Dynamic Carbon Emission Management[*]

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Abstract

The control of carbon emissions by policymakers poses the corporate challenge of developing an optimal carbon management policy. We provide a unified model that characterizes how firms should optimally manage emissions through production, green investment, and the trading of carbon credits, as well as the implications for asset prices. We show that carbon regulation induces firms to tilt towards more immediate yet transient types of green investment—such as abatement as opposed to innovation—as it becomes more costly to comply. Perhaps surprisingly, firms with a large stock of carbon credits are less committed to curbing emissions. Lastly, even if more polluting firms command a higher risk premium, carbon regulation need not reduce firm value.

Keywords: Carbon Emissions, Carbon Abatement, Green Innovation, Carbon Credits, Carbon Trading, Carbon Tax, Asset Prices.

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1 Introduction

Keeping carbon emissions under control is one of the greatest challenges of our time. In this context, economic research started addressing the questions concerning the macroeconomic implications of climate change as well as the welfare consequences of different regulatory regimes.[1] Perhaps surprisingly, while streamlining the firm's problem to solve for aggregate implications, the existing literature has not yet addressed a fundamental question that naturally surfaces from the *corporate* perspective: Considering all the tools available to businesses, how should firms best manage their carbon emissions? The urgency in the corporate community to address this question is palpable: Looking ahead, firms are increasingly implementing carbon accounting and investing to either fulfill or comply with ambitious climate goals.[2]

Starting from the canonical premise in finance that corporate decisions should maximize shareholder value, we thus study the tradeoffs posed by carbon regulation on firms. We develop a novel theoretical framework that characterizes firms' optimal carbon management policy under alternative regulatory systems as well as the ensuing implications for asset prices. Corporate carbon management is dynamic in nature, as virtually all forms of carbon regulation track the stock of a firm's carbon emissions over time. Accordingly, we propose a continuous-time dynamic model in which the firm best handles carbon emissions by balancing out the scale of production, the engagement in immediate abatement measures to offset emissions, and the investment in innovative solutions to permanently switch to more sustainable technologies. Importantly, unlike previous studies, firms are not exogenously polluters or cleansers: In our model, the firms' carbon footprint is endogenous and may switch sign over time as management maximizes shareholder value.

A notable feature of our framework is being broad enough to characterize optimal carbon management policies under the three most prevalent regulatory frameworks: laissez-faire, a carbon credit trading system (henceforth, carbon trading system), and a carbon tax system.

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[1] See, for instance, the seminal models by Acemoglu, Aghion, Bursztyn, and Hemous (2012) and Acemoglu, Akcigit, Hanley, and Kerr (2016), or the policy considerations by Pindyck (2021) and Stavins (2022).

[2] See the reports by McKinsey & Company (2021, 2022) and The World Bank (2022).

The carbon trading system is a market-based approach by which carbon credits give firms the right to deploy a fixed volume of carbon emissions into the atmosphere and, thus, produce.[3] In this system, carbon credits are tradable: Firms in need of credits can buy them in the carbon markets, whereas firms with excess of credits may instead choose to sell them. By contrast, under the carbon tax system, a central authority sets a predetermined price that emitters pay for each ton of emissions, incentivizing firms to reduce their carbon footprint to minimize their tax liability. Naturally, the key tradeoffs determining a firm's policies are affected by the regulatory environment. Firms continuously evaluate the most cost-effective policies to curtail emissions by managing production, investing in long-term green innovation, engaging in more immediate abatement projects, or trading carbon credits.

We start by investigating the optimal carbon emission management under the carbon trading system. The firm optimally sells credits on carbon markets when its balance exceeds an (endogenous) target level, and buys credits when running out of them. In this system, we show that the firm manages production and green investment in a precautionary fashion: The firm aims at preserving its stock of credits to avoid resorting to the costly carbon credit market. Hence, the credit trading system makes the firm effectively *risk averse* to unexpected shocks to its emission profile.[4] When the firm has a low credit balance, the firm's effective risk aversion is relatively higher. The firm then optimally cuts on production to reduce its consumption of credits and, additionally, increases its investment in abatement and green innovation. Conversely, when the firm has a large carbon credit balance, the firm's effective risk aversion is relatively lower. Thus, the firm increases production—and, thus, its emissions—and reduces its engagement in green investment. The model then reveals that having a large balance of carbon credits actually *reduces* the firm's commitment to curb emissions. Consistent with De Jonghe, Mulier, and Schepens (2020), a firm with a

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[3] In different jurisdictions, this scheme has been labeled as "emissions trading scheme" (ETS) or "cap-and-trade." The most established ETS is the European Union's one (EU ETS). Other major ETS are those in China, the UK, Switzerland, and South Korea. In North America, there are subregional schemes in California, Quebec, and in Northeastern United States (so called "Regional Greenhouse Gas Initiative")

[4] This result aligns with previous findings in dynamic inventory models, such as Bolton, Chen, and Wang (2011) and Décamps, Mariotti, Rochet, and Villeneuve (2011).

large stock of carbon credits is not "greener," as it produces (and, thus, pollutes) more and invests less in abatement and green innovation.

A number of important implications follow. Compared to laissez-faire, the carbon trading system is effective at reducing carbon emissions, consistent with the evidence in Fowlie, Holland, and Mansur (2012) or Martin, Muuls, and Wagner (2016). This happens through a twofold channel: (1) The firm produces less compared to laissez-faire, which naturally curbs its gross emissions, and (2) the firm invests in abatement, to clean up at least part of its emissions. That is, when the ability to emit is contingent on carbon credits, the optimal production decision is *forward-looking*: The firm optimally internalizes that production generates emissions that, by eroding the stock of carbon credits, affect firm value. Whereas advocates of the carbon trading system typically emphasize that the ability to buy credits leaves production uncapped—as it allows firms to carry on producing beyond their maximum emission allowance by buying credits on the market—we show instead that the firm under-produces in this system compared to the laissez-faire benchmark.[5]

The most notable result of our analysis is yet that the carbon trading system affects the firm's optimal mix of green investment. As the firm's credit balance decreases, the firm puts more emphasis on abatement projects—which, while having a transient effect, immediately reduce the firm's carbon footprint by offsetting emissions—and less emphasis on green innovation—which, while having a long-lasting impact and a greater upside, have a longer gestation period and an uncertain outcome. Hence, because abatement is more effective than innovation in reducing net emissions in the short-term, the firm substitutes innovation with abatement when its credit balance is low, to avoid having to resort to the costly carbon credit market. A similar pattern arises if the purchasing price of carbon credits is high: The firm shifts its focus from green innovation to abatement to more immediately offset its emissions.

More generally, our analysis reveals that the carbon trading system does not reduce firm value *unconditionally*, in spite of the long-standing perceived conflict between the interests

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[5] In a counterfactual setup in which the firm needs to hold credits to emit but cannot trade them, production attains its laissez-faire scale if the firm's carbon credit balance is sufficiently large.

of environmental regulators and those of businesses. This prediction is consistent with recent evidence showing the heterogeneous effects on firms of climate regulation (among others, see Martin, de Preux, and Wagner, 2014; Bolton, Lam, and Muuls, 2023) or climate considerations (Sautner, van Lent, Vilkov, and Zhang, 2023). For firms with positive net emissions—i.e., net polluters, or firms that choose not to fully offset their emissions via abatement projects—the carbon trading system is largely a cost, which reduces firm value and investment in green innovation compared to laissez-faire. In turn, for firms with negative net emissions—i.e., net cleansers, or firms that optimally more than offset their emissions via abatement—the carbon trading system yields gains from selling credits in expectation. For these firms, the carbon trading system can raise firm value as well as the engagement in innovation compared to laissez-faire. We further show that, all else equal, a firm can dynamically be a net polluter or cleanser depending on its credit balance. While being a polluter when its credit balance is sufficiently large, a firm may become a net cleanser if its credit balance is low, to avoid resorting to the costly credit market.

We next analyze optimal carbon management under a carbon tax system. Similar to the carbon trading system, we find that the carbon tax system leads the firm to optimally reduce its net emissions relative to laissez-faire, to limit its tax liability. In particular, the firm both reduces its scale of production relative to laissez-faire and engages in abatement projects.[6] Also, a greater carbon tax incentivizes firms to invest more on abatement projects—which indeed can decrease firm's net emissions though have a transient impact—than on long-term, green innovation. Under reasonable parameterization, we show that this effect is stronger under the carbon tax than under the carbon trading system.

Considering jointly our findings on the carbon trading and the carbon tax systems, our analysis thus sheds light on a previously unexplored effect of carbon pricing. To minimize the cost of carbon emissions, firms put more emphasis on short-term, transient measures to combat pollution rather than on long-term green innovation. Yet, as emphasized e.g. by Acemoglu, Aghion, Barrage, and Hemous (2019) and Aghion, Boneva, Breckenfelder,

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[6] Unlike the carbon trading system, however, the firm does not develop risk aversion to unexpected shocks to its emission profile.

Laeven, Olovsson, Popov, and Rancoita (2022), the reduction in carbon emissions necessary to limit global warming requires the application of innovative technologies.[7] We then investigate whether the provision of ex-ante subsidies to green innovation can milden this effect. We find that subsidies indeed boost a firm's investment in green innovation, and slightly decrease their engagement in abatement. Overall, because the increase in green innovation is more sizable than the decline in abatement, subsidies effectively lead to an increase in corporate green investment.

Lastly, we investigate the asset pricing implications of our theory. We study whether the efforts (or lack thereof) that firms put into reducing their carbon footprint have a material impact on their risk premia. Crucially, we are able to reproduce the positive relation between firm's emissions and risk premia observed empirically, as reported by Bolton and Kacperczyk (2021). In our model, production is not only associated with greater carbon emissions, but also with greater exposure to systematic risk. Risk premia are also increasing in the firm's emission *intensity*, in support of the evidence in Hsu, Li, and Tsou (2022). Polluting firms command a higher risk premium when subject to carbon regulation relative to the laissez-faire benchmark, consistent with the evidence in Meng (2017). Nonetheless, and aligned with the prediction in our model that carbon regulation does not necessarily reduce firm value, we also show that firms with sufficiently negative net emissions exhibit *lower* risk premia compared to the laissez-faire benchmark.

Related literature Our model relates to the growing climate finance literature. In particular, our model relates to theoretical studies investigating how firms can be incentivized to internalize the social cost of emissions. Pioneering this literature, Heinkel, Kraus, and Zechner (2001) assess how exclusionary ethical investing affects corporate behavior and firm's cost of capital. Broccardo, Hart, and Zingales (2021) and Oehmke and Opp (2021) study the conditions under which socially-responsible investors affect firm behavior. Landier

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[7] In the context of the shale gas revolution in the United States, Acemoglu et al. (2019) show that providing a cheaper way to firms to limit their carbon emissions displaces green innovation and, thus, can trap the economy to continue using fossil fuels by postponing the switch towards green innovation.

and Lovo (2020) study how socially-responsible funds can induce firms to reduce their toxic emissions. Hong, Wang, and Yang (2021) study a model featuring the gradual accumulation of decarbonization capital by firms that, while nonproductive, reduces their cost of capital. Gans (2012) questions the ability of carbon regulation to effectively spur green innovation. Heider and Inderst (2022) introduce emission externalities and industry equilibrium into a Holmstrom-Tirole like model to examine environmental policies with costly external financing, whereas Ramadorai and Zeni (2022) study the impact of firms' beliefs on emission abatement. Our contribution is to study optimal carbon management policies by taking a comprehensive look at the tools that firms can undertake to curb emissions and transition to a cleaner economy, also investigating the ensuing asset pricing implications.

Our paper then provides new testable implications and theoretical grounds to the growing empirical literature studying the relation between corporate finance and pollution. Consistent with our predictions, Bushnell, Chong, and Mansur (2013) and Martin, de Preux, and Wagner (2014) show that carbon regulation in the European Union has an ambiguous effect on firm performance. Looking at different carbon trading systems, Fowlie, Holland, and Mansur (2012) and Martin, Muuls, and Wagner (2016) conclude that they are effective at curbing emissions—while rationalizing this finding, we analyze the channels through which this happens. Derrien, Krueger, Landier, and Yao (2021) show that negative environmental, social, and governance news trigger significant downgrades in earnings forecasts at all horizons, suggesting that consumers value sustainability, a feature that we feed into our model. Last, Bolton, Kacperczyk, and Wiedemann (2023) illustrate that more polluting firms are less engaged in green innovation, a feature that our model can reproduce.

Methodologically, our paper relates to the dynamic inventory models in corporate finance. Previous contributions in this strand have focused on dynamic cash management, spurred by the increase in corporate cash holdings since the Eighties. Notable contributions include Bolton, Chen, and Wang (2011, 2013), Décamps et al. (2011), Hugonnier, Malamud, and Morellec (2015), Décamps, Gryglewicz, Morellec, and Villeneuve (2017), Malamud and Zucchi (2019), and Della Seta, Morellec, and Zucchi (2020). Our paper characterizes instead a novel type of corporate inventory management: The dynamic management of carbon credits, which can be accumulated, bought, and sold. Relatedly, Bustamante and Zucchi (2023b) characterize how carbon emission management interacts with financing constraints.

Lastly, our paper relates to the literature that investigates the asset pricing implications of environmental policies. In this strand, our model yields predictions aligned with the studies by Meng (2017), Bolton and Kacperczyk (2021) and Hsu, Li, and Tsou (2022), showing empirically that "dirtier" firms command a higher risk premium. Other related papers include the empirical study by Chava (2014), who finds that investors demand significantly higher expected returns on stocks excluded by environmental screens, and Bolton and Kacperczyk (2022), who find higher stock returns associated with higher levels and growth rates of carbon emissions across sectors and economies.

The paper is organized as follows. Section 2 describes the model. Section 3 analyzes optimal policies in a laissez-faire benchmark. Section 4 studies optimal policies under the carbon trading scheme. Section 5 analyzes instead policies in a carbon tax system. Section 6 investigates the impact of innovation subsidies within the carbon trading or the carbon tax systems. Section 7 investigates the asset pricing implications of the firm's carbon emission management. Section 8 concludes. Technical developments and proofs are gathered in the Appendix.

2 The model

We design a dynamic model for a firm that manages its carbon emissions. We begin by assuming that the firm operates in a carbon trading system—yet, as we illustrate later in the text, our setup is amenable to be adapted to a laissez-faire environment (see Section 3) and to a carbon tax system (see Section 5). Under the carbon trading system, a carbon credit gives firms the right to emit a fixed volume of carbon emissions. The firm then has incentives to accumulate credits, denoted by C_t, to be able to emit and, thus, produce. Because credits can be traded but are costly, the firm has economic incentives to keep its emissions in check.

We define the firm's "greenness" g_t as the degree of technological sustainability of the firm. The firm can increase its greenness by investing in green innovation.

Time is continuous, and the economy admits a constant risk free rate denoted by r.

Production and Carbon Emissions Through its production process, the firm generates carbon emissions. Denoting by Y_t the firm's endogenous scale of production, the ensuing flow of carbon emissions is described by the following dynamics:

$$dE_t = (\nu Y_t - \xi g_t - s_t g_t)dt + \sigma Y_t dW_t. \tag{1}$$

This specification implies that emissions are greater and more volatile if the firm's scale of production is larger.[8] In this equation, W_t is a standard Brownian motion, and σ is a positive constant representing the volatility of the firm's emissions per unit of production. Moreover, ν represents an industry-specific parameter gauging the emission intensity per unit of production. The term ξg_t implies that the higher the firm's greenness—which the firm can improve through green innovation breakthroughs—the lower the firm's net emissions for a given scale of production Y_t. In turn, the endogenous quantity s_t is the firm's engagement in emission abatement (on which we elaborate more below). Importantly, we do not impose any restrictions on the sign of the drift $(\nu Y_t - \xi g_t - s_t g_t)$, meaning that the firm endogenously decides whether to be a polluter or a cleanser. If the drift is positive, the firm is a net polluter, and does not fully offset its emissions. If negative, the firm is a net cleanser, and more than offsets its emissions.

The firm's choice of production affects output prices. In particular, we assume that the firm faces the following inverse demand function:

$$p(Y_t) = a - b\frac{Y_t}{g_t} \quad a > 0\,, b > 0. \tag{2}$$

Notably, the sensitivity of prices to Y_t is scaled by greenness g_t to capture that the greener

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[8] In a similar vein, Hong, Wang, and Yang (2021) assume that emissions are proportional to capital.

the firm is, the greater the amount of product demanded by consumers for a given price level. The assumption is consistent with the growing evidence that consumers reward greener firms (see, e.g., Derrien et al. (2021)).[9] For simplicity, we consider that the firm is a monopolist, and we normalize the cost of production to zero.

Carbon credits Carbon credits are akin to permission slips that allow the firm to release carbon emissions into the atmosphere. Once the firm emits a fixed quantity of carbon emissions (typically, a ton of carbon dioxide or the equivalent in other greenhouse gases), one credit is retired. Accumulating credits has then a benefit, as carbon credits guarantee the firm the ability to carry on producing. At the same time, carbon credits entail a maintenance/storage cost χ, which is proportional to the firm's accumulated credits.[10]

As in the real world, we assume that the credit system is nested into a trading scheme. Namely, if the firm exhausts its credits, it either has to stop emitting pollutants into the atmosphere, or it can buy credits from other firms willing to sell them. In turn, if the firm finds itself with an excess of credits, it can sell them to firms willing to buy them. Consistent with the regulatory trend observed in recent years, we assume that trading is centralized and the firm buys its credits on a carbon credit platform at the price $\gamma > 0$. In turn, the firms can sell credits at the cost $\gamma(1-\psi)$, with $\psi > 0$ representing the compensation of the platform.[11]

The dynamics of the firm's carbon credits then satisfy:

$$dC_t = -dE_t + dP_t - dO_t + dI_t. \tag{3}$$

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[9] Derrien et al. (2021) show that analysts significantly downgrade earnings forecasts on a firm following negative ESG news on such firm. They show that the negative revision of earnings forecasts reflects expectation of lower sales rather than higher future costs. See also Choi, Gao, and Jiang (2020). More broadly, our demand function is consistent with the argument that the marginal utility of households depends on the quality of the environment, already present in Acemoglu et al. (2012).

[10] In several jurisdictions, carbon credits require maintenance because they need to be certified.

[11] Effectively, platforms ease the needs of firms with an excess of credits (sellers) and firms with a shortfall of credits (buyers). The assumption that the fee is charged to sellers is consistent with the functioning of major carbon credit platforms.

The above equation implies that the firm's emission flow (defined in equation (1)) depletes the firm's stock of carbon credits. The process P_t represents the purchased credits, which increase the firm's credit stock. Conversely, O_t is the process representing the credits that the firm sells to other firms, which deplete the firm's credit stock. Finally, the process I_t is the inflow of credits when a firm attains a breakthrough stemming from its investment in green innovation, as we describe in the next paragraph.[12]

Abatement and green innovation The specification in equation (1) implies that, as in the real world, the firm can limit its net carbon emissions by investing in abatement projects. Examples of abatement projects are, for instance, investment in (international) carbon offset projects, carbon capture and storage, or afforestation or reforestation projects. That is, while making it up for the effect of at least part of the firm's carbon emissions in the short-run, these projects do not impact the long-term sustainability of the firm. Denoting the firm's engagement in carbon abatement by s_t, we assume that the cost associated with such projects is given by the quadratic specification $\frac{s_t^2}{2}\theta g_t$, where θ is a positive parameter.[13]

On top of investing in abatement, the firm also invests in green innovation. Our modeling of innovation builds on the endogenous growth literature (see Aghion, Akcigit, and Howitt (2014) for a survey). Namely, if the firm spends the quadratic cost $\frac{z_t^2}{2}\zeta g_t$, then a green breakthrough arrives at Poisson rate ϕz_t. The innovation rate z_t is an endogenous choice of the firm—a greater z_t entails greater cost, but increases the likelihood of attaining a green breakthrough. When a breakthrough happens, the firm's greenness g_t increases by a factor $\lambda > 1$. Furthermore, we assume that if the firm has already had many breakthroughs (i.e., g_t is higher), the cost of innovation increases.

A higher level of greenness g_t brings along long-term benefits to the firm. First, for a given scale of production, the firm exhibits lower net emissions—and, thus, it is less

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[12] This assumption is consistent with the idea that innovative firms are rewarded some credits for free. For instance, in the EU ETS, manufacturing industries receive a share of their emission allowances for free, based on benchmarks that reward most efficient installations in each sector.

[13] We assume a quadratic formulation for the abatement cost, similar to the study by Hong, Wang, and Yang (2021) on the dynamics of decarbonization capital.

polluting—regardless of the firm's engagement in abatement. Second, it leads to an increase in the demand for the firm's product (see equation (2)).[14] On top of these long-term benefits, a green breakthrough also brings along a short-term benefit. Namely, because green innovation effectively improves the sustainability of its technology, the firm is awarded a lumpy amount of credits at no charge that replenishes the firm's credit balance.[15]

Whereas both abatement and green innovation aim at making the firm more sustainable, the key difference between the two types of green investment is the horizon of their impact. Abatement aims at reducing the expected flow of emissions in the present—basically, it cleans up (some of) the firm's gross emissions. Yet, it has a short-lived impact as it does not change the firm's technology. Conversely, green innovation has long-lasting effects as it leads to a permanent increase in the sustainability of the firm's technology, which in turn makes the firm permanently less polluting for a given scale of production.

Optimality The firm maximizes its value by managing production Y_t, its engagement in abatement s_t, its green innovation rate z_t, and its stock of carbon credits C_t—the latter entails choosing the target level of credits (which we denote by C^*) as well as the optimal buy/sell strategy (P_t and O_t).

To better single out the effect of the carbon trading system on firm's choices, we start by investigating the laissez-faire benchmark case in the next section, in which the firm does not hold nor manage carbon credits. We then solve for optimal policies and value in the carbon trading scheme in Section 4.

3 Laissez-faire benchmark

As a benchmark, we start by considering a laissez-faire environment in which the firm does not accumulate carbon credits.[16] In this case, firm value is solely a function of greenness,

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[14] As we show, however, the ensuing increase in production does not lead to higher net emissions.

[15] For simplicity, we assume that upon a breakthrough, the firm can replenish its balance all the way to its endogenous target level, which we define in the model solution.

[16] This may be either because the firm can pollute unboundedly, or because credits are available for free.

i.e. $V^B(g_t)$. Following standard argument, firm value satisfies the following equation:

$$rV^B = \max_{Y^B, z^B, s^B} \ \phi z^B \left(V^B(\lambda g) - V^B(g)\right) + \left(Y^B p(Y^B) - \frac{(z^B)^2}{2} g\zeta - \frac{(s^B)^2}{2} g\theta\right). \tag{4}$$

The left-hand side of this equation is the return required by the investors. The right-hand side is the change in firm value on each time interval. In particular, the first term is the effect of a green breakthrough. The last term is the expected net cash flow to shareholders on each time interval. Notably, the firm does not internalize the impact of its emissions—i.e., the firm's emission rate in equation (1) does not impact firm dynamics.

To obtain the firm's optimal policies and value, we conjecture and verify that firm value scales with greenness, i.e., $V^B(g_t) = g_t v^B$, where we denote by v^B the scaled firm value in this benchmark. We also denote by $y^B = Y_t^B / g_t$ the scaled production quantity. Substituting into equation (4) and maximizing with respect to y gives the optimal production quantity:

$$y^B = \frac{a}{2b}. \tag{5}$$

This expression implies that, in the laissez-faire benchmark, the optimal production rate is independent of the firm's emissions. Moreover, the firm has no incentives to invest in abatement—as illustrated by equation (4), abatement projects entail a cost but do not bring any upside in the laissez-faire environment. So, the optimal abatement strategy is zero in this case ($s^B = 0$). In turn, the firm invests in green innovation, and its optimal innovation rate is:

$$z^B = \frac{\phi}{\zeta} v^B \left(\lambda - 1\right). \tag{6}$$

This expression illustrates that the firm invests more in green innovation if the Poisson coefficient ϕ (affecting the likelihood of a breakthrough) is higher, if the surplus upon a breakthrough λ is greater, or if the cost of innovation ζ is smaller. Notably, the only reason why the firm innovates in the laissez-faire benchmark is that a higher level of greenness increases the firm's demand for its product (see equation (2)).

Plugging equations (5) and (6) back into the (scaled) HJB equation gives firm value:

$$v^B = \zeta \frac{r - \sqrt{r^2 - \frac{a^2}{b}\frac{\phi^2}{2\zeta}\left(\lambda - 1\right)^2}}{\phi^2\left(\lambda - 1\right)^2}. \tag{7}$$

This expression shows that firm value does not depend on the expected flow nor on the volatility of carbon emissions. As we show next, this is not the case when firms need to accumulate carbon credits to have the right to emit pollutants.

4 Model solution

We now analyze the environment in which the firm accumulates and trades carbon credits. Given that credits are costly, the firm does not buy them until its credit stock is depleted. In turn, the firm does not sell credits as long as the marginal value of credits inside the firm is greater than the marginal gain from selling them. As a result, we conjecture that there is a region $[0, C^*]$ in which the firm accumulates credits and does not trade them. At $C = 0$, the firm buys credits. Conversely, at the target cash level C^*, the firm sells credits.

In the region $[0, C^*]$, the dynamics of firm value satisfy the following Hamilton-Jacobi-Bellman (HJB) equation:

$$\begin{aligned} rV(C,g) = \max_{z,Y,s} \quad & -(Y\nu - \xi g - sg)V_C + \frac{\sigma^2}{2}Y^2 V_{CC} + \phi z\left(V(C^*, \lambda g) - V(C,g)\right) \\ & + \left(Yp(Y) - \frac{z^2}{2}g\zeta - \frac{s^2}{2}g\theta - \chi C\right). \end{aligned} \tag{8}$$

The left-hand side of this equation is the return required by the investors. The right-hand side is the expected change in firm value on each time interval. The first term is the effect of a change in the credit balance on firm value. If $Y\nu - \xi g - sg > 0$, the firm depletes its credit balance in expectation, and faster if it produces more or if it invests less in abatement. Namely, the the more the firm's investment in abatement or the greater the firm's greenness, the lower the firm's expected net emissions. The second term on the right hand side is the effect of carbon emission volatility on firm value. The third term is the effect of a green innovation breakthrough, in which case the firm's greenness as well as its credit balance increase. The last term is the expected net cash flow to shareholders on each time interval. Differently from equation (4), equation (8) illustrates that, under the carbon trading scheme, the firm's emissions impact its dynamics.

Similar to Section 3, we define quantities scaled by greenness. Namely, we define

$$V(C,g) = g\,v\left(\frac{C}{g}\right) \equiv gv(c) \tag{9}$$

where $v(c)$ denotes firm value scaled by greenness and c denotes scaled credits so that $C = gc$. The scaled target level of credits is denoted by $c^* = C^*/g$. We also define scaled production by $y = Y/g$. Substituting into equation (8) gives the scaled HJB (see equation (22) in Appendix A.2), which we differentiate with respect to y to get the optimal production quantity:

$$y(c) = \frac{a - \nu v'(c)}{2b - \sigma^2 v''(c)}. \tag{10}$$

Comparing this expression with the optimal production quantity in the laissez-faire benchmark (i.e., equation (5)) reveals that, under the carbon trading scheme, production becomes a function of the firm's expected emission intensity and volatility (i.e., the parameters ν and σ). That is, the carbon trading system makes the firm internalize its emissions by giving them financial value. Equation (10) implies that the greater the marginal value of a carbon credit (v') or the greater the firm's emission intensity (ν), the lower the firm's production.

Consider next the optimal abatement policy, obtained by differentiating the scaled HJB equation with respect to s:

$$s(c) = \frac{v'(c)}{\theta}. \tag{11}$$

This expression implies that the greater the marginal value of carbon credits, the more the firm will engage in abatement to reduce its net emissions. Compared to the laissez-faire benchmark, the firm *does* invest in abatement because the firm needs credits to produce, and credits are costly. Thus, to slow down the depletion of carbon credits due to production, the firm invests in abatement.

Finally, maximizing the scaled HJB equation with respect to z gives the optimal green innovation rate:

$$z(c) = \frac{\phi}{\zeta}(\lambda v(c^*) - v(c)). \tag{12}$$

This equation shows that the firm invests more in innovation if the surplus associated with a breakthrough $(\lambda v(c^*) - v(c))$ is higher, if the cost of green innovation ζ is smaller, or if the likelihood of a green breakthrough is greater thanks to a higher Poisson coefficient ϕ. On top of the incentives at play in the laissez-faire case, the firm operating in the carbon trading system invests in innovation to additionally increase the effectiveness of its abatement strategies and to replenish its credit balance.

Substituting equations (10), (11), (12) into the HJB equation gives an ordinary differential equation (ODE), which we solve subject to the following boundary conditions. First, when the firm runs out of credits at $c = 0$, it resorts to the credit market. Recall that buying credits entails a proportional cost γ. Thus, the following boundary condition holds:

$$v'(0) = \gamma, \tag{13}$$

which implies that, when the firm buys credits, the marginal value of credits inside the firm equals their marginal cost.[17] Next, the firm sells credits exceeding the endogenous target threshold c^*. Hence, the following boundary condition holds at c^*:

$$v'(c^*) = \gamma(1 - \psi). \tag{14}$$

That is, at c^*, the marginal value of credits inside the firm is equal to the gain associated

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[17] Realistically, we assume that buying credits does not put at stake the viability of the firm—i.e., it does not push the firm below its liquidation value. I.e., the following inequality: $v(0) > \ell$ holds, where we denote by ℓ the firm's liquidation value.

with selling them.[18] The threshold c^* is pinned down by the super-contact condition:

$$v''(c^*) = 0, \tag{15}$$

which guarantees that the threshold is optimally chosen (Dumas, 1991).

4.1 Model analysis

In this section, we investigate the firm's value and optimal policies analytically. We start by investigating firm value as a function of the stock of carbon credits c.

Proposition 1 *Under the carbon trading scheme, firm value is increasing and concave in its stock of carbon credits c.*

Proposition 1 shows that, under the carbon trading scheme, the firm is effectively risk averse. Because carbon credits are costly, the firm optimally engages in the precautionary accumulation of carbon credits to ensure continuity of production and, simultaneously, avoid having to resort to the costly carbon credit market too often. Hence, keeping a non-negative carbon credit balance acts as an operating constraint that enables the firm to produce.[19] As we show in the next proposition, the stock of accumulated credits shapes the firm's optimal decisions.

Proposition 2 *Under the carbon trading scheme:*

(1) Production $y(c)$ increases with the stock of credits c, and so do revenues $y(c)\,(a - by(c))$;

(2) Abatement $s(c)$ and green innovation $z(c)$ decrease with the stock of credits c;

(3) The firm's expected net emissions $(\nu y(c) - \xi - s(c))$ increase with the stock of credits c.

Proposition 2 shows that the larger the firm's stock of carbon credits, the greater its production rate. That is, when the stock of credits is low, the firm decreases production so to

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[18] For any $c > c^*$, firm value is linear, as the firm sells all the credits exceeding such threshold.

[19] Despite our stock variable is carbon credits (rather than cash), this result is similar to the finding of previous inventory models (see Bolton, Chen, and Wang, 2011; Décamps et al., 2011; Hugonnier, Malamud, and Morellec, 2015), in which financially constrained firms find it optimal to accumulate cash to buffer cash flow shocks and mitigate exposure to costly or uncertain external funding.

curtail its emissions and reduce its consumption of carbon credits, in order to avoid having to resort to the costly carbon credit market. By contrast, the firm increases production as its stock of carbon credits is larger. As a result, revenues also increase with c.

Proposition 2 also illustrates that the firm's abatement and innovation policies are complementary to its production choices in keeping emissions in check. Namely, when c is low, the firm actively reduces its emissions not only by scaling down production but also by investing more in abatement and green innovation, to avoid having to resort to the costly carbon credit market. In turn, as c increases, the firm's incentives to invest in abatement or in green innovation decrease. Overall, the proposition suggests that the firm manages production, abatement, and innovation policies to steer its carbon credit balance away from the $c = 0$ boundary, at which point it has to buy costly credits in order to sustain production.

As a direct implication of the monotonicity of s and y with respect to the stock of credits, Proposition 2 additionally highlights that the firm's expected net emissions increase as c increases. That is, a firm with a large stock of credits is not "cleaner" from an environmental perspective. In fact, because a larger stock of credits gives the right to the firm to emit more pollutants into the atmosphere, the firm takes advantage of it by increasing production and, hence, polluting more. In other words, having a large carbon credit balance relaxes the firm's commitment to curb pollution. This result is consistent with De Jonghe, Mulier, and Schepens (2020), who document that, within the EU ETS, polluting firms have weaker incentives to become greener if they have more emission allowances.

In addition, results (2) and (3) of Proposition 2 jointly imply that firms exhibiting the higher emissions are less engaged in green innovation—controlling for technological differences across firms, such as their emission intensity ν or the impact of innovation-driven sustainability ξ.[20] This result is consistent with the evidence reported by Bolton, Kacperczyk, and Wiedemann (2023) that, controlling for firm fixed effects, firms with higher emissions are less engaged in green innovation.

Next, we compare policies and firm value in the carbon trading system and in the laissez-

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[20] We elaborate more on the impact of these aspects on innovation later in this section. See Figure 4 and Figure 5.

faire benchmark. This comparison allows us to understand how the carbon trading scheme affects the firm's incentives to curb emissions.

Proposition 3 *Compared to the laissez-faire benchmark, the carbon trading scheme:*
(1) Leads the firm to produce below the laissez-faire level ($y(c) < y^B$) *and to earn lower revenues for any* $c \leq c^*$*;*
(2) Incentivizes the firm to invest in abatement;
(3) Has an ambiguous effect on the optimal innovation rate—i.e., $z(c)$ *can be both higher or smaller than* z^B*—but a positive effect on the firm's R&D ratio, i.e.,* $z(c)/v(c) > z^B/v^B$*;*
(4) Has an ambiguous effect on valuations, i.e., $v(c)$ *can be both lower or higher than* v^B*.*

Recall from Proposition 1 that, under the carbon trading scheme, the firm is effectively risk averse for any $c < c^*$ (i.e., $v'' < 0$). Moreover, for any $c < c^*$, the marginal value of credits inside the firm is always greater than the marginal gain associated with selling them (i.e., $v'(c) > \gamma(1 - \psi)$). As a result, equation (10) implies that the firm's scale of production always falls below that associated with the laissez-faire benchmark, which is consistent with the empirical evidence in Martin, Muuls, and Wagner (2016). To avoid having to resort to buying costly credits, the firm reduces its "consumption" of credits by reducing its production and, thus, its emissions. Hence, while the proponents of the carbon trading system typically emphasize that the ability to buy credits allows the firm to carry on producing even when it reaches its maximum emission allowance (i.e., when its credit balance is depleted), our model shows that this system gives rise to a precautionary behavior: The firm always produces below its laissez-faire benchmark y^B for any level c of its credit balance.

Proposition 3 also shows that, in contrast with the laissez-faire benchmark, the firm *does* invest in abatement under the carbon trading system. Indeed, the firm seeks to reduce its net emissions and, hence, its need to resort to the costly carbon credit market. Moreover, whereas the firm's optimal investment in innovation can be either greater or smaller than in the laissez-faire benchmark, the firm's optimal R&D ratio ($z(c)/v(c)$) is always greater in the credit scheme. The next proposition pins down a condition for the innovation rate to be greater in the credit system than in the laissez-faire benchmark.

Proposition 4 *The rate of green innovation is greater under the carbon trading system than in the laissez-faire benchmark if* $v(c^*) > v^B$*—i.e., firm value in the carbon trading system exceeds its counterpart in the laissez-faire benchmark. This can hold if the firm's expected net emissions under the carbon trading system are sufficiently negative.*

As summarized by the result (4) in Proposition 3, the credit system can represent both a gain and a cost for firms: Firm value can be higher or lower than in the laissez-faire benchmark. This prediction is consistent with recent evidence showing that the effects of climate considerations and regulations on firms are heterogeneous, see e.g. Bolton, Lam, and Muuls (2023), Martin, de Preux, and Wagner (2014), or Sautner et al. (2023). In particular, for a firm whose net emissions are sufficiently negative—meaning that the firm is a net cleanser—the carbon trading system represents an opportunity for profits. In expectation, such firm accumulates (as opposed to consumes) credits, which the firm will eventually sell when the credit balance reaches its target level c^*. The present value of such gain increases firm value beyond its laissez-faire benchmark, which also inflates the surplus from innovation and, hence, the firm's optimal innovation rate.

The next result is a straightforward implication of Proposition 3.

Corollary 5 *The firm is expected to pollute less under the carbon trading system than under the laissez-faire benchmark, i.e.,*

$$y\nu - \xi - s < y^B\nu - \xi - s^B.$$

As shown in Proposition 3, the firm exhibits a higher scale of production in the laissez-faire environment and, at the same time, it does not invest in abatement. Therefore, the firm pollutes more in expectation in such environment than under the carbon trading system. We conclude that the carbon trading system is indeed effective at curbing emissions, as firms internalize the environmental impact of their production activity. The reduction in emissions triggered by carbon trading systems has been empirically documented, e.g., by Fowlie, Holland, and Mansur (2012) or Martin, Muuls, and Wagner (2016).

4.1.1 Numerical implementation

Calibration Table 1 reports our baseline parameterization. We assume that the risk free rate is equal to 0.02. We normalize ϕ to 1 as in other innovation models—hence, z effectively represents the arrival rate of a green breakthrough.[21] We assume that λ is equal to 1.065, which is consistent with the innovation step size estimated by Acemoglu et al. (2016). We assume that the cost ζ is four times larger than α, meaning that innovation is much costlier than abatement. We assume that the emission intensity ν is equal to 0.3, so to match the average emissions around the world per purchasing power parity of GDP as reported by the World Bank. Moreover, we assume that the volatility coefficient σ is 0.3, which implies that the volatility of emissions is about 15%. We assume that the price of carbon credits γ is equal to 0.3, the maintenance cost χ is 0.025, whereas the transaction fee ψ is 15%. Specifically, we set the price of carbon credits γ so that our baseline firm expects to exhibit a positive emission flow.[22] The value for ψ is consistent with the information on trading fees reported by carbon exchanges.[23] We set ξ to 0.001, a conservative baseline, but we also explore higher values later in our comparative statics. We set the parameters of the inverse demand function to $a = 0.5$ and $b = 0.4$, which give a markup of around 30% at c^* (consistent with Hall, 2018) and a return on assets around 11% (consistent with that of dividend-paying firms in Farre-Mensa and Ljungqvist, 2016).

The dynamics of optimal choices and outcomes Figure 1 shows the firm's optimal choices in the carbon trading system compared to the laissez-faire benchmark. Confirming

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[21] See, for instance, Akcigit, Hanley, and Serrano-Velarde (2021), Akcigit and Kerr (2018), or Bustamante and Zucchi (2023a).

[22] The price of carbon varies widely within and across jurisdictions. As a result, we assess the robustness of our results by extensively analyzing comparative statics with respect to γ.

[23] The fees reported by exchanges live in a broad range, up to 20%. In general, smaller exchanges charge larger fees. Moreover, tax treatment of the proceeds from selling credits effectively increases the value of ψ.

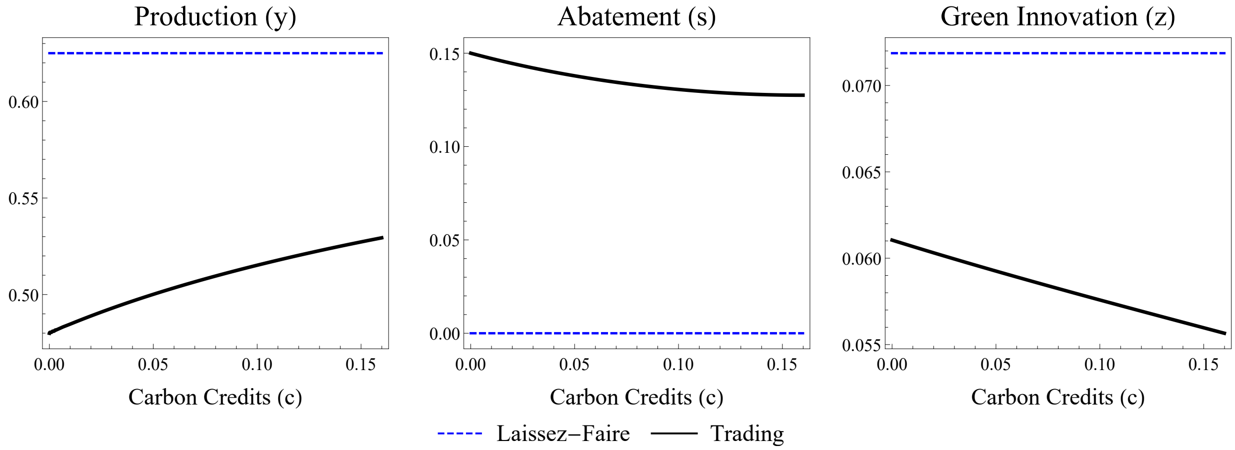

Figure 1: Optimal choices. The figure shows the firm's optimal production $y(c)$, abatement $s(c)$, and innovation $z(c)$ policies as a function of the firm's credit balance c. The solid black line represents the carbon credit system, whereas the dashed blue line represents the laissez-faire benchmark.

our result in Proposition 3, the left panel shows that production is consistently lower under the carbon trading system than under the laissez-faire benchmark. Moreover, as shown in the middle panel, the firm engages in abatement under the carbon trading system—and more so when c is smaller—whereas it does not in the laissez-faire benchmark. Moving to innovation, the right panel shows that, under our baseline parameterization, the firm invests more in green innovation in the laissez-faire benchmark.[24] At the same time, Proposition 3 shows that the innovation ratio $(z(c)/v(c))$ is always higher under the carbon trading system. That is, the firm has fewer resources to spare on innovation under the carbon trading system because it has lower revenues (as shown in Proposition 3) and because it invests in abatement. While this leads to a lower innovation rate in absolute terms, the innovation ratio is actually greater under the carbon trading system.

The left panel of Figure 2 shows the ratio $z(c)/(s(c)+z(c))$, which denotes the weight of innovation (as opposed to abatement) in the firm's engagement in combating pollution—in the following, we denote this ratio as the "innovation share." The innovation share is strictly

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[24] As we explain in Proposition 4 and numerically show below, this result can flip under parameterizations in which the firm makes sufficiently negative net emissions in expectation.

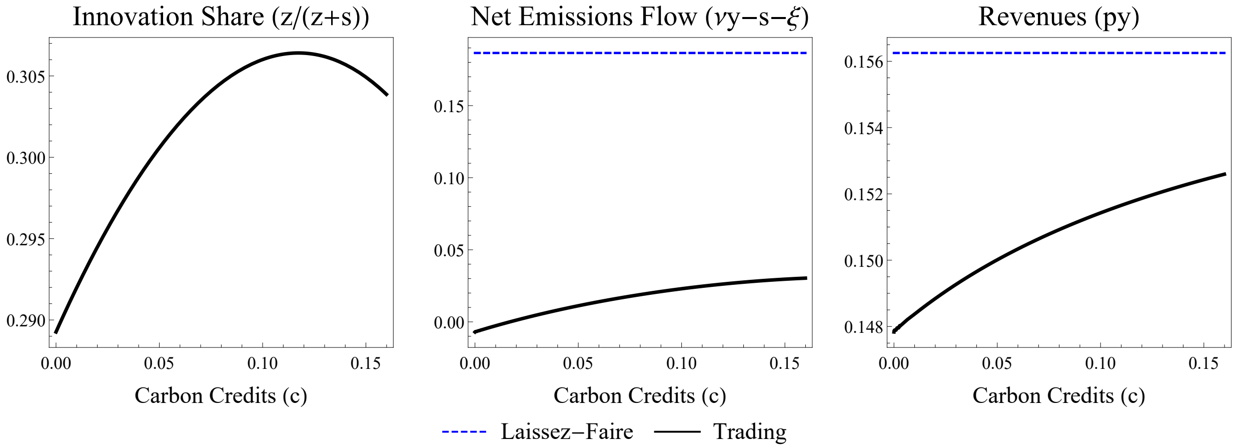

Figure 2: Innovation share, net emissions flow, and revenues. The figure shows the firm's innovation share, the expected net emission flow, and revenues as a function of the firm's credit balance c. The solid black line represents the carbon credit scheme, whereas the dashed blue line represents the laissez-faire benchmark.

lower than one and, thus, strictly below the innovation share in the laissez-faire benchmark.[25] Furthermore, under our baseline parameterization, the innovation share is non-monotonic in c. Namely, when c approaches zero, the innovation share decreases. The reason is that the likelihood that the firm will have to resort to buying credits increases. As a result, the firm puts more emphasis on abatement than on innovation, because abatement more effectively helps the firm decrease the pace of credit depletion. In fact, whereas innovation has a higher upside, it is more uncertain. Thus, when the credit balance shrinks, the firms focuses more on immediate though transient measures (such as abatement) to curb pollution, rather than on long-lasting ones (such as innovation). The plot also shows that, when c approaches its target level, the innovation share decreases, as the surplus from innovation shrinks as c gets closer to its target level.

The remaining charts in Figure 2 further illustrate outcome quantities related to pollution and profitability. Confirming our result in Corollary 5, the figure shows that the expected net emission flow is always greater in the laissez-faire benchmark. That is, by giving financial value to emissions, the carbon trading system makes the firm internalize

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[25] Because the firm does not invest in abatement in the laissez-faire benchmark, then the innovation share is always equal to 1.

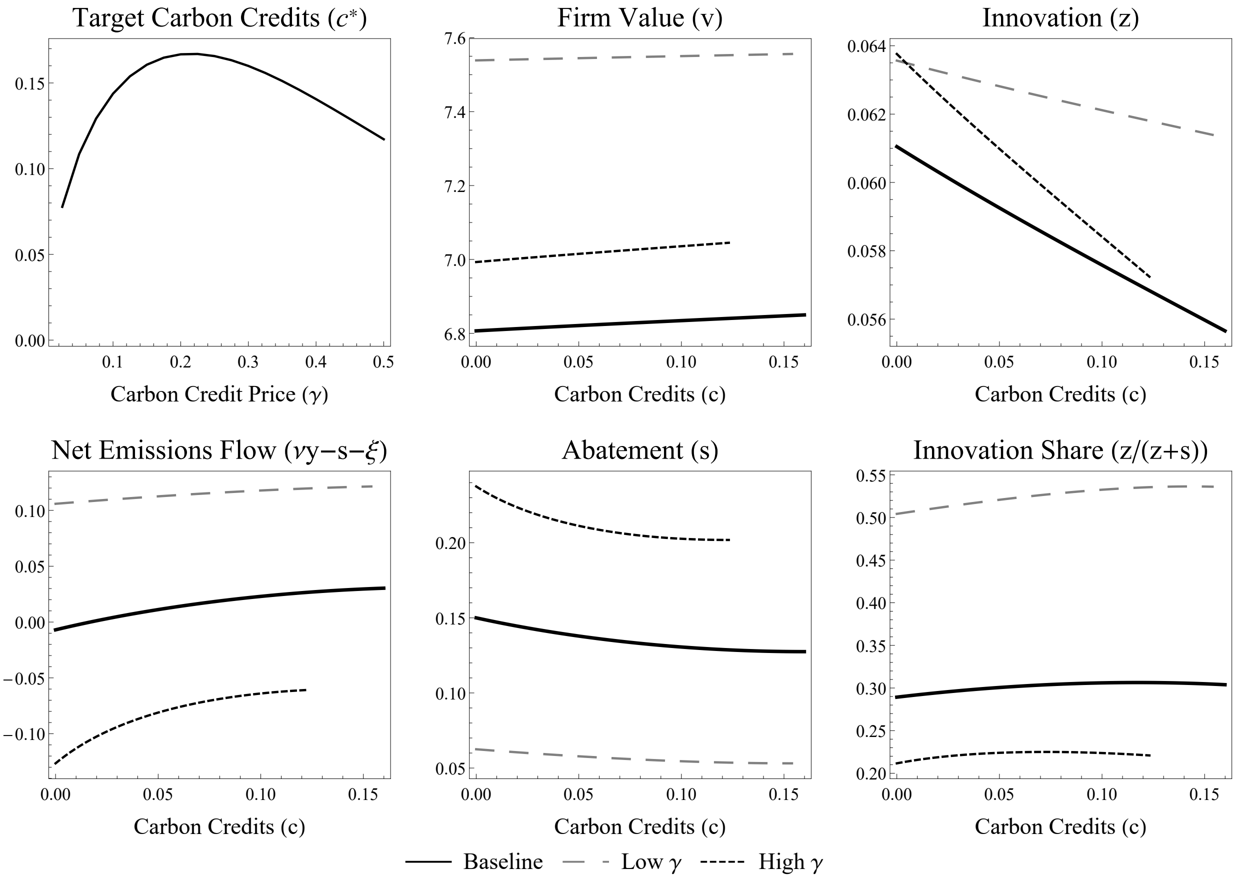

Figure 3: Sensitivity to the cost of carbon credits. The figure shows the firm's target credit balance c^* as a function of γ, as well as firm value, the optimal innovation rate, the net emission flow, the optimal abatement rate, and the innovation share as a function of the stock of carbon credits c for different levels of γ. "Low γ" corresponds to 0.2, whereas "high γ" to 0.45.

(at least partially) its carbon footprint, which leads to a reduction in its flow of net emissions. However, because the firm produces less in the carbon credit scheme, it exhibits lower revenues under the credit system.

The dual role of the carbon credit price Figure 3 investigates the sensitivity of several endogenous quantities to the price of carbon credits, γ. The top panel shows that the target level of credits c^* is hump-shaped in γ. The reason is that γ has a dual role from the firm's perspective—it can represent both a loss or a gain depending on whether the firm is in the market to buy or sell credits. Thus, a greater γ implies that the cost of buying credits is larger—which should lead the firm to keep a larger credit balance—but also implies that the gain from selling credits is greater too—then leading the firm to keep fewer credits.

The dual nature of γ (being both a loss and a gain) also explains why this parameter has a non-monotonic impact on firm value and green innovation, as illustrated in the top middle and right panels of Figure 3. This non-monotonicity is driven by the sign of the expected net emission flow, as shown in the bottom left panel. If the firm exhibits positive net emissions in expectation—i.e., $\nu y(c) - s(c) - \xi > 0$, which is the case when γ is sufficiently low—it expects to be in need of buying credits. As such, an increase in γ has a negative impact on firm value and, thus, on innovation. On the other hand, if the firm exhibits negative net emission—i.e., $\nu y(c) - s(c) - \xi < 0$, which is the case when γ is sufficiently high—it expects to accumulate credits in expectation, making it more likely that it will eventually be able to sell them. Thus, an increase in γ has a positive impact on both firm value and innovation.

By contrast, the bottom middle panel of Figure 3 shows that the firm's abatement policy consistently increases with γ. Recall that abatement increases with the marginal value of carbon credits ($v'(c)$), as per equation (11). Because the marginal value of carbon credits increases with γ, so does the firm's optimal engagement in abatement. Overall, driven by the increasing pattern of s in γ, the innovation share decreases with γ—i.e., the more costly credits are, the more the firm shifts from innovation to abatement as a way to curb pollution. This is a previously unexplored consequence of carbon pricing—i.e., while the carbon trading system effectively leads the firm to reduce its carbon emissions, it does so through short-term measures (such as abatement) that nonetheless cannot help the transition to more sustainable technologies.[26]

Endogenous Carbon Footprint In contrast with models that *assume* that firms exhibit either positive or negative emissions (i.e., Acemoglu et al., 2016), our model endogenizes the firm's emission profile. Notably, a firm can dynamically be a net polluter or cleanser depending on its credit balance. Consistent with our result on firms undertaking precaution-

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[26] See Acemoglu et al. (2019), Aghion et al. (2022), or De Haas and Popov (2023), among others.

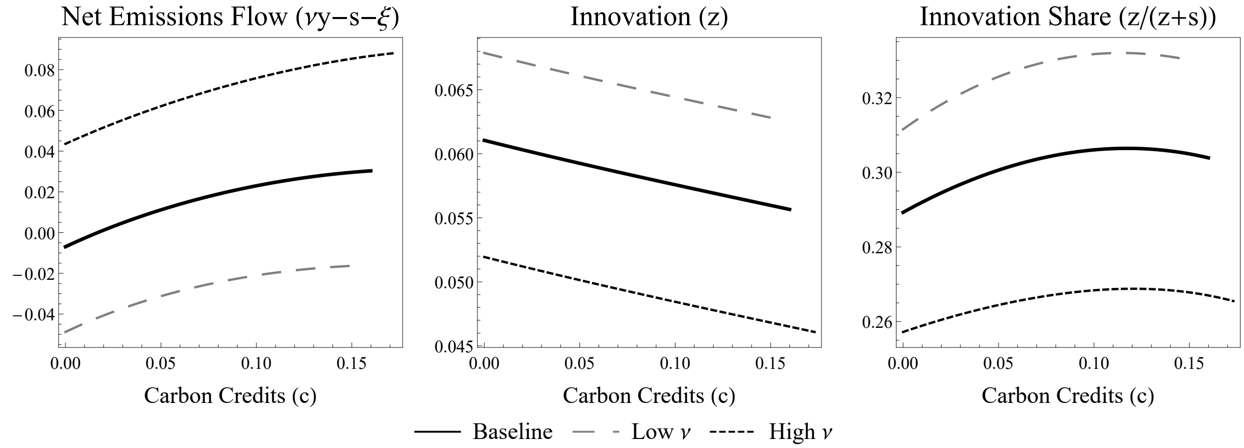

Figure 4: Emission intensity and green investment. The figure shows the net carbon emissions $(\nu y - s)$, innovation $(z(c))$, and the innovation share $(z(c)/(z(c)+s(c)))$ as a function of carbon credits for three different levels of ν. "Low ν" corresponds to 0.2, whereas "high ν" to 0.45.

ary policies when running out of credits, the left panel of Figure 4 shows that, while being a net polluter when its credit balance is sufficiently large, a firm might be a net cleanser if its credit balance is low, in the attempt to avert having to resort to the costly carbon credit market (see the "baseline" case, in which the net emission flow is negative when c is close to zero).[27] The middle and right panels of Figure 4 further illustrate that, all else equal, firms with a higher emission intensity reduce their engagement in innovation. Because these firms intrinsically face a higher expected cost of carbon trading, they reduce their engagement in long-term albeit uncertain measures to combat pollution (such as innovation) and increase their engagement in immediate though transient measures (such as abatement).

The comparative statics with respect to ξ complement those for ν while studying firms' incentives to curb emissions through green innovation. In our model, ν describes the emission intensity of the industry in which the firm operates (e.g., an oil company will structurally have a high ν than a service company), whereas ξ represents how much a firm can improve its sustainability through green innovation. Figure 5 shows that the greater ξ, the more the firm invests in innovation, both in absolute terms and relative to total green investment. In

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[27] The figure also shows that the target level of carbon credits c^* increases with ν—if the firm's emissions per production unit are larger, the firm responds by keeping a larger credit buffer.

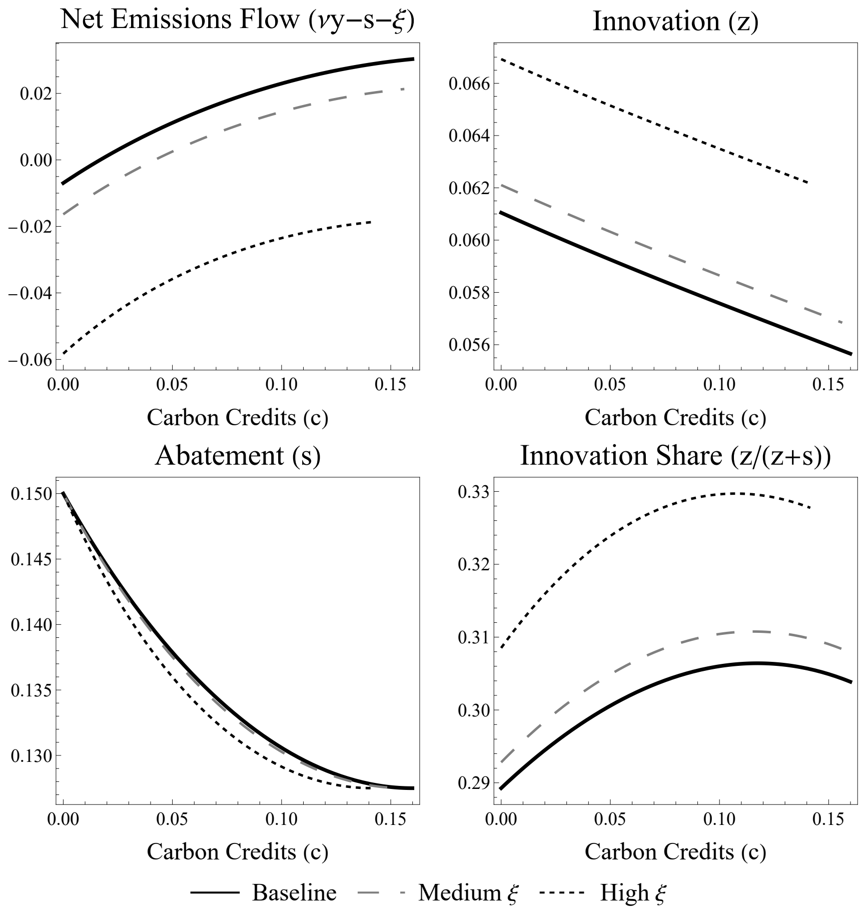

Figure 5: Sustainability and green investment. The figure shows the net carbon emissions, innovation, abatement and innovation share for different levels of parameter ξ, capturing the extent to which green innovation reduces carbon emissions in the long run. "Medium ξ" corresponds to 0.01, whereas "high ξ" we consider 0.05.

turn, abatement slightly decreases as an effect of an increase in ξ: Firms are more willing to tap green innovation as opposed to abatement. Through the lens of our model, recent findings in Cohen, Gurun, and Nguyen (2022) suggest that industries with higher ν typically gain more from green innovation, and thus also have higher levels of ξ. Figure 4 and Figure 5 jointly imply, then, that it is not a priori obvious whether industries with higher ν should necessarily exhibit lower levels of green innovation.

4.2 The impact of credit trading

The carbon trading system requires firms to engage in two tasks: The first is the accumulation of carbon credits, and the second is the trading of such credits. In this section, we aim to disentangle the impact of these tasks on firm value by considering an environment in which the firm accumulates credits but cannot buy and sell them. We assume that, if the firm runs out of credits, it has to pay a penalty or fine. We denote this penalty by ω, which is paid on emissions that are not backed by carbon credits. To focus on the sensible case, we assume that $\omega > \gamma$—i.e., being fined is more expensive than buying credits.

In this setting, the dynamics of firm value still satisfy equation (8), and the optimal production, abatement, and innovation decisions are exactly as in the case with no credits. Yet, the resulting ODE (see equation (23)) is solved subject to different boundary conditions. First, at $c = 0$, the following boundary condition holds:

$$v'(0) = \omega \tag{16}$$

as the firm pays the penalty ω when it runs out of credits. Second, the firm stops accumulating credits when the marginal benefit of credits is zero. We denote such level of credits as $\tilde{c}$, at which the following boundary condition holds:

$$v'(\tilde{c}) = 0. \tag{17}$$

The threshold $c = \tilde{c}$ is optimal, and thus identified by the following super-contact condition:

$$v''(\tilde{c}) = 0. \tag{18}$$

We have the following result.

Proposition 6 *If carbon credits cannot be traded:*

(1) The firm attains its optimal scale of production at $\tilde{c}$, *i.e.,* $y(\tilde{c}) = y^B = \frac{a}{2b}$.

(2) Abatement is greater compared to the case in which trading is allowed when the credit

balance is sufficiently close to $c = 0$*, but it decreases to zero as* $c \to \tilde{c}$*.*

Whereas the firm never attains its unconstrained production capacity y^B when it can trade carbon credits (see Proposition 3), Proposition 6 shows that the firm can attain it absent credit trading. In fact, absent trading, the firm has greater incentives to accumulate carbon credits to avoid having to pay the penalty fee. Moreover, the firm does not gain from selling credits, which further boosts the firm's incentives to accumulate them. Accumulating more credits allows the firm to emit (and, thus, produce) more. At $\tilde{c}$, the firm eventually attains the unconstrained production rate y^B.

Proposition 6 also suggests that banning trading has a non-monotonic effect on the firm's engagement in abatement. When the firm cannot purchase credits, it invests more in abatement when the credit balance is low. In other words, the possibility of purchasing credits reduces the firm's commitment to take steps aimed at reducing emissions. At the same time, when the credit balance is sufficiently large, the firm's investment in abatement eventually goes to zero absent trading. Because the firm accumulates a larger credit balance absent trading, it adopts laxer abatement policies when close to the target $\tilde{c}$.

Overall, our analysis shows that banning trading has a non-monotonic impact on the firm's incentives to reduce its carbon footprint—it boosts them when the firm has a small balance of credits, but it hinders them when the firm has a large balance.

5 Optimal policies in a carbon tax system

As an alternative policy tool to the carbon trading scheme, a number of countries around the world have contemplated carbon taxes—i.e., taxes levied on a firm's net emissions—as well as subsidies to firms with negative net emissions. We now investigate how such provisions would affect the firm's optimal policies, including its production as well as its incentives to invest in abatement and green innovation.

We assume that the carbon tax rate is levied on the expected flow of net emissions. If the firm's expected net emission flow is positive, then the firm is effectively charged the carbon tax. If, instead, the firm's expected net emission flow is negative—i.e., the firm more than offsets its emissions through abatement—then, the firm enjoys a subsidy. For simplicity, we assume that the tax and the subsidy have the same magnitude. Under these assumption, we denote firm value by $V^{\tau}(g)$, and denote the carbon tax/subsidy rate by κ. Using standard arguments, the dynamics of firm value satisfy:

$$rV^{\tau}(g) = \max_{Y^{\tau},z^{\tau},s^{\tau}} \ \phi z^{\tau}\left[V^{\tau}(\lambda g) - V^{\tau}(g)\right] + \left[Y^{\tau}p(Y^{\tau}) - \frac{(z^{\tau})^2}{2}g\zeta - \frac{(s^{\tau})^2}{2}g\theta - \kappa(Y^{\tau}\nu - \xi g - s^{\tau}g)\right]. \tag{19}$$

This equation differs from equation (4) in the last term on the right-hand side, which represents the impact of the tax/subsidy. To solve for the firm's optimal policies, we again use the scaling property (see Appendix A.3 for details). Differentiating the resulting equation with respect to z gives the same expression for the innovation rates as in the laissez-faire benchmark analyzed in Section 3. In turn, the optimal production quantity satisfies

$$y^{\tau} = \frac{a - \kappa\nu}{2b}. \tag{20}$$

This expression implies that the larger the carbon tax rate, the lower the firm's production rate, and even more so if the emission intensity ν is larger. Moreover, the firm's optimal abatement rate satisfies

$$s^{\tau} = \frac{\kappa}{\theta}, \tag{21}$$

which implies that the firm invests more in abatement if the carbon tax rate is larger. That is, the carbon tax makes the firm internalize the impact of its emissions on its optimal policies. The next proposition follows.

Proposition 7 *In the presence of the carbon tax:*
(1) The firm never attains its unconstrained production rate $y^B = \frac{a}{2b}$*,*
(2) If $\kappa = \gamma$*, the firm invests more in abatement in the carbon tax system than in the carbon trading system for any* $c > 0$*, whereas they coincide in the two regimes at* $c = 0$*. If* $\kappa > \gamma$*,*

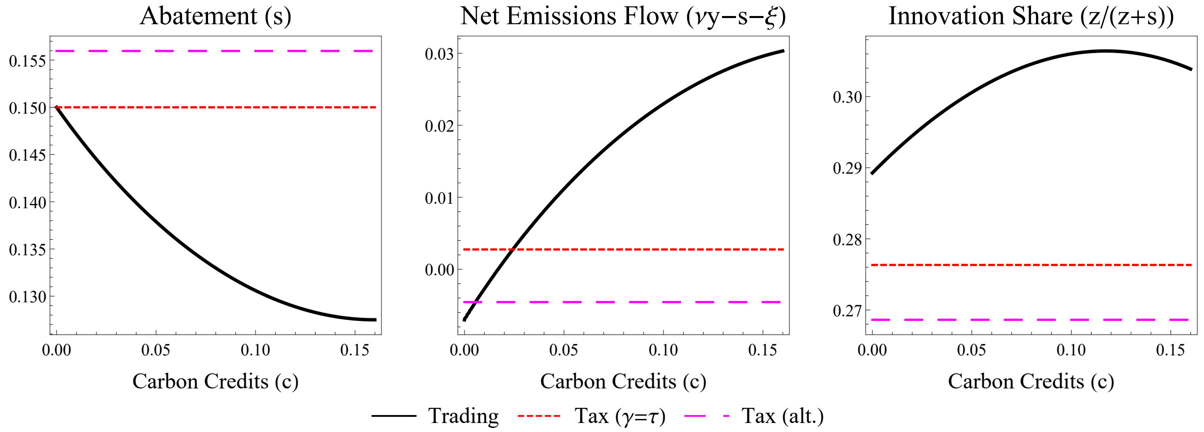

Figure 6: Green investment and emissions under different regulatory schemes. The figure shows the firm's abatement policy, its net emission flow, and the innovation share under the credit trading system (solid black line) and under the carbon tax system (the red dotted line represents the case $\kappa = \gamma$ whereas the magenta dashed line represents the alternative calibration discussed in the text).

the firm invests uniformly more in abatement in the carbon tax system than in the carbon trading system. Conversely, if $\kappa < \gamma(1-\psi)$*, the firm invests uniformly less in abatement than in the carbon trading system.*

Notably, the carbon tax system alone does not trigger the firm to become effectively risk averse. Nonetheless, because the firm's scale of production directly impacts how much the firm is levied, Proposition 7 shows that y^{τ} is consistently below the laissez-faire (unconstrained) level y^{B}. Moreover, whereas the firm's optimal abatement policy is contingent on its credit balance under the carbon trading system, it is constant under the carbon tax system. Proposition 7 also shows that the firm's engagement in abatement can be greater or smaller than under the carbon trading scheme depending on the magnitude of the tax vis-à-vis the cost of carbon credits. Interestingly, if $\kappa = \gamma$—i.e., the tax equals the cost of purchasing carbon credits—the firm invests more in abatement under the tax system than under the trading system for any $c > 0$.

Figure 6 compares the optimal policies under the tax system with those under the carbon trading system. The red dotted lines depict the carbon tax system under the assumption that $\kappa = \gamma$. The left panel shows that, in the carbon tax scheme, the firm invests more in abatement than under the carbon trading scheme but at $c = 0$ to reduce its tax liability, consistent with Proposition 7. In fact, investment in abatement affects its net emissions, which in turn directly impacts its carbon tax liability. The middle panel shows that the firm exhibits higher net emissions in the carbon trading system that in the carbon tax system whenever c is sufficiently high. At the same time, the carbon tax has an ambiguous impact on innovation if $\kappa = \gamma$. The firm invests more in innovation under the trading system if its credit balance is sufficiently low, whereas the relation flips if the credit balance is sufficiently high. Overall, the innovation share $z/(z + s)$ is lower under the tax system, as shown in the right panel. That is, under the tax scheme, the firm favors abatement over innovation even more, as abatement leads to an immediate reduction in the firm's tax liability (whereas, as discussed, innovation takes time and has an uncertain outcome).

To gauge the robustness of our results, we compare the carbon tax versus the carbon trading systems when varying κ. Namely, we set κ so that the firm's scale of production (and, thus, the firm's gross emissions νy) under the tax scheme is equal to those of a cross-section of firms with credits uniformly distributed between 0 and c^* under the credit system.[28] The magenta dashed lines in Figure 6 depict this alternative calibration. The figure confirms our results discussed above. Under the carbon tax, the firm invests uniformly more in abatement and exhibits lower net emissions. Also, the innovation share is lower than under the carbon trading scheme.

6 Subsidies to green innovation

A key result of our analysis is that carbon regulation—being a carbon trading system or a carbon tax system—leads firms to tilt towards more immediate albeit transient policies to reduce their carbon emissions. A natural question then arises as to whether there are complementary tools that regulatory bodies can harness to partly offset this effect and encourage the investment in long-term, green innovation—whose outcome can lead to per-

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[28] The assumption of a uniformly-distributed cross section is also adopted in the cash management model of Hugonnier, Malamud, and Morellec (2015).

sistent improvements in firms' greenness.

In this section, we focus on one such tool: Subsidies to green innovation. To this end, we assume that subsidies cover a portion ι of the firm's cost of innovation, which then decreases to $(1-\iota)\frac{z^2}{2}g_t\zeta$. Therefore, ι represents the magnitude of the subsidy, decreasing the effective cost of innovation. While the expression of the optimal abatement and production policies remain the same in this case, the optimal green innovation policy is

$$z(c) = \frac{\phi}{\zeta(1-\iota)}(\lambda v(c^*, \iota) - v(c, \iota))$$

in the carbon trading system, whereas it is

$$z^\tau = \frac{\phi}{\zeta(1-\iota)} v^\tau(\iota)\,(\lambda - 1)$$

in the carbon tax system. These equations suggest that because innovation subsidies decrease the effective costs of green innovation, the firm's optimal innovation rate should increase. Moreover, the subsidy obviously impacts firm value in both systems, so it should affect innovation expenditures through this channel too.

Yet, an important difference sets the carbon trading system and the carbon tax systems apart. Namely, under the carbon trading system, ex-ante subsidies to innovation affect the marginal value of credits, which in turn affects the marginal benefit from abatement (as illustrated by the numerator of equation (11)). Through this channel, the firm's optimal abatement—and, thus, the firm's net emissions—are also affected by the ex-ante subsidy. Differently, under the carbon tax system, the marginal benefit of abatement is fixed and equal to the tax rate κ. As a result, the subsidy does not bear any implications for the firm's optimal engagement in abatement.

Figure 7 then investigates quantitatively the impact of the ex-ante subsidy under the carbon trading system. The figure shows that the subsidy effectively increases the firm's investment in innovation as well as the innovation share. At the same time, the subsidy slightly affects the firm's abatement policy too—namely, it leads firms to slightly reduce

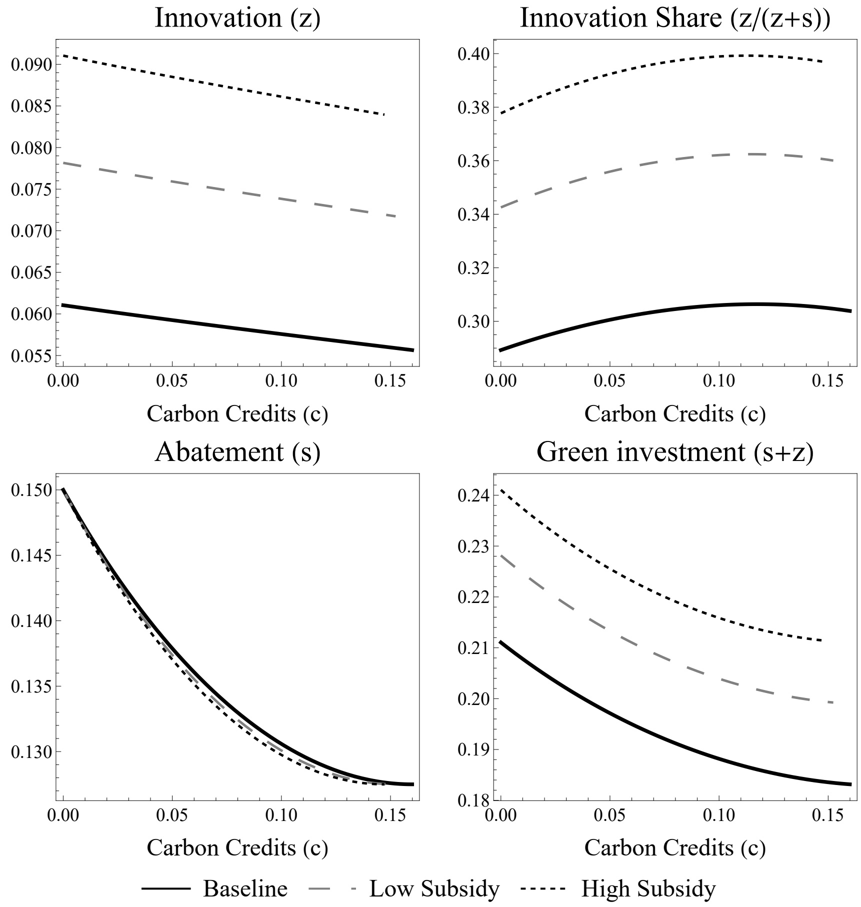

Figure 7: The impact of innovation subsidies. The figure shows the firm's optimal investment in innovation, its innovation share, the optimal abatement policies, and the firm's green expenditures (sum of abatement and innovation) as a function of carbon credits, under the baseline case with no subsidy (solid black line), or in the presence of a either low (dashed line, equal to 20%) or high subsidy to innovation (dotted line, equal to 30%).

their abatement rate. That is, the subsidy effectively leads the firm to shift from abatement to green innovation. Overall, the subsidy spurs the total amount that firms spend on becoming green. That is, the increase in green innovation more than compensates the decline in abatement, as corporate green expenditures (green innovation and abatement) unambiguously increase.

7 Implications for asset prices

We now examine the asset pricing implications of our theory. Namely, following recent empirical works (see, e.g., Bolton and Kacperczyk, 2021; Hsu, Li, and Tsou, 2022), we investigate whether investors indeed demand a premium for more polluting firms in the cross section, by looking at how risk premia relate to firm emissions. As we elaborate in Appendix A.4, we derive the firm's risk premium $\mathcal{R}$ by comparing the HJB equations under the physical and risk-neutral measures, as in Bolton, Chen, and Wang (2013). We assume a single source of systematic risk that is priced, with which the firm's cash flows are imperfectly correlated by a factor $\rho > 0$.

We begin by comparing a firm's risk premium under the carbon trading system, $\mathcal{R}(c)$, vis-à-vis the laissez-faire benchmark, $\mathcal{R}^B$.

Proposition 8 *Under the carbon trading system, the risk premium can be lower than in the laissez-faire benchmark (i.e.,* $\mathcal{R}(c)/\mathcal{R}^B < 1$*) if expected net emissions are negative (i.e., if* $\nu y(c) - s(c) < 0$*).*

Proposition 8 reveals that the carbon trading scheme can either inflate or deflate risk premia, as $\mathcal{R}(c)/\mathcal{R}^B$ can be greater or smaller than one. This result is consistent with our analysis on valuations in Section 4. As shown in Proposition 4, a firm may be more valuable under the carbon trading system than under the laissez-faire benchmark, and this outcome can arise if the firm exhibits net emissions which are sufficiently negative in expectation. Similarly, the carbon trading system has an ambiguous impact on firm's risk premia depending on whether the firm is a net polluter or a net cleanser.

The left and middle panels of Figure 8 illustrate the main insights of the model with respect to risk premia under the carbon trading system compared to the laissez-faire benchmark. Under the baseline parametrization, the left panel shows that the ratio $\mathcal{R}(c)/\mathcal{R}^B$ increases with the firm's credit balance c under the carbon trading scheme, as $\mathcal{R}(c)$ does. Simultaneously, as we have shown in Section 4, the firm exhibits a greater net emission flow when its credit balance is larger. The corresponding prediction is that more polluting firms

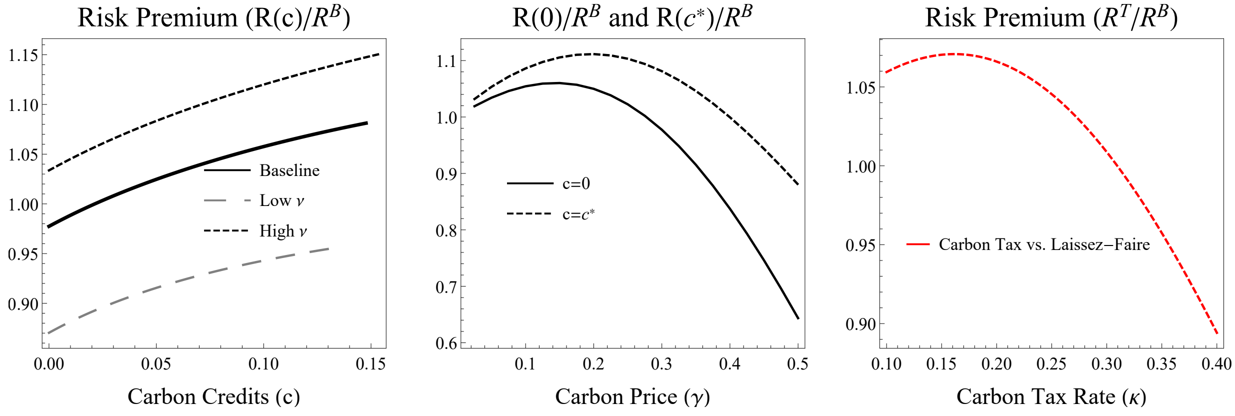

Figure 8: RISK PREMIA. The left panel illustrates the ratio $\mathcal{R}(c)/\mathcal{R}^B$ under the carbon trading system as a function of carbon credits c for different levels of the firm's emission intensity ν. The middle panel illustrates the ratio $\mathcal{R}(c)/\mathcal{R}^B$ under the carbon trading system, evaluated at $c = 0$ and at $c = c^*$, as a function of the price of carbon credits (γ). The right panel illustrates the ratio $\mathcal{R}^T/\mathcal{R}^B$ under the carbon tax system as a function of the carbon tax rate κ.

earn higher expected returns, aligned with the empirical evidence in Bolton and Kacperczyk (2021). In our model, emissions and risk premia are both related to the firm's scale of production: If the firm expands production, not only its carbon emissions rise, but also its exposure to systematic risk increases, resulting in a higher risk premium.

Figure 8 further illustrates the comparative statics of the ratio $\mathcal{R}(c)/\mathcal{R}^B$ with respect to the price of carbon credits (γ) and the emission intensity (ν).[29] The left panel shows that firms with higher emission intensity ν have higher risk premia, which is aligned with the evidence in Hsu, Li, and Tsou (2022). Furthermore, $\mathcal{R}(c)$ is non-monotonic with respect to γ. This result is consistent with valuations also being non-monotonic in γ (see Section 4), as the firm switches from being a net polluter (at lower values of γ) to a net cleanser (with sufficiently high values of γ).

We also explore the asset pricing implications under the carbon tax system. The right panel of Figure 8 illustrates the risk premium under the carbon tax scheme, $\mathcal{R}^\tau$, relative to the laissez-faire benchmark, $\mathcal{R}^B$. Similar to the effect of γ in the carbon trading scheme,

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[29] Keeping in mind that $\mathcal{R}^B$ is independent of these parameters (as emissions no not affect firm dynamics in the laissez-faire benchmark), the figure ultimately reveals the comparative statics of $\mathcal{R}(c)$.

the net emissions of the firm switch sign as the tax rate κ increases. A higher κ prompts the firm to invest more in abatement, which can lead the firm from being a net polluter to be a net cleanser. Hence, the risk premium is non-monotonic in κ.

8 Concluding remarks

The systematic control of firms' carbon emissions by regulators poses a new challenge in the corporate world, which involves maximizing shareholder wealth by developing an optimal carbon management policy. This paper provides a unified model to study, precisely, how firms should optimally manage carbon emissions through production, green investment (both abatement and green innovation), and the management of carbon credits. Our theoretical framework is broad enough to allow a comparison of optimal corporate policies under the three main regulatory schemes observed internationally—the carbon trading system, the carbon tax system, and a laissez-faire benchmark.

The main takeaway of our model is that carbon pricing leads firms to put more emphasis on more immediate albeit transient forms of green investment, such as abatement as opposed to green innovation. We show that subsidies to innovation can partly undo this effect and, overall, boost the firm's engagement to becoming greener. Our model also shows that, under a carbon trading scheme, firms adopt precautionary policies such as under-producing compared to the laissez-faire benchmark. This result challenges the conventional wisdom that the carbon trading scheme preserves a firm's ability to produce (and, thus, pollute) even when it reached its maximum allowance. Perhaps surprisingly, we find that firms with a large stock of carbon credits are less committed to reducing emissions. Last, we conclude that carbon regulation is not necessarily a cost for corporations, as firms can actually attain higher value and profits than under laissez-faire if their net emissions are sufficiently negative.

In sum, our paper delivers new insights that speak to the long- and short-term effects of carbon regulation—a macroeconomic concern—but more importantly to the microeconomic and financial aspect of corporate carbon management. Our analysis can be extended in many ways—which we intend to examine in future research—to further uncover how firms' new need to manage emissions alters stylized predictions on corporate decision making.

A Appendix

A.1 Proof of the results in Section 3

Using the scaling property gives the scaled HJB equation:

$$rv^B = \max_{y^B, z^B, s^B} \quad \phi z^B v^B (\lambda - 1) + \left(y^B \left(a - b y^B \right) - \frac{(z^B)^2}{2}\zeta - \frac{(s^B)^2}{2}\theta \right).$$

Plugging equations (5) and (6) into this equation gives:

$$\frac{\phi^2}{2\zeta} (\lambda - 1)^2 (v^B)^2 - rv^B + \frac{a^2}{4b} = 0$$

which we solve with respect to v^B and obtain equation (7).

A.2 Proof of the results in Section 4

To solve the firm's problem when it accumulates credits, we use the homogeneity property explained in the main text. That is, we have:

$$V_C(C, g) = v'(c) \quad V_{CC}(C, g) = \frac{v''(c)}{g},$$

which we substitute into equation (8) and get the scaled HJB:

$$rv(c) = \max_{z,y,s} \ (s + \xi - y\nu)v' + \frac{\sigma^2}{2} y^2 v'' + \phi z \left(\lambda v(c^*) - v(c) \right) + \left(y(a - by) - \frac{z^2}{2}\zeta - \frac{s^2}{2}\theta - \chi c \right). \tag{22}$$

Differentiating equation (22) gives the optimal policies reported in equations (10), (11), and (12). Plugging these policies into (22) gives the following equation:

$$rv(c) = \frac{\phi^2}{2\zeta} (\lambda v(c^*) - v(c))^2 + \frac{(v'(c))^2}{2\theta} + \frac{(a - \nu v')^2}{2(2b - \sigma^2 v'')} - \chi c + \xi v'. \tag{23}$$

At $c = c^*$, this equation boils down to

$$\frac{\phi^2}{2\zeta}(\lambda-1)^2 v^2(c^*) - rv(c^*) + \frac{\gamma^2(1-\psi)^2}{2\theta} + \frac{(a-\nu\gamma(1-\psi))^2}{4b} - \chi c^* + \xi = 0,$$

which we solve with respect to $v(c^*)$:

$$v(c^*) = \frac{\zeta}{\phi^2(\lambda-1)^2}\left[r - \sqrt{r^2 - 2\frac{\phi^2}{\zeta}(\lambda-1)^2\left[\frac{\gamma^2(1-\psi)^2}{2\theta} + \frac{(a-\nu\gamma(1-\psi))^2}{4b} - \chi c^* + \xi\right]}\right]$$

Next we show Proposition 1.

Proof of Proposition 1 $v''(c) \le 0$ means that v' is monotonically decreasing for any c. Towards a contradiction, suppose that there are two credit levels $0 < c_1 < c_2 < c^*$ such that $v'(c_1) = v'(c_2) = \Gamma > \gamma(1-\psi)$. This means that there is a $\bar{c} \in [c_1, c_2]$ at which v' attains a local maximum—i.e., $v''(\bar{c}) = 0$, $v''(c_1) > 0$, and $v''(c_2) < 0$. At c_1, firm value satisfies:

$$rv(c_1) = \frac{\phi^2}{2\zeta}\left(\lambda v(c^*) - v(c_1)\right)^2 + \frac{(v'(c_1))^2}{2\theta} + \frac{(a-\nu v'(c_1))^2}{2(2b-\sigma^2 v''(c_1))} - \chi c_1 + \xi v'(c_1). \tag{24}$$

whereas at c_2:

$$rv(c_2) = \frac{\phi^2}{2\zeta}\left(\lambda v(c^*) - v(c_2)\right)^2 + \frac{(v'(c_2))^2}{2\theta} + \frac{(a-\nu v'(c_2))^2}{2(2b-\sigma^2 v''(c_2))} - \chi c_2 + \xi v'(c_2). \tag{25}$$

Subtracting (25) from (24) gives:

$$\begin{aligned} r\left[v(c_1) - v(c_2)\right] =& \frac{\phi^2}{2\zeta}\left[\left(\lambda v(c^*) - v(c_1)\right)^2 - \left(\lambda v(c^*) - v(c_2)\right)^2\right] \\ &+ \frac{(a-\nu v'(c_1))^2}{2}\left[\frac{1}{(2b-\sigma^2 v''(c_1))} - \frac{1}{(2b-\sigma^2 v''(c_2))}\right] - \chi(c_1 - c2). \end{aligned} \tag{26}$$

Because v' is positive over the interval $[c_1, c_2]$, then $v(c_1) < v(c_2)$. Therefore, the left-hand side of this equation is negative. In turn, the first term on the right-hand side is positive, because $c_1 < c_2$ and $v(c)$ is increasing in c. Because $v''(c_1) > 0$ and $v''(c_2) < 0$, the second term is positive too. Finally $-\chi(c_1 - c_2) > 0$ too. Thus, equation (26) does not hold. It then means such two levels c_1 and c_2 do not exist. v' then monotonically decreases in c, and the claim holds. ■

We now turn to prove Proposition 2.

Proof of Proposition 2 We start by showing part (1) of the claim. Differentiating the optimal policy gives:

$$y'(c) = \frac{-v''(c)\nu + \sigma^2 y(c) v'''(c)}{2b - \sigma^2 v''} \tag{27}$$

As $v'' \leq 0$ (by Proposition 1) and $y \geq 0$, then to prove the claim we simply need to show that $v''' > 0$ for any c. Differentiating equation (23) gives:

$$rv'(c) = \; -\frac{\phi^2}{\zeta}(\lambda v(c^*) - v(c))v'(c) + \frac{v'(c)v''(c)}{\theta} + \frac{a - \nu v'}{2b - \sigma^2 v''}\left(-\nu v''(c) + \frac{\sigma^2 (a - \nu v')v'''}{2(2b - \sigma^2 v'')}\right) - \chi + \xi v''.$$

At $c = c^*$, the equation becomes:

$$rv'(c^*) = \; -\frac{\phi^2}{\zeta}(\lambda v(c^*) - v(c^*))v'(c^*) + \frac{a - \nu v'(c^*)}{2b}\left(\frac{\sigma^2(a - \nu v'(c^*))v'''(c^*)}{2(2b)}\right) - \chi.$$

Because the left-hand side is positive (recall that $v'(c^*) = \gamma(1-\psi)$) whereas the first and last terms on the right-hand side are negative, $v'''(c^*)$ must be positive for the equation to hold. Toward a contradiction, suppose this is not the case and there is a point c^{**} at which $v'''(c^{**}) = 0$. Hence, there are two levels $c_1 < c^{**} < c_2$ such that $v''(c_1) = v''(c_2)$ (and negative by Lemma 1), and $v'''(c_1) < 0$ and $v'''(c_2) > 0$. Thus, we have at c_1

$$\begin{aligned} rv'(c_1) = &-\frac{\phi^2(\lambda v(c^*) - v(c_1))v'(c_1)}{\zeta} + \frac{v'(c_1)v''(c_1)}{\theta} \\ &+ \frac{a - \nu v'(c_1)}{2b - \sigma^2 v''(c_1)}\left(-\nu v''(c_1) + \frac{\sigma^2(a - \nu v'(c_1))v'''(c_1)}{2(2b - \sigma^2 v''(c_1))}\right) - \chi + \xi v''(c_1) \end{aligned} \tag{28}$$

and at c_2

$$\begin{aligned} rv'(c_2) = & -\frac{\phi^2(\lambda v(c^*) - v(c_2))v'(c_2)}{\zeta} + \frac{v'(c_2)v''(c_1)}{\theta} \\ & + \frac{a - \nu v'(c_2)}{2b - \sigma^2 v''(c_1)} \left(-\nu v''(c_1) + \frac{\sigma^2(a - \nu v'(c_2))v'''(c_2)}{2(2b - \sigma^2 v''(c_1))} \right) - \chi + \xi v''(c_2). \end{aligned} \tag{29}$$

Subtracting (28) from (29) gives

$$\begin{aligned} r(v'(c_2) - v'(c_1)) = & \frac{\phi^2}{\zeta} \left[(\lambda v(c^*) - v(c_1))v'(c_1) - (\lambda v(c^*) - v(c_2))v'(c_2) \right] + \frac{(v'(c_2) - v'(c_1))v''(c_1)}{\theta} \\ & + \frac{\nu^2 v''(c_1)(v'(c_2) - v'(c_1))}{2b - \sigma^2 v''(c_1)} + \sigma^2 \frac{(a - \nu v'(c_2))^2 v'''(c_2) - (a - \nu v'(c_1))^2 v'''(c_1)}{2(2b - \sigma^2 v''(c_1))^2}. \end{aligned}$$

By Proposition 1, the left-hand side is negative, whereas the first, second, and third terms on the right-hand side are positive. Moreover, under the conjecture, $v'''(c_2) > 0$ and $v'''(c_1) < 0$ hold, and the last term is positive too. Hence, the ODE would not hold so it must be that v''' does not switch sign, so $y(c)$ is monotonically increasing. The result about the monotonicity of revenues stems from the monotonicity of $y(c)$. In fact, revenues are $y(c)(a - by(c))$, whose derivative is $(a - 2by(c))$, which is indeed positive.

Part (2) of the claim stems from Proposition (1), as $s'(c) = v''(c)/\theta$. In turn, $z'(c) = -\frac{\phi}{\zeta}v'(c)$–as $v'(c) \geq \gamma(1 - \psi) > 0$ for any c.

Part (3) of the proof stems from the monotonicity of $y(c)$ and $s(c)$. The claim follows. ■

Proof of Proposition 3 Part (1) follows from Proposition 1. I.e., the numerator of $y(c)$ (see equation (10)) is smaller than a, as $v' \geq \gamma(1 - \psi)$. In turn, because $v'' < 0$, the denominator of $y(c)$ is equal or greater than $2b$ as $v'' \leq 0$. As a result, because the quantity $y(a - by)$ is non decreasing in y up to $y = a/(2b)$, then revenues are also uniformly smaller in the trading system vis-à-vis the laissez-faire environment.

Part (2) of the proof simply stems from the fact that the firm does not invest in abatement in the laissez-faire benchmark, whereas $s(c) \geq \frac{\gamma(1-\psi)}{\theta} > 0$ for any c.

Part (3) relies on Part (4), which we prove first. Namely, $v^B > v(c^*)$ if the following inequality holds:

$$\frac{a^2}{2b} > \frac{(a-\nu\gamma(1-\psi))^2}{2b} + \frac{\gamma^2(1-\psi)^2}{\theta} - 2\chi c^* + 2\xi$$

which boils down to

$$\frac{\nu\gamma(1-\psi)\left(\nu\gamma(1-\psi)-2a\right)}{2b} + \frac{\gamma^2(1-\psi)^2}{\theta} - 2\chi c^* + 2\xi < 0.$$

A sufficient condition for the above to hold is that

$$\frac{\nu\gamma(1-\psi)\left(\nu\gamma(1-\psi)-2a\right)}{2b} + \frac{\gamma^2(1-\psi)^2}{\theta} + 2\xi < 0 \Rightarrow a > \frac{\nu\gamma(1-\psi)}{2} + \frac{\gamma(1-\psi)b}{\theta\nu} + \frac{2\xi b}{\nu\gamma(1-\psi)}.$$

If instead, $\chi = 0$ and $a < \frac{\nu\gamma(1-\psi)}{2} + \frac{\gamma(1-\psi)b}{\theta\nu} + \frac{2\xi b}{\nu\gamma(1-\psi)}$, then $v^B < v(c^*)$ holds.

To prove part (3), recall that, at c^*, the optimal innovation rate satisfies:

$$z(c^*) = \frac{\phi}{\zeta}\left(\lambda - 1\right) v(c^*)$$

which is similar to the expression of z^B up to v^B. It then follows that the optimal innovation rate can be either greater or smaller under the carbon trading system vis-à-vis the laissez-faire benchmark. At the same time, the R&D ratio is always greater in the case in which the firm holds credits, as

$$\frac{z(c)}{v(c)} = \frac{\phi}{\zeta}\left(\frac{\lambda v(c^*)}{v(c)} - 1\right) > \frac{\phi}{\zeta}\left(\lambda - 1\right) = \frac{z^B}{v^B} \tag{30}$$

as $\frac{v(c^*)}{v(c)} > 1$ given that firm value increases with c. ■

Proof of Proposition 4 Because $z(c)$ decreases with c, then $z(c) > z^B$ holds for any c if it is true at $c = c^*$. At $c = c^*$, this is guaranteed if $v(c^*) > v^B$. The inequality $v(c^*) > v^B$

boils down to:

$$\frac{\nu^2\gamma(1-\psi)}{2b} - \frac{a\nu}{b} + \frac{\gamma(1-\psi)}{\theta} - 2\chi c^* \frac{1}{\gamma(1-\psi)} + \frac{2\xi}{\gamma(1-\psi)} > 0.$$

and, by calculations, we get

$$a - \gamma(1-\psi)\left[\frac{\nu}{2} + \frac{b}{\theta\nu} - 2\chi c^* \frac{b}{\gamma^2(1-\psi)^2\nu} + \frac{2\xi b}{\gamma^2(1-\psi)^2\nu}\right] < 0. \tag{31}$$

Note that

$$a-\gamma(1-\psi)\left[\frac{\nu}{2} + \frac{b}{\theta\nu} + \frac{2\xi b}{\gamma^2(1-\psi)^2\nu}\right] < a-\gamma(1-\psi)\left[\frac{\nu}{2} + \frac{b}{\theta\nu} + \frac{2\xi b}{\gamma^2(1-\psi)^2\nu} - 2\chi c^* \frac{b}{\gamma^2(1-\psi)^2\nu}\right] \tag{32}$$

Equivalently, this equation implies that $v(c^*) > v^B$ if a is sufficiently low.

Consider now the firm's net emissions. Because they are monotonic increasing in c, if they are negative at c^*, they are negative for all c. Namely, emissions are negative at c^* if the following inequality holds:

$$\nu y(c^*) - s(c^*) - \xi = \frac{a\nu - \nu^2\gamma(1-\psi)}{2b} - \frac{\gamma(1-\psi)}{\theta} - \xi < 0$$

By calculations, this boils down to

$$a - 2\gamma(1-\psi)\left[\frac{\nu}{2} + \frac{b}{\theta\nu} + \frac{b\xi}{\nu\gamma(1-\psi)}\right] < 0$$

That is, if a is sufficiently small, then emissions are negative for all c. Hence, there is a joint interval for a such that emissions are negative and firm value is higher than under laissez-faire. The claim follows ■

A.2.1 Proof of the results in Section 4.2 (banning trading)

Proof of Proposition (6) Part (1) of the claim simply stems from the expression of $y(c)$ (which is exactly as in the setup with trading) and the boundary conditions at $c = \tilde{c}$.

Part (2) of the claim stems from the expression of $s(c)$ (which is exactly as in the setup with trading) and the boundary conditions at $c = 0$ and at $c = c^*$.

■

A.3 The model with the carbon tax

To solve for the firm's optimal policies in the carbon tax system, we again use the scaling property and get:

$$rv^\tau = \max_{y^\tau, z^\tau, s^\tau} \; \phi z^\tau (\lambda - 1) v^\tau + \left(y^\tau (a - by^\tau) - \frac{(z^\tau)^2}{2}\zeta - \frac{(s^\tau)^2}{2}\theta - \kappa(y^\tau \nu - s^\tau - \xi) \right). \quad (33)$$

Plugging the optimal policies into the above equation we have

$$\frac{\phi^2}{2\zeta}(\lambda - 1)^2 (v^\tau)^2 - rv^\tau + \frac{(a - \kappa\nu)^2}{4b} + \frac{\kappa^2}{2\theta} + \kappa\xi = 0.$$

so we solve for firm value

$$v^\tau = \frac{r - \sqrt{r^2 - \frac{\phi^2}{\zeta}(\lambda - 1)^2 \left[\frac{(a-\kappa\nu)^2}{2b} + \frac{\kappa^2}{\theta} + \kappa\xi \right]}}{\frac{\phi^2}{\zeta}(\lambda - 1)^2} \quad (34)$$

Proof of Proposition 7 Part (1) of the proposition is straightforward given that $\kappa > 0$ and $\nu > 0$. Part (2) stems from the fact that, in the carbon trading system, $s(c) \in \left[\frac{\gamma(1-\psi)}{\theta}, \frac{\gamma}{\theta}\right]$. ■

A.4 Deriving the firm's risk premium

To derive the asset pricing implications of our model, we add some assumptions to our setup. Namely, we assume that there is a systematic source of risk in the economy. This risk is priced and affects the dynamics of the stochastic discount factor, denoted by ξ_t, which satisfies:

$$\frac{d\xi_t}{\xi_t} = -rdt - \eta d\widetilde{B}_t. \tag{35}$$

In this equation, r is the constant risk-free rate of the economy. $d\widetilde{B}_t$ is a standard Brownian motion representing the systematic source of diffusion risk, and η represents the associated constant market price of risk.

Now, we assume that the firm's cash flow is volatile and imperfectly correlated with the source of systematic risk. Namely, we assume that cash flow dynamics are given by:

$$\left[y_t(a - by_t)g_t - \frac{z_t^2}{2}\zeta g_t - \frac{s_t^2}{2}\theta g_t - c_t g_t \chi\right] dt + \sigma_\pi y_t g_t \left[\rho d\widetilde{B}_t + \sqrt{1-\rho^2} d\widetilde{B}_t^{F\perp}\right].$$

The first term represents the initiator's profits from production net of abatement and green innovation expenditures, as well as the cost of maintaining credits. The second term represents the volatility of the firm's cash flows, which increases with the firm's production rate. The parameter σ_π is a positive constant, and $\widetilde{B}^F$ is a standard Brownian motion under the physical probability measure. The Brownian motion $\widetilde{B}^F$ is correlated with the aggregate shock $\widetilde{B}$ by a factor $\rho \geq 0$. That is, $\widetilde{B}^F$ can be decomposed into the orthogonal components $\widetilde{B}_t$ and $\widetilde{B}_t^{F\perp}$ through ρ, where $\widetilde{B}_t^{F\perp}$ captures idiosyncratic risk independent of the aggregate (priced) risk $\widetilde{B}_t$.

A heuristic derivation of risk premia involves a comparison of the HJB equations under the physical and risk-neutral measures, as in Bolton, Chen, and Wang (2013) and Bustamante and Zucchi (2023a). We define risk premia as expected returns in excess of the risk free rate r:

$$\mathcal{R}_t \equiv \rho\eta \frac{\sigma_\pi y_t}{v_t}. \tag{36}$$

Next we show Proposition 8.

Proof of Proposition 8 Using the definition for risk premia, we have

$$\frac{\mathcal{R}(c)}{\mathcal{R}^B} = \frac{y(c)}{y^B}\frac{v^B}{v(c)} \tag{37}$$

By Proposition (3), the first ratio on the right-hand side is lower than one, i.e., $\frac{y(c)}{y^B} < 1$. Conversely, $\frac{v^B}{v(c)}$ can be either greater or smaller than one. In particular, it is smaller than one (i.e., $v^B < v(c)$) if net emissions are negative, as per Proposition 4. ■

In our numerical analysis, we follow standard assumptions in the literature and consider $\sigma_\pi = 0.25$, $\eta = 0.25$, and $\rho = 0.5$. Furthermore, we impose $a \equiv a' - \eta\rho\sigma_\pi$, where a refers to our baseline parametrization in the body of the paper, and a' represents the new value of the scale parameter in the inverse demand function in equation 2.

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Table 1: Baseline parameters.

Parameter	Description	Value
r	Risk-free rate	0.020
ν	Emission intensity	0.300
σ	Emission volatility coefficient	0.300
θ	Abatement cost coefficient	2.000
ζ	R&D cost coefficient	8.000
ϕ	Poisson coefficient of green breakthroughs	1.000
λ	Jump upon a green breakthrough	1.065
ξ	Effect of sustainability on emissions	0.001
γ	Price of carbon credits	0.300
ψ	Fee on carbon sales	0.150
χ	Maintenance cost	0.025
a	Maximum clearing price	0.500
b	Slope of the inverse demand function	0.400