# Whatever It Takes? Market Maker of Last Resort and its Fragility

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**Abstract**

We provide a theoretical framework to analyze the market maker of last resort (MMLR) role of central banks. Central bank announcement to purchase assets in case of distress promotes private agents' willingness to make markets, which immediately restores liquidity to prevent disorderly sales. This, in turn, decreases the future need for the central bank to intervene. Here, the central bank can reduce the expected usage of the facility by announcing a large capacity, that is, it can end up buying less ex-post by committing to do more ex-ante. However, this beneficial feature comes with potential downsides. First, the central bank may not achieve the intended outcome due to the possibility of multiple self-fulfilling equilibria, which may arise if it does not intervene with sufficient aggression or if market participants have doubts about its commitment. Second, public liquidity provision may crowd out private liquidity if the MMLR access becomes permanent and make the intervention ineffective.

**Keywords:** market maker of last resort, liquidity, asset purchase program, multiple equilibria, time inconsistency

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*"A properly constructed MMLR must have a large capacity, but might need to do little. ... The classic is Mario Draghi's "whatever it takes," where the ECB provided a backstop for euro area sovereigns but ended up buying nothing"* (Cecchetti and Tucker 2021)

*"The ECB's efforts rely to an important extent on a bluff, in the sense that nobody knows what would happen if OMT were actually required"* (Krugman 2014)

# 1 Introduction

Central banks traditionally acted as a lender of last resort (LoLR) to support financial stability by providing emergency loans to illiquid banks against safe collateral. While this liquidity backstop helped maintain funding stability in the banking sector, they recently had to reinvent themselves as the financial system transitioned from bank-based to market-based, where stable market liquidity became crucial for systemic soundness. In particular, they started acting as a market maker of last resort (MMLR) to address liquidity shortages in specific markets by outrightly purchasing illiquid assets.

Several such interventions achieved remarkable success. The announcement of MMLR operations immediately stabilized financial markets, even without accompanying actual purchases. A good example would be the Outright Monetary Transactions (OMT) program of the ECB that provided a backstop for Euro area sovereign debt amid the European debt crisis, where Mario Draghi famously promised to do "whatever it takes" but ended up buying nothing. Other examples include the Bank of England's 2009 MMLR operations in sterling corporate bonds and the Federal Reserve's Secondary Market Corporate Credit Facilities (SMCCF) and Municipal Liquidity Facility (MLF) in response to the Covid-19 pandemic. These public backstops instantly reinstated private liquidity upon the introduction and, consequently, the central banks did very little, if anything.[1]

However, some are skeptical about whether these would always work, questioning their robustness (see, e.g., the opening quote by Krugman 2014). After all, the implementation of OMT was

[1] While authorization was for $750 billion for the SMCCF, the Federal Reserve's holdings of corporate bonds and exchange traded funds only peaked at $14 billion. A number of recent studies document that the Fed's launch of these programs restored market liquidity quickly, even without any actual intervention (Brunnermeier and Krishnamurthy, 2020; Boyarchenko et al., 2021; Haddad et al., 2021; Kargar et al., 2021; O'Hara and Zhou, 2021; Vissing-Jorgensen, 2021).

not without friction.[2] In fact, despite being implemented by the same central banks, more recent attempts were much less effective. The Bank of England introduced its emergency gilt-buying program in September 2022 to mitigate disruptions in the gilt market that followed the government tax cut announcement and the resulting firesales by pension funds. ECB also announced its Transmission Protection Instrument (TPI) in July 2022 in response to fragmentation in the European sovereign debt market due to a switch to tighter monetary policy. In contrast with their precedents, these measures' announcements did not promptly restore market liquidity. Rather, they aroused skepticism about the central banks' credibility because the asset purchases would inevitably accompany a sharp increase in the money supply when they claimed to constrain it to dampen inflation.

How should we reconcile these conflicting outcomes of the prior interventions? What would be the features of a successful facility that achieves the stability objective effectively? Also, should this unconventional asset-purchase operation remain in the central bank's permanent toolkit?[3] To answer these questions, we need a theoretical framework that characterizes the mechanism of MMLR and, more importantly, its possible fragility and downsides. However, to this date, academic and policy literature does not have a well-developed model to analyze these critical issues.

This study aims to fill that void in the literature and provides a theoretical model of MMLR. We first characterize the MMLR's "announcement effect," where asset prices increase immediately following the announcement of future liquidity provision, even without accompanying actual asset purchases. We also show that the central bank can expect to buy less ex-post by committing to buy more ex-ante. Here, more audacious actions paradoxically lead to more conservative outcomes, which is beneficial for a central bank that wishes to constrain its balance sheet expansion and money supply while attempting to prevent disruptions in the financial market. We then present the optimal policy and discuss potential fragilities in implementing this policy due to multiple self-fulfilling equilibria. The multiplicity may arise if the central bank does not intervene with sufficient aggression or if market participants have doubts about its commitment. Lastly, we examine distortions in private incentives that may arise if the MMLR access becomes permanently available.

[2]For instance, critics raised concerns about the program's legality. See *German government defends ECB bonds after first day in court*, available at https://www.dw.com/en/german-government-defends-ecb-bonds-after-first-day-in-court/a-16875177

[3]See the interview with Paul Tucker for the debate regarding the MMLR, available at https://www.moneyandbanking.com/commentary/2015/3/4/interview-with-paul-mw-tucker.

Our three-period model considers interactions among long-term investors (including banks, mutual funds, or pension funds, referred to as "insiders"), liquidity providers (dealers, referred to as "outsiders"), and a central bank. At $t = 0$, insiders receive a liquidity shock requiring a cash injection, which they meet by selling their assets to market-making outsiders (Duffie, 2010). The amount of assets insiders need to liquidate depends on market liquidity, that is, the price bid by outsiders. Specifically, since more assets need to be sold to create the necessary cash when with a lower liquidation price, the scale of $t = 0$ liquidations becomes more significant if outsiders bid a lower purchasing price.

While providing liquidity to make markets, outsiders are not efficient users of the assets. Hence, they acquire the assets to sell back to more efficient buyers later at $t = 1$, rather than hold them until $t = 2$ when they mature. Therefore, with market competition, their willingness to pay at $t = 0$ depends on the perspective of the future price they would receive at $t = 1$ to break even in expectation. Here, future liquidity outlook affects dealers' cost of immediacy provision.

Insiders receive some funds later at $t = 1$, whose amount is randomly distributed.[4] As efficient users of the assets, they use the new cash to buy back the assets from outsiders. Here, the price of the assets outsiders would receive from insiders depends on how much funds insiders have. That is, for a given amount of inventory assets held by outsiders, the purchasing price would be equal to the fundamental value when insiders have enough cash inflows to buy the entire outsider inventories at that price, but fall below it with insufficient cash available in the market, resulting in cash-in-the-market pricing (Allen and Gale, 1994, 1998). Hence, future asset prices are also affected by the scale of outsiders' inventory, which becomes larger if insiders have sold more assets due to the lack of immediacy provision.

Given these features, an interrelation arises between immediate liquidity and future liquidity. Outsiders' expectation about the future price at $t = 1$ influences their willingness to pay at $t = 0$, which subsequently affects the scale of early liquidations at $t = 0$. At the same time, the scale of liquidations at $t = 0$ affects the future price at $t = 1$ due to potential cash-in-the-market pricing. With this interdependence, the asset price in equilibrium comprises a fixed point.

[4] Alternatively, we can interpret this as a random arrival of capital that can run the asset efficiently but is slow-moving (Mitchell et al., 2007; Duffie, 2010).

Note that a negative spiral can arise to exacerbate liquidity dry-up if outsiders anticipate the future price of their inventories to be low. The negative prospects limit their willingness to provide immediacy, which leads to more firesales by insiders. Larger liquidations subsequently increase outsiders' inventory and depress future prices, further constraining their market-making incentives to cause a sharp decrease in asset prices and disorderly liquidations. To prevent such disruptions, the central bank can step in as a market maker of last resort by introducing a liquidity backstop through an asset purchasing facility.

Specifically, at $t = 0$, the central bank announces a capacity of the facility denoted as $L$, where it promises to inject up to $L$ units of liquidity to purchase assets from outsiders at $t = 1$. This intervention can result in a strong announcement effect that instantely supports the price at $t = 0$, restraining disorderly liquidations. The effect comes from two channels reflecting the interrelation between $t = 0$ and $t = 1$ liquidity. First, the intervention directly affects the future asset price with increased cash in the market at $t = 1$. The prospect of higher future prices immediately increases outsiders' willingness to pay at $t = 0$, which reduces early liquidations. In addition, an indirect effect arises to amplify the direct effect. Smaller liquidations at $t = 0$, in turn, reduce the scale of outsiders' inventory and further improve their prospects of selling them at a better price at $t = 1$. This again promotes their market-making incentives at $t = 0$, generating a positive spiral. Therefore, the scale of the announcement effect depends on the scales of these direct and indirect effects.

Interestingly, we show that when the facility capacity $L$ is sufficiently large, the central bank can reduce the expected usage of the facility at $t = 1$ by announcing a larger capacity at $t = 0$. Hence, the central bank can expect to buy less ex-post by showing a stronger willingness to do more ex-ante. This outcome benefits a central bank that must limit its money supply or balance sheet expansion while maintaining financial stability. Note that, as a last resort, the central bank does not need to do anything if there arrives enough insider liquidity in the market to sustain the asset price at the fundamental value. With cash-in-the-market pricing, the likelihood of this event arising at $t = 1$ increases if fewer assets are sold at $t = 0$. This is because, with fewer inventory assets held by the outsiders, it becomes more likely that the newly arriving private liquidity can sufficiently prop up the $t = 1$ price to the fundamental value without the (or with a small) injection of public liquidity. As the central bank's commitment $L$ increases, the expected future price therefore rises, which in turn promotes the current asset price and limits disorderly sales by insiders. When the facility capacity $L$ exceeds a certain threshold, further expansion reduces the number of $t = 1$ states that would require public liquidity injections and thus decreases the usage of the facility. In this case, we observe a negative association between the initial commitment and the expected usage of the facility.[5] This is exactly what would constitute a successful facility, as mentioned in the opening quote by Cecchetti and Tucker 2021.

Despite this beneficial feature, we argue that the MMLR intervention can have certain drawbacks and may not be suitable for all central banks. While the central bank can economize on the expected usage of the facility owing to the positive spiral amplifying the announcement effect, that exact feedback effect may result in multiple self-fulfilling equilibria. In the "good" equilibrium, outsiders actively make markets at $t = 0$ in anticipation of high future prices, which instantly calms markets, and the central bank ends up doing very little as intended. In the "bad" equilibrium, on the contrary, outsiders are somehow pessimistic about future prices, which constrains their market-making incentives. This leads to substantial asset liquidations at $t = 0$, which force the central bank to buy more at $t = 1$. Yet, the asset price at $t = 1$ remains low with significant outsider inventories being sold, making the pessimistic belief self-fulfilling. With this fragility, the central bank may not achieve the intended outcome that restrains both fire-sales and central bank purchases. Instead, the policy can result in significant disorderly liquidations and public liquidity injections.

We first show that multiple equilibria can arise if the central bank does not intervene with sufficient aggression, that is, when the facility capacity $L$ is not large enough. This suggests that the central bank may sometimes adopt an *overly* aggressive strategy (such as a promise to do "whatever it takes") to eliminate bad equilibria and avoid fragility, even if it is not the first-best option. We also show that the fragility can arise if the central bank's commitment becomes an issue due to certain factors such as time inconsistency or political pressures. For example, outsiders might doubt

[5] In her speech *Liquidity Shocks: Lessons Learned from the Global Financial Crisis and the Pandemic* delivered on August 11, 2021, Lorie Logan made a similar point: "If intermediaries or end investors are confident that liquidity will be available in the future, either in the form of funding or asset purchases, they may perceive market-making and investing as less risky today—restoring the flow of transactions before any central bank operations are conducted. ... To the extent that announcements of central bank actions can reduce that liquidity demand and encourage a return to normal investing and market-making activity, they can significantly improve conditions even with little or no actual activity."

whether the central bank would indeed inject substantial liquidity ex-post if inflation pressures made a larger money supply more costly.[6] Hence, to avoid this fragility, central banks should intervene at a large enough scale, and market participants should not doubt whether the central bank will honor the commitment.[7]

The MMLR intervention could also distort private incentives if it becomes part of central banks' permanent toolkit. In our extension, we show that the availability of public liquidity can crowd out private liquidity because the intervention makes such hoarding of private liquidity less attractive. This nullifies the benefit of the public backstop and, in that case, MMLR would simply replace private liquidity, forcing the central bank to use the facility more often while little promoting overall market liquidity. These results suggest that a priori, the MMLR option should avail itself of access only during exceptional systemic events; at the same time, the central bank should act with sufficient aggression in case it decides to utilize the MMLR.

The paper is related to the vast literature on central bank interventions during liquidity crises that date back to Thornton (1982) and Bagehot (1873). The literature has primarily focused on the LoLR role of central banks in the traditional bank-based system, which provides a backstop for funding liquidity to contain bank runs.[8] The modern financial system, on the other hand, is more market-based with the substantial growth of non-bank intermediaries, where dealers' provision of market liquidity in the presence of fire-sales is of central importance for financial stability (Brunnermeier and Pedersen, 2009; Tucker, 2009; Duffie, 2010). More recent studies, particularly following the Global Financial Crisis, analyzed the role of public interventions on market liquidity and financial stability (e.g., Acharya and Yorulmazer 2007, Acharya et al. 2010, Diamond and Rajan 2011, Acharya

[6]When announcing its emergency gilt-buying program in September 2022, the Bank of England indicated that the intervention was temporary and would unwind the purchased assets upon the program termination to avoid conflicting with its effort to constrain inflation. Besides the central bank's balance sheet constraints due to its monetary policy objectives, the commitment problem may arise from its reluctance to get exposed to certain types of credit risk and political concerns between central banks and governments.

[7]In discussing the Fed's response to the pandemic, Brunnermeier and Krishnamurthy (2020) note that "(o)ur conjecture is that the Fed's announcement has been viewed by the market as a "whatever it takes" moment. That is, the commitment to act aggressively in the high yield bond market has been taken as a signal of the Fed's willingness to defuse future episodes of financial instability in the broad credit market. This commitment has removed a bad equilibrium and reduced market tail risk. If our conjecture is correct, then the Fed does not currently need to make good on its promise and activate the corporate bond purchase program at this point in time. The important aspect of the Fed's announcements has been the signal of its willingness to act if dislocations arise, and reinforcing this commitment is all that is needed at present."

[8]See, e.g., Bordo (1990), Santos (2006) and Ennis (2016) for the surveys of prior studies.

et al. 2012, and Stein 2012). This paper differs from them in that it theoretically formalizes the mechanism of the newly introduced MMLR operation that provides a liquidity backstop for private dealers. While the interventions examined in the prior studies require an actual liquidity injection through direct lending or asset purchasing, the MMLR facility, like the LoLR, may not be used after all if the public backstop successfully reinstates market liquidity (Tucker 2009, Mehrling 2010). To our best knowledge, this paper is the only article that provides a theoretical framework to delve into the efficacy of this new tool.

The MMLR interventions attracted much attention recently with their remarkable success during the Covid-19 pandemic. A number of studies empirically document how they instantly restored liquidity upon the introduction when the dealers' market-making capabilities were constrained (see, e.g., Brunnermeier and Krishnamurthy 2020; Boyarchenko et al. 2021; Haddad et al. 2021; Kargar et al. 2021; Ma et al. 2021; O'Hara and Zhou 2021; Vissing-Jorgensen 2021). However, more recent interventions by the Bank of England and the ECB in late 2022 were much less effective than the precedents, questioning their robustness. Our model provides important policy implications by presenting fragilities in implementing the MMLR policy to reconcile these conflicting outcomes, also discussing possible distortions to arise should this unconventional measure be included in the central bank's permanent toolkit.

More broadly, this paper is related to the literature on the effect that intermediary frictions impose on market liquidity (see, e.g., Gromb and Vayanos 2002; Brunnermeier and Pedersen 2009; Duffie 2010), focusing on the cost of dealers' immediacy provision (see, e.g., Grossman and Miller 1988; Bao et al. 2018; Bessembinder et al. 2018; Goldberg and Nozawa 2021; He et al. 2022; Choi et al. 2023). We contribute to this literature by examining the role of public liquidity backstops and their downsides.

The rest of the paper is organized as follows. Section 2 introduces the model. Section 3 presents the positive results. Section 4 discusses the optimal policy and its potential fragility. Section 5 extends the baseline model to examine private incentive distortions. Section 6 concludes.

# 2 Model

In this section, we introduce the model, the agents and the timeline, and define the equilibrium of the model.

## 2.1 Agents and asset markets

We consider a model with three dates: $t = 0, 1, 2$. The economy has insiders (banks/mutual funds), outsiders (dealers), and a central bank. There is a continuum of insiders with measure 1, each endowed with one unit of an asset that matures at $t = 2$ and generates a return of $R$ when run by insiders. Insiders get a liquidity shock at $t = 0$ and need to sell some of their assets to generate the funds needed. They later receive some funds at $t = 1$ that they can use to buy back (some of) the assets sold at $t = 0$.

Outsiders do not have any projects to invest in but have deep pockets to purchase assets when they are up for sale. However, outsiders are not the efficient users of these assets, that is, they can generate only $R - \Delta$ per unit of the asset when they run and hold the asset until maturity. Hence, insiders value the asset higher than outsiders, and the outsiders acquire the assets as temporary market makers, with an intention to sell back to insiders afterward.[9] We assume that the outsiders are risk-neutral with discount rate equal 1.

Insiders are hit by a liquidity shock at $t = 0$, which forces them to sell some of their assets. We assume that this shock is system-wide so that there is no financial capacity within the insiders to acquire the assets, and thus the assets need to be sold to outsiders at $t = 0$. The amount of assets sold by the insiders, denoted by $\alpha$, depends on their liquidation price $P_0$. We assume (i) $\alpha'(P_0) \leq 0$, that is, when the price is lower, more assets need to be sold, and (ii) $\alpha''(P_0) \geq 0$, that is, sales increase in a weakly convex fashion as the price $P_0$ decreases.[10] Since outsiders are not efficient in running the assets, their willingness to pay at $t = 0$ depends on the price they anticipate to sell the

[9] See, e.g., Stoll (1978), Amihud and Mendelson (1980), and Grossman and Miller (1988) for models of market makers providing immediacy.

[10] This would be the case if, e.g., insiders need to raise $c$ at $t = 0$ by liquidating the assets at the price $P_0$, which implies $\alpha = \frac{c}{P_0}$ satisfying $\alpha'(P_0) \leq 0$ and $\alpha''(P_0) \geq 0$. Negative association between $\alpha$ and $P_0$ can also arise from, e.g., fire sales (Shleifer and Vishny, 1992) and cash-in-the-market pricing (Allen and Gale, 1994, 1998). Furthermore, a number of empirical studies (e.g., Chen et al., 2010; Goldstein et al., 2017; Falato et al., 2021; Ma et al., 2021) document that decreased asset price due to illiquidity induces further mutual fund redemptions. In such cases, the asset managers need to raise more funds as the price declines, which amplifies fire-sales and results in the convexity.

assets at $t = 1$. Note that outsiders prefer to sell the asset back to an insider at $t = 1$ for any price greater than $R - \Delta$. We assume that the asset market at $t = 0$ is competitive where outsiders break even in equilibrium.

Insiders receive some funds at $t = 1$, which we denote as $I_1$ and is randomly distributed with a continuous pdf $f(.)$ (and cdf $F(.)$) over the interval $[0, \bar{I}]$ as of $t = 0$.[11] We assume this to be uniformly distributed for expositional simplicity. As the efficient users of the assets, they use their cash $I_1$ to buy back the assets they sold to outsiders. The price at $t = 1$, denoted as $P_1$, depends on the amount of cash insiders have, following cash-in-the-market pricing (Allen and Gale, 1994, 1998). That is, given the amount of assets held by outsiders, the asset price would equal the fundamental value $R$ when insiders have sufficient cash inflows, but fall below it with limited cash available in the market. We elaborate on this in Section 2.2 below.

## 2.2 Central bank intervention

Limited liquidity at $t = 1$ would add downward pressure on the asset price $P_1$. The prospect of low future prices in turn diminishes outsiders' willingness to provide liquidity at $t = 0$. This leads to a low price $P_0$ and more fire-sales at $t = 0$, which further depresses future prices. To prevent such a negative spiral, the central bank can step in as a market maker of last resort by providing a liquidity backstop.

Suppose that the central bank introduces a facility with capacity $L$, that is, it promises to inject up to $L$ units of liquidity to purchase assets at $t = 1$.[12] When the central bank injects $L$ at $t = 1$, the total liquidity available for asset purchases is $I_1 + L$. Since $P_1$ given $\alpha$ depends on the amount of available liquidity, we have:

- For $I_1 + L \geq \alpha R$, there is enough liquidity in the market to sustain the price at the fundamental value $R$ for all assets.

[11] This could also be interpreted as an arrival of new insiders with slow-moving capital (Mitchell et al., 2007; Duffie, 2010; Acharya et al., 2013). We assume that the maximum liquidity $\bar{I}$ insiders can have is sufficiently large so that $\bar{I} \geq \bar{\alpha}\Delta$, where $\bar{\alpha}$ is the maximum amount of early liquidations. This technical assumption is for simplicity and is not critical for any of our main results.

[12] The optimal MMLR capacity, of course, should be chosen based on specific objectives of central banks. For now, we treat the MMLR capacity as given, discussing the optimal choice in Section 4.

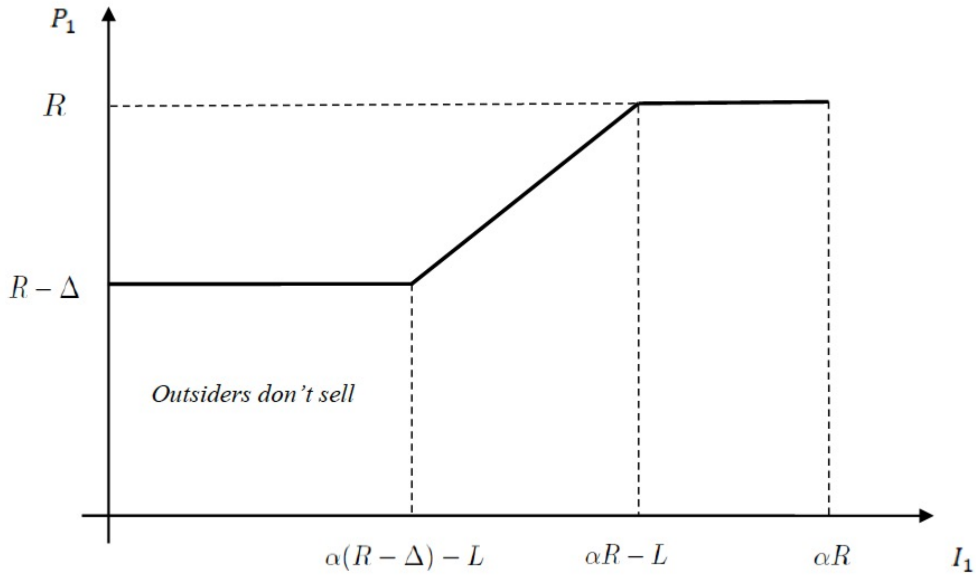


Figure 1: Price $P_1$ as a function of insider capital $I_1$.

- For $\alpha(R-\Delta) \leq I_1 + L < \alpha R$, the price of the asset is determined by the available liquidity in the market, that is, $P_1 = \frac{I_1+L}{\alpha}$, which we refer to as cash-in-the-market pricing (CIMP).

- For $I_1 + L < \alpha(R-\Delta)$, we have $P_1 = R - \Delta$ and outsiders would not sell the asset since they can generate $R - \Delta$ by holding the asset until maturity.

Hence, the market-clearing price $P_1$ at $t = 1$ can be written as:

$$P_1 = \begin{cases} R & \text{for} \quad I_1 + L \geq \alpha R \\ \frac{I_1+L}{\alpha} & \text{for} \quad \alpha(R-\Delta) \leq I_1 + L < \alpha R \\ R - \Delta & \text{for} \quad I_1 + L < \alpha(R-\Delta) \end{cases}, \tag{1}$$

where Figure 1 illustrates $P_1$ as a function of $I_1$, given the facility capacity $L$ and outsiders' inventory $\alpha$.

### 2.3 Timeline and equilibrium

The timeline of the model is given as follows. At $t = 0$, the central bank announces the capacity of the facility $L$. Outsiders then choose $P_0$, the price they are willing to pay for the asset, which determines $\alpha$. At $t = 1$, $I_1$ realizes and the central bank injects additional liquidity to acquire the assets from the outsiders. At $t = 2$, the return from the asset is realized.

Next, we define the equilibrium of the model. Given that the asset market at $t = 0$ is competitive and outsiders are risk-neutral with discount rate equal 1, outsiders' willingness to pay at $t = 0$ equals $E[P_1]$. Note that $P_0$ is the only choice variable given the capacity $L$ of the MMLR facility. Here, $P_0$ is a rational expectations equilibrium if it satisfies

$$P_0 = E[P_1] \tag{2}$$

where $P_1$, given in (1), is a function of $\alpha$ and $L$. Since $\alpha$ is a function of $P_0$, this can be written as $P_0 = E[P_1(\alpha(P_0), L)]$, where the equilibrium $P_0$ is the fixed point of this equation.

## 3 Positive results

In this section, we examine the effect of the central bank facility on the equilibrium price $P_0$ and the expected usage of the facility. We start by characterizing $P_0$ and its response to changes in the size of the facility $L$, where changes in $P_0$ lead to changes in asset sales $\alpha$ at $t = 0$. The actual liquidity injection by the central bank at $t = 1$, denoted as $\tilde{L}$, depends on the amount of liquidity $I_1$ insiders have, as well as the amount of assets $\alpha$ that have been sold at $t = 0$. This is because as a last resort, the central bank does not need to do anything if there arrives enough insider liquidity in the market to sustain the asset price at the fundamental value. In other words, $\tilde{L}$ is a random variable as of $t = 0$ and the usage of the facility at $t = 1$ can be smaller than the facility's capacity $L$ when private liquidity $I_1$ turns out to be large or outsider inventory $\alpha$ is small. We characterize the expected usage of the facility $E[\tilde{L}]$, which also reflects the expected increase in central bank money supply, and show that the expected usage can decrease in the size of the facility $L$ when $L$ is greater than a certain threshold. Hence, an aggressive central bank commitment can lead to fewer asset sales at

$t = 0$ and, in turn, lower usage of the facility at $t = 1$.

### 3.1 Price $P_0$ and the announcement effect

Next, we examine how the price $P_0$ responds as the capacity of the facility $L$ increases, which we refer to as an "announcement effect".

Note that $P_0 = E[P_1(\alpha(P_0), L)]$ in equilibrium, and thus, we have

$$\frac{dP_0}{dL} = \frac{\partial E[P_1]}{\partial L} + \left[\frac{\partial E[P_1]}{\partial \alpha} \times \frac{\partial \alpha}{\partial P_0}\right] \times \frac{dP_0}{dL},$$

which gives us

$$\frac{dP_0}{dL} = \frac{\frac{\partial E[P_1]}{\partial L}}{1 - \left[\frac{\partial E[P_1]}{\partial \alpha} \times \frac{d\alpha}{dP_0}\right]}, \tag{3}$$

where we assume $\frac{\partial E[P_1]}{\partial \alpha} \times \frac{d\alpha}{dP_0} < 1$ to guarantee a stable fixed point. The numerator reflects the direct effect of the central bank facility on the asset price, whereas the denominator reflects a feedback effect that amplifies the direct effect. In particular, the expectation of higher future prices promotes outsiders' willingness to bid higher prices $P_0$ at $t = 0$, which reduces early liquidations $\alpha$. Smaller $\alpha$, in turn, improves outsiders' prospects to sell the assets they acquired at a high price $P_1$ at $t = 1$, which again increases $P_0$ to generate a positive spiral. The scale of the marginal announcement effect $\frac{dP_0}{dL}$ in equilibrium depends on these direct and indirect effects, that is, $\frac{\partial E[P_1]}{\partial L}$ and $\frac{\partial E[P_1]}{\partial \alpha}$.

Next, we characterize the expected price $E[P_1]$ as of $t = 0$. From equation (1) that characterizes $P_1$, we know that the price $P_1$ can take three different cases depending on the available liquidity $L + I_1$ in the market at $t = 1$: (a) the lower bound $R - \Delta$ for low levels of liquidity; (b) CIMP given by $\frac{(L+I_1)}{\alpha}$ for intermediate levels of liquidity; and (c) the fundamental value $R$ for high levels of liquidity. Hence, $E[P_1]$ will be the expected value out of these possible three cases, and depending on the facility capacity $L$ we have:

- For $L \leq \alpha(R - \Delta) - \bar{I}$, we always have $P_1 = R - \Delta$ at $t = 1$ so that $E[P_1] = R - \Delta$.

- For $\alpha(R - \Delta) - \bar{I} < L < \alpha R - \bar{I}$, $P_1$ can have the value $R - \Delta$ for low values of $I_1$ and also

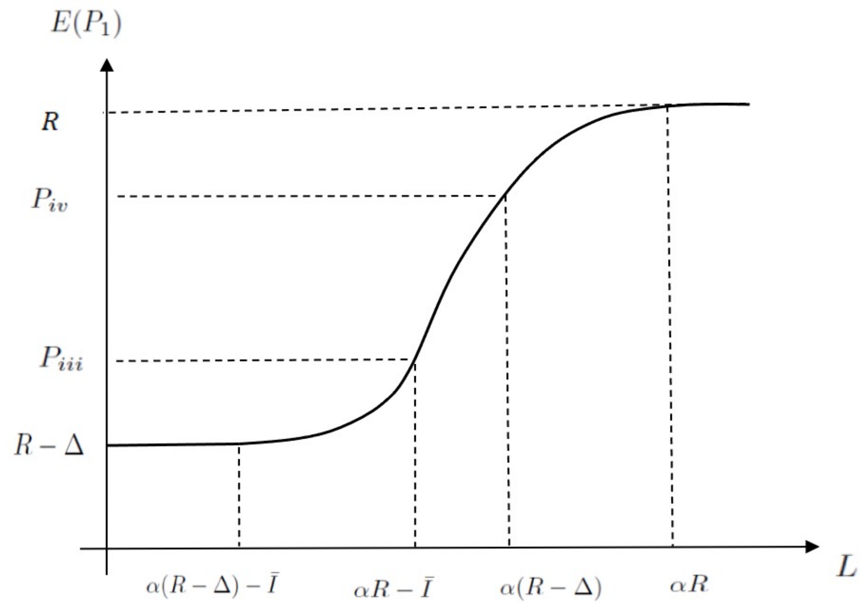


Figure 2: Figure illustrates the expected price $E(P_1)$ as a function of the capacity of the facility $L$ for a uniform distribution for insider liquidity $I_1$.

CIMP at $t = 1$ for large enough $I_1$.

- For $\alpha R - \bar{I} < L < \alpha(R - \Delta)$, $P_1$ can take all three possible cases.
- For $\alpha(R - \Delta) < L < \alpha R$, $P_1$ is given by CIPM for low values of $I_1$ or the fundamental value $R$ for large $I_1$.
- For a sufficiently large $L$ with $L \geq \alpha R$, we always have $P_1 = R$ so that $E[P_1] = R$.

This gives us:

$$E[P_1] = \begin{cases} R - \Delta & \text{(i) if } L < \alpha(R-\Delta) - \overline{I} \\ (R-\Delta)F(\alpha(R-\Delta)-L) + \int_{\alpha(R-\Delta)-L}^{\overline{I}} \frac{(L+I_1)}{\alpha} f(I_1)dI_1 & \text{(ii) if } \alpha(R-\Delta)-\overline{I} < L < \alpha R - \overline{I} \\ (R-\Delta)F(\alpha(R-\Delta)-L) + \int_{\alpha(R-\Delta)-L}^{\alpha R - L} \frac{(L+I_1)}{\alpha} f(I_1)dI_1 + R\left[1 - F(\alpha(R-L)\right] & \text{(iii) if } \alpha R - \overline{I} < L < \alpha(R-\Delta) \\ \int_0^{\alpha R - L} \frac{(L+I_1)}{\alpha} f(I_1)dI_1 + R\left[1 - F(\alpha(R-L)\right] & \text{(iv) if } \alpha(R-\Delta) < L < \alpha R \\ R & \text{(v) if } L > \alpha R \end{cases}, \quad (4)$$

which is illustrated in Figure 2.

We now derive $\frac{\partial E[P_1]}{\partial L}$, the direct effect in equation (3). We know that $E[P_1]$ is constant in cases (i) and (v) so that $\frac{\partial E[P_1]}{\partial L} = 0$. For the other three intermediate cases, using the Leibniz integral rule, we obtain:

$$\frac{\partial E[P_1]}{\partial L} = \begin{cases} \frac{1}{\alpha}\left[1 - F(\alpha(R-\Delta) - L)\right] & \text{(ii) if } \alpha(R-\Delta) - \overline{I} < L < \alpha R - \overline{I} \\ \frac{1}{\alpha}\left[F(\alpha R - L) - F(\alpha(R-\Delta) - L)\right] & \text{(iii) if } \alpha R - \overline{I} < L < \alpha(R-\Delta), \\ \frac{1}{\alpha}\left[F(\alpha R - L)\right] & \text{(iv) if } \alpha(R-\Delta) < L < \alpha R \end{cases} \tag{5}$$

which is strictly positive. Hence, an increase in the capacity of the facility directly increases the expected price $E[P_1]$ with more cash in the market, except for cases (i) and (v) with too little or too much cash in the market, respectively.

We next derive $\frac{\partial E[P_1]}{\partial \alpha}$, which determines the feedback effect in equation (3). Again, $E[P_1]$ is constant in cases (i) and (v) so that $\frac{\partial E[P_1]}{\partial \alpha} = 0$. For the other three cases, using the Leibniz integral rule, we obtain:

$$\frac{\partial E[P_1]}{\partial \alpha} = \begin{cases} -\int_{\alpha(R-\Delta)-L}^{\overline{I}} \frac{(L+I_1)}{\alpha^2} f(I_1) dI_1 & \text{(ii) if } \alpha(R-\Delta) - \overline{I} < L < \alpha R - \overline{I} \\ -\int_{\alpha(R-\Delta)-L}^{\alpha R - L} \frac{(L+I_1)}{\alpha^2} f(I_1) dI_1 & \text{(iii) if } \alpha R - \overline{I} < L < \alpha(R-\Delta), \\ -\int_{0}^{\alpha R - L} \frac{(L+I_1)}{\alpha^2} f(I_1) dI_1 & \text{(iv) if } \alpha(R-\Delta) < L < \alpha R \end{cases} \tag{6}$$

which is strictly negative. Hence, a decrease in asset liquidations at $t = 0$ promotes the future asset price at $t = 1$ with a smaller inventory of assets to sell by outsiders. Therefore, we have $\frac{\partial E[P_1]}{\partial L} \geq 0$, $\frac{\partial E[P_1]}{\partial \alpha} \leq 0$, and $\frac{d\alpha}{\partial P_0} < 0$. These in (3) give us our first main result $\frac{dP_0}{dL} \geq 0$, that is, as the capacity $L$ of the central bank facility increases, the price $P_0$ of assets at $t = 0$ increases resulting in fewer sales $\alpha$ at $t = 0$.

**Proposition 1.** *We have* $\frac{dP_0}{dL} \geq 0$ *and* $\frac{d\alpha}{dL} \leq 0$.

In sum, a possible intervention by the central bank would directly increase the expected future price with more cash in the market. This, in turn, promotes outsiders' liquidity provision at $t = 0$ and thus increases $P_0$. Furthermore, the indirect effect amplifies this direct effect. That is, a higher $P_0$ results in fewer asset liquidations $\alpha$ at $t = 0$, and with fewer assets purchased by outsiders at

$t = 0$, fewer assets will be sold at $t = 1$ resulting in a further increase in $E[P_1]$. This again increases $P_0$ and the subsequent feedback amplifies the announcement effect.

Examining how the marginal effect changes as the capacity $L$ increases, we have the following result.

**Corollary 1.** *$\frac{dP_0}{dL}$ is continuous and: (i) 0 if $L < \alpha(R-\Delta)-\overline{I}$; (ii) increasing in $L$ if $\alpha(R-\Delta)-\overline{I} < L < \alpha R - \overline{I}$; (iii) constant if $\alpha R - \overline{I} < L < \alpha(R-\Delta)$; (iv) decreasing in $L$ if $\alpha(R-\Delta) < L < \alpha R$; (v) 0 if $L > \alpha R$.*

In other words, the marginal effect $\frac{dP_0}{dL}$ is not very strong when $P_0$ is near its lower bound $R-\Delta$ but becomes more significant as the capacity $L$ increases to promote the liquidation price. As the capacity further expands, the incremental effect starts to decline since the demand for market liquidity gets saturated to bring the price near its fundamental value $R$.

### 3.2 Usage of the facility

In this section, we analyze the usage of the facility at $t = 1$, denoted as $\tilde{L}$. Focusing on how the expected usage (and thus central bank money supply) responds to an increase in the facility capacity $\frac{dE[\tilde{L}]}{dL}$, we argue that rather surprisingly, the central bank can reduce the expected usage of the facility by announcing a larger capacity ex ante if the announcement effect is sufficiently strong.

Next, we characterize $\frac{dE[\tilde{L}]}{dL}$. Recall that at $t = 0$, the central bank announces to use up to $L$ at $t = 1$ to purchase assets through its facility. First, note that from (1), we have $P_1 < R$ with probability 1 when the capacity of the facility is small with $L < \alpha R - \overline{I}$. In that case, the central bank would always have to intervene up to its full capacity at $t = 1$ regardless of $I_1$. Therefore, we simply have $E[\tilde{L}] = L$ and $\frac{dE[\tilde{L}]}{dL} = 1$, where an increase in the facility capacity is always associated with an increase in the expected usage.

When $L$ is sufficiently large with $L > \alpha R$, we have $P_1 = R$ with probability 1 from (1). In that case, there is no unmet demand for liquidity and increasing the capacity $L$ will not have any effect on the usage of the facility, that is, $\frac{dE[\tilde{L}]}{dL} = 0$.

With an intermediate capacity such that $\alpha R - \bar{I} < L < \alpha R$, the usage of the facility depends on the availability of insider liquidity $I_1$. Specifically, for $I_1 \geq \alpha R$, insiders have enough cash to

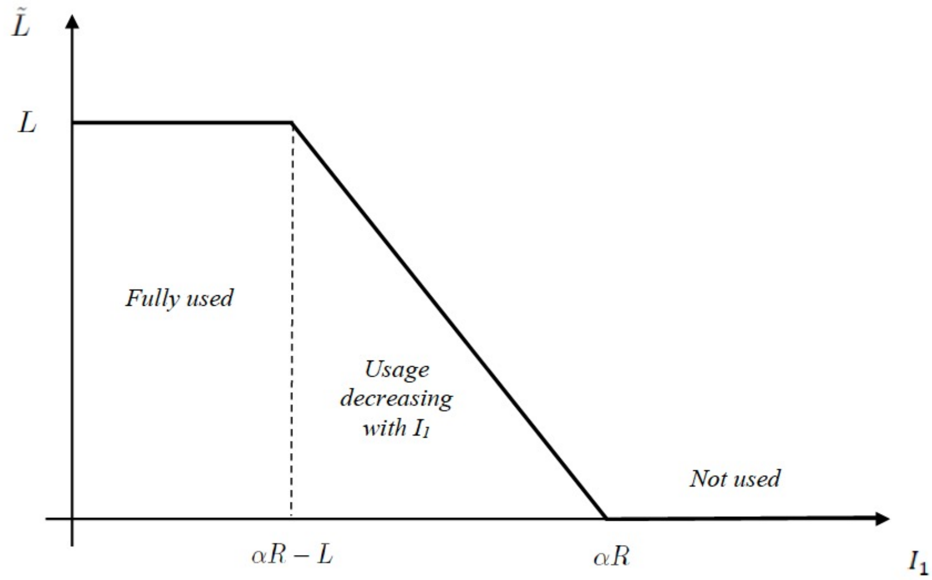


Figure 3: Usage of the facility as a function of insider liquidity $I_1$.

pay $R$ for all liquidated assets at $t = 1$ and the facility is not used at all, that is, $\tilde{L} = 0$. For $\alpha R - L \leq I_1 < \alpha R$, the price of the asset is $R$, where the facility is only partially used with $\tilde{L} = \alpha R - I_1$. For $I_1 < \alpha R - L$, the facility is fully utilized with $\tilde{L} = L$ but, even then, the price of the asset cannot be sustained at $R$. This gives us:

$$\tilde{L} = \begin{cases} 0 & \text{for} \quad I_1 > \alpha R \\ \alpha R - I_1 & \text{for} \quad \alpha R - L \leq I_1 < \alpha R \\ L & \text{for} \quad I_1 < \alpha R - L \end{cases} .$$

Figure 3 illustrates the usage of facility $\tilde{L}$ as a function of $I_1$ in this case.

Therefore, we can characterize the expected usage of the facility when $\alpha R - \overline{I} < L < \alpha R$ as follows:

$$E[\tilde{L}] = \int_0^{\alpha R - L} L f(I_1) dI_1 + \int_{\alpha R - L}^{\min\{\alpha R, \overline{I}\}} (\alpha R - I_1) f(I_1) dI_1 + \int_{\min\{\alpha R, \overline{I}\}}^{\overline{I}} 0 \times f(I_1) dI_1.$$

Note that the capacity of the facility has a direct effect (through $L$) and an indirect effect (through

$\alpha$). Using the Leibniz integral rule, we obtain:

$$\frac{dE[\tilde{L}]}{dL} = \underbrace{F(\alpha R - L)}_{>0,\text{ greater usage}} + \underbrace{\frac{d\alpha}{dL} R\left[F\left(\min\{\alpha R, \overline{I}\}\right) - F(\alpha R - L)\right]}_{<0,\text{ less usage}}. \tag{7}$$

The first term is positive, reflecting the greater amount of liquidity injection required in the states with limited insider liquidity. As $L$ becomes larger, the central bank would need to inject additional liquidity in the future states with $P_1 < R$, which arises with probability $F(\alpha R - L)$. Note that this likelihood decreases in $L$. Thus, this positive effect on the expected usage monotonically weakens in $L$. The second term is negative since the prospect of more aggressive interventions results in higher $E[P_1]$, which, in turn, increases $P_0$ resulting in fewer liquidations $\alpha$ at $t = 0$. As a result, fewer assets get sold at $t = 1$ resulting in a smaller amount of liquidity injection through the facility at $t = 1$ to have $P_1 = R$. This happens when $\alpha R - L < I_1 < \min\{\alpha R, \overline{I}\}$, which becomes more likely with larger $L$. In these states, the central bank can get to spend less due to a smaller $\alpha$, and this negative effect on the expected usage becomes stronger when $|\frac{d\alpha}{dL}|$ is larger. We can summarize $\frac{dE[\tilde{L}]}{dL}$ as:

$$\frac{dE[\tilde{L}]}{dL} = \begin{cases} 1 & \text{for } L < \alpha R - \overline{I} \\ F(\alpha R - L) + R\frac{d\alpha}{dL}\left[F\left(\min\{\alpha R, \overline{I}\}\right) - F(\alpha R - L)\right] & \text{for } \alpha R - \overline{I} < L < \alpha R. \\ 0 & \text{for } L > \alpha R \end{cases} \tag{8}$$

Note that $\frac{dE[\tilde{L}]}{dL}$ is continuous in $L$, and thus is positive with small enough $L$. For larger $L$, the expected usage of the facility can decrease in the capacity of the facility if the second negative effect in (7) dominates the first positive effect. That is, for $\alpha R - \overline{I} < L < \alpha R$, we have $\frac{dE[\tilde{L}]}{dL} < 0$ if

$$\frac{d\alpha}{dL} < -\frac{F(\alpha R - L)}{R\left[F\left(\min\{\alpha R, \overline{I}\}\right) - F(\alpha R - L)\right]}. \tag{9}$$

Note that $\frac{d\alpha}{dL} = \frac{d\alpha}{dP_0} \times \frac{dP_0}{dL}$ ($\leq 0$), and thus $\frac{d\alpha}{dL}$ is smaller if $\frac{dP_0}{dL}$ is larger — the expected usage of the facility can decrease in $L$ if the announcement effect $\frac{dP_0}{dL}$ is significant enough to satisfy (9).

Also, recall from Corollary 1 that $\frac{dP_0}{dL}$ is small when $P_0$ is very low, but becomes larger as $L$ (and thus $P_0$ ) increases. This suggests that (9) may not hold when with a low liquidation price $P_0$ , but start to hold as the capacity $L$ increases. Elaborating on this further, using (3) we can write (9) as:

$$\frac{d\alpha}{dP_0} \times \frac{\frac{\partial E[P_1]}{\partial L}}{1 - \frac{\partial E[P_1]}{\partial \alpha} \times \frac{d\alpha}{dP_0}} < -\frac{F(\alpha R - L)}{R\left[F\left(\min\{\alpha R, \overline{I}\}\right) - F(\alpha R - L)\right]},$$

that is,

$$\frac{d\alpha}{dP_0} < -\left[\frac{\partial E[P_1]}{\partial L} \times \frac{R\left[F\left(\min\{\alpha R, \overline{I}\}\right) - F(\alpha R - L)\right]}{F(\alpha R - L)} - \frac{\partial E[P_1]}{\partial \alpha}\right]^{-1}, \tag{10}$$

which gives the required minimum sensitivity $|\frac{d\alpha}{dP_0}|$ to have $\frac{dE[\tilde{L}]}{dL} < 0$. Recall from (3) that a larger $|\frac{d\alpha}{dP_0}|$ implies a stronger indirect effect that amplifies the direct effect of the capacity expansion on the asset price, which results in a stronger announcement effect $\frac{dP_0}{dL}$. When $\frac{dP_0}{dL}$ is large, public provision of liquidity backstop reinstates private liquidity instantly, which in turn makes the future intervention unnecessary.

Note here that the right hand side (RHS) of (10) is continuous and monotonically increasing in $L$ from $\frac{\partial E[P_1]}{\partial L}$ given in (5) and $\frac{\partial E[P_1]}{\partial \alpha}$ in (6). Hence, we obtain the following result.

**Proposition 2.** *The RHS of (10) is negative and continuously increasing in $L$, with the minimum $\underline{\alpha}'$ and the maximum $\bar{\alpha}'$ for $\alpha R - \overline{I} < L < \alpha R$.*

It is obvious from $\frac{dP_0}{dL} \geq 0$ that the left hand side (LHS) of (10) is also continuous and weakly increasing in $L$. Figure 4 illustrates the case where the expected usage of the facility declines as its capacity increases if (and only if) the capacity is larger than a certain threshold. Here, $\frac{dE(\tilde{L})}{dL} > 0$ if $L$ is smaller than $L^{'}$, but $\frac{dE(\tilde{L})}{dL} < 0$ if $L$ is greater than $L^{'}$.

In sum, the central bank being aggressive in market making would reduce outsiders' concerns at $t = 0$ since they should be able to sell their inventories at a decent price $P_1$ to the insiders or the central bank at $t = 1$. This increases outsiders' willingness to act as temporary market makers and increase their bidding price $P_0$ at $t = 0$. The higher price at $t = 0$ leads to fewer sales $\alpha$, and with fewer assets held by the outsiders, it becomes more likely that the insider liquidity on its own is sufficient to prop up the price $P_1$ to the fundamental value $R$ at $t = 1$ without the (or with a

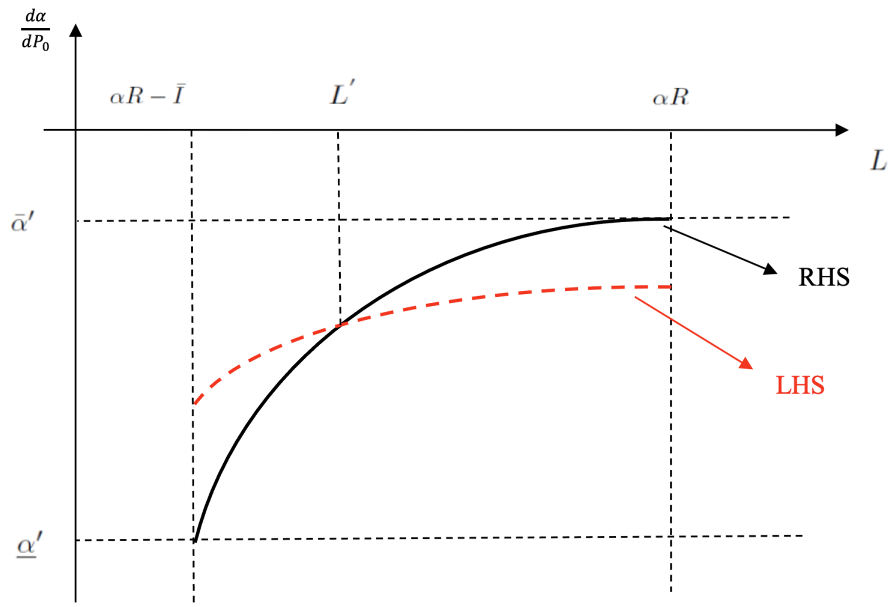


Figure 4: Figure illustrates the condition for expected usage of the facility to decrease (i.e., $LHS > RHS$) with the capacity of the facility. Expected usage increases in the capacity $L$ for $L < L^{'}$ and declines in the capacity for $L > L^{'}$.

small) assistance from the central bank. In this case, the central bank can expect to buy less and limit its money supply by showing stronger willingness to buy more in case of necessity – seemingly audacious decisions can lead to more conservative outcomes.

## 4 Optimal policy and fragility

In this section, we characterize the optimal policy of the central bank and analyze the potential fragility in its implementation due to multiple self-fulfilling equilibria. We also analyze commitment problems that may arise and discuss how this can impair the implementation of the optimal policy by the central bank.

### 4.1 Optimal policy

Here, we characterize the optimal choice of the MMLR capacity. Since the optimal decision depends on the policy objectives that are specific to individual central banks, we adopt a reduced-form objective function and focus on presenting the major trade-offs.

Specifically, we consider a central bank that (i) aims at limiting liquidations $\alpha$ at $t = 0$, but also (ii) attempts to economize its scale of interventions $E(\tilde{L})$. Hence, we assume that the central bank chooses the capacity of the facility $L$ at $t = 0$ to minimize the loss function given by

$$\mathcal{L} = \gamma(\alpha) + E[\tilde{L}], \tag{11}$$

where $\gamma$ increases in a weakly convex fashion in $\alpha$ such that more asset sales at $t = 0$ leads to a higher cost for the central bank. The central bank does not like asset sales $\alpha$ as they can lead to misallocation costs and welfare losses when they are disorderly. It also does not like to use the facility excessively as this may require the central bank to manage the assets when it is not the most efficient runner of the assets. In addition, it may require the central bank to expand its balance sheet and money supply, which may conflict with its monetary policy objectives such as inflation targeting. Moreover, as a market maker, the central bank intends to hold the assets only temporarily till the market recovers. Hence, keeping a larger inventory can become costly.[13]

We can write the FOC for the interior solution as follows:

$$\frac{d\mathcal{L}}{dL} = \underbrace{\gamma^{'}(\alpha)\frac{d\alpha}{dL}}_{<0} + \underbrace{\frac{dE[\tilde{L}]}{dL}}_{>0 \text{ or } <0} . \tag{12}$$

The first term in the RHS is negative, as long as the market price responds to the liquidity injection, that is, $\frac{dP_0}{dL} > 0$. The central bank in this case can limit the costs of disorderly liquidations by increasing $L$.

The second term in the RHS can take both signs as discussed in Section 3.2. When the second term has a positive sign, an increase in the facility capacity would pose a trade-off. On the one hand, a larger capacity decreases asset liquidations at $t = 0$, which has a desirable effect for the central bank objective. On the other hand, a larger capacity increases the expected usage of the facility,

[13] MMLR operations therefore differ from the asset purchasing programs that do not intend to unwind the purchased assets quickly (e.g., QE). When launching its emergency gilt-buying program in September 2022, the Bank of England made clear that it was a temporary measure and would unwind the purchased assets timely upon the program termination, so as not to conflict with its effort to curb inflation.

which is costly. When $\frac{dE[\tilde{L}]}{dL} > 0$ for all $L$, the central bank will choose the optimal capacity $L^*$ that balances the trade-off between the decrease in early liquidations, that is, $\gamma'(\alpha)\frac{d\alpha}{dL}$, and the increase in the usage of the facility, that is, $\frac{dE[\tilde{L}]}{dL}$.

However, this trade-off disappears when we have $\frac{dE[\tilde{L}]}{dL} < 0$. In that case, a further expansion of the capacity of the facility is evidently desirable as it limits asset liquidations at $t = 0$ and, at the same time, decreases the expected usage of the facility, which *always* reduces the loss function $\mathcal{L}$.[14] It is obvious that any $L$ with $\frac{dE[\tilde{L}]}{dL} < 0$ cannot be the optimal solution – the central bank should always increase its facility capacity in such cases.

This implies that the optimal capacity $L^*$ may not have an interior solution. In Figure 4, for instance, we have $\frac{dE[\tilde{L}]}{dL} < 0$ for all $L' < L < \alpha R$ so that $\frac{d\mathcal{L}}{dL}$ is also negative in that region and there is no trade-off arising from increasing the capacity $L$. Hence, the central bank should increase the capacity of the facility up to $L = \alpha R$. Note that $\frac{d\mathcal{L}}{dL} = 0$ for all $L > \alpha R$ since the market is fully saturated with liquidity and any further increase in $L$ will not have any additional effect, that is, we have $\frac{d\alpha}{dL} = 0$ and, thus, $\frac{dP_0}{dL} = 0$. Therefore, any $L > \alpha R$, which would never leave any liquidity demand unmet, can be an optimal capacity for the central bank objective with the identical loss $\mathcal{L}$ from (11).[15] Nonetheless, some central banks deliberately declare that they would intervene in an *overly* aggressive way, or announce a "whatever it takes" policy. Next, we discuss how such a strong aggression may make a difference in the presence of multiple equilibria by eliminating the potential bad equilibria.

## 4.2 Multiple equilibria and MMLR capacity

Next, we analyze the potential fragility that may arise in implementing the MMLR policy. We specifically argue that multiple equilibria may exist when the central bank does not intervene with sufficient capacity.

[14] Specifically, using $\frac{dE[\tilde{L}]}{dL}$ in equation (7), note that $\frac{d\mathcal{L}}{dL} < 0$ can be written as:

$$\frac{d\alpha}{dP_0} < -\Big[\frac{\partial E[P_1]}{\partial L} \times \frac{R\left[F\left(\min\{\alpha R, \bar{I}\}\right) - F\left(\alpha R - L\right) + \gamma'(\alpha)\right]}{F\left(\alpha R - L\right)} - \frac{\partial E[P_1]}{\partial \alpha}\Big]^{-1}, \tag{13}$$

where the *RHS* again increases continuously in $L$. This is a weaker condition than (10) since $\gamma'(\alpha) < 0$ so that $\frac{dE[\tilde{L}]}{dL} < 0$ becomes a sufficient condition for $\frac{d\mathcal{L}}{dL} < 0$.

[15] The optimality would hold if $\mathcal{L}$ with $L = \alpha R$ is smaller than the local minimum of $\mathcal{L}$ for $0 \leq L \leq L'$.

In our analysis in section 4.1, we implicitly assumed that the central bank can always have the desired outcome with the minimized loss function by choosing $L^*$ optimally. Note, however, that once the MMLR capacity $L^*$ is chosen, any $P_0$ satisfying (2) can become an equilibrium outcome. In other words, multiple equilibria arise given the capacity $L$ when there exist multiple fixed points satisfying $P_0 = E[P_1(\alpha(P_0), L)]$. In that case, the central bank may not achieve the intended outcome that minimizes the loss function.

In investigating multiple equilibria, we begin by examining how $E[P_1] \equiv E[P_1(\alpha(P_0), L)]$ changes with $P_0$. Technically, we have a fixed point when $E[P_1]$ as a function of $P_0$ intersects the 45-degree line. We can write the derivative as

$$\frac{\partial E[P_1]}{\partial P_0} = \frac{\partial E[P_1]}{\partial \alpha} \times \frac{d\alpha}{dP_0}, \tag{14}$$

where $\frac{\partial E[P_1]}{\partial \alpha} \leq 0$ as we derived in equation (6) and $\frac{d\alpha}{dP_0} < 0$, hence $\frac{\partial E[P_1]}{\partial P_0} \geq 0$. That is, $E(P_1)$ is weakly increasing in $P_0$. Note that an increase in $P_0$ leads to fewer sales $\alpha$ at $t = 0$, and thus fewer assets get sold by outsiders at $t = 1$ for the same level of liquidity $L + I_1$ in the market, which results in a higher expected price $E(P_1)$ at $t = 1$. However, to get an idea about the potential fixed points and, hence, the multiple equilibria, we need to get an idea about the shape of $E(P_1)$ as a function of $P_0$ vis-a-vis the 45-degree line.

Ignoring the third order effect $\alpha'''(P_0) \approx 0$ for expositional purposes,[16] using $\alpha''(P_0) \geq 0$ and $\frac{\partial E[P_1]}{\partial \alpha}$ from equation (6), we get the following result.

**Proposition 3.** *Given $L$, there exists $\tilde{P}_0(L)$ such that $\frac{\partial^2 E[P_1]}{\partial P_0^2} \leq 0$ for all $P_0 > \tilde{P}_0(L)$, and $\frac{\partial^2 E[P_1]}{\partial P_0^2} \geq 0$ for all $P_0 < \tilde{P}_0(L)$. The inflection point $\tilde{P}_0(L)$ weakly decreases in $L$.*

In other words, $E[P_1]$ increases in a concave fashion in $P_0$, except when $E[P_1]$ is close to the lower bound $R - \Delta$ where it becomes convex in $P_0$ (see the modest policy in Figure 5). The concave (convex) region becomes larger (smaller) as $L$ increases, where, for sufficiently large $L$, the convex region disappears and $E(P_1)$ always increases in a concave way (see the aggressive policy in Figure 5). Intuitively, $E[P_1]$ being close to its lower bound $R - \Delta$ implies that $P_1$ would be $R - \Delta$ in

[16] A weaker sufficient condition is $\alpha''(P_0)$ being monotone in $P_0$.

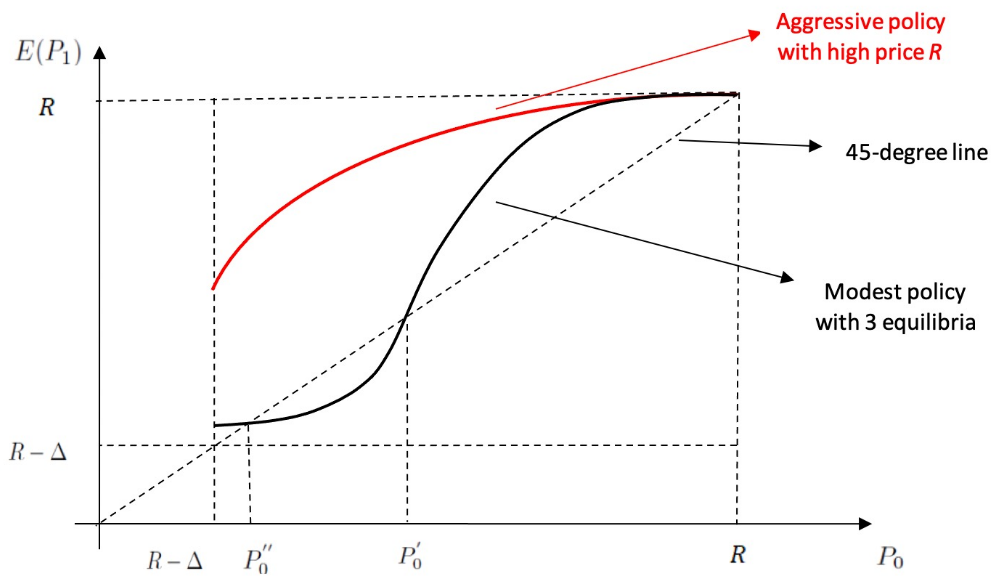


Figure 5: Figure illustrates how multiple equilibria can exist with a modest facility whereas an aggressive policy such as "whatever it takes" can eliminate multiple equilibria and achieve the good equilibrium.

most of the states at $t = 1$. Thus, most of the assets are likely to be held by the outsiders with limited market liquidity. There, a marginal increase in $P_0$ that reduces $\alpha$ would increase *both* the asset price $\frac{L+I_1}{\alpha}$ and the likelihood of CIMP, which results in the convexity when $E[P_1]$ is near $R - \Delta$. In contrast, when $E[P_1]$ is large, a further increase in $P_0$ with a smaller $\alpha$ would increase the likelihood of the $t = 1$ states where the market is saturated with liquidity such that $P_1 = R$, in which case a marginal decrease in $\alpha$ would have no additional effect on the asset price $P_1$. This results in the concavity when $E[P_1]$ is high enough. In addition, $\alpha(P_0)$ decreases in $P_0$ in a convex fashion, which makes $\frac{\partial E[P_1]}{\partial P_0}$ smaller for larger $P_0$. These give us the shape of $E(P_1)$ as a function of $P_0$ as characterized in Proposition 3.

**Corner solution – whatever it takes.** We now discuss why the central bank may choose to be overly aggressive by announcing the "whatever it takes" policy. At the end of Section 4.1, we concluded that for the case illustrated in Figure 4, any $L \geq \alpha R$ would optimally saturate the demand for liquidity in the market to have $P_0 = R$ and become an optimal capacity. Still, given the potential fragility from multiple equilibria, the central bank may wish to announce a considerably large $L$ to avoid the unintended sub-optimal outcomes arising in the bad equilibrium. Figure 5 compares two different capacities, $L_H$ and $L_M$ with $L_H \gg L_M > \alpha R$. Note that any fixed point $P_0^*$ satisfying $P_0^* = E[P_1(P_0^*)]$ can become an equilibrium price. Under the aggressive policy with the large capacity $L_H$, we only have a single fixed point $P_0 = R$, where we have the intended outcome with minimized loss $\mathcal{L}$. However, under the modest policy with the capacity $L_M$, we can additionally have $P_0^{'}$ and $P_0^{''}$ as an equilibrium outcome, where the central bank ends up having more asset liquidations $\alpha$ at $t = 0$ *and* greater usage of the facility. In this case, the central bank should announce the aggressive policy with the large capacity $L_H$ to affect off-the-equilibrium beliefs and eliminate the bad equilibria.

**Interior solution – overly aggressive announcement.** Similar arguments can be made when we have an interior solution for the optimal capacity of the facility, that is, when $L^* < \alpha R$ with $P_0^* < R$. Figure 6 illustrates three cases with different optimal capacities $L_1 > L_2 > L_3$. Suppose that the optimal capacity is large with $L^* = L_1$. In that case, the facility would sufficiently support the market price $P_1$ at $t = 1$, and we have a unique equilibrium with a high $P_0$ and a low $\alpha$. When the optimal capacity is modest with $L^* = L_2$, we have multiple equilibria that could be Pareto ranked – instead of the intended outcome with the high $P_0$ with a small loss $\mathcal{L}$, we may end up with worse outcomes with a greater loss $\mathcal{L}$, where the central bank faces a lower $P_0$ and a higher $\alpha$, as well as higher expected usage of the facility $E[\tilde{L}]$. When $L^*$ is significantly small such that $L^* = L_3$, we again have a unique equilibrium but the MMLR policy does not seem very "effective" — the outcome is close to the "bad" equilibrium for the case of $L_2$ with large loss $\mathcal{L}$. Also, note from (3) and (14) that a steeper slope $\frac{\partial E[P_1]}{\partial P_0}$ implies a larger announcement effect $\frac{dP_0}{dL}$. Here, while the strong indirect effect allows the central bank to spend less as it increases the capacity of the facility, that exact feedback effect can also cause the multiple equilibria to arise.

This raises an interesting discussion about what the central bank would do in the presence of multiple equilibria. Suppose, from the objective function of the central bank, we obtain $L^* = L_2$ as the optimal policy that minimizes $\mathcal{L}$. However, as we discussed, the central bank may suffer from the multiplicity of equilibria and may end up with an unintended outcome in this case. A cautious central bank may instead follow a robust strategy that would resemble a maximin strategy, where the

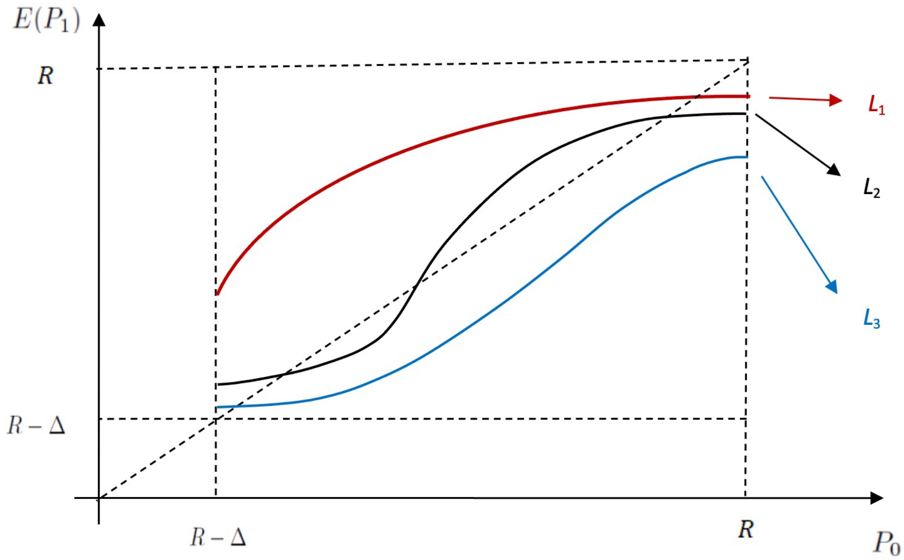


Figure 6: Figure compares the possible equilibrium outcomes for $L^* < \alpha R$ when $L^*$ is large ($L_1$), moderate ($L_2$), or small ($L_3$).

central bank maximizes the worst outcome. In that case, even though $L_1$ is not the optimal outcome from the FOC of the objective function, the central bank may still choose to implement it to prevent the potential bad equilibria from implementing $L_2$ when the loss from the bad equilibrium is larger than the loss from implementing $L_1$. Hence, to prevent the fragility arising from multiple equilibria, the central bank may choose the second-best policy, and be *overly* aggressive and implement $L_1$ with the unique equilibrium.

In sum, for not too large $L^*$, it is possible to have "good" and "bad" equilibria that are both self-fulfilling. In the good equilibrium, outsiders are willing to bid a high price anticipating they could later sell the acquired assets at a high price. Since outsiders provide more liquidity at $t = 0$, fewer fire-sales arise. The central bank may not need to intervene much at $t = 1$ since liquidity within the insiders would be sufficient to buy back these assets from the outsiders, which results in a small loss $\mathcal{L}$ for the central bank. In contrast, in a bad equilibrium, outsiders in anticipation of low future prices bid a low price, causing substantial fire-sales at $t = 0$. The central bank then needs to inject high levels of liquidity at $t = 1$ in more number of states, yet the prices in those states can still be low. This is the bad self-fulfilling equilibrium with a large loss $\mathcal{L}$ for the central bank.

Importantly, the central bank can eliminate the bad equilibria by announcing a facility with a large capacity that can provide a substantial amount of liquidity in times of necessity. In that case, the central bank would *surely* be propping up the future price $P_1$, which encourages outsiders to provide liquidity at $t = 0$. The liquidity provision by outsiders at $t = 0$ limits liquidations $\alpha$, which makes the pessimistic belief nonviable and eliminates the bad equilibria. On the contrary, the perspective of an intervention with a lesser capacity would sustain the pessimistic belief, making the bad equilibria self-fulfilling.

### 4.3 Commitment problems and multiple equilibria

We previously argued that central banks that are ready to intervene aggressively can eliminate multiple equilibria, thus achieving the intended outcome while getting to intervene less at the end. However, this is only possible when the central bank can indeed intervene at $t = 1$ as announced at $t = 0$, with no commitment problem arising from issues such as time inconsistency or political pressures, etc. Some are skeptical about whether these policies would always work as intended, questioning their robustness. Speaking of the OMT, Krugman says that "the ECB's efforts rely to an important extent on a bluff, in the sense that nobody knows what would happen if OMT were actually required." In announcing its gilt-purchase program in September 2022, the Bank of England needed to make clear that this would be a temporary measure with a set termination date since there was significant concern about inflationary pressures requiring a tighter money supply. Next, we analyze the fragility that may arise when the central bank has certain constraints in the ex-post implementation of its policy.

**Whatever it takes, revisited.** Let us revisit the two policies illustrated in Figure 5. The "whatever it takes" policy showed how the central bank's strong commitment can affect the outcome of the intervention. As we discussed in the previous section, the central bank can surely achieve the intended outcome with a small loss $\mathcal{L}$ having low $\alpha$ and low $E(\tilde{L})$ by announcing the large capacity $L^* = L_H$, but may have a bad outcome with the modest capacity $L_M$ due to the multiplicity of equilibria.

Now, suppose that the central bank has announced the large capacity $L_H$ but, in fact, it cannot spend more than $L_M$ at $t = 1$.[17] If outsiders believe in the central bank's commitment, then the only equilibrium would be the good equilibrium with $P_0 = R$ and a small $\alpha$, where the central bank facing a small $\alpha$ does not need to intervene much at $t = 1$ — that is, $\tilde{L} < L_M$ with probability 1 and the "bluff" would work. However, if outsiders have doubts about the central bank's actual capability to intervene, they may choose $P_0'$ or $P_0''$ instead, and the "bluff" can fail with a large $\alpha$ — since the central bank would only intervene up to $L_M$ ex-post, outsiders' concern becomes self-fulfilling. Hence, the lack of central bank's commitment can lead to unintended sub-optimal outcomes.

**Bluffing under time inconsistency.** We can also consider a case where the central bank faces a constraint on the amount of assets it can acquire. For instance, the central bank may not be an efficient user of these assets, where it can only generate $R - \Delta_{CB}$ from the assets. Denoting $\alpha_{CB}$ as the unit of assets the central bank acquires, suppose that $\Delta_{CB}$ is increasing in $\alpha_{CB}$, that is, as the central bank acquires more assets, it starts to acquire assets it is less and less efficient in running.[18] Also, running a large portfolio of assets can require additional resources from the central bank and can distract its main efforts in sustaining price and financial stability. Unlike quantitative easing, MMLR intends to buy assets temporarily and sell later when private markets recover, which would be harder to rewind with larger inventories.[19] There may also be political pressures from purchasing too many assets since the central bank would be criticized for "replacing" the private market. For all these reasons, it may not be ex-post efficient or even implementable for the central bank to acquire more than $\hat{\alpha}_{CB}$ units of assets. Note that this would change the central bank loss function as follows:

$$\mathcal{L} = \gamma(\alpha) + E[\tilde{L}] + \delta(\alpha_{CB}) \times \mathbf{1}_{\alpha_{CB} > \hat{\alpha}_{CB}}, \tag{15}$$

where $\delta$ is positive and increasing, and $\mathbf{1}$ is the indicator function that equals 1 for $\alpha_{CB} > \hat{\alpha}_{CB}$, and 0 otherwise.

As with $\tilde{L}$, $\alpha_{CB}$ also depends on the amount of asset liquidations $\alpha$ at $t = 0$ and the insider

[17] This practical limit can come directly from certain central bank objectives (e.g., inflation targeting) but can also be exogenously given outside of the model such as political pressures or legislative restrictions.

[18] We can even have $\Delta_{CB} > \Delta$ for $\alpha_{CB}$ greater than a certain threshold $\hat{\alpha}_{CB}$, in which case the central bank would prefer having outsiders run some of the assets rather than acquiring more.

[19] For instance, the Bank of England in ther news release declared that their gilt market operation is "strictly time limited" and "(t)he purchases will be unwound in a smooth and orderly fashion once risks to market functioning are judged to have subsided." See `https://www.bankofengland.co.uk/news/2022/september/bank-of-england-announces-gilt-market-operation`.

liquidity $I_1$ at $t = 1$. In particular, we have:

- For $I_1 \geq \alpha R$, insiders have enough cash to pay $R$ for all the assets at $t = 1$. Hence, all assets are acquired by the insiders and $\alpha_{CB} = 0$.
- For $\alpha R - L \leq I_1 < \alpha R$, the price is $P_1 = R$, where insiders acquire $\frac{I_1}{R}$ units and the rest is acquired by the central bank, that is, $\alpha_{CB} = \alpha - \frac{I_1}{R}$.
- For $\alpha(R - \Delta) - L \leq I_1 < \alpha R - L$, the price is $P_1 = \frac{L+I_1}{\alpha}$, which gives $\alpha_{CB} = \frac{L}{P_1} = \frac{\alpha L}{L+I_1}$.
- For $0 \leq I_1 < \alpha(R - \Delta) - L$, the price is $P_1 = R - \Delta$ even with the fully utilized central bank facility and we have $\alpha_{CB} = \frac{L}{R-\Delta}$.

This gives us:

$$\alpha_{CB} = \begin{cases} \frac{L}{R-\Delta} & \text{for} \quad 0 \leq I_1 < \alpha(R - \Delta) - L \\ \frac{\alpha L}{L+I_1} & \text{for} \quad \alpha(R - \Delta) - L < I_1 < \alpha R - L \\ \alpha - \frac{I_1}{R} & \text{for} \quad \alpha R - L \leq I_1 < \alpha R \\ 0 & \text{for} \quad I_1 > \alpha R \end{cases}. \tag{16}$$

Note that given $L$ and $I_1$, $\alpha_{CB}$ is increasing in $\alpha$ — all else equal, the central bank would need to purchase more assets at $t = 1$ when more assets get liquidated at $t = 0$.

Figure 7 presents the commitment problem that would arise when the optimal policy is an interior solution $L_1$ as in Figure 6. Suppose the central bank has optimally chosen $L^* = L_1$ to minimize the loss $\mathcal{L}$. If the central bank can commit to implementing this, we would have the unique equilibrium $P_0^*$ along with the corresponding $\alpha^*$, and suppose that this is small enough to satisfy $\alpha^* < \hat{\alpha}_{CB}$. Here, the two loss functions given in (11) and (15) become equivalent with $\alpha^* < \hat{\alpha}_{CB}$ – the central bank might have bluffed but ex post it worked well due to the commitment since it never had to acquire more than $\hat{\alpha}_{CB}$.

Now suppose that the central bank cannot commit, and would need to restrict its asset acquisition ex post with the upper bound $\hat{\alpha}_{CB}$. This changes the shape of $E[P_1(P_0)]$ as in Figure 7. Since $\alpha(P_0)$ decreases continuously in $P_0$, there exists $\hat{P}_0$ such that $\alpha(P_0) = \hat{\alpha}_{CB}$ for $P_0 = \hat{P}_0$. The central bank would not need to acquire more than $\hat{\alpha}_{CB}$ units at $t = 1$ if $P_0 > \hat{P}_0$, but for $P_0 < \hat{P}_0$, it may be

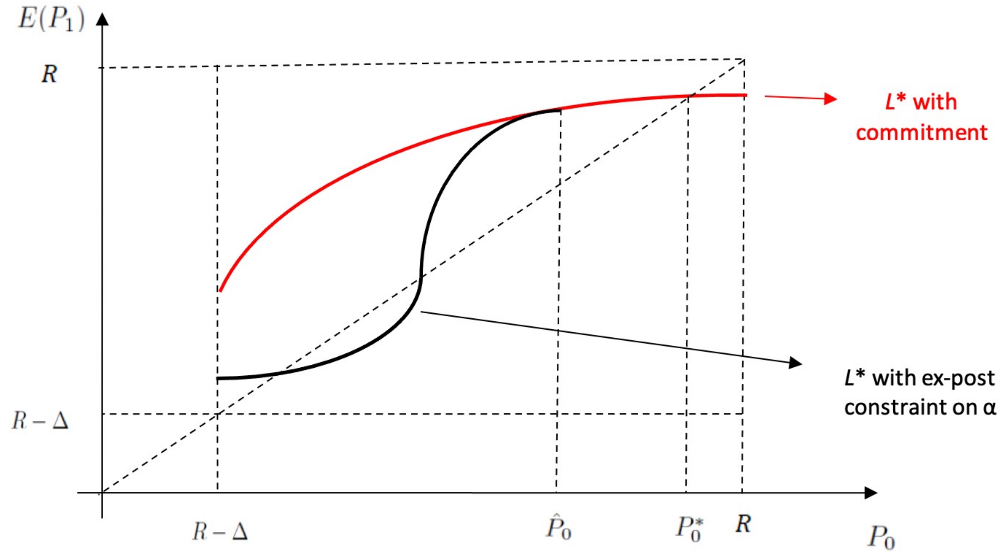


Figure 7: Figure illustrates the ex-post commitment problem with the interior solution $L_1$ as in Figure 6 and how the lack of commitment may impair its implementation.

forced to limit its intervention below the announced capacity and thus, we see the deviation of the two curves below $\hat{P}_0$.

As Figure 7 illustrates, multiple fixed points can arise when the central bank has the ex-post constraint and cannot commit credibly ex-ante. As in the previous section, we have the "bad" equilibria in addition to the "good" equilibrium. In the good equilibrium, high $P_0$ leads to low $\alpha$, which allows the central bank not to deviate from its announcement at $t = 1$. In the bad equilibrium, however, low $P_0$ leads to large liquidations $\alpha$, which tests central bank's commitment and forces the central bank to deviate from its announcement at $t = 1$.

## 5 Incentive Distortions

We next extend our baseline model to discuss whether the MMLR facility should remain in the central bank's permanent toolkit, like discount window lending, or make itself available only in specific circumstances. We argue that unconditional availability of the facility may distort private incentives to hoard liquidity, which in turn nullifies the benefit of the public backstop and increases the social cost.

In our baseline model, we take the random arrival of insider liquidity at $t = 1$ (i.e., distribution of $I_1$) as exogenously given. Suppose that this ex-post liquidity is affected by private agents' ex-ante decision. That is, liquidity providers choose how much liquidity to hoard at $t = -1$, and their decisions affect the $t = 1$ distribution of $I_1$ such that more liquidity would arrive ex-post if more is hoarded ex-post. Also, suppose that they choose to set more funds aside when anticipating higher expected return for liquidity hoarding, denoted as $\overline{r}$, which can be defined as

$$\overline{r} \equiv E\left[\frac{R}{P_1}\right] - 1.$$

Therefore, higher $\overline{r}$ results in more liquidity hoarding, which in turn shifts $F_{\overline{r}}$ (cdf of $I_1$) to the right; for any two $\overline{r} = \overline{r}_H, \overline{r}_L$ with $\overline{r}_H > \overline{r}_L$, $F_{\overline{r}_H}$ is first-order stochastic dominant (FOSD) over $F_{\overline{r}_L}$.

If the liquidity hoaders anticipate the liquidity intervention to be initiated at $t = 0$, the MMLR facility, regardless of its capacity $L$, has the following perverse effect.

**Proposition 4.** *For any $L$, we always have the same asset price $P_0$. The expected usage of the facility $E[\tilde{L}]$ and the central bank loss function $\mathcal{L}$ increases in $L$.*

In other words, the central bank's willingness to provide liquidity constrains the profitability of private liquidity provision, impairing incentives to hoard liquidity ex-ante (e.g., Gale and Yorulmazer 2013; Choi et al. 2016). In equilibrium, public liquidity *replaces* private liquidity such that the same amount of assets get liquidated at $t = 0$. Without affecting the scale of fire-sales, the central bank simply spends more when announcing a larger facility capacity.

This result implies that although the MMLR intervention should be aggressive enough when triggered, it needs to make itself available only in "unexpected" situations rather than being part of the central bank's permament toolkit. Like the case of deposit insurance, certain "constructive ambiguity" helps prevent ex-ante incentive distortions of the private sector.

# 6 Conclusion

After the crisis of 2007-2009, and of course, with the pandemic, the MMLR role of central banks attracted significant attention. Several central banks, including the Band of England, ECB, and the Federal Reserve, introduced the MMLR facility, and market liquidity promptly recovered even if the actual asset purchases were minimal. However, when the same interventions were announced by the Bank of England and ECB late in 2022, they did not work as before because these central banks were also expected to constrain their money supply to meet the inflation target.

This paper developed a theoretical framework to formalize the functioning of MMLR intervention on market liquidity, which allows us to examine its optimal design and robust implementation. Our results have the following policy implications. First, the MMLR intervention must be aggressive enough to obtain the intended outcome. Second, the central bank may get to intervene less ex-post when it announces more aggressive intervention ex-ante. Third, lukewarm interventions or lack of commitment by the central bank may result in multiple self-fulfilling equilibria to bring fragilities. Fourth, rather than remaining in the central bank's permanent toolkit, the MMLR facility should make itself available only in exceptional circumstances to prevent ex-ante incentive distortion for the private liquidity providers.

The paper's model would enrich our understanding of this new policy option and form a basis for future work that analyzes the success and fragility of such facilities both theoretically and empirically. As commitment becomes critical, a pre-determined rule may help central banks to resolve time-inconsistency problems. However, this may distort ex-ante incentives, where "constructive ambiguities" are necessary to prevent moral hazard. Besides distorting the private liquidity provision incentives that this paper discussed, containing disorderly liquidations may undermine the disciplining role of runs (Calomiris and Kahn, 1991; Diamond and Rajan, 2001). The affected banks and nonbanks may rely excessively on the central bank and hold inadequate levels of liquidity (Repullo, 2005). In addition, by providing these institutions an option, MMLR may delay and prevent the cleaning up of their balance sheets (Diamond and Rajan, 2011; Acharya and Tuckman, 2014). These are important issues that deserve further research.

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## Appendix

**Proof of Proposition 2:**

It is obvious that $-\Big[\frac{\partial E[P_1]}{\partial L}\times\frac{R\big[F\big(\min\{\alpha R,\overline{I}\}\big)-F(\alpha R-L)\big]}{F(\alpha R-L)}-\frac{\partial E[P_1]}{\partial\alpha}\Big]^{-1}$ is continuous in $L$ because $\frac{\partial E[P_1]}{\partial L}$ and $\frac{\partial E[P_1]}{\partial\alpha}$ are continuous. We show that for the uniformly distributed $I_1$, this is increasing in $L$ when $\alpha R-\overline{I}<L<\alpha R$. Note that here we have case (iii) for smaller $L$ with $\alpha R-\overline{I}<L<\alpha(R-\Delta)$, and case (iv) for larger $L$ with $\alpha$(R-$\Delta$)$<L<\alpha R$.

We first analyze case (iii). From (5) and (6), we have $\frac{\partial E[P_1]}{\partial L}=\frac{\Delta}{\overline{I}}$ and $\frac{\partial E[P_1]}{\partial\alpha}=-\frac{2\Delta R-\Delta^2}{2\overline{I}}$ in this case. The RHS of (10) hence becomes

$$RHS=-\Big[\frac{\Delta}{\overline{I}}\times R\frac{\min\{L,\overline{I}-(\alpha R-L)\}}{\alpha R-L}+\frac{2\Delta R-\Delta^2}{2\overline{I}}\}\Big]^{-1},$$

which is increasing in $L$.

We next analyze case (iv) where we have $\frac{\partial E[P_1]}{\partial L}=\frac{\alpha R-L}{\alpha\overline{I}}$ and $\frac{\partial E[P_1]}{\partial\alpha}=-\frac{(\alpha R+L)(\alpha R-L)}{2\alpha^2\overline{I}}$. Hence, the RHS of (10) becomes

$$\begin{aligned}RHS&=-\alpha\overline{I}\Big[R\times\min\{L,\overline{I}-(\alpha R-L)\}+\frac{(\alpha R+L)(\alpha R-L)}{2\alpha}\Big]^{-1}\\&=-\alpha\overline{I}\Big[-\frac{(\alpha R-L)^2}{2\alpha}+\alpha R^2+R\times\min\{0,\overline{I}-\alpha R\}\Big]^{-1},\end{aligned}$$

which is again increasing in $L$ with $L<\alpha R$. The maximum and minimum come from the monotonically and continuity. ■

**Proof of Proposition 3:**

Note that we had different cases (i.e., case (i) to (v)) depending on the size of $L$. Since we would like to consider $E[P_1]$ as a function of $P_0$, we now need to solve for the boundaries for each case with respect to $P_0$. We can do this by first solving with respect to $\alpha$, and then with respect to $P_0$ using the inverse function $P_0=\alpha^{-1}$. We therefore have

$$\frac{\partial E[P_1]}{\partial P_0} = \begin{cases} 0 & \text{(i) if } P_0 < \alpha^{-1}\left(\frac{L+\bar{I}}{R-\Delta}\right) \\ -\int_{\alpha(R-\Delta)-L}^{\bar{I}} \frac{(L+I_1)\frac{d\alpha}{dP_0}}{\alpha^2} f(I_1)dI_1 & \text{(ii) if } \alpha^{-1}\left(\frac{L+\bar{I}}{R-\Delta}\right) < P_0 < \alpha^{-1}\left(\frac{L+\bar{I}}{R}\right) \\ -\int_{\alpha(R-\Delta)-L}^{\alpha R-L} \frac{(L+I_1)\frac{d\alpha}{dP_0}}{\alpha^2} f(I_1)dI_1 & \text{(iii) if } \alpha^{-1}\left(\frac{L+\bar{I}}{R}\right) < P_0 < \alpha^{-1}\left(\frac{L}{R-\Delta}\right) \\ -\int_{0}^{\alpha R-L} \frac{(L+I_1)\frac{d\alpha}{dP_0}}{\alpha^2} f(I_1)dI_1 & \text{(iv) if } \alpha^{-1}\left(\frac{L}{R-\Delta}\right) < P_0 < \alpha^{-1}\left(\frac{L}{R}\right) \\ 0 & \text{(v) if } P_0 > \alpha^{-1}\left(\frac{L}{R}\right) \end{cases},$$

and for the uniform $I_1$, this becomes

$$\frac{\partial E[P_1]}{\partial P_0} = \begin{cases} 0 & \text{(i) if } P_0 < \alpha^{-1}\left(\frac{L+\bar{I}}{R-\Delta}\right) \\ -\frac{d\alpha}{dP_0}\frac{1}{\alpha^2 \bar{I}}\left[L\bar{I} + \frac{\bar{I}^2}{2} - \frac{(\alpha(R-\Delta)+L)(\alpha(R-\Delta)-L)}{2}\right] & \text{(ii) if } \alpha^{-1}\left(\frac{L+\bar{I}}{R-\Delta}\right) < P_0 < \alpha^{-1}\left(\frac{L+\bar{I}}{R}\right) \\ -\frac{d\alpha}{dP_0}\frac{\Delta}{\bar{I}}\left[R - \frac{\Delta}{2}\right] & \text{(iii) if } \alpha^{-1}\left(\frac{L+\bar{I}}{R}\right) < P_0 < \alpha^{-1}\left(\frac{L}{R-\Delta}\right) \\ -\frac{d\alpha}{dP_0}\frac{1}{2\bar{I}}\left[R^2 - \left(\frac{L}{\alpha}\right)^2\right] & \text{(iv) if } \alpha^{-1}\left(\frac{L}{R-\Delta}\right) < P_0 < \alpha^{-1}\left(\frac{L}{R}\right) \\ 0 & \text{(v) if } P_0 > \alpha^{-1}\left(\frac{L}{R}\right) \end{cases}.$$

Here it is clear that $\frac{\partial^2 E[P_1]}{\partial P_0^2} < 0$ for cases 3 and 4. Also, for linear $\alpha$ where $d\alpha/dP_0$ is constant, it's straightforward to show $\frac{\partial^2 E[P_1]}{\partial P_0^2} > 0$ for case 2. Considering the general case with $\frac{d^2\alpha}{dP_0^2} > 0$, for simplicity we assume $\alpha'''(P_0) \approx 0$, that is, $\frac{d^2\alpha}{dP_0^2}$ is constant.

We focus on case 2. Note that

$$\frac{\partial^2 E[P_1]}{\partial P_0^2} = \frac{\partial^2 E[P_1]}{\partial \alpha^2} \times \frac{d\alpha}{dP_0} + \left[\frac{\partial E[P_1]}{\partial \alpha} \times \frac{d^2\alpha}{dP_0^2}\right] \equiv A \times B + C \times D \tag{17}$$

where $A \equiv \frac{\partial^2 E[P_1]}{\partial \alpha^2} = \frac{d\alpha}{dP_0} \times 4\alpha\bar{I}[\bar{I}^2 + 2L\bar{I} - L^2] < 0$; $B \equiv \frac{d\alpha}{dP_0} < 0$; $C \equiv \frac{\partial E[P_1]}{\partial \alpha} \leq 0$; $D \equiv \frac{d^2\alpha}{dP_0^2} \equiv \kappa > 0$.

We now consider how these changes in $P_0$ for $\alpha^{-1}\left(\frac{L+\bar{I}}{R-\Delta}\right) < P_0 < \alpha^{-1}\left(\frac{L+\bar{I}}{R}\right)$. For the lowest $P_0$ with which $\alpha = \frac{L+\bar{I}}{R-\Delta}$, we have $P_0 = R - \Delta$ and thus $C = 0$. Hence, since $E[P_1]$ increases in $P_1$, we know $\frac{\partial^2 E[P_1]}{\partial P_0^2} > 0$ at its lower bound with the lowest $P_0 = R - \Delta$. Now, note that an increase in $P_0$ (and thus smaller $\alpha$) would make (a) $|A|$ smaller, (b) $|B|$ smaller, (c) $|C|$ larger ($\because |\frac{\partial E[P_1]}{\partial \alpha}|$ decreases in $\alpha$, see case 2 for $\frac{\partial E[P_1]}{\partial \alpha}$ ), (d) $|D|$ constant and unchanged. Hence, as $P_0$ increases, (17) decreases monotonically in this region. We can define $\hat{P}_0$ as the price that satisfies $A \times B + C \times D = 0$. If $A \times B + C \times D > 0$ for all $\alpha^{-1}\left(\frac{L+\overline{I}}{R-\Delta}\right) < P_0 < \alpha^{-1}\left(\frac{L+\overline{I}}{R}\right)$, then $\hat{P}_0$ is defined from $\alpha(\hat{P}_0) = \frac{L+\overline{I}}{R}$ (i.e., threshold between cases 2 and 3).

Now we show that $\hat{P}_0$ decreases in $L$. Note that $B$ and $D$ in (17) are independent of $L$. Also, note that $|A| = |\frac{d\alpha}{dP_0} \times 4\alpha\overline{I}[\overline{I}^2 + 2L\overline{I} - L^2]|$ decreases in $L$ and $|C|$ increases in $L$. Therefore, for larger $L$, we need larger $|\frac{d\alpha}{dP_0}|$ at $P_0 = \hat{P}_0$ to have $A \times B + C \times D = 0$. Hence, $\hat{P}_0$ decreases in $L$. ■