Asset-side Bank Runs and Liquidity Rationing: A Vicious Cycle*

Zongbo Huang

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Abstract

I analyze runs on credit lines in an infinite-horizon banking model, focusing on the strategic complementarity between bankers and credit line borrowers. Panic drawdowns by borrowers and bank liquidity rationing can reinforce each other, creating a vicious cycle. Using data from U.S. banks, I estimate the model and quantify the amplification effect arising from strategic complementarity. This amplification effect accounted for two-thirds of the contraction of total bank credit during the 2008-2009 crisis. Moreover, I show that policies targeting borrowers have a crowding-in effect and can effectively contain credit contraction.

Keywords: Credit lines; liquidity rationing; strategic complementarity; amplification.
JEL codes: E44, E50, G01, G21, G28

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1 Introduction

The global financial crisis was fueled by panics that arose in unprecedented circumstances. These panics were triggered by disruptions in short-term liabilities, such as margin calls and rollover freezes, and by the run-like behavior of asset-side credit line borrowers. Empirical research (Ivashina and Scharfstein, 2010; Campello, Graham and Harvey, 2010) provide evidence supporting the credit line run narrative that borrowers strategically draw down credit lines as bank liquidity tightens.[1]

Despite extensive empirical evidence, important questions about the credit line run mechanism remain unanswered. How does it exacerbate credit contraction, and is it quantitatively significant? Additionally, what are the policy implications? To address these gaps, I estimate a dynamic banking model using data on U.S. banks. The model features dynamic interactions between borrowers' credit line run behavior and bankers' liquidity rationing, which I refer to as *banker–borrower strategic complementarity*.

My analysis focuses on the amplification effect due to strategic complementarity rather than multiple equilibria emphasized in deposit-run models. Using the estimated model, I investigate the quantitative importance of the amplification effect and find that it accounted for two-thirds of the U.S. banks' credit contraction during the crisis. Furthermore, policies that target borrowers can effectively contain credit contraction and may lead to a larger effect when combined with policies targetting banks.

The strategic complementarity arises from the contingency of credit line contracts and the bank balance sheet channel. On the one hand, credit lines are a form of contingent liquidity (Sufi, 2009; Acharya et al., 2021). Bankers can cut (stop rolling over or revoke) credit lines, which creates incentives for borrowers to strategically draw down credit lines to preserve funds and avoid separation from their credit lines. On the other hand, in downturns when intermediation costs increase, financing strategic drawdowns becomes costly, and the balance sheet channel (Bernanke et al., 1999) predicts that bankers will deleverage and ration liquidity, mainly through credit line cuts (Chodorow-Reich and Falato, 2022). The two strategic responses reinforce each other: credit

[1] Figure 1 reproduces the evidence. Bank credit contracted severely during the global financial crisis. Meanwhile, borrowers drew down a larger fraction of credit lines during the crisis, causing the loan-to-credit ratio to shoot up as drawn credit lines showed up on the balance sheet as loans.

line cuts induce strategic drawdowns, leading to more deleveraging pressure and further credit line cuts, creating a feedback process that amplifies adverse shocks during crises.

To quantify the importance of this amplification mechanism, I develop an infinite-horizon model in Section 2 that captures the banker–borrower strategic complementarity. In the model, a representative banker provides credit lines to a continuum of borrowers. Borrowers operate projects subject to liquidity shocks in the spirit of Holmstrom and Tirole (1998). The more borrowers rely on credit lines to ensure against shocks, the more strongly they react to potential separation from credit lines. To capture the bank balance sheet channel, I impose a leverage ratio requirement and an intermediation cost sensitive to bank leverage.

In this setting, a hike in intermediation cost triggers the amplification spiral between strategic drawdowns and credit line cuts. The magnitude of the amplification reflects the intensity of strategic complementarities that depend on the borrowers' reliance on credit lines, the banker's cost sensitivity to leverage, and the cost to cut credit lines. Moreover, because credit line contracts are long-term, the dynamic strategic complementarity arises from the continuation values.

In Section 3, I estimate the model using the simulated method of moments. Estimating models with strategic complementarities is challenging due to the potential for simultaneity and omitted–variable biases. Instead of estimating the strategic responses directly, I draw insights from the social interaction literature (Glaeser, Sacerdote and Scheinkman, 1996) that the stronger the strategic interactions are, the larger the amplification is when adverse shocks hit banks. Therefore, I compare credit line drawdowns and credit growth at the bank level before and during the 2008-2009 crisis to reveal the strength of strategic complementarity.

The estimated model has a good in-sample fit, replicates the time-series empirical patterns, and generates acyclical leverage because banks can deleverage by cutting credit lines without firesales. Furthermore, although I do not target any cross-sectional moment in the estimation, the estimated model captures the empirical associations between credit contraction and pre-crisis leverage accurately.

Next, I use the estimated model to quantify the amplification mechanism in Section 4. I start by plotting the impulse response functions to a shift from normal times to crises and show that dampening the response of either the bankers or borrowers contains the amplification effect. In section 4.2, I study a counterfactual with no strategic drawdowns, where the borrower's strategy is fixed and does not respond to the banker's strategy, leading to the collapse of the banker–borrower strategic complementarity. Comparing this scenario with the estimated model quantifies the amplification effect due to the strategic complementarity. The results indicate that the credit contraction is much less significant in the counterfactual. The amplification effect accounts for two-thirds of the credit contraction in the estimated model: bank credit contracts by 1.54% every quarter in the estimated model and by 0.54% in the counterfactual without amplification. To assess the robustness of the quantitative result, I follow Andrews, Gentzkow and Shapiro (2017) and compute the sensitivities to estimation moments in Section 4.3. The diagnostic suggests a limited misspecification bias.

Finally, I discuss policy implications in Section 5. Ex-ante prudential policies, such as the leverage ratio requirements, are effective because they enhance bank lending capacity during crises. Specifically, I consider a set of countercyclical policies: fix the leverage ratio requirement in crises and increase that in normal times from 6%, the baseline, to 6.4%. In the counterfactual with a requirement of 6.4%, both credit contraction and strategic drawdowns reduce by approximately half.

I then investigate two ex-post policies: the funding for lending (FFL) scheme, which aims to reduce bank intermediation costs, and the corporate credit facilities (CCFs), which provide direct liquidity to borrowers. While policies targeting banks, such as FFL, have been traditionally used to support bank credit to the real economy, policies targeting real businesses, such as CCFs, have gained attention and were implemented during the Covid-19 crisis due to concerns about potential bank malfunction.

My analysis unveils a compelling argument in support of policies like CCFs that target borrowers. Although such policies do not directly target banks, they can effectively contain credit contraction and stimulate bank credit due to the banker–borrower strategic complementarity. Furthermore, such policies may generate an even greater impact when combined with FFL schemes.

My paper connects to several strands of the bank run literature (Allen and Gale, 1998; Cooper and Ross, 1998; Rochet and Vives, 2004; Goldstein and Pauzner, 2005; Vives, 2005; Liu, 2016). Previous studies suggest that liquidity management strategies, such as suspension of deposit convertibility (Engineer, 1989; Cipriani, Martin, McCabe and Parigi, 2014; Ennis and Keister, 2009) and fund cash-rebuilding (Zeng, 2017; Shek, Shim and Shin, 2018; Morris, Shim and Shin, 2017), may exacerbate, rather than mitigate, runs in a dynamic framework. While this work usually focuses on creditors' reactions to liquidity adjustments, I combine the insight with the bank balance sheet channel and study the strategic complementarity between bankers and borrowers.

My study also contributes to the literature on how capacity constraints lead to strategic complementarities (Arellano and Kocherlakota, 2014; Drozd and Serrano-Padial, 2018; Bond and Rai, 2009; Infante and Vardoulakis, 2021). However, unlike these studies, the constraint in my model emerges endogenously from the banker's dynamic problem. In addition, my paper complements studies on dynamic coordination failures (He and Xiong, 2012; Cheng and Milbradt, 2012; Schroth, Suarez and Taylor, 2014). In my model, dynamic strategic complementarities emerge because credit lines are contingent rather than due to a staggered debt structure.

Furthermore, my paper contributes to the macro-finance literature on financial frictions and recent works that study optimal regulations using dynamic models by introducing a novel amplification mechanism. Works by Gertler and Kiyotaki (2015) and Gertler et al. (2020) connect the bank balance sheet channel with bank runs (see also Benhabib, Miao and Wang (2016) and Miao and Wang (2015)). They characterize runs as self-fulfilling rollover freezes following Cole and Kehoe (2000). In contrast, I model runs due to liquidity mismatch in the spirit of Diamond and Dybvig (1983), and the model features realistic partial runs instead of a sudden banking sector collapse, thus connecting more closely to empirical evidence.

Finally, my paper builds on the empirical literature on credit lines (Chava and Roberts, 2008; Sufi, 2009; Roberts and Sufi, 2009a; Nini et al., 2009; Ippolito et al., 2019; Acharya et al., 2021; Chodorow-Reich and Falato, 2022). These studies show that credit lines, as contingent liquidity, depend on bank soundness and that firms run on credit lines in downturns. However, little is known about the quantitative importance of these findings in exacerbating adverse shocks. Complementing these studies, I structurally estimate a dynamic banking model to quantify the amplification arising from this contingency.

2 The Infinite Horizon Model

In this section, I develop an infinite-horizon model that incorporates three key features of credit lines: (1) banks can ration liquidity through credit line cuts (Roberts and Sufi, 2009a; Sufi, 2009), (2) banks use this discretion based on their liquidity soundness (Ippolito et al., 2019; Chodorow-Reich and Falato, 2022), and (3) credit line drawdowns surge when borrowers are worried about their future access to credit lines (Ivashina and Scharfstein, 2010; Ippolito et al., 2015).

The model considers a representative banker who lends to a continuum of credit line and passive term loan borrowers. All agents are risk-neutral with the same discount rate of $1-\beta$. I use $\mathbf{S} = [E, \Phi, L, s]$ to represent the current values of the payoff-relevant states, which include bank equity E, credit limit Φ, term loans L, and exogenous state s.

The timing of each period is as follows. First, the exogenous state of the economy is realized and publicly observed. The state $s \in \{n, c\}$ captures normal and crisis periods and follows a Markov process with transition probabilities $\pi(s', s)$. In other words, the occurrence of a crisis is an exogenous shock to the economy rather than an endogenous outcome or a result of equilibrium selection. Then, borrowers choose their dividends and investment. Next, each borrower privately observes a liquidity shock $\lambda \in \{0, 1\}$ and a return shock $\kappa \in [0, 1]$, both of which are identically and independently distributed over time and across borrowers. Each borrower decides how much to draw down from credit lines based on these shocks. At the end of the period, the banker decides on liquidity provision.

2.1 Borrowers' Problem

Each period, a borrower starts with a net worth ω and a credit line with a limit of ϕ. The borrower has three choices to make: dividend payout c, investment k, and drawdown l. In the spirit of Holmstrom and Tirole (1998), borrowers face liquidity shocks with probability Λ.[2] If a liquidity shock occurs, the borrower needs to inject an additional ρk to prevent costly liquidation. Therefore, the borrower keeps enough buffer such that $(1+\rho)k \leq \omega - c + \phi$. I assume that borrowers retrieve the injected funds ρk fully at the end of the period to preserve the size of their business.

The ex-ante value function before the borrower makes any choices and observes the realizations

[2] The probability does not depend on the exogenous state. I discuss empirical evidence supporting this assumption in Section 3 and perform a robustness check in Section D of the Internet Appendix.

of the idiosyncratic shocks is denoted by $v(\omega, \phi; \mathbf{S})$. After observeing the idiosyncratic shocks but before the drawdown decision, the borrower's value function is denoted by $u(\omega - c, \phi, k, \lambda, \kappa; \mathbf{S})$. Thus,

$$v(\omega, \phi; \mathbf{S}) = \max_{c \geq 0, k \geq 0, (1+\rho)k \leq \omega - c + \phi} c + (1 - \beta)\mathbb{E}_{\lambda,\kappa}[u(\omega - c, \phi, k, \lambda, \kappa; \mathbf{S})]. \tag{1}$$

Equation (2) characterizes how the borrower's net worth evolves. The investment technology is modeled as linear. The borrower receives a gross return of r_k while paying a loan rate of $r(s)$ on the drawdown and a constant maintenance fee r_m on the limit. The loan rate is dependent on the exogenous state s as it is determined as a constant spread over benchmark interest rates, which change significantly during crises. On the other hand, the maintenance fee, a fixed amount specified in the contract, is independent of the state. The return r_k is state-independent as well. Nevertheless, as discussed later, this assumption does not affect strategic drawdowns since the borrower's value functions are separable in net worth and credit limit.

In periods without a liquidity shock ($\lambda = 0$), the borrower can choose to take a positive drawdown and invest the funds, earning an idiosyncratic return $\kappa r(s)$. The parameter κ follows a cumulative distribution function $F(\cdot)$, which I assume to be uniform between zero and one in the empirical analysis.

$$\omega' = \omega - c + r_k k - r_m \phi - r(s)l + (1 - \lambda)\kappa r(s)l. \tag{2}$$

The return shock measures how efficiently borrowers can utilize the funds they have drawn. If $\kappa = 0$, borrowers can only hold the funds without being able to invest them. On the other hand, if $\kappa = 1$, borrowers can invest the funds and earn the same rate as what they pay. This return shock creates differences in the cost of drawdowns in period absence of a liquidity shock. While the benefit of drawing down funds is homogenous across borrowers, only those with sufficiently high return shocks will choose to do so. Consequently, the model exhibits partial runs on credit lines, which is consistent with the behavior observed in real-world data.

The optimal investment and dividend decisions are corner solutions in the case of risk-neutrality and linear technology. Specifically, if the investment return is higher than the expected cost of credit lines, the borrower will invest as much as possible, which is $(1 + \rho)k = \omega - c + \phi$. On the other hand, if the investment return is lower than the expected cost of credit lines, the borrower will not use the credit line at all. Assumption 1 ensures that credit lines are relevant and the formal case holds.

Assumption 1 *The expected net return of investment is positive; that is,* $r_k - (1+\rho)r(s)\Lambda > 0$.

Additionally, the borrower always chooses to delay dividend payouts when the expected return of net worth is greater than one, and pays out all net worth immediately when the expected return is less than one. However, these cases do not reflect realistic dividend policies. To address this, I follow the financial accelerator literature (Gertler and Kiyotaki, 2015) and introduce an exogenous probability ξ, beyond Assumption 2, that the borrower exits. It is worth noting that exits are distinct from defaults, which typically result in the renegotiation and continued operation (Cooley and Quadrini, 2001). When the borrower exits, all net worth is paid out as dividends, and a new borrower is brought in to maintain the lending relationship with the banker.

Assumption 2 *The return of net worth is greater than 1; that is,* $\beta\left(1+\frac{r_k}{1+\rho}\right) \geq 1$.

Next, I outline the law of motion for the credit limit, focusing on the scenarios where the banker rations liquidity, as this represents the main tension in the model. Borrowers face the risk of losing their unused credit line with a probability p, which is endogenous and the same for all borrowers. Notably, this probability is independent of privately observed shocks that are unknown to the banker. Furthermore, since the banker cannot cut drawn credit lines, the credit limit for the next period either drops to the drawn amount l with probability p or remains at its previous level ϕ with probability $1-p$.

When the banker extends new credit lines without rationing liquidity, the model assumes that the credit limit increases proportionally with the borrower's net worth, subject to availability. Furthermore, if the credit lines provided by the banker grow faster than the net worth of existing borrowers, the excess credit limit is distributed to newly entered borrowers based on their net worth. Further details are available in Section B.1 in the Internet Appendix.

After introducing the two laws of motion, I analyze the drawdown choice of borrowers. Since the borrower invests the maximum amount such that $(1+\rho)k = \omega - c + \phi$, she must fully draw down the credit line to avoid liquidation when $\lambda = 1$. She solves

$$u(\omega, \phi, k, 1, \kappa; \mathbf{S}) = \max_{l} \mathbb{1}_{[l=\phi]} \left\{ \xi\omega' + (1-\xi)\mathbb{E}_{\mathbf{S}'|\mathbf{S}}[v(\omega', \phi; \mathbf{S}')] \right\} + \mathbb{1}_{[l<\phi]}\, v^L, \tag{3}$$

where ξ is the exit rate, which is acyclical (Evans, 1987; Cooley and Quadrini, 2001; Lee and Mukoyama, 2015). The liquidation value, denoted as v^L, is set low enough to ensure that the borrower chooses to fully draw down the credit line to avoid liquidation. Consequently, the banker cannot cut the borrower's credit line.

In the absence of a liquidity shock, a critical strategic choice emerges for the borrower. She faces a trade-off: by drawing down credit lines, she incurs loan costs that reduce her net worth but can secure the credit line and avoid potential separation from the banker. The borrower solves

$$u(\omega, \phi, k, 0, \kappa; \mathbf{S}) = \max_{l} \xi\omega' + (1-\xi)\mathbb{E}_{\mathbf{S}'|\mathbf{S}} \left[(1-p)v(\omega', \phi; \mathbf{S}') + pv^E(\omega', l, \phi; \mathbf{S}') \right]. \tag{4}$$

where $v^E(\omega, l, \phi; \mathbf{S})$ denotes the continuation value after separation, and $v(\omega, \phi; \mathbf{S}) - v^E(\omega, l, \phi; \mathbf{S})$ represents the loss from separation.

Suppose a borrower is separated from her previous credit lines and can only invest using the drawn funds l, which are below the previous credit limit ϕ. The borrower can search for a substitute, and with an exogenous probability of η in each period, she may obtain a new credit line with the previous credit limit ϕ and exits the separated status. The probability of substitution reflects the friction in the search process. As discussed later, since the banker only cuts credit lines in crises, η is only relevant in crises. For a formal presentation of the problem after separation, please refer to Section B.2 in the Internet Appendix.

Finally, I show in Section B.3 in the Internet Appendix that the value functions are linear: $v(\omega, \phi; \mathbf{S}) = v_1(\mathbf{S})\omega + v_2(\mathbf{S})\phi$ and $v^E(\omega, l, \phi; \mathbf{S}) = v_1^E(\mathbf{S})\omega + v_2^E(\mathbf{S})l + v_3^E(\mathbf{S})\phi$. The linearity implies that the correlation between the credit limit and net worth is irrelevant to the aggregate dynamics. Moreover, when $\lambda = 0$, the borrower's drawdown decision degenerates to a cutoff rule: $l = \phi$ if the return shock κ exceeds a state-dependent threshold $\bar{\kappa}(\mathbf{S})$ that makes the net cost of drawdowns small, and $l = 0$ otherwise.

As a result, the model features partial runs. Given that the total credit limit of all borrowers equals the credit limit granted by the banker, liquidity-driven drawdowns amount to $\Lambda\Phi$, and the strategic drawdowns are represented by $(1-\Lambda)\Phi\left[1 - F(\bar{\kappa}(\mathbf{S}))\right]$. The resulting aggregate drawdown, denoted as L_D and defined in Equation (5), is the sum of these two types of drawdowns.

$$L_D = \Lambda\Phi + (1-\Lambda)\Phi\big[1 - F(\bar{\kappa}(\mathbf{S}))\big]. \tag{5}$$

2.2 Banker's Problem

In each period, the banker begins with equity E, term loans L extended to firms outside the model, and credit lines with a total limit Φ. The banker raises deposits to finance both term loans and used credit lines with state-dependent loan rates $r(s)$ and deposit rates $z(s)$. Given the used credit lines L_D, the per-period profit is

$$\Pi = (1-\tau)\big[r_m\Phi + r(s)(L+L_D) - z(s)(L+L_D-E) - G(E, L+L_D, s)\big], \tag{6}$$

where τ is the tax rate and r_m is the maintenance fee. The last term $G(E, L+L_D, s)$ represents the operating cost, including wage and communication costs, which generally captures intermediation costs that increase with bank leverage (see, e.g., Berger et al. (2017) and Gambacorta and Shin (2018)). In the empirical analysis, I set $G(E,L,s) = a_n(L+L_D)$ in the normal state and $\big(a_c - \psi\frac{E}{L+L_D}\big)(L+L_D)$ in the crisis state, where ψ parameterizes the sensitivity to leverage.

After borrowers draw down credit lines, the banker chooses how much liquidity to provide. He can ration liquidity through credit line cuts.[3] Importantly, the liquidity rationing strategy cannot depend on borrowers' idiosyncratic shocks that the banker does not observe. This implies that the banker faces ex-ante identical borrowers and is indifferent to cutting the credit lines of any two borrowers. Consequently, the banker's strategy degenerates to the total size of credit limit cuts Δ, which is limited to the size of unused credit lines $\Phi - L_D(\mathbf{S})$.

In addition, the banker can issue new credit in the form of credit lines or term loans, denoted as N_C and N_L, respectively; however, since the model focuses on situations where the banker rations liquidity, the choice between credit lines and term loans does not significantly impact the model's implications. Specifically, I assume the banker has a predetermined target ratio between credit lines and term loans, where credit lines account for 56% of bank credit, and term loans

[3] Motivated by the observation that net issuance of term loans was positive during the 2008-2009 crisis in the DealScan data, I assume that the banker only rations liquidity only through credit lines but not term loans. Chodorow-Reich et al. (2022) also find that credit lines are the main margin of liquidity rationing.

account for 44%. If credit line cuts result in a ratio below the target, the banker will issue new credit lines to restore the ratio to the target level. Otherwise, if the ratio is at the target level, the banker will issue new credit lines and term loans proportionally to maintain the target ratio.[4]

Moreover, to ensure a well-defined bank problem, I assume that the banker exits with probability ξ^b in each period.[5] If the banker exits, he sells the bank to an entrant and pays out equity E' as dividends. The banker can also pay out dividends d before exit, but I do not allow the banker to issue new equity since banks issued little equity during the 2008-2009 crisis. The banker's problem is

$$V(E,\Phi,L,s) = \max_{d,\Delta,N_C,N_L\geq 0} d + (1-\beta)\left[\xi^b E' + (1-\xi^b)\,\mathbb{E}_{s'|s}\,V(E',\Phi',L',s')\right] \tag{7}$$

$$\text{s.t.} \quad E' = E + \Pi - d - C(\Delta, N_C, N_L, \Phi + L) \tag{8}$$

$$E' \geq \zeta(\Phi' + L') \tag{9}$$

$$\Delta \leq \Phi - L_D, \tag{10}$$

$$\Phi' = \Phi - \Delta + N_C, \tag{11}$$

$$L' = L + N_L, \tag{12}$$

where inequality (9) represents a leverage ratio requirement, where the equity-to-credit ratio must exceed ζ. I choose to impose the leverage ratio requirements instead of the capital requirement as it takes into account the unused credit lines and provides a more direct connection to the model.

The term $C(\Delta, N_C, N_L, \Phi + L)$ in equation (8) represents loan adjustment costs, which cause a slow response in bank variables. These costs play a similar role to those in Q-theory models of investment (Hayashi, 1982) and in the financial accelerator literature (He and Krishnamurthy, 2013; Brunnermeier and Sannikov, 2014; Gertler, Kiyotaki and Prestipino, 2020). The costs can also be explained by asymmetric information, which includes screening costs, legal expenses, and opportunity cost of time (Roberts and Sufi, 2009b; Berlin, Nini and Edison, 2020; Anderson,

[4] Mian and Santos (2018) find that the average drawdown rate was 57% between 1988 and 2010, ranging from 50% to 65% in different years. Additionally, Chodorow-Reich and Falato (2022) report that the mean drawdown rate during 2006–07 was 53% for the whole sample, 56% for the sample with covenant information, and 55% for the lender–covenant sample. Based on these drawdown rates and balance sheet data presented in Table 1, credit lines accounted for approximately 56% of bank credit before the crisis.

[5] Similar to the perpetual youth models, the effective discount rate is $\beta(1-\xi^b)$. A well-defined problem requires it to be larger than bank credit growth; otherwise, the bank value becomes unbounded.

Hachem and Zhang, 2021).

In the empirical analysis, I follow De Nicolò, Gamba and Lucchetta (2014) and set $C(\Delta, N_C, N_L, \Phi + L) = [\frac{\gamma^-}{2}\Delta^2 + \frac{\gamma^+}{2}(N_C + N_L)^2]/(\Phi + L)$. This means that the banker incurs screening and monitoring per-unit costs γ^+ when issuing new credit and per-unit rationing costs γ^- when cutting credit lines.[6]

The banker aims to achieve optimal leverage, which depends on the exogenous state s. During normal periods, the profit is linear in assets and liabilities and hence does not depend on leverage. However, the banker still targets an interior solution for optimal leverage due to the leverage ratio requirement and the potential for distress in crises. In crisis periods, the banker targets higher leverage as the profit decreases and becomes sensitive to leverage.

At the optimal leverage, the marginal value of the next period's equity is equal to one over the discount rate β. As the banker's value is concave, he pays out dividends whenever the leverage exceeds the state-dependent optimal level.

At the normal steady state, which is when the exogenous state remains normal for an extended period, the banker's leverage stays at the optimal level. Moreover, the new credit issuance is chosen such that the marginal cost equals the marginal value of additional credit, similar to the Q theory. If the leverage is below the optimal level, the banker issues less new credit to accumulate equity more quickly. In normal periods, the banker will not cut any credit lines.[7]

During crises, cutting credit lines is an effective way for the banker to deleverage. The optimal size of credit line cuts is determined such that the benefit from deleveraging is equal to the cost of downsizing plus the rationing costs.

2.3 Equilibrium and Banker–Borrower Strategic Complementarity

Bankers adjust both prices and quantities in practice to reduce their exposure to credit lines. However, either price adjustments or quantity adjustments create the same incentives for borrowers to strategically draw down their credit lines, which can increase their bargaining power and avoid

[6]The per-unit costs may depend on the exogenous state s, but in the model, the banker only cuts credit lines during crises and extends new credit in normal periods. Hence, it is sufficient to have two cost parameters. Additionally, I do not distinguish between issuing new credit lines and new term loans, as both incur screening and monitoring costs.

[7]When the leverage ratio binds and the banker's profit is insufficient to reduce leverage, the banker may cut credit lines to meet the leverage ratio requirement in normal periods. However, this case is not relevant given the empirical moments in Section 3. Furthermore, this aligns with empirical evidence that banks overlook covenant violations and do not cut credit lines in normal times.

adverse adjustments. Therefore, strategic complementarity is not tied to any particular type of adjustment. Nevertheless, due to the information asymmetry in the financial market (Stiglitz and Weiss, 1981) and to ensure tractability, I focus on quantity adjustments and analyze pure liquidity rationing.

In this scenario, the banker randomly restricts the unused credit lines of borrowers that are identical ex-ante, and each borrower faces the same likelihood of separation. The probability of separation, represented by p, equals the size of credit line cuts (Δ) divided by the total unused credit lines ($\Phi - L_D$). Importantly, since the borrower's value function is linear, random separation is equivalent to a reduction in the borrower's credit limit.

I consider the Markov perfect equilibrium, where value functions, the banker's policy function, and aggregated borrower drawdowns are functions of the payoff-relevant states $\mathbf{S} = [E, \Phi, L, s]$. Formally,

Definition 1 *A Markov perfect equilibrium is given by value functions for borrowers and the banker* $\{v, V\}$*, policy functions* $\{c, k, l\}$ *and* $\{d, \Delta, N_C, N_L\}$*, and probability* p *such that*

1. $\{v, V\}$ *solve problems* (7) *and* (1)-(4) *given the policy functions;*
2. $\{c, k, l\}$ *are optimal given value function* v *and the banker's policy functions;*
3. $\{d, \Delta, N_L, N_C\}$ *are optimal for the banker given value function* V *and the drawdown policy;*
4. *The total credit limit of all borrowers equals the credit limit granted by the banker* Φ*. The aggregate drawdown is given by* (5)*. The separation probability is* $p = \Delta/(\Phi - L_D)$*.*

The model incorporates a dynamic strategic complementarity between borrowers' drawdown decisions and the banker's liquidity rationing. An increase in aggregate drawdown leads to a mechanical increase in bank leverage. Since operating costs increase with leverage during crises, the banker faces stronger deleveraging pressure and will ration liquidity more aggressively. The more sensitive operating costs are to leverage, the more responsive the banker's rationing policy will be to aggregate drawdowns.

Meanwhile, anticipating more stringent rationing, borrowers have a greater incentive to strategically draw down their credit lines. The probability of substitution, η, determines the sensitivity of strategic drawdowns to the liquidity rationing policy. The higher this probability is, the smaller the borrowers' response will be. In this way, the strategies of the banker and the borrowers reinforce each other, leading to a vicious cycle that amplifies adverse shocks.

2.4 Numerical Solution

I solve the model numerically using global methods. I briefly sketch the algorithm below and discuss the implementation details in Section C in the Internet Appendix. To approximate value and policy functions, I use piecewise linear functions evaluated at a state vector grid and iterate them until they converge. To update each agent's optimal policies and value functions in each round, I follow the algorithm proposed by Pakes and McGuire (1994) and the approximated value functions and the other agents' policy functions from the previous iteration.

A key challenge in solving the model is the kinks in value and policy functions, which arise due to borrowers' drawdown decisions responding only to the banker's strategy when the banker rations liquidity. These kinks propagate through strategic interactions to the entire state space and substantially complicate the solution. To deal with this issue, I follow Iskhakov et al. (2017) and introduce random noise, which can be a structural uncertainty shock or a smoothing device. I then use simple moving averages to smooth out the kinks and ensure consistent solutions under a set of parameters in the estimation.

Another challenge is that strategic complementarity may lead to multiple equilibria. However, since the data do not suggest clustering into a small finite number of distributions, as implied by multiple equilibria, I focus on the amplification effect and partial runs instead. To avoid the issue of multiplicity, I impose an upper bound on the degree of strategic complementarity during the estimation process. After estimating the model, I verify numerically that the equilibrium is unique using different initial guesses of the value and policy functions.

2.5 Model Discussion

The amplification mechanism in the model is robust to alternative modeling choices. First, while the assumption that borrowers can draw down credit lines before the banker moves is based on the observation that banks renegotiate credit lines with lags due to information frictions, the mechanism remains valid under alternative assumptions. In particular, credit line runs can still occur even if the banker moves first, as long as credit line cuts are gradual, as indicated in the data. More broadly, although I rely on empirical evidence to motivate the features of credit line contracts, they can also arise as an optimal design in the presence of realistic contractual and informational frictions (Payne, 2018).

Second, while the borrower side is stylized in my model, the linearity of the value function permits richer heterogeneities. For instance, introducing the covenant violation status and assuming that the banker can only cut credit lines with violations allows for different borrower responses depending on their violation status. Similarly, I do not distinguish between retirement and revocation, but it is possible that borrowers respond less to retirement because banks have more bargaining power over maturing credit lines. The model accommodates such heterogeneities among borrowers and other idiosyncratic shocks that occur after drawdowns. What matters for aggregate dynamics is the average response. For aggregate dynamics, estimating the response of aggregate drawdowns to the separation probability suffices, as it reflects a weighted average of individual responses. Furthermore, the model can incorporate autocorrelated shocks and shocks realized before drawdowns by introducing extra state variables and expanding the strategic space.

Finally, it is worth noting that the model is a partial equilibrium model. While loan and deposit rates depend on the exogenous state that captures monetary policies outside the model, they do not depend on endogenous state variables. However, it is possible to endogenize the deposit rates and micro-found the operating costs by incorporating an elastic deposit supply. Additionally, loan rates can also be recognized as a spread that depends only on the exogenous state, above an endogenous base rate. In contrast, endogenizing loan rates for each credit line is not tractable as it would significantly expand the state and strategy space. This would complicate the banker's problem; for example, the liquidity rationing policy would depend on the endogenous rate and its distribution across borrowers.

Besides prices, the model can be extended to include other general equilibrium features. For instance, one possible extension is to endogenize the probability of substitution, making it dependent on the aggregate liquidity provision and search frictions. This extension introduces another externality among identical banks, leading to further amplification. However, estimating the search process would require additional data. Another potential extension is to model where funds flow after being drawn down strategically. If more funds flow back to the banking system, the intermediation cost will decrease, attenuating the amplification mechanism. Nevertheless, this force is likely to be small in magnitude. During the 2008-2009 crisis, borrowers tended to invest funds in assets with an explicit government guarantee, such as Treasury bonds (Acharya and Mora, 2015). Although some funds may flow back to banks, they are more likely to go through the wholesale market, which incurs higher costs (Drechsler et al., 2021).

3 Estimation

Estimating models that involve strategic complementarities presents a challenge due to the endogenous nature of agents' strategies and the potential impact of common shocks. To address this issue, I adopt an approach from the social interaction literature and estimate the model at the group (bank) level rather than the individual (loan) level. Specifically, I draw on the insights of Glaeser, Sacerdote and Scheinkman (1996). They argue that the high variance in crime rates across time and space, such as the significant decline in U.S. homicide rates from 1933 to 1961, cannot be fully explained by observable factors, suggesting the importance of strategic interactions. Similarly, I examine the time-series variance in bank credit growth and credit line drawdowns before and during the 2008-2009 crisis. As banks did not ration liquidity before the crisis, the strategic complementarity between bankers and borrowers only emerged during the crisis. Thus, contrasting these periods reveals the degree of strategic complementarity. Specifically, a stronger strategic complementarity leads to more severe credit contractions and aggressive credit line drawdowns.

One potential concern is that the exogenous shock that hit banks during the crisis and the amplification from strategic complementarity can both cause bank credit to contract. To isolate the two effects, I follow Glaeser, Sacerdote and Scheinkman (1996) and control for observed differences in loan rates, deposit rates, and net non-interest expenses across time. Additionally, I impose a specific structure motivated by the bank balance sheet channel to capture amplifications. Specifically, I assume that the marginal cost of intermediation is increasing in bank leverage. Thus, once borrowers draw down more credit lines, bank leverage increases, and banks have greater incentives to cut credit lines, causing more severe credit contraction.

Another similar concern is the increase in credit line drawdowns during the crisis may reflect higher demand for operating liquidity rather than strategic interactions. However, several pieces of evidence suggest that this alternative explanation is unlikely. First, according to the Federal Reserve's Senior Loan Officer Opinion Survey on Bank Lending Practices, banks reported a broad reduction, rather than an increase, in demand for new loans during the crisis. Second, the loan-to-credit ratio barely increases for banks with positive credit growth, which suggests that borrowers with these banks do not draw down more. Moreover, the empirical literature finds little assortative matching between banks and borrowers (Chodorow-Reich and Falato, 2022). Thus, the aggregate demand for operating liquidity is likely similar for all banks, which cannot account for the observed increase in drawdowns. Third, Chodorow-Reich and Falato (2022) show that only borrowers who violate credit line covenants and borrow from less healthy banks used a larger fraction of credit lines in 2008-2009 than in 2006-07. This finding is consistent with the credit line runs narrative rather than a higher demand for operating liquidity.

To further address this concern, I conduct a robustness analysis in Section D of the Internet Appendix by re-estimating the model with different assumptions about the demand for operating liquidity during crises. The results of this analysis indicate that the magnitudes of the amplification effect remain similar, supporting the hypothesis that strategic complementarity is the main driver of the observed bank-level outcomes.

3.1 Data and Summary Statistics

I collect quarterly balance sheet information of all U.S. commercial banks from the Consolidated Report of Condition and Income (Call Reports). This data set contains a consistent series of information on credit lines from 1990 to 2011. As credit line lending started gradually in the 1990s, I focus on the sample after the 2001-2002 crisis in the estimation. Then, I aggregate the bank-level data to the bank-holding company level and focus on the 25 largest bank-holding companies based on total assets in each quarter.

Table 1 presents summary statistics for balance sheet ratios. Comparing the statistics before and after 2008Q3, the equity-to-asset and deposit-to-asset ratios remain similar. The loan-to-asset ratio increases from 63% to 66%, while the unused credit line-to-asset ratio drops from 28% to 23%, suggesting that borrowers drew down a larger fraction of credit lines during the crisis, which showed up as loans. For more detailed data description and institutional details, please refer to

Section A in the Internet Appendix.

3.2 Estimation Procedure

I denote by $\theta \in \Theta$ the vector of model parameters and partition it into two groups:

$$\theta_1 = [\pi(b,g), \pi(g,b), r_k, \rho, \xi, \Lambda, r(g), r(b), z(g), z(b), \tau, \zeta, r_m, \xi^b]$$
$$\theta_2 = [\beta, \gamma^+, \eta, \psi, \gamma^-, a_n, a_c]$$

To estimate the mode, I first calibrate the parameters $[\pi(b,g), \pi(g,b), \beta, \zeta]$ before the estimation and quantify the remaining parameters in θ_1 using the average statistics of the large U.S. bank sample constructed in Section 3.1. Nest, I estimate θ_2 by using the simulated method of moments, which selects parameter values that minimize the distance between moments from actual data and their analogs generated by the model simulation. Section C in the Internet Appendix provides further details of the estimation procedure.

I use eight moments to jointly identify the seven parameters in θ_2. Specifically, I map the pre-2008Q3 period to the model's normal steady state and the crisis periods (2008Q3-2009Q2) to the transition path from the normal steady state into a crisis. In addition, I define credit as the sum of assets and unused credit lines in actual and simulated moments. Appendix Table AI provides details on how to construct the moments, and Figures E1-E7 offer visual evidence of how the targeted moments vary with model parameters. Table 2 then presents the elasticities of each moment to the estimated parameters. Ideally, these moments must be insensitive to rich heterogeneities in the data but outside the model.

The first three moments—credit growth, equity-to-credit ratio, and Tobin's Q in normal times—identify the discount rate parameter β and the issuance cost γ^+. The discount rate disciplines the banker's intertemporal tradeoff. A lower β represents a smaller discount and better growth opportunity. Consequently, the bank grows faster and takes lower leverage, leading to a higher Tobin's Q. The discount rate also indirectly pins down the dividend rate in normal times.[8] In addition, the issuance cost γ^+ controls how fast banks can expand in normal times: the higher the

[8] I do not use the dividend rate as a moment because it can be derived from other moments. At the normal steady state, the budget constraint requires that net interest income minus net non-interest expense equal dividends plus new equity as bank credit grows.

cost is, the slower the credit grows. Table 2 confirms that the moments are sensitive to β and γ^+.

The next set of moments are derived from crisis periods and include credit growth, equity-to-credit ratio, and asset-to-credit ratio. These moments are used to identify the probability of substitution η, leverage sensitivity parameter ψ, and the rationing cost γ^-. A lower probability of substitution η makes borrowers' drawdown decisions more sensitive to the banker's rationing policy, leading to a higher asset-to-credit ratio in crises. Similarly, a higher ψ incentivizes bankers to deleverage more aggressively, predicting a higher equity-to-credit ratio in crises. Finally, the credit growth helps pin down the rationing costs γ^-. The higher the costs are, the more slowly the bank credit contracts.

The remaining two moments serve to identify the operating cost parameters a_n and a_c. In normal times, the net non-interest expense corresponds to bank profit in the model. Therefore, an increase in a_n results in higher expenses and lower bank profits. Similarly, the net non-interest expense during crises determines the operating cost a_c. Notably, since all parameters are jointly identified, the two moments on cash flows are also sensitive to other parameters, as demonstrated in Table 2.

3.3 Estimation Results

Table 3 presents the set of parameters that are determined before the estimation process. I set the transition probabilities of the state of the economy based on the NBER business cycle dates.The parameters of borrower technology are determined using the Federal Reserve's Financial Accounts data. I set $r_k = 2.04\%$ to the average after-tax return of nonfinancial corporation business and $\xi = 0.9\%$ to the average dividend payout ratio. These average statistics remained similar before and during the crisis. Moreover, I set $\rho = 0.925$ using the average statistics and a model-consistent formula. For further details, please refer to Section A in the Internet Appendix.

Next, I quantify $[r(g), r(b), z(g), z(b), \tau, \xi_b]$ directly to their empirical counterparts. Studies using the Shared National Credit data (Mian and Santos, 2018; Chodorow-Reich et al., 2022) suggest that the average drawdown rate of credit lines before the crisis is 0.55, and thus, I set Λ to 0.55. Using this value along with the unused credit line-asset ratio from Table 1, term loans account for 44%, and credit lines account for 56% of bank credit. Additionally, in normal times, the asset-to-credit ratio is $56\% \times 0.55 + 44\% = 0.748$.

For the required leverage ratio ζ, I follow the Basel Accords, which mandate a ratio of 0.06 for the U.S. insured bank-holding companies. As for the maintenance fee r_m, I utilize the average all-in-undrawn fee of 8.86 basis points from the DealScan data.

Then, I turn to the parameters estimated using the simulated method of moments. Panel A of Table 4 reports the estimation results with standard errors. All estimates are statistically significant. The quarterly discount rate estimate is 0.0182. Although higher than the average federal funds rate, it is comparable to the estimates in the literature (1.88% in Tian (2022) and 2.57% in Corbae and D'Erasmo (2021)) and also in line with the annual return on equity for U.S. banks of 7.63% reported in Atkeson et al. (2019).

The estimated probability of substitution ($\eta = 0.317$) suggests that approximately 50% borrowers obtain a new source of liquidity within two quarters after separation, and 78% borrowers do so within four quarters. This partial substitution aligns with the empirical work on bond versus bank financing (Adrian et al., 2013; Becker and Ivashina, 2014), while the evidence on the speed at which firms can find a substitute remains limited.

Furthermore, my analysis reveals a positive cost sensitivity to leverage during crises, which is consistent with the reduced-form evidence on bank equity and profitability (Berger, 1995). To support this finding, I also regress equity against net non-interest expense using the large U.S. bank sample. Specifically, I follow the linear specification below that includes bank and quarter fixed effects. The coefficient of equity is -0.148 (standard error 0.0367), which is statistically significant and not far from my estimate ($-\psi = -0.105$).

$$\text{Net Non-int. Exp.}_{i,t} = \iota_1 \text{Equity}_{i,t} + \iota_2 \text{Asset}_{i,t} + \text{Bank}_i + \text{Quarter}_t + \varepsilon_{i,t}.$$

The estimates of adjustment costs vary widely in the literature depending on model specifications and samples used. My estimate of rationing cost (0.116) is comparable to the estimates of adjustment costs for banks in the literature (0.1 in De Nicolò, Gamba and Lucchetta (2014) and 0.188 in Tian (2022)). While the estimate of the issuance cost is much higher, it is still smaller than the estimates using the Q-theoretical approach for non-financial firms (Gilchrist and Himmelberg, 1995). Similar to the neoclassical models, a larger cost coefficient is required to match Tobin's Q

(1.09) in the data.

Finally, the estimated operating cost parameters indicate a significant countercyclical non-interest expense beyond deposit rates. In normal times, one dollar of intermediated asset costs the banker 33 basis points (a_n) beyond the deposit rates, which is approximately 40% of the net interest margin (loan rate minus deposit rate). In crises, the marginal operating cost ($a_c - \psi \frac{E}{L+L_D}$) amounts to 103 basis points, which is approximately two times larger than that in normal times.

3.4 Model Fit

The model reproduces the empirical moments used in the estimation. Panel B of Table 4 compares the empirical and model-implied moments. Their differences are statistically insignificant. In the data and the model, bank credit contracts significantly during crisis periods at a rate of approximately 6% per year. Moreover, the asset-to-credit ratio increases from 0.748 in normal times to 0.772 in crises, as borrowers draw down more credit lines. Tobin's Q in the simulated data is slightly smaller than the actual data, possibly due to the imprecise measurement of banks' market values in the actual data.

The model replicates the time-series patterns discussed in Section 3.1. This is unsurprising since I target the credit growth and asset-to-credit ratios in the estimation. The model also generates acyclical leverage: leverage increases slightly from 10.0 before the crisis to 10.4. The weak association between asset growth and book equity growth is consistent with the empirical evidence documented by Adrian and Shin (2011). While leverage is countercyclical in most macro-finance models, contingent credit lines allow banks to shrink their balance sheets less costly and more promptly. As a result, leverage does not increase significantly in crises.

As an external validity check, I examine a conditional moment that is directly connected to the policy functions in the model but not used in the estimation. Specifically, I consider the coefficient that associates credit contraction during the 2008-2009 crisis with bank leverage immediately before the crisis. Since the model has limited cross-sectional variation, I start from the normal steady state, perturb the equity-to-credit ratio, and simulate responses of credit growth with different equity-to-credit ratios. I then compare the regression coefficient with the following specification in actual and simulated data.

$$\text{Credit Growth}_i = \iota_1 \text{Pre-crisis Equity-to-credit Ratio}_i + \varepsilon_{i,t}.$$

The coefficient should be positive since, with lower leverage before a crisis, banks face less deleveraging pressure and should experience smaller credit contraction. The coefficient, using actual data, is 0.280 with a standard error of 0.117, while that using simulated data is 0.346. Therefore, even though I do not target this conditional moment in the estimation process, the estimated model captures the cross-sectional correlations in the actual data well.

4 The Amplification of Strategic Complementarity

In this section, I present plots of impulse responses to illustrate the amplification mechanism and quantify the amplification effect due to the banker-borrower strategic complementarity.

4.1 Impulse Response Functions

Figure 2 plots the impulse responses resulting from a shift from the normal steady state to the crisis state. During a crisis, the intermediation cost hikes, and the banker targets a much lower leverage ratio and rations liquidity by cutting credit lines. As a result, bank credit (the upper panel) drops by 1.5% in the first quarter. The cumulative credit contraction amounts to 6.0% during the first year and 11.7% during the first two years.

The right panel plots the impulse response of the borrowers. As the banker cuts credit lines, the borrowers face a threat of separation. They respond by drawing down credit lines strategically, with approximately 8% of those who do not need liquidity still drawing down their credit lines to secure them.

The responses of the banker and the borrowers reinforce each other. More strategic drawdowns lead to higher leverage and intermediation costs and more aggressive liquidity rationing through the bank balance sheet channel. Therefore, dampening the response of either side can restrict the amplification effect. To illustrate this point, Figure 2 includes two counterfactual impulse responses: a case with a higher probability of substitution ($\eta = 0.35$) and one with a higher rationing cost ($\gamma^- = 0.13$). A higher η makes borrowers less reliant on credit lines and respond less to credit line cuts. Moreover, when bankers face a higher rationing cost, they respond less to drawdowns and ration liquidity more gradually. In either case, the amplification effect is contained, resulting in less severe credit contraction and fewer strategic drawdowns.

4.2 Quantifying the Amplification Effect

This section studies two counterfactuals to quantify the amplification effect. First, I consider a benchmark model where the banker can commit to his strategy.

The baseline model in Section 2 is time-inconsistent. From the ex-ante perspective, the banker prefers to cut fewer credit lines to discourage borrowers from making strategic drawdowns. However, after observing the aggregate drawdown, the banker's future self would respond optimally ex-post instead of sticking to the ex-ante strategy. This time-inconsistent problem arises because current regulations prohibit credit line contracts from being contingent on bank balance sheet strength, which prevents the banker from committing credibly to the ex-ante strategy.

Enabling the banker to commit resolves the time-inconsistent problem and results in a constrained-efficient equilibrium. Specifically, I alter the model timing to allow the banker to announce liquidity provisions, such as credit line cuts or new credit issuance, at the beginning of each period and then stick to the announcement. In this benchmark with commitment, the banker understands that aggregate drawdown will increase if he commits to more credit line cuts. As a result, he internalizes this adverse consequence and becomes more hesitant to do so.

Table 5 compares the counterfactual against the estimated baseline model. With commitment, the banker largely refrains from rationing liquidity unless the leverage ratio requirement is binding. As a result, in the first year of a crisis, the banker chooses not to cut credit lines at all, resulting in zero credit contraction. Consequently, borrowers choose not to make strategic drawdowns. Moreover, both the banker and the borrowers become better off. The borrowers benefit because bank commitment makes their credit lines more secure. At the same time, the banker's welfare improves because he can at least commit to the equilibrium strategy in the baseline model without commitment. Consistently, Tobin's Q (the sum of bank value V and liabilities divided by assets) and the value of credit lines ($v_2(\mathbf{S})$ in the model) in normal times become higher than in the baseline model.

Second, I shut down the strategic complementarity and study a counterfactual with no strategic drawdowns. In this case, borrowers only draw down when hit by the liquidity shock ($\lambda = 1$) while keeping the probability of substitution η the same.

To quantify the amplification effect, I compare the second counterfactual with the baseline model. Row (3) of Table 5 shows that in this counterfactual, bank credit falls by 0.536% per quarter in the first four quarters into a crisis, which is approximately one-third of that in the baseline model. Thus, the banker–borrower strategic complementarity accounts for two-thirds of the overall credit contraction. Moreover, Tobin's Q is higher than that in the model with commitment because the banker faces no pressure from strategic drawdowns. In addition, the value of credit lines is lower than the baseline model because, on the one hand, the banker cuts fewer credit lines, which boosts the value of credit lines, and on the other hand, borrowers cannot strategically draw down even if the cost of doing so is low.

4.3 Sensitivity to Estimation Moments

In this section, I examine the robustness of the quantified amplification effect by computing the sensitivity matrix proposed in Andrews, Gentzkow and Shapiro (2017). Let X be a statistic of interest. Suppose the model is misspecified; the estimate $\hat{X}$ is biased. The bias consists of two components: how a given alternative model specification would impact the moments and how changes in moments would affect estimated parameters and $\hat{X}$.

The first component is often straightforward and depends on the context, while the sensitivity matrix captures the second component and is locally given by:

$$\hat{X} - X = (\nabla X)' \underbrace{\left[-\left(\mathbf{J}'W\mathbf{J}\right)^{-1}\mathbf{J}'W\right]}_{\text{sensitivity matrix}}, \tag{13}$$

where $\hat{X}$ is the estimate (e.g., the magnitude of the amplification) and X is its true value, W is the SMM weighting matrix, $\mathbf{J}$ is the Jacobian matrix of the parameters with respect to moments, and ∇X is the gradient of the estimate X with respect to parameters. Roughly speaking, the bias depends on two sensitivities: how sensitive parameters are to moments (captured by the Jacobian matrix **J**) and how sensitive the statistic of interest is to parameters (captured by the gradient ∇X). The larger (smaller) the first (second) sensitivity is, the smaller the bias is.

Table 6 presents the sensitivity matrix in Panel A and the gradient-adjusted sensitivity matrix in Panel B. To better illustrate the results, consider a hypothetical scenario where the average asset-to-credit ratio in the crisis is mismeasured by 0.00849 (one standard deviation in the actual data). The sensitivity matrix shows that this mismeasurement would lead to a bias of 0.0211% in the quantified amplification effect on credit contraction, which is $1.54\% - 0.536\% = 1.004\%$ according to Table 5. Similarly, a mismeasurement of Tobin's Q by 0.0147 (one standard deviation in the actual data) leads to a bias of 0.195%. These results suggest that the quantified amplification effect is robust and not significantly affected by misspecification bias.

4.4 The Strategic Interactions among Borrowers

While my analysis focuses on the banker–borrower strategic complementarity, strategic interactions among borrowers are also at play. In this section, I decompose the separation probability to show that strategic interactions among borrowers have a limited impact. Specifically, I rewrite the separation probability as follows:

$$\frac{\Delta}{\Phi - L_D} = \frac{\Delta}{\Phi - \Lambda} + \frac{\Delta}{\Phi - \Lambda} \times \frac{L_D - \Lambda}{\Phi - L_D}. \tag{14}$$

The first term on the right-hand side, the credit line cuts divided by total unused credit lines, comes directly from liquidity rationing. It captures the separation probability a single borrower faces, assuming all other borrowers choose not to draw down their credit lines strategically. The second term accounts for borrowers' strategic interactions. When other borrowers draw down their credit lines strategically, each borrower faces a higher separation probability because the size of the unused credit lines reduces. With a fixed Δ, the second term increases with the number of borrowers who make strategic drawdowns $L_D - \Lambda$. In other words, as more borrowers run, borrowers who have not yet drawn down their credit lines are subject to a higher separation probability.

Using the estimated model, I compute the two terms and find that 92.3% of the separation probability comes from the first term, while only 7.7% comes from the second term. This indicates that the strategic interactions among borrowers only play a secondary role in the amplification mechanism.

5 Policy Implications

This section uses the estimated model to evaluate ex-ante financial regulations and ex-post policies aimed at supporting bank lending during crises. Prudential regulations, such as the leverage ratio requirement, enhance bank lending capacity, while ex-post fiscal and monetary policies, such as FFL schemes and CCFs, incentivizes banks to lend to borrowers.

5.1 Leverage Ratio Requirements

I first examine the impact of prudential regulations that impose tighter quantity requirements, specifically the leverage ratio requirement. The leverage ratio requirement requires the ratio of Tier 1 capital to banks' leverage exposure, including off-balance-sheet exposure such as unused credit lines, must exceed a certain threshold. For example, in the United States, the Federal Reserve Bank mandates a minimum ratio of 6% for insured bank-holding companies.

The financial accelerator literature has demonstrated that quantity requirements can address the distortions caused by pecuniary externalities. Likewise, in my model, the banker fails to internalize the impact of liquidity rationing on borrowers' drawdown decisions, leading to excessive leverage. Therefore, a tighter leverage ratio requirement can induce the banker to maintain lower leverage in normal times, helping to dampen the vicious cycle by keeping more dry powder available. Furthermore, an effective leverage ratio requirement should be countercyclical, inducing lower leverage without imposing additional pressure on banks during crises (Davydiuk, 2017; Malherbe, 2020).

I implement changes in leverage ratio requirements as follows. First, I use the equity-to-credit ratio in the model as the leverage ratio. Second, I keep the threshold of 6% in crises and adjust the threshold in normal periods (from 6% to 6.1%, 6.2%, 6.3%, and 6.4%, respectively). Finally, I solve the model with various leverage ratio requirements and compare the results for credit contraction and strategic drawdowns. It is worth noting that the steady states differ in those counterfactuals. Under tighter leverage ratio requirements, banks feature lower leverage in the normal steady states.

Panel B in Table 5 presents the results. A tighter requirement is effective. Compared with the baseline model, an increase in the threshold from 6% to 6.4% reduces credit contraction and strategic drawdowns by approximately half. Moreover, because a tighter requirement also reduces credit growth in normal times, its effect on Tobin's Q of normal periods is not monotonic. Additionally, conditional on having a credit line, the value of the line increases as the requirement tightens and banks cut fewer credit lines.

5.2 FFL and CCF

Next, I examine ex-post policies to support bank lending: FFL schemes and CCFs. FFL schemes are widely used to provide funds at cheaper rates than the market. For example, the Bank of England and the Eurosystem adopted such schemes during the Global Financial Crisis. They share with asset purchase programs, such as quantitative easing, the same goal of reducing banks' intermediation costs. I implement the FFL schemes as a reduction in the intermediation cost parameter a_c in the model.

Different from FFL schemes, CCFs provide liquidity to corporations directly. There is a longstanding concern that whether banks can effectively channel funds to real economies; in particular, banks may hoard liquidity due to precautionary motives, or banks may channel funds only to large borrowers but not small and medium businesses. Therefore, policymakers have called on alternative policies to support corporations directly. For example, during the recent Covid crisis, major economies implemented CCFs to support large firms and small and medium businesses. I implement the CCFs in the model as an increase in the probability of substitution η after borrowers' credit lines are cut.

This paper provides a new argument for direct support to corporations. Given the banker–borrower strategic complementarity, even if only the banker is hit directly during crises, policies targeting borrowers can be effective in boosting bank credit supply. Suppose borrowers can get access to alternative liquidity, such as CCFs. The marginal benefit of strategic drawdowns reduces. Thus, the total drawdown decreases, the banker faces less pressure to deleverage and cuts fewer credit lines.

To quantify this crowding-in effect of CCFs, I simulate the policy counterfactuals in two steps. First, I solve the model with the estimated parameters adjusted for police experiments. Specifically, I increase the probability of substitution η by 0.5% to 4% (from 31.7% in the benchmark model). From a back-of-envelope calculation, a one percentage point increase corresponds to approximately ten billion dollars of direct liquidity provision, comparable to the size of CCFs adopted in the Covid crisis.[9] In the second step, I calculate the impulse responses to a shift from the normal steady state in the estimated model (with estimated parameters and no policy adjustment) to the crisis state. Starting from the same steady state captures the ex-post nature of the policies and allows a close comparison. Note that I also assume that policymakers are committed to the policies until the crisis ends and in future crises.

Figure 3 displays the reductions in credit contraction in counterfactuals relative to the baseline model (the dashed blue line with circles). For example, if the probability of substitution increases by 4% (from 31.7% to 35.7%), quarterly credit contraction will decrease from 1.54% in the baseline model to 1.3%. Furthermore, I consider a second set of counterfactuals implementing FFL to gauge the economic significance of the crowding-in effect. Specifically, I reduce the intermediation cost a_c by 0.2 to 1.6 basis points.[10] The dotted dark red line with triangles in Figure 3 plots the reductions in credit contraction in this second set of counterfactuals. The results show that a 0.04 increase in the probability of substitution and a 1.6 basis-point reduction in the intermediation cost have similar effects in boosting credit growth in crises. It is tempting to further compare the effects per dollar spent in CCFs versus FFL schemes, yet, they are not comparable. CCFs work through loan quantities, while FFL schemes focus on intermediation costs (prices) and provide a subsidy below the market.

A related debate is whether to adopt both policies simultaneously. Given that both are effective in spurring credit growth, it seems that the marginal contribution of either policy will diminish when the other policy is present, which does not support the simultaneous adoption. In contrast, it

[9] The total bank credit in 2008 was approximate \$9000 billion. Banks cut credit by 1.5%, and borrowers find a substitution with a probability of 0.317 every quarter. Thus, in the first year into a crisis, the direct liquidity provision for a 0.01 increase in the probability is $9000\times0.01\times10.9\%$=9.8 billion. As of May 31, 2021, the outstanding amount under the CCF was \$13.8 billion, that under the Main Street Lending Program was \$16.3 billion, and that under the Paycheck Protection Program was \$84.2 billion.

[10] The sizes, 0.8 to 6.4 basis points annually, are smaller but comparable to the 25 basis-point reductions in 1-year annual treasury yield around quantitative easing announcements during the 2008-2009 financial crisis (Krishnamurthy and Vissing-Jorgensen, 2012).

turns out that the combined effects (the solid orange line with diamonds in Figure 3) can be larger than the sum of the two individual effects. Across all comparisons, the combined effect ranges from 0.65 to 1.4 times the sum of the two individual effects. In particular, in the counterfactual where the cost reduces by 1.2 basis points and the probability of substitution increases by 3%, the combined effect (0.33% reduction in credit contraction) is larger than the sum of two individual effects (0.11% for CCFs and 0.12% for FFL schemes).

This intriguing result justifies the adoption of CCFs and FFL schemes simultaneously. Intuitively, it is because the banker' and the borrowers' strategies are nonlinear. Implementing either policy can contain the amplification, leading to fewer credit line cuts and strategic drawdowns. Meanwhile, the degree of complementarity may change in either direction because of the nonlinear strategies. If the complementarity strengthens, the second policy will introduce a larger marginal effect when the first policy is present than absent.

6 Conclusion

Asset–side bank runs can happen even absent any threat of bank failure. Contingent credit lines create incentives for borrowers to strategically draw down credit lines to secure liquidity for future use. To demonstrate this run mechanism, I have developed a dynamic model that integrates the bank balance sheet channel with credit line runs. A novel banker–borrower strategic complementarity arises where liquidity rationing in the form of credit line cuts induces strategic drawdowns, leading to deleveraging pressure and further liquidity rationing. This process repeats and forms a vicious cycle that amplifies adverse shocks during crises.

I have estimated the model using data on large U.S. banks before and during the 2008-2009 crisis. Counterfactual experiments reveal that the amplification effect accounts for two-thirds of the credit contraction, approximately 1% shortfalls quarterly in the first year into a crisis. I have also used the estimated model to study policy experiments. Ex-ante prudential policies, such as countercyclical leverage ratio requirements, enhance bank lending capacity during crises. In addition, I have analyzed policies that reduce bank intermediation costs and those providing direct liquidity to corporations. Because of the strategic complementarity, the latter policies can effectively contain credit contraction even if they do not directly affect banks.

The quantitative significance of the amplification mechanism raises several important questions for future research. First, it is important to investigate ways to improve the design of credit line contracts. My results suggest that allowing banks to commit to credit lines can mitigate the amplification effect. However, how to achieve this warrants further scrutiny. Intriguingly, credit lines with less strict covenants and split controls have become popular since the 2008-2009 crisis (Berlin et al., 2020). Second, it would be valuable to explore how strategic complementarity affects aggregate investment and output. To achieve this, a complete characterization of credit line borrowers would be necessary while keeping the analysis tractable. It is also important to consider the allocation of liquidity across different firms, as Greenwald et al. (2021) and Chodorow-Reich et al. (2022) point out. Finally, my framework speaks to dynamic strategic complementarities in other contexts, such as fund redemption restrictions, swing pricing, and partial defaults on sovereign debt (Li et al., 2021; Hanson et al., 2015; Cipriani et al., 2014).

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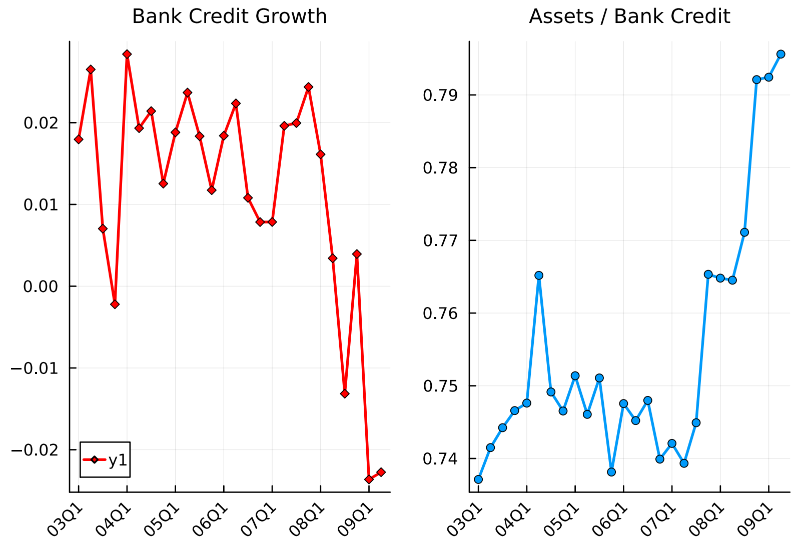

Figure 1: **Bank Credit and Asset-to-Credit Ratio**

The left panel plots the median quarterly growth in bank credit (assets plus unused credit lines) of the 25 largest banks in each quarter from 2003Q1 to 2009Q2. The right panel plots the median asset-to-credit ratio.

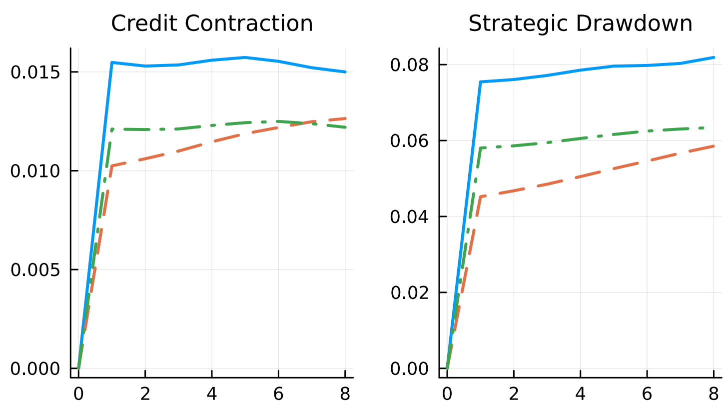

Figure 2: **Impulse Responses**

The figure displays the impulse responses to a shift from the normal steady state to the crisis state that are discussed in Section 4.1. It includes the dynamics of credit contraction $\Delta/(\Phi + L)$ in the left panel and strategic drawdown in the right panel. Strategic drawdown is defined as the fraction of borrowers who draw down their credit lines despite not needing liquidity, which is represented as $(L_D - \Lambda\Phi)/(\Phi - \Lambda\Phi)$. In addition to the responses in the estimated model (the solid blue lines), the figure depicts the impulse responses in two counterfactuals, one with a higher probability of substitution ($\eta = 0.35$, the dashed red lines) and one with a higher rationing cost ($\gamma^- = 0.13$, the dash-dotted green lines).

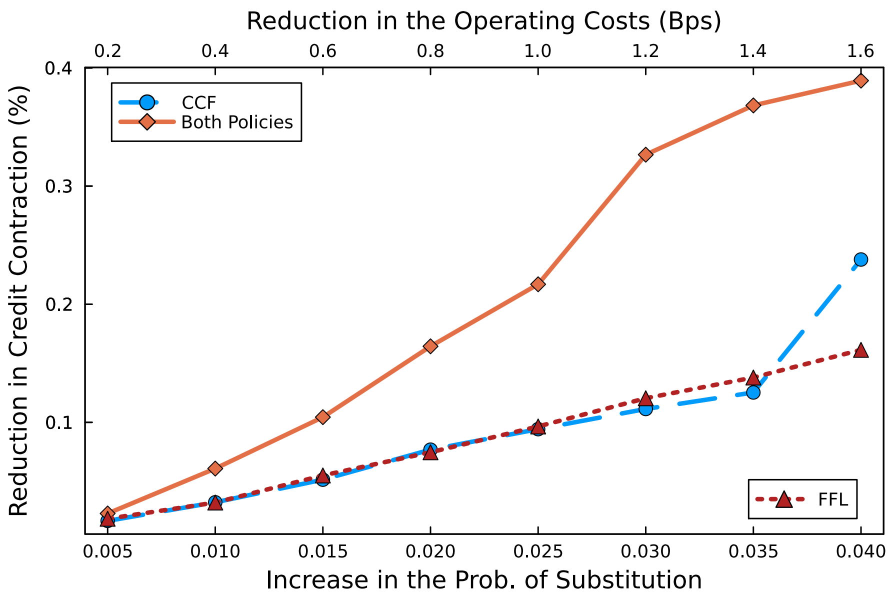

Figure 3: **Policy Counterfactuals**

The figure illustrates three sets of policy counterfactuals that are discussed in Section 5.2. The first set of counterfactuals (dashed blue line with circles) implements CCFs by increasing the probability of substitution of borrowers after being separated from their credit lines (x-axis at the bottom). The second set of counterfactuals (dotted dark red line with triangles) implements FFL schemes by reducing banks' operating costs (x-axis at the top). The third set of counterfactuals (solid orange line with diamonds) implements both policies simultaneously. The vertical axis plots the differences in credit contraction in crises between the counterfactuals and the estimated model.

Table 1: **Summary Statistics Before and During the 2008-2009 Crisis**

	2002Q1-2008Q2				2008Q-2009Q2			
	mean	p25	p50	p75	mean	p25	p50	p75
Loan/asset	0.6282	0.5651	0.6592	0.7200	0.6623	0.6234	0.6905	0.7422
Unused credit lines/asset	0.2826	0.2045	0.2740	0.3577	0.2272	0.1618	0.2279	0.2864
Equity/asset	0.0999	0.0813	0.0932	0.1079	0.1038	0.0854	0.1000	0.1155
Deposit/asset	0.6719	0.6166	0.6742	0.7258	0.6898	0.6422	0.6911	0.7502

Table 2: **Elasticity of Moments to Parameters**

This table presents the elasticity of simulated moments with respect to the estimated structural parameters. Specifically, for each parameter ω, I hold all other parameters fixed and vary ω around the SMM estimate $\hat{\omega}$ to compute the elasticity of moment n to parameter k. I compute the elasticity as

$$\mathcal{E}_{n,k} = \frac{m_n^+ - m_n^-}{\omega^+ - \omega^-}\left|\frac{\hat{\omega}}{\hat{m}_n}\right|,$$

where $\hat{\omega}_k$ is the parameter value at the SMM estimate and $\hat{m}_n$ the corresponding value for moment n. ω_k^+ (respectively ω_k^-) is the parameter value located right above (resp. below) on the grid used to plot Appendix Figures E1 - E6. m_n^+ (resp. m_n^-) is the corresponding simulated moment obtained using parameter ω_k^+ (resp. ω_k^-), keeping the other parameters at their SMM estimate. Table AI presents the definitions of the moments.

(Normal)	Credit Growth	Equity/Credit	Tobin's Q	Net Non-int. Expense
β	-1.57	-5.78	-0.078	-0.537
γ^+	-1.21	-0.201	-0.0105	-0.242
η	-0.016	0.145	-0.000791	-0.00547
ψ	0.31	4.61	0.016	0.106
γ^-	-0.0288	0.417	-0.00132	-0.00988
a_n	-1.07	-1.74	-0.0545	0.671
a_c	-1.26	-6.36	-0.0628	-0.431
(Crisis)	**Credit Growth**	**Equity/Credit**	**Asset/Credit**	**Net Non-int. Expense**
β	-39.0	-6.01	1.12	8.84
γ^+	-1.72	-0.197	0.0497	0.326
η	0.7	0.164	-0.0465	-0.266
ψ	33.5	4.88	-0.966	-8.32
γ^-	1.96	0.47	-0.0529	-0.597
a_n	-11.2	-1.85	0.31	2.64
a_c	-48.7	-6.84	1.4	12.4

Table 3: **Parameters Determined with External Information**

This table presents the parameters that are determined before the estimation process. I calibrate the first six parameters from related data, as described in Section 3.2. Additional details can be found in Internet Appendix A. The remaining parameters are quantified based on the conditional means of their empirical counterparts.

Parameter	**Description**	**Value**
$\pi(c,n)$	Transition prob. from normal state to crisis state	0.025
$\pi(n,c)$	Transition prob. from crisis state to normal state	0.167
r_k	After-tax return of nonfinancial corporations	0.0204
ρ	Size of liquidity shocks	0.925
ξ	Firm's exit rate	0.009
Λ	Prob. of liquidity Shock	0.55
$r(n)$	Loan interest rates (normal state)	0.0124
$r(c)$	Loan interest rates (crisis state)	0.0107
$z(n)$	Bank deposit rates (normal state)	0.0052
$z(c)$	Bank deposit rates (crisis state)	0.00378
τ	Bank tax rate	0.329
ξ_b	Bank's exit rate	0.016
ζ	The leverage ratio requirement	0.06
r_m	Maintenance fee (bps)	8.86

Table 4: **Estimation Results**

This table presents the results of the simulated method of moments estimation. The estimation procedure is described in Section 3 in the main text and Section C in the Internet Appendix. Panel A presents the estimated parameters along with their standard errors. Panel B displays the simulated moments alongside the actual moments and the *t*-statistics of the pairwise differences. Further details regarding the definitions of the moments can be found in Table AI in the Internet Appendix.

Panel A: Parameters Estimated via SMM			
Parameters	Description	Value	Std. Error
β	Discount Rate	0.0182	0.00219
γ^{+}	Issuance Cost	2.68	0.573
η	Probability of Substitution	0.317	0.158
ψ	Sensitivity to Leverage	0.105	0.0388
$\gamma-$	Rationing Cost	0.116	0.0481
a_n	Operating Cost	0.00330	0.000321
a_c	Operating Cost	0.0204	0.00393

Panel B: Moment Conditions			
Moments	Model	Data	t-stat
Credit Growth (normal)	0.0143	0.0167	-1.42
Equity/Credit (normal)	0.0745	0.0745	0.00114
Tobin's Q (normal)	1.06	1.09	-2.64
Net Non-int. Expense $\times 100$ (normal)	0.238	0.238	0.011
Credit Growth (crisis)	-0.0154	-0.0143	-0.327
Equity/Credit (crisis)	0.0744	0.0768	-0.559
Asset/Credit (crisis)	0.772	0.776	-0.509
Net Non-int. Expense $\times 100$ (crisis)	0.746	0.690	0.557
Over-identification test			$\chi^2 = 8.12$

Table 5: **Quantifying the Amplification Effect and Policy Counterfactuals**

Panel A presents the results of counterfactual experiments conducted in Section 4.2 to quantify the amplification effect due to the banker-borrower strategic complementarity. Columns (1) and (2) report Tobin's Q (the sum of bank value V and liabilities divided by assets) and credit line value ($v_2(\mathbf{S})$ in the model) in the normal steady state. Columns (3) and (4) report the quarterly average credit contraction ($\Delta/(\Phi + L)$) and the fraction of borrowers drawing down strategically ($(L_D - \Lambda\Phi)/(\Phi - \Lambda\Phi)$) in the first four quarters into a crisis in the simulation. Row (1) is the baseline model, row (2) is the model variant with bank commitment, and row(3) is the counterfactual with no strategic drawdowns.
Panel B presents the results of counterfactual experiments conducted in Section 5.1, where I keep the required ratio of 6% in crises and increase the required ratio in normal periods from 6% to 6.1%, 6.2%, 6.3%, and 6.4%, respectively. The results are presented in rows (1) to (4) for each counterfactual experiment.

Panel A: Quantifying the Amplification				
	Tobin's Q	Credit Line Value	Credit Contraction (%)	Strategic Drawdown (%)
(1) Baseline	1.05563	0.104291	1.54	7.68
(2) With commitment	1.05778	0.105154	0.0	0.0
(3) No Strategic Drawdown	1.0589	0.103829	0.536	0.0
Panel B: Leverage Ratio Requirements (Baseline $\zeta = (0.06, 0.06)$)				
	Tobin's Q	Credit Line Value	Credit Contraction (%)	Strategic Drawdown (%)
(1) $\zeta = (0.061, 0.06)$	1.05562	0.104302	1.53	7.64
(2) $\zeta = (0.062, 0.06)$	1.0556	0.10434	1.37	6.77
(3) $\zeta = (0.063, 0.06)$	1.05542	0.104551	0.998	4.85
(4) $\zeta = (0.064, 0.06)$	1.05569	0.104621	0.776	3.91

Table 6: **Sensitivity to Moments**

This table presents the sensitivity of estimates of the amplification effect to moments, following Andrews et al. (2017). Panel A reports the sensitivity matrix $\left(\mathbf{J}'W\mathbf{J}\right)^{-1}\mathbf{J}'W$, where $\mathbf{J}$ is the estimated Jacobian matrix, and W is the SMM weighting matrix. Panel B reports the gradient-adjusted sensitivities of the estimated amplification effect to moments, where ∇X is the gradient of the estimated effect to parameters. Table AI in the Internet Appendix presents the definitions of the moments.

Panel A: Sensitivity Matrix $\left(\mathbf{J}'W\mathbf{J}\right)^{-1}\mathbf{J}'W$				
(Normal)	Credit Growth	Equity/Credit	Tobin's Q	Net Non-int. Expense
Info: β	0.231	0.00131	-0.125	-3.66
γ^+	-188.0	-57.4	47.0	-12.0
η	0.00492	10.2	2.54	-2.11
ψ	2.29	5.2	-2.62	-43.4
γ^-	1.4	-0.896	-0.345	53.2
a_n	-0.0384	0.018	-0.00903	1.34
a_c	0.213	0.693	-0.237	-4.04
(Crisis)	Credit Growth	Equity/Credit	Asset/Credit	Net Non-int. Expense
β	-0.0419	-0.106	0.0282	-1.34
γ^+	13.0	52.5	-2.33	125.0
η	-26.6	-6.32	-14.4	-29.3
ψ	0.506	-5.0	0.0231	1.09
γ^-	-4.67	6.87	2.45	-32.9
a_n	-0.00408	-0.0165	0.00072	-0.0392
a_c	0.0211	-0.526	-0.0182	1.09

Panel B: Gradient-adjusted Sensitivity Matrix $(\nabla X)'\left(-\mathbf{J}'W\mathbf{J}\right)^{-1}\mathbf{J}'W$				
(Normal)	Credit Growth	Equity/Credit	Tobin's Q	Net Non-int. Expense
Amp. Effect (%)	0.0639	-0.194	-0.195	-0.112
(Crisis)	Credit Growth	Equity/Credit	Asset/Credit	Net Non-int. Expense
Amp. Effect (%)	0.127	0.172	0.0211	-0.41

Internet Appendix for ”Asset-side Bank Run and Liquidity Rationing: A Vicious Cycle”

A Data and Institutional Background

Bank balance sheet. ”I obtained the Call Reports data from Wharton Research Data Services and aggregated the bank-level data to the bank holding company level (RSSD9348). To ensure data quality, I filtered the dataset by dropping observations with loans (RCFD1400) that were less than 20% of assets (RCFD2170). I also excluded observations with quarterly asset growth above 10% and those with quarterly asset growth below -10% and annual asset growth above 10% at the same time. Additionally, I removed Morgan Stanley and Goldman Sachs from the dataset, as they both became bank-holding companies during the 2008-2009 crisis.

I identify the 25 largest bank holding companies each quarter based on their total domestic assets (RCON2170). To compute the actual moments in estimations, I require each bank to have at least 25 observations for the 30-quarter sample period from 2002Q1 to 2009Q2. I also collect loan-level data from the DealScan dataset and stock price data from the Center for Research in Security Price (CRSP). For a complete list of variables and their definitions, please refer to Table AI.

Bank credit issuance. The DealScan dataset covers the syndicated loan market. I only consider the observations in which the lender is a lead arranger (LeadArrangerCredit==“YES”) and match the facility-level data with the issuance bank using the file that links DealScan Lender IDs to Call Report BHC-level IDs by Chakraborty et al. (2020). If amounts committed to a facility by some lenders are missing, I assign unattributed amounts equally among those lenders. To further refine the data, I limit the dataset to U.S.-based banks and facilities with the U.S. dollar as the currency.

Using this dataset, I compute quarterly net issuance rates by the 25 largest banks, which is the issued amount minus the matured amount divided by the total amount of existing loans. To differentiate between credit lines and term loans, I utilize the LoanType variable and categorize loans as credit lines if they fall into one of the four types— ”Revolver/Line$<$ 1 Yr.”, ”Revolver/Line $>=$ 1 Yr.”, ”364-Day Facility”, and ”Revolver/Term Loan.” The remaining types are classified as

Table AI: **Variable Definitions**

Variables	**Details of construction**
Unused credit lines $\Phi - L_D$	RCFD3814+RCFD3816+RCFD3817+RCFD3818 +RCFD3411 +RCFD6550 (Acharya and Mora, 2015)
Bank credit $\Phi + L$	Assets (RCFD2170) plus unused credit lines.
Asset-to-credit ratio $(L + L_D)/(\Phi + L)$	Assets (RCFD2170) / bank credit.
Equity-to-credit ratio $E/(\Phi + L)$	Equity (RCFD3210) / bank credit.
Tobin's Q $(V + L + L_D - E)/(L + L_D)$	Bank value (market cap) plus liabilities (RCFD2170-RCFD3210), then divided by assets (RCFD2170)
Net non-interest expensen $[G(E, L{+}L_D, s){+}C(\Delta, N_C, N_L, \Phi{+}L)]\ /(\Phi{+}L)$	Non-interest expense (RIAD4093+RIAD4230) minus non-interest income (RIAD4079), then divided by bank credit.
Tax rate τ	Applicable income taxes (RIAD4302) / before-tax income (RIAD4301).
Loan interest rate $r(s)$	Interest income (RIAD4107) / assets (RCFD2170).
Bank deposit rate $z(s)$	Interest expense (RIAD4073) / liabilities (RCFD2170-RCFD3210).

term loans. The average quarterly net issuance rate of term loans is 0.3% from 2008Q3 to 2009Q2.

In addition, I compute the maintenance fee, which corresponds to the all-in-undrawn fee in the dataset. The all-in-undrawn fee is similar before and during the 2008-2009 crisis. Thus, in the estimation, I set the maintenance fee to the average all-in-undrawn fee of all facilities issued between 2002 and 2010.

Borrower balance sheet. I obtain the annual balance sheet information of nonfinancial corporate businesses from the Z.1 Statistical Release as of Jun 06, 2019. Variables include nonfinancial assets excluding inventories, net worth, loans, and net operating surplus.

I calculate the after-tax return r_k as the net operating surplus divided by nonfinancial assets, excluding inventories. To pin down the size of liquidity shocks ρ, I assume an average drawdown ratio of 0.55 based on previous studies of the Shared National Credit data. I then use the following model-consistent formula.

$$\frac{\text{net worth} + \text{loans} + \text{unused credit}}{1 + \rho} = \text{nonfin. assets excl. inventories.} \tag{IA.1}$$

Institutional background of credit lines. Credit lines are contingent liquidity sources for borrowers, as they are subject to rollover risk and revocations. Banks have full discretion in rolling over credit lines, while revocations entail several institutional details.

Most commercial loan contracts contain non-pricing terms, known as loan covenants, which restrict the actions that borrowers can take or specify minimum or maximum thresholds for balance sheet variables. If borrowers violate any covenant, the lender obtains the right to renegotiate and modify the contract. Moreover, most credit lines have a material adverse change covenant, which allows lenders to determine whether a borrower's credit quality has deteriorated significantly enough to trigger a violation. Chodorow-Reich and Falato (2022) find that over one-third of loans in their SNC sample breached a covenant during the 2008-2009 crisis, providing lenders with an opportunity to force a renegotiation. While banks typically overlook covenant violations in normal times, they exercised their discretion to modify credit limits and loan rates in the 2008-2009 crisis when they themselves were in distress.

However, once borrowers draw down their credit lines, banks cannot force them to repay the funds. Therefore, borrowers can draw down strategically to secure funds for future use. Ivashina and Scharfstein (2010) is the first to point out that the increase in total lending in 2008Q4 is due to increased drawdowns by existing credit line borrowers. They also provide evidence that borrowers do so strategically by examining SEC filings and coined the term "credit line runs."

To identify the effect of the expected decline in liquidity supply on credit line usage, Ippolito et al. (2015) compare drawdowns by the same firm from different banks. They find that higher exposure to the interbank market leads to more drawdowns. Moreover, the 2008Q4 CFO survey explicitly asked about firms' reasons for drawing down credit lines (Campello, Graham and Harvey, 2010). Seventeen percent of constrained firms and 8% of unconstrained firms reported that they drew down credit lines in case the bank restricted credit line access in the future. Unfortunately, this question only appears in the 2008Q4 survey, so we cannot tell whether firms drew down strategically beyond the crisis. This fact, nevertheless, demonstrates that firms indeed have incentives to run on credit lines.

B Model Derivations

B.1 Credit Limit Dynamics When the Banker Extends New Credit

The law of motion of the credit limit depends on whether the banker issues new credit lines or cuts existing ones. While borrowers move first in each period, they can anticipate the banker's decision, which depends on the state vector $\mathbf{S}$ at the equilibrium.

When the banker extends new credit lines, I assume that credit limits increase proportionally with net worth. Specifically, I assume that $\phi' = \phi\omega'/\omega$. If the aggregate credit limit summed over all borrowers falls short of the credit limit provided by the banker, I allow new entrants to take the remaining credit limit. Otherwise, if the aggregate credit limit exceeds that provided by the banker, a constant multiplier $\iota \in [0,1]$ is imposed on the credit limit growth of each borrower. In this case, $\phi' = \iota\phi\omega'/\omega$ such that the aggregate credit limit matches the credit limit provided by the banker. A property of this law of motion is that if $\phi \leq \rho\omega$ holds in the initial period, it holds for all subsequent periods.

The primary purpose of having this law of motion is to match aggregate quantities when the banker issues new credit lines. It does not significantly affect the model's implications, as the banker–borrower strategic complementarity only comes into play when the banker rations liquidity. With this law of motion, the borrower's problem, after observing the liquidity shock, becomes

$$u(\omega,\phi,k,1,\kappa;\mathbf{S}) = \max_{l} \mathbb{1}_{[l=\phi]}\left\{\xi\omega' + (1-\xi)\mathbb{E}_{\mathbf{S}'|\mathbf{S}}[v(\omega',\phi';\mathbf{S}')]\right\} + \mathbb{1}_{[l<\phi]}\, v^L, \tag{IB.2}$$

$$u(\omega,\phi,k,0,\kappa;\mathbf{S}) = \max_{l} \xi\omega' + (1-\xi)\mathbb{E}_{\mathbf{S}'|\mathbf{S}}\left[v(\omega',\phi';\mathbf{S}')\right]. \tag{IB.3}$$

This differs from the borrower's problem (3) and (4) in two ways. First, the law of motion for net worth is different. Second, borrowers have no incentive to strategically draw down their credit lines since the banker does not ration liquidity.

B.2 Borrowers' Problem After Separation

I denote the continuation value after separation as $v^E(\omega,\phi,\bar{\phi};\mathbf{S})$, where ϕ is the actual credit limit after separation, and $\bar{\phi}$ is the credit limit before separation. $\bar{\phi}$ is relevant because, with probability η, the borrower can obtain a new credit line with the same credit limit as before separation.

Similar to the problem before separation (1) - (4), I denote the value function before observing the idiosyncratic shocks as $v^E(\omega, \phi, \bar{\phi}; \mathbf{S})$ and the value function afterward but before the drawdown decision as $u^E(\omega, \phi, \bar{\phi}, k, \lambda, \kappa; \mathbf{S})$. Thus,

$$v^E(\omega, \phi, \bar{\phi}; \mathbf{S}) = \max_{c \geq 0, k \geq 0, (1+\rho)k \leq \omega - c + \phi} c + \beta \mathbb{E}_{\lambda, \kappa} \left[u^E(\omega - c, \phi, \bar{\phi}, k, \lambda, \kappa; \mathbf{S}) \right].$$

The law of motion of net worth remains the same as in Equation (2).

When facing a liquidity shock ($\lambda = 1$), the borrower always draws down her credit line fully to avoid liquidation. Moreover, with probability η, the borrower obtains a new credit line and leaves the separation status. Therefore, the continuation value becomes $v(\omega', \bar{\phi}; \mathbf{S}')$, the same as the value before separation. The value function satisfies

$$u^E(\omega, \phi, \bar{\phi}, k, 1, \kappa; \mathbf{S}) = \xi \omega' + (1 - \xi) \mathbb{E}_{\mathbf{S}'|\mathbf{S}} \left[\eta v(\omega', \bar{\phi}; \mathbf{S}') + (1 - \eta) v^E(\omega', \phi, \bar{\phi}; \mathbf{S}') \right].$$

When there is no liquidity shock ($\lambda = 0$), the borrower faces the same tradeoff as before the separation. The continuation value depends on whether the credit limit is cut further (with probability p) and whether the borrower obtains a new credit line (with probability η).

$$u^E(\omega, \phi, \bar{\phi}, k, 0, \kappa; \mathbf{S}) =$$
$$\max_{l} \xi \omega' + (1 - \xi) \mathbb{E}_{\mathbf{S}'|\mathbf{S}} \left[(1-p) \left[\eta v(\omega', \bar{\phi}; \mathbf{S}') + (1 - \eta) v^E(\omega', \phi, \bar{\phi}; \mathbf{S}') \right] + p v^E(\omega', l, \bar{\phi}; \mathbf{S}') \right].$$

At the equilibrium, since the borrower's drawdown choice follows a cut-off rule, the credit limit always drops to zero after a separation. Therefore, with $\phi = 0$, the borrower's problem becomes

$$u^E(\omega, 0, \bar{\phi}, k, 1, \kappa; \mathbf{S}) = \xi \omega' + (1 - \xi) \mathbb{E}_{\mathbf{S}'|\mathbf{S}} \left[\eta v(\omega', \bar{\phi}; \mathbf{S}') + (1 - \eta) v^E(\omega', 0, \bar{\phi}; \mathbf{S}') \right],$$
$$u^E(\omega, 0, \bar{\phi}, k, 0, \kappa; \mathbf{S}) = \xi \omega' + (1 - \xi) \mathbb{E}_{\mathbf{S}'|\mathbf{S}} \left[\eta v(\omega', \bar{\phi}; \mathbf{S}') + (1 - \eta) v^E(\omega', 0, \bar{\phi}; \mathbf{S}') \right].$$

B.3 Linear Value Functions

In this section, I guess and verify that $v(\omega,\phi;\mathbf{S}) = v_1(\mathbf{S})\omega + v_2(\mathbf{S})\phi$ and $v^E(\omega,l,\phi;\mathbf{S}) = v_1^E(\mathbf{S})\omega + v_2^E(\mathbf{S})l + v_3^E(\mathbf{S})\phi$. The linear value functions have two important properties. First, the continuation value must be continuous at $l = \phi$. Thus, $v_2^E(\mathbf{S}) = v_2(\mathbf{S}) - v_3^E(\mathbf{S})$. Second, the value of net worth remains the same after separation; that is, $v_1(\mathbf{S}) = v_1^E(\mathbf{S})$. When $\lambda = 1$, borrowers always choose to drawdown fully,

$$u(\omega,\phi,k,1,\kappa;\mathbf{S}) = \xi\omega' + (1-\xi)\mathbb{E}_{\mathbf{S}'|\mathbf{S}}[v_1(\mathbf{S}')\omega' + v_2(\mathbf{S}')\phi'], \tag{IB.4}$$

subject to the law of motion (2).

When $\lambda = 0$, the drawdown decisions depend on the following tradeoff: the benefit of strategic drawdowns is to avoid the loss from separation $v_2(\omega',\phi;\mathbf{S}') - v_3^E(\omega',l,\phi;\mathbf{S}')$, while the cost of doing so is that the net worth decreases by $(1-\kappa)r(s)l$. With the linear value functions, the optimal choice is a corner solution. Specifically, borrowers draw down fully ($l = \phi$) if κ exceeds a threshold value $\bar{\kappa}$, and they do not draw down at all ($l = 0$) otherwise. The threshold depends on the state $\mathbf{S}$.

$$\bar{\kappa}(\mathbf{S}) = 1 - \frac{\mathbb{E}\big[(1-\xi)p(v_2(\mathbf{S}') - v_3^E(\mathbf{S}'))|\mathbf{S}\big]}{r(s)\mathbb{E}\big[\xi + (1-\xi)v_1(\mathbf{S}')\big|\mathbf{S}\big]}. \tag{IB.5}$$

Then, I derive the equilibrium drawdown decisions characterized by the threshold. I assume that κ follows a uniform distribution on $[0,1]$ and focuses on empirically relevant solutions that are stable and have a separation probability below 1. Proposition 1 demonstrates that there at most one such solution exists.

Proposition 1 *If κ follows a uniform distribution on $[0,1]$, when the separation probability is below 1, there exists at most one stable solution of Equations* (1) - (5).

Proof. Since $p = \Delta/(\Phi - L_D)$, $\bar{\kappa}(\mathbf{S})$ depends on the aggregate drawdown L_D from Equation (IB.5). Rewrite Equation (5) and take $[\Lambda + (1-\Lambda)(1-F(\bar{\kappa}(\mathbf{S}))]\Phi - L_D$ as a function of L_D. If κ follows a uniform distribution on [0,1], the function is strictly convex. Therefore, it has at most two roots. It follows that there are at most two solutions of Equations (1) - (5), and only one of them is stable.

■

Finally, I rewrite the borrowers' problem when the banker rations liquidity as follows. It is then straightforward to verify that the value functions are linear.

$$
\begin{aligned}
v_1(\mathbf{S})\omega+v_2(\mathbf{S})\phi &= \Lambda\beta\xi\mathbb{E}[\omega'(\mathbf{S})|\lambda=1,\kappa] + (1-\Lambda)\beta\xi\mathbb{E}[\omega'(\mathbf{S})|\lambda=0,\kappa] \\
&\quad + \Lambda\beta(1-\xi)\Big\{\mathbb{E}[v_1(\mathbf{S}')]\mathbb{E}[\omega'(\mathbf{S})|\lambda=1,\kappa]+\mathbb{E}[v_2(\mathbf{S}')]\phi\Big\} \\
&\quad +(1-\Lambda)\beta(1-\xi)\Big\{F(\bar{\kappa}(\mathbf{S}))p\Big[\mathbb{E}[v_1^E(\mathbf{S}')]\mathbb{E}[\omega'(\mathbf{S})|\lambda=1,\kappa]+\mathbb{E}[v_3^E(\mathbf{S}')]\phi\Big] \\
&\quad +\big[(1-F(\bar{\kappa}(\mathbf{S})))+F(\bar{\kappa}(\mathbf{S}))(1-p)\big]\Big[\mathbb{E}[v_1(\mathbf{S}')]\mathbb{E}[\omega'(\mathbf{S})|\lambda=1,\kappa]+\mathbb{E}[v_2(\mathbf{S}')]\phi\Big]\Big\}.
\end{aligned}
$$

Because the drawdown choice follows a cut-off rule, the credit limit always drops to zero after a separation. With $\phi=0$, we have

$$
\begin{aligned}
v_1^E(\mathbf{S})\omega+v_3^E(\mathbf{S})\bar{\phi} &= \beta\xi\mathbb{E}[\omega'(\mathbf{S})|\lambda,\kappa] + \eta\beta(1-\xi)\Big\{\mathbb{E}[v_1(\mathbf{S}')]\mathbb{E}[\omega'(\mathbf{S})|\lambda,\kappa]+\mathbb{E}[v_2(\mathbf{S}')]\bar{\phi}\Big\} \\
&\quad + (1-\eta)\beta(1-\xi)\Big\{\mathbb{E}[v_1^E(\mathbf{S}')]\mathbb{E}[\omega'(\mathbf{S})|\lambda,\kappa]+\mathbb{E}[v_3^E(\mathbf{S}')]\bar{\phi}\Big\},
\end{aligned}
$$

where $\omega'(\mathbf{S})=\omega+r_k\omega/(1+\rho)$.

C Computational Algorithm and Estimation Details

This appendix describes the algorithms used to solve and estimate the model. A key challenge is that there are kinks in the value functions and policy functions. Following Iskhakov et al. (2017), I introduce random noise and take moving averages to smooth out kinks in the policy functions. The noise can be viewed as a structural uncertainty shock or simply a smoothing device.

Solving the model. I solve the model using value function iteration with linear interpolation. This robust approach helps to alleviate concerns about the multiplicity of equilibria and non-smooth policy functions. To implement the leverage ratio requirement, I assume the bank goes through reconstruction and assign bank equity as the banker's value whenever the requirement is violated. The algorithm is as follows:

1. Place a grid on the state space. I normalize the banker's problem by bank credit, such that

the payoff-relevant state vector degenerates to $\mathbf{S} = [\frac{E}{L+\Phi}, \frac{L}{L+\Phi}, s]$. The grid of $\frac{E}{L+\Phi}$ contains 101 grid points with equal space between 0.06, the required leverage ratio, and 0.16. The grid of $\frac{L}{L+\Phi}$ contains seven grid points spanning 0.44 to 0.5. The exogenous state s has two grid points, 1 (normal) or 2 (crisis). Increasing the size of the grid does not significantly affect the solution.

2. Initialize value functions $V^{(0)}(\mathbf{S})$ and $v^{(0)}(\omega, \phi; \mathbf{S})$.
3. Initialize policy function $\Delta^{(0)}(\mathbf{S}) = 0$ and $L_D^{(0)}(\mathbf{S}) = \Lambda$. A negative $\Delta(\mathbf{S})$ represents the issuance of new credit.
4. Starting with the functions $V^{(n)}(\mathbf{S})$, $v^{(n)}(\omega, \phi; \mathbf{S})$, $\Delta^{(n)}(\mathbf{S})$, and $L_D^{(n)}(\mathbf{S})$ from the n-th iteration or from the initial guess ($n = 0$), iterate the value functions and the policy functions as follows:
   (a) Solve for the borrowers' optimal policy L_D^* and value function v^* given $\Delta^{(n)}$ and $v^{(n)}$.
   (b) Smooth L_D^* and v^* by taking the moving averages with a window width of three grid points of $\frac{E}{L+\Phi}$ and three grid points of $\frac{L}{L+\Phi}$.
   (c) Update the policy function and the value function such that $L_D^{(n+1)} = L_D^*$ and $v^{(n+1)} = 0.5v^* + 0.5v^{(n)}$.
   (d) Solve for the banker's optimal policy Δ^* and value function V^* given $L_D^{(n+1)}$.
   (e) Smooth V^* by taking the moving averages with a window width of three grid points and update the policy function and the value function such that $\Delta^{(n+1)} = \Delta^*$ and $V^{(n+1)} = 0.025V^* + (1 - 0.025)V(n)$.
5. Go back to Step 4 until the value functions and policy functions from two consecutive iterations are close enough.

Estimation. I estimate the parameters θ_2 by the simulated method of moments (SMM). SMM chooses parameter values that minimize the distance between moments from real data and their analogs generated by the model simulation. Once I have solved the model for a given set of parameters θ_2, I simulate the normal steady state and a transition path from the normal steady state into crisis periods. Both the actual and the simulated moments are computed in a simple way as described in Section 3.3.

Denote M as the vector of moments from the actual data and $\mathbf{M}(\theta_2)$ as the moments generated by the model with parameters θ_2. The SMM procedure searches the set of parameters that minimizes the weighted deviations between the actual and the simulated moments; that is,

$$(M - \mathbf{M}(\theta_2))' W (M - \mathbf{M}(\theta_2)), \tag{IC.6}$$

I choose the weight matrix W as the inverse of the variance-covariance matrix of the actual moments estimated by bootstrapping with replacement at the bank level on the actual data. This adjustment accounts for scale differences and correlations between the moments.

I use the covariance matrix adaptation evolution strategy (CMA-ES) algorithm to minimize the distance. The CMA-ES algorithm is a derivative-free evolutionary algorithm for nonlinear optimization problems. I draw off-springs in the parameter space in each round. The off-springs are normally distrusted around the mean inherited from the last round. Then, I compute the distance between simulated data with each off-spring and actual data as its fitness. I rank the off-springs by their fitness and update the mean and variance–covariance matrix for the next round. I iterate this procedure until the offsprings are close enough to the mean.

To compute the standard errors, I use the standard formula for the variance–covariance matrix of parameter estimates (Strebulaev and Whited, 2012). Since there is no cross-sectional heterogeneity in the model, the simulation error is absent. The covariance matrix of the estimates is given by

$$\left(\mathbf{J}' W \mathbf{J}\right)^{-1}, \tag{IC.7}$$

where W is the inverse of the variance–covariance matrix of data moments and $\mathbf{J} = \frac{\partial \mathbf{M}(\theta_2)}{\partial \theta_2}$ is the Jacobian matrix around the SMM estimate. I approximate the Jacobian matrix using forward finite differences, identical to the computation in Table 2.

Finally, I follow Andrews et al. (2017) and compute the sensitivity of model estimators (e.g., the amplification effect) to moments. The sensitivity is given by

$$(\nabla X)' \left(- \mathbf{J}' W \mathbf{J}\right)^{-1} \mathbf{J}' W, \tag{IC.8}$$

where W is the inverse of the variance–covariance matrix of data moments, $\mathbf{J}$ is the Jacobian matrix, and ∇X is the gradient of the estimator with respect to structural parameters, all computed at the

SMM estimates.

D Robustness Analysis

This section explores the robustness of the estimation and quantitative results. As discussed in Section 3, empirical evidence suggests that the hike in drawdown rate is due to the bank–borrower strategic complementarity rather than shocks on borrowers' demand for operating liquidity. To further examine this issue, I estimate two models with a state-dependent probability of liquidity shocks.

In particular, I set $\Lambda = 0.55$ in normal times, as in the estimation in Section 3.3, and $\Lambda = 0.565$ or 0.58 in crises, rather than 0.55. I then estimate the parameters θ_2 and compute the magnitude of the amplification effect as in Section 4.2.

Table DII presents the results, which suggest that the amplification effect is robust to changes in state-dependent demand for operating liquidity. Specifically, when the demand for operating liquidity increases in crises, the estimation delivers a higher probability of substitution and fewer strategic drawdowns. This results in larger standard errors for the estimates of substitution probability compared to the baseline estimation in column (1). However, the other estimated parameters remain similar.

Interestingly, even though there are fewer strategic drawdowns, the amplification effect is close to that in the baseline estimation. For example, comparing columns (2) versus (1), strategic drawdowns reduce by 24

These findings suggest that the amplification effect is not driven by the correlation between drawdowns and the exogenous state but rather by the positive association between drawdowns and bank leverage at the equilibrium. This positive association creates incentives for bankers to ration liquidity and deleverage, even in the absence of state-dependent demand.

Table DII: **Robustness Analysis**

This table represents the estimation results of models with state-dependent liquidity demand. Column (1) replicates the results in Table 4 in the main text, with $\Lambda = 0.55$ in both normal and crisis periods. For columns (2) and (3), I set $\Lambda = 0.565$ and 0.58 in crisis periods and estimate two models, respectively. The upper panel displays the estimated parameters, with standard errors in parentheses calculated in the same way as in the main text. The middle panel shows the estimated moments: credit contraction and strategic drawdown in crises. In the lower panel, I consider the counterfactual with no strategic drawdowns and compute the magnitude of the amplification effect.

Parameters	(1) $\Lambda = 0.55$	(2) $\Lambda = 0.565$	(3) $\Lambda = 0.58$
Discount Rate β	0.0182	0.0181	0.0175
	(0.00219)	(0.00696)	(0.00294)
Issuance Cost γ^+	2.68	2.68	2.73
	(0.574)	(0.464)	(0.602)
Probability of Substitution η	0.317	0.367	0.512
	(0.158)	(0.262)	(0.292)
Sensitivity to Leverage ψ	0.105	0.108	0.109
	(0.0388)	(0.0376)	(0.0179)
Rationing Cost γ^-	0.116	0.106	0.0567
	(0.0481)	(0.0619)	(0.24)
Operating Cost a_n	0.00330	0.00329	0.00333
	(0.000321)	(0.000309)	(0.000393)
Operating Cost a_c	0.0204	0.0206	0.0209
	(0.00393)	(0.00323)	(0.00231)
Baseline Model			
(1a) Credit contraction (%)	1.54	1.41	1.40
(2a) Strategic Drawdown (%)	7.68	5.82	3.83
No Strategic Drawdown			
(1b) Credit contraction (%)	0.536	0.521	0.184
(2b) Strategic Drawdown (%)	0.0	0.0	0.0
Amp. Effects: row (1a) − row (1b)	1.00	0.889	1.22

E Additional Figures

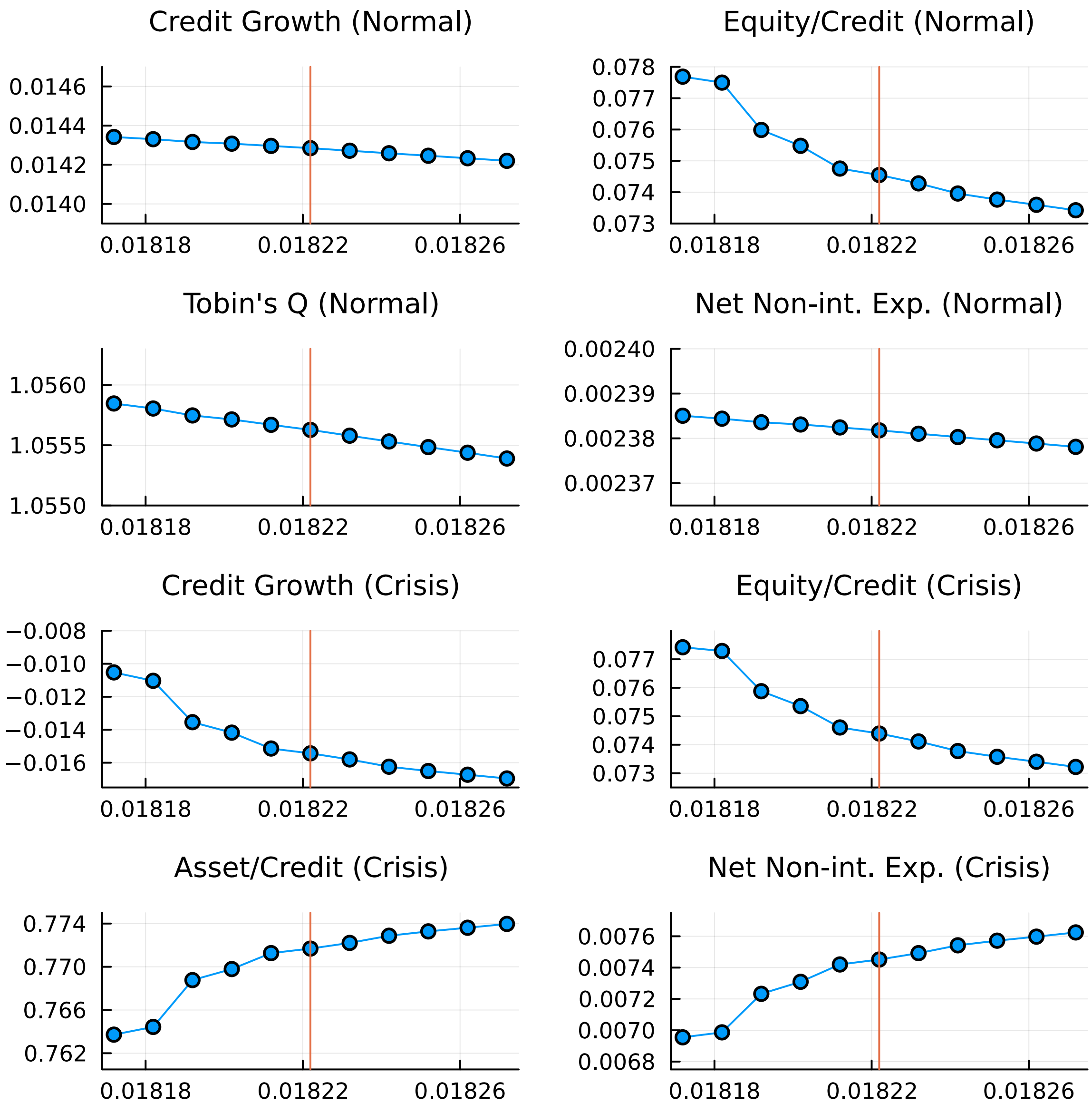

Figure E1: Sensitivity of moments to discount rates β

Note: I set all estimated parameters at the SMM estimate. Then, I perturb β. For each value of β, I solve the model, simulate the data, and compute the moments. Each panel corresponds to one moment. The red vertical line corresponds to the SMM estimate of β.

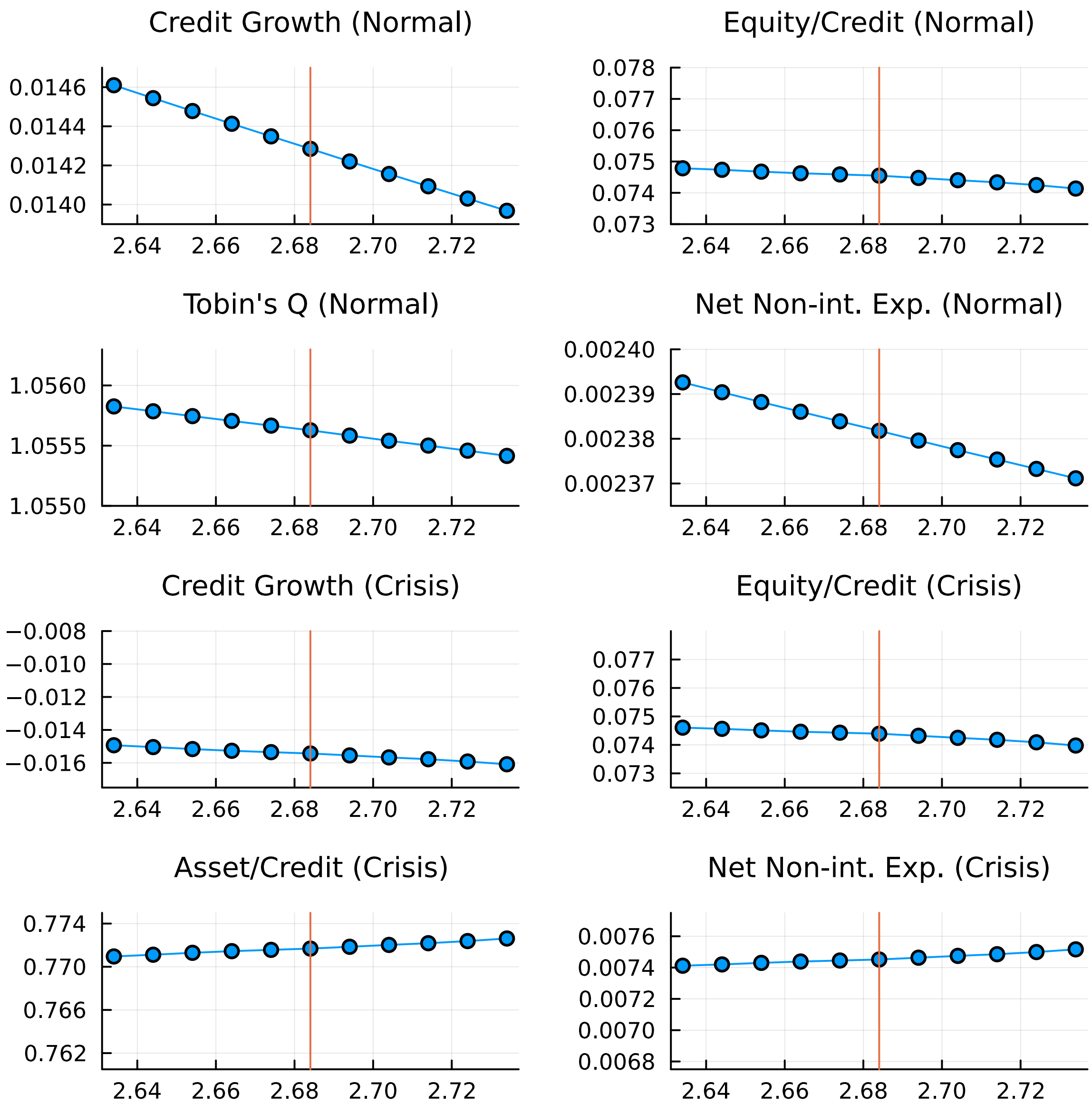

Figure E2: Sensitivity of moments to issuance costs γ^+

Note: I set all estimated parameters at the SMM estimate. Then, I perturb γ^+. For each value of γ^+, I solve the model, simulate the data, and compute the moments. Each panel corresponds to one moment. The red vertical line corresponds to the SMM estimate of γ^+.

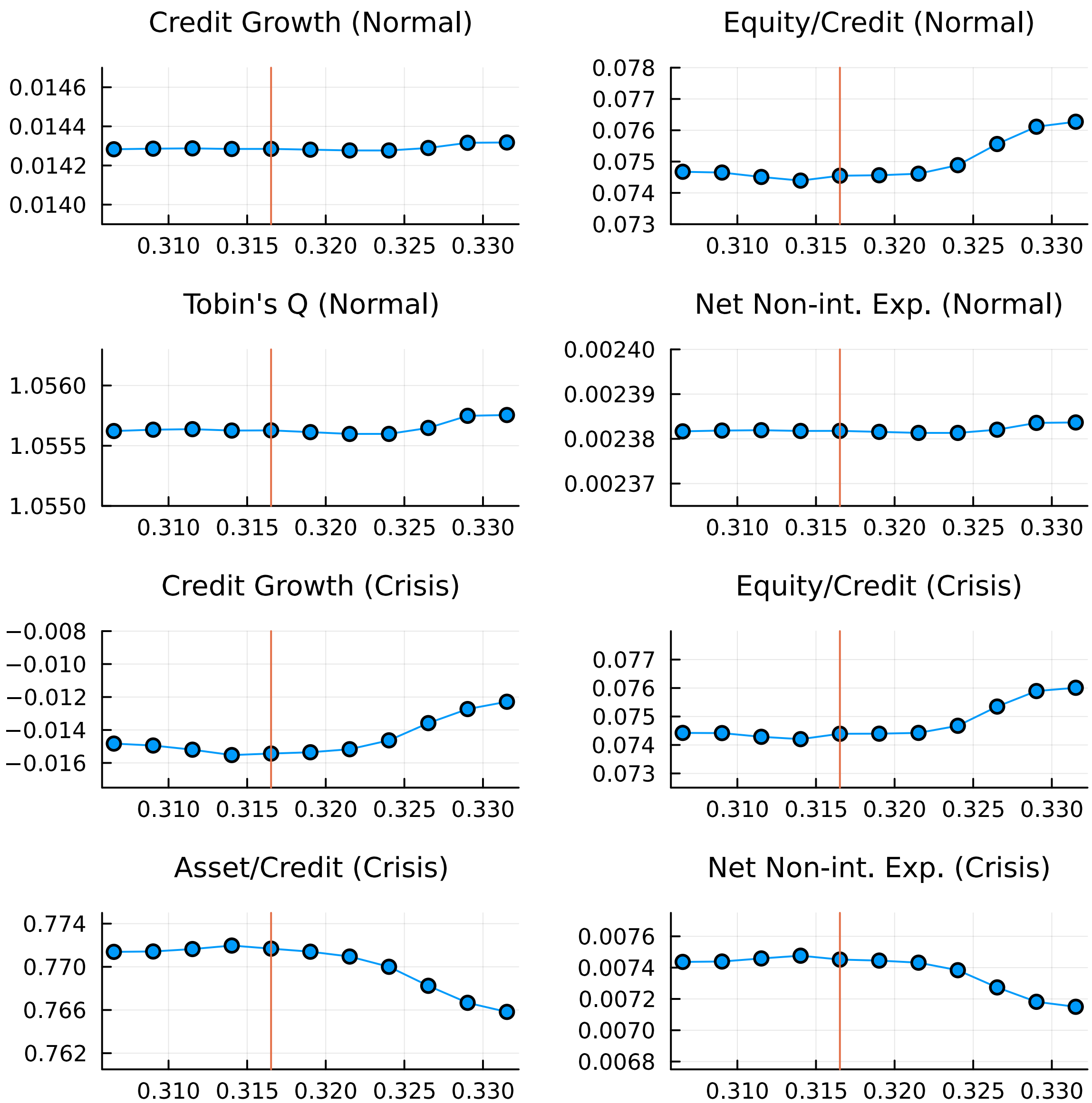

Figure E3: Sensitivity of moments to substitution probability η

Note: I set all estimated parameters at the SMM estimate. Then, I perturb η. For each value of η, I solve the model, simulate the data, and compute the moments. Each panel corresponds to one moment. The red vertical line corresponds to the SMM estimate of η.

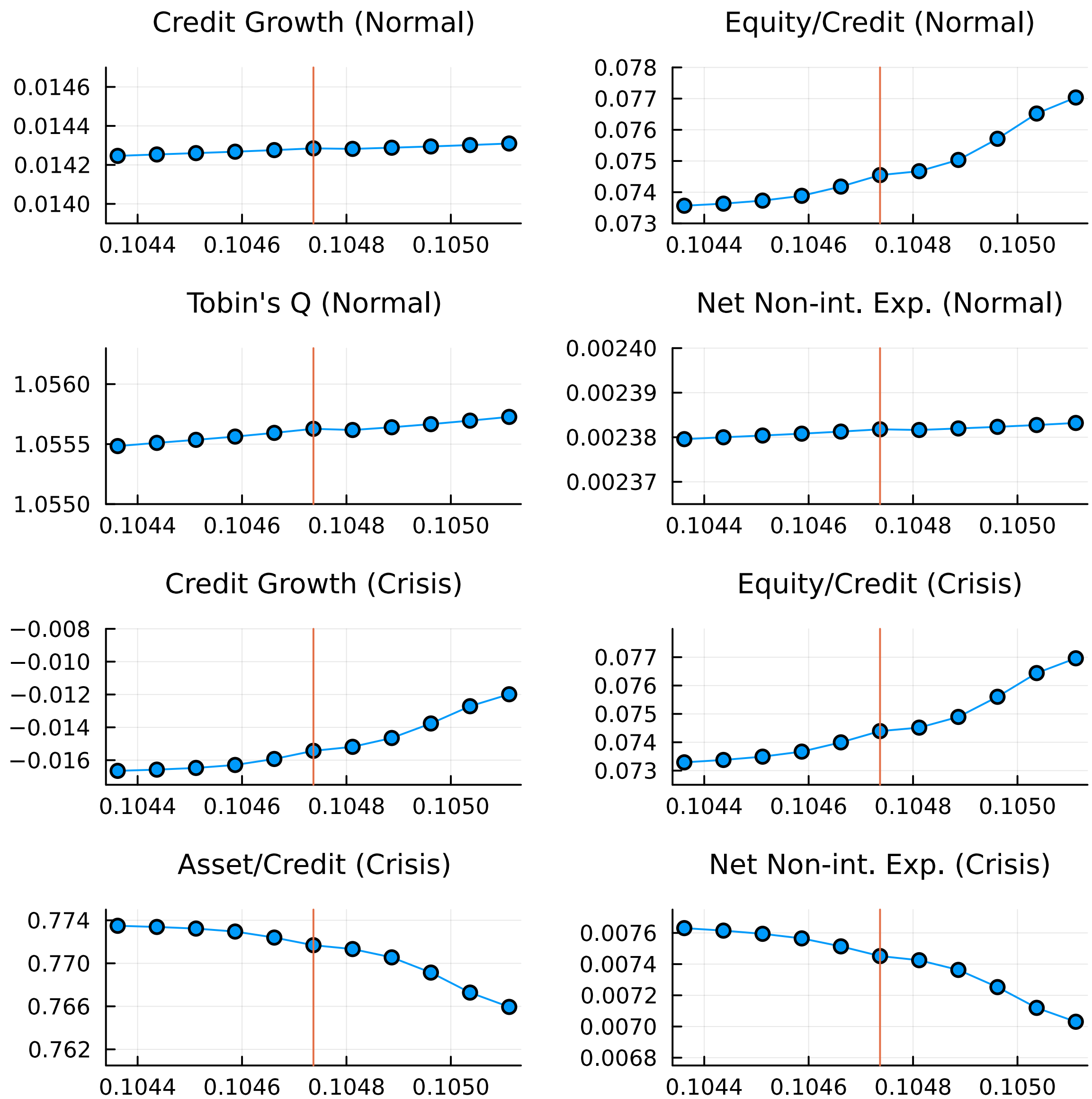

Figure E4: Sensitivity of moments to leverage sensitivity ψ

Note: I set all estimated parameters at the SMM estimate. Then, I perturb ψ. For each value of ψ, I solve the model, simulate the data, and compute the moments. Each panel corresponds to one moment. The red vertical line corresponds to the SMM estimate of ψ.

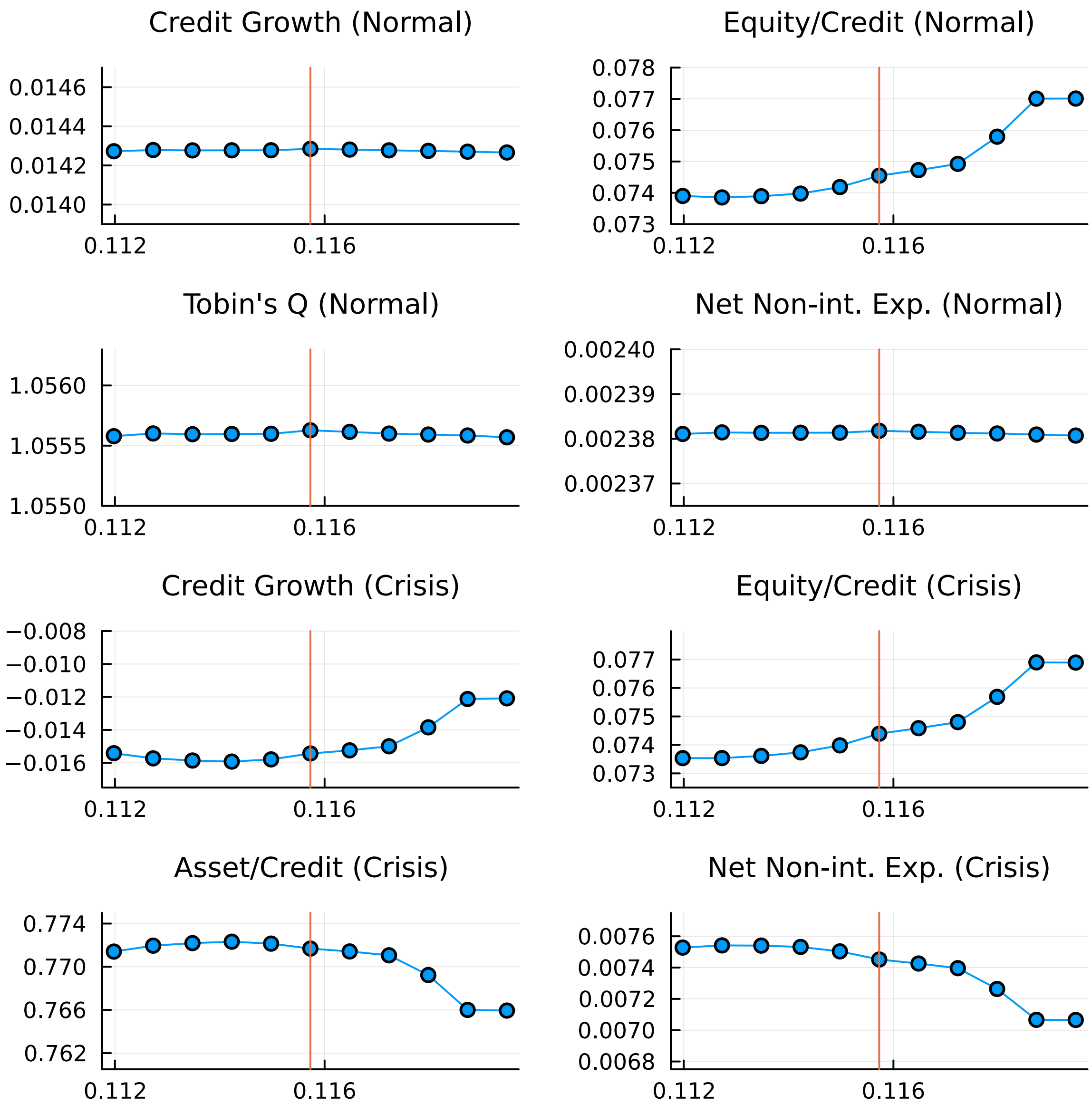

Figure E5: Sensitivity of moments to rationing costs γ^-

Note: I set all estimated parameters at the SMM estimate. Then, I perturb γ^-. For each value of γ^-, I solve the model, simulate the data, and compute the moments. Each panel corresponds to one moment. The red vertical line corresponds to the SMM estimate of γ^-.

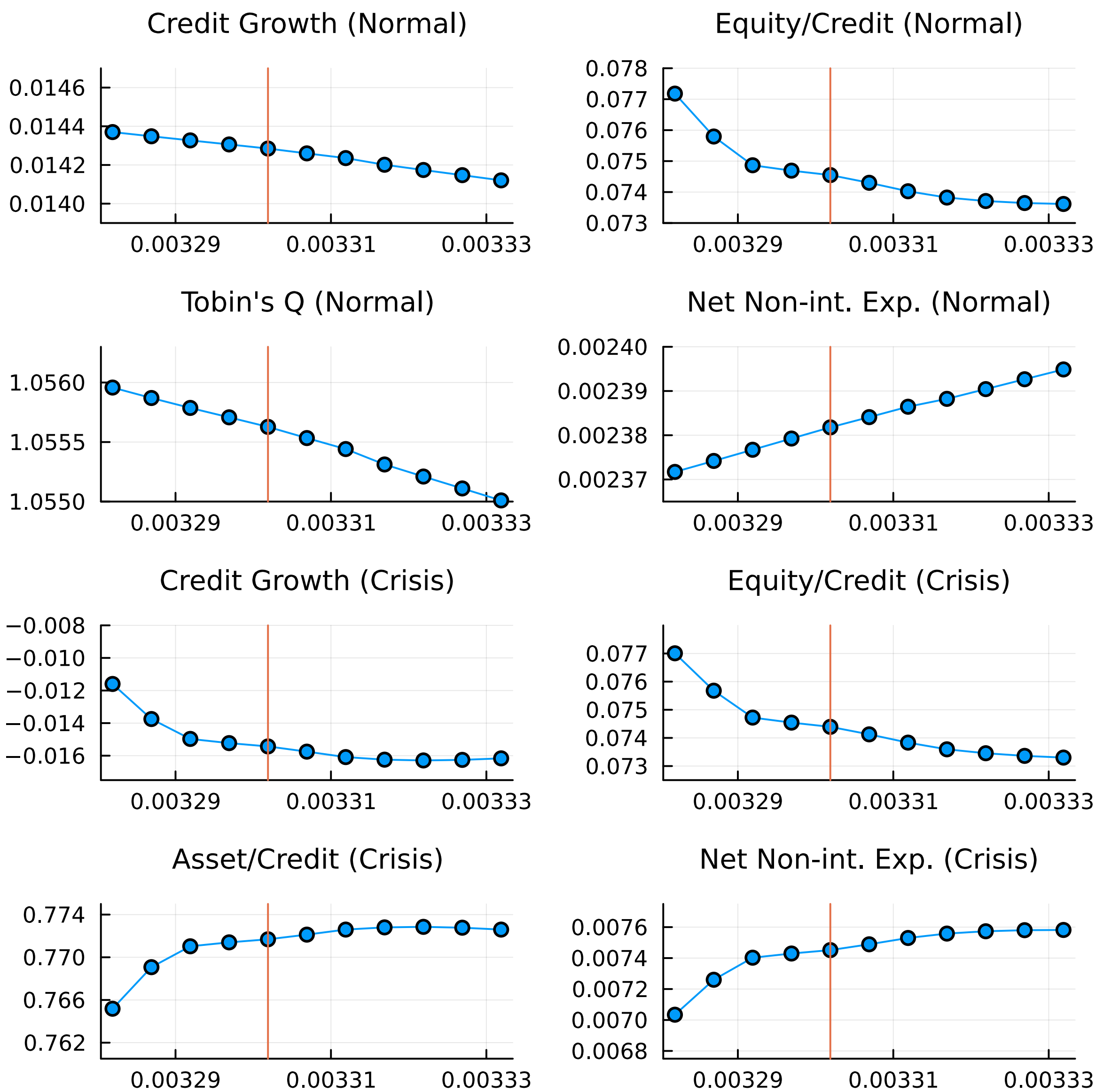

Figure E6: Sensitivity of moments to operating costs a_n

Note: I set all estimated parameters at the SMM estimate. Then, I perturb a_n. For each value of a_n, I solve the model, simulate the data, and compute the moments. Each panel corresponds to one moment. The red vertical line corresponds to the SMM estimate of a_n.

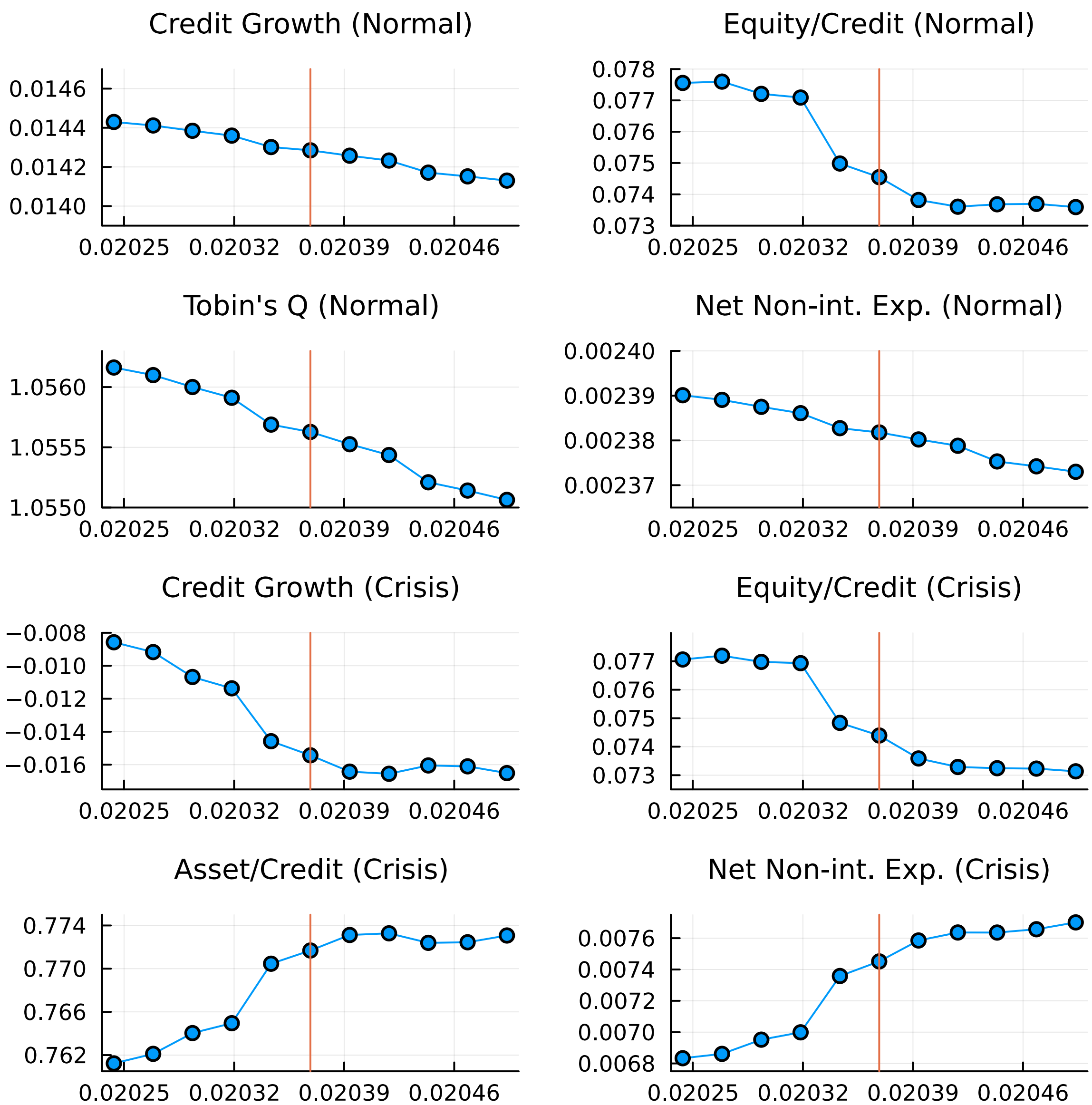

Figure E7: Sensitivity of moments to operating costs a_c

Note: I set all estimated parameters at the SMM estimate. Then, I perturb a_c. For each value of a_c, I solve the model, simulate the data, and compute the moments. Each panel corresponds to one moment. The red vertical line corresponds to the SMM estimate of a_c.