Financing Cycles and Maturity Matching*

Thomas Geelen† Jakub Hajda‡ Erwan Morellec§ Adam Winegar¶

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Abstract

Capital ages and must eventually be replaced. We propose a theory of financing in which firms finance new capital with debt and optimally deleverage to free up debt capacity as their capital ages, thereby generating debt cycles. Concurrently, firms shorten the maturity of their debt to match the remaining life of their capital, generating maturity cycles. We provide time series and cross-sectional evidence that strongly supports these predictions and highlights the key roles of capital age and asset life for both debt dynamics and debt maturity choices.

Keywords: capital age, maturity matching, debt cycles, maturity cycles.

JEL Classification: E32, G31, G32.

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†Copenhagen Business School and Danish Finance Institute. E-mail: tag.fi@cbs.dk

‡HEC Montréal. E-mail: jakub.hajda@hec.ca

§Corresponding author. EPFL, Swiss Finance Institute, and CEPR. E-mail: erwan.morellec@epfl.ch

¶BI Norwegian Business School. E-mail: adam.w.winegar@bi.no

Capital ages and must eventually be replaced (Feldstein and Rothschild, 1974). As an example, in 2011 American Airlines ordered 460 airplanes to replace its ageing fleet.[1] Large, planned replacement investments are not exclusive to airlines, but are a hallmark of real-world business operations. For instance, aggregate replacement investments of U.S. public firms amounted to $1.27tn in 2019—representing around 21% of their capital stock. In this paper, we argue that planned replacement investments are an important driver of financing choices that lead to financing cycles and to a matching of debt maturity with asset maturity.

To demonstrate how planned replacement investments fundamentally affect firm financing, we proceed in two steps. We first develop a dynamic model of investment and financing in which capital ages and firms can choose not only the amount of debt to issue but also the maturity of this debt. In this model, firms borrow to finance investment and optimally deleverage to free up debt capacity as capital ages, allowing them to issue new debt when old capital needs to be replaced. To achieve these dynamics, firms issue debt with a maturity that matches the useful life of new assets and a repayment schedule that reflects the need to free up debt capacity as capital ages. These dynamics lead to debt cycles and to a matching between the maturity of the debt contract and that of the asset it finances. They additionally imply that leverage and debt maturity are negatively related to capital age while debt maturity and the length of financing cycles are positively related to the useful life of assets. We then test these predictions on a large sample of listed U.S. firms over the 1975–2018 period and, as hinted by Figure 1, find strong support for all of them in the data.

Our model builds on prior dynamic models of firm investment and financing (Gomes, 2001; Hennessy and Whited, 2005; DeAngelo, DeAngelo, and Whited, 2011). But it differs in that capital has a finite useful life instead of being geometrically depreciated, as in, e.g., Arrow (1964), Rogerson (2008), Rampini (2019) or Livdan and Nezlobin (2021).[2] Just as any non-geometric form of depreciation would, a finite useful life makes capital age relevant for

[1] See the Financial Times of July 7 2012, Procurement: Dependent on vision and strategy.

[2] The standard assumption of geometric depreciation makes capital age irrelevant for the firm's problem since a capital's future productivity (and value) can be perfectly described by its current productivity. Subsection I.E shows that our results are robust to alternative forms of depreciation. The key force underlying our results and predictions is that the firm replaces ageing capital via large, planned investments.

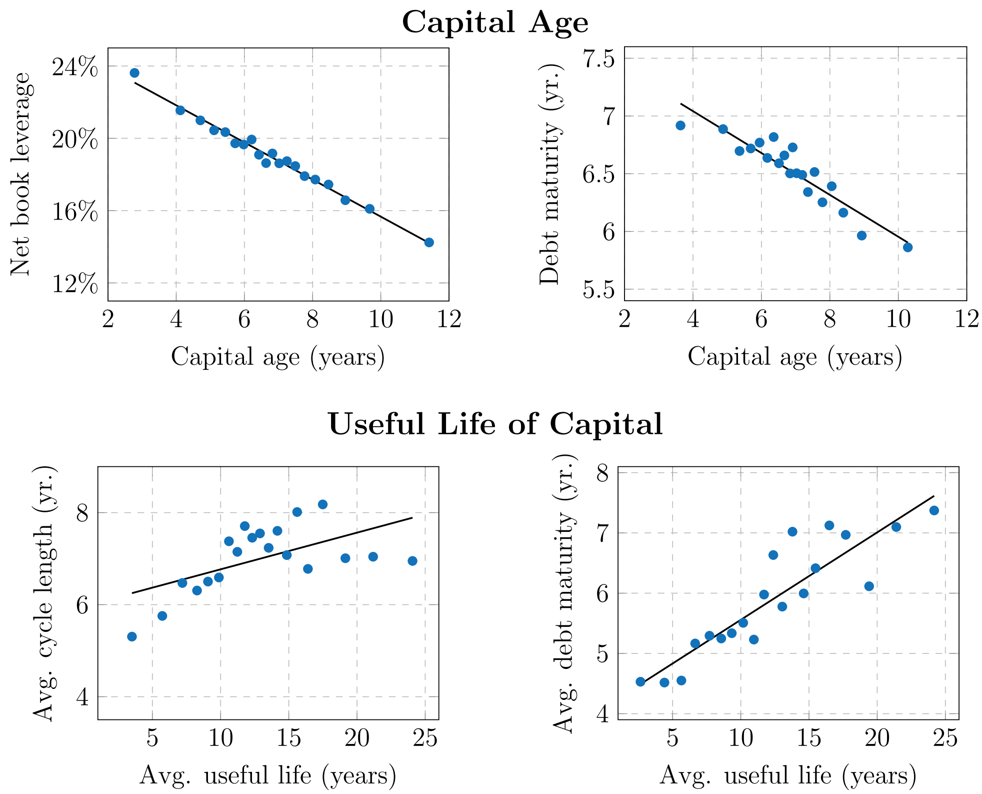


Figure 1: **Debt financing, capital age, and useful life**. The top panels control for firm fixed effects. Each dot corresponds to $1/20^{th}$ of the sample firms. The sample period is from 1975 to 2018. Variables are defined in Table A.1.

investment and financing decisions. A finite useful life of assets means that the productivity of capital, but not its value, remains constant over its lifespan after which it needs to be replaced—a good approximation for many forms of capital.[3] As an example, consider two airlines with the same number of airplanes. One airline utilizes airplanes which are, on average, older than the airplanes of the other airline. Geometric depreciation of the airplanes would imply that the airline with younger airplanes should fly more passengers, as its capital is younger and therefore more productive. However, since the airlines have the same number of airplanes, they fly roughly the same number of passengers. In our model, as in the airline example, the firm knows it needs to make replacement investments in the future due to the

[3]As will become clear, our results get mechanically stronger if profits decrease with capital age, for example due to lower utilization (Benmelech and Bergman, 2011) or increasing maintenance costs.

finite life of its assets (airplanes). That is, the firm faces large, planned investments.

In the model, the firm has an incentive to finance investment with debt because creditors are more patient than shareholders, which is equivalent to debt providing tax benefits. But since it faces a borrowing constraint (Lian and Ma, 2021) and the cost of investment exceeds profits, the firm manages its leverage (or net worth) keeping in mind future funding needs. Therefore, the firm initially levers up when buying new capital. However, it progressively reduces its net debt as its capital ages to free up debt capacity that will be used to finance future replacement investments. In addition, because issuing debt is costly (Altınkılıç and Hansen, 2000; Yasuda, 2005), the firm only issues debt when buying capital—instead of e.g. rolling-over one-period debt—to minimize issuance costs. To do so, the firm issues debt with a maturity that matches the useful life of new assets and with a repayment schedule that progressively frees up debt capacity. By doing so, the firm ensures that the repayment of maturing debt provides enough financial slack to finance replacement investments.

These net debt and maturity dynamics arise in our model from the fact that capital ages and has a finite useful life, leading the firm to predictably replace existing capital in lumps and to match the maturity of new debt to that of the asset to be financed. These dynamics generate debt cycles and maturity cycles, imply that firms have inherently unstable leverage (DeAngelo and Roll, 2015), and rationalize the *pro-active* leverage declines documented in Denis and McKeon (2012) and DeAngelo, Gonçalves, and Stulz (2018).[4] We demonstrate that they also imply (*i*) a negative relation between capital age and both leverage and debt maturity, in line with the patterns highlighted in the top row of Figure 1, and (*ii*) a positive relation between the useful life of assets and both the length of debt cycles and debt maturity, in line with the patterns outlined in the bottom row of Figure 1.

A key feature of our model is that the borrowing constraint takes the form of a cash-flow based constraint, and not a collateral constraint. Indeed, recent empirical research (e.g. Lian and Ma (2021) and Block, Jang, Kaplan, and Schulze (2023)) has shown that borrowing constraints are overwhelmingly cash-flow based. We show that cash-flow based constraints

[4]Notably, DeAngelo et al. (2018) find that this deleveraging reflects the decision to repay debt and retain earnings as opposed to exogenous shocks that drive stock-market prices up and leverage ratios down.

lead to debt cycles when investment is lumpy, as firms need to free up debt capacity towards the end of the life of old assets to buy new assets. In particular, cash-flow based constraints imply that both the ratio of net debt over EBITDA and debt maturity decrease with capital age.[5] We also show in our empirical analysis that these predictions find support in the data and that capital tangibility, which proxies for the collateral channel, has no meaningful effect on the relation between capital age and either leverage or debt maturity.

We test the time-series and cross-sectional predictions of the model using data on U.S. public firms and produce two main findings. First, in line with the model predictions, we find in time-series regressions that capital age is a significant determinant of both leverage and debt maturity, even after conditioning on a standard set of leverage and maturity controls. In addition, when examining the importance of different factors in explaining leverage ratios, respectively debt maturity, as in Frank and Goyal (2009), we find that capital age is the factor with the most, respectively second most, explanatory power. In separate tests aimed at exploring the mechanism, we show that the effects of capital age on leverage and debt maturity are stronger when investment is more lumpy, when the return on investment is lower, or when the firm is smaller, in line with our predictions. Second, we find in cross-sectional tests that the useful life of assets is a significant determinant of both the length of debt cycles and average debt maturity. Notably, firms with longer-lived assets follow longer debt cycles and have a higher average debt maturity, in line with our predictions.

Importantly, by highlighting the distinct roles of capital age and useful life of assets in explaining debt maturity choices, our paper allows us to rationalize the conflicting findings of prior empirical studies on the "maturity matching principle."[6] Notably, Stohs and Mauer (1996) run pooled regressions without firm fixed-effects (using primarily cross-sectional vari-

[5]Because the value of the asset falls as its remaining life is reduced, debt has to *mechanically* come down with a collateral constraint. Collateral constraints do not imply however that net debt over EBITDA should decrease with capital age (as we show in our empirical analysis), only that net debt should.

[6]This mixed empirical support is puzzling given that (*i*) Graham and Harvey (2001) find in their survey of corporate managers that the desire to match debt maturity to asset maturity is the most important factor in the debt maturity choice and (*ii*) standard textbooks, such as Brealey, Myers, and Allen (2020), Ross, Westerfield, Jaffe, and Jordan (2019), and Berk and DeMarzo (2019), present maturity matching as an important principle of financial management, noting that financing long-term assets by rolling-over short-term debt would be risky (due to fluctuations in short-term rates) and costly (due to refinancing costs).

ation to identify coefficients) and document a positive relation between asset maturity and debt maturity. Custódio, Ferreira, and Laureano (2013) run panel regressions with firm fixed-effects (relying on time-series variation to identify the regression coefficients) and find no effect of asset maturity on debt maturity. We demonstrate that maturity matching implies that capital age—which is a dynamic variable—should predict debt maturity choices in time-series regressions. By contrast, the useful life of (new) assets—which is primarily a time-invariant firm characteristic—should explain cross-sectional differences in debt maturity choices. Consistent with our model predictions and the results in these studies, we find that asset maturity is a significant determinant of debt maturity in the cross-section but not in the time-series. Our results instead show that capital age is the key driver of debt maturity in the time-series, as predicted by our theory.

We perform various robustness tests to confirm the validity of our results, using alternative proxies for capital age and the useful life of assets, alternative measures of debt maturity, and alternative industry definitions. All these robustness tests confirm our findings.

Our paper makes several contributions. First, we develop a framework in which investment cycles lead to endogenous debt and maturity cycles. This framework brings together the literature on vintage capital (Arrow, 1964; Ramey and Shapiro, 2001; Rogerson, 2008; Rampini, 2019; Livdan and Nezlobin, 2021) and the literature on lumpy investment (Cooper and Haltiwanger, 1993; Caballero and Engel, 1999; Cooper, Haltiwanger, and Power, 1999; Winberry, 2021). While existing papers focus on investment dynamics, our paper instead articulates the effects of vintage capital and lumpy investment on financing decisions. Notably, our paper is the first to show that lumpy investment leads to debt and maturity cycles and to shed light on the implications of capital age for debt maturity and dynamics.

Second, our paper advances the literature studying dynamic financing and investment decisions (Gomes, 2001; Hennessy and Whited, 2005; Clementi and Hopenhayn, 2006; Nikolov, Schmid, and Steri, 2019) by highlighting the role of capital age and asset life in determining not only debt dynamics but also debt maturity choices. In this literature, our model shares several features with DeAngelo et al. (2011) in that investment spikes are accompanied by leverage spikes and firms deleverage progressively to free up debt capacity. However, our analysis is distinctive because of *i*) the roles it assigns to capital age and asset life, *ii*) the associated implications it derives for firm-level cycles, and *iii*) its analysis of debt maturity. Our model is also related to Rampini and Viswanathan (2013) and Rampini (2019), who investigate the consequences of asset-based borrowing constraints for firm financing. In these studies, the market for physical capital is frictionless so that capital only affects the firm's future through its residual value. In addition, investment is smooth and firms only issue one-period debt so that there is no notion of debt cycles or maturity matching.

Third, we contribute to the literature on debt maturity by proposing a theory in which firms match the maturity of their assets and debt liabilities.[7] We show that the maturity structure linkage emerges naturally in a world in which *i*) firms borrow to meet funding needs for immediate investment and *ii*) subsequently deleverage to have debt capacity when assets in place reach the end of their useful life. In this literature, our paper is most closely related to Myers (1977) and Hart and Moore (1994). In Myers (1977), firms with more growth options shorten debt maturity to reduce debt overhang. Instead, our theory ties the debt maturity choice to the useful life of assets in place. This allows us to show that optimal financing is characterized by cycles and to generate unique predictions relating capital age and the useful life of assets to leverage and debt maturity. Hart and Moore (1994) consider a model in which a firm can invest in a single asset with finite life and argue that managers' ability to withdraw their human capital imposes a constraint on how much the firm can borrow as a function of the present value of future cash flows. As this present value mechanically goes down with capital age, debt is structured in such a way that its value decreases over time and its maturity matches asset maturity. In our model, firms are infinitely lived and do not face collateral constraints. Yet maturity matching is optimal. Maturity and leverage choices are driven by future—not past—investment and the need to free up debt capacity as capital ages. Consistent with our mechanism, we find that financing cycles are stronger for firms with more investment lumpiness and with a lower return on investment.

Lastly, we leverage our theoretical analysis to contribute to the large empirical literatures on capital structure (Leary and Roberts, 2005; Lemmon, Roberts, and Zender, 2008; Frank

[7]See e.g. Cheng and Milbradt (2012), Diamond and He (2014), He and Milbradt (2016), or Huang, Oehmke, and Zhong (2019) for recent contributions on the debt maturity choice.

and Goyal, 2009) and debt maturity (Stohs and Mauer, 1996; Custódio et al., 2013; Choi, Hackbarth, and Zechner, 2018). We do so by showing that our mechanism for the formation of debt cycles (DeAngelo et al., 2018) is consistent with the dynamics of leverage around investment peaks (Bargeron, Denis, and Lehn, 2018) and the incidence of large, proactive increases in leverage (Denis and McKeon, 2012; DeAngelo and Roll, 2015). Our analysis also brings out the key roles of capital age and asset life in the dynamics of leverage and debt maturity and provides cross-sectional and time series evidence that strongly supports the proposed mechanism. An additional empirical contribution of our paper is to use net debt to EBITDA to measure leverage, with our model generating predictions specifically related to this measure. While Graham (2022) finds in his survey of CFOs that this is by far the most commonly used measure of leverage in practice, he also notes that "in The Journal of Finance articles published since 2015 that mention leverage [...] none use debt/EBITDA".

I Model

We first consider a dynamic model of investment and financing in which firms can invest in a single asset with constant productivity to highlight the mechanism driving maturity and leverage dynamics in the simplest possible setting. Section I.E shows that our results are robust to introducing shocks, multiple assets, or alternative types of economic depreciation.

A Assumptions

Time is discrete and indexed by $t \in \{0, 1, 2, ...\}$. We consider a representative firm owned by a risk-neutral entrepreneur who discounts cash flows at rate $r > 0$. The firm has cash reserves C_0 at time $t = 0$. Each period, it can use one unit of capital to produce one unit of the final good in the next period, which yields a profit $\pi > 0$. The firm can acquire a unit of new capital, which is delivered immediately, for a price K. Capital cannot be sold—investment is irreversible—and has a finite useful life.[8] Notably, capital has a constant

[8] In this respect, we depart from most existing work, which relies on geometric depreciation of capital following Hayashi (1982). There exists ample empirical evidence that geometric depreciation does not fully

productive capacity over a finite number n of periods after which it needs to be replaced. That is, capital has a constant productivity over its lifespan but a declining value. This type of economic depreciation is known as one-hoss-shay depreciation (see Arrow, 1964; Laffont and Tirole, 2001; Rampini, 2019; Livdan and Nezlobin, 2021) and is largely used in practice. Livdan and Nezlobin (2021) note for example that firm-level data on capital goods, such as property, plant, and equipment (PP&E), is prepared in practice almost exclusively under the assumption that the efficiency of capital goods is constant over a finite useful life.[9]

We assume that investment is positive net present value (NPV) (Appendix A provides the exact parameter restriction). The present value of the cash flows of a firm that always produces goods is then given by

$$\sum_{t=1}^{\infty} \frac{1}{(1+r)^t}\pi - \sum_{i=0}^{\infty} \frac{1}{(1+r)^{i*n}}K = \frac{\pi}{r} - \frac{(1+r)^n K}{(1+r)^n - 1}.$$

Figure 2 shows the cash flows of a firm that produces each period and replaces capital at the end of its useful life. Under this policy, capital replacement leads to investment spikes, as seen in the data (Doms and Dunne, 1998; Cooper and Haltiwanger, 2006; Whited, 2006).

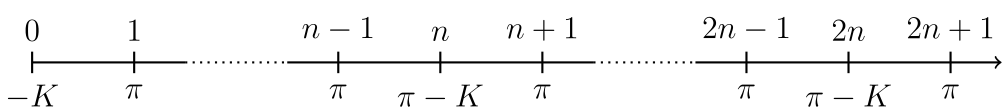


Figure 2: **Firm cash flows**. Each period, the firm produces and capital generates a profit of π the next period. Each n periods, new capital is bought at a price K.

As in Rampini and Viswanathan (2010), the firm finances investment with cash (retained earnings) and debt. Creditors are more patient than the entrepreneur and discount cash flows at a rate $\rho_D < r$, which generates an incentive for the firm to issue debt. This assumption is

reflect reality (Feldstein and Rothschild, 1974; Harper, 1982; Ramey and Shapiro, 2001; Rogerson, 2008) and that depreciation is backloaded (Giandrea, Kornfeld, Meyer, and Powers, 2021). In our setting, depreciation of capital can take the form of physical depreciation and/or (expected) technological obsolescence.

[9] One could argue that firms purchase many different types of capital and therefore geometric depreciation is a good approximation of their actual productive capacity. But as in the example given, there exists substantial within-firm variation in capital age in the data, and therefore depreciation of capital productivity≠depreciation of capital value inside the firm, which is required to use geometric depreciation.

standard in discrete time dynamic financing and investment models (e.g., DeAngelo et al., 2011), and is equivalent to the existence of tax benefits of debt $\rho_D = (1-\tau)r < r$, where $\tau \in (0, 1)$ is the corporate tax rate.

The most common approaches for modeling borrowing constraints in capital structure models is to consider either cash-flow based constraints (Clementi and Hopenhayn, 2006) or asset-based constraints (Rampini and Viswanathan, 2010). In our model, financing cycles arise with *either type of constraint*. In a recent study, Lian and Ma (2021) show that 80% of debt contracts in the U.S. are associated with cash-flow-based borrowing constraints (see also Griffin, Nini, and Smith, 2019; Block et al., 2023) while only 20% of debt contracts are associated with asset-based borrowing constraints. We therefore assume that when the firm produces the final good at time t, it can issue debt up to the cash-flow-based constraint:

$$D_t \leq \phi \times \pi,$$

where D_t is total debt at time t and $\phi \in [\underline{\phi}, \bar{\phi})$ is a constant multiple. The lower bound $\underline{\phi} > 0$ ensures that the firm can purchase the asset. The upper bound $\bar{\phi}$ ensures that debt is risk-free irrespective of the fraction of their principal creditors recover in default. Appendix A provides the exact parameter restrictions. Subsection I.E shows that asset-based borrowing constraints mechanically strengthen our result that firms lower net debt as capital ages since the collateral value declines as capital ages.

In practice, issuing debt is costly (Altınkılıç and Hansen, 2000; Yasuda, 2005).[10] We consider that the firm incurs debt issuance costs $\epsilon > 0$ that are proportional to the amount of debt raised. We allow the firm to have multiple debt issues outstanding at the same time with (possibly) different maturities. Interest on debt is paid each period. We study the situation in which debt issuance costs become small $\epsilon \to 0$. To make sure that the firm does not have permanent debt in its capital structure, we assume that capital investment cannot be fully financed by debt and current period profits, $K > \phi\pi + \pi$.[11]

[10] In fact, the surveys of Graham and Harvey (2001) and Graham (2022) show that transaction costs and fees come much before bankruptcy costs or personal taxes as a determinant of capital structure.

[11] Our results on leverage do not depend on this assumption. Our results on debt maturity apply to the

The firm earns a return $\rho_C \in (0, \rho_D)$ on its cash holdings, implying that the firm never holds both cash and debt (as in, e.g., Hennessy and Whited, 2005; DeAngelo et al., 2011) and has no incentives to retain more cash than is needed to fund investment.

B Equity Value

At time t, the firm has cash reserves C_t and invests I_t in new capital (if at all). Dividends then given by the budget constraint

$$\begin{aligned} Div_t =& \pi \mathbb{I}_{\{firm\ produces\}} - I_t + C_{t-1}(1+\rho_C) - C_t + D_t - D_{t-1}(1+\rho_D) \\ & - \epsilon \max\{D_t - D_{t-1}, 0\} \\ =& \pi \mathbb{I}_{\{firm\ produces\}} - I_t + ND_t - ND_{t-1}\left(1 + \rho_D \mathbb{I}_{\{ND_{t-1} \geq 0\}} + \mathbb{I}_{\{ND_{t-1} < 0\}} \rho_C\right) \\ & - \epsilon \max\{\min\{ND_t, ND_t - ND_{t-1}\}, 0\}, \end{aligned} \tag{1}$$

where $ND_t = D_t - C_t$ is the firm's net debt, which summarizes its financing policy, and $\mathbb{I}_{\{x \geq y\}}$ is the indicator function of the event $x \geq y$.

Management maximizes the present value of future dividends by choosing investment I_t and financing ND_t policies. That is, equity value solves

$$E_0 = \sup_{\{I_t, ND_t\}_{t \in \{0,1,2,\ldots\}}} \sum_{t \geq 0}^{\infty} \frac{Div_t}{(1+r)^t},$$

where dividends follow from the budget constraint in equation (1) and are non-negative, and net debt satisfies the borrowing constraint $ND_t \leq \phi \times \pi$.

C No Debt Issuance Cost

To structure the analysis, we first examine the firm's financing and investment dynamics when there are no debt issuance costs $\epsilon = 0$, which makes the debt maturity choice irrelevant.

non-permanent part of the capital structure if this assumption is violated.

In Subsection I.D, we allow for debt issuance cost and show that this leads to maturity matching between assets and debt liabilities.

In our model, investment is positive NPV. Furthermore, given the firm's borrowing constraint, management has no incentive to abscond with the debt proceeds since this would imply it has to forgo future investment opportunities. Finally, time discounting implies that management has no incentive to replace existing capital early and incur the investment cost early. As a consequence, we have that (see the Appendix for all proofs):

Proposition 1 (Firm Investment)**.** *The firm never defaults on its debt and replaces existing capital when it reaches the end of its useful life and never before.*

Next, let $a \in \{0, 1, ..., n-1\}$ be the age of the firm's current capital. With a slight abuse of notation, we also use a as a time index. ND_a will therefore refer to net debt given that the firm has capital with age a. Given that the return on cash is lower than the return on debt $\rho_C < \rho_D$, the firm never holds both cash and debt at the same time. Therefore, financing policies are summarized by the firm's net debt ND_a. Given debt's lower required rate of return $\rho_D < r$, the firm wants to maximize its borrowing while still being able to replace capital when it reaches the end of it useful life. It does so by raising the maximum amount of debt when it invests $ND_0 = \phi\pi$. As capital ages, the firm then optimally starts repaying debt to create financial slack. This financial slack allows the firm to invest in new capital by issuing new debt when existing capital reaches the end of its useful life. The firm delays lowering its net debt as long as possible to maximize debt benefits without sacrificing its ability to replace ageing capital. The following theorem formalizes this result:

Theorem 1 (Debt Cycles)**.** *As capital ages, the firm frees up debt capacity to finance replacement investments, in that*

$$ND_{a+1} \leq ND_a.$$

Figure 3 shows the optimal dynamics of investment and financing. The firm finances investment by increasing net debt because of the benefits of debt financing, i.e. $\rho_D < r$.

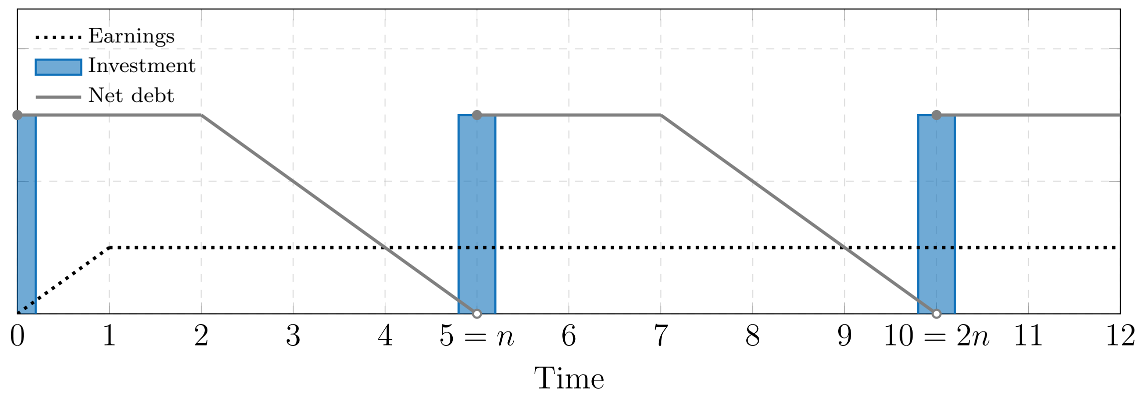


Figure 3: **Earnings, investment, and net debt dynamics.**

It then optimally lowers net debt. The firm does so to free up debt capacity to be able to finance its replacement investment, which is positive NPV. These dynamics generate debt cycles that are driven by the firm's ageing capital.

In our model, as in many recent dynamic capital structure models such as Strebulaev (2007), Morellec, Nikolov, and Schürhoff (2012), or DeMarzo and He (2021), the firm makes financing decisions with the objective of managing its net debt to earnings ratio. This is consistent with industry practice. For example, in a survey of corporate CEOs Graham (2022) documents that debt/EBITDA is by far the most popular measure of debt usage. Indeed, the corporate credit market has norms about debt relative to earnings and, when firms issue debt, they generally cannot surpass the reference level of debt to EBITDA that lenders use. Also, when debt contracts include cash-flow based borrowing constraints, firms are explicitly subjected to specific debt to EBITDA ratios.[12] Thus, both in practice and in our model, firms actively manage their net debt to earnings ratio.

Importantly, the debt cycles depicted in Figure 3 are consistent with several empirical findings: *i*) Denis and McKeon (2012) find that firms lever up to finance investment, which occurs in our model due to firms financing the replacement of ageing capital with debt; *ii*) Denis and McKeon (2012) and DeAngelo et al. (2018) find that firms significantly decrease

[12] Griffin et al. (2019) show that debt/EBITDA is included in the most commonly used covenant packages and that there is an increasing use of cash flow-based covenants in recent years.

leverage after reaching a peak, which occurs in our model because firms want to free up debt capacity to finance the eventual replacement of ageing capital; *iii*) DeAngelo and Roll (2015) find that corporate capital structure is inherently unstable, which is consistent with our debt cycles leading to inherently unstable firm leverage even in the absence of uncertainty.

In addition to rationalizing prior findings, the model generates unique cross-sectional and time-series predictions for leverage. Within a firm, the model predicts that

Prediction 1. *Capital age and the ratio of net debt to earnings are negatively related.*

This negative relation arises because of the need to free up debt capacity as capital ages (Theorem 1). Across firms, the model predicts that

Prediction 2. *The duration of debt cycles is positively related to the useful life of assets.*

Our model also allows us to study the effects of lumpiness in investment and profitability on debt cycles. In our model, the cost of investment is given by K while its benefits are reflected in π. For a given level of cash flows π, a greater cost of investment K implies both that investment is more lumpy, as the firm needs to spend more whenever it invests, and that the return on investment, defined as $\frac{\pi}{K}$, is lower. In the Appendix, we show that:

Proposition 2 (Debt Cycles, Lumpy Investment, and Return on Investment)**.** *As the cost of investment increases $K' > K$ the effects of capital age on net debt become more pronounced:*

$$|ND_{t+1} - ND_t| \leq |ND'_{t+1} - ND'_t|.$$

The more expensive capital becomes the more financial slack the firm needs to finance investment. As a result, as shown by Proposition 2, debt cycles become more pronounced as the cost of investment in physical capital K increases. This leads to the following prediction:

Prediction 3. *The effects of capital age on leverage, as measured by net debt over earnings, are more pronounced in firms with more lumpy investment and lower return on investment.*

D Maturity Matching

With debt issuance costs $\epsilon \to 0$, the firm implements the same net debt dynamics as in Theorem 1 but only issues debt when buying capital to minimize issuance costs. As a result, the debt maturity choice has no bearing on the debt cycles. To achieve these debt dynamics, the firm issues debt with a maturity that approximately matches the useful life of new assets and with a repayment schedule that progressively frees up debt capacity. This way, the firm makes sure that by repaying maturing debt it creates enough financial slack to finance replacement investments. The following theorem formalizes this result.

Theorem 2 (Long-Term Debt Financing)**.** *With debt issuance costs, the firm only issues debt when buying new capital and optimally issues long-term debt with a repayment schedule such that net debt follows the same cycles as in Theorem 1.*

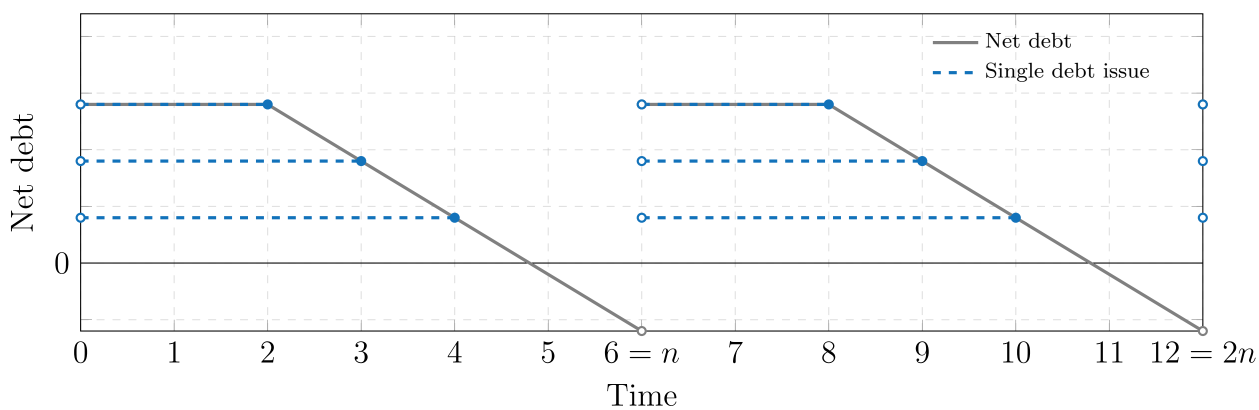


Figure 4: **Firm financing and optimal debt maturity.** The figure shows the optimal financing and debt maturity given an asset maturity of $n = 6$.

Figure 4 considers the case of a firm with assets that have a useful life of 6 years. This firm optimally issues three bonds whenever it replaces existing assets. The first bond has a maturity of three years (the top bond issue in Figure 4 disappears in year three), the second bond has a maturity of four years, and the third bond has a maturity of five years. Given this debt issuance strategy, net debt dynamics are optimal, in that the firm optimally frees up debt capacity, while issuance cost are minimized (and shareholder value is maximized).

Let M_a be the average maturity of outstanding debt given that capital age is a. When $ND_a \leq 0$, the firm has no debt outstanding and $M_a = 0$. When $ND_a > 0$, we have that

$$M_a = \sum_{i=a}^{n-1} \mathbb{I}_{\{ND_i > 0\}} (i + 1 - a) \frac{ND_i - \max\{ND_{i+1}, 0\}}{ND_a}.$$

We can then show that capital age and average debt maturity are negatively related.

Proposition 3 (Debt Maturity Cycles)**.** *Average debt maturity is decreasing in capital age:*

$$M_{a+1} \leq M_a.$$

Figure 5 shows how average debt maturity evolves through time when assets have a useful life of 6 years and the firm implements the optimal debt maturity structure at issuance. The firm only issues debt when buying new capital. Debt issuance leads to an increase in the average debt maturity which then decreases as capital ages until the firm invests again. Therefore, capital ageing not only leads to debt cycles but also to maturity cycles.[13]

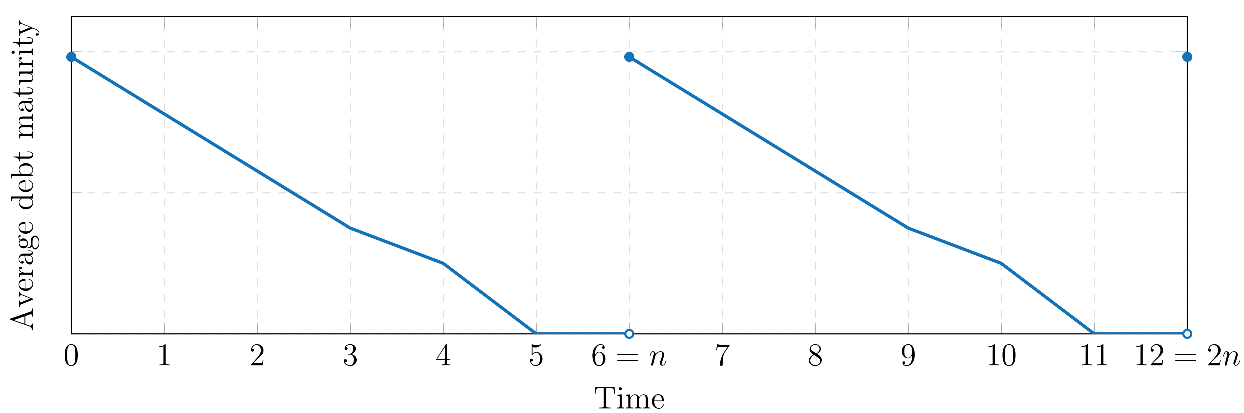


Figure 5: **Average debt maturity.** This figure shows the average debt maturity of a firm with assets that have a useful life of 6 years.

An implication from the optimal financing policy is that the firm can postpone deleverag-

[13]Optimal financing can be implemented by either issuing amortising debt or by issuing multiple debt issues with staggered maturities.

ing when assets have a greater useful life and does so by issuing debt with a longer maturity.

Theorem 3 (Maturity Matching)**.** *Increasing the useful life of assets increases average debt maturity in that* $\frac{\Delta M_a}{\Delta n} \geq 0$.

The model generates both cross-sectional and time-series predictions for debt maturity. Within a firm, the model predicts that (see Proposition 3)

Prediction 4. *Capital age and debt maturity are negatively related.*

While cross-sectionally, the model predicts that (see Theorem 3)

Prediction 5. *Average debt maturity is positively related to the useful life of assets.*

E Robustness

I Other Forms of Capital Depreciation

Our model assumes that the efficiency of capital goods follows a one-hoss shay pattern, as in e.g. Arrow (1964), Rogerson (2008), Rampini (2019), or Livdan and Nezlobin (2021). This form of capital efficiency allows us to generate crisp empirical predictions on financing decisions and debt maturity choices. An important question is whether this form of capital efficiency is necessary for our results. *The short answer is no.* Debt cycles are generated by large replacement investments financed with debt. Thus, any form of economic depreciation that leads to large investments suffices as we show in Proposition 4 of the Internet Appendix.

II Investment and Debt Dynamics

In the baseline model, the firm invests in one unit of capital that is replaced every n periods. Assume now that the firm has multiple capital units of different vintages. Propositions 5 and 6 of the Internet Appendix show that in this case both the ratio of net debt to earnings and debt maturity are weakly decreasing until the next time the firm invests. In addition, the firm's capital stock ages when it does not invest, leading to a negative relation both between capital age and net debt over earnings and between capital age and debt maturity. Furthermore, increasing the time to the next investment date leads to an increase in debt maturity, consistent with the "maturity matching principle". As the Propositions and Figures 6 and 7 highlight, a higher investment frequency leads to less pronounced and shorter cycles. Therefore, the higher the frequency of investment the shorter the average debt maturity and the shorter the periods over which leverage and debt maturity decrease.

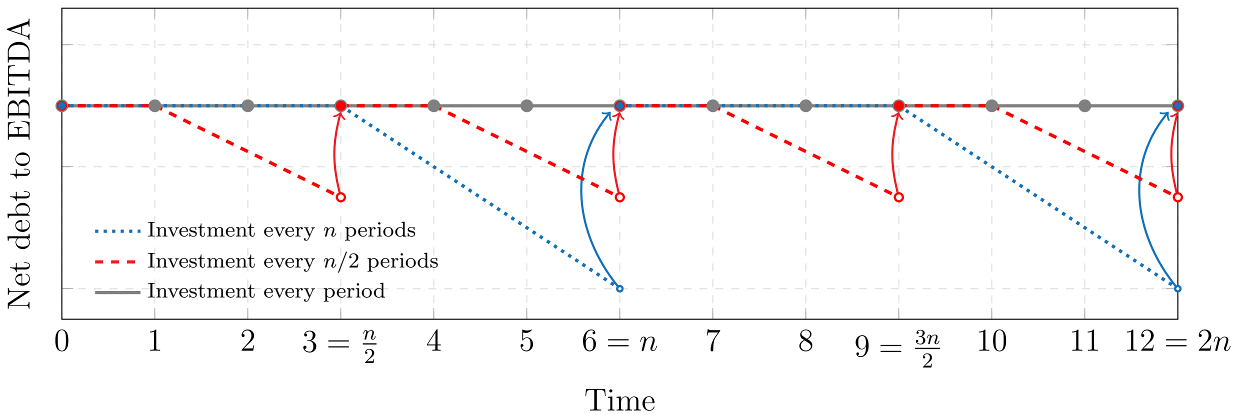


Figure 6: **Investment frequency and debt cycles.** Arrows indicate points in time when the firm invests and starts a new debt cycle.

III Finite Useful Life versus Fixed Investment Costs

In our model, the indivisibility of assets leads to lumpiness in investment and the finite life of assets leads to predictability in the timing of investment. The combination of the two then leads to financing cycles and maturity matching. But of course, and as shown for example by Cooper and Haltiwanger (1993) and Cooper et al. (1999), fixed investment costs will also lead to lumpiness in investment when assets are perfectly divisible. And if there are no shocks, then the timing of investment also becomes predictable (capital just needs to depreciate sufficiently) and therefore it is possible to issue debt that matures exactly when the firm needs to invest. In this case, financing will follow cycles and debt maturity will decrease with capital age, as in our model.

Enriching the model with TFP shocks would lead to a divergence between the predictions of our model versus a model based on fixed investment costs à la Cooper et al. (1999). Indeed,

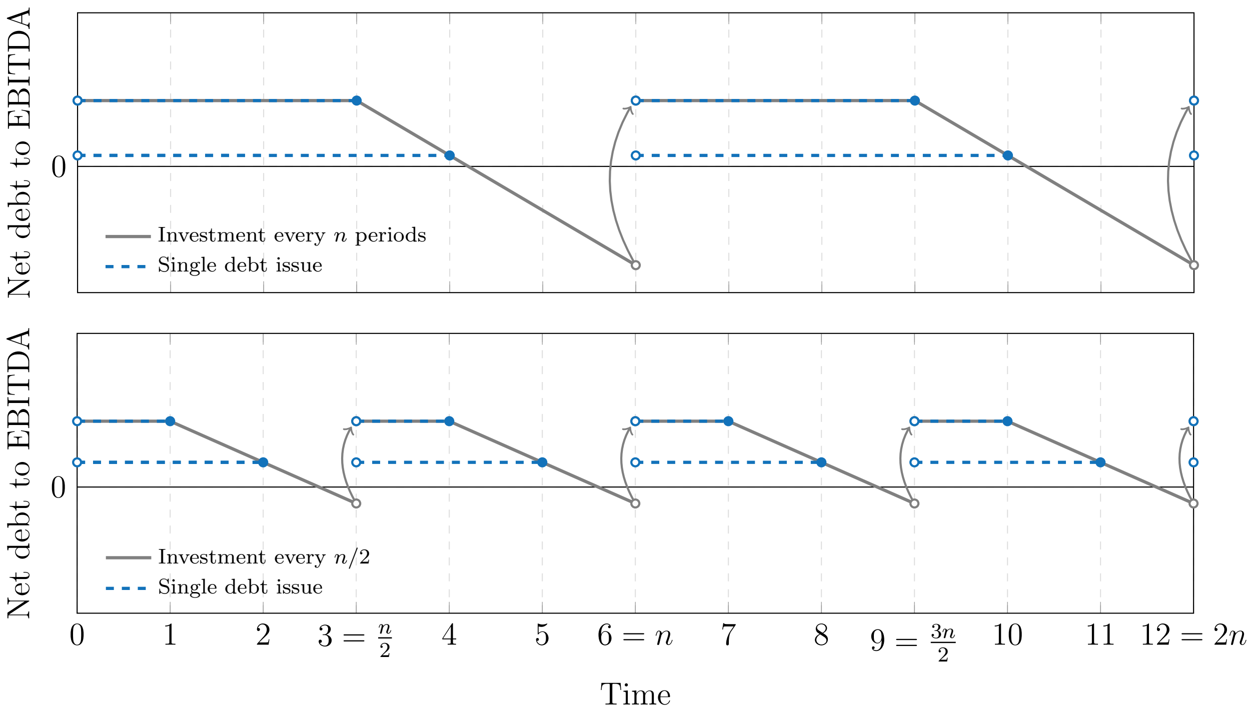


Figure 7: **Leverage dynamics for different investment frequencies.** The higher the frequency of investment, the less pronounced and the shorter the cycles are.

consider a firm with one unit of capital. Our mechanism based on the finite useful life of assets guarantees that the firm knows exactly when it needs to replace this unit of capital. Thus the firm knows when it needs to have enough debt capacity to replace the asset. This mechanism also makes the duration matching between assets and liabilities optimal when there are costs of issuing debt. What happens if we introduce TFP shocks in this model? A large positive TFP shock will not lead to early replacement of capital. The firm may however want to invest in additional assets. In this case, the firm will again want to match the maturity of the new debt contract with the life of the assets it finances. A large negative TFP shock could cause the firm to sell its assets. In this case, debt covenants will require the firm to repay debt. That is, the replacement date of existing capital (if it occurs) is perfectly predictable when assets have a finite useful life and maturity matching still arises.

Consider next a model with fixed adjustment costs and geometric depreciation. When the firm is subject to TFP shocks, investment timing depends not only on how much capital has been depreciated but also on the actual path of the TFP shock process. In this case, the replacement date of existing capital (if it occurs) becomes stochastic (as in, e.g., Abel and Eberly, 1994) and it is no longer possible to exactly match debt maturity with the useful life of assets. That is, even though one can compute the expected replacement date of existing capital, the replacement date is a random variable and having a debt contract that matches the expected replacement date no longer guarantees that the firm has freed up enough debt capacity to invest when it is optimal to do so.[14] Thus, a model with fixed adjustment costs, geometric depreciation, and TFP shocks will not generate maturity matching.

IV Shocks

Our baseline model considers a deterministic environment. While solving a general dynamic financing and investment model with shocks would be computationally infeasible,[15] we can derive additional results on shocks and financing cycles by specializing the model further. In the Internet Appendix, we study two extensions of the baseline model. In the first extension, the firm has multiple divisions that face correlated shocks, which is equivalent to a model where the firm faces large but infrequent shocks. In the second one, the firm has multiple divisions that face of uncorrelated shocks, which is equivalent to a model where the firm faces frequent but small shocks. As Proposition 7 in the Internet Appendix shows, financing cycles arise when firms face large and (relatively) infrequent shocks, leading to lumpy investment, while these cycles are smoothed out by small and frequent shocks, leading to smooth investment. When the firm is only subject to small shocks, capital age is counterfactually constant through time. Indeed, we find in the data that capital age is time-varying and correlates with leverage and debt maturity. We also find that the effects we document are larger when investment is more lumpy, in line with our model predictions.

[14] If the firm faces a large positive TFP shock then it expands its capital base thereby incurring the fixed investment cost. Once the firm has incurred the fixed cost, it also replaces any depreciated existing capital. As a result, the replacement date of existing capital is stochastic and depends on the realizations of TFP.

[15] The reason is the large number of state variables. We would need to keep track of TFP, all the capital vintages, and the firm's net debt. In the data, useful life is 13.1 years on average so this would imply that we would have 13 different capital vintages and therefore 15 state variables.

V Cash-Flow Versus Asset-Based Borrowing Constraints

While both asset-based and cash-flow-based borrowing constraints are observed in practice, recent empirical research shows that cash-flow-based constraints are most prevalent. For instance, Lian and Ma (2021) document that 80 percent of the value of U.S. corporate debt is based on constraints related to earnings, as assumed in our model, whereas 20 percent is asset-based lending. They also show that cash-flow based lending has become more prevalent over time. Relatedly, Block et al. (2023) note in their survey of U.S. and European investors with private debt assets under management that "the absence of asset-based loans indicate that private debt funds, both in the U.S. and Europe, resemble banks in their preference for priority rights over firms' cash flows."

The Internet Appendix shows that if debt levels are tied to the value of assets—an asset-based borrowing constraint—and the value of assets decreases through time (because of depreciation for the book value or because of ageing for the market value), then debt levels will mechanically decrease over time until assets are replaced. That is, debt cycles are mechanically driven by the constraint. Indeed, asset-based borrowing constraints force firms to deleverage because they become tighter as capital ages, which does not happen with a cash-flow based constraint. One would expect asset-based borrowing constraints to be more prevalent in firms that have more assets to borrow against. Yet, we find in Section II that asset tangibility—which proxies for the importance of the asset-based borrowing channel—does not affect the relation between capital age and either leverage or debt maturity, suggesting that asset-based borrowing constraints do not drive our results.

II Empirical Analysis

This section provides empirical support for the model predictions, with a main focus on the time-series Predictions 1, 3, and 4 relating capital age to leverage and debt maturity.

A Data and Variables

Our empirical analysis is based on a sample of U.S. public firms from annual Compustat over the period of 1975–2018. We use a sample selection procedure similar to that in Peters and Taylor (2017) and Lin, Palazzo, and Yang (2020). In particular, we exclude firms whose SIC code is between 4900 and 4999 (utility or regulated firms), between 6000 and 6999 (financial firms), or greater than 9000 (government agencies etc.). We also exclude firms operating in R&D–intensive sectors (SIC codes 737, 384, 382, 367, 366, 357, and 283).[16] We winsorize all variables at 1% and 99% levels to mitigate the impact of outliers. We drop all observations with missing values on one or more variables of interest. We remove observations with a market-to-book ratio larger than 20, negative book equity or negative EBITDA. Our final sample consists of 68,833 firm-year observations with 6,001 unique firms.

Our model predicts that leverage and debt maturity should decrease with capital age (a), while the length of debt cycles and average debt maturity should increase with the useful life of assets (n). To test these predictions, we need to measure capital age and the useful life of assets. We follow prior research when constructing these measures. In particular, we construct our measure of capital age similar to Salvanes and Tveteras (2004) and Lin et al. (2020). Specifically, we first calculate net and gross investment for firm i at time t as:

$$I_{i,t}^{net} = ppent_{i,t+1} - ppent_{i,t} \qquad and \qquad I_{i,t}^{gross} = \delta_{i,t+1} ppent_{i,t} + I_{i,t}^{net},$$

where $ppent_{i,t}$ refers to net PP&E and $\delta_{i,t}$ is the BEA industry economic depreciation rate assigned to firm i at time t.[17] Capital age $CA_{i,t}$ is then defined as:

$$CA_{i,t} = \begin{cases} (CA_{i,t-1} + 1) \times \frac{(1-\delta_{i,t}) ppent_{i,t-1}}{ppent_{i,t}} + \frac{I_{i,t-1}^{gross}}{ppent_{i,t}} & if\ I_{i,t-1}^{gross} > 0, \\ CA_{i,t-1} + 1 & otherwise. \end{cases}$$

[16] Our empirical results are robust to including R&D-intensive industries; see Subsection II.D.

[17] We use the depreciation rates from the *Implied Rates of Depreciation of Private Nonresidential Fixed Assets* table, available at https://apps.bea.gov/national/FA2004/Details/xls/DetailNonres_rate.xlsx. We match the depreciation rates to Compustat using the linking table provided by the BEA, which exploits the NAICS industry classification. In Subsection II.D, we recompute our measure of capital age using the accounting depreciation from Compustat, i.e. $\delta_{i,t} = dpc_{i,t}/ppent_{i,t}$ and obtain similar results.

When the firm had positive gross investment in the previous period, capital age is calculated as a weighted average of the old capital, which ages one year, and new capital, which is one year old. The weights of old and new capital, $(1-\delta_{i,t})ppent_{i,t-1}/ppent_{i,t}$ and $I^{gross}_{i,t-1}/ppent_{i,t}$, reflect the respective shares of the old and new capital in this period's total capital. When gross investment is negative, we assume that all capital vintages are disposed of in an equal way so that capital ages by one year. We initialize the measure of capital age by calculating the ratio of accumulated depreciation and amortization ($dpact_{i,0}$) to current depreciation and amortization ($dpc_{i,0}$) from Compustat. Subsection II.D shows that our main results are robust to using alternative measures of capital age.

To measure the useful life of assets, we follow the empirical literature which relies on deflating gross PP&E by current depreciation (Stohs and Mauer, 1996; Custódio et al., 2013; Livdan and Nezlobin, 2021). We proxy the useful life of firm i's assets at time t by

$$UL_{i,t} = \left\| \frac{ppegt_{i,t} + ppegt_{i,t-1}}{2dpc_{i,t}} \right\|,$$

where $ppegt_{i,t}$ refers to gross PP&E, $dpc_{i,t}$ is current depreciation and amortization, and $\|\cdot\|$ rounds to the nearest integer. The measure is justified by the observation that firms largely use straight-line depreciation rule for their fixed assets and reflects the number of years needed to fully depreciate the capital stock. As in Livdan and Nezlobin (2021), we cap the measure at 25 years.[18] Subsection II.D shows that our main results are robust to using alernative measures of useful life.

We measure financial leverage using net debt to EBITDA, net book leverage, and net market leverage. Net debt to EBITDA is the main variable of interest as our model generates specific predictions with respect to this measure of indebtedness, which is also the most commonly used measure in practice (Lian and Ma, 2021; Graham, 2022). We additionally

[18]We calculate our measure of capital age and useful life before applying the data filters to maximize the number of observations in our sample. Furthermore, the measure of useful life of assets is calculated using a different variable for depreciation than that used in the measure of capital age. This is because we want to be as close as possible to the measures used in the literature. Moreover, the BEA reports geometric depreciation rates. Since assets never fully depreciate under geometric depreciation, imputing the useful life of assets requires additional assumptions relative to using straight-line depreciation rates from Compustat, which allow for a direct computation of useful life.

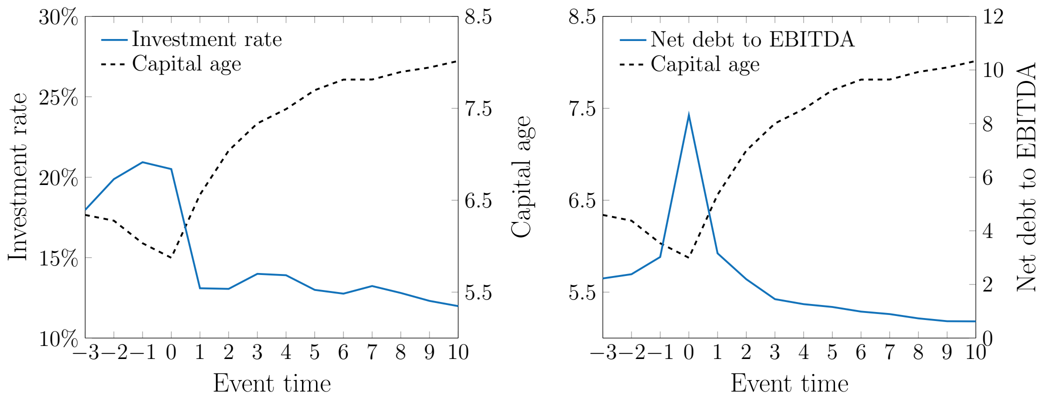
present results when using net book leverage and net market leverage to verify that our mechanism also applies to measures of leverage commonly used in the academic literature. We test the predictions regarding debt maturity using the ratios of debt maturing in more than 3 and 5 years to total debt (as in Custódio et al., 2013) and debt maturity from Capital IQ (as in Choi et al., 2018). Table 1 presents the summary statistics of our measures of capital age and asset life and of the dependent variables. Appendix C provides the definitions and summary statistics of all the variables used in the paper.

[Table 1 about here.]

Panel A of Table 1 shows that average capital age in our sample equals 6.8 years, which is close to the value of 5.7 years in Lin et al. (2020). Moreover, capital age exhibits substantial variation across firms, with a standard deviation of 3.2 years. The average useful life of assets is 13 years, similar to the value of 12.6 years in Livdan and Nezlobin (2021), which suggests that average capital age equals half of the useful life of assets, as in our model.[19] Sample firms have an average net debt to EBITDA ratio of 2.2, net book leverage ratio of 19% and net market leverage ratio of 22.3%. On average, 52% (32.8%) of their debt matures in more than 3 (5) years. The average debt maturity from Capital IQ is 6.51 years, in line with prior studies (e.g., Choi et al., 2018). Notably, average debt maturity is also close to average capital age. Panel B of Table 1 shows the within-firm correlations between the variables of interest. As indicated by Figure 1, net leverage and debt maturity are negatively correlated with capital age while debt maturity is positively correlated with the useful life of assets.

Before formally testing the model's predictions, we illustrate our mechanism with Figure 8, which shows the evolution of capital age, net debt to EBITDA, and investment around leverage peaks. Event time $t = 0$ indicates the peak of the debt cycle, defined for each firm as the year in which net debt to EBITDA reaches it maximum value (DeAngelo et al., 2018). Capital age is the lowest after a peak in leverage, indicating that firms have replaced old capital. Over time, capital age increases while net debt to EBITDA decreases. Leverage peaks occur after investment peaks have led to the replacement of old capital.

[19] If new capital is bought every $n = 13$ years, the time-series average capital age is 6 years in our model.



(a) Investment (b) Net debt to EBITDA

Figure 8: **Debt cycles: Peak to trough**. Dynamics of capital age, investment, and net debt to EBITDA around a net debt to EBITDA peak. Event time $t = 0$ indicates the net debt to EBITDA peak. We include debt cycles with at least 3 years from peak to trough, defined as the year in which net debt to EBITDA is at its minimum value for each firm. All variables are defined in Table A.1.

B Within-Firm Evidence

To formally test Prediction 1 stipulating that leverage and capital age are negatively related, we estimate fixed-effect leverage regressions in which we control for the standard determinants of leverage. Notably, we run regressions of the form

$$Lev_{i,j,t+1} = \phi CA_{i,t} + \beta X_{i,t} + \eta_i + \gamma_{t+1} + \kappa_{j,t+1} + \varepsilon_{i,t+1},$$

where $Lev_{i,j,t+1}$ is the net leverage of firm i in industry j, and the vector of controls $X_{i,t}$ includes profitability, size, market-to-book, tangibility, cash flow volatility, R&D, and firm age (Lemmon et al., 2008). All specifications include firm fixed effects η_i and year fixed effects γ_t to account for time-invariant firm heterogeneity and time-varying factors common to all firms, respectively. Some specifications additionally include industry-year fixed effects $\kappa_{j,t}$ to control for industry-level shocks that can drive investment and leverage, where we use the Hoberg-Phillips fixed industry classification with 100 industries (Hoberg and Phillips, 2010, 2016). We cluster standard errors at the firm level. The main parameter of interest in these tests is ϕ, which we expect to be negative according to Prediction 1.

Table 2 presents the estimates of ϕ for net debt to EBITDA (columns 1 to 3), net book leverage (columns 4 to 6) and net market leverage (columns 7 to 9). The results confirm the sign of the univariate correlations from Table 1: Capital age is negatively associated with leverage, even when including standard explanatory variables and controlling for fixed effects. In particular, a one standard deviation increase in capital age is associated with a 0.403 drop in net debt to EBITDA ratio, which corresponds to a 18% reduction relative to the mean. Columns 6 and 9 shows that it is also associated with a 3.2 percentage point lower net book leverage ratio and a 3.3 percentage point lower net market leverage ratio, corresponding to a reduction of 16.4% and 14.8% relative to their mean, respectively.

[Table 2 about here.]

In unreported results, we find that capital age provides substantial incremental explanatory power for leverage even when taking into account its standard determinants. Specifically, the adjusted within R^2 increases by 9%, 24% and 9% for net debt to EBITDA, net book leverage, and net market leverage, respectively, when including capital age in the specification. Additionally, in Panel A of Table IA.1 in the Internet Appendix we carry out an analysis of the importance of different determinants of leverage similar to that in Frank and Goyal (2009). Our results suggest that capital age is by and large the most important determinant of leverage in terms of explanatory power.

To test Prediction 4 suggesting that there is a negative relation between debt maturity and capital age, we follow the approach of Custódio et al. (2013) and Choi et al. (2018) and estimate maturity regressions of the form

$$Mat_{i,j,t+1} = \phi CA_{i,t} + \beta X_{i,t} + \eta_i + \gamma_{t+1} + \kappa_{j,t+1} + \varepsilon_{i,t+1},$$

where $Mat_{i,j,t+1}$ is the maturity of the debt of firm i in industry j, $X_{i,t}$ is the vector of controls, and η_i, γ_t, $\kappa_{j,t}$ are firm, year, and industry-year fixed effects. Here again, the main parameter of interest is the parameter ϕ, which we expect to be negative.

Table 3 presents the resulting estimates for the share of debt maturing in more than 3 years (columns 1 to 3), the share of debt maturing in more than 5 years (columns 4 to

6), and debt maturity from Capital IQ (columns 7 to 9). In line with Prediction 4, our results indicate that capital age is negatively associated with debt maturity. A one standard deviation increase in capital age is associated with a 0.51 year lower debt maturity and with a 3 (respectively 2.1) percentage point lower share of debt maturing in 3 (respectively 5) years. Furthermore, the economic effect is significant, as capital age also provides additional explanatory power: the adjusted within R^2 respectively increases by 30%, 48%, and 62% for debt maturing in more than 3 years, 5 years, and for debt maturity from Capital IQ. We also analyze the importance of all the determinants used in our debt maturity regressions, following Frank and Goyal (2009). We find that capital age is the second most important determinant of debt maturity (see Panel B of Table IA.1 in the Internet Appendix).

[Table 3 about here.]

Unlike capital age, asset maturity is not a statistically significant determinant of debt maturity in the regressions of Table 3. This lack of significance is due to controlling for firm fixed effects in these regressions. Asset maturity is essentially a time-invariant firm characteristic and, therefore, should only have explanatory power in cross-sectional regressions. Firm fixed effects explain roughly 80% of the variation in asset maturity in Table 3 and, as a result, asset maturity has negligible explanatory power. When running cross-sectional regressions (see Subsection II.E), we find a positive and statistically significant relation between asset maturity and debt maturity, as predicted by our theory.[20] This intuition and findings help us rationalize the conflicting evidence in Stohs and Mauer (1996)—positive effect of asset maturity on debt maturity—and Custódio et al. (2013)—no effect of asset maturity on debt maturity. Notably, Stohs and Mauer (1996) run pooled regressions without firm fixed-effects and therefore primarily use cross-sectional variation to identify their regression coefficients. They find a positive and statistically significant coefficient on asset maturity. Custódio et al. (2013) instead run panel regressions with firm fixed-effects and therefore rely on time-series

[20]Panel B of Table IA.1 in the Internet Appendix confirms that asset maturity (or useful life in unreported regressions) is not a significant within-firm determinant of debt maturity and has negligible adjusted within R^2. Thus, the inclusion of correlated control variables does not drive the results for Asset Maturity in Table 3. In unreported results, without including firm fixed effects, we find that both asset maturity and useful life are significant determinants of debt maturity and have a substantial adjusted R^2.

variation to identify the regression coefficients. They find no effect of asset maturity on debt maturity in this specification. Consistent with our model predictions and the results in these studies, we find that asset maturity/useful life is a robust determinant of debt maturity in the cross-section but not in the time-series. Our results instead show that capital age is the key driver of debt maturity in the time-series, as predicted by our theory.

C Exploring the Mechanism

Having established that capital age plays an important role in explaining within-firm variation in net leverage and debt maturity (Predictions 1 and 4), we further analyze our mechanism by investigating how it is affected by the lumpiness of investment, the return on investment, firm size, and asset tangibility.

[Table 4 about here.]

We first analyze the role of investment lumpiness. According to Prediction 3, we expect that financing is more sensitive to capital age when investment is lumpier. To test the hypothesis, we split firms into terciles based on two proxies of investment lumpiness—the firm-level skewness and kurtosis of investment. We then run regressions of net leverage and debt maturity on lagged capital age interacted with indicators for each tercile.[21] Panel A and B of Table 4 present the resulting estimates and confirm the negative relations between capital age and both leverage and debt maturity documented in Tables 2 and 3. In addition, they confirm the implications of Prediction 3 by showing that the effects become monotonically stronger as investment lumpiness increases. In fact, the effects are more than doubled when moving from the lowest to the highest tercile with a difference that is statistically significant. For example, when measuring lumpiness with skewness, a one standard deviation increase in capital age is associated with a 0.604 drop in net debt to EBITDA when investment is more lumpy, but only a 0.203 drop in net debt to EBITDA when it is less lumpy.

[21] We do not run these interactive tests on average maturity because we only have observations for this variable for a substantially smaller subset of firms and thus the tests would not be comparable when doing the tercile splits across the different specifications.

We next turn to analyzing the effects of the return on investment. In line with Prediction 3, we expect that leverage is less sensitive to capital age when firms have a higher return on investment. We test this prediction by running regressions of net leverage and debt maturity on lagged capital age interacted with indicators for firms split into terciles based on their return on investment. The resulting estimates are presented in Panel C of Table 4 and show that the effects of capital age on firm financing are monotonically decreasing in the return on investment, in line with Prediction 3. For example, specifications (1) and (2) of Panel C show that roughly two-thirds of the effect is removed when moving from the lowest to the highest tercile and that the difference is statistically significant. In particular, a one standard deviation increase in capital age is associated with a 0.642 drop in net debt to EBITDA when the return on investment is low, but only a 0.191 drop in net debt to EBITDA when the return on investment is high.

[Table 5 about here.]

The model predicts that firms with a less diversified and divisible asset base are more exposed to our mechanism because they have lumpier planned investments. We investigate this prediction by using firm size to proxy for the divisibility of the asset base. To illustrate this mechanism, consider a scenario where a smaller firm possesses only one unit of capital, while a larger firm possesses ten units of capital with different vintages. Due to the indivisible nature of the smaller firm's capital, it would face relatively larger planned replacement investments as its capital ages. In contrast, the larger firm's replacement investments are spread out over time, leading to a smoother investment pattern and a weaker relationship between capital age and financing (see Figure 7). Consequently, we expect to observe a stronger negative relationship between capital age and both net leverage and debt maturity in the subset of smaller firms. In line with this intuition, the results in Panel A of Table 5 show that the effects of capital age on leverage and debt maturity are indeed the strongest among the smallest firms. Specifically, the effect of capital age diminishes monotonically as firm size increases in seven out of eight specifications, with the effect being approximately half the magnitude for larger firms as compared to smaller firms.

Lastly, we examine the extent to which tangible capital, which proxies for the importance of the collateral channel, interacts with capital age with the objective of determining whether our mechanism is stronger when firms have higher tangible capital intensity. To do so, we split firms into terciles every year based on the intensity of capital tangibility and run regressions of net leverage and debt maturity on lagged capital age interacted with indicators for each tercile. The resulting estimates of these regressions are presented in Panels B and C of Table 5. These estimates suggest that capital tangibility has no meaningful effect on the relation between capital age and either leverage or debt maturity. In Panel B, we use property, plant and equipment over total assets to proxy for capital tangibility (Frank and Goyal, 2009). Columns (1) and (2) show that firms with more tangible assets appear to be more (respectively less) affected by capital age in the specification without (respectively with) controls. In both regressions, the estimates are not statistically significant. The other six columns show similar results, except for two specifications, without controls, in which the coefficients capturing the effects of tangibility are weakly significant. In Panel C, we use an alternative measure of capital tangibility from Almeida and Campello (2007), which also accounts for account receivables and inventory. Columns (1) and (2) show that for our main leverage measure, the signs of the estimates are opposite to the signs predicted by the collateral channel driving our mechanism. That is, firms with more tangible capital are less affected by capital age. However, the coefficients are not statistically significant. Net book leverage regressions show marginally significant results in the specification without controls. No clear picture emerges from the debt maturity regressions in columns (5) to (8). The results in Panel B and C therefore indicate that capital tangibility has no meaningful effect on the relation between capital age and either leverage or debt maturity, which suggests that the collateral channel does not drive our results.

D Robustness

We conduct several robustness tests by examining how our results are affected by the sample composition, the definition of depreciation, and the measure of capital age. In each robustness test, we replicate the regression models from Subsection II.B while controlling for all the determinants of net leverage and debt maturity as well as firm and industry-year fixed effects (i.e., the comparable results can be found in columns 3, 6 and 9 in Tables 2 and 3).

[Table 6 about here.]

First, in Panel A of Table 6, we show that the effect of capital age on net leverage and debt maturity remains quantitatively similar when including firms that operate in R&D–intensive sectors (SIC codes 737, 384, 382, 367, 366, 357, and 283) in the sample.

Second, in Panels B and C of Table 6, we document that the effect of capital age on net leverage and debt maturity remains quantitatively similar when changing the definition of the depreciation rate. We do so by calculating capital age using the accounting depreciation rate implied by Compustat instead of the BEA industry economic depreciation rate. In Panel B we compute the depreciation rate as $\delta^1_{i,t} = dpc_{i,t}/ppent_{i,t}$, that is the depreciation expense over net property, plant and equipment. In Panel C, we calculate the depreciation rate as $\delta^2_{i,t} = (dpc_{i,t} - am_{i,t})/ppent_{i,t}$, i.e. the depreciation expense minus amortization of intangibles over net property, plant and equipment. The results presented in Table 6 indicate that using the accounting depreciation rate from Compustat rather than the economic depreciation rate from BEA does not materially affect our results, neither statistically nor economically.

[Table 7 about here.]

Third, in Table 7 we show that the results are robust to using different measures of capital age. We consider three alternative measures. In Panel A, we modify our baseline measure by assuming that firms first dispose of the oldest capital vintages when disinvesting. In contrast, our baseline measure assumes that *all* vintages are equally affected (Lin et al., 2020) by disinvestment. In Panel B, we proxy capital age by the ratio of accumulated (*dpact*) to current depreciation (*dpc*). In Panel C, we follow Ai, Croce, and Li (2012) and use the weighted average age of firms' capital vintages over the past $T = 7$ years to measure capital age. As suggested by the summary statistics in Table A.2, all alternative measures of capital age have means and standard deviations comparable to those of our original measure. Moreover, the pairwise correlation coefficient between the baseline and alternative measures ranges from 0.44 to 0.79.[22] Overall, the results in Table 7 illustrate that changing capital age proxy does not materially affect the economic and statistical significance of the results.

Finally, in Table IA.2 of the Internet Appendix, we show that our results are robust to changing the industry definition, by using the Hoberg-Phillips fixed industry classification with 50 industries or the Fama-French industry classification with 49 industries.

E Cross-Sectional Evidence

We next turn to testing the cross-sectional predictions of the model indicating that firms with longer-lived assets should follow longer debt cycles (Prediction 2) and have a higher average debt maturity (Prediction 5). We proxy for the useful life of assets using the ratio of the book value of physical assets to depreciation costs as in Livdan and Nezlobin (2021). This measure captures the economic useful life of assets and does not directly depend on capital adjustment costs. Indeed, the measure corresponds to the number of years needed to fully depreciate the capital stock and does not rely on the timing of the replacement investment. For robustness, we also use alternative measures of asset life including the average of the capital age and the asset maturity capped at 25 years, calculated as in Stohs and Mauer (1996) and Custódio et al. (2013).

To test the first prediction relating the useful life of assets to the duration of debt cycles, we need to obtain a measure of the length of a firm's financing cycle. To do so, we define a leverage spike as an instance in which the firm's net debt to EBITDA ratio exceeds its firm-specific median by one standard deviation. The length of the cycle is then the number of years between the first observation and the first spike, between consecutive leverage spikes, or between the last spike and the end of the sample period for the given firm, conditional on a minimum cycle length of three years, similar to Cooper et al. (1999).[23] Firms that do not have at least one spike are excluded.[24] We then calculate the average useful life of assets and

[22]The measure of Ai et al. (2012) differs the most as it requires at least 7 years of continuous investment data to calculate capital age. This reduces the overall sample size. While calculating this measure with $T = 10$ or $T = 15$ yields a capital age proxy with a mean closer to that of the remaining measures, it results in having substantially fewer observations, which affects the statistical power of our tests.

[23]Table IA.4 in the Internet Appendix shows that the results are robust to using a 5-year filter.

[24]We cannot calculate the cycle length for roughly 48% of firms in our full sample. Most of these firms

the average as well as the maximum length of the debt cycle for each firm in our sample.

To formally test Prediction 2, we run cross-sectional regressions of the form

$$Cycle_i = \alpha + \phi UL_i + \beta X_i + \varepsilon_i,$$

where $Cycle_i$ is either the maximum or the average length of the cycle of firm i, UL_i is the average useful life of firm i's asset, and X_i is a vector of average firm-level controls analogous to the controls in the within-firm tests in Table 2. We cluster standard errors at the industry level using the Hoberg-Phillips fixed industry classification with 100 industries. The main parameter of interest is the parameter ϕ, which we expect to be positive.

Table 8 presents the resulting estimates for the maximum debt cycle lengths (columns 1 to 2) and the average debt cycle length (columns 3 to 4). In specifications 2 and 4 we control for all independent variables from Table 2. The results suggest a strong positive association between the cycle length and the firm's average asset life, consistent with Prediction 2, and are robust to controlling for common determinants of leverage. A one-year increase in asset life is associated with a roughly one-month increase in the average debt cycle length, depending on the specification. Moreover, the results are similar and robust to using other alternative measures of asset life (Panels B and C). Thus, consistent with Prediction 2, firms with longer-lived assets have longer debt cycles.

[Table 8 about here.]

To test Prediction 5 indicating that firms match the maturity of their debt to that of their assets, we regress the firm-level average debt maturity on the average useful life of assets. Formally, we run cross-sectional regressions of the form

$$Mat_i = \alpha + \phi UL_i + \beta X_i + \varepsilon_i,$$

where Mat_i is the average debt maturity for firm i, and X_i is a vector of average firm-level

do not have any debt cycle due to insufficient data. 56% of the excluded firms are only in our data for a maximum spell of 3-years and 91% have less than 10 years of consecutive observations. Thus, the majority of excluded firms are due to their short-spells in the data rather than a lack of lumpiness.

controls analogous to the controls in the within-firm tests in Table 3. Here again, ϕ is the main parameter of interest and we expect it to be positive based on Prediction 5.

Table 9 presents the resulting estimates for the average % debt maturing in more than 3 years (columns 1 and 2) and 5 years (columns 3 and 4), and the average debt maturity from Capital IQ (columns 5 and 6). In specifications 2, 4 and 6 we control for all independent variables from Table 3, except for asset maturity. The results document a positive and significant relation between average debt maturity and average useful life in all specifications and are thus consistent with Prediction 5 that firms with longer-lived assets have longer debt maturities, which corresponds to maturity matching. Moreover, the results are robust to using alternative measures of asset life (Panels B and C). As previously noted, the results suggest that asset maturity is a significant determinant of debt maturity in the cross-section while capital age is a key driver of debt maturity in the time-series (Table 3), consistent with our model.

[Table 9 about here.]

As a robustness test, we show in Table IA.3 in the Internet Appendix that our cross-sectional results for debt cycles are robust to defining them using net book leverage rather than net debt to EBITDA. Additionally, we show in Table IA.4 that our cross-sectional results for debt cycles are robust to having a minimum of five years between spikes.

III Conclusion

Capital ages and must eventually be replaced. This paper develops a dynamic investment and financing model to study how ageing capital generates variation in financing decisions. In this model, firms issue debt to finance investment. As capital ages, they deleverage to free up debt capacity, which allows them to replace old capital by issuing new debt. To achieve these dynamics, firms issue debt with a maturity that matches the useful life of new assets and an amortization schedule that reflects the need to free up debt capacity as capital ages. These debt dynamics lead to debt cycles and to a maturity matching theory of debt. They also imply that both leverage and debt maturity should be negatively related to capital age while both the duration of debt cycles and debt maturity should be positively related to the useful life of assets. We take the model predictions to the data and find that all our measures of leverage and debt maturity are negatively related to capital age while all measures of the duration of debt cycles or debt maturity are positively related to the useful life of assets, as predicted by the model. In addition, we find that the effects of capital age on leverage and maturity are stronger in smaller firms, firms with more lumpy investment, and with a lower return on investment, in line with the model predictions. Overall, our results indicate that capital age is an important driver of firms' financing dynamics.

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Panel A: Summary statistics

	Capital age	Useful life	ND/ EBITDA	Net book leverage	Net mkt. leverage	% debt mat.$> 3y$	% debt mat.$> 5y$	Debt mat. (yr.)
Mean	6.804	13.048	2.236	0.189	0.223	0.520	0.328	6.514
Standard deviation	3.215	5.709	4.205	0.226	0.264	0.328	0.302	4.861
Q1	4.408	9.000	0.312	0.050	0.041	0.228	0.005	3.337
Median	6.367	13.000	1.441	0.200	0.199	0.584	0.292	5.288
Q3	8.700	17.000	3.033	0.340	0.397	0.795	0.569	8.109
N	68833	66380	68833	68833	68833	68833	68833	16905

Panel B: Within-firm pairwise correlations

	Capital age	Useful life	ND/ EBITDA	Net book leverage	Net mkt. leverage	% debt mat.$> 3y$	% debt mat.$> 5y$	Debt mat. (yr.)
Capital age	1							
Useful life	0.243	1						
ND/EBITDA	-0.041	0.001	1					
Net book lev.	-0.144	-0.074	0.507	1				
Net mkt. lev.	-0.121	-0.058	0.518	0.843	1			
% debt mat.$> 3y$	-0.098	0.005	0.032	0.139	0.078	1		
% debt mat.$> 5y$	-0.120	0.013	0.016	0.085	0.049	0.640	1	
Debt mat. (yr.)	-0.065	0.008	0.005	0.031	0.004	0.202	0.250	1

Table 1: **Summary statistics: capital age and financing**. The table contains the summary statistics of capital age, the useful life of assets, and the financing variables. These include three measures of net leverage: net debt to EBITDA, net book leverage, net market leverage and three measures of debt maturity: the ratios of debt maturing in more than 3 or 5 years to total debt as well as the debt maturity from Capital IQ. Panel A contains the summary statistics and Panel B contains the within-firm pairwise correlations between the respective variables. All variables are defined in Table A.1.

	ND/EBITDA			Net book leverage			Net market leverage		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Capital age	-0.425***	-0.355***	-0.403***	-0.041***	-0.032***	-0.032***	-0.042***	-0.031***	-0.033***
	(-11.46)	(-8.48)	(-7.55)	(-17.26)	(-11.57)	(-9.54)	(-14.97)	(-9.91)	(-9.09)
Profitability		-0.817***	-0.745***		-0.032***	-0.027***		-0.048***	-0.045***
		(-22.90)	(-17.11)		(-18.20)	(-12.38)		(-24.11)	(-18.25)
Size		0.484***	0.516***		0.050***	0.062***		0.085***	0.100***
		(4.50)	(3.38)		(6.85)	(7.15)		(10.82)	(10.48)
Market-to-book		-0.049	-0.044		-0.009***	-0.012***		-0.026***	-0.023***
		(-1.57)	(-1.21)		(-4.39)	(-5.01)		(-12.74)	(-10.40)
Tangibility		0.371***	0.392***		0.039***	0.040***		0.048***	0.045***
		(5.24)	(4.15)		(7.90)	(7.14)		(8.95)	(7.26)
Cash flow volatility		-0.042	-0.020		-0.003**	-0.001		-0.003**	-0.000
		(-1.28)	(-0.55)		(-2.13)	(-0.55)		(-1.97)	(-0.11)
R&D		-0.116**	-0.010		-0.007*	-0.001		-0.008**	-0.001
		(-2.14)	(-0.17)		(-1.79)	(-0.32)		(-2.11)	(-0.24)
Firm age		0.140	0.772		-0.024	0.033		-0.020	0.020
		(0.29)	(1.57)		(-0.62)	(0.96)		(-0.53)	(0.54)
Year FE	Yes	Yes	No	Yes	Yes	No	Yes	Yes	No
Firm FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Ind.-Year FE	No	No	Yes	No	No	Yes	No	No	Yes
Observations	56707	48261	32499	56707	48261	32499	56707	48261	32499
Adj. within R^2	0.0063	0.0450	0.0411	0.0303	0.0847	0.0812	0.0206	0.1179	0.1163

Table 2: **Capital age and leverage – within-firm regressions**. This table presents estimates from regressions of net debt to EBITDA and net leverage ratios on lagged capital age. The dependent variable is *Net debt to EBITDA* in columns 1 to 3; *Net book leverage* in columns 4 to 6 and *Net market leverage* in columns 7 to 9. Each explanatory variable is standardized by its full-sample standard deviation. The models in columns 3, 6 and 9 include industry-year fixed effects created using Hoberg-Phillips fixed industry classification with 100 industries. All variables are defined in Table A.1. t-statistics are reported in parentheses. Standard errors are clustered at the firm level. We use * $p < 0.10$, ** $p < 0.05$, and *** $p < 0.01$ to indicate p-values.

	% debt maturing > 3*y*			% debt maturing > 5*y*			Debt maturity (yr.)		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Capital age	-0.039***	-0.030***	-0.030***	-0.031***	-0.025***	-0.021***	-0.390***	-0.413***	-0.510***
	(-13.42)	(-8.96)	(-6.57)	(-10.21)	(-7.00)	(-4.50)	(-2.97)	(-2.94)	(-3.35)
Size		0.105***	0.189***		0.023	0.070**		1.919**	1.694
		(3.94)	(5.31)		(0.91)	(2.22)		(1.97)	(1.44)
Size squared		-0.051**	-0.128***		0.023	-0.009		-1.230	-0.944
		(-2.07)	(-3.90)		(0.96)	(-0.30)		(-1.20)	(-0.79)
Market-to-book		0.007**	0.007*		0.004	0.001		0.077	0.068
		(2.39)	(1.96)		(1.57)	(0.41)		(0.78)	(0.60)
Asset maturity		0.006	0.003		0.007*	0.006		0.212	0.181
		(1.40)	(0.66)		(1.65)	(0.97)		(1.53)	(1.12)
Abnormal earnings		0.002**	0.001		0.003***	0.003**		0.046**	0.064**
		(2.13)	(0.84)		(2.70)	(2.28)		(2.05)	(2.04)
Cash flow volatility		-0.001	0.002		-0.002	0.002		0.005	0.002
		(-0.46)	(0.62)		(-0.74)	(0.76)		(0.07)	(0.03)
R&D		-0.007	-0.010		-0.007	-0.008		0.162	0.034
		(-1.33)	(-1.52)		(-1.31)	(-1.08)		(0.92)	(0.17)
Net book leverage		0.032***	0.040***		0.015***	0.016***		0.084	0.150
		(9.43)	(9.02)		(4.45)	(3.69)		(0.88)	(1.41)
Firm age		-0.062	-0.031		-0.094*	-0.048		2.461	2.830
		(-1.28)	(-0.61)		(-1.71)	(-0.76)		(1.30)	(1.43)
Observations	56707	47027	31502	56707	47027	31502	14054	12754	11318
Adj. within R^2	0.0088	0.0191	0.0200	0.0065	0.0108	0.0088	0.0026	0.0060	0.0075

Table 3: **Capital age and debt maturity – within-firm regressions**. The table presents estimates from regressions of debt maturity on lagged capital age. The dependent variable is *% of debt maturing in > 3 years* in columns 1 to 3; *% of debt maturing in > 5 years* in columns 4 to 6; and *Debt maturity (yr.)* in columns 7 to 9. Each explanatory variable is standardized by its full-sample standard deviation. Models in columns 3, 6 and 9 include industry-year fixed effects created using Hoberg-Phillips fixed industry classification with 100 industries. All variables are defined in Table A.1. t-statistics are reported in parentheses. Standard errors are clustered at the firm level. We use * $p < 0.10$, ** $p < 0.05$, and *** $p < 0.01$ to indicate p-values.

	ND/EBITDA		Net book leverage		% debt mat. > 3y		% debt mat. > 5y	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Panel A: Investment lumpiness – Skewness								
Capital age	-0.304***	-0.203***	-0.035***	-0.024***	-0.033***	-0.020***	-0.022***	-0.011
	(-4.17)	(-2.62)	(-7.47)	(-4.86)	(-4.81)	(-2.67)	(-2.97)	(-1.43)
Capital age × Middle	-0.163*	-0.137	-0.006	-0.005	-0.009	-0.007	-0.007	-0.008
	(-1.74)	(-1.46)	(-1.04)	(-0.85)	(-0.96)	(-0.79)	(-0.78)	(-0.85)
Capital age × High	-0.410***	-0.401***	-0.016**	-0.017***	-0.019**	-0.019**	-0.015	-0.019*
	(-3.94)	(-3.82)	(-2.48)	(-2.63)	(-2.06)	(-1.97)	(-1.61)	(-1.93)
Controls	No	Yes	No	Yes	No	Yes	No	Yes
Observations	33996	32056	33996	32056	33996	31075	33996	31075
Adj. within R^2	0.0104	0.0437	0.0330	0.0827	0.0080	0.0205	0.0047	0.0092
Panel B: Investment lumpiness – Kurtosis								
Capital age	-0.288***	-0.231***	-0.033***	-0.023***	-0.041***	-0.030***	-0.024***	-0.015*
	(-3.62)	(-2.74)	(-6.89)	(-4.70)	(-5.96)	(-3.98)	(-3.32)	(-1.88)
Capital age × Middle	-0.174*	-0.076	-0.010	-0.006	0.004	0.008	-0.006	-0.006
	(-1.85)	(-0.80)	(-1.64)	(-0.89)	(0.42)	(0.90)	(-0.60)	(-0.55)
Capital age × High	-0.434***	-0.369***	-0.018***	-0.018***	-0.009	-0.007	-0.010	-0.012
	(-4.05)	(-3.40)	(-2.80)	(-2.73)	(-0.99)	(-0.72)	(-1.12)	(-1.23)
Controls	No	Yes	No	Yes	No	Yes	No	Yes
Observations	33996	32056	33996	32056	33996	31075	33996	31075
Adj. within R^2	0.0106	0.0435	0.0332	0.0827	0.0080	0.0205	0.0047	0.0091
Panel C: Return on investment								
Capital age	-0.711***	-0.642***	-0.047***	-0.037***	-0.041***	-0.031***	-0.034***	-0.028***
	(-9.87)	(-8.15)	(-13.09)	(-9.54)	(-8.51)	(-5.84)	(-6.62)	(-5.09)
Capital age × Middle	0.295***	0.323***	0.002	0.002	-0.003	0.003	0.003	0.008
	(4.95)	(5.17)	(0.71)	(0.95)	(-0.69)	(0.68)	(0.62)	(1.58)
Capital age × High	0.413***	0.451***	0.014***	0.015***	0.001	0.005	0.010*	0.015**
	(5.92)	(6.10)	(3.73)	(3.85)	(0.23)	(0.73)	(1.80)	(2.58)
Controls	No	Yes	No	Yes	No	Yes	No	Yes
Observations	34713	32499	34713	32499	34713	31502	34713	31502
Adj. within R^2	0.1125	0.1213	0.0627	0.0967	0.0095	0.0217	0.0056	0.0100

Table 4: **Exploring the mechanism – investment lumpiness and profitability**. This table presents estimates from regressions of net leverage variables and debt maturity on lagged capital age interacted with indicators for firms split into terciles by the proxies of investment lumpiness (the firm-level investment skewness, Panel A; and firm-level investment kurtosis, Panel B) and profitability proxied by return on investment (EBITDA divided by book assets, Panel C). *Middle* and *High* indicate the middle and highest terciles of each splitting variable. The dependent variables are *Net debt to EBITDA*, *Net Book Leverage*, *% of debt maturing in > 3 years*, and *% of debt maturing in > 5 years*. Each explanatory variable is standardized by its full-sample standard deviation. Specifications 2, 4, 6 and 8 control for all independent variables from Tables 2 for net leverage and 3 for debt maturity. All specifications include indicators for the middle and high terciles. All models include firm and industry-year fixed effects created using the Hoberg-Phillips fixed industry classification with 100 industries. t-statistics are reported in parentheses. Standard errors are clustered at the firm level. We use * $p < 0.10$, ** $p < 0.05$, and *** $p < 0.01$ to indicate p-values.

	ND/EBITDA		Net book leverage		% debt mat. > 3y		% debt mat. > 5y	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Panel A: Firm size								
Capital age	-0.601***	-0.592***	-0.050***	-0.044***	-0.044***	-0.038***	-0.031***	-0.030***
	(-7.26)	(-6.83)	(-10.06)	(-8.51)	(-6.62)	(-5.20)	(-5.16)	(-4.49)
Capital age × Middle	0.223**	0.282***	0.016***	0.017***	0.010	0.012	0.007	0.008
	(2.56)	(3.18)	(3.04)	(3.09)	(1.27)	(1.36)	(1.01)	(0.96)
Capital age × High	0.213**	0.324***	0.019***	0.023***	0.020**	0.023***	0.022***	0.027***
	(2.28)	(3.41)	(3.29)	(3.79)	(2.39)	(2.61)	(2.63)	(3.06)
Controls	No	Yes	No	Yes	No	Yes	No	Yes
Observations	34713	32499	34713	32499	34713	31502	34713	31502
Adj. within R^2	0.0126	0.0434	0.0496	0.0914	0.0180	0.0259	0.0153	0.0165
Panel B: Capital tangibility								
Capital age	-0.437***	-0.444***	-0.037***	-0.033***	-0.039***	-0.028***	-0.022***	-0.015**
	(-6.26)	(-5.85)	(-8.74)	(-7.39)	(-6.22)	(-4.10)	(-3.40)	(-2.09)
Capital age × Middle	-0.055	0.065	-0.003	0.004	-0.007	-0.006	-0.008	-0.008
	(-0.73)	(0.85)	(-0.62)	(0.87)	(-1.02)	(-0.88)	(-1.03)	(-1.01)
Capital age × High	-0.111	0.044	-0.009*	-0.001	-0.005	0.002	-0.016*	-0.011
	(-1.22)	(0.47)	(-1.66)	(-0.11)	(-0.61)	(0.30)	(-1.90)	(-1.23)
Controls	No	Yes	No	Yes	No	Yes	No	Yes
Observations	34681	32494	34681	32494	34681	31500	34681	31500
Adj. within R^2	0.0102	0.0415	0.0410	0.0837	0.0078	0.0200	0.0048	0.0088
Panel C: Alternative measure of capital tangibility								
Capital age	-0.532***	-0.436***	-0.038***	-0.031***	-0.044***	-0.032***	-0.023***	-0.016**
	(-7.95)	(-6.14)	(-8.44)	(-6.43)	(-7.97)	(-5.57)	(-3.76)	(-2.43)
Capital age × Middle	0.032	0.018	-0.006	-0.005	-0.001	-0.001	-0.012*	-0.010
	(0.51)	(0.28)	(-1.54)	(-1.22)	(-0.24)	(-0.15)	(-1.93)	(-1.62)
Capital age × High	0.031	0.028	-0.008*	-0.006	0.002	0.004	-0.012*	-0.011
	(0.39)	(0.33)	(-1.72)	(-1.15)	(0.26)	(0.51)	(-1.65)	(-1.42)
Controls	No	Yes	No	Yes	No	Yes	No	Yes
Observations	34180	32044	34180	32044	34180	31193	34180	31193
Adj. within R^2	0.0126	0.0434	0.0577	0.0978	0.0110	0.0241	0.0090	0.0136

Table 5: **Exploring the mechanism – size and tangibility**. This table presents estimates from regressions of net leverage variables and debt maturity on lagged capital age interacted with indicators for firms split into terciles by firm size (book assets, Panel A) and by the tangibility of assets (proxied by the net property plant and equipment over total assets in Panel B and by the asset tangibility from Almeida and Campello (2007) in Panel C). The lowest tercile is the baseline group. *Middle* and *High* indicate the middle and highest terciles of each splitting variable. The dependent variables are *Net debt to EBITDA*, *Net Book Leverage*, *% of debt maturing in > 3 years*, and *% of debt maturing in > 5 years*. Each explanatory variable is standardized by its full-sample standard deviation. Specifications 2, 4, 6 and 8 control for all independent variables from Tables 2 for net leverage and 3 for debt maturity. All specifications include indicators for the middle and high terciles. All models include firm and industry-year fixed effects created using the Hoberg-Phillips fixed industry classification with 100 industries. t-statistics are reported in parentheses. Standard errors are clustered at the firm level. We use * $p < 0.10$, ** $p < 0.05$, and *** $p < 0.01$ to indicate p-values.

Panel A: Including R&D-intensive industries

	ND/EBITDA	Net book lev.	Net market lev.	% debt mat. > $3y$	% debt mat. > $5y$	Debt mat. (yr.)
Capital age	-0.406***	-0.032***	-0.032***	-0.031***	-0.022***	-0.507***
	(-8.41)	(-10.47)	(-9.84)	(-7.33)	(-5.14)	(-3.62)
Observations	41316	41316	41316	40182	40182	14249
Adj. within R^2	0.0354	0.0846	0.1000	0.0183	0.0082	0.0056

Panel B: Capital age calculated using Compustat depreciation rate

	ND/EBITDA	Net book lev.	Net market lev.	% debt mat. > $3y$	% debt mat. > $5y$	Debt mat. (yr.)
Capital age	-0.328***	-0.028***	-0.028***	-0.026***	-0.018***	-0.391***
	(-6.12)	(-8.01)	(-7.56)	(-5.82)	(-3.74)	(-2.82)
Observations	32702	32702	32702	31737	31737	11357
Adj. within R^2	0.0401	0.0752	0.1123	0.0189	0.0080	0.0060

Panel C: Capital age calculated using Compustat depreciation rate excluding amortization

	ND/EBITDA	Net book lev.	Net market lev.	% debt mat. > $3y$	% debt mat. > $5y$	Debt mat. (yr.)
Capital age	-0.337***	-0.029***	-0.028***	-0.028***	-0.019***	-0.430***
	(-6.22)	(-8.22)	(-7.67)	(-6.25)	(-3.88)	(-2.95)
Observations	32702	32702	32702	31737	31737	11357
Adj. within R^2	0.0403	0.0758	0.1126	0.0194	0.0082	0.0063

Table 6: **Capital age and financing – alternative sample and different definition of depreciation rates**. This table presents estimates from regressions of net debt to EBITDA, net leverage ratios and debt maturity on lagged capital age when changing the sample construction by keeping R&D-intensive firms (Panel A) and when capital age is calculated using alternative definitions of the depreciation rate (depreciation expense over net property, plant and equipment in Panel B and depreciation expense minus amortization of intangibles over net property, plant and equipment in Panel C). We control for all independent variables from Table 2 in leverage regressions and from Table 3 in debt maturity regressions. Each explanatory variable is standardized by its full-sample standard deviation. All specifications include firm, and industry-year fixed effects, created using Hoberg-Phillips fixed industry classification with 100 industries. All variables are defined in Table A.1. t-statistics are reported in parentheses. Standard errors are clustered at the firm level. We use * $p < 0.10$, ** $p < 0.05$, and *** $p < 0.01$ to indicate p-values.

Panel A: Capital age calculated by disposing of oldest capital vintages first

	ND/EBITDA	Net book lev.	Net market lev.	% debt mat. $> 3y$	% debt mat. $> 5y$	Debt mat. (yr.)
Capital age	-0.281***	-0.026***	-0.026***	-0.021***	-0.018***	-0.129
	(-6.28)	(-9.08)	(-8.88)	(-4.75)	(-4.03)	(-0.85)
Observations	31107	31107	31107	30162	30162	10781
Adj. within R^2	0.0400	0.0765	0.1154	0.0176	0.0083	0.0046

Panel B: Capital age calculated as accumulated to current depreciation

	ND/EBITDA	Net book lev.	Net market lev.	% debt mat. $> 3y$	% debt mat. $> 5y$	Debt mat. (yr.)
Capital age	-0.271***	-0.025***	-0.026***	-0.015***	-0.013**	-0.091
	(-5.81)	(-8.69)	(-8.01)	(-2.97)	(-2.42)	(-0.67)
Observations	32217	32217	32217	31268	31268	11223
Adj. within R^2	0.0395	0.0780	0.1145	0.0176	0.0074	0.0043

Panel C: Capital age proxy based on Ai et al. (2012) with $T = 7$

	ND/EBITDA	Net book lev.	Net market lev.	% debt mat. $> 3y$	% debt mat. $> 5y$	Debt mat. (yr.)
Capital age	-0.118***	-0.005***	-0.006**	-0.007**	-0.005	-0.219**
	(-3.14)	(-2.58)	(-2.44)	(-2.08)	(-1.44)	(-2.39)
Observations	26523	26523	26523	25735	25735	9629
Adj. within R^2	0.0354	0.0638	0.1022	0.0164	0.0063	0.0050

Table 7: **Capital age and financing – alternative measures of capital age**. This table presents estimates from regressions of net debt to EBITDA, net leverage ratios and debt maturity on alternative measures of lagged capital age (by assuming that firms dispose of oldest capital vintages first in Panel A, by proxying capital age as the ratio of accumulated (*dpact*) to current depreciation (*dpc*) in Panel B and by calculating capital age as the weighted average age of firms' capital vintages as in Ai et al. (2012) over the past $T = 7$ years in Panel C). We control for all independent variables from Table 2 in leverage regressions and from Table 3 in debt maturity regressions. Each explanatory variable is standardized by its full-sample standard deviation. All specifications include firm, and industry-year fixed effects, created using Hoberg-Phillips fixed industry classification with 100 industries. All variables are defined in Table A.1. t-statistics are reported in parentheses. Standard errors are clustered at the firm level. We use * $p < 0.10$, ** $p < 0.05$, and *** $p < 0.01$ to indicate p-values.

	Max debt cycle		Avg. debt cycle	
	(1)	(2)	(3)	(4)
Panel A: Useful life				
Useful life	0.187***	0.104***	0.116***	0.078***
	(6.13)	(4.06)	(5.05)	(3.96)
Controls	No	Yes	No	Yes
Observations	2401	2390	2401	2390
Adj. R^2	0.027	0.244	0.017	0.168
Panel B: Average capital age				
Capital age	0.680***	0.176***	0.427***	0.119***
	(11.47)	(4.50)	(11.51)	(3.82)
Controls	No	Yes	No	Yes
Observations	2402	2391	2402	2391
Adj. R^2	0.089	0.244	0.057	0.167
Panel C: Asset maturity				
Asset maturity	0.090**	0.101***	0.051**	0.061**
	(2.63)	(3.09)	(2.17)	(2.61)
Controls	No	Yes	No	Yes
Observations	2362	2351	2362	2351
Adj. R^2	0.010	0.242	0.005	0.165

Table 8: **Asset life and debt cycles – cross-sectional regressions**. The dependent variable is *Maximum debt cycle length* in columns 1 and 2, and *Average debt cycle length* in columns 3 and 4. Firms with no leverage spike have a cycle length that is undefined and are dropped from the sample. We require a minimum of three years between subsequent spikes. In specifications 2 and 4 we control for all independent variables from Table 2. t-statistics are reported in parentheses. Standard errors are clustered at the industry level using Hoberg-Phillips fixed industry classification with 100 industries. We use * $p < 0.10$, ** $p < 0.05$, and *** $p < 0.01$ to indicate p-values.

	% debt mat. > 3y		% debt mat. > 5y		Debt maturity (yr.)	
	(1)	(2)	(3)	(4)	(5)	(6)
Panel A: Useful life						
Useful life	0.015***	0.010***	0.014***	0.009***	0.150***	0.105***
	(11.07)	(8.66)	(8.65)	(8.56)	(5.82)	(5.08)
Controls	No	Yes	No	Yes	No	Yes
Observations	4360	4240	4360	4240	2436	2408
Adj. R^2	0.088	0.421	0.097	0.355	0.044	0.205
Panel B: Average capital age						
Capital age	0.021***	0.008***	0.022***	0.008***	0.236***	-0.005
	(6.72)	(2.85)	(9.40)	(3.64)	(5.80)	(-0.12)
Controls	No	Yes	No	Yes	No	Yes
Observations	4376	4247	4376	4247	2441	2411
Adj. R^2	0.041	0.388	0.060	0.321	0.024	0.186
Panel C: Asset maturity						
Asset maturity	0.012***	0.009***	0.010***	0.008***	0.115***	0.105***
	(8.54)	(10.26)	(5.62)	(10.92)	(4.43)	(7.24)
Controls	No	Yes	No	Yes	No	Yes
Observations	4301	4175	4301	4175	2397	2368
Adj. R^2	0.086	0.439	0.080	0.359	0.042	0.220

Table 9: **Asset life and debt maturity – cross-sectional regressions**. The dependent variable is the average of each firm's *% of debt maturing in > 3 years* in columns 1 to 2; *% of debt maturing in > 5 years* in columns 3 to 4; and *Debt maturity (yr.)* in columns 5 to 6. In specifications 2, 4 and 6 we control for all independent variables from Table 3, except for asset maturity. t-statistics are reported in parentheses. Standard errors are clustered at the industry level using the Hoberg-Phillips fixed industry classification with 100 industries. We use * $p < 0.10$, ** $p < 0.05$, and *** $p < 0.01$ to indicate p-values.

Appendix

The first part of the appendix derives the results for the baseline model. The second part derives the debt maturity results. The third part defines the variables used in the empirical analysis.

A Baseline Model

We impose the following parameter restrictions. First we assume that

$$\pi > rK\left(1+\frac{1}{r}\frac{\rho_C}{(1+\rho_C)^n-1}\right), \tag{A.1}$$

which ensures that investing is positive NPV for an unlevered firm. Second, we assume that

$$\phi \geq \underline{\phi} = \frac{\max\{K-C_0,0\}}{\pi}, \tag{A.2}$$

$$\phi < \bar{\phi} = \min\left\{\frac{1}{r}-\frac{K}{\pi}\left(1+\frac{1}{r}\frac{\rho_C}{(1+\rho_C)^n-1}\right),\frac{1}{\rho_D}\left(1-\frac{K}{\pi}\frac{\rho_C(1+r)^n+r(1+\rho_C)^n-r}{(1+\rho_C)^n-1}\right)\right\}. \tag{A.3}$$

As we show below, the upper bound on ϕ ensures that debt is risk-free. The lower bound on ϕ ensures that the firm can initially purchase the asset.

The results are organized as follows. First, we show that investing is positive NPV when investment is internally financed (Lemma 1). Second, we show that this is also true when the firm can issue debt and that the firm has no incentive to default (Proposition 1). Having established that the firm invests and does not default, we derive the firm's optimal financing policy (Theorem 1). We then establish that the firm pays dividends in period $t+1$ only if the borrowing constraint binds in period t (Lemma 2) and that the borrowing constraints binds when the firm invests (Lemma 3).

Lemma 1 (Benchmark Firm Value)**.** *The value of a firm that retains profits to finance investment internally is given by*

$$C_0+\frac{\pi}{r}-K\left(1+\frac{1}{r}\frac{\rho_C}{(1+\rho_C)^n-1}\right).$$

Proof. If the firm saves s today and for the next $n-1$ periods and earns a rate ρ_C on its cash balances, then the future value of its savings in $n-1$ periods is

$$\sum_{i=0}^{n-1} s(1+\rho_C)^i = s\frac{(\rho_C+1)^n-1}{\rho_C}.$$

As a result, the firm has enough savings to finance investment after n periods if

$$s = K\frac{\rho_C}{(\rho_C+1)^n - 1}.$$

The firm earns enough to save for investment if

$$\pi - s = \pi - K\frac{\rho_C}{(\rho_C+1)^n - 1} \geq 0,$$

This is guaranteed by restriction (A.1). The value of a firm that saves to finance investment is then given by

$$C_0 - K + \sum_{t=1}^{\infty} \frac{\pi - s}{(1+r)^t} = C_0 + \frac{\pi}{r} - K\left(1 + \frac{1}{r}\frac{\rho_C}{(1+\rho_C)^n - 1}\right),$$

which is bigger than C_0 given the restriction on K. □

Proof of Propostion 1. We want to show that the firm always invests when assets reach the end of their useful life and has no incentive to default. To do so, we assume that creditors always believe that the firm will not default and therefore charge an interest rate ρ_D on debt. We then show that, given this belief, the firm has no incentive to default and always invests so that the belief is consistent and constitutes an equilibrium.

Since the firm holds cash $C_0 > 0$ and there is no debt payment due, the firm never defaults at time $t = 0$. Furthermore, the firm never defaults when it holds a positive amount of cash as net debt is negative. Therefore, we assume in this lemma that net debt is positive, in that $ND_t > 0$. Assume now that the firm does not invest at time $t = 0$ and defaults at $t = 1$. This is suboptimal since

$$\underbrace{C_0 + D_0}_{\text{Value of firm that defaults at } t=1} \leq C_0 + \phi\pi < \underbrace{C_0 + \frac{\pi}{r} - K\left(1 + \frac{1}{r}\frac{\rho_C}{(1+\rho_C)^n - 1}\right)}_{\text{Value of an internally financed firm}} \leq E_0$$

where the first inequality follows from the borrowing constraint and the second inequality follows from the restrictions on ϕ; see equations (A.2) and (A.3).[25] As a result, default can only happen for $t > 1$.

Assume that the firm has net debt $ND_t > 0$ at time $t > 0$ and defaults at time $t+1 > 1$. If the firm has capital installed at time t and therefore produces the final good at time $t+1$, we have that $\rho_D ND_t \leq \rho_D \phi\pi < \pi$ (see equation (A.3)). Therefore, the firm can make the

[25] We need (A.2) to hold since it ensures that the firm has enough resources to invest at time zero.

interest payment $\rho_D ND_t$ and a positive dividend payment

$$Div_{t+1} \geq \pi - \rho_D \phi \pi > 0$$

if it chooses $ND_{t+1} = ND_t$ and defaults at $t+2$. As a result, the firm will not default if it produces the good at $t+1$.

Assume next that the firm has no (more) installed capital at time t and does not invest so that it does not produce the good at $t+1 > 1$ and therefore defaults at $t+1$. Clearly, each period since the last time it invested $t' \geq t-n$ it must be that leverage is $ND_{t'} = \phi\pi$. Otherwise, the firm would benefit from increasing leverage and bringing dividend payments forward in time since $\rho_C < \rho_D < r$ and $\rho_D \phi \pi < \pi$. This also implies that the firm pays a dividend of $Div_{t'} = \pi - \rho_D \phi \pi$ for the n-periods $t' \in [t-n+1, t]$.

Our objective is now to show that there is a profitable deviation for the firm's shareholders, namely to save for the n-periods $t' \in [t-n+1, t]$ and invest at time t and thereby avoid default at $t+1$. If instead of paying dividends, the firm saves $s < \pi - \rho_D \phi \pi$ each period after the last time it invested ($t' \in [t-n+1, t]$) and puts this money in a savings account, then its savings at time t amount to:

$$\sum_{a=0}^{n-1} s(1+\rho_C)^{n-1-a} = s\frac{(1+\rho_C)^n - 1}{\rho_C}.$$

Instead, paying out s each period generates a value at time t of

$$\sum_{a=0}^{n-1} s(1+r)^{(n-1-a)} = s\frac{(1+r)^n - 1}{r}.$$

The firm saves enough to finance investment if

$$s = K\frac{\rho_C}{(1+\rho_C)^n - 1}$$

We need that the firm generates enough profits to save this amount. That is, we need

$$\pi(1-\rho_D\phi) > K\frac{\rho_C}{(1+\rho_C)^n - 1}, \tag{A.4}$$

which holds under restriction (A.3). The firm prefers saving over paying dividends if

$$\underbrace{s\frac{(1+r)^n - 1}{r}}_{\textit{Pay dividends}} = K\frac{\rho_C}{(1+\rho_C)^n - 1}\frac{(1+r)^n - 1}{r} < \underbrace{\frac{\pi - \rho_D\phi\pi}{r} - K\left(1 + \frac{1}{r}\frac{\rho_C}{(1+\rho_C)^n - 1}\right)}_{\textit{Internally financed firm with debt obligations } \phi\pi}.$$

The firm that would save for investment is worth at least as much as the internally financed firm that makes coupon payments on its debt forever.[26] This condition can be written as

$$\phi < \frac{1}{\rho_D}\left(1 - \frac{K}{\pi}\frac{\rho_C(1+r)^n + r(1+\rho_C)^n - r}{(1+\rho_C)^n - 1}\right),$$

which holds under restriction (A.3).

A direct implication of the fact that the firm never defaults is that it always replaces capital at the end of its useful life. The firm also never replaces capital early. If it would do so, then it could increase its firm value by delaying replacement and yield a return of $\rho_C K > 0$ on the cost of capital, which could be paid out as a dividend while leaving all other policies and cash flows unchanged. □

Proof of Theorem 1. We want to show that the firm's net debt is weakly decreasing in capital age. To establish this result, we first need to show that the firm only pays dividends when the borrowing constraint binds in the previous period.

We know from Proposition 1 that the firm always replaces capital when it reaches the end of its useful life and that the debt is risk-free. Assume that for some t, $Div_{t+1} > 0$ while $ND_t < \phi\pi$. Define ΔDiv_t as

$$\Delta Div_t = \min\left\{\frac{Div_{t+1}}{1+\rho_D}, \phi\pi - ND_t\right\}.$$

Increasing dividends at time t to $Div'_t = Div_t + \Delta Div_t$ by using debt financing would imply that $Div'_{t+1} \geq Div_{t+1} - (1+\rho_D)\Delta Div_t$. The inequality follows from the fact that the interest rate is lower if net debt was negative before $ND_t < 0$.[27] This change in policy would increase shareholder value since its effect on equity value (at time t) is at least

$$\Delta Div_t - \frac{(1+\rho_D)\Delta Div_t}{1+r} > 0.$$

As a result, if $ND_t < \phi\pi$, then $Div_{t+1} = 0$ and therefore if $Div_{t+1} > 0$ then $ND_t = \phi\pi$.

[26] Observe that the value of the internally financed firm is actually a lower bound since some of the savings can be used to temporarily lower net debt, which yields a rate of return $\rho_D > \rho_C$.

[27] Indeed, if $ND_t < 0$ and $ND_t + \Delta Div_t \leq 0$ then the discount rate is ρ_C and the change in the amount that needs to be repaid at $t+1$ is

$$(1+\rho_C)(ND_t + \Delta Div_t) - (1+\rho_C)ND_t = (1+\rho_C)\Delta Div_t < (1+\rho_D)\Delta Div_t.$$

If $ND_t < 0$ and $ND_t + \Delta Div_t > 0$, this change is

$$(1+\rho_D)(ND_t + \Delta Div_t) - (1+\rho_C)ND_t = (1+\rho_D)\Delta Div_t + ND_t(\rho_D - \rho_C) < (1+\rho_D)\Delta Div_t.$$

Instead, if $ND_t > 0$ this change is $(1+\rho_D)\Delta Div_t$.

Assume $a > 0$ and $ND_{a-1} < ND_a \leq \phi\pi$. If $ND_{a-1} > 0$ then

$$Div_a = \pi + ND_a - ND_{a-1}(1 + \rho_D) > \rho_D \phi\pi - \rho_D ND_{a-1} + (ND_a - ND_{a-1}) > 0$$

because $\phi < \frac{1}{\rho_D}$, see equation (A.3). While if $ND_{a-1} < 0$

$$Div_a = \pi + ND_a - ND_{a-1}(1 + \rho_C) > 0.$$

But this contradicts the previous result and therefore $ND_{a-1} \geq ND_a$.

□

Lemma 2. *If $Div_{t+1} > 0$ then $ND_t = \phi\pi$.*

Proof. This result follows directly from the proof of Theorem 1. □

Lemma 3. $ND_{a=0} = \phi\pi$.

Proof. We want to show that $ND_{a=0} = \phi\pi$. We do so by showing that $ND_{a=0} < \phi\pi$ can never occur. Assume that for some $t' \geq 0$ with $a = 0$ we have $ND_{t'} < \phi\pi$. Let $t'' > t'$ be the next time that $ND_{t''} = \phi\pi$ and $a = 0$. Assume that t'' does not exist. In this case, and owing to Theorem 1 and Lemma 2, the firm never pays dividends for $t > t'$ since $ND_t < \phi\pi$. Therefore, equity value is zero. But this cannot be the optimal strategy since investment is positive NPV (Proposition 1) and therefore generates a surplus that can be distributed to shareholders, which would yield a positive equity value. As a result, t'' must exist. We know that $ND_{t''-n} < \phi\pi$ since $t' \leq t'' - n < t''$. Given that Theorem 1 implies that net debt is weakly decreasing within a cycle and $ND_{t''-n} < \phi\pi$, we have that $ND_t < \phi\pi$ for $t \in [t'' - n, t'' - 1]$ because of the definition of t' and t''. From Lemma 2, it then follows that the firm does not pay any dividends over the interval $t \in [t'' - n + 1, t'']$ where $t'' - n + 1 > 0$.

Each period t, the firm has a cash flow of π but needs to pay interest. The firm can save at least $s = K\frac{\rho_C}{(1+\rho_C)^n - 1}$ since equation (A.4) holds. Therefore, the firm lowers net debt by at least s each period over this time interval and as a result net debt decreases by at least

$$\sum_{a=0}^{n-1} s(1 + \rho_C)^a = s\frac{(1 + \rho_C)^n - 1}{\rho_C} = K.$$

As a result, we have that

$$\pi - \left(1 + \rho_D \mathbb{I}_{\{ND_{t''-1} \geq 0\}} + \rho_C \mathbb{I}_{\{ND_{t''-1} < 0\}}\right) ND_{t''-1} > \pi - \rho_D \phi\pi - ND_{t''-1} > K - ND_{t''-n+1}.$$

This implies that the dividend at time t'', which follows from the budget constraint, is

$$\begin{aligned} Div_{t''} &= \pi - K + ND_{t''} - \left(1 + \rho_D \mathbb{I}_{\{ND_{t''-1} \geq 0\}} + \rho_C \mathbb{I}_{\{ND_{t''-1} < 0\}}\right) ND_{t''-1} \\ &> K - K + ND_{t''} - ND_{t''-n+1} = \phi\pi - ND_{t''-n+1} \\ &> 0. \end{aligned}$$

This makes it impossible that $ND_{t''-1} < \phi\pi$ owing to Lemma 2. This result in combination with Theorem 1 then implies that $ND_{t''-n} = \phi\pi$ but this contradicts the fact that $ND_t < \phi\pi$ for $t \in [t''-n, t-1'']$. This rules out that $ND_{a=0} < \phi\pi$ so that we must have $ND_{a=0} = \phi\pi$. □

Proof of Proposition 2. We show using backward induction that higher investment costs $K' > K$ lead to stronger leverage cycles.

Assume $K \leq \pi - \rho_D \phi\pi$. In that case, the firm always keep its net debt at $\phi\pi$ and invests using retained earnings. As a consequence,

$$|ND_a - ND_{a-1}| = 0 \leq |ND'_a - ND'_{a-1}|.$$

Assume next that $K > \pi - \rho_D \phi\pi$ so that $K' > \pi - \rho_D \phi\pi$. In that case, the firm needs debt capacity $ND_{a=n-1} < \phi\pi$ to finance investment and we know from Lemma 2 that $Div_{a=0} = 0$. Furthermore, Lemma 3 implies that $ND_{a=0} = \phi\pi$. From the budget constraint it then follows that

$$0 = \pi - K + \phi\pi - \left(1 + \rho_D \mathbb{I}_{\{ND_{a=n-1} \geq 0\}} + \rho_C \mathbb{I}_{\{ND_{a=n-1} < 0\}}\right) ND_{a=n-1}.$$

There is a unique $ND_{a=n-1}$ that solves this equation. Furthermore, this $ND_{a=n-1}$ is decreasing in K. These results also hold true for $ND'_{a=n-1}$ and imply that

$$0 \leq ND_{a=0} - ND_{a=n-1} = \phi\pi - ND_{a=n-1} < \phi\pi - ND'_{a=n-1} = ND'_{a=0} - ND'_{a=n-1}$$

and therefore

$$|ND_{a=0} - ND_{a=n-1}| \leq |ND'_{a=0} - ND'_{a=n-1}|.$$

We are going to show the result for $a > 0$ using backwards induction. We have just shown that $ND_{a=n-1} \geq ND'_{a=n-1}$. Assume now that $ND_a \geq ND'_a$ and $a > 0$. We want to show that $ND_{a-1} \geq ND'_{a-1}$ and the proposition's result. There are three cases.

1. Assume $ND_{a-1} < \phi\pi$ and $ND'_{a-1} < \phi\pi$ then we have that $Div_a = Div'_a = 0$, see

Lemma 2. Assume $ND_{a-1} < ND'_{a-1}$ then the budget constraint implies that

$$\begin{aligned}
0 &= \pi + ND_a - \left(1 + \rho_D \mathbb{I}_{\{ND_{a-1}\geq 0\}} + \rho_C \mathbb{I}_{\{ND_{a-1}<0\}}\right) ND_{a-1} \\
&= \pi + ND'_a - \left(1 + \rho_D \mathbb{I}_{\{ND'_{a-1}\geq 0\}} + \rho_C \mathbb{I}_{\{ND'_{a-1}<0\}}\right) ND'_{a-1}, \\
ND_a - ND'_a &= \left(1 + \rho_D \mathbb{I}_{\{ND_{a-1}\geq 0\}} + \rho_C \mathbb{I}_{\{ND_{a-1}<0\}}\right) ND_{a-1} \\
&\quad - \left(1 + \rho_D \mathbb{I}_{\{ND'_{a-1}\geq 0\}} + \rho_C \mathbb{I}_{\{ND'_{a-1}<0\}}\right) ND'_{a-1} \\
&< 0,
\end{aligned}$$

This contradicts the fact that $ND_a \geq ND'_a$. Thus, we must have $ND_{a-1} \geq ND'_{a-1}$. We still need to show the proposition's result. We know that the budget constraint

$$0 = \pi + ND_a - \left(1 + \rho_D \mathbb{I}_{\{ND_{a-1}\geq 0\}} + \rho_C \mathbb{I}_{\{ND_{a-1}<0\}}\right) ND_{a-1}$$

holds. From this budget constraint it directly follows that

$$\begin{aligned}
0 \leq ND_{a-1} - ND_a &= \pi - \left(\rho_D \mathbb{I}_{\{ND_{a-1}\geq 0\}} + \rho_C \mathbb{I}_{\{ND_{a-1}<0\}}\right) ND_{a-1} \\
&\leq \pi - \left(\rho_D \mathbb{I}_{\{ND'_{a-1}\geq 0\}} + \rho_C \mathbb{I}_{\{ND'_{a-1}<0\}}\right) ND'_{a-1} \\
&= ND'_{a-1} - ND'_a.
\end{aligned}$$

The inequality follows from the fact that $ND_{a-1} \geq ND'_{a-1}$. Therefore

$$|ND_{a-1} - ND_a| \leq |ND'_{a-1} - ND'_a|.$$

2. Assume $ND_{a-1} < \phi\pi$ and $ND'_{a-1} = \phi\pi$ then we have that $Div_a = 0$ from Lemma 2. The budget constraint then implies that

$$\begin{aligned}
0 &= -Div_a + \pi + ND_a - \left(1 + \rho_D \mathbb{I}_{\{ND_{a-1}\geq 0\}} + \rho_C \mathbb{I}_{\{ND_{a-1}<0\}}\right) ND_{a-1} \\
&= -Div'_a + \pi + ND'_a - (1+\rho_D) ND'_{a-1} \\
&\leq \pi + ND'_a - (1+\rho_D) ND'_{a-1}.
\end{aligned}$$

As a consequence,

$$Div_a \geq (ND_a - ND'_a) - \left(1 + \rho_D \mathbb{I}_{\{ND_{a-1}\geq 0\}} + \rho_C \mathbb{I}_{\{ND_{a-1}<0\}}\right) ND_{a-1} + (1+\rho_D) ND'_{a-1} > 0,$$

which is a contradiction. Therefore, this case cannot arise.

3. Assume $ND_{a-1} = \phi\pi$ and $ND'_{a-1} \leq \phi\pi$. This case directly implies that $ND_{a-1} \geq$

ND'_{a-1}. If $ND'_{a-1} = \phi\pi$ then

$$0 \leq ND_{a-1} - ND_a = \phi\pi - ND_a \leq \phi\pi - ND'_a = ND'_{a-1} - ND'_{a-1}.$$

If $ND'_{a-1} < \phi\pi$ then $Div'_a = 0$ by Lemma 2. From the budget constraint it then follows that

$$\begin{aligned} 0 \leq & ND_{a-1} - ND_a = -Div_a + \pi - \rho_D ND_{a-1} \\ \leq & \pi - \left(\rho_D \mathbb{I}_{\{ND'_{a-1} \geq 0\}} + \rho_C \mathbb{I}_{\{ND'_{a-1} < 0\}}\right) ND'_{a-1} = ND'_{a-1} - ND'_a. \end{aligned}$$

Therefore,

$$|ND_{a-1} - ND_a| \leq |ND'_{a-1} - ND'_a|.$$

These steps recursively establish our result.

□

B Debt Maturity

We first establish the optimal debt issuance strategy (Theorem 2). We then show that average debt maturity is decreasing in capital age (Proposition 3) and increasing in asset maturity (Theorem 3).

Proof of Theorem 2. We first show that the net debt dynamics are the same when $\epsilon \to 0$ as when debt issuance is frictionless. These net debt dynamics allow us to show the absence of permanent debt and derive the optimal debt issuance strategy.

Let $E_0(\epsilon)$ be the equity value given issuance costs ϵ. Without issuance costs, debt maturity is irrelevant as any long-term debt contract can be implemented by a sequence of short-term contracts. Furthermore, $E_0(0) \geq E_0(\epsilon)$ since issuance cost depress firm value. As a result, the net debt and investment dynamics are the same as in the baseline model when $\epsilon \to 0$. If this was not the case, then we would have $\lim_{\epsilon \downarrow 0} E_0(\epsilon) < E_0(0)$ and using the one-period debt implementation from the baseline model would dominate for sufficiently small issuance costs $\epsilon \to 0$.

Given these net debt dynamics, the firm wants to issue debt that minimizes issuance costs. Observe that cash generates a lower return than debt $\rho_C < \rho_D$ and given that debt issuance costs are small $\epsilon \to 0$, the firm only has debt outstanding when $ND_t > 0$ and only cash in hand when $ND_t < 0$.

Because the firm always invests when assets reach the end of their useful life (Proposition 1), we have that $ND_{a=n-1} < 0$ since it needs both cash and debt to finance investment since $\phi\pi + \pi < K$. As a result, the firm does not issue debt with a maturity longer than n-periods.

To minimize issuance costs the firm only issues debt when it invests with a maturity that matches the net debt dynamics during the capital's lifetime. □

Proof of Proposition 3. We first establish that average debt maturity has a recursive structure that depends on the ratio of this and next period's net debt. We then establish that the ratio of this and next period's net debt can be ordered, which allows us to show that average debt maturity declines as capital ages.

Define $\hat{a}$ as the largest capital age such that debt is positive

$$\hat{a} = \sup\{a | ND_a > 0\}.$$

Given that $K > \phi\pi + \pi$, we know that $ND_{n-1} < 0$ and therefore that $\hat{a} < n-1$. Furthermore, from Theorem 1 we have that $ND_a \leq 0$ for $a > \hat{a}$. Therefore average debt maturity is $M_a = 0$ for $a > \hat{a}$.

We can write the average debt maturity as

$$\begin{aligned}
M_a &= \sum_{i=a}^{n-1} \mathbb{I}_{\{ND_i > 0\}} (i + 1 - a) \frac{ND_i - \max\{ND_{i+1}, 0\}}{ND_a} \\
&= \sum_{i=a}^{\hat{a}} (i + 1 - a) \frac{ND_i - \max\{ND_{i+1}, 0\}}{ND_a} \\
&= \frac{1 * ND_a - 1 * ND_{a+1} + 2 * ND_{a+1} - ... - (\hat{a} - a)ND_{\hat{a}} + (\hat{a} + 1 - a)ND_{\hat{a}}}{ND_a} \\
&= \frac{ND_a + ... + ND_{\hat{a}}}{ND_a} = 1 + \frac{ND_{a+1} + ... + ND_{\hat{a}}}{ND_a} \\
&= 1 + \frac{ND_{a+1}}{ND_a} M_{a+1}.
\end{aligned}$$

Define $B_a = \frac{ND_{a+1}}{ND_a}$ for $a < \hat{a}$. The above equation can be rewritten as

$$M_a = 1 + B_a M_{a+1}.$$

From Theorem 1 and the definition of $\hat{a}$ it follows that $B_a \in (0, 1]$.

We want to show that $B_{a+1} \leq B_a$ for $a < \hat{a} - 1$. Assume first that $ND_{a+1} = \phi\pi$. In this case, we have $B_{a+1} \leq 1 = \phi\pi/\phi\pi = ND_{a+1}/ND_a = B_a$ (Theorem 1). Assume next that $ND_{a+1} < \phi\pi$. Then we also have $ND_{a+2} \leq ND_{a+1} < \phi\pi$ (Theorem 1). From the budget constraint in equation (1), the fact that $ND_{a+2} \geq ND_{\hat{a}} > 0$ (Theorem 1), and the fact that the firm pays no dividends at $a + 2$ since $ND_{a+1} < \phi\pi$ (Lemma 2), it then follows that

$$ND_{a+2} = ND_{a+1}(1 + \rho_D) - \pi$$

and therefore

$$B_{a+1} = (1+\rho_D) - \frac{\pi}{ND_{a+1}}.$$

If $ND_a < \phi\pi$ then the same argument implies that

$$B_a = (1+\rho_D) - \frac{\pi}{ND_a}.$$

Since ND_a is weakly decreasing in a (Theorem 1), we then have that $B_{a+1} \leq B_a$.

If $ND_a = \phi\pi$ the same argument implies that

$$ND_{a+1} = Div_{a+1} + ND_a(1+\rho_D) - \pi \geq ND_a(1+\rho_D) - \pi$$

and therefore

$$B_a \geq (1+\rho_D) - \frac{\pi}{ND_a}.$$

and we get that $B_{a+1} \leq B_a$

As a consequence

$$1 \geq B_0 \geq B_1 \geq ... \geq B_{\hat{a}-1} > 0.$$

It is easy to see that $M_{\hat{a}} = 1$ and therefore

$$M_{\hat{a}-1} = 1 + B_{\hat{a}-1} M_{\hat{a}} \geq 1 = M_{\hat{a}}.$$

We can now establish our result using backward induction. Assume that $M_{\hat{a}-i-1} \geq M_{\hat{a}-i} \geq 0$. We then know that

$$M_{\hat{a}-i-2} = 1 + B_{\hat{a}-i-2} M_{\hat{a}-i-1} \geq 1 + B_{\hat{a}-i-1} M_{\hat{a}-i-1} \geq 1 + B_{\hat{a}-i-1} M_{\hat{a}-i} = M_{\hat{a}-i-1} \geq 0,$$

which recursively establishes that the debt maturity is decreasing in a. □

Proof of Theorem 3. We first show that increasing asset life by a year yields the same net debt dynamics just one year lagged. This result in combination with Proposition 3 allows us to show that average debt maturity weakly increases with asset life.

Define the function

$$d(ND_{a-1}, ND_a) = \pi - K\mathbb{I}_{\{a=0\}} + ND_a - \left(1 + \mathbb{I}_{\{ND_{a-1} \geq 0\}}\rho_D + \mathbb{I}_{\{ND_{a-1} < 0\}}\rho_C\right) ND_{a-1},$$

which is the "*dividend*" the firm would pay when capital has age a and debt levels are ND_{a-1}

and ND_a, see equation (1). Observe that

$$\frac{\partial d(ND_{a-1}, ND_a)}{\partial ND_{a-1}} < 0. \tag{A.5}$$

Given ND_a, if the firm pays no dividends then the net debt from the previous period ND_{a-1} solves

$$d(ND_{a-1}, ND_a) = 0,$$

which has a unique solution that we call $\hat{ND}(ND_a)$. Given ND_a, if the firm pays dividends $Div_a > 0$, then the net debt from the previous period ND_{a-1} solves

$$d(ND_{a-1}, ND_a) = Div_a,$$

which has a unique solution that we call $\tilde{ND}(ND_a, Div_a)$. Equation (A.5) implies that

$$\tilde{ND}(ND_a, Div_a) < \hat{ND}(ND_a). \tag{A.6}$$

Let $ND_a(n)$ be the net debt of a firm with asset maturity n and capital age a with other quantities made dependent on n in a similar way. We first want to establish that $ND_a(n) = ND_{a+1}(n+1)$ for $a \geq 0$. We do so using backward induction. Lemma 3 implies that $ND_0(n) = ND_0(n+1) = \phi\pi$. We additionally know that $ND_{a=n-1}(n) < 0 < \phi\pi$ and similarly that $ND_{a=n}(n+1) < 0 < \phi\pi$ as otherwise the firm cannot finance investment since $\phi\pi + \pi < K$. This together with Lemma 2 implies that $Div_0(n) = Div_0(n+1) = 0$. Therefore,

$$ND_{a=n-1}(n) = ND_{a=n}(n+1) = \hat{ND}(\phi\pi).$$

We can now establish recursively that $ND_a(n) = ND_{a+1}(n+1)$. Indeed assume that $ND_a(n) = ND_{a+1}(n+1)$. There are two cases to consider.

Case 1: If $\phi\pi \geq \hat{ND}(ND_a(n))$ then $\phi\pi \geq \hat{ND}(ND_a(n)) > \tilde{ND}(ND_a(n), Div_a)$ for any $Div_a > 0$, see equation (A.6), and it cannot be the case that the firm pays dividends at time a because in that case the debt level at $a-1$ would have been $\phi\pi > \tilde{ND}(ND_a(n), Div_a)$, which violates Lemma 2. As a result, when $\phi\pi \geq \hat{ND}(ND_a(n))$ then $ND_{a-1}(n) = \hat{ND}(ND_a(n))$ and via the same reasoning $ND_a(n+1) = \hat{ND}(ND_{a+1}(n+1)) = \hat{ND}(ND_a(n))$. Therefore,

$$ND_{a-1}(n) = ND_a(n+1) = \hat{ND}(ND_a(n)).$$

Case 2: If $\phi\pi < \hat{ND}(ND_a(n))$ then it must be that the firm pays dividends since otherwise the debt level in the previous period would violate the borrowing constraint. Given that the firm pays dividends and Lemma 2, we must have that

$$ND_{a-1}(n) = ND_a(n+1) = \phi\pi.$$

This recursively establishes that $ND_a(n) = ND_{a+1}(n+1)$ for $a \geq 0$. Furthermore, we have $ND_0(n+1) = \phi\pi = ND_0(n) = ND_1(n+1)$; see Lemma 3.

A firm with assets that have a useful life of $n+1$ periods that issues debt with a maturity that is one year longer than a firm with assets that have a useful life of n, has net debt dynamics $ND_{a+1}(n+1) = ND_a(n)$ for $a \geq 0$ with $ND_0(n+1) = ND_1(n+1) = \phi\pi$, which we just showed is the optimal net debt level when the useful life of assets is $n+1$. This in turn implies that $M_{a+1}(n+1) = M_a(n)$ and, in combination Proposition 3, leads to the desired result. □

C Data Definitions and Summary Statistics

I Capital IQ Maturity Data

We supplement the firm-level debt maturity proxy derived from Compustat with a more detailed measure from Capital IQ security issuance data, which covers the period of 2002 to 2018. To merge the security- and firm-level data, we use the most recent filing dates and remove any observations with the same ID/date, description, maturity, and interest rate. We further remove all securities with missing `gvkey` and drop entries for credit lines that reflect the drawdown limit only, as opposed to actual utilisation. We drop all observations with missing or negative maturity values. We then compute the firm-level maturity as the weighted average of individual-security maturities weighted by their notional amounts. As the final data filter, we drop observations for which the total debt in Capital IQ is greater than Compustat by more than 10%, as in Colla, Ippolito, and Li (2013) and Choi et al. (2018).

II Definitions of Variables

The variables used in the paper are defined in Table A.1.

Variable	Definition
Capital age	See Subsection II.A
Useful life	See Subsection II.A
Net debt to EBITDA	Ratio of total debt (`dltt+dlc`) less cash (`che`) over EBITDA (`ebitda`); set to missing when EBITDA is negative

Table A.1: **Definitions of variables**. The table contains the definitions of all variables used throughout the paper (in order of appearance).

Variable	Definition
Net market leverage	Ratio of total debt (dltt+dlc) less cash (che) over total debt plus market value of equity (prcc_f*csho)
Net book leverage	Ratio of total debt (dltt+dlc) less cash (che) over total assets (at)
% debt maturing > 3y	Ratio of long-term debt (dltt) minus debt maturing in 2- and 3-years (dd2+dd3) over total debt (dlc+dltt)
% debt maturing > 5y	Ratio of long-term debt (dltt) minus debt maturing in 2-, 3-, 4-, and 5-years (dd2+dd3+dd4+dd5) over total debt (dlc+dltt)
Debt maturity (yr.)	Average maturity of outstanding bonds and loans from Capital IQ, weighted by their notional amounts
Investment	Capital expenditures (capx) over lagged installed capital (l.ppegt)
Profitability	Operating income (oibdp) over total assets (at)
Size	Natural log of real sales (log(sale/defl)), where defl is the CPI deflator
Tangibility	Ratio of property, plant and equipment (ppent) to total assets (at)
Market-to-book	Ratio of the sum of market value of equity (prcc_f*csho) and book value of debt (at-ceq) to total assets (at)
Cash flow volatility	Moving 3-year standard deviation of profitability
R&D	Ratio of R&D expenditure (xrd) to sales (sale), missing values replaced with zero
Firm age	Time since listing (defined as the first appearance of each firm in CRSP) in years
Asset maturity	Gross property, plant and equipment over depreciation and amortization (ppegt/dp) times the proportion of property, plant and equipment in total assets (ppegt/at), plus current assets over the cost of goods sold (act/cogs) times the proportion of current assets in total assets (act/at); we cap it at 25 years
Abnormal earnings	The difference between the income before extraordinary items, adjusted for common stock equivalents (ibadj-l.ibadj) over the market value of equity used in calculating earnings per share (prcc_f*cshpri)

Table A.1: **Definitions of variables**. The table contains the definitions of all variables used throughout the paper (in order of appearance).

Variable	Definition
Investment skewness (firm-level)	The firm-level skewness of investment, measured as the ratio of capital expenditures (`capx`) over lagged installed capital (`l.ppegt`); we require at least 5 observations per firm
Investment kurtosis (firm-level)	The firm-level kurtosis of investment, measured as the ratio of capital expenditures (`capx`) over lagged installed capital (`l.ppegt`); we require at least 5 observations per firm
Return on investment	EBITDA (`ebitda`) over total assets (`at`)
Alternative tangibility	Ratio of the sum of 0.715 times receivables (rect), 0.547 times inventory (invt), and 0.535 times property, plant, and equipment (ppent) to book assets (at)
Debt cycle length	Number of years to the first leverage spike, between subsequent leverage spikes, or after the last spike, conditional on a minimum cycle length of 3 years
Alternative capital age (1)	Capital age calculated as in Subsection II.A, except that, when the firm disinvests, the oldest vintages are disposed of first, rather than all vintages equally
Alternative capital age (2)	Accumulated (`dpact`) to current (`dpc`) depreciation expense
Alternative capital age (3)	The weighted average of capital vintages, when averaging over the past T and where more weight is put on younger vintages, following Ai et al. (2012) with $T = 7$
Alternative depreciation rate	Depreciation expense (`dpc`) over net plant, property and equipment (`ppent`), winsorized at 1% and 99% levels before calculating capital age
Alternative capital age (4)	Capital age calculated as in Subsection II.A using the depreciation rate from Compustat
Alternative depreciation rate excluding amortization	Depreciation expense (`dpc`) minus amortization of intangibles (`am`) over net plant, property and equipment (`ppent`), missing amortization values replaced with zero, winsorized at 1% and 99% levels before calculating capital age
Alternative capital age (5)	Capital age calculated as in Subsection II.A using the depreciation rate exluding amortization from Compustat

Table A.1: **Definitions of variables**. The table contains the definitions of all variables used throughout the paper (in order of appearance).

III Summary Statistics

Table A.2 contains the summary statistics of all the variables used in the paper which were not provided in Table 1.

	Mean	Std. dev.	Q1	Median	Q3	N
Depreciation rate (BEA)	0.085	0.029	0.068	0.081	0.098	68833
Profitability	0.142	0.073	0.091	0.134	0.183	68833
Size	5.469	1.978	4.093	5.468	6.825	68833
Market-to-book	1.454	0.767	0.982	1.223	1.650	68833
Tangibility	0.362	0.230	0.179	0.317	0.520	68790
Cash flow volatility	0.040	0.039	0.016	0.029	0.050	60424
R&D	0.008	0.017	0.000	0.000	0.007	68833
Firm age	19.198	17.280	6.674	14.085	25.674	66739
Asset maturity	10.154	7.198	4.390	8.416	14.634	67109
Abnormal earnings	0.006	0.188	-0.020	0.009	0.035	66938
Inv. skewness	0.992	0.867	0.391	0.918	1.515	4387
Inv. kurtosis	3.755	2.692	2.116	2.903	4.446	4387
Alternative tangibility	0.418	0.111	0.353	0.443	0.503	67800
Return on investment	0.142	0.073	0.091	0.134	0.183	68833
Alternative capital age (1)	5.753	2.931	3.564	5.336	7.506	66206
Alternative capital age (2)	5.782	3.471	3.351	5.173	7.448	68125
Alternative capital age (3)	3.350	0.758	2.907	3.361	3.796	52697
Depreciation rate (Compustat)	0.196	0.260	0.099	0.138	0.202	74039
Alternative capital age (4)	5.511	2.779	3.493	5.116	7.031	74039
Depreciation rate excl. amortization	0.170	0.176	0.096	0.132	0.185	74039
Alternative capital age (5)	5.639	2.748	3.628	5.250	7.159	74039

Table A.2: **Summary statistics**. The table contains the summary statistics of the variables used in the regression models of net leverage and debt maturity. The sample period is from 1975 to 2018. All variables are winsorized at 1% and 99% levels and defined in Table A.1.

Internet Appendix to: Financing Cycles

Thomas Geelen[†] Jakub Hajda[‡] Erwan Morellec[§] Adam Winegar[¶]

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This appendix contains:

1. Additional results related to capital depreciation, investment and debt dynamics, cash-flow versus asset-based borrowing constraints, and the impact of shocks.

2. Additional robustness tests that support the model predictions.

Other Forms of Capital Depreciation

Our model assumes that the efficiency of capital goods follows a one-hoss shay pattern, as in e.g. Arrow (1964), Rogerson (2008), Rampini (2019), or Livdan and Nezlobin (2021). This form of capital efficiency keeps the model tractable since capital age a is a sufficient statistic for the state of the firm when $t > 0$. This in turn allows us to generate crisp empirical predictions on financing decisions and debt maturity choices.

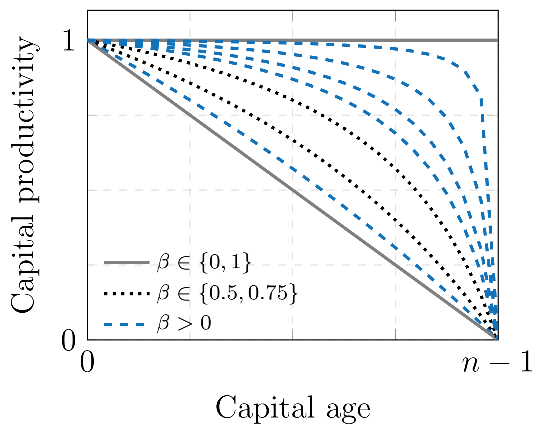
An important question is whether this form of capital efficiency is necessary for our results. *The short answer is no.* Debt cycles are generated by large replacement investments financed with debt. Thus, any form of depreciation that leads to large replacement investments suffices (see Proposition 4). But what forms of depreciation have this feature?

The U.S. Bureau for Labor Statistics (BLS) estimates the productivity of capital in place relative to the productivity of new capital (or, equivalently, the productivity of capital a years after it has been installed) using the function

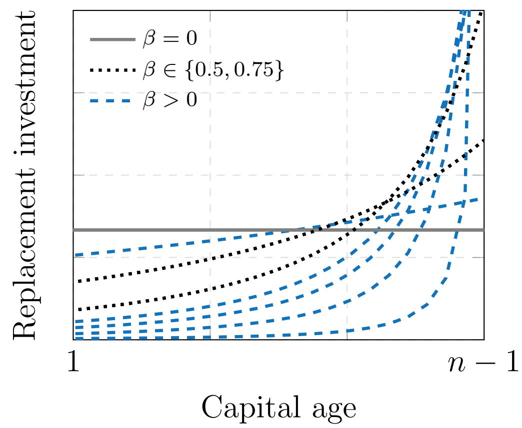
$$S(a|\beta) = \mathbb{I}_{\{a \leq n-1\}} \frac{n-a}{n-\beta a},$$

where $\beta \in [0, 1]$; see Giandrea et al. (2021). Our model with capital that has a finite useful life represents the case in which $\beta = 1$. The case $\beta = 0$ corresponds to a linear decrease in asset productivity. Figure IA.1a shows intermediate cases $\beta \in (0, 1)$. A linear decrease in productivity implies that the replacement investment needed to compensate for the lost productivity of the original capital is constant, in that $S(a-1|0) - S(a|0) = \frac{1}{n}\mathbb{I}_{\{a \leq n\}}$. By

[†] Copenhagen Business School and Danish Finance Institute, Denmark
[‡] HEC Montréal, Canada
[§] EPF Lausanne, Swiss Finance Institute, and CEPR, Switzerland
[¶] BI Norwegian Business School, Norway



(a) Capital productivity



(b) Replacement investment

Figure IA.1: **Depreciation and replacement investments.** The figure shows the productivity of capital $S(a|\beta)$ as it ages and the replacement investment $S(a-1|\beta) - S(a|\beta)$ necessary due to the depreciation of the original capital.

contrast, any form of depreciation with $\beta > 0$ back loads the replacement investment leading to large planned replacement investments in capital, as shown by Figure IA.1b. The U.S. Bureau for Labor Statistics uses $\beta = 0.75$ for structures and $\beta = 0.5$ for equipment (see Giandrea et al., 2021). With $\beta = 0.75$ and $n = 4$ (respectively $n = 5$), the firm makes 57.1% (respectively 50%) of its replacement investments in the last useful year of the asset.

We now allow for arbitrary depreciation schedules of capital assuming that capital fully depreciates in n periods, where n can be arbitrarily large. Let π_t be the firm profits at time t. To keep the analysis tractable, we make the following two assumptions:

1. The borrowing constraint is time-invariant

$$D_t \leq \Phi.$$

2. Given the optimal policies, the firm generates enough profits to make interest payments

$$\pi_t > \rho_D \Phi \geq \rho_D D_t.$$

Under these restrictions, we can establish that given an arbitrary form of capital depreciation and an arbitrary distribution of the firm's capital age:

Proposition 4 (Ageing Capital and Leverage with Arbitrary Capital Depreciation)**.** *Let time $T > t$ be the next time that the firm invests. Then for $t' \in \{t, ..., T-2\}$, capital ages* *while net debt weakly declines*

$$ND_{t'+1} \leq ND_{t'}.$$

Proof of Proposition 4. We first establish that the firm never defaults and that the borrowing constraint must bind at time t for the firm to pay dividends at time $t+1$ (the equivalent of Lemma 2). We then show that once net debt starts decreasing it does so (at least) until the firm invests. As a consequence, during this period, capital ages while net debt declines.

The second condition ensures that the firm never defaults since it can always make interest payments and therefore the rate of return on debt is ρ_D.

We want to show that the firm's net debt is weakly decreasing in capital age at least until the firm invests. To obtain this result, we first need to show that the firm only pays dividends when the borrowing constraint binds in the previous period. We show below that it is suboptimal for the firm to pay dividends at time $t+1$ if the borrowing constraint does not bind at time t. Therefore, the borrowing constraint must bind at time t if the firm pays dividends at time $t+1$.

We first establish that if the firm has net debt $ND_t < \Phi$ then $Div_{t+1} = 0$. For this purpose, assume that we have $Div_{t+1} > 0$ while $ND_t < \Phi$ for some t. Define ΔDiv_t as

$$\Delta Div_t = \min \left\{ \frac{Div_{t+1}}{1+\rho_D}, \Phi - ND_t \right\}.$$

Increasing dividends at time t to $Div'_t = Div_t + \Delta Div_t$ using debt financing implies that $Div'_{t+1} \geq Div_{t+1} - (1+\rho_D)\Delta Div_t$, where the inequality follows from the fact that the interest rate is lower if net debt is negative (i.e. if $ND_t < 0$); see footnote 27. This change in policy would increase shareholder value since its effect on equity value (at time t) is at least

$$\Delta Div_t - \frac{(1+\rho_D)\Delta Div_t}{1+r} > 0,$$

which contradicts optimality of the firm's policies. Therefore, if $Div_{t+1} > 0$ then $ND_t = \Phi$.

Next, we show that net debt weakly decreases over time at least until the firm invests. Let $t' \in \{t, T-2\}$ where T is the next time the firm invests. There are two cases. First, if $ND_{t'} = \Phi$ then $ND_{t'} \geq ND_{t'+1}$ because of the borrowing constraint. Second, if $ND_{t'} < \Phi$ then the firm does not pay dividends at time $t'+1$ since $ND_{t'} < \Phi$. Furthermore, $ND_{t'} > ND_{t'+1}$ since $\pi_{t'+1} > \rho_D \Phi > \rho_D ND_{t'}$. Finally, from t' to $t'+1$ installed capital becomes a year older since there is no investment while net debt weakly decreases. □

Investment and Debt Dynamics

In the baseline model, the firm can invest in one unit of capital and replaces it every n periods. Assume now that the firm has N units of capital of different vintages, each of which generates a profit π per period for n periods. Furthermore, assume the firm replaces capital when it reaches the end of its useful life. Therefore, the firm invests at times $n * i - a_k^0$, $\forall i \in \mathbb{N}$ and $\forall k \in \{1, ..., N\}$, where a_0^k is the age of capital unit $k \in \{1, ..., N\}$ at time zero. We can then show that:

Proposition 5 (Debt Cycles with Multiple Assets)**.** *Let time $T > t$ be the next time that the firm invests. Then for $t' \in \{t, ..., T-2\}$, capital ages while net debt weakly declines*

$$ND_{t'+1} \leq ND_{t'}.$$

Proof of Proposition 5. Since the firm invests by replacing capital at the end of its useful life we have that:

1. The borrowing constraint is time-invariant

$$D_t \leq N\phi\pi.$$

2. Given the optimal policies and the parameter restriction on ϕ, the firm generates enough profits to make interest payments

$$N\pi > \rho_D N\phi\pi \geq \rho_D D_t.$$

The proof is then the same as the proof of Proposition 4. □

We can also show that:

Proposition 6 (Maturity Cycles with Multiple Assets)**.** *Let time $T > t$ be the next time that the firm invests. Assume that net debt at time $T-1$ is negative, i.e. $ND_{T-1} < 0$. Then for $t' \in \{t, ..., T-2\}$, debt maturity weakly declines in that*

$$M_{t'+1} \leq M_{t'}.$$

Furthermore, keeping ND_{T-1} constant, we have that for $t' \in \{t, ..., T-1\}$ the debt maturity is longer if investment (date T) gets delayed in that

$$\frac{\partial M_{t'}}{\partial T} \geq 0.$$

Proof. Similar arguments as in the proof of Proposition 3 imply $M_{t'}(T) \geq M_{t'+1}(T)$ for $t' \in \{t, ..., T-1\}$ where T indicates the next investment date.

Backward induction using the fact that the firm only pays dividends when its debt is at $ND_t = N\phi\pi$ and the fact that $ND_T(T) = ND_{T+1}(T+1)$ implies that $ND_{t'}(T) = ND_{t'+1}(T+1)$ for $t' \in \{t, ..., T-1\}$. This in combination with the fact that $ND_T(T) = ND_{T+1}(T+1) < 0$ implies that $M_{t'}(T) = M_{t'+1}(T+1)$ for $t' \in \{t, ..., T-1\}$ and as a consequence for $t' \in \{t, ..., T-1\}$ $M_{t'}(T+1) \geq M_{t'+1}(T+1) = M_{t'}(T)$. □

Cash-Flow Versus Asset-Based Borrowing Constraints

While in the paper, we rely on a cash-flow based borrowing constraints (Lian and Ma, 2021). As we now show, debt cycles would mechanically be stronger with an asset-based borrowing constraint (Rampini and Viswanathan, 2022).

Let V_a be the residual value of capital, which we define as the present value of future cash flows that capital with age a generates:

$$V_a = \frac{\pi}{1+r} + ... + \frac{\pi}{(1+r)^{n-a}} = \sum_{t=a+1}^{n} \frac{\pi}{(1+r)^{t-a}}.$$

We assume that the replacement value of capital $\tilde{V}_a = \frac{K}{V_0} V_a$ is proportional to the residual value of capital V_a such that for new capital the replacement value is equal to the purchase price of new capital $\tilde{V}_0 = K$.

Assuming the firm is producing and with a debt repayment due next period of $D_a(1+\rho_D)$, an asset-based borrowing constraint would restrict debt to be less than some fraction $\chi \in \left[0, \frac{1}{1+\rho_D}\right]$ of the capital's residual value

$$D_a < \chi \tilde{V}_a.$$

Since the replacement value of assets $\tilde{V}_a$ decreases with capital age, such a constraint can only strengthen the debt cycles identified in Theorem 1. The reason is that firms are forced to deleverage because the borrowing constraint becomes tighter as capital ages, which does not happen with a cash-flow based borrowing constraint.

Shocks

While solving a general dynamic financing and investment model with shocks would be computationally infeasible, we can derive interesting results on shocks and financing cycles by specializing the model further. Indeed, assume that the firm has a unit mass of divisions $i \in [0, 1]$ where each division i is the same as a firm in our baseline model in that it can use one unit of capital to generate a cash flow π against which it can borrow. Assume that $\rho_C \to r$ and $\phi\pi + \pi < K$. Furthermore, assume that each division i faces shocks such that division i at time t cannot replace assets (for example because of a credit supply shock, a supply chain disruption leading to time-to-build, or the departure of the division manager or of key personnel). We assume that the state of the new capital market for division i follows a Markov chain. With probability $\mathbb{P}_{A \to U} = q$, new capital becomes unavailable for a single period $\mathbb{P}_{U \to A} = 1$. We assume that q is sufficiently small.

The assumption that $\rho_C \to r$ ensures that the firm does not alter its investment decisions to optimise its capital structure since the benefits from changes in capital structure are small compared to any cost of speeding up/delaying investment. Therefore, each division replaces capital once it reaches the end of its useful life and new capital is available. As the following proposition shows, financing dynamics are driven by investment dynamics, which depend on the correlation of the shocks across divisions.

Proposition 7 (Shock Correlation and Financing Dynamics)**.** *In the stationary solution of the model:*

- *If* ***shocks are perfectly correlated*** *across divisions then all capital has the same age and firm financing is characterized by* ***cycles*** *as in the baseline model (Theorem 1 and 2, and Proposition 3) in that for* $a \in \{0, ..., n-2\}$*:*[28]

$$\begin{aligned} ND_{a+1} &\leq ND_a, \\ M_{a+1} &\leq M_a. \end{aligned}$$

- *If* ***shocks are uncorrelated*** *across divisions then average capital age is constant through time, the firm has* ***constant leverage*** *equal to* $ND_t = \phi\pi(1-q)$*, and has a constant debt maturity.*

Proof of Proposition 7. First, observe that tax benefits of debt are small because $\rho_D \to r$ (since $\rho_C < \rho_D$ and $\rho_C \to r$)

$$\frac{\phi\pi}{\rho_D} - \frac{\phi\pi}{r} \to 0.$$

As a consequence, two firms that do not default and have the same investment dynamics but make different financing choices will have a negligible difference in value since $\rho_C \approx \rho_D \approx r$.

[28] What happens to the net debt dynamics when financing is not available. If at time t, the firm should replace capital then $0 = \pi - K + \phi\pi - (1+\rho_C)ND_{t-1}$. If at time t no financing is available then the firm will replace new capital at $t+1$ without generating any income and therefore $0 = -K + \phi\pi - (1+\rho_C)ND_t$. As consequence, in this situation $ND_t = \frac{\phi\pi - K}{1+\rho_C} \leq \frac{\phi\pi + \pi - K}{1+\rho_C} = ND_{t-1} < 0$. Furthermore, $M_{t-1} = M_t = 0$ since $ND_t \leq ND_{t-1} < 0$.

Therefore, a firm that does not default will not alter its investment strategy to accommodate a different financing strategy since changes in the investment strategy (delaying or speeding up investment) face first-order costs while changes in financing do not. It is therefore optimal for the firm to invest when capital reaches the end of its useful life or the period thereafter in case capital is unavailable since investment is positive NPV and $\rho_C \to r$.

Before we can continue, we first need to show that $\pi(1-q) - K/n > \rho_D \phi \pi (1-q)$, i.e. that the firm can repay debt in a model with uncorrelated shocks. From equation (A.3) and $\rho_C \to r$ it follows that

$$\phi < \frac{1}{r} - \frac{K}{\pi}\left(1 + \frac{1}{r}\frac{r}{(1+r)^n - 1}\right).$$

Observe that

$$\left(1 + \frac{1}{(1+r)^n - 1}\right) - \frac{1}{nr} = \frac{nr(1+r)^n - (1+r)^n + 1}{nr\left((1+r)^n - 1\right)}.$$

This equation has the same sign as $nr(1+r)^n - (1+r)^n + 1$ and by direct calculation for $n = 1$ this equation is positive. Furthermore,

$$\{nr(1+r)^n - (1+r)^n + 1\} - \left\{(n+1)r(1+r)^{n+1} - (1+r)^{n+1} + 1\right\} = -(n+1)r^2(1+r)^n < 0,$$

which allows us to recursively establish that for $n > 1$

$$\left(1 + \frac{1}{(1+r)^n - 1}\right) - \frac{1}{nr} > 0.$$

and therefore

$$\rho_D \phi \pi \leq r\phi\pi < \pi - Kr\left(1 + \frac{1}{(1+r)^n - 1}\right) < \pi - \frac{K}{n}.$$

Uncorrelated shocks: Assume the firm does not default. Then, in a stationary equilibrium the firm spends K/n on new capital each period and average capital age is constant throughout time in a model with uncorrelated shocks. The firm prefers to have debt outstanding because $\rho_D < r$. Always having the maximum amount of debt $\phi\pi(1-q)(\to \phi\pi)$ outstanding is feasible since the firm's operating profits minus investment cost exceed debt payments $\pi(1-q) - K/n > \rho_D \phi\pi(1-q)$. Therefore, the firm keeps its leverage at the constraint $\phi\pi(1-q)$. Furthermore, the firm has no incentive default since dividends are positive each period $Div = \pi(1-q) - K/n - \rho_D\phi\pi(1-q) > 0$. The firm issues perpetual debt to minimize issuance cost. Note also that our setup implies that $\epsilon \to 0$ before $\rho_C \to r$ and as a consequence debt issuance costs are always smaller than the benefits to debt issuance.

Perfectly correlated shocks: Assume the firm does not default. Then, in a stationary equilibrium, the firm spends K on new capital when old capital reaches the end of its useful life and new financing is available. In a model with perfectly correlated shocks, this implies that all capital has the same age.

The firm tries to maximize the amount of debt it has conditional on being able to invest when capital reaches the end of its useful life. This implies that the firm must follow the same financing strategy as in Theorem 1. Indeed, if the firm issued more debt, then it would be unable to invest when capital reaches the end of its useful life, which cannot be optimal since the benefits to debt issuance are small while investment is positive NPV. Therefore, it must be that $ND_{a=n-1} < 0$ since $\phi\pi + \pi < K$. Assume now that new financing is unavailable and the assets have reached the end of their useful life, then the firm can delay investment by one period since $ND_{a=n-1} < 0$. The firm will then have enough funds to invest next period since $\rho_C > 0$.

From Theorem 1 it follows that net debt is decreasing in capital age as long as the firm has productive capital.Suppose that the shock hits when the firm has to invest at time t, forcing the the firm to postpone investment. If at time t, the firm should replace capital then from the budget constraint we have that $(1 + \rho_C)ND_{t-1} = \pi - K + \phi\pi$. Because the firm has to postpone investment to $t + 1$ and generates no income, we also have that $(1 + \rho_C)ND_t = -K + \phi\pi$. We thus get that $ND_t = \frac{\phi\pi - K}{1+\rho_C} \leq \frac{\phi\pi+\pi-K}{1+\rho_C} = ND_{t-1} < 0$.

Similar arguments as in Theorem 1, in combination with the fact that q is small, imply that when new financing is available the firm has no incentive to default. When new financing is not available and the asset has not reached the end of its useful life, then the structure of the Markov chain and the same arguments as above imply that the firm has no incentive to default. When new financing is not available and the asset has reached the end of its useful life, the firm holds cash and cannot borrow since it generates no profits. Furthermore, $\rho_C \to r$ and future investment opportunities are positive NPV. Therefore, the firm has no incentive to default.

Finally, the same arguments as in Theorem 2 and Proposition 3 imply that $M_{a+1} \leq M_a$. Notably, if at time t no new financing is available and the firm wants to invest then $M_{t-1} = M_t = 0$ since $ND_t \leq ND_{t-1} < 0$ and our result thus implies that the maturity is weakly decreasing until the next date the firm invests (as in Proposition 6). □

Robustness and Additional Results

1. Table IA.1 documents the importance of capital age and all the other factors used in the financing and debt maturity regressions following the approach of Frank and Goyal (2009).

2. Table IA.2 presents estimates from regressions of financing and debt maturity on lagged capital age for different definitions of industries.

3. Table IA.3 presents estimates from regressions of firm-level maximum and average debt cycle lengths based on net book leverage spikes on measures of asset life.

4. Table IA.4 presents estimates from regressions of firm-level maximum and average debt cycle lengths requiring five years between leverage spikes on measures of asset life.

Panel A: Net leverage (net book leverage)

			Adjusted within R^2	
Explanatory variable	Coef.	t-stat.	Individual	Cumulative
Capital age	-0.045***	-14.17	0.034773	0.034773
Profitability	-0.031***	-14.8	0.025098	0.058086
Market-to-book	-0.026***	-11.53	0.016957	0.06327
Tangibility	0.049***	8.62	0.015885	0.07083
Size	0.074***	9.25	0.015319	0.081152
Cash flow volatility	-0.007***	-3.36	0.001159	0.081143
Firm age	0.044	1.14	0.000147	0.081202
R&D	0.001	0.21	-0.000029	0.081183

Panel B: Debt maturity (% of debt maturing in > 3 years)

			Adjusted within R^2	
Explanatory variable	Coef.	t-stat.	Individual	Cumulative
Net book leverage	0.047***	11.07	0.011149	0.011149
Capital age	-0.044***	-10.18	0.007835	0.016054
Size	0.093***	7.8	0.005774	0.018239
Size squared	0.067***	5.82	0.003287	0.019673
R&D	-0.012*	-1.81	0.000212	0.019831
Asset maturity	-0.009*	-1.67	0.000166	0.019824
Cash flow volatility	-0.003	-0.96	0.000028	0.019825
Abnormal earnings	-0.001	-0.65	-0.000022	0.01982
Market-to-book	0.001	0.2	-0.000035	0.020063
Firm age	-0.008	-0.17	-0.000037	0.020046

Table IA.1: **Capital age and financing – importance of individual determinants**. This table presents estimates from regressions of net book leverage (for comparability to Frank and Goyal (2009); Panel A) debt maturity (% of debt maturing in > 3 years; Panel B) on lagged controls from Table 2 in Panel A and from Table 3 in Panel B. We obtain the coefficient estimates, t-statistic and the individual adjusted within R^2 by regressing net book leverage or debt maturity on each explanatory variable. We then sort the variables by their individual adjusted within R^2 and regress net book leverage by consecutively adding explanatory variables, which allows to obtain the cumulative adjusted within R^2. All regressions include firm and industry-year fixed effects, created using the Hoberg-Phillips fixed industry classification with 100 industries, and are run on the same sample as the regression model in column (3) in Table 3. All variables are defined in Table A.1. We use * $p < 0.10$, ** $p < 0.05$, and *** $p < 0.01$ to indicate p-values.

Panel A: Net leverage

	ND/EBITDA		Net book leverage		Net market leverage	
	(1)	(2)	(3)	(4)	(5)	(6)
Capital age	-0.386***	-0.386***	-0.033***	-0.033***	-0.032***	-0.032***
	(-7.25)	(-8.71)	(-9.58)	(-11.66)	(-8.89)	(-10.01)
Ind.-Yr. FE (HP50)	Yes	No	Yes	No	Yes	No
Ind.-Yr. FE (FF49)	No	Yes	No	Yes	No	Yes
Controls	Yes	Yes	Yes	Yes	Yes	Yes
Firm FE	Yes	Yes	Yes	Yes	Yes	Yes
Observations	32763	47944	32763	47944	32763	47944
Adj. within R^2	0.0414	0.0418	0.0806	0.0838	0.1148	0.1107

Panel B: Debt maturity

	% debt maturing $> 3y$		% debt maturing $> 5y$		Debt maturity (yr.)	
	(1)	(2)	(3)	(4)	(5)	(6)
Capital age	-0.029***	-0.030***	-0.021***	-0.024***	-0.439***	-0.365***
	(-6.60)	(-8.28)	(-4.60)	(-6.38)	(-2.92)	(-2.62)
Firm FE	Yes	Yes	Yes	Yes	Yes	Yes
Ind.-Yr. FE (HP50)	Yes	No	Yes	No	Yes	No
Ind.-Yr. FE (FF49)	No	Yes	No	Yes	No	Yes
Observations	31778	46715	31778	46715	11483	12530
Adj. within R^2	0.0195	0.0183	0.0082	0.0097	0.0054	0.0052

Table IA.2: **Capital age and financing – alternative industry definitions**. This table presents estimates from regressions of net leverage (Panel A) and debt maturity (Panel B) on lagged capital age for different definitions of industries. The dependent variables in Panel A are *Net debt to EBITDA* in columns 1 and 2, *Net book leverage* in columns 3 and 4 and *Net market leverage* in columns 5 and 6. The dependent variables in Panel B are *% of debt maturing in > 3 years* in columns 1 and 2; *% of debt maturing in > 5 years* in columns 3 and 4; and *Debt maturity (yr.)* in columns 5 and 6. We control for all independent variables from Table 2 in the net leverage regressions and from Table 3 in the debt maturity regressions. Each explanatory variable is standardized by its full-sample standard deviation. All models include industry-year fixed effects created using the Hoberg-Phillips fixed industry classification with 50 industries (*HP50*) and the Fama-French 49 industries (*FF49*). All variables are defined in Table A.1. t-statistics are reported in parentheses. Standard errors are clustered at the firm level. We use * $p < 0.10$, ** $p < 0.05$, and *** $p < 0.01$ to indicate p-values.

	Max debt cycle		Avg. debt cycle	
	(1)	(2)	(3)	(4)
Panel A: Useful life				
Useful life	0.225***	0.100***	0.151***	0.073***
	(6.36)	(4.17)	(5.96)	(3.56)
Controls	No	Yes	No	Yes
Observations	2021	2013	2021	2013
Adj. R^2	0.034	0.254	0.023	0.176
Panel B: Average capital age				
Capital age	0.667***	0.121**	0.464***	0.107**
	(9.69)	(2.04)	(9.83)	(2.33)
Controls	No	Yes	No	Yes
Observations	2024	2016	2024	2016
Adj. R^2	0.080	0.253	0.057	0.176
Panel C: Asset maturity				
Asset maturity	0.121***	0.102***	0.077***	0.066**
	(3.25)	(3.05)	(2.99)	(2.45)
Controls	No	Yes	No	Yes
Observations	1999	1991	1999	1991
Adj. R^2	0.017	0.254	0.010	0.175

Table IA.3: **Asset life and net book leverage cycles – cross-sectional regressions**. The dependent variable is *Maximum debt cycle length* in columns 1 and 2, and *Average debt cycle length* in columns 3 and 4, where cycle length depends on spikes in net book leverage. Firms with no leverage spike have a cycle length that is undefined and are dropped from the sample. We require a minimum of three years between subsequent spikes. In specifications 2 and 4 we control for all independent variables from Table 2. t-statistics are reported in parentheses. Standard errors are clustered at the industry level using Hoberg-Phillips fixed industry classification with 100 industries. We use * $p < 0.10$, ** $p < 0.05$, and *** $p < 0.01$ to indicate p-values.

	Max debt cycle		Avg. debt cycle	
	(1)	(2)	(3)	(4)
Panel A: Useful life				
Useful life	0.135***	0.071***	0.089***	0.047*
	(5.25)	(2.68)	(3.86)	(1.94)
Controls	No	Yes	No	Yes
Observations	1802	1797	1802	1797
Adj. R^2	0.013	0.225	0.008	0.152
Panel B: Average capital age				
Capital age	0.626***	0.177***	0.451***	0.153***
	(9.43)	(4.04)	(7.94)	(3.76)
Controls	No	Yes	No	Yes
Observations	1802	1797	1802	1797
Adj. R^2	0.074	0.227	0.051	0.155
Panel C: Asset maturity				
Asset maturity	0.050*	0.067*	0.032	0.033
	(1.69)	(1.97)	(1.41)	(1.04)
Controls	No	Yes	No	Yes
Observations	1774	1769	1774	1769
Adj. R^2	0.003	0.224	0.001	0.150

Table IA.4: **Asset life and debt cycles – five-year filter**. The dependent variable is *Maximum debt cycle length* in columns 1 and 2, and *Average debt cycle length* in columns 3 and 4. Firms with no leverage spike have a cycle length that is undefined and are dropped from the sample. We require a minimum of five years between spikes. In specifications 2 and 4 we control for all independent variables from Table 2, except for asset maturity. t-statistics are reported in parentheses. Standard errors are clustered at the industry level using Hoberg-Phillips fixed industry classification with 100 industries. We use * $p < 0.10$, ** $p < 0.05$, and *** $p < 0.01$ to indicate p-values.