Incentives for Information Acquisition and Voting by Shareholders*

Adam Meirowitz
Yale University

Shaoting Pi
Iowa State University

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Abstract

In extant work on information acquisition and governance, the value of information is related only to the impact of a shareholder's vote on corporate policies. We consider a setting where shareholders can vote and trade. The incentives to acquire information are higher but the opportunity to generate trading rents distorts voting incentives and reduces the quality of governance for any fixed level of information acquisition. These negative incentives are stronger when more voters are informed and eventually dominate the gains from more information acquisition by shareholders. As a result, the quality of firm governance is eventually decreasing in the fraction of shareholders that become informed. One take-away is that concerns that proxy advisors may crowd out information acquisition and reduce governance quality seem overstated. Turning to the role of transparency, we show that if the market can only learn whether a motion passed as opposed to the exact voting tally, then opportunities to trade do not cause these distortions, and governance is dramatically improved. Accordingly, the analysis provides a rationale for reducing transparency in governance.

KEYWORDS: strategic voting; common values voting; shareholder voting, corporate governance, information aggregation, Condorcet Jury Theorem

*Preliminary Draft. PLEASE DO NOT CITE OR CIRCULATE WITHOUT PERMISSION

1 Introduction

The justification for reliance on shareholders to vote on corporate policy (or provide a mandate for decisions by management) hinges largely on the idea that shareholders will obtain information that is germane to the decision at hand. To a first approximation, however, it is believed that only a small fraction of shareholders actually acquire information and use it to guide any level of informed oversight or policy choice. The presence of substantial investors who passively follow proxy advisors' recommendations to vote has not escaped regulators. SEC Commissioner Daniel M. Gallagher writes:

I have grave concerns as to whether investment advisers are indeed truly fulfilling their fiduciary duties when they rely on and follow recommendations from proxy advisory firms. Rote reliance by investment advisers on advice by proxy advisory firms in lieu of performing their own due diligence with respect to proxy votes hardly seems like an effective way of fulfilling their fiduciary duties and furthering their clients' interests. (Source: SEC Website)

Of course, reliance on proxy recommendations need not be a cause for grave concern if the recommendations are based on an unbiased and nuanced assessment of quality information. However, a second concern for many observers is the possibility of bias,[1] as proxy advisors may not seek to maximize the same objectives as investors.

[1]For example, Ma and Xiong (2021)find that even an unconflicted proxy advisor tends to skew its recommendations based on its clients' beliefs or preferences. Malenko, Malenko, and Spatt (2021) find that a profit-maximizing proxy advisor, who derives revenue from selling information to shareholders, has incentives to bias its recommendations in order to create controversy in shareholder voting. Levit and Tsoy (2022) develop a more general model applicable to proxy advisors and find that such advisors benefit from providing one-size-fits-all recommendations, which may not align with shareholders' interests, in order to enhance their influence. Matsusaka and Shu (2021) find that proxy advisors might slant their advice toward biased shareholders, consequently steering corporate elections away from value maximization.

The concern, though, is not just that proxy recommendations are biased. Some worry that they also crowd out unbiased quality information that would be able to offset this bias. This then leads to concerns that in addition to standard free-riding problems, which typically result in underinvestment in information for collective choice, the provision of cheap information by proxy advisors may crowd out the information acquisition that is crucial to a well-functioning process of shareholder governance.

Recent innovative theoretical work provides support for this intuition. In the setting of a common values problem where shareholders simply evaluate whether a given policy increases firm value, Malenko and Malenko (2019) develop a model that captures some of these concerns. Their analysis highlights a natural intuition; a shareholder is willing to gather information up to the point where the marginal cost of additional information balances the expected impact the voter's information will have. This expected impact is the product of the likelihood that the shareholder's vote is pivotal and the stakes (i.e., how much share value is impacted by making the correct choice). In settings with many voters, this so-called pivot probably is quite small.

Our point of departure is the recognition that shareholders see themselves as involved in more than just governance, and thus the value of information is not limited to its impact on voting. As evidenced by recent empirical studies, shareholders produce and process information for voting items (e.g., Gao and Huang (2022) and Iliev and Lowry (2015)) and their voting strategies and trading strategies interact (e.g., Li, Maug, and Schwartz-Ziv (2022) and Fos and Holderness (2021)). We thus worry that work on how information acquisition impacts governance, which ignores the fact that information also impacts trading and trading and voting can interact, may miss important effects. Few would argue with the idea that in addition to realizing the value of information on governance by voting informatively, informed shareholders may also capitalize on their information by trading more wisely and extracting informational rents from markets.

If the point were just that information levels are uniformly higher once one accounts for opportunities to use information for both voting and trading, our prognosis for corporate governance would be unambiguously more positive than extant theoretical work. We find, however, that things are more complicated. Building off the voting and trading model of Meirowitz and Pi (2022), we consider settings where some shareholders acquire private information and others do not. All shareholders then vote and are free to trade in a market. The value of information and, therefore, incentives to acquire it depend on margins from both voting and trading. We find that when too many shareholders acquire information, voting must be very uninformative, and governance is of low quality. Accordingly, the likely effect of reducing information acquisition costs is ambiguous: if one could fix how shareholders vote, adding additional informed shareholders would improve governance. However, in equilibrium, adding these informed shareholders may cause equilibrium voting strategies to become less informative. Thus, instead of improving the informativeness of the governance decision, adding more informed voters may actually reduce the probability of selecting the correct corporate policy. The non-monotonic relationship between the number of informed shareholders and the efficiency of governance implies that the optimal fraction of informed shareholders is less than one and that governance is not always improved by reforms aiming at increasing information acquisition by shareholders.

This claim is even stronger if one rightfully accounts for the possible costs of information acquisition by more shareholders. This logic then undermines the concern that by crowding out the acquisition of higher-quality information, proxy advisors necessarily harm governance. Reducing information acquisition has an upside, as it may improve the degree to which information that is acquired is used to select desirable policies.

2 Overview of Findings

First, we find that there are two types of informational advantages shareholders may have over the market after voting. The first type of informational advantage occurs when a shareholder acquires information, and the market cannot perfectly infer her private information from the voting results. Moreover, we find that an informed shareholder is able to capitalize on her private signal by trading after every realization of the public voting outcomes (even if she is not the pivotal voter). This implies that the value of acquiring private information does not hinge on the probability of being pivotal. This feature is contrary to the conclusion commonly drawn by the previous literature. The second source of informational advantage is knowing if a vote is correlated with a private signal. Since the market cannot tell if a vote is from an informed shareholder and thus is informative or from an uninformed shareholder and thus simply noise, the share prices after voting can be thought of as based on an average level of voting informativeness. This point allows us to see the second type of informational advantage which accrues to shareholders who do not acquire information (referred to as "uninformed shareholders" below). An uninformed shareholder knows that her vote is less informative than the market. Accordingly, she recognizes that market prices that treat her vote as having average informativeness are distorted. An uninformed voter thinks the firm is overpriced (underpriced) and thus wants to sell (buy) if she finds that the alternative she votes for is chosen (not chosen).

Second, we find that how informed shareholders utilize their private information depends on the number of shareholders acquiring information. Each informed shareholder is willing to vote for the alternative favored by her information only if the number of shareholders acquiring information is less than a threshold. If the number of informed shareholders is larger than this threshold, every informed shareholder votes with a mixed strategy and votes against her information with positive probability. This reduces the quality of information aggregation given the information that is acquired. Moreover, as the number of informed shareholders increases, the equilibrium votes of informed shareholders become less correlated with their information.

This second finding introduces friction between the ratio of informed shareholders to all shareholders and the probability of making the correct decision through voting. On the one hand, as the number of informed shareholders increases, more information is added to the voting process. This will increase the efficiency of voting if behavior remains constant. On the other hand, as the number of informed shareholders increases, each informed shareholder's vote becomes less informative. This will decrease the likelihood of making the correct decision through voting. As a result, the overall effect of having more informed shareholders on the quality of the decision made by voting depends on which effect dominates. We find that in equilibrium, the second effect eventually dominates the first one. When the ratio of informed shareholders to all shareholders is higher than a certain threshold, having more informed shareholders actually reduces the probability that the correct policy is chosen.

We are able to implicitly characterize the information cost that supports acquisition by the optimal number of shareholders as a function of the total number of shareholders and primitive signal quality. Asymptotic analysis allows us to present a clean characterization of the optimal cost when the number of shareholders is large. Not surprisingly, the optimal cost is increasing in signal quality. More subtly, the optimal cost does not vanish with the number of shareholders. In other words, even though the impact that information acquisition can have on an agent's ability to impact the policy choice or market prices through voting vanishes, the value of information does not vanish. This is because an informed shareholder uses her private information to decide whether to bet with or against the firm after the policy choice is made. When private signals are informative, getting trading decision right has non-vanishing value for a shareholder. The value of information then remains proportional to the shareholders' stakes from seeing the firm making the correct decision. We take this as strong confirmation that extant work is missing the first-order effects and that interactions between market and voting behavior should not be ignored. Finally, while uninformed shareholders are able to extract informational rents from trading (precisely because they know their vote is less informative than the market thinks it was), this benefit does vanish as the number of shareholders gets large.

To assess whether limits on trading enhance governance, we compare a model with voting and trading to a baseline with only voting. Here, we observe that due to the opportunities to extract informational rents from trading, the equilibrium levels of information acquisition are higher when trading is possible. Thus, despite the distortions to voting that trading opportunities create, sometimes the increase in information acquisition from the opportunity to trade results in a higher equilibrium probability of making the correct choice. So, governance can be improved by trading opportunities after voting.

Finally, to connect with the general tendency of regulators and pundits to favor transparency, we analyze a model in which the market can only observe whether shareholders approved the policy change. In this model, the vote tally is unobservable, and thus the opportunities to trade do not lead to distortions in voting. Shareholders still extract some informational rents from the market, and so reduced transparency can increase incentives to gather information compared to a model with no trading. This also improves corporate governance, given the level of information that is acquired.

3 Closely Related Literature

This paper bridges two recent theoretical contributions. As mentioned, Malenko and Malenko (2019) endogenize information acquisition decisions. In particular, they look at how shareholders choose between idiosyncratic private signals and a common signal sold by a proxy advisor. The focus is on whether the latter crowds out the former. Meirowitz and Pi (2022) take the distribution of private information as fixed and add a trading stage to the classic voting problem. The key finding here is that shareholder voting is generally less informative because, in equilibrium, voters must balance incentives to steer firm policy in the correct decision with incentives to mislead the market about their assessment of the value of the shares and then trade on these informational asymmetries. Of course, both effects can be small for a given share, but equilibrium requires balancing these kinds of margins.

A secondary strand of theoretical work is also relevant. A few papers take a mechanism design approach to studying information acquisition and decision-making in committees or collective choice bodies. The focus of these papers is finding optimal ways to acquire and use the information to make a policy decision in common values setting. Here the most relevant papers are Persico (2004) and Gerardi and Yariv (2008). Persico (2004) focuses on characterizing the best equilibrium given a fixed voting rule. Gerardi and Yariv (2008) do not take the institution as fixed and characterize the optimal mechanism. Two important insights come from these papers. First, in contrast to the approach taken by Malenko and Malenko (2019), where the focus is on equilibria in which each voter uses the same mixed strategy in information acquisition decisions, Persico (2004) finds that efficiency requires that voters use asymmetric information acquisition strategies. Some voters opt to acquire information in pure strategies, and some opt to remain uninformative in pure strategies. We build off this insight and focus on equilibria in which information acquisition strategies are degenerate and asymmetric because this involves an efficiency gain over the approach in Malenko and Malenko (2019). But, more importantly, we take this approach because many accounts of shareholder voting leave room for different kinds of investors. It is widely believed that some investors are active hands-on participants. They learn what they can and are typically involved voters. On the other hand, there are investors that routinely take a hands-off approach. The fact that one investor generally stays in one of these two categories as opposed to switching her level of engagement from vote to vote justifies this equilibrium selection.[2]

A second insight, which comes out of the analysis of Gerardi and Yariv (2008) is that once the incentives to acquire information and reveal this information are both considered, an optimal institution will involve inefficient governance given the information that is acquired. This is of course true in our equilibrium as well. It is worth noting that we stop well short of the mechanism design approach. In fact, we don't even select the optimal equilibrium to the game as we do not seek to find equilibria where voters that opt to remain uninformed vote in pure strategies in ways that minimize their contribution to variance in the policy chosen. We choose to proceed in this way because we follow the empirical literature in asserting that the voting behavior of retail investors is not particularly predictable.

[2]The presence of different kinds of investors could owe to exogenous difference in the players or it can be an equilibrium phenomena resulting from play of asymmetric equilibria to a symmetric game. We explore the latter and offer an explanation for the emergence of this form asymmetric behavior. In a richer dynamic setting, this endogenous/equilibrium sorting in a symmetric environment could then lead to other differences. Players that choose to acquire information might then take actions that lower their costs to acquiring information.

4 Model

We consider a firm that has n (an odd number) of shareholders, and each of them has 1 share. A fraction $\frac{k}{n}$ of the shareholders are assumed to receive private signals. Shareholders vote and then trade. [3]

Voting consists of making a decision $x \in 0, 1$ under the simple majority rule. The shareholders face uncertainty about which decision is better for the firm. Formally speaking, we denote the underlying state by $\omega \in 0, 1$, with the interpretation that if $x = \omega$, each share has value 1, and if $x \neq \omega$, each share has value 0. The common prior is that $Pr(\omega = 1) = \frac{1}{2}$.

At the beginning of the game, k of the shareholders receive a private signal s_i about the underlying state ω where $s_i \in 0, 1$. Private signals are imperfectly informative, with $Pr(s_i = \omega|\omega) = q > \frac{1}{2}$, and conditionally independent. If a shareholder does not receive a signal, we denote her information set with $s_i = \emptyset$. It is convenient to record whether i has received a signal by a_i, with $a_i = 1$ corresponding to receiving a signal and $a_i = 0$ corresponding to not obtaining a signal. We assume that k as well as the identity of the agents obtaining a signal is common knowledge. In Section 7, we endogenize k.

In the first period, shareholders cast ballots $v_i \in 0, 1$. The publicly available vote tally is denoted by $t = \sum_{i=1}^{n} v_i$. Whichever policy receives more votes is selected.

$$x = \begin{cases} 1 \text{ if } t \geq \frac{n-1}{2} \\ 0 \text{ otherwise} \end{cases} \tag{1}$$

[3] In Section 7, we endogenize the acquisition decision by assuming a common cost to information and examining equilibria with asymmetric but deterministic acquisition decisions. Other approaches to endogenize information acquisition can be considered and would build upon our basic results on voting and trading with k informed shareholders.

It is convenient to also describe the tally from shareholders other than i, denoted $t_{-i} = \sum_{j \in n-\{i\}} v_j$.

In the second period, after observing the policy x, the vote counts t, and the common price $P_x(t)$ each trader submits an order $b_i \in \{-1, 0, 1\}$ with the interpretation that $b_i = -1$ denotes selling their share, $b_i = 0$ denotes holding and $b_i = 1$ denotes buying an additional share. Trades are executed at the common price $P_x(t)$, which is assumed to satisfy a no-arbitrage condition,

$$P_x(t) = E[1_{x=\omega}|t]$$

where the expectation is taken over a version of the conditional probability that is based on a correct conjecture of the joint probability $Pr(t|s)$. As long as strategies are measurable, we may conveniently write,

$$P_x(t) = Pr(\omega = x|x, t)$$

where the conditional probability satisfies Bayes' rule given correct conjectures of the voting strategies. Note that because shareholders can compute the price based on public information, it does not matter whether we assume that the price is posted before or after orders are submitted.[4] Meirowitz and Pi (2022) extend the model to allow orders to impact prices and show that qualitatively the logic from the posted price model carries over. For reasons of traceability, we retain this feature from their baseline model.

[4]To focus incentives on how information acquisition and governance can depend on the anticipation of optimal trading, we do not explicitly include pre-voting trade. What matters is that at the time of voting, previous market transactions do not fully reveal the private information of the shareholders. The presence of noise traders is sufficient to ensure this feature.

Finally, the state is observed, and the value of the share is realized. One interpretation is that the firm provides a one-time dividend of either 1 or 0 for each share, and the game ends. Thus, at the end of the game, the value of each share is given by

$$v(x, \omega) = \begin{cases} 1 \text{ if } \omega = x \\ \\ 0 \text{ otherwise} \end{cases} \tag{2}$$

and an agent that bought a share obtains payoff $2v(x, \omega) - P_x(t)$, an agent that sold a share receives payoff $P_x(t)$ and an agent that made no trades obtains payoff $v(x, \omega)$.

Note that we assume that only aggregate vote counts are observed by shareholders and the market maker.

5 Benchmark: Acquiring Information and Voting

We begin with a natural benchmark in which there is no trading opportunity after voting. We ultimately characterize the condition on cost for there to exist an equilibrium in which $k \leq n$ shareholders buy information and the remaining $n - k$ shareholders do not buy information. We first flesh out the incentives in the voting stage, taking k as fixed and then turning to endogenizing k in Section 7.

Proposition 1 (Voting Strategy Without Trading)**.** *Given that k shareholders acquire information, the following voting strategy constitutes a mutual best response: each informed shareholder sincerely votes their signal, $v_i(s_i) = s_i$ for $s_i \in \{0, 1\}$, and every uninformed shareholder votes for each policy with equal probability, $Pr(v_i(s_i = \emptyset) = 1) = Pr(v_i(s_i = \emptyset) = 0) = \frac{1}{2}$.*

The proof is in the appendix.

Of course, there are other mutual best responses. For example, holding fixed the acquisition strategies, a profile where all voters cast the same ballot is a mutual best response since no voter is ever pivotal.

We now examine how the number of informed shareholders affects the probability that the above voting strategy makes a correct decision.

Proposition 2 (Information Aggregation Without Trading)**.** *The probability that the above voting strategy selects the correct policy is strictly increasing in the number of informed shareholders, k.*

$$\frac{dPr^*(x=\omega)}{dk} > 0$$

The proof is in the appendix.

Proposition 2 shows that without considering the market, having more informed shareholders improves the probability of selecting the correct policy via shareholder voting. This is because an informed shareholder's vote is more likely to be correct than an uninformed shareholder's vote, and the probability an informed voter votes correctly does not depend on the number of informed voters. In the following section, we will see that if the probability that an informed shareholder's vote is correct decreases (as it does in equilibrium when there is also trading), then when more shareholders acquire information, the probability of selecting the correct policy does not necessarily increase. Our next result hinges on the fact that in the baseline model, the value of acquiring a private signal hinges on a shareholder being pivotal.

Proposition 3 (Information Acquisition Strategies and Information Cost)**.** *To support an equilibrium in which k shareholders buy information, the cost of acquiring* *information, c, must satisfy*

$$(q-\frac{1}{2})\underbrace{\sum_{i=0}^{\frac{n-1}{2}}\binom{k}{i}q^i(1-q)^{k-i}\binom{n-k-1}{\frac{n-1}{2}-i}(\frac{1}{2})^{n-k-1}}_{Pr(pivotal\ as\ an\ UNinformed\ shareholder)} \leq c \leq (q-\frac{1}{2})\underbrace{\sum_{i=0}^{\frac{n-1}{2}}\binom{k-1}{i}q^i(1-q)^{k-1-i}\binom{n-k}{\frac{n-1}{2}-i}(\frac{1}{2})^{n-k}}_{Pr(pivotal\ as\ an\ INformed\ shareholder)} \tag{3}$$

The proof is in the appendix.

The takeaway here is that more information is always better when there is no liquidity. We thus are tempted to call for reducing the price of information.

6 Voting and Trading

Now, we consider the full model in which shareholders can trade after they vote. Because votes may reveal information and influence inferences that impact trading, we seek Perfect Bayesian Equilibrium. In the information acquisition period, k shareholders acquire private information, while the remaining $n-k$ shareholders do not. We focus on equilibria with type-symmetric voting strategies, in which, during the voting period, each informed shareholder votes for her private signal with probability $m \in [\frac{1}{2}, 1]$. In such an equilibrium, the probability that an informed shareholder's vote is correct conditional on the underlying state is $z := Pr(v_i = \omega|\omega) = mq + (1-m)(1-q)$. Additionally, we focus on equilibria in which the uninformed shareholders vote for each policy with equal probability. Recall that in the model in which trading is possible, we assume that in the trading period, every shareholder can choose to buy one share, hold, or sell one share, $b_i \in -1, 0, 1$.

We analyze the model starting with subforms in which trading occurs.

6.1 Trading Period

The stock price in the trading period depends on the probability that voting selects the correct policy.[5] Given a selected policy, the price depends on the number of votes for the selected policy. For example, when $x = 1$, a larger voting tally t implies that more informed shareholders are receiving the signal of 1, and thus the stock price based on a larger t is higher than the stock price based on a smaller t.

The following lemma gives the pricing function that describes how the price changes with x and t.

Lemma 1 (Stock Price After Voting). *The price after voting depends on the chosen policy x and voting tally t.*

$$P_x(t) = E[v(x,\omega)|x,t] = \begin{cases} Pr(\omega = 1|t), & \text{if } x = 1 \\ 1 - Pr(\omega = 1|t), & \text{if } x = 0 \end{cases} \tag{4}$$

where

$$Pr(\omega = 1|t) = \frac{Pr(t|\omega = 1)}{Pr(t|\omega = 1) + Pr(t|\omega = 0)} \tag{5}$$

$$Pr(t|\omega = 1) = \sum_{i=0}^{t} \binom{k}{i} z^i (1-z)^{k-i} \binom{n-k}{t-i} \left(\frac{1}{2}\right)^{n-k} \tag{6}$$

and

$$Pr(t|\omega = 0) = \sum_{i=0}^{t} \binom{k}{i} (1-z)^i z^{k-i} \binom{n-k}{t-i} \left(\frac{1}{2}\right)^{n-k}. \tag{7}$$

Proof. The expression is obtained by substituting into the Bayes' rule. ■

[5]To simplify the model without losing intuitions, we assume that the price does not depend on the number of trading orders. Meirowitz and Pi (2022) show that an informed shareholder's trade-off between voting for the policy she thinks is best and voting against it to maximize trading rents still holds when the pricing function also depends on net trading orders from shareholders and noise traders.

In the trading stage, each shareholder buys (sells) one share if her expectation of firm value is higher (lower) than the price. After shareholder voting, how a shareholder interprets the voting results (x, t) and thus her expectation of firm value depends on whether the shareholder buys information or not, how she votes, and her private information (if she has any). Therefore, it is natural that shareholders take various sides of the market after observing the voting results. In particular, different information positions ($a_i = 0$ versus $a_i = 1$) cause uninformed shareholders and informed shareholders to take different trading strategies, and informed shareholders with different private information ($s_i = 1$ versus $s_i = 0$) also trade differently. But as we now show, the driving equilibrium is stark. Uninformed shareholders bet against their vote because they recognize that the market will interpret their vote for 1 (0) as weak evidence in favor of 1 (0), while they know the vote is based on a coin toss only. Informed shareholders bet in line with their signal, as they recognize that their signal is not fully capitalized into market prices due to the fact that in equilibrium voting is not fully informative.

Proposition 4 (Trading Strategy)**.** *At the trading stage, each uninformed shareholder buys one share if $x \neq v_i$ and sells one share if $x = v_i$. Every informed shareholder buys one share if $x = s_i$ and sells one share if $x \neq s_i$.*

The proof is in the appendix.

In equilibrium, each uninformed shareholder compares her vote and the chosen policy, and then buys (sells) if her vote is different from (the same as) the chosen policy. Every informed shareholder compares her private signal and the chosen policy, and then buys (sells) if her private signal is the same as (different from) the chosen policy. We now move back to the voting period.

6.2 Voting Period

Each informed shareholder's voting strategy depends on the difference between $EU(v_i = s_i|s_i)$ and $EU(v_i \neq s_i|s_i)$ where these expected utilities correctly anticipate equilibrium trading strategies and market price as a function of t. We have

$$\begin{aligned}
&EU(v_i = s_i|s_i) - EU(v_i \neq s_i|s_i) \\
&= \underbrace{Pr(t' = \frac{n-1}{2}|s_i = 1)(2Pr(\omega = 1|t' = \frac{n-1}{2}, s_i = 1) - 1)}_{Pivotal\ Effect} \\
&- \underbrace{\sum_{t'=0}^{n-1} Pr(t'|s_i = 1)(Pr(\omega = 1|t' + 1) - Pr(\omega = 1|t'))}_{Signaling\ Effect}
\end{aligned} \tag{8}$$

Since the first term measures the gains from voting for one's signal over voting against one's signal when an informed shareholder i is pivotal, we call it the "Pivotal Effect". The second term sums the gains from voting against one's signal over voting for one's signal and trading to capitalize on this informational advantage in all cases. Thus, we call the second term the "Signaling Effect".

An informed shareholder's voting strategy depends on which effect dominates. If *Signaling Effect(z)-Pivotal Effect(z)* ≤ 0 when $z = q$, then voting sincerely is a best response for an informed shareholder. However, if *Signaling Effect(z)-Pivotal Effect(z)* ≥ 0 when $z = q$, then voting against one's signal is a best response for an informed shareholder. If at z^* both inequalities hold with equality, then mixing is a best response, and we can support the mixed strategy m with $z^* = mq + (1-q)m$. In particular, z^* is determined by the following indifference condition.

$$\begin{aligned}
&\sum_{t'=0}^{n-1} Pr(t'|s_i = 1)(Pr(\omega = 1|t' + 1) - Pr(\omega = 1|t')) \\
&= Pr(t' = \frac{n-1}{2}|s_i = 1)(2Pr(\omega = 1|t' = \frac{n-1}{2}, s_i = 1) - 1)
\end{aligned} \tag{9}$$

Obviously, both the Signaling Effect and Pivotal Effect are a function of the number of informed shareholders, k. Thus, in equilibrium, the number of informed shareholders, k, affects the strength of the pivotal effect and signaling effect and thus influences informed shareholders' voting strategies.

We may gain traction on how $\frac{k}{n}$ impacts voting and information aggregation by considering the case of large n. Accordingly, we close this section by assuming n is large and employing asymptotic methods. In the next subsection, we help fix ideas by presenting a small n example.

Proposition 5 (Voting Strategies when Trading Possible)**.** *Let $n \to \infty$. For each n, there is a threshold $\kappa(n)$ which converges to $\frac{\sqrt{\frac{2}{\pi}}}{\sqrt{n-1}erf\left(\frac{\sqrt{n-1}}{\sqrt{2}}\right)}$ for which in an equilibrium voting strategies of the informed shareholders are as follows.*

If $\frac{k}{n} \leq \kappa(n)$, then every informed shareholder sincerely votes for her signal. If $\frac{k}{n} > \kappa(n)$, every informed shareholder uses a non-degenerate mixed voting strategy. The probability that each informed shareholder's vote is correct converges to

$$z^*(n,k) = Pr(v_i = \omega|\omega) = \frac{1}{2} + \frac{n(2q-1)}{\sqrt{2\pi}k\sqrt{n-1}erf\left(\frac{\sqrt{n-1}}{\sqrt{2}}\right)}$$

In both cases, every uninformed shareholder votes for each policy with a probability of one half.

The proof is in the appendix.

6.3 Information Aggregation with Trading

The growth of the number of informed shareholders brings two effects. On the one hand, more signals are being added to the voting game, which helps with information aggregation. On the other hand, the informativeness of informed shareholders' votes is decreasing ($\frac{\partial z}{\partial k} < 0$), which hurts information aggregation. Thus, the overall effect of having more informed shareholders on information aggregation depends on which effect dominates.

Proposition 6 (Information Aggregation with Trading)**.** *If n is large enough, then when $\frac{k}{n} \leq \kappa(n)$ so that informed shareholders sincerely vote ($z = q$) the probability of making the correct choice, $Pr(x = \omega)$, increases with k. However, when $\frac{k}{n} > \kappa(n)$, so that informed shareholders mix ($z < q$), $Pr(x = \omega)$ decreases with k. The value k^* maximizing $Pr(x = \omega)$ is asymptotically given by*

$$k^* = \frac{n\sqrt{\frac{2}{\pi}}}{\sqrt{n-1}erf\left(\frac{\sqrt{n-1}}{\sqrt{2}}\right)}$$

Furthermore, in the limit the maximal $Pr^*(x = \omega)$ obtained at k^* is

$$Pr^*(x = \omega) = \Phi\left(\sqrt{\frac{2}{\pi}}(2q-1)\right)$$

which is strictly smaller than q.

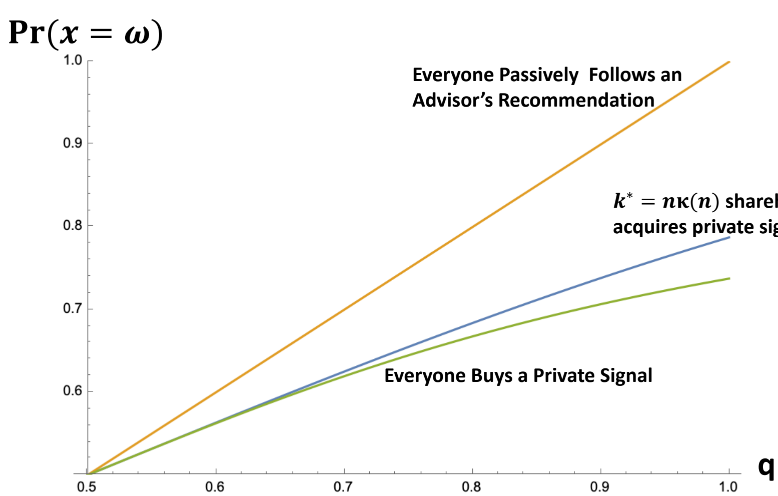


Figure 1: Probability that Voting Selects The Correct Policy Under Three Situations

The proof appears in the appendix.

7 Endogenizing the Number of Informed Shareholders

In this section, we first provide the condition to endogenize the number of informed shareholders. Then, we characterize the optimal information cost that can sustain an equilibrium with the optimal level of information acquisition in the limit. Finally, we demonstrate that when information is excessively expensive, no one wants to buy it. However, if everyone votes with a public signal, then the probability of selecting the correct policy is even higher than when the optimal number of shareholders acquires private information. This remains true even if the public signal is less accurate than a private signal.

So far, we have treated k as exogenous. While heterogeneous information acquisition strategies can emerge in a variety of settings, one parsimonious extension of our model involves adding an initial stage where each shareholder can choose to acquire the private signal at a common cost, c. Supporting an equilibrium to the larger game in which k shareholders obtain signals in pure strategies and $n-k$ do not buy signals in pure strategies involves characterizing a pair of conditions on cost, c, to support investment in acquiring information by k and only k shareholders. As we will see, some values of c can support multiple equilibrium values of k, while other values of c cannot support any equilibria of this form.

To sustain an equilibrium in which k shareholders buy information and the rest $n-k$ shareholders do not buy information, the cost of acquiring a signal must satisfy the condition that none of the k informed shareholders wants to deviate by being uninformed, and none of the $n-k$ uninformed shareholders want to deviate by being informed. In working through the calculations, one key feature that surfaces is that a shareholder can realize some value from their private signal for any realization of t. That is to say, information provides some benefits even if the shareholder is not pivotal.

Proposition 7 (Information Value and Information Cost)**.** *In an equilibrium in which k shareholders buy information and $n-k$ shareholders do not buy information, the cost of information must satisfy*

$$\underbrace{EU(a_i = 0 \xrightarrow{d} 1) - EU(a_i = 0)}_{\underline{c}(k)} \leq c \leq \underbrace{EU(a_i = 1) - EU(a_i = 1 \xrightarrow{d} 0)}_{\bar{c}(k)} \tag{10}$$

, where

$$\underbrace{EU(a_i = 0) - EU(a_i = 0 \xrightarrow{d} 1)}_{\underline{c}(k)}$$

$$= \sum_{t'=0}^{\frac{n-1}{2}-1} Pr_U(t')(2Pr_U(\omega = 0|t') - P_0(t'+1)) + \sum_{t'=\frac{n-1}{2}}^{n-1} Pr_U(t')P_1(t'+1)$$

$$- \max\{\sum_{t'=0}^{t'=\frac{n-1}{2}-1} Pr_U(t'|s_i = 1)P_0(t'+1) + \sum_{t'=\frac{n-1}{2}}^{n-1} Pr_U(t'|s_i = 1)(2Pr_U(\omega = 1|t', s_i = 1) - P_1(t'+1))$$

$$, \sum_{t'=0}^{t'=\frac{n+1}{2}} Pr_U(t'|s_i = 1)P_0(t'+1) + \sum_{t'=\frac{n-1}{2}}^{n-1} Pr_U(t'|s_i = 1)(2Pr_U(\omega = 1|t', s_i = 1) - P_1(t'+1))\}$$

and

$$\underbrace{EU(a_i = 1) - EU(a_i = 1 \xrightarrow{d} 0)}_{\bar{c}(k)}$$

$$= \sum_{t'=0}^{\frac{n-1}{2}-1} Pr_I(t'|s_i = 1)P_0(t'+1) + \sum_{t'=\frac{n-1}{2}}^{n-1} Pr_I(t'|s_i = 1)(2Pr_I(\omega = 1|t', s_i = 1) - P_1(t'+1))$$

$$- \left(\sum_{t'=0}^{\frac{n-1}{2}-1} Pr_I(t')(2Pr_I(\omega = 0|t') - P_0(t'+1)) + \sum_{t'=\frac{n-1}{2}}^{n-1} Pr_I(t')P_1(t'+1) \right)$$

The proof is in Appendix.

Proposition 7 gives the minimum and maximum costs that can sustain an equilibrium in which exactly k informed shareholders buy information. Specifically, the cost cannot be cheaper than $\underline{c}(k)$; otherwise, an uninformed shareholder would deviate from the equilibrium and buy information. Similarly, the cost cannot be more expensive than $\bar{c}(k)$; otherwise, an informed shareholder would deviate from the equilibrium and not invest in acquiring information.

We give a numerical solution. Suppose $n = 9$, $q = \frac{4}{5}$, and $b = \frac{1}{2}$, we calculate each interval of costs $[\underline{c}(k), \bar{c}(k)]$ that can sustain the equilibrium in which $k = \{0, 1, 2....7, 8, 9\}$ shareholders buy information.

k	$EU(a_i = 1)$	$EU(a_i = 1 \overset{\text{d}}{\to} 0)$	$\bar{c}(k)$	$EU(a_i = 0)$	$EU(a_i = 0 \overset{\text{d}}{\to} 1)$	$\underline{c}(k)$
0	-	-	-	0.5000	0.8199	0.3199
1	0.8487	0.5333	0.3154	0.6154	0.9187	0.3034
2	0.8795	0.6442	0.2354	0.7256	0.9356	0.2099
3	0.8998	0.7317	0.1681	0.8026	0.9417	0.1391
4	0.9117	0.7460	0.1657	0.7970	0.9415	0.1445
5	0.9182	0.7534	0.1648	0.7932	0.9412	0.1481
6	0.9223	0.7579	0.1643	0.7904	0.9410	0.1506
7	0.9250	0.7610	0.1640	0.7885	0.9408	0.1524
8	0.9270	0.7632	0.1638	0.7870	0.9407	0.1537
9	0.9285	0.7648	0.1637	-	-	-

Then, we characterize c^* when $n \to \infty$ and $k = k^* = \frac{n\sqrt{\frac{2}{\pi}}}{\sqrt{n-1}\mathrm{erf}\left(\frac{\sqrt{n-1}}{\sqrt{2}}\right)}$. That's to say; we seek the cost that can sustain the equilibria with the optimal level of information acquisition when there are a lot of shareholders. After obtaining the form of c^*, we analyze how q affects c^*.

Proposition 8 (Optimal Information Cost). *When $n \to \infty$, the cost that can sustain the equilibrium with the optimal level of information acquisition is given by*

$$\lim_{n\to\infty} c^* = (2q-1)\Phi\left(\sqrt{\frac{2}{\pi}}(2q-1)\right)$$

, which is monotonically increasing with q.

Proof. Recall that to sustain the equilibrium in which k^* shareholders buy information, the information costs must satisfy

$$\underbrace{EU(a_i = 0 \overset{\text{d}}{\to} 1) - EU(a_i = 0)}_{\underline{c}(k^*)} \leq c^* \leq \underbrace{EU(a_i = 1) - EU(a_i = 1 \overset{\text{d}}{\to} 0)}_{\bar{c}(k^*)} \tag{11}$$

We show that both $\underline{c}(k^*)$ and $\bar{c}(k^*)$ converge to $(2q-1)\Phi\left(\sqrt{\frac{2}{\pi}}(2q-1)\right)$ when $n \to \infty$. First, let us focus on $\bar{c}(k^*)$ in which we have

$$\begin{aligned}
&\lim_{n\to\infty} EU(a_i = 1)\\
&= \lim_{n\to\infty} \sum_{t'=0}^{\frac{n-1}{2}-1} Pr_I(t'|s_i = 1)P_0(t'+1) + \lim_{n\to\infty} \sum_{t'=\frac{n-1}{2}}^{n-1} Pr_I(t'|s_i = 1)(2Pr_I(\omega = 1|s_i = 1, t') - P_1(t'+1))\\
&= \lim_{n\to\infty} \sum_{t'=\frac{n-1}{2}}^{n-1} Pr_I(t'|s_i = 1)2Pr_I(\omega = 1|s_i = 1, t') + \lim_{n\to\infty} E[P_0(t'+1)|s_1 = 1] - \lim_{n\to\infty} E[P_1(t'+1)|s_i = 1]\\
&= \lim_{n\to\infty} 2q \sum_{t'=\frac{n-1}{2}}^{n-1} Pr_I(t'|\omega = 1)
\end{aligned} \tag{12}$$

and

$$
\begin{aligned}
&\lim_{n\to\infty} EU(a_i = 1 \to 0) \\
&= \lim_{n\to\infty} \sum_{t'=0}^{t'=\frac{n-1}{2}-1} Pr_I(t')(2Pr_I(\omega = 0|t') - P_0(t'+1)) + \lim_{n\to\infty} \sum_{t'=\frac{n-1}{2}}^{n-1} Pr_I(t')P_1(t'+1) \\
&= \lim_{n\to\infty} \sum_{t'=0}^{\frac{n-1}{2}-1} Pr_I(t')2Pr_I(\omega = 0|t') + \lim_{n\to\infty} E[P_1(t'+1)] - \lim_{n\to\infty} E[P_0(t'+1)] \\
&= \lim_{n\to\infty} \sum_{t'=0}^{\frac{n-1}{2}-1} Pr_I(t'|\omega = 0)
\end{aligned}
\tag{13}
$$

Thus,

$$
\begin{aligned}
&\lim_{n\to\infty} \bar{c}(k^*) \\
&= \lim_{n\to\infty} EU(a_i = 1) - \lim_{n\to\infty} EU(a_i = 1 \to 0) \\
&= \lim_{n\to\infty} \sum_{t'=\frac{n-1}{2}}^{n-1} Pr_I(t'|\omega = 1) - \lim_{n\to\infty} \sum_{t'=0}^{\frac{n-1}{2}-1} Pr_I(t'|\omega = 0) \\
&= (2q-1) \lim_{n\to\infty} \sum_{t'=\frac{n-1}{2}+1}^{n-1} Pr_I(t'|\omega = 1) + \lim_{n\to\infty} qPr_I(t' = \frac{n-1}{2}|\omega = 1) \\
&= (2q-1) \lim_{n\to\infty} \left(1 - \Phi\left(\frac{\frac{n+1}{2} - ((k^*-1)(1-q) + (n-k^*)\frac{1}{2})}{\sqrt{(k^*-1)q(1-q) + (n-k^*)\frac{1}{4}}}\right)\right) \\
&= (2q-1)\Phi\left(\sqrt{\frac{2}{\pi}}(2q-1)\right)
\end{aligned}
\tag{14}
$$

Now we focus on $\underline{c}(k^*)$. Similarly, we have

$$\begin{aligned}
&\lim_{n\to\infty} \underline{c}(k^*) \\
&= \lim_{n\to\infty} EU(a_i = 0 \to 1) - EU(a_i = 0) \\
&= \lim_{n\to\infty} (2q-1) \sum_{t'=\frac{n-1}{2}+1}^{n-1} Pr_{UI}(t'|\omega = 1) + \lim_{n\to\infty} qPr_{UI}(t' = \frac{n-1}{2}|\omega = 1) \\
&= (2q-1) \lim_{n\to\infty} \left(1 - \Phi\left(\frac{\frac{n+1}{2} - (k^*(1-q) + (n-k^*-1)\frac{1}{2})}{\sqrt{k^*q(1-q) + (n-k^*-1)\frac{1}{4}}}\right)\right) \\
&= (2q-1)\Phi\left(\sqrt{\frac{2}{\pi}}(2q-1)\right)
\end{aligned} \tag{15}$$

Note that

$$\lim_{n\to\infty} \underline{c}(k^*) = \lim_{n\to\infty} \bar{c}(k^*) = (2q-1)\Phi\left(\sqrt{\frac{2}{\pi}}(2q-1)\right)$$

. Recall that

$$\underline{c}(k^*) \leq c^* \leq \bar{c}(k^*) \tag{16}$$

Because of Sandwich Theorem, we have

$$\lim_{n\to\infty} c^* = (2q-1)\Phi\left(\sqrt{\frac{2}{\pi}}(2q-1)\right) \tag{17}$$

Obviously, this is increasing in q.

■

The figure below plots an informed shareholder's and an uninformed shareholder's expected utility in the equilibrium with the optimal level of information acquisition.

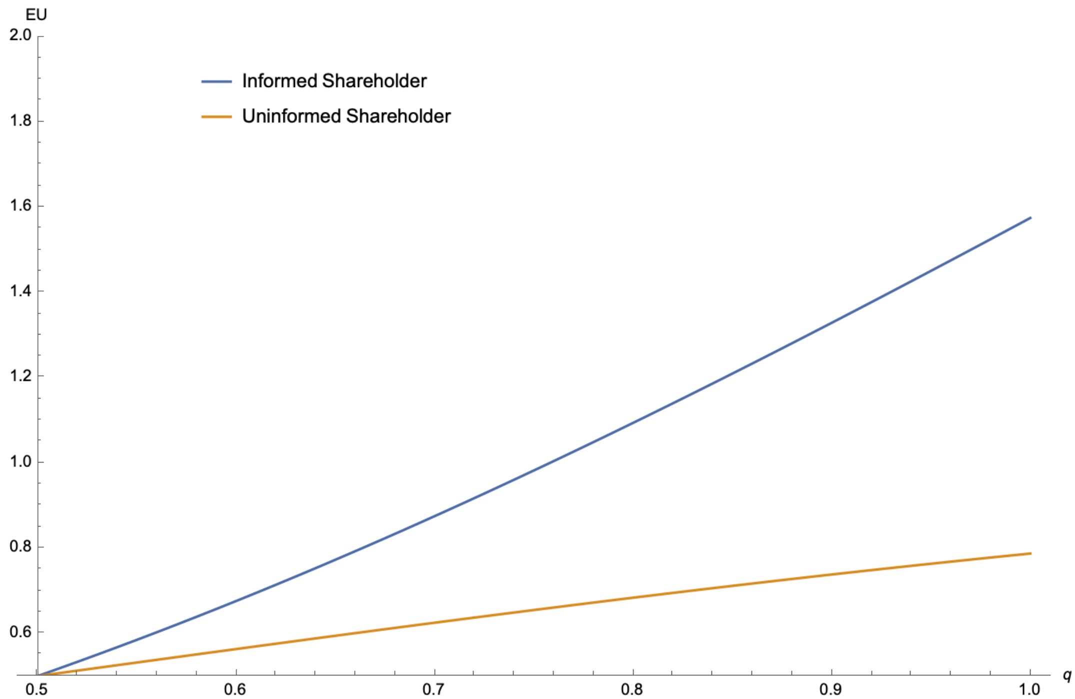


Figure 2: Shareholders' Expected Utility in the Equilibrium with the Optimal Level of Information Acquisition

Finally, we show that when the acquisition costs are too high, no one wants to buy a private signal. But if there is a public signal (e.g., voting recommendations from management or proxy advisors) and everyone relies on the public signal while voting, the probability of selecting the correct policy is actually higher compared to the situations where the optimal number of shareholders acquire a private signal. Remarkably, this remains true even when the public signal is less accurate than a private signal.

Proposition 9 (No Information Acquisition and Voting with Less Accurate Public Information Can Be Better)**.** *If c is weakly higher than $\sum_{t'=\frac{n-1}{2}}^{n-1} \binom{n-1}{t'} \left(\frac{1}{2}\right)^{n-1} (2q-1)$, no shareholder wants to acquire information in equilibrium. When no one acquires private information and all rely on a public signal (less accurate than a private signal)* *to vote, the probability of selecting the correct policy can actually be higher compared to situations where the optimal number of shareholders acquire private information.*

Proof. Consider a shareholder deviating from not acquiring information. Note that in equilibria where no one buys information, the market does not respond to t. So, the shareholder's expected utility from the deviation is

$$\begin{aligned}
&\sum_{t'=0}^{\frac{n-1}{2}-1} Pr(t'|s_i=1)P_0(t'+1) + \sum_{t'=\frac{n-1}{2}}^{n-1} Pr(t'|s_i=1)(2Pr(\omega=1|t',s_i=1) - P_1(t'+1) \\
&= \sum_{t'=0}^{\frac{n-1}{2}-1} Pr(t'|s_i=1)\frac{1}{2} + \sum_{t'=\frac{n-1}{2}}^{n-1} Pr(t'|s_i=1)(2q-\frac{1}{2}) \\
&= \sum_{t'=0}^{\frac{n-1}{2}-1} Pr(t')\frac{1}{2} + \sum_{t'=\frac{n-1}{2}}^{n-1} Pr(t')(2q-\frac{1}{2})
\end{aligned} \tag{18}$$

The last step is because none of the other $n-1$ shareholders buys a signal.

If she does not deviate, her expected payoff is $\frac{1}{2}$. Thus, if

$$c \geq \sum_{t'=\frac{n-1}{2}}^{n-1} Pr(t')(2q-1) = \sum_{t'=\frac{n-1}{2}}^{n-1} \binom{n-1}{t'} \left(\frac{1}{2}\right)^{n-1} (2q-1)$$

, then no shareholder wants to deviate from not acquiring information.

Now, consider there is a public signal. The informativeness of the public signal is $Pr(s_i=\omega|\omega) = q_{public}$. When every shareholder relies on the public signal to vote, the probability of the correct policy being selected is q_{public}. On the other hand, as shown by Proposition 6, when there are the optimal number of shareholders acquiring private information, the probability of selecting the correct policy is $\Phi\left(\sqrt{\frac{2}{\pi}}(2q-1)\right) < q$. Therefore, when $\Phi\left(\sqrt{\frac{2}{\pi}}(2q-1)\right) < q_{public} < q$, the governance is actually better when no one buys private information and everyone one simply votes with a less accurate public signal than when the optimal number of shareholders acquire private information.

■

8 How does trading affect the incentives to acquire information and thus information aggregation?

The above analysis has shown that the opportunity to trade after voting induces shareholders to strategically vote against their information and thus may distort the policy choice. Yet, this section shows that the probability of selecting the correct policy via voting may be higher when trading is possible than when there is no trading, $Pr(x = \omega; Trading) > Pr(x = \omega; No\ Trading)$.

When there is no trading, a shareholder can benefit from investing in information only if her vote is pivotal. In contrast, when there is trading, a shareholder can realize informational rents in all cases; In addition to increasing $Pr(x = w)$ when she happens to be pivotal, she can always capitalize on her private information through strategic trading on the market when she is not pivotal. Thus, trading, as the other channel to extract information rents, encourages more shareholders to invest in information than when there is no trading. This sometimes increases the probability of selecting the correct policy via voting.

Formally speaking, the opportunity to trade generates two effects. First, given a fixed information cost, we should see more shareholders invest in information when trading is allowed. Second, recall that the opportunity of trading incentives shareholders to strategically vote against their information, and thus the informativeness of informed shareholders' votes may be lower when trading is possible than when trading is impossible. As a result of these two effects, $Pr(x = \omega; Trading)$ can be larger (smaller) than $Pr(x = \omega; No\ Trading)$ if the first effect (second effect) dominates.

Before rigorously analyzing this, we illustrate the phenomena by continuing the example. In particular, suppose that shareholders cannot trade, we find the minimum cost and maximum cost that supports an equilibrium in which $k \in \{0, 1, 2...7, 8, 9\}$ shareholders acquire information and calculate the corresponding $Pr(x = \omega; No\ Trading)$. The results are shown in the left three columns of the table below.

k	Trading			No Trading		
	Max Cost	Mini Cost	$Pr(x = \omega)$	Max Cost	Mini Cost	$Pr(x = \omega)$
0	-	0.31989	0.50000	-	0.08203	0.50000
1	0.31537	0.30336	0.58203	0.08203	0.08203	0.58203
2	0.23538	0.20994	0.66406	0.08203	0.07781	0.66406
3	0.16815	0.13909	0.72655	0.07781	0.06938	0.74188
4	0.16575	0.14446	0.72301	0.06938	0.05763	0.81125
5	0.16480	0.14809	0.72030	0.05763	0.04440	0.86888
6	0.16432	0.15058	0.71837	0.04440	0.03187	0.91328
7	0.16402	0.15237	0.71694	0.03187	0.02150	0.94515
8	0.16383	0.15371	0.71586	0.02150	0.01376	0.96666
9	0.16369	0.00000	0.71502	0.01376	0.00000	0.98042

Table 1: Cost Ranges and $Pr(x = \omega)$ When Trading is Possible/Impossible

Consistent with our expectations, these numerical results show that the opportunity to trade largely incentives shareholders to buy information. As long as $0.082031 \leq c \leq 0.16369$, all of the shareholders want to buy information when trading is allowed. However, when trading is impossible, none of the shareholders want to buy information at this cost. Thus, as long as $0.082031 \leq c \leq 0.16369$, $Pr(x = \omega; Trading) = 0.71502$, which is larger than $Pr(x = \omega; No\ Trading) = 0.5$. But if the information is very expensive ($c > 0.31989$), then, no matter whether trading exists or not, no shareholder wants to acquire information. Therefore, when $c > 0.31989$, we have $Pr(x = \omega; Trading) = Pr(x = \omega; No\ Trading) = 0.5$. In addition, if the information is very cheap ($c \leq 0.01376$), then all shareholders want to buy information regardless of the existence of trading. In this case, we have we have $Pr(x = \omega; Trading) = 0.71502$, which is smaller than $Pr(x = \omega; No\ Trading) = 0.98042$.

Proposition 10 (Information Aggregation With/Without Trading)**.** *When information is sufficiently cheap ($c \to 0$), we have $Pr(x = \omega; No\ Trading) > Pr(x = \omega; Trading)$. When the information cost is exorbitant ($c \to \infty$), then $k(Trading) = k(No\ Trading) = 0$, and we have $Pr(x = \omega; No\ Trading) = Pr(x = \omega; Trading) = \frac{1}{2}$.*

Furthermore, there exists a cost threshold c_T such that when the information cost is $c \geq c_T$, we must have $Pr(x = \omega; Trading) > Pr(x = \omega; No\ Trading)$, which means for a non-empty set of information costs information aggregation is better when trading is possible than when trading is not possible.

The proof appears in the appendix.

9 Disclosure Policy: Less Transparency Can Improve Governance

In this section, we want to understand how the transparency of voting outcomes affects equilibria. In particular, we consider making the voting tally t unobservable.

When t is not observable but x is known, the market sets the price as follows.

$$
\begin{aligned}
& P_x \\
& = Pr(x = \omega | x) \\
& = \begin{cases} \sum_{t=0}^{\frac{n-1}{2}} Pr(\omega = 0|t)Pr(t), & x = 0 \\ \sum_{t=\frac{n+1}{2}}^{n} Pr(\omega = 1|t)Pr(t), & x = 1 \end{cases}
\end{aligned} \tag{19}
$$

Proposition 11 (Trading Strategies When t is Unobservable)**.** *When t is not observable, both informed shareholders and uninformed shareholders still have informational advantages over the market and thus want to trade after voting.*

Their trading strategies are the same as those when t is observable. In particular, informed shareholders want to buy one share when $x = s_i$ but sell one share when $x \neq s_i$. Uninformed shareholders want to buy one share when $x \neq v_i$ but sell one share when $x = v_i$.

Proof. Consider an informed shareholder who has a signal of 1. When $x = 1$, her expectation of the share value is $Pr(\omega = 1|s_i = 1, x = 1)$, which is higher than the price, $Pr(\omega = 1|x = 1)$. When $x = 0$, her expectation of the share value is $Pr(\omega = 0|s_i = 1, x = 1)$, which is smaller than the price, $Pr(\omega = 1|x = 0)$. Thus, the informed shareholder wants to buy one share when $x = s_i$ but sell one share when $x \neq s_i$.

Then, consider an uninformed shareholder voting for 1. When $x = 1$, her expectation of share value is $Pr(\omega = 1|x = 1, v_i = 1, a_i = 0) = \sum_{t'=\frac{n-1}{2}}^{n-1} Pr(\omega = 1|t')Pr(t')$, while the price set by the market can be written as

$$Pr(\omega = 1|x = 1)$$

$$\sum_{t=\frac{n+1}{2}}^{n} Pr(\omega = 1|t)Pr(t)$$

$$= \sum_{t'=\frac{n-1}{2}}^{n-1} Pr(\omega = 1|t')Pr(t')Pr(a_i = 0) + Pr(\omega = 1|t', v_i(a_i = 1) = 1)Pr(t')Pr(a_i = 1) \tag{20}$$

Since $Pr(\omega = 1|t', v_i(a_i = 1) = 1) > Pr(\omega = 1|t')$ for $t' \in [\frac{n-1}{2}, n-1]$, we have $Pr(\omega = 1|x = 1, v_i = 1, a_i = 0) < P_{x=1}$. Thus, she wants to sell one share.

When $x = 0$, the uninformed shareholder's expectation of share value is $Pr(\omega = 0|x = 0, v_i = 1, a_i = 0) = \sum_{t'=0}^{\frac{n-1}{2}-1} Pr(\omega = 0|t')Pr(t')$. The share price is

$$Pr(\omega = 0|x = 0)$$

$$\sum_{t=0}^{\frac{n-1}{2}} Pr(\omega = 1|t)Pr(t)$$

$$= \sum_{t'=0}^{\frac{n-1}{2}-1} Pr(\omega = 1|t')Pr(t')Pr(a_i = 0) + Pr(\omega = 0|t', v_i(a_i = 1) = 1)Pr(t')Pr(a_i = 1) \tag{21}$$

Since $Pr(\omega = 0|t', v_i(a_i = 1) = 1) < Pr(\omega = 0|t')$ for $t' \in [\frac{n-1}{2}, n-1]$, we know $Pr(\omega = 0|x = 0, v_i = 1, a_i = 0) < P_{x=0}$. Thus, she wants to buy one share.

■

Proposition 12 (Voting Strategies t is Unobservable)**.** *When t is not observable, informed shareholders sincerely vote for their information, while uninformed shareholders randomly vote for each policy with the probability of $\frac{1}{2}$.*

Proof. First, we prove that the informed shareholders want to sincerely vote for her signal in equilibria. Note that when she is not the pivotal voter, she is indifferent between voting for her signal and voting against her signal. This is because her vote cannot change the policy, and the price only depends on the policy now (as the voting tally t is unobservable). So, we can focus on the event of her being pivotal. When she is pivotal, her payoff from voting her signal is

$$\begin{aligned} &U_{pivotal}(v_i = 1|s_i = 1) \\ &= Max\{2Pr(\omega = 1|t' = \frac{n-1}{2}, s_i = 1) - Pr(\omega = 1|x = 1), Pr(\omega = 1|x = 1)\} \end{aligned} \tag{22}$$

, yet her payoff from voting against her signal is

$$\begin{aligned} &U_{pivotal}(v_i = 0|s_i = 1) \\ &= Max\{2Pr(\omega = 0|t' = \frac{n-1}{2}, s_i = 1) - Pr(\omega = 0|x = 0), Pr(\omega = 0|x = 0)\} \end{aligned} \tag{23}$$

Because of the symmetry of the binomial distributions and voting strategies, we have $Pr(\omega = 1|x = 1) = Pr(\omega = 0|x = 0)$. In other words, the share price set by the market is the same regardless of which policy is selected. Obviously, $Pr(\omega = 1|t' = \frac{n-1}{2}, s_i = 1) > Pr(\omega = 0|t' = \frac{n-1}{2}, s_i = 1)$. Thus, if $U_{pivotal}(v_i = 1|s_i = 1) = 2Pr(\omega = 1|t' = \frac{n-1}{2}, s_i = 1) - Pr(\omega = 1|x = 1)$, we must have $U_{pivotal}(v_i = 1|s_i = 1) \geq U_{pivotal}(v_i = 0|s_i = 1)$. On the other hand, if $U_{pivotal} = Pr(\omega = 1|x = 1)$, we also must have $U_{pivotal}(v_i = 1|s_i = 1) \geq U_{pivotal}(v_i = 0|s_i = 1)$. Thus, the informed shareholder has no profitable deviations from sincerely voting for her signal.

Second, we prove that uninformed shareholders randomly vote in equilibria.

■

Then we show that contrary to what regulators believe, less transparency can lead to better corporate governance.

One complication in assessing how the choice to make t observable or unobservable impacts the equilibrium probability of making the correct decision is that for a fixed cost the values of k that can be supported in equilbiria are different depending on whether t is observed by the market or not. Moreover, in either case, sometimes multiple values of k can be supported for a fixed cost. Fortunately, however, we can order the levels of information aggregation for the interesting cases when the fraction of informed shareholders is not too small.

Proposition 13. *Consider two games one with t observable and k informed shareholders and n shareholders and the other with t unobservable and k' informed shareholders and n shareholders. When $\frac{k'}{n} > \kappa(n)$, $Pr(x = \omega; t \text{ is unobservable with } k', n) > Pr(x = \omega; t \text{ is observable with } k, n)$.*

Proof.

If $\frac{k'}{n} \geq \kappa(n)$, informed shareholders strategically vote when t is observable, and we have shown that $Pr(x = \omega; t \text{ is observable})$ begins to decrease with $\frac{k}{n}$. However, when t is not observable, informed shareholders always sincerely vote for their information, and thus $Pr(x = \omega; t \text{ is unobservable})$ increases with $\frac{k'}{n}$. Thus $Pr(x = \omega; t \text{ is observable with } k, n) \leq Pr(x = \omega; t \text{ is observable with } \kappa(n), n) = Pr(x = \omega; t \text{ is unobservable with } \kappa(n), n) \leq Pr(x = \omega; t \text{ is unobservable with } k', n)$

■

It is worth noting that $\frac{\kappa(n)}{n}$ converges to 0 as n grows and so our ordering provides the relevant result if we are interested in a large number of shareholders in which a non-negligible fraction obtain information.

10 Proxy Advisor

In this section, we examine equilibrium voting and information aggregation when shareholders choose between buying a signal from a proxy advisor or investing in information acquisition on their own or acquiring no information in the information acquisition stage. To fix ideas we assume that the signal from the proxy advisor is imperfectly informative, $p = Pr(s^p = \omega|\omega) \in (\frac{1}{2}, 1)$. We further assume that a shareholder only acquires one type of information, either a private signal or the proxy advisor's signal, but not both. Of course acquiring no signal is still feasible in this model.

Solving the model numerically, we show that all of the main insights from the main model carry over. In particular, we confirm that z^* in equilibrium decreases with the number of shareholders who buy private information, and thus $Pr(x = \omega)$ can decrease with the number of shareholders who buy private information. We also confirm that uninformed shareholders can still extract information rents from voting and trading, as they privately know which policy they voted for and whether their vote has any informational value.

k	k_p	uninformed	z	z_p	$Pr(x = \omega)$
0	3	6	-	0.55	0.5391
1	3	5	0.80	0.55	0.5438
2	3	4	0.80	0.55	0.5468
3	3	3	0.73	0.55	0.5470
4	3	2	0.67	0.55	0.5467
5	3	1	0.64	0.55	0.5466
6	3	0	0.61	0.55	0.5465

Table 2: Voting Strategies and $Pr(x = \omega)$ with a Proxy Advisor's Signal

We continue the example above, considering $n = 9$ and $q = \frac{4}{5}$. We focus on the case in which 3 shareholders buying the proxy advisor's signal, $k_p = 3$, and the informativeness of the proxy advisor's signal is $p = 0.55$. Table 2 considers various information acquisition strategies and shows that the probability that a vote from a shareholder with a private signal is correct, z, decreases, as the number of shareholders buying private signals increases from $k = 0$ to $k = 6$. When $k \geq 3$, shareholders with private signals vote strategically, which causes the probability of selecting the correct policy to decrease with the number of informed shareholders. For the sake of brevity, we do not explicitly consider the equilibrium acquisition decisions for particular values of the costs. Instead, we simply show how voting and aggregation vary with different acquisition strategies. Recovering the cost values that support these patterns is a mechanical exercise. The key robustness check here is that in the presence of a proxy advisor (consumed by a third of the shareholders it can be beneficial to reduce the acquisition of private signals. This challenges the crowding out intuition.

11 Ruling out equilibria in which uninformed shareholders deterministically split their votes when $k > 1$

We now show that in fact the focus on equilibria in which uninformed shareholders vote in mixed strategies is natural. In a profile in which the uninformed voters use degenerate voting strategies, their are strong incentives to deviate and extract larger informational rents. Consider that k is an odd number and greater than 1. Suppose by contradiction that there exists equilibria in which half of the uninformed shareholders vote for 1, half of the uninformed shareholders vote for 0, and k informed shareholders take symmetric voting strategies (either mixed or pure).

It's critical to note that the voting strategies of the conjectured equilibria allow everyone, including informed shareholders, uninformed shareholders, and the market, to infer the number of informed shareholders who vote for policy 1. In particular, since half of the uninformed shareholders always vote for policy 1 in equilibria, everyone knows the number of informed shareholder voting for policy 1 is $v_I = t - \frac{n-k}{2}$. This eliminates uninformed shareholders' informational rents from trading in the market.

Proposition 14. *In the conjectured equilibria, uninformed shareholders do not have informational advantages over the market. An informed shareholder's expected payoff in equilibria is*

$$\sum_{j=0}^{k} Pr(v_I = j)Pr(\omega = x|v_I = j)$$

Proof. For any $v_I \in [0, k]$, the voting t must be $\frac{n-k}{2} + v_I$ in equilibria. Because the price is only affected by the tally of informed shareholders' votes, the market can set the price directly on v_i so that $Pr(\omega = x|v_I)$ for any $v_I \in [0, k]$.

Since an uninformed shareholder also knows that $\frac{n-k}{2}$ uninformed shareholders always vote for 1 as well, her expectation of share value is also $Pr(\omega = x|v_I)$. So, her expected payoff in equilibria is

$$\sum_{j=0}^{k} Pr(v_I = j)Pr(\omega = x|v_I = j)$$

■

Now consider an uninformed shareholder who is expected to vote for 0 in the equilibria deviates and votes for 1 instead.

The uninformed shareholder buys (sells) one share if her expectation of the share value is greater (lower) than the share price. If $\frac{n-1}{2} \leq v_I < k-1$, then $t = v_I + 1 + \frac{n-k}{2}$ and the selected policy is 1. Since the market does not know the informed shareholder deviates, the market is mislead to believe $t - \frac{n-k}{2} = v_I + 1$ informed shareholders vote for 1. So, the price is $Pr(\omega = 1|v_I + 1)$. But the uninformed shareholders privately know that her deviation (voting for 1), so her expectation of share value is $Pr(\omega = 1|v_I)$. So, she thinks the firm is overpriced and thus wants to sell. On the other hand, if $v_I < \frac{n-1}{2}$, then the selected policy is 0. Again, since the market is misled to believe that $v_I + 1$ informed shareholders votes for 1, the market sets the price to be $Pr(\omega = 0|v_I + 1)$. But the uninformed shareholder's expectation of share value is $Pr(\omega = 0|v_I)$. So, she thinks the firm is underpriced, and thus wants to buy one share.

There is an off-equilibrium path case. In particular, when all uninformed shareholders happen to vote for 1 ($v_I = k$), the voting tally t will be $\frac{n-k}{2} + k + 1$. But, in equilibrium, the maximum t is $\frac{n-k}{2} + k$ In this case, we assume that the market can infer that one uninformed shareholder who should vote for 0 deviates and actually votes for 1. So, the market sets the price to $Pr(\omega = 1|v_I = k)$.

So, the uninformed shareholder's expected payoff from the deviation is

$$\begin{aligned}
&EU_d(v_i = 0 \to 1) \\
&= Pr(v_I = k)Pr(\omega = 1|v_I = k) \\
&+ \sum_{j=\frac{k+1}{2}}^{k-1} Pr(v_I = j)Pr(\omega = 1|v_I = j + 1) \\
&+ Pr(v_I = \frac{k-1}{2})Pr(\omega = 1|v_I = \frac{k-1}{2} + 1) \\
&+ \sum_{j=0}^{\frac{k-1}{2}-1} Pr(v_I = j)(2Pr(\omega = 0|v_I = j) - Pr(\omega = 0|v_I = j + 1))
\end{aligned} \tag{24}$$

Thus the difference between her expected payoff from the derivation, $EU_d(v_i = 0 \to 1)$ and her expected payoff from the equilibria $EU(v_i = 0)$ is

$$\begin{aligned}
&EU_d(v_i = 0 \to 1) - EU(v_i = 0) \\
&= Pr(v_I = k)(Pr(\omega = 1|v_I = k) - Pr(\omega = 1|v_I = k)) \\
&+ \sum_{j=\frac{k+1}{2}}^{k-1} Pr(v_I = j)(Pr(\omega = 1|v_I = j+1) - Pr(\omega = 1|v_I = j) \\
&+ Pr(v_I = \frac{k-1}{2})(Pr(\omega = 1|v_I = \frac{k-1}{2} + 1) - Pr(\omega = 0|v_I = \frac{k-1}{2})) \\
&+ \sum_{j=0}^{\frac{k-1}{2}-1} Pr(v_I = j)(2Pr(\omega = 0|v_I = j) - Pr(\omega = 0|v_I = j+1) - Pr(\omega = 0|v_i = j))
\end{aligned} \tag{25}$$

Note that $Pr(\omega = 1|v_I = \frac{k-1}{2} + 1) = Pr(\omega = 0|v_I = \frac{k-1}{2})$ due to the symmetry of binomial distribution. So, we have

$$\begin{aligned}
&EU_d(v_i = 0 \to 1) - EU(v_i = 0) \\
&= \sum_{j=\frac{k+1}{2}}^{k-1} Pr(v_I = j)(Pr(\omega = 1|v_I = j+1) - Pr(\omega = 1|v_I = j) \\
&+ \sum_{j=0}^{\frac{k-1}{2}-1} Pr(v_I = j)(Pr(\omega = 0|v_I = j) - Pr(\omega = 0|v_I = j+1))
\end{aligned} \tag{26}$$

, which is greater than 0, because $Pr(\omega = 1|v_I)$ increases with v_I and $Pr(\omega = 0|v_I)$ decreases with v_I.

How to interpret this equation? It is essential to note that the above equation is exactly the distortions of share price caused by the uninformed shareholder's vote, which takes a similar form as the Signaling Effect, $Pr(\omega = 1|v_I = j+1) - Pr(\omega = 1|v_I = j)$ or $Pr(\omega = 0|v_I = j) - Pr(\omega = 0|v_I = j+1)$. More specifically speaking, when $v_I > \frac{n+1}{2}$, her votes make the firm overpriced. When $v_I < \frac{n-1}{2} - 1$, her votes make the firm underpriced. By selling the firm when it is s overpriced and buying the firm when it is underpriced, the uninformed shareholder obtains a higher expected payoff than staying in conjectured equilibria. Thus, the conjectured equilibria do not exist.

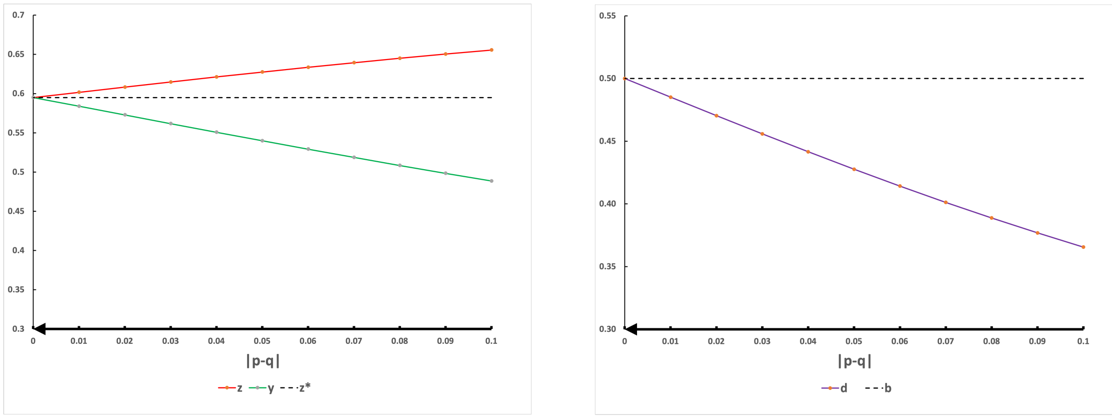
12 Checking Robustness of the Asymmetric Information Environment

We now show relaxing the symmetry in the informational environment does not alter our key findings. In this section, we consider that the information quality is different conditional on different states. In particular, $Pr(s_i = 1|\omega = 1) = p$ and $Pr(s_i = 1|\omega = 1) = q$, while $p \neq q$. The voting strategies conditional on $s_i = 1$, $s_i = 0$, and $a_i = 0$ are denoted with $r =: Pr(v_i = 1|s_i = 1)$, $l =: Pr(v_i = 0|s_i = 0)$, and $d =: Pr(v_i = 1|a_i = 0)$. Accordingly, we have $z =: Pr(v_i = 1|\omega = 1, a_i = 1) = pr + (1-p)(1-l)$ and $y =: Pr(v_i = 0|\omega = 0, a_i = 1) = ql + (1-q)(1-r)$.

We prove the theorem below.

Proposition 15. *As $|p-q| \to 0$, $z \to z^*$, $y \to z^*$, and $d \to \frac{1}{2}$ is equilibria.*

Note that z^* is the equilibrium voting strategy when $p = q$. That is to say, as the difference in information quality across different states vanishes, the equilibria when $p \neq q$ converge to the equilibria when $p = q$. The proof involves three steps. First, we derive the pricing function when $p \neq q$. Second, we find the equilibrium conditions given the pricing function. Third, we show that when $|p-q| \to 0$, $z \to z^*$, $y \to z^*$, and $d \to \frac{1}{2}$ satisfies the equilibrium conditions. The detailed proof appears in the appendix. Below is a numerical example. Suppose that $n = 9$, $k = 7$, $q = 0.75$. The figure show that as $|p-q| \to 0$, both z and y at equilibria converge to z^* and d converges to $\frac{1}{2}$.



(a) z and y converge to z^* (b) d converges $\frac{1}{2}$

Figure 3: As $|p - q| \to 0$, equilibria converge to equilibria of main model where $p = q$

Proof. First, we derive the pricing function and equilibrium conditions when $p \neq q$. The pricing function becomes

$$P(x,t) = E[v(x,\omega)|x,t] = \begin{cases} Pr(\omega = 1|t), & \text{if } x = 1 \\ 1 - Pr(\omega = 1|t), & \text{if } x = 0 \end{cases} \tag{27}$$

where,

$$Pr(\omega = 1|t) = \frac{Pr(t|\omega = 1)}{Pr(t|\omega = 1) + Pr(t|\omega = 0)} \tag{28}$$

,

$$Pr(t|\omega = 1) = \sum_{i=0}^{t} \binom{k}{i} z^i (1-z)^{k-i} \binom{n-k}{t-i} d^{t-i}(1-d)^{n-k-t+i} \tag{29}$$

, and

$$Pr(t|\omega = 0) = \sum_{i=0}^{t} \binom{k}{i} (1-y)^i y^{k-i} \binom{n-k}{t-i} d^{t-i}(1-d)^{n-k-t+i}. \tag{30}$$

Second, we derive the equilibrium conditions. Note that shareholders' trading strategies are the same as Proposition 4. So, the voting strategies at equilibria depend on the following three equations.

$$
\begin{aligned}
&EU(v_i = 1|s_i = 1) - EU(v_i = 1|s_i = 1) \\
&= Pr(t' = \frac{n-1}{2}|s_i = 1)(2Pr(\omega = 1|s_i = 1, t' = \frac{n-1}{2}) - 1) \\
&- \sum_{t'=0}^{n-1} Pr(t'|s_i = 1)(Pr(\omega = 1|t' + 1) - Pr(\omega = 1|t'))
\end{aligned} \tag{31}
$$

,

$$
\begin{aligned}
&EU(v_i = 1|s_i = 0) - EU(v_i = 1|s_i = 0) \\
&= Pr(t' = \frac{n-1}{2}|s_i = 0)(2Pr(\omega = 1|s_i = 0, t' = \frac{n-1}{2}) - 1) \\
&- \sum_{t'=0}^{n-1} Pr(t'|s_i = 1)(Pr(\omega = 1|t' + 1) - Pr(\omega = 1|t'))
\end{aligned} \tag{32}
$$

, and

$$
\begin{aligned}
&EU(v_i = 1|s_i = \emptyset) - EU(v_i = 0|s_i = \emptyset) \\
&= \sum_{t'=0}^{\frac{n-1}{2}-1} Pr(t')(2Pr(\omega = 0|t') - Pr(\omega = 0|t' + 1)) + \sum_{t'=\frac{n-1}{2}}^{n-1} Pr(t')Pr(\omega = 1|t' + 1) \\
&- \sum_{t'=0}^{\frac{n-1}{2}} Pr(t')Pr(\omega = 0|t' - \sum_{t'=\frac{n-1}{2}+1}^{n-1} Pr(t')(2P(\omega = 1|t') - Pr(\omega = 1|t'))
\end{aligned} \tag{33}
$$

Third, we prove that Equation 31 and Equation 32 both converge to Equation (8) and Equation 33 converges to Equation XXX, when $q \to p$, $z \to z^*$, $y \to z^*$, and $d \to \frac{1}{2}$. To prove this, we show that all components of the above three equations and the pricing function (e.g., $Pr(t'|s_i)$, $Pr(\omega|s_i, t')$, $Pr(\omega = 1|s_i, t')$, $Pr(t|\omega)$) converge to their corresponding expressions in the maim model where $p = q$. This step involves very long algebra manipulations. So, we only exhibit $\lim_{p \to q, z, y \to z^*, d \to \frac{1}{2}} Pr(t'|s_i =$

1) $\rightarrow Pr(t'|s_i = 1)$ when $p = q$ and $z = z^*$ as an example here. Note that

$$
\begin{aligned}
&Pr(t'|s_i = 1) \\
&= \frac{Pr(t'|\omega = 1)Pr(\omega = 1|s_i = 1) + Pr(t'|\omega = 0)Pr(\omega = 0|s_i = 1)}{Pr(s_i = 1)} \\
&= \sum_{i=0}^{t'} \binom{k-1}{i} z^i (1-z)^{k-1-i} \binom{n-k}{t'-i} d^{t'-i}(1-d)^{n-k-t'+i} p \\
&+ \sum_{i=0}^{t'} \binom{k-1}{i} (1-y)^i y^{k-1-i} \binom{n-k}{t'-i} d^{t'-i}(1-d)^{n-k-t'+i}(1-q)
\end{aligned}
\tag{34}
$$

Then,

$$
\begin{aligned}
&\lim_{p \rightarrow q, z, y \rightarrow z^*, d \rightarrow \frac{1}{2}} Pr(t'|s_i = 1) \\
&= \sum_{i=0}^{t'} \binom{k-1}{i} (z^*)^i (1-z^*)^{k-1-i} \binom{n-k}{t'-i} \frac{1}{2}^{n-k} q \\
&+ \sum_{i=0}^{t'} \binom{k-1}{i} (1-z^*)^i (z^*)^{k-1-i} \binom{n-k}{t'-i} \frac{1}{2}^{n-k} (1-q)
\end{aligned}
\tag{35}
$$

,which is equal to $Pr(t'|s_i = 1)$ in the main model where $p = q$. Similarly, we can prove other components of Equation 31 to Equation 32 also converge to their corresponding expressions in the maim model where $p = q$. As a result, when $|p - q| \rightarrow 0$, $z \rightarrow z^*$, $y \rightarrow z^*$, and $d \rightarrow \frac{1}{2}$ is equilibria. ■

13 Discussion

The idea that investors would have incentives to acquire information about the firms they invest in is entirely standard, if not obvious, to scholars of finance. This channel has been ignored in theoretical studies of governance. The standard approach when considering voting or governance by investors is to consider the value of information only in impacting governance. We move beyond this narrow perspective by developing a model of information acquisition and governance in which shareholders can also realize rents from trading. The equilibrium analysis illustrates that, in general, information acquisition will be higher once investing opportunities are considered because shareholders can extract information rents from trading in addition to the rents they extract from potentially helping to select the value-enhancing policy. We also document subtle spillover effects. The opportunity to extract informational rents creates governance distortions. Here we determine that overall these distortions need not dominate the benefits of having a more informed group voting on firm policy. In general, we find that for some costs of acquiring information, governance is better because of the opportunities to generate trading rents, despite the fact that there are distortions to voting behavior. Moreover, we find that even if information acquisition were free, it is not always socially valuable. In equilibrium, there can be too much information acquisition in the sense that so many investors are informed that distortions on voting behavior are too strong, and the likelihood of selecting the value-enhancing policy is lower than it would be in equilibrium if fewer people acquired information. This then justifies the conclusion that governance and the welfare of investors are not always enhanced by regulations that make information easy to obtain. The optimal number of informed investors is generally well short of all investors, and thus the optimal cost of acquiring information is not 0. Accordingly, whether reforms that increase acquisition are good for firm governance depends on the features of the governance environment.

Importantly, we confirm that, in fact, the dominant channel for information to impact shareholders must be through the fact that information guides trading, and so previous work that misses this channel is perhaps importantly incomplete. We find that when the number of shareholders gets large, at an optimal information cost, the effect of information on voting must vanish, but the impact of information on trading rents does not. Thus, inclusion of this channel is of first order importance, and understanding the strategic spillovers between trading rents and voting rents seems central to understanding this complex strategic setting.

A final substantive takeaway stems from an extension in which we reduce governance transparency. When the market can only see whether a motion passes but not the vote tally, then distortions to voting incentives are absent, and more information is unambiguously better. Thus, this type of reduction in transparency can dramatically improve the quality of governance and also negates the concerns about too much information that we highlight in the main model.

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A Proofs

A.1 Proof of Proposition 1

Proof. We verify $v_i(s_i) = s_i$ for $s_i \in \{0, 1\}$ and $Pr(v_i(s_i = \emptyset) = 1) = Pr(v_i(s_i = \emptyset) = 0) = \frac{1}{2}$ are mutual best responses.

Consider a shareholder with the signal of $s_i = 1$. Conjecturing that she is pivotal, her expected payoff from voting for policy 1 is

$$\begin{aligned} & EU(v_i = 1|pivotal, s_i = 1) \\ & = Pr(\omega = 1|pivotal, s_i = 1) \\ & = \frac{Pr(t' = \frac{n-1}{2}|\omega = 1)q}{Pr(t' = \frac{n-1}{2}|\omega = 1)q + Pr(t' = \frac{n-1}{2}|\omega = 0)(1-q)} \\ & = \frac{1}{1 + \frac{Pr(t' = \frac{n-1}{2}|\omega=0)}{Pr(t' = \frac{n-1}{2}|\omega=1)} \frac{1-q}{q}} \end{aligned} \tag{36}$$

, where t' denotes the voting tally of all the other shareholders.

If she votes for policy 0, her expected payoff is

$$\begin{aligned} & EU(v_i = 0|pivotal, s_i = 1) \\ & Pr(\omega = 0|pivotal, s_i = 1) \\ & = \frac{Pr(t' = \frac{n-1}{2}|\omega = 0)(1-q)}{Pr(t' = \frac{n-1}{2}|\omega = 0)(1-q) + Pr(t' = \frac{n-1}{2}|\omega = 1)q} \\ & = \frac{1}{1 + \frac{Pr(t' = \frac{n-1}{2}|\omega=1)}{Pr(t' = \frac{n-1}{2}|\omega=0)} \frac{q}{1-q}} \end{aligned} \tag{37}$$

Note that $Pr(t' = \frac{n-1}{2}|\omega = 0) = Pr(t' = \frac{n-1}{2}|\omega = 1)$ because of the symmetry of voting strategies. Then, since $\frac{1}{2} < q < 1$, we have $\frac{q}{1-q} > \frac{1-q}{q}$, and thus we have $EU(v_i = 1|pivotal, s_i = 1) > EU(v_i = 0|pivotal, s_i = 1)$.

Similarly, when $s_i = 0$, we have $EU(v_i = 0|pivotal, s_i = 0) > EU(v_i = 1|pivotal, s_i = 0)$. So, for each informed shareholder, voting sincerely is the best response.

Now consider a shareholder without a signal. The uninformed shareholder's expected payoff from voting for policy 1 is

$$\begin{aligned} & EU(v_i = 1|pivotal, s_i = \emptyset) \\ &= \frac{Pr(t' = \frac{n-1}{2}|\omega = 1)}{Pr(t' = \frac{n-1}{2}|\omega = 1) + Pr(t' = \frac{n-1}{2}|\omega = 0)} \end{aligned} \tag{38}$$

Her expected payoff from voting for 0 is

$$\begin{aligned} & EU(v_i = 0|pivotal, s_i = \emptyset) \\ &= \frac{Pr(t' = \frac{n-1}{2}|\omega = 0)}{Pr(t' = \frac{n-1}{2}|\omega = 0) + Pr(t' = \frac{n-1}{2}|\omega = 1)} \end{aligned} \tag{39}$$

Since we have $Pr(t' = \frac{n-1}{2}|\omega = 0) = Pr(t' = \frac{n-1}{2}|\omega = 1)$, the uninformed shareholder is indifferent with voting for each policy. Thus, $Pr(v_i(s_i = \emptyset) = 1) = Pr(v_i(s_i = \emptyset) = 0) = \frac{1}{2}$ is a best response ■

A.2 Proof of Proposition 2

Proof.

$$\begin{aligned} & Pr(\omega = x) \\ &= Pr(x = 1|\omega = 1)Pr(\omega = 1) + Pr(x = 0|\omega = 0)Pr(\omega = 0) \\ &= \frac{1}{2}(Pr(t \geq \frac{n+1}{2}|\omega = 1) + Pr(t \leq \frac{n-1}{2}|\omega = 0)) \end{aligned} \tag{40}$$

Note that $Pr(t \geq \frac{n+1}{2}|\omega = 1) = Pr(t \leq \frac{n-1}{2}|\omega = 0))$ due to the symmetry of voting strategies. We have

$$
\begin{aligned}
&Pr(\omega = x) \\
&= \sum_{t=\frac{n+1}{2}}^{n} Pr(t|\omega = 1) \\
&= \sum_{t=\frac{n+1}{2}}^{n} \sum_{i=0}^{t} \binom{k}{i} q^i (1-q)^{k-i} \binom{n-k}{t-i} (\frac{1}{2})^{n-k} \\
&= 1 - \Phi\left(\frac{\frac{n+1}{2} - (kq + (n-k)\frac{1}{2})}{\sqrt{(kq(1-q) + (n-k)\frac{1}{4})}} \right)
\end{aligned}
\tag{41}
$$

Because $\frac{d\frac{\frac{n+1}{2}-(kq+(n-k)\frac{1}{2})}{\sqrt{(kq(1-q)+(n-k)\frac{1}{4})}}}{dk} < 0$, we have $Pr(\omega = x)$ increases in k. ■

A.3 Proof of Proposition 3

Proof. To sustain an equilibrium in which k shareholders buy information, we must rule out two deviations, one additional shareholder acquiring information and one less shareholder acquiring information. To rule out the first we need c to exceed the change in the probability that an uninformed voter's vote is correct if she acquires a signal and votes with it instead of randomizing in her vote, $q - \frac{1}{2}$ weighted by the odds of being pivotal if she is not supposed to acquire a signal when k of n voters acquire signals,

$$
(q - \frac{1}{2}) \underbrace{\sum_{i=0}^{\frac{n-1}{2}} \binom{k}{i} q^i (1-q)^{k-i} \binom{n-k-1}{\frac{n-1}{2} - i} (\frac{1}{2})^{n-k-1}}_{Pr(\text{pivotal as an UNinformed shareholder})} \leq c \tag{42}
$$

To rule out the second, we need the cost to be less than the change in probability of voting correctly weighted by the odds a voter acquiring a signal is pivotal

$$c \leq (q - \frac{1}{2}) \underbrace{\sum_{i=0}^{\frac{n-1}{2}} \binom{k-1}{i} q^i (1-q)^{k-1-i} \binom{n-k}{\frac{n-1}{2} - i} (\frac{1}{2})^{n-k}}_{Pr(\text{pivotal as an INformed shareholder})}$$

■

A.4 Proof of Proposition 4

Proof. Suppose without the loss of generality that $x = 1$. When $x = 1$, the price is given by $Pr(\omega = 1|t)$. Each shareholder's trading strategy depends on the compassion between her expectation of the firm value and the price.

First, consider an uninformed shareholder who votes for 0. It is easy to see that $Pr(\omega = 1|v_i = 0, t, s_i = \emptyset) > Pr(\omega|t)$, because the uninformed shareholder knows that t voters among k informed shareholders and $n - k - 1$ uninformed shareholders vote for 1, yet the market thinks that t voters among k informed shareholders and $n - k$ uninformed shareholders vote for 1. So, the uninformed shareholder privately knows that each vote for policy 1 has a higher chance of being cast by an informed shareholder than what the market believes ($\frac{k}{n-1}$ v.s $\frac{k}{n}$). Therefore, the uninformed shareholder wants to buy one share.

Second, we prove that an uninformed shareholder who votes for 1 wants to sell. To do this we show that $Pr(\omega = 1|t, v_i = 1, s_i = \emptyset) \leq Pr(\omega = 1|t)$ for $t \in [\frac{n+1}{2}, n]$.[6] When $t = \frac{n+1}{2}$, shareholder i knows that she is the pivotal voter, which implies that

[6] Perhaps interestingly, the above arguments proving $Pr(\omega = 1|v_i = 0, t, s_i = \emptyset) > Pr(\omega|t)$ cannot directly prove $Pr(\omega = 1|v_i = 1, t, s_i = \emptyset) > Pr(\omega|t)$. If the uninformed shareholder vote for 1 and observes t, she knows that $t - 1$ voters among k informed shareholders and $n - k - 1$ uninformed shareholders. Thus, although the uninformed shareholder still believes each vote has a higher probability of being cast by an informed shareholder, the uninformed shareholder also realizes there may be fewer informed voters voting for policy 1. It is easier to notice this via an example. Suppose that $t \geq k$. Then, given t and $v_i = 0$, it is possible that t informed shareholders vote for 1. But, if given t and $v_i = 1$, there are at most $t - 1$ shareholders voting for 1.

there are exactly half of the other $n-1$ vote for 1 and half of them vote for 0. Due to the symmetry of voting strategies, the pivotal event is not informative to shareholder i. Since she does not have private information, we must have

$$Pr(\omega = 1|t = \frac{n+1}{2}, v_i = 1, s_i = \emptyset) = \frac{1}{2}.$$

On the other hand, we also know that the price

$$Pr(\omega = 1|t = \frac{n+1}{2}) > \frac{1}{2}$$

because of the existence of k informative votes ($z > \frac{1}{2}$).

Note that $\frac{d|Pr(\omega=1|t)-Pr(\omega=1|t,v_i=1,s_i=\emptyset)|}{dt} < 0$ for $t \in [\frac{n+1}{2}, n]$. This is because both the market's posterior beliefs and shareholder i' posterior beliefs are affected by the public information t. As the public signal becomes more convincing, the differences in agents' beliefs shrink. The voting tally t is most noise when it is around $\frac{n+1}{2}$ and becomes more convincing as it approaches to the two tails, 0 and n. Thus, when t is between $\frac{n+1}{2}$ and n, the difference between the market's Bayesian posterior belief and shareholder i' Bayesian posterior beliefs, $|Pr(\omega = 1|t) - Pr(\omega = 1|t, v_i = 1, s_i = \emptyset)|$, monotonically decreases with t.

Then, we focus on $t = n$. When $t = n$, both the market and shareholder i know that every shareholder, including shareholder i, must vote for 1. Therefore, when $t = n$, the uniformed shareholder and the market essentially have the same information set, and thus $Pr(\omega = 1|t) = Pr(\omega = 1|t, v_i = 1, s_i = \emptyset)$ when $t = n$.

Therefore, we know that $Pr(\omega = 1|t) > Pr(\omega = 1|t, v_i = 1, s_i = \emptyset)$ when $t \in [\frac{n+1}{2}, n]$. As a result, the uninformed shareholder wants to sell when she votes for 1 and observes that $x = 1$.

Now consider an informed shareholder who buys information and has $s_i = 0$. If she votes for 1, we know that $Pr(\omega = 1|t = \frac{n+1}{2}, v_i = 1, s_i = 0) = 1 - q < Pr(\omega =$

$1|t = \frac{n+1}{2}) = \frac{1}{2}$. We also know that $Pr(\omega = 1|t = n, v_i = 1, s_i = 0) < Pr(\omega = 1|t = n)$, because the informed shareholder has private information that her vote for 1 actually is strategically against her signal 0. Recall that the difference between the market's belief and the shareholder's belief shrinks when t increases from $t = \frac{n+1}{2}$ to n. Therefore, we know that $Pr(\omega = 1|t, v_i = 1, s_i = 0) < Pr(\omega = 1|t)$ for $t \in [\frac{n+1}{2}, n]$, and thus the shareholder wants to sell. Now consider the case that the informed shareholder has $s_i = 0$ but votes for 0. When observing t, both the market maker and the informed shareholder know that $n - t$ shareholders votes for 0. For every vote for policy 0, the market maker cannot tell the type of shareholder, $s_i \in \{\emptyset, 0, 1\}$, casting it. However, the shareholder knows that her vote for policy 0 is based on her private signal of s_i. Thus, for every $t \in [\frac{n+1}{2}, n-1]$, the shareholder's expectation of firm value is lower than the price. So, the shareholder with the signal of $s_i = 0$ still wants to sell if she votes for 0.

■

A.5 Proof of Proposition 5

Proof. We first simplify the signaling effect and pivotal effect in algebra.

$$
\begin{aligned}
&\text{Signaling Effect}\\
&=\sum_{t'=0}^{n-1} Pr(t'|s_i=1)(P_{x=1}(t'+1)-P_{x=1}(t'))\\
&=\sum_{t'=0}^{n-1}(Pr(t'|\omega=1)q+Pr(t'|\omega=0)(1-q))(P_{x=1}(t'+1)-P_{x=1}(t'))\\
&=q\sum_{t'=0}^{n-1}(Pr(t'|\omega=1)(P_{x=1}(t'+1)-P_{x=1}(t'))\\
&+(1-q)\sum_{t'=0}^{n-1} Pr(t'|\omega=0)(P_{x=1}(t'+1)-P_{x=1}(t'))\\
&=\sum_{t'=0}^{n-1}(Pr(t'|\omega=1)(P_{x=1}(t'+1)-P_{x=1}(t'))
\end{aligned}
\tag{43}
$$

The last step is because $\sum_{t'=0}^{n-1} Pr(t'|\omega=1)(P_{x=1}(t'+1)-P_{x=1}(t')) = \sum_{t'=0}^{n-1} Pr(t'|\omega=0)(P_{x=1}(t'+1)-P_{x=1}(t'))$ due to the symmetry of binomial distribution.

$$
\begin{aligned}
&\text{Pivotal Effect}\\
&=Pr(t'=\frac{n-1}{2}|s_i=1)(2Pr(\omega=1|t'=\frac{n-1}{2},s_i=1)-1)\\
&=(Pr(t'=\frac{n-1}{2}|\omega=1)q+Pr(t'=\frac{n-1}{2}|\omega=0)(1-q))(2q-1)\\
&=Pr(t'=\frac{n-1}{2}|\omega=1)(2q-1)
\end{aligned}
\tag{44}
$$

Thus, the indifference condition implying z^* is

$$
\sum_{t'=0}^{n-1} Pr(t'|\omega=1)(P_{x=1}(t'+1)-P_{x=1}(t'))=Pr(t'=\frac{n-1}{2}|\omega=1)(2q-1) \tag{45}
$$

Dividing both sides by $Pr(t'=\frac{n-1}{2}|\omega=1)$, we have

$$
\sum_{t'=0}^{n-1}\frac{Pr(t'|\omega=1)}{Pr(t'=\frac{n-1}{2}|\omega=1)}(P_{x=1}(t'+1)-P_{x=1}(t'))=2q-1 \tag{46}
$$

Note that conditional on $\omega = 1$, the voting tally t is the convolution of two binomial distributions with different success rates (z and $\frac{1}{2}$). When n is large, this convolution approximates to a normal distribution with the mean of $kz + (n-k)\frac{1}{2}$ and the variance of $kz(1-z) + (n-k)\frac{1}{4}$. Similarly, conditional on $\omega = 1$ ($\omega = 0$), t' is normally distributed with with the mean of $(k-1)z + (n-k)\frac{1}{2}$ $((k-1)(1-z) + (n-k)\frac{1}{2})$ and the variance of $(k-1)z(1-z) + (n-k)\frac{1}{4}$. Thus, the left-hand side of the indifference condition becomes

$$\int_{t'=0}^{n-1} \frac{\phi(t';\mu_1,\sigma_1)}{\phi(\frac{n-1}{2};\mu_1,\sigma_1)} \cdot \left(\frac{\phi(t'+1;\mu_2,\sigma_2)}{\phi(t'+1;\mu_2,\sigma_2) + \phi(t'+1;\mu_3,\sigma_3)} - \frac{\phi(t';\mu_2,\sigma_2)}{\phi(t';\mu_2,\sigma_2) + \phi(t';\mu_3,\sigma_3)} \right) dt' \tag{47}$$

, where

$\mu_1 = (k-1)z + (n-k)\frac{1}{2}$, $\sigma_1 = \sqrt{(k-1)z(1-z) + (n-k)\frac{1}{4}}$
$\mu_2 = kz + (n-k)\frac{1}{2}$, $\sigma_2 = \sqrt{kz(1-z) + (n-k)\frac{1}{4}}$
$\mu_3 = k(1-z) + (n-k)\frac{1}{2}$, $\sigma_3 = \sqrt{kz(1-z) + (n-k)\frac{1}{4}}$

We view the equation in the integral symbols as a function of z and denote it as $f(z)$. We then take Taylor expansions of of $f(z)$ around $z = \frac{1}{2}$.[7] Then, we have

$$\begin{aligned} & f(z) \\ & = f(\frac{1}{2}) + \frac{f'(\frac{1}{2})}{1!})(z - \frac{1}{2}) + \frac{f'(\frac{1}{2})}{2!})(z - \frac{1}{2})^2 + \ldots\ldots \\ & = 0 + \frac{2k\left(z-\frac{1}{2}\right)e^{-\frac{(n-2t-1)^2}{2(n-1)}}}{n} - \frac{4\left(z-\frac{1}{2}\right)^2\left((k-1)ke^{-\frac{(n-2t-1)^2}{2(n-1)}}(n-2t-1)\right)}{(n-1)n} + O\left(\left(z-\frac{1}{2}\right)^3\right) \end{aligned} \tag{48}$$

[7] There are two reasons for expanding at $z = \frac{1}{2}$. First, as the number of informed shareholders, k, increases to n (the case that every shareholder has private information), our model becomes to the model of Meirowitz and Pi (2022). According to Propostion XXX of Meirowitz and Pi (2022), we know that we must have $\lim_{n(k)\to\infty} z \to \frac{1}{2}$. Second, note that every term contains $(z - \frac{1}{2})^i$. Recall that $z \in (\frac{1}{2}, q)$. Then, we must have $0 < z - \frac{1}{2} < q - \frac{1}{2} < 1$. Since $0 < z - \frac{1}{2} < 1$, we know that, for any z in $(\frac{1}{2}, q)$, $(z - \frac{1}{2})^i$ must exponentially vanish towards 0 as i increases to ∞.

Accordingly, we have

$$\frac{\sqrt{\frac{\pi}{2}}k\sqrt{n-1}(2z-1)\text{erf}\left(\frac{\sqrt{n-1}}{\sqrt{2}}\right)}{n} = 2q-1 \tag{49}$$

In an equilibrium in which informed shareholders sincerely vote for their information ($z = q$), it must be the signaling effect is weakly smaller than the pivotal effect when $z = q$. This implies that

$$\frac{k}{n} \leq \frac{\sqrt{\frac{2}{\pi}}}{\sqrt{n-1}\text{erf}\left(\frac{\sqrt{n-1}}{\sqrt{2}}\right)}$$

That is to say, to ensure that all informed shareholders sincerely vote for the information they own in voting ($z = q$), we cannot have too many informed shareholders. In particular, the ratio between the amounts of informed shareholders (k) and the amounts of all shareholders (n) cannot be too high.

On the other hand, if $\frac{k}{n} > \frac{\sqrt{\frac{2}{\pi}}}{\sqrt{n-1}\text{erf}\left(\frac{\sqrt{n-1}}{\sqrt{2}}\right)}$, then we have equilibria in which informed shareholders strategically vote with $z < q$. In particular, z is given by

$$\frac{1}{2} + \frac{n(2q-1)}{\sqrt{2\pi}k\sqrt{n-1}\text{erf}\left(\frac{\sqrt{n-1}}{\sqrt{2}}\right)}$$

■

A.6 Proof of Proposition 6: Information Aggregation with Trading

Proof. From above, we have in the limit

$$Pr(x=\omega) = 1 - \Phi\left(\frac{\frac{n+1}{2} - (kz + (n-k)\frac{1}{2})}{\sqrt{kz(1-z) + (n-k)\frac{1}{4}}}\right) \tag{50}$$

We first consider the case of $\frac{k}{n} \leq \kappa(n)$ in which informed shareholders sincerely vote. Since all informed shareholders sincerely vote ($z = q$), the probability of selecting the correct policy increases as the number of informed increases. To see this, note that

$$\frac{d \frac{\frac{n+1}{2} - (kz + (n-k)\frac{1}{2})}{\sqrt{kz(1-z) + (n-k)\frac{1}{4}}}}{d\ k} = \frac{(2q-1)\left(k(1-2q)^2 - 2n + 2q - 1\right)}{2\left(n - k(1-2q)^2\right)^{3/2}} < 0$$

Thus, we know $Pr(x = \omega)$ is increasing in k when $\frac{k}{n} \leq \kappa(n)$.

Then, we consider the case of $\frac{k}{n} > \kappa(n)$. Recall that an informed shareholder takes a mixed strategy, $z = \frac{1}{2} + \frac{n(2q-1)}{\sqrt{2\pi} k \sqrt{n-1} \mathrm{erf}\left(\frac{\sqrt{n-1}}{\sqrt{2}}\right)}$ when $\frac{k}{n} > \kappa(n)$. As the number of informed shareholders increases, two effects happen simultaneously. First, as k increases, there are more informative votes ($z > \frac{1}{2}$), which helps information aggregation. Second, as k increases, each informative vote becomes less informative ($\frac{dz}{dk} < 0$), which hurts information aggregation. As a result, the aggregate effect of having more informed shareholders on information aggregation efficiency depends on which effects dominate.

After we substitute $z = \frac{1}{2} + \frac{n(2q-1)}{\sqrt{2\pi} k \sqrt{n-1} \mathrm{erf}\left(\frac{\sqrt{n-1}}{\sqrt{2}}\right)}$ into $Pr(x = \omega)$, we get

$$Pr(x = \omega) = 1 - \Phi\left(\frac{\sqrt{\pi}\sqrt{n-1}\mathrm{erf}\left(\frac{\sqrt{n-1}}{\sqrt{2}}\right) + \sqrt{2}n(1-2q)}{\sqrt{n-1}\mathrm{erf}\left(\frac{\sqrt{n-1}}{\sqrt{2}}\right)\sqrt{n\left(\pi - \frac{2n(1-2q)^2}{k(n-1)\mathrm{erf}\left(\frac{\sqrt{n-1}}{\sqrt{2}}\right)^2}\right)}}\right)$$

Note that

$$\sqrt{\pi}\sqrt{n-1}\mathrm{erf}\left(\frac{\sqrt{n-1}}{\sqrt{2}}\right) + \sqrt{2}n(1-2q) < 0$$

. Then, as k increases, $Pr(\omega = x)$ decreases in the number of informed shareholders.

Overall, when $\frac{k}{n} \leq \kappa(n)$, informed shareholders sincerely vote ($z = q$), and $Pr(x = \omega)$ increases with k. However, when $\frac{k}{n} \leq \kappa(n)$, informed shareholders strategically vote ($z < q$), and $Pr(x = \omega)$ decreases with k. Thus, $Pr(\omega = x)$ is a concave function of k.

To find the maximum $Pr(x = \omega)$, we substitute $k = \frac{n\sqrt{\frac{2}{\pi}}}{\sqrt{n-1}\text{erf}\left(\frac{\sqrt{n-1}}{\sqrt{2}}\right)}$ and $z = q$ into the equation (50).

$\frac{\sqrt{\frac{2}{\pi}}}{\sqrt{n-1}\text{erf}\left(\frac{\sqrt{n-1}}{\sqrt{2}}\right)}$

$$
\begin{aligned}
&Pr(x = \omega) \\
&= 1 - \Phi\left(\frac{\sqrt{n-1}\text{erf}\left(\frac{\sqrt{n-1}}{\sqrt{2}}\right) + \sqrt{\frac{2}{\pi}}n(1-2q)}{\sqrt{n-1}\text{erf}\left(\frac{\sqrt{n-1}}{\sqrt{2}}\right)\sqrt{n - \frac{\sqrt{\frac{2}{\pi}}n(1-2q)^2}{\sqrt{n-1}\text{erf}\left(\frac{\sqrt{n-1}}{\sqrt{2}}\right)}}}\right)
\end{aligned}
\tag{51}
$$

Since

$$
\begin{aligned}
&\lim_{n\to\infty} \frac{\sqrt{n-1}\text{erf}\left(\frac{\sqrt{n-1}}{\sqrt{2}}\right) + \sqrt{\frac{2}{\pi}}n(1-2q)}{\sqrt{n-1}\text{erf}\left(\frac{\sqrt{n-1}}{\sqrt{2}}\right)\sqrt{n - \frac{\sqrt{\frac{2}{\pi}}n(1-2q)^2}{\sqrt{n-1}\text{erf}\left(\frac{\sqrt{n-1}}{\sqrt{2}}\right)}}} \\
&= \sqrt{\frac{2}{\pi}}(1-2q)
\end{aligned}
\tag{52}
$$

, we have

$$
\begin{aligned}
&\lim_{n\to\infty} Pr(x = \omega) \\
&= 1 - \Phi\left(\sqrt{\frac{2}{\pi}}(1-2q)\right) \\
&= \Phi\left(\sqrt{\frac{2}{\pi}}(2q-1)\right)
\end{aligned}
\tag{53}
$$

, which is strictly smaller than q.

■

A.7 Proof of Proposition 7: Information Costs with Trading

Proof. First, we find the condition under which none of the k informed shareholders wants to deviate by becoming uninformed.

An informed shareholder's equilibrium payoff is

$$
\begin{aligned}
&EU(a_i = 1) - c \\
&= \frac{1}{2}Max\{EU(v_i = 1|s_i = 1), EU(v_i = 0|s_i = 1)\} + \frac{1}{2}Max\{EU(v_i = 1|s_i = 0), EU(v_i = 0|s_i = 0)\} \\
&- c
\end{aligned}
\tag{54}
$$

If informed shareholders vote sincerely in equilibrium, then it must be $EU(v_i = 1|s_i = 1) \geq EU(v_i = 0|s_i = 1)$. If informed shareholders play mixed strategies, then it must be $EU(v_i = 1|s_i = 1) = EU(v_i = 0|s_i = 1)$. So, we have $Max\{EU(v_i = 1|s_i = 1), EU(v_i = 0|s_i = 1)\} = EU(v_i = 1|s_i = 1)$. Similarly, we have $Max\{EU(v_i = 1|s_i = 0), EU(v_i = 0|s_i = 0)\} = EU(v_i = 0|s_i = 0)$. Thus,

$$
\begin{aligned}
&EU(a_i = 1) - c \\
&= \frac{1}{2}EU(v_i = 1|s_i = 1) + \frac{1}{2}EU(v_i = 0|s_i = 0) - c
\end{aligned}
\tag{55}
$$

Note that $EU(v_i = 1|s_i = 1) = EU(v_i = 0|s_i = 0)$ due to the symmetry of informed shareholders' strategies. That's to say, for an informed shareholder, no particular signal should have an additional value than the other signal in equilibrium.

So, we have

$$EU(a_i = 1) - c$$
$$= EU(v_i = 1|s_i = 1) - c$$
$$= \sum_{t'=0}^{\frac{n-1}{2}-1} Pr_I(t'|s_i = 1)P_0(t'+1) + \sum_{t'=\frac{n-1}{2}}^{n-1} Pr_I(t'|s_i = 1)(2Pr_I(\omega = 1|t', s_i = 1) - P_1(t'+1))$$
$$- c \tag{56}$$

To simplify notations, we define the function $f_I(t, z, k)$.

$$f_I(t, z, k) := \sum_{i=0}^{t} \binom{k-1}{i} z^i (1-z)^{k-1-i} \binom{n-k}{t-i} \left(\frac{1}{2}\right)^{n-k} \tag{57}$$

Then,

$$Pr_I(t'|s_i = 1) = f_I(t', z, k)q + f_I(t', 1-z, k)(1-q) \tag{58}$$

and

$$Pr_I(\omega = 1|t', s_i = 1) = \frac{qf_I(t', z, k)}{qf_I(t', z, k) + (1-q)f_I(t', 1-z, k)} \tag{59}$$

Now we turn to find out the informed shareholders' payoff if she deviates from acquiring information to not acquiring information. Since no particular policy is better than the other policy when shareholder i does not have information about the underlying state, we can conveniently assume that she votes for policy 1, $v_i = 1$.

$$\begin{aligned} &EU(a_i = 1 \xrightarrow{\text{d}} 0) \\ &= \sum_{t'=0}^{\frac{n-1}{2}-1} Pr_I(t')(2Pr_I(\omega = 0|t') - P_0(t'+1)) + \sum_{t'=\frac{n-1}{2}}^{n-1} Pr_I(t')P_1(t'+1) \end{aligned} \tag{60}$$

, where

$$Pr_I(t') = f_I(t', z, k)\frac{1}{2} + f_I(t', 1-z, k)\frac{1}{2} \tag{61}$$

and

$$Pr_I(\omega = 1|t') = \frac{f_I(t', z, k)}{f_I(t', z, k) + f_I(t', 1-z, k)} \tag{62}$$

The cost that can make the deviation from acquiring information to not acquiring information unprofitable must satisfy

$$c \leq \underbrace{EU(a_i = 1) - EU(a_i = 1 \xrightarrow{\text{d}} 0)}_{\bar{c}(k)} \tag{63}$$

, where

$$\begin{aligned}
&\underbrace{EU(a_i = 1) - EU(a_i = 1 \xrightarrow{\text{d}} 0)}_{\bar{c}(k)} \\
&= \sum_{t'=0}^{\frac{n-1}{2}-1} Pr_I(t'|s_i = 1)P_0(t'+1) + \sum_{t'=\frac{n-1}{2}}^{n-1} Pr_I(t'|s_i = 1)(2Pr_I(\omega = 1|t', s_i = 1) - P_1(t'+1)) \\
&- \left(\sum_{t'=0}^{\frac{n-1}{2}-1} Pr_I(t')(2Pr_I(\omega = 0|t') - P_0(t'+1)) + \sum_{t'=\frac{n-1}{2}}^{n-1} Pr_I(t')P_1(t'+1) \right)
\end{aligned} \tag{64}$$

Second, we find the condition under which none of the $n-k$ uninformed shareholders wants to deviate by acquiring a signal.

In equilibrium, an uninformed shareholder's expected payoff is

$$EU(a_i = 0) = Max\{EU(v_i = 1|s_i = \emptyset), EU(v_i = 0|s_i = \emptyset)\} \tag{65}$$

Recall that $EU(v_i = 1|s_i = \emptyset) = EU(v_i = 0|s_i = \emptyset)$. Then, we have

$$\begin{aligned}
&EU(a_i = 0) \\
&= EU(v_i = 1|s_i = \emptyset) \\
&= \sum_{t'=0}^{\frac{n-1}{2}-1} Pr_U(t')(2Pr(\omega = 0|t') - P_0(t'+1)) + \sum_{t'=\frac{n-1}{2}}^{n-1} Pr(t')P_1(t'+1)
\end{aligned} \tag{66}$$

To simplify notations, we define the function $f_U(t, z, k)$.

$$f_U(t, z, k) := \sum_{i=0}^{t} \binom{k}{i} z^i (1-z)^{k-i} \binom{n-k-1}{t-i} \left(\frac{1}{2}\right)^{n-k-1} \tag{67}$$

Then,

$$\begin{aligned} &Pr_U(t') \\ &= Pr(t'|\omega = 1)Pr(\omega = 1) + Pr(t'|\omega = 0)Pr(\omega = 0) \\ &= \frac{1}{2} f_U(t', z, k) + \frac{1}{2} f_U(t', 1-z, k) \end{aligned} \tag{68}$$

and

$$\begin{aligned} &Pr_U(\omega = 0|t') \\ &= \frac{Pr_U(t'|\omega = 0)Pr(\omega = 0)}{Pr_U(t'|\omega = 1)Pr(\omega = 1) + Pr_U(t'|\omega = 0)Pr(\omega = 0)} \\ &= \frac{f_U(t, 1-z, k)}{f_U(t, z, k) + f_U(t, 1-z, k)} \end{aligned} \tag{69}$$

On the other hand, if an uninformed shareholder deviates by acquiring information. Then, her expected payoff from the deviation is

$$\begin{aligned} &EU(a_i = 0 \xrightarrow{\text{d}} 1) - c \\ &= \max\{EU(v_i = 1|s_i = 1), EU(v_i = 0|s_i = 1)\} - c \\ &= \max\{ \sum_{t'=0}^{t'=\frac{n-1}{2}-1} Pr_U(t'|s_i = 1)P_0(t'+1) + \sum_{t'=\frac{n-1}{2}}^{n-1} Pr_U(t'|s_i = 1)(2Pr_U(\omega = 1|t', s_i = 1) - P_1(t'+1)) \\ &, \sum_{t'=0}^{t'=\frac{n+1}{2}} Pr_U(t'|s_i = 1)P_0(t'+1) + \sum_{t'=\frac{n-1}{2}}^{n-1} Pr_U(t'|s_i = 1)(2Pr_U(\omega = 1|t', s_i = 1) - P_1(t'+1))\} \\ &- c \end{aligned} \tag{70}$$

, where

$$\begin{aligned} & Pr_U(t'|s_i = 1) \\ & = Pr_U(t', s_i = 1|\omega = 1) + Pr_U(t', s_i = 1|\omega = 0) \\ & = q f_U(t', z, k) + (1-q) f_U(t', 1-z, k) \end{aligned} \tag{71}$$

and

$$\begin{aligned} & Pr(\omega = 1|t', s_i = 1) \\ & = \frac{Pr(t', s_i = 1|\omega = 1)Pr(\omega = 1)}{Pr(t', s_i = 1|\omega = 1)Pr(\omega = 1) + Pr(t', s_i = 1|\omega = 0)Pr(\omega = 0)} \\ & = \frac{q f_U(t', z, k)}{q f_U(t', z, k) + (1-q) f_U(t', 1-z, k)} \end{aligned} \tag{72}$$

To prevent the deviation from not acquiring, we must have

$$c \geq EU(a_i = 0) - EU(a_i = 0 \xrightarrow{\text{d}} 1) \tag{73}$$

, where

$$\begin{aligned} & \underbrace{EU(a_i = 0) - EU(a_i = 0 \xrightarrow{\text{d}} 1)}_{\underline{c}(k)} \\ & = \sum_{t'=0}^{\frac{n-1}{2}-1} Pr_U(t')(2Pr_U(\omega = 0|t') - P_0(t'+1)) + \sum_{t'=\frac{n-1}{2}}^{n-1} Pr_U(t')P_1(t'+1) \\ & - \max\{ \sum_{t'=0}^{t'=\frac{n-1}{2}-1} Pr_U(t'|s_i = 1)P_0(t'+1) + \sum_{t'=\frac{n-1}{2}}^{n-1} Pr_U(t'|s_i = 1)(2Pr_U(\omega = 1|t', s_i = 1) - P_1(t'+1)) \\ & , \sum_{t'=0}^{t'=\frac{n+1}{2}} Pr_U(t'|s_i = 1)P_0(t'+1) + \sum_{t'=\frac{n-1}{2}}^{n-1} Pr_U(t'|s_i = 1)(2Pr_U(\omega = 1|t', s_i = 1) - P_1(t'+1))\} \end{aligned} \tag{74}$$

Overall, we must have

$$\underbrace{EU(a_i = 0) - EU(a_i = 0 \xrightarrow{\text{d}} 1)}_{\underline{c}(k)} \leq c \leq \underbrace{EU(a_i = 1) - EU(a_i = 1 \xrightarrow{\text{d}} 0)}_{\bar{c}(k)} \tag{75}$$

■

A.8 Proof of Proposition 10: Information Aggregation With/Without Trading

Proof. When the cost of information is tiny ($c \to 0$), then we can have an equilibrium in which all shareholders buy information regardless of whether trading exists or not. When $k(Trading) = k(No\ Trading) = n$, we know $Pr(x = \omega; No\ Trading) > Pr(x = \omega; Trading)$. This is because all informed shareholders sincerely vote when there is no trading, while informed shareholders will strategically vote when there is trading.[8]

The other trivial case is that the information is very exorbitant ($c \to \infty$). Then, we can only have equilibria in which no shareholder wants to acquire information, no matter whether trading is possible or not. When $k(Trading) = k(No\ Trading) = 0$, $Pr(x = \omega; No\ Trading) = Pr(x = \omega; Trading) = \frac{1}{2}$.

Now we turn to prove that there must exist $c_T \in (0, \infty)$ such that $Pr(x = \omega; Trading) > Pr(x = \omega; No\ Trading)$. We first focus on the case in which shareholders cannot trade. To sustain the equilibrium in which no shareholder wants to buy information, the cost of information must satisfy

$$EU(not\ buy, k = 0, No\ Trading) \leq EU_D(not\ buy \to buy, k : 0 \to 1, No\ Trading) - c$$

[8] Meirowitz and Pi (2021) prove the impossibility of an equilibrium in which all shareholders are informed and vote sincerely.

Because when trading is not allowed, shareholders get payoffs if and only if the chosen policy x is the same as the underlying state ω, we have

$$EU(not\ buy, k = 0, No\ Trading) = Pr(x = \omega, k = 0) = \frac{1}{2}$$

Suppose one shareholder i deviates from the equilibrium by choosing to buy information, then her expected payoff is

$$\begin{aligned}
&EU_D(not\ buy \to buy, k : 0 \to 1, No\ Trading) - c \\
&= Pr(x = \omega, k = 1, No\ Trading) - c \\
&= Pr(s_i = 1)(Pr(\omega = 1|s_i = 1)Pr(t' \geq \frac{n-1}{2}|\omega = 1) + Pr(\omega = 0|s_i = 1)Pr(t' < \frac{n-1}{2}|\omega = 0)) \\
&+ Pr(s_i = 0)(Pr(\omega = 1|s_i = 0)Pr(t' > \frac{n-1}{2}|\omega = 1) + Pr(\omega = 0|s_i = 0)Pr(t' \leq \frac{n-1}{2}|\omega = 0)) \\
&- c \\
&= qPr(t' \geq \frac{n-1}{2}) + (1-q)Pr(t' < \frac{n-1}{2}) - c
\end{aligned} \tag{76}$$

Hence, we establish the lemma below.

Lemma 2. *When there is no trading, to sustain an equilibrium in which no shareholder want to buy information, the cost of information must be*

$$c \geq qPr(t' \geq \frac{n-1}{2}) + (1-q)Pr(t' < \frac{n-1}{2}) - \frac{1}{2}$$

Now we turn to the case in which shareholders can trade after voting. To sustain an equilibrium in which no shareholder invests in information acquisition, the cost of the information must satisfy

$$EU(not\ buy, k = 0, Trading) \leq EU_D(not\ buy \to buy, k : 0 \to 1, Trading) - c$$

Suppose that an uninformed shareholder i votes for 1, then her expected payoff at the equilibrium is

$$\begin{aligned}
&EU(not\ buy, k=0, Trading) \\
&= Pr(t' < \frac{n-1}{2})(2Pr(x=1|t') - P_0(t'+1)) + Pr(t' \geq \frac{n-1}{2})P_1(t'+1)
\end{aligned} \tag{77}$$

Note that when no one buys information in equilibrium, the voting tally is not informative at all, and thus the price after voting is always $\frac{1}{2}$ regardless of t. So, we have

$$\begin{aligned}
&EU(not\ buy, k=0, Trading) \\
&= Pr(t' < \frac{n-1}{2})(2 \cdot \frac{1}{2} - \frac{1}{2}) + Pr(t' \geq \frac{n-1}{2})\frac{1}{2} \\
&= \frac{1}{2}
\end{aligned} \tag{78}$$

If the uninformed shareholder i deviates from the equilibrium (buying information), her expected payoff from the deviation is

$$\begin{aligned}
&EU_D(not\ buy \to buy, k: 0 \to 1, Trading) - c \\
&= Pr(s_i=1)Pr(t' < \frac{n-1}{2}|s_i=1)P_0(t'+1) + Pr(t' \geq \frac{n-1}{2})(2Pr(\omega=1|s_i=1,t') - P_1(t'+1)) \\
&+ Pr(s_i=0)Pr(t' \leq \frac{n-1}{2}|s_i=0)(2Pr(\omega=0|s_i=0,t') - P_0(t')) + Pr(t' \geq \frac{n+1}{2})P_1(t') \\
&- c \\
&= Pr(t' < \frac{n-1}{2})\frac{1}{2} + Pr(t' \geq \frac{n-1}{2})(2q - \frac{1}{2}) - c
\end{aligned} \tag{79}$$

Thus, we obtain the lemma below.

Lemma 3. *When there is no trading, to sustain the equilibrium in which no one wants to invest in information, the cost of information must be sufficiently high such that*

$$c \geq Pr(t' < \frac{n-1}{2})\frac{1}{2} + Pr(t' \geq \frac{n-1}{2})(2q - \frac{1}{2}) - \frac{1}{2}$$

Combining Lemma 2 and Lemma 3, we know that if the cost of information satisfies that

$$qPr(t' \geq \frac{n-1}{2})+(1-q)Pr(t' < \frac{n-1}{2})-\frac{1}{2} < c < Pr(t' < \frac{n-1}{2})\frac{1}{2}+Pr(t' \geq \frac{n-1}{2})(2q-\frac{1}{2})-\frac{1}{2}$$

, then we know that no shareholder wants to buy information in equilibrium when there is no trading (thus $Pr(x = \omega; No\ Trading) = \frac{1}{2}$) and that at least one shareholder wants to buy information in equilibrium when there is trading (thus $Pr(x = \omega; No\ Trading) > \frac{1}{2}$).

Note that the set

$$\{c|qPr(t' \geq \frac{n-1}{2})+(1-q)Pr(t' < \frac{n-1}{2})-\frac{1}{2} < c < Pr(t' < \frac{n-1}{2})\frac{1}{2}+Pr(t' \geq \frac{n-1}{2})(2q-\frac{1}{2})-\frac{1}{2}\} \tag{80}$$

is not empty, because of $\frac{1}{2} > 1-q$ and $2q - \frac{1}{2} > q$.

Thus, when c_T is in the set above, we must have $Pr(x = \omega; Trading) > Pr(x = \omega; No\ Trading)$, because no shareholders buy information when trading does not exist but at least one shareholder buys information when trading is allowed.

■