proposing a model which enables the use of patient data to determine the geometry of the root model, yet retaining the computational speed of the analytical models. This is useful where the root geometry for the patient can be obtained from CT imaging, allowing a closer approximation of the real root to be triangulated.

Methods

The PDL is assumed to be very thin, have uniform thickness, and have homogeneous, isotropic and linear elastic material properties. The boney socket and the tooth root are assumed to be rigid, and rotations of the PDL about each axis are assumed to be small. The surface is discretized into triangular elements. If the global displacements are applied to the crown, the strain in each element can be calculated since the geometric and material properties are defined in the literature ([1],[2]). Global stresses are calculated from summing the stiffness of all elements across the surface (Equation 1)

$$F = \Sigma \, (AE/\delta) \, K \, u \qquad (1)$$

where K is the stiffness matrix, F is a vector of forces and moments, u is a vector of displacements and rotations, a is the surface area, E is the Young's Modulus and δ is the thickness of the PDL. Hydrostatic stress (σ_{hyd}), the average of the three normal stress components, can be obtained from the strains that were calculated for each element, as defined by Equation 2.

$$\sigma_{hyd} = (E \, \varepsilon) \, / \, (3 \, (1 - 2v)) \qquad (2)$$

where ε is the normal strain on the element and v is the Poisson's ratio. The semi-analytical model was written in MATLAB and a parabolic surface replicating the surface in [1] was imported as an STL file.

Results

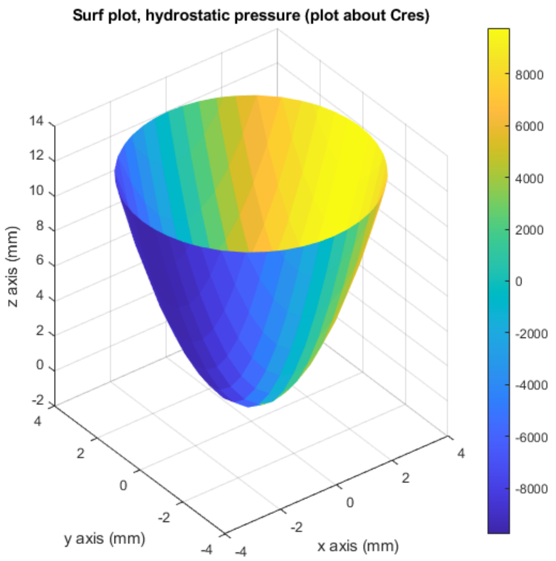

Figure 1 Triangulated paraboloid used in semi-analytical model. Pressure presented as colour scale; units in Pascals

The maximum hydrostatic pressure was compared to the model used in [1]. The analytical result for the maximum hydrostatic pressure was 9.200 kPa. The semi-analytical result was 9. 183 kPa, a difference of 0.19%. The variation of the hydrostatic pressure across the surface can be seen in Figure 1.

Discussion

The analytical model for the stresses in the PDL under loading was derived using models from [1] and [2]. The semi-analytical model described in this paper improves on both papers by enabling the user to input image data of real teeth to determine the PDL stiffness.. The results presented here closely matched the analytical solution; small discrepancies are expected due to discretization of surface.

References

1. Provatidis, International Journal of Engineering Science, 39:1361–81, 2001.
2. Van Schepdael et al, Medical Engineering and Physics, 35:403–10, 2013.