REMODELING OF 3D MATERIALS VIA A COMBINATION OF THE HOMOGENIZED CONSTRAINED MIXTURE THEORY WITH PLASTICITY

Felipe Sempertegui, Stephane Avril

Mines Saint-Étienne, Univ. Jean Monnet, INSERM, U 1059 Sainbiose, F-42023, Saint-Étienne, France

Introduction

The Homogenized Constrained Mixture Theory (H-CMT) [1] is a cost-effective alternative to its classical counterpart [2]. Even though both frameworks predict remodeling of soft tissues, the former requires less memory storage than the latter. This advantage is achieved through a homogenization in time, which results in an expression involving a stress rate tensor $\dot{\boldsymbol{\sigma}}$. The author [1] used this expression as the basis to formulate a ready-to-use equation that is applicable to 1D-like constituents only.

However, if one wishes to predict the remodeling of 3D materials (e.g., dispersed & anisotropic fibers [3]), an equation in tensorial format should be developed instead. In this work, we propose a H-CMT framework, which can be applied to 3D materials. It is formulated by intersecting the founding tensorial expression of the H-CMT [1] with finite plasticity. By re-interpreting the remodeling variable as a plastic term, we were able to predict the remodeling of 3D-like materials based on standard "return mapping" algorithms.

Methods

We assume that the homeostatic stress only triggers remodeling isochorically, that remodeling is incompressible and that the strain energy density function can be split into a volumetric and a deviatoric component. By integrating the expression containing $\dot{\boldsymbol{\sigma}}$ with the backward euler method and by choosing the deviatoric stress invariant J_2 as to reduce that resulting tensor expression to a scalar, we obtain

$$f = 0.5(\boldsymbol{\sigma}' - \boldsymbol{\sigma}_r'):(\boldsymbol{\sigma}' - \boldsymbol{\sigma}_r') = 0, \qquad (1)$$

where $\boldsymbol{\sigma}$ is the current stress, $\boldsymbol{\sigma}_r$ is a backstress variable that emerges from the aforementioned assumptions and the superscript " $'$ " indicates that only the isochoric component should be considered.

The scalar in Eq. 1 can be re-interpreted as a standard yield criterion for kinematic hardening, which is commonly used in finite plasticity.

Results

The intersection between the H-CMT and plasticity allowed us to adapt already existing "return mapping" algorithms and test them on anisotropic materials. We chose the strain energy function proposed by [3] to be our test case and we implemented it in our in-house Finite Element Method framework. Fig. 1 shows the evolution of stresses occurring in a completely constrained model. The stress components of the fiber develop such as to approach a homeostatic target.

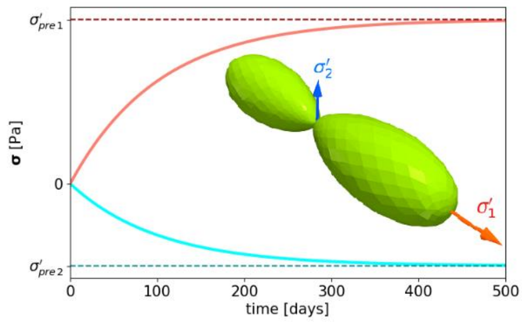

Figure 1: Evolution of the stress components of the anisotropic material proposed by [3]. The green ellipsoids represent the von Mises distribution of the fibers. The subscript "1" indicates the main direction of the fiber, "2" indicates its perpendicular direction and "pre" refers to the components of the homeostatic stress.

Discussion

We proposed an extended version of the H-CMT, which is built upon the overlap between the work of [1] and plasticity. This can be achieved by modifying the original stress rate tensor expression in [1] and by using a deviatoric stress invariant as an equivalent yield criterion.

The stresses in Fig. 1 converge towards homeostasis in the long term and their development is physically consistent. Due to these preliminary results, now the research community has access to a ready-to-use framework that predicts remodeling of 3D materials.

References

1. Cyron, C. J. et al., A homogenized constrained mixture (and mechanical analog) model for growth and remodeling of soft tissue. Biomechanics and modeling in mechanobiology, 15, 1389-1403, 2016Somebody et al, J Biomech, 45:1-10, 2014.
2. Humphrey, J. D. et al., A constrained mixture model for growth and remodeling of soft tissues. Mathematical models and methods in applied sciences, 12(03), 407-430, 2002
3. Gasser, T. C. et al., Hyperelastic modelling of arterial layers with distributed collagen fibre orientations. Journal of the royal society interface, 3(6), 15-35, 2006

Acknowledgements

This project is funded by the European Union's Horizon 2020 research and innovation programme (Grant Agreement 101017523).