Can the Fed Control Inflation? Stock Market Implications

Daniel Andrei* Michael Hasler†

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Abstract

This paper investigates the stock market implications of the Federal Reserve's ability to control inflation, focusing on investor uncertainty and learning about it. Investor uncertainty about the Fed's ability to control inflation heightens stock market volatility and risk premium, particularly during pronounced monetary tightening and easing cycles. Moreover, investor learning generates an asymmetry that amplifies the impact of inflation surprises when the Fed tightens or loses its inflation control credibility, causing particularly high volatility and risk premium. Empirical tests support our model's predictions, highlighting the importance of investors learning about the Fed's ability to control inflation in shaping financial markets.

Keywords: Asset Pricing, Inflation, Federal Reserve, Rate Hikes
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*McGill University, Desautels Faculty of Management, 1001 Sherbrooke Street West, Office 549, Montréal, Québec H3A 1G5, daniel.andrei@mcgill.ca, danielandrei.info.

†University of Texas at Dallas, Naveen Jindal School of Management, 800 W Campbell Rd, Richardson, TX 75080, USA; michael.hasler@utdallas.edu; https://sites.google.com/michaelhasler

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1 Introduction

Inflation, a key economic indicator, can disrupt economies and significantly affect people's well-being. The COVID-19 pandemic led to a resurgence of inflation as governments worldwide implemented drastic measures. Lockdowns forced people to stay home and businesses to halt operations, prompting governments and central banks to introduce lenient fiscal and monetary policies to assist firms and consumers. However, reduced output and a sharp rise in the money supply created an imbalance, sparking inflation. The United States Consumer Price Index (CPI) saw a significant jump one year after lockdowns started, with year-over-year growth hitting 2.6% in March 2021 and exceeding 8.5% in March 2022. In response, the Federal Reserve (henceforth, "Fed") began raising interest rates in March 2022.

This raises an important question: Can the Fed effectively control inflation? The Fed's ability to control inflation greatly influences financial markets, investor trust, and overall economic stability. Without the Fed's inflation-fighting credibility, inflation is at risk of becoming self-perpetuating. This paper explores the Fed's credibility problem from investors' perspective, suggesting that they are uncertain about the Fed's ability to manage inflation and must therefore gather information from observed inflation data. Incorporating this learning process into a general equilibrium model reveals that the market risk premium and volatility increase when the Fed loses its credibility in fighting inflation. Furthermore, when the Fed responds to high inflation data by raising interest rates, it leads to a stock market downturn, heightened volatility, and a surge in the market risk premium.

While scholars and policymakers have long debated the Fed's ability to effectively control inflation (Bernanke and Mishkin, 1997; Svensson, 1999; Clarida, Gali, and Gertler, 2000), and emphasized the importance of maintaining credibility to keep inflation at bay (Kydland and Prescott 1977; Alesina and Summers 1993; Barro and Gordon 1983; Walsh 2017), our study introduces a fresh perspective. We agree that credibility is paramount, but we also focus on the role of investor uncertainty and learning in shaping the relationship between the Fed's credibility and its ability to control inflation. By integrating these elements into our analysis, we explore the following question: What are the stock market implications of investors' uncertainty and learning about the Fed's ability to manage inflation?

Our analysis employs a general equilibrium economy model (Lucas, 1978), featuring a representative agent with Epstein and Zin (1989) preferences who consumes the aggregate output. The nominal price of the consumption good acts as a proxy for the consumer price index. In this setting, the Fed adjusts the nominal interest rate based on the Taylor rule, increasing rates in response to inflation growth or signs of overheating. Meanwhile, the representative agent observes inflation data and updates their beliefs about the Fed's ability to control inflation via interest rate hikes. As the Fed raises interest rates and a subsequent decline in inflation is observed, the agent's confidence in the Fed's ability grows. However, if inflation persists, the agent loses faith in the Fed's ability, realizing that interest rate hikes are insufficient to curb inflation, ultimately eroding the Fed's credibility.

The analysis uncovers two novel effects that set our study apart from existing literature. First, we find that uncertainty about the Fed's ability to control inflation results in higher stock market volatility and risk premium, especially during intense monetary tightening or easing periods. As inflation strays from its target, the Fed's credibility is called into question, prompting the stock market to react strongly to new information. For instance, a high inflation reading during aggressive tightening may cause a significant market decline, similar to a stock market crash. Conversely, a low inflation reading in the same context could trigger a substantial market rally.

The second effect relates to the representative agent's valuation of monetary policy. Assuming a preference for early resolution uncertainty (Bansal and Yaron, 2004), monetary policy is valuable for the agent because it reduces long-run risk. The Fed tightens during overheating and eases during weakening, stabilizing economic cycles. However, this desirable stabilizing force comes at a cost to the stock market, which negatively correlates with the Fed's actions: the market falls when the Fed tightens and rises when the Fed eases. In asset-pricing terms, the stock market is considered a "bad" asset due to its negative correlation with a "good" risk, leading the agent to demand a risk premium to hold it. This effect is asymmetric, depending on the cycle type (tightening or easing). The agent demands a higher risk premium during tightening because learning magnifies the impact of inflation surprises (positive or negative). For instance, a positive inflation surprise during tightening weakens the Fed's credibility, resulting in doubly bad news. In contrast, the agent requires a lower risk premium during easing, as learning reduces the impact of inflation surprises. As a result, investor learning leads to a higher risk premium during tightening periods.

To quantify these effects, we estimate the parameters of the model using Maximum Likelihood, employing data on U.S. real GDP, Federal funds rate, and inflation rate from 1955 to 2021. The estimated parameter values yield asset-pricing moments that are in line with the data. Specifically, the model predicts a real interest rate of 1%, nominal interest rate of 4.5%, market risk premium of 8%, market return volatility of 19%, and market Sharpe ratio of 0.43.

We empirically test the model's predictions using the S&P 500 as a proxy for the market. Our methodology unfolds in several stages. First, we calculate the empirical market risk premium by fitting a regression of future S&P 500 excess return on the current S&P 500 dividend yield (Fama and French, 1989; Cochrane, 2008) and the realized S&P 500 return variance (French, Schwert, and Stambaugh, 1987; Guo, 2006). Next, we determine the empirical market return volatility by fitting an Exponential GARCH model (Nelson, 1991) to the S&P 500 excess return, capturing the asymmetric response of volatility to return shocks of different signs. We also compute the empirical price-dividend ratio by dividing the S&P 500 price by the cumulative dividend paid by the index over the past twelve months. The empirical real interest rate is computed as the Federal funds rate minus the inflation rate. Lastly, the empirical expected output growth rate is obtained by fitting an Autoregressive-Moving-Average model to the real GDP growth rate. We then derive the model-implied market risk premium, market return volatility, market price-dividend ratio, real interest rate, and expected output growth rate by inputting the state variables extracted from the Maximum Likelihood estimation into our theoretical framework.

Our model aligns well with the data, showing positive and statistically significant relations between empirical and model-implied quantities. As predicted by the model, we observe an increase in the empirical real interest rate, expected output growth rate, market risk premium, and market return volatility when the Fed tightens, alongside a drop in the empirical market price-dividend ratio. These relations are statistically significant at the 1% level. The empirical real interest rate, expected output growth rate, and market price drop as inflation increases, consistent with the predictions of the model. Moreover, we find that a decrease in the Fed's ability to control inflation leads to a statistically significant increase in the empirical market risk premium and market return volatility, in line with the model. Overall, our empirical findings support the theoretical predictions of the model, providing further evidence of its validity.

Our paper builds on previous studies examining the Fed's role in controlling inflation and maintaining credibility, as mentioned earlier, along with additional contributions (Bernanke, Laubach, Mishkin, and Posen, 1998; Woodford, 2003). It also relates to studies on investor learning and its impact on market outcomes, such as asset pricing, volatility, and risk premia (Timmermann, 1993; Pastor and Veronesi, 2009). Moreover, we connect our work with the literature on the effects of monetary policy on financial markets and risk premia (Bernanke and Kuttner, 2005; Rigobon and Sack, 2004; Gürkaynak, Sack, and Swanson, 2004). Lastly, the model we develop stems from the general equilibrium literature (Lucas, 1978) and explores the interaction between incomplete information (Detemple, 1986), inflation (Xiong and Yan, 2010; Cochrane, 2011), interest rates (Buraschi and Jiltsov, 2005; Wachter, 2006), and asset prices, thereby expanding the literature on asset prices in monetary economies (Danthine and Donaldson, 1986; Bakshi and Chen, 1996; Gallmeyer, Hollifield, Palomino, and Zin, 2007).

In related work, Bauer, Pflueger, and Sunderam (2022) analyze how professional forecasters perceive the Federal Reserve's monetary policy rule and how these perceptions impact asset prices and monetary policy transmission. They demonstrate that the perceived dependence of the federal funds rate on economic conditions is time-varying and cyclical, with forecasters updating their beliefs in response to monetary policy actions. Their study highlights the importance of understanding public perceptions of monetary policy and their implications for policy effectiveness, which is relevant to our research on the Fed's ability to control inflation. Our paper extends this line of research by incorporating investor uncertainty and learning in a monetary economy, further exploring the relationship between the Fed's credibility and its capacity to effectively manage inflation.

Although our study focuses on investor learning and the Fed's ability to control inflation, we acknowledge that our analysis simplifies the complex economic dynamics by omitting aspects such as fiscal policy (Sargent and Wallace, 1981; Leeper, 1991), international trade and exchange rates (Obstfeld and Rogoff, 1995; Calvo and Reinhart, 2002; Gali and Monacelli, 2005), or the role of a financial intermediation sector in shaping inflation and asset prices (Bernanke, Gertler, and Gilchrist, 1999; Gertler and Kiyotaki, 2010; Corhay and Tong, 2021). Nevertheless, by emphasizing the role of investor learning, our research complements the existing literature and encourages further exploration of the interplay between these areas.

The paper proceeds as follows: Section 2 presents our model and its main implications; Section 3 describes the estimation of model parameters; Section 4 reports the empirical tests and results; and Section 5 concludes with a summary of our findings and potential avenues for future research.

2 Model

The economy is defined over a continuous-time infinite horizon and consists of a single representative agent who derives utility from consumption. The agent has Kreps-Porteus preferences (Epstein and Zin, 1989; Weil, 1990) with a subjective discount rate ρ, relative risk aversion γ, and elasticity of intertemporal substitution ψ. The agent's indirect utility function is given by

$$J_t = \mathbb{E}_t \left[\int_t^\infty h(C_s, J_s) ds \right],$$

where the aggregator h is defined as in Duffie and Epstein (1992):

$$h(C, J) = \frac{\rho}{1 - 1/\psi} \left(\frac{C^{1-1/\psi}}{[(1-\gamma)J]^{1/\theta - 1}} - (1-\gamma)J \right), \quad \text{with } \theta \equiv \frac{1-\gamma}{1-1/\psi}.$$

The aggregate consumption in the economy, denoted by δ, follows the dynamic process

$$\frac{d\delta_t}{\delta_t} = \mu_{\delta,t}dt + \sigma_\delta dB_{\delta,t}, \tag{1}$$

where $\mu_{\delta,t}$ is the expected consumption growth rate, $\sigma_\delta > 0$ is a known constant, and B_δ is a one-dimensional Brownian motion. The expected consumption growth rate varies over time and is determined endogenously from the agent's optimality conditions, as we will demonstrate below.

The consumption price level, p_t, evolves according to

$$dp_t/p_t = \pi_t dt,$$

where π_t is the expected rate of inflation, which follows the mean-reverting process:

$$d\pi_t = \lambda_\pi \left(\overline{\pi}_t - \pi_t\right) dt + \sigma_\pi dB_{\pi,t}. \tag{2}$$

In equation (2), $\lambda_\pi > 0$ is a known constant and represents the mean-reversion speed of inflation, $\sigma_\pi > 0$ is a known constant, and B_π is a one-dimensional Brownian motion uncorrelated with B_δ. The mean of inflation, $\overline{\pi}_t$, varies over time according to

$$\overline{\pi}_t = \breve{\pi} - a_t(r_{N,t} - \overline{r}_N),$$

where $r_{N,t}$ is the nominal interest rate, whose long-term mean is $\overline{r}_N$, $\breve{\pi}$ is the long-term mean of inflation under neutral interest rates (when $r_{N,t} = \overline{r}_N$), and a_t is a parameter that governs how inflation responds to deviations of the nominal rate from its long-term mean.

Our model's central assumption is that the Fed governs the mean of inflation $\overline{\pi}_t$ by setting the nominal interest rate $r_{N,t}$. In doing so, the Fed modifies the gap $\overline{\pi}_t - \pi_t$ in equation (2), which governs the reversion of inflation towards its mean. Importantly, the Fed controls the mean of inflation, $\overline{\pi}_t$, and not directly inflation, creating a lag between rate changes and the inflation's response to those changes.

Consider an example where, without loss of generality, the expected inflation π_t is high, and the Fed is tightening ($r_{N,t} - \overline{r}_N > 0$). Then, a positive value for the parameter a_t implies a low $\overline{\pi}_t$ and a faster reversion of inflation to lower levels. That is, positive values for the parameter a_t imply that the Fed can control inflation by increasing its mean-reversion speed. Conversely, a negative value for the parameter a_t weakens the Fed's ability to bring down inflation, meaning that inflation remains sticky and the Fed cannot control it effectively.

The focus of our paper is on the parameter a_t, which reflects the Fed's *ability* to control inflation. We assume that the representative agent does not observe a_t. That is, the agent is unsure whether the Fed can bring inflation back down in the near future when it has become excessively high. The parameter a_t follows a hidden diffusion process

$$da_t = -\lambda_a a_t dt + \sigma_a dB_{a,t}, \tag{3}$$

where $\lambda_a > 0$ and $\sigma_a > 0$ are known constants, and B_a is a one-dimensional Brownian motion, uncorrelated with B_δ and B_π.

The representative agent observes the process of aggregate consumption δ_t, nominal interest rates $r_{N,t}$ set by the Fed, and consumption prices p_t. Since consumption prices are observable, so is the expected inflation process (2). The history of the expected inflation process together with the history of nominal interest rates allows the agent to learn about Fed's ability to control inflation, i.e., about a_t. Defining $\mathcal{F}_t^{\pi,r_N}$ the information set of the agent at time t, standard filtering theory (Liptser and Shiryaev, 2001) implies that the agent's posterior mean, $\widehat{a}_t \equiv \mathbb{E}[a_t|\mathcal{F}_t^{\pi,r_N}]$, and the posterior variance, $\nu_{a,t} \equiv \mathbb{E}[(a_t - \widehat{a}_t)^2|\mathcal{F}_t^{\pi,r_N}]$, follow

$$d\widehat{a}_t = -\lambda_a \widehat{a}_t dt - \frac{(r_{N,t} - \overline{r}_N)\lambda_\pi \nu_{a,t}}{\sigma_\pi} d\widehat{B}_{\pi,t}, \tag{4}$$

$$d\nu_{a,t} = \left[\sigma_a^2 - 2\lambda_a \nu_{a,t} - \left(\frac{(r_{N,t} - \overline{r}_N)\lambda_\pi \nu_{a,t}}{\sigma_\pi}\right)^2\right] dt, \tag{5}$$

where $\widehat{B}_\pi$ is a Brownian motion under agent's filtration and represents a surprise change in expected inflation. Post-filtering, the agent perceives the expected inflation process as

$$d\pi_t = \lambda_\pi \left[\breve{\pi} - \widehat{a}_t(r_{N,t} - \overline{r}_N) - \pi_t\right] dt + \sigma_\pi d\widehat{B}_{\pi,t}. \tag{6}$$

The agent's updating of beliefs in Equation (4) depends on the difference $r_{N,t} - \overline{r}_N$. To fix ideas, assume that the Fed is tightening, meaning that $r_{N,t} > \overline{r}_N$. Then a positive surprise change in expected inflation ($d\widehat{B}_{\pi,t} > 0$, or an inflationary shock) lowers the agent's estimate $\widehat{a}_t$. The agent's confidence in Fed's ability to control inflation decreases after the inflationary shock because inflation keeps rising despite the Fed's tightening. If, on the contrary, the agent observes a negative surprise change in expected inflation—a deflationary shock—then $\widehat{a}_t$ increases, restoring the agent's confidence in Fed's ability to fight inflation.

We observe an asymmetric response of $\widehat{a}_t$ to inflation surprises. When the Fed is tightening, a positive inflation surprise not only represents bad news but also lowers the agent's estimate $\widehat{a}_t$ or their perception of the Fed's ability to control inflation. Conversely, in the case of an easing episode, the same positive surprise in inflation leads the agent to perceive an improvement in the Fed's ability to bring inflation back to its long-term mean. This asymmetric response of $\widehat{a}_t$ will be essential for some of our asset pricing results.

The posterior uncertainty $\nu_{a,t}$ evolves locally deterministically over time as described in (5). It tends to increase when interest rates are close to being neutral ($r_{N,t} \approx \overline{r}_N$) because valuable information about the Fed's ability to control inflation can only be observed when the Fed tries to either fight inflation ($r_{N,t} > \overline{r}_N$) or increase inflation ($r_{N,t} < \overline{r}_N$). Importantly, the posterior uncertainty never vanishes since the agent learns about a moving target, which evolves as in (3). As shown below, $\nu_{a,t}$ is the channel through which the agent's confidence in the Fed's ability to control inflation generates novel asset pricing results.

The Fisher equation states that the nominal interest rate $r_{N,t}$ must equal the sum of the real interest rate $r_{R,t}$ and the expected inflation rate:

$$r_{N,t} = r_{R,t} + \pi_t. \tag{7}$$

The Fed uses the Taylor rule to guide its response to deviations in inflation and economic growth. The rule relies on two positive and known constants, namely β_π and β_μ. If the recent history of inflation and economic growth exceed their target levels, the Fed increases the nominal interest rate according to:

$$r_{N,t} = \overline{r}_N + \beta_\pi \left(\phi_{\pi,t} - \overline{\pi}\right) + \beta_\mu \left(\phi_{\mu,t} - \overline{\mu}_\delta\right). \tag{8}$$

The Taylor rule considers the difference between the current *inflation index* $\phi_{\pi,t}$ and the targeted inflation rate $\overline{\pi}$, and the difference between the current *consumption growth index* $\phi_{\mu,t}$ and the natural expected consumption growth rate $\overline{\mu}_\delta = \frac{\mathbb{E}(d\delta_t/\delta_t)}{dt}$. The inflation index, $\phi_{\pi,t}$, is based on the history of observations of the price level, while the consumption growth index, $\phi_{\mu,t}$, is based on the history of observations of the aggregate consumption:

$$\phi_{\pi,t} = \omega_\pi \int_0^t e^{-\omega_\pi(t-s)} \frac{dp_s}{p_s}, \tag{9}$$

$$\phi_{\mu,t} = \omega_\mu \int_0^t e^{-\omega_\mu(t-s)} \frac{d\delta_s}{\delta_s}. \tag{10}$$

To understand the meaning of the indices $\phi_{\pi,t}$ and $\phi_{\mu,t}$, note first that (9) and (10) imply the following dynamics:

$$d\phi_{\pi,t} = \omega_\pi(\pi_t - \phi_{\pi,t})dt, \tag{11}$$

$$d\phi_{\mu,t} = \omega_\mu(\mu_{\delta,t} - \phi_{\mu,t})dt + \omega_\mu \sigma_\delta dB_{\delta,t}, \tag{12}$$

where it can be shown that the unconditional means of $\phi_{\pi,t}$ and $\phi_{\mu,t}$ are respectively $\overline{\pi}$ and $\overline{\mu}_\delta$. Consider now a discretization of (11) with time steps Δt:

$$\phi_{\pi,t} = \left(1 - e^{-\omega_\pi \Delta t}\right) \sum_{n=0}^{\infty} e^{-\omega_\pi n \Delta t} \pi_{t-n\Delta t}. \tag{13}$$

The expression (13) resembles an exponential moving average, with the parameter ω_π driving the weight associated with the present relative to the past. If ω_π is large, the past price growth influences to a small degree the index, causing it to closely represent current price growth. On the other hand, if ω_π is small, the past history of price growth influences the index to a greater extent. This logic also applies to the index $\phi_{\mu,t}$, with the added impact of B_δ shocks, reminiscent of an ARMA model. In fact, discretizing (12) produces:

$$\phi_{\mu,t} = \left(1 - e^{-\omega_\mu \Delta t}\right) \sum_{n=0}^{\infty} e^{-\omega_\mu n \Delta t} \mu_{\delta,t-n\Delta t} + \omega_\mu \sigma_\delta \sqrt{\frac{1 - e^{-\omega_\mu \Delta t}}{2\omega_\mu}} \sum_{n=0}^{\infty} e^{-\omega_\mu n \Delta t} Z_{t-n\Delta t},$$

where Z is the $N(0,1)$ discrete counterpart of the Brownian B_δ. The parameter ω_μ controls the weight associated with the present consumption growth relative to the past, with a higher ω_μ giving more weight to recent data.

To summarize, the indices $\phi_{\pi,t}$ and $\phi_{\mu,t}$ allow the Fed to base its interest rate decision not only on the latest estimates of inflation and expected consumption growth but on their entire history. The Taylor rule is thus fairly general and can be specialized to cases when $\phi_{\pi,t} = \pi_t$ and $\phi_{\mu,t} = \mu_{\delta,t}$, as well as cases when the past observations are given more weight. The parameter values ω_π, ω_μ, β_π, and β_μ will be estimated from the data in Section 3.

It is worth noting that when $\omega_\pi = \omega_\mu \equiv \omega$, the dynamics of the process

$$\phi_t \equiv \beta_\pi \left(\phi_{\pi,t} - \overline{\pi}\right) + \beta_\mu \left(\phi_{\mu,t} - \overline{\mu}_\delta\right), \tag{14}$$

which enters the Taylor rule in (8), do not include the two indices $\phi_{\pi,t}$ and $\phi_{\mu,t}$:

$$d\phi_t = \omega[\beta_\mu(\mu_{\delta,t} - \overline{\mu}_\delta) + \beta_\pi(\pi_t - \overline{\pi}) - \phi_t]dt + \omega \beta_\mu \sigma_\delta dB_{\delta,t}. \tag{15}$$

The dynamics in (15) show that ϕ_t mean-reverts at speed ω towards its stochastic mean, which is determined by the weighted sum of the inflation and consumption growth deviations from their targets. This reduction of $\phi_{\pi,t}$ and $\phi_{\mu,t}$ into a single state variable ϕ_t when $\omega_\pi = \omega_\mu \equiv \omega$ not only simplifies the numerical method but also facilitates the interpretation of ϕ_t. Specifically, high values of ϕ_t indicate tightening, and low values indicate easing.

In Section 3, we will provide evidence that the estimated values of the mean-reversion speeds ω_π and ω_μ are nearly identical. Since eliminating one state variable simplifies the numerical solution of the equilibrium and helps us to interpret our findings more easily, we assume going forward that $\omega_\pi = \omega_\mu \equiv \omega$. This implies that ϕ_t satisfies (14) and that the nominal interest rate is determined by:

$$r_{N,t} = \overline{r}_N + \phi_t, \tag{16}$$

with the dynamics of ϕ_t provided in (15).

We observe that the process $\phi_t = r_{N,t} - \overline{r}_N$ has a direct impact on the agent's updating of beliefs in equation (4). This establishes a clear connection between the Fed's decisions, as governed by (16), and the agent's learning process regarding the Fed's ability to control inflation, as described in (4).

Solving for the equilibrium in this economy involves writing the HJB equation:

$$\max_C \{h(C, J) + \mathcal{L}J\} = 0, \tag{17}$$

with the differential operator $\mathcal{L}J$ following from Itô's lemma. In keeping with existing work (e.g., Benzoni, Collin-Dufresne, and Goldstein, 2011), we guess the following value function:

$$J(C, \pi, \widehat{a}, \phi, \nu_a) = \frac{C^{1-\gamma}}{1-\gamma} \left[\rho e^{I(x_t)}\right]^\theta, \tag{18}$$

where $I(x_t)$ is the log wealth-consumption ratio and $x_t \equiv [\pi_t \ \widehat{a}_t \ \phi_t \ \nu_{a,t}]^\top$ denotes the state vector. (Note that the state vector does not include $\mu_{\delta,t}$, which in our model will be endogenously determined in equilibrium as a function of the other state variables.)

Substituting the guess (18) into the HJB Equation (17) and imposing the market-clearing condition $C_t = \delta_t$, yields a partial differential equation for the log wealth-consumption ratio. We numerically solve this equation using Chebyshev polynomials (Judd, 1998). Appendix A describes the solution method and details the numerical procedure.

Equilibrium market price of risk and real risk-free rate Following Duffie and Epstein (1992), the state price density in this economy is given by

$$\xi_t = \exp\left[\int_0^t h_J(C_s, J_s)ds\right] h_C(C_t, J_t) = \exp\left[\int_0^t \left(\frac{\theta-1}{e^{I(x_s)}} - \rho\theta\right) ds\right] \rho^\theta C_t^{-\gamma} (e^{I(x_t)})^{\theta-1}.$$

A two-dimensional Brownian vector, $\widehat{B}_t \equiv [B_{\delta,t} \ \widehat{B}_{\pi,t}]^\top$, drives the state variables in this economy. As a result, the market price of risk in this economy is also two-dimensional, denoted as $m_t \equiv [m_{\delta,t} \ m_{\pi,t}]^\top$. Both the market price of risk and the real risk-free rate $r_{R,t}$

result from the dynamics of the state price density,

$$\frac{d\xi_t}{\xi_t} = -r_{R,t}dt - m_t^\top d\widehat{B}_t, \tag{19}$$

Itô's Lemma yields the market prices of risk for B_δ and $\widehat{B}_\pi$:

$$m_{\delta,t} = \gamma\sigma_\delta + (1-\theta)\sigma_\delta\beta_\mu\omega I_\phi, \tag{20}$$

$$m_{\pi,t} = (1-\theta)\left[\sigma_\pi I_\pi - \frac{\lambda_\pi \nu_{a,t}}{\sigma_\pi} I_{\widehat{a}}(r_{N,t} - \overline{r}_N)\right], \tag{21}$$

where we denote by I_z the partial derivative of the log wealth-consumption ratio with respect to the state variable $z \in \{\pi, \widehat{a}, \phi, \nu_a\}$.

Focusing on the market price of risk $m_{\delta,t}$, the Fed's monetary policy plays an important role in mitigating growth fluctuations caused by B_δ. The Fed tightens when facing an overheating economy (high ϕ_t), leading to an expected negative sign for I_ϕ. Conversely, the Fed eases when facing a weak economy, also implying $I_\phi < 0$. (We will confirm the assumed signs of the partial derivatives of $I(x_t)$ in Section 4.) The agent values the Fed's stabilizing force through the long-run risk channel, with $1-\theta$ measuring the preference for early resolution of uncertainty. From the long-run risk agent's perspective, the Fed's response to changes in ϕ_t reduces long-run risk and, with it, $m_{\delta,t}$. This effect is stronger as σ_δ (the scale of economic fluctuations), β_μ (the output gap coefficient in the Taylor rule), and ω (the weight given to recent growth data) increase.

For the market price of risk $m_{\pi,t}$, we expect a negative I_π (as higher inflation reduces expected real consumption growth and the wealth-consumption ratio) and a positive $I_{\widehat{a}}$ (since greater trust in the Fed's inflation control ability raises the wealth-consumption ratio). Assuming the Fed is tightening ($r_{N,t} - \overline{r}_N > 0$) and considering the signs $I_\pi < 0$ and $I_{\widehat{a}} > 0$, we obtain a negative $m_{\pi,t}$. Consequently, the agent is willing to pay a premium for assets whose returns covary positively with inflation. The magnitude of the price of risk $m_{\pi,t}$ grows with a strong preference for early resolution of uncertainty (large $1-\theta$), with higher inflation volatility (large σ_π), and, crucially, with higher uncertainty in the Fed's inflation control ability (large $\nu_{a,t}$).

Itô's Lemma applied to (19) yields the equilibrium real risk-free rate:

$$r_{R,t} = \rho + \frac{\mu_{\delta,t}}{\psi} - \frac{\gamma(1+\psi)}{2\psi}\sigma_\delta^2 - \frac{1-\theta}{2}\left(\sigma_{W,t}^2 - \sigma_\delta^2\right), \tag{22}$$

where $\sigma^2_{W,t}$ is the instantaneous variance of wealth,

$$\sigma^2_{W,t} \equiv \sigma^2_\delta + 2\sigma^2_\delta \beta_\mu \omega I_\phi + \sigma^2_\pi I^2_\pi + \frac{\lambda^2_\pi \nu^2_{a,t}(r_{N,t} - \overline{r}_N)^2}{\sigma^2_\pi} I^2_{\widehat{a}} + \sigma^2_\delta \beta^2_\mu \omega^2 I^2_\phi - 2\lambda_\pi \nu_{a,t}(r_{N,t} - \overline{r}_N) I_\pi I_{\widehat{a}}.$$

The first two terms in (22) are familiar drivers of the real risk-free rate: the time preference rate and the expected growth rate of consumption. The last two terms result from precautionary saving and represent an adjustment for risk, which includes consumption risk and excess wealth risk. The last term vanishes in the CRRA case ($\theta = 1$).

Replacing $r_{R,t}$ in the Fisher equation (7), then fixing $r_{N,t} = \overline{r}_N$ and taking unconditional expectations on both sides determines the neutral level of interest rates, $\overline{r}_N$, as a known function of the other parameters:

$$\overline{r}_N = \rho + \overline{\pi} + \frac{\overline{\mu}_\delta}{\psi} - \frac{\gamma(1+\psi)}{2\psi}\sigma^2_\delta - \frac{1-\theta}{2}\left(2\sigma^2_\delta \beta_\mu \omega \overline{I}_\phi + \sigma^2_\pi \overline{I}^2_\pi + \sigma^2_\delta \beta^2_\mu \omega^2 \overline{I}^2_\phi\right),$$

where $\overline{I}_\pi$ and $\overline{I}_\phi$ are the values of the partial derivatives of the log wealth-consumption ratio measured when all state variables are at their long-term means: $\pi = \overline{\pi}$, $\widehat{a} = 0$, $\phi = 0$, and $\nu_a = \overline{\nu}_a$.

In our economy, the expected growth rate of consumption is endogenously determined in equilibrium and depends on monetary policy. Equation (22), together with the Fisher equation (7), lead to an equilibrium expected growth rate:

$$\mu_{\delta,t} = \psi(r_{N,t} - \pi_t - \rho) + \frac{\gamma(1+\psi)}{2}\sigma^2_\delta + \frac{\psi(1-\theta)}{2}(\sigma^2_{W,t} - \sigma^2_\delta). \tag{23}$$

Equation (23) determines the expected growth given a real interest rate, with the latter being a function of the nominal rate and expected inflation. Meanwhile, equation (6) describes the inflation path based on the agent's perceived impact of the Fed's decisions. In order to close the model, these two equations are supplemented with the Taylor rule (8), which determines the nominal interest rate $r_{N,t}$. Collectively, these three equations imply that the real consumption's equilibrium path depends on monetary policy, making the expected consumption growth $\mu_{\delta,t}$ endogenous and monetary policy non-neutral.

In equation (23), the nominal interest rate does not move one-for-one with expected inflation[1], resulting in fluctuations in the real interest rate. These changes in the real interest rate, in turn, impact consumption since the representative agent adjusts her expected future

[1] Applying Itô's Lemma to the Taylor rule equation (8) shows that the nominal interest rate depends on the Brownian B_δ, while expected inflation π_t depends on the Brownian B_π as given in equation (2). Therefore, based on the Fisher equation (7), a change in expected inflation results in a change in the real interest rate. In other words, monetary policy is non-neutral.

consumption growth to align with the new real interest rate level. To illustrate this, let us consider the log level of real consumption, denoted by $c_t = \log(\delta_t)$. By discretizing equation (1) and using equation (23), we can write $\mathbb{E}_t[c_{t+1}] - c_t = \mu_{\delta,t} - \sigma_\delta^2/2$, which implies:

$$c_t - \mathbb{E}_t[c_{t+1}] = \psi(\rho + \pi_t - r_{N,t}) - \left(\frac{\gamma(1+\psi)}{2\psi} + \frac{1}{2}\right)\sigma_\delta^2 - \frac{\psi(1-\theta)}{2}(\sigma_{W,t}^2 - \sigma_\delta^2). \tag{24}$$

The optimality condition (24), arising from the representative agent's first-order condition for consumption today versus consumption tomorrow, aligns with conditions found in standard monetary policy frameworks (e.g., Galí, 2015, Chapter 3, p. 54). According to this condition, the agent consumes more today relative to tomorrow when either the subjective discount rate ρ or the inflation rate π_t is high, and consumes less today relative to tomorrow when the nominal interest rate $r_{N,t}$ is high. As anticipated, by raising the nominal interest rate, the Fed curbs current consumption.

The final term in Equation (24) acts as the "exogenous preference shifter" in monetary economies. A change in this term can be interpreted as a discount rate shock (Galí, 2015, Chapter 3). A key difference in our model is that this shock is endogenous and driven by the excess variance of wealth, $\sigma_{W,t}^2 - \sigma_\delta^2$. An increase in the excess variance of wealth results in lower consumption today relative to tomorrow because the representative agent prefers early resolution of uncertainty. As such, the excess variance of wealth boosts precautionary saving and discourages current consumption.

Equilibrium asset prices As in Bansal and Yaron (2004), we will now consider an asset (the "market") that pays an aggregate dividend, which follows the dynamic process

$$\frac{dD_t}{D_t} = [(1-\alpha)\overline{\mu}_\delta + \alpha\mu_{\delta,t}]dt + \sigma_D dB_{D,t}, \tag{25}$$

where B_D is a one-dimensional Brownian motion uncorrelated with $\{B_\delta, \widehat{B}_\pi\}$, α is the dividend leverage on expected consumption growth (Abel, 1999), and σ_D helps calibrate the volatility of dividends which in the data is larger than that of consumption. Assuming non-zero correlations between B_D and $\{B_\delta, \widehat{B}_\pi\}$ is possible but not necessary to achieve our main objective of isolating the impact of learning about the Fed on asset prices. In equation (25), the expected growth rate of dividends is an affine function of the economy's expected growth rate, $\mu_{\delta,t}$. As inflation and monetary policy impact $\mu_{\delta,t}$, we will analyze how asset pricing reflects this impact. Lastly, the constant $(1-\alpha)\overline{\mu}_\delta$ in the drift of (25) ensures that the average dividend growth rate is equal to the average consumption growth rate, $\overline{\mu}_\delta$.

Denote the log price-dividend ratio by $\Pi(x_t)$, which solves a partial differential equation we relegate to Appendix A. The diffusion of market returns is a vector with three elements:

$$s_{\delta,t} = \sigma_\delta \beta_\mu \omega \Pi_\phi, \tag{26}$$

$$s_{\pi,t} = \sigma_\pi \Pi_\pi - \frac{\lambda_\pi \nu_{a,t}}{\sigma_\pi} \Pi_{\widehat{a}} (r_{N,t} - \overline{r}_N), \tag{27}$$

$$s_{D,t} = \sigma_D.$$

Multiplying each of the market prices of risk in (20)-(21) with the corresponding diffusions in (26)-(27), then taking the sum, yields the market risk premium (the market price of risk for B_D is zero, and thus σ_D does not enter the risk premium):

$$\begin{aligned} RP_t = {} & \gamma \sigma_\delta^2 \beta_\mu \omega \Pi_\phi + (1-\theta) \sigma_\delta^2 \beta_\mu^2 \omega^2 \Pi_\phi I_\phi + (1-\theta) \sigma_\pi^2 \Pi_\pi I_\pi \\ & - (1-\theta) \nu_{a,t} (\Pi_\pi I_{\widehat{a}} + \Pi_{\widehat{a}} I_\pi) \lambda_\pi (r_{N,t} - \overline{r}_N) + (1-\theta) \frac{\lambda_\pi^2 \nu_{a,t}^2}{\sigma_\pi^2} \Pi_{\widehat{a}} I_{\widehat{a}} (r_{N,t} - \overline{r}_N)^2. \end{aligned} \tag{28}$$

In line with our analysis of the log wealth-consumption ratio, we hypothesize—and confirm in Section 4—that: $\Pi_\phi < 0$ (the Fed tightens during an overheating economy and eases during a weakening economy, resulting in a negative relationship between ϕ_t and asset prices); $\Pi_{\widehat{a}} > 0$ (confidence in the Fed's ability to control inflation boosts asset prices); and $\Pi_\pi < 0$ (inflation reduces growth and negatively affects asset prices).

Two primary factors influence the risk premium. First, for the long-run risk agent, the Fed's monetary policy lowers the market price of B_δ risk—refer to our discussion of equation (20)—resulting in $I_\phi < 0$. Consequently, the term $(1-\theta)\sigma_\delta^2 \beta_\mu^2 \omega^2 \Pi_\phi I_\phi$ in (28) is positive. In other words, the Fed's tightening or easing policy reduces long-run risk and is thus favorable. However, the market declines when the Fed tightens (when ϕ increases) and rises when the Fed eases (when ϕ decreases), creating a negative correlation between B_δ and the market, which leads to a positive risk premium. The magnitude of this effect on the risk premium depends on the agent's perceived confidence in the Fed's ability to control inflation, $\widehat{a}_t$. Suppose $\widehat{a}_t$ is positive and large. In that case, the Fed's strong ability to control inflation lowers the risk premium, as the Fed's actions today will promptly bring back inflation to its long-term mean. This weakens the impact of long-run risk and thus the risk premium. Further discussion on this effect can be found in Section 4.

The uncertainty channel $\nu_{a,t}$ is the second factor affecting the market risk premium. It is represented by the second-row terms in equation (28), which form a quadratic expression in $(r_{N,t} - \overline{r}_N)$. The product $\Pi_{\widehat{a}} I_{\widehat{a}}$ is positive, and thus the quadratic term generates a U-shape. This means that uncertainty about the Fed's ability to control inflation increases the risk premium when the Fed deviates from a neutral monetary policy. Moreover, the linear term in $(r_{N,t} - \overline{r}_N)$ leads to an asymmetric response. Since $\Pi_\pi I_{\widehat{a}} + \Pi_{\widehat{a}} I_\pi < 0$, the risk premium is higher during a tightening cycle than during an easing cycle. This asymmetry follows from equation (4), which shows that learning amplifies the impact of inflation surprises during tightening episodes and dampens it during easing episodes. Finally, the risk premium is magnified by the term $(1-\theta)\sigma_\pi^2 \Pi_\pi I_\pi$, which is positive when both the aggregate wealth and the market decrease with inflation, in other words, when $\Pi_\pi < 0$ and $I_\pi < 0$. All these effects are more pronounced when there is high uncertainty, the economy is in a more extreme tightening or easing cycle, or the agent strongly prefers early resolution of uncertainty.

These two forces driving the risk premium reflect our paper's main contributions. The first force is based on the idea that the Fed's monetary policy stabilizes aggregate fluctuations and is therefore desirable in a long-run risk economy. However, the market bears a cost in the form of a risk premium, especially when the Fed's ability to control inflation, $\widehat{a}_t$, is low or negative. The second force is based on the idea that the agent is uncertain about the Fed's ability to control inflation. This uncertainty increases the risk premium when the Fed deviates from a neutral monetary policy, creating concerns that the Fed may not be able to bring inflation back to target, particularly during tightening periods.

Turning now to stock market variance, the process (25) together with the log price-dividend ratio $\Pi(x_t)$ imply the instantaneous stock return variance in this economy:

$$\sigma_t^2 = \sigma_D^2 + \sigma_\delta^2 \beta_\mu^2 \omega^2 \Pi_\phi^2 + \sigma_\pi^2 \Pi_\pi^2 - 2\lambda_\pi \nu_{a,t} \Pi_{\widehat{a}} \Pi_\pi (r_{N,t} - \overline{r}_N) + \frac{\lambda_\pi^2 \nu_{a,t}^2}{\sigma_\pi^2} \Pi_{\widehat{a}}^2 (r_{N,t} - \overline{r}_N)^2. \quad (29)$$

The last two terms in the stock return variance are novel, and are caused by the uncertainty about the Fed's ability to control inflation. These terms show that the stock return variance increases when the Fed deviates from a neutral monetary policy ($\overline{r}_N \neq r_{N,t}$). The term linear in $(r_{N,t} - \overline{r}_N)$ is positive during tightening episodes and negative during easing episodes, which creates an asymmetry that follows from the agent's learning. As a result, we observe an asymmetric U-shaped pattern for stock return variance, with uncertainty about the Fed's ability to control inflation becoming more important during tightening cycles.

3 Parameter Estimation

We estimate the model's parameters by Maximum Likelihood using U.S. real Gross Domestic Product (GDP) data, Federal funds rate (Fed Funds rate) data, and Consumer Price Index (CPI) data. Appendix B provides details about the Maximum Likelihood estimation. Real GDP is from NIPA tables, while the Fed funds rate and the CPI are from FRED. The data is at the monthly frequency from January 1955 to December 2021. The log real GDP

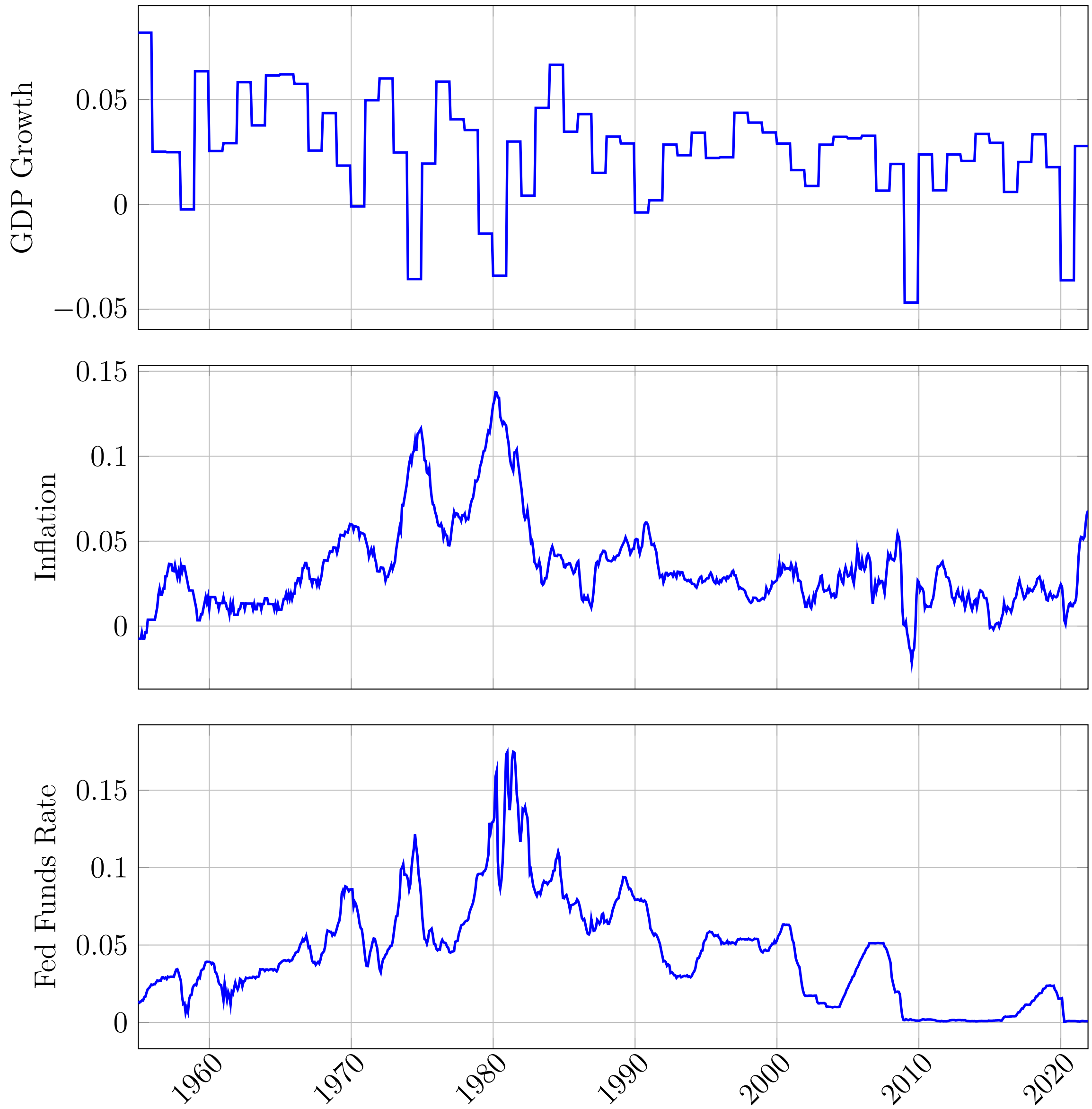

Figure 1: GDP Growth, Inflation, and Federal Funds Rate.
This figure plots the observed annualized U.S. real GDP growth rate (top panel), CPI inflation rate (middle panel), and Federal Funds rate (bottom panel).

growth rate, log CPI growth rate, and continuously compounded Fed funds rate are used as proxies for the real log output growth rate $\log(\delta_{t+\Delta}/\delta_t)$, the inflation rate π_t, and the nominal interest rate r_{Nt}, respectively. These time series are depicted in Figure 1. The bottom panel reveals that the Federal funds rate exceeded 10% in the mid-1970s and early 1980s to combat soaring inflation, as shown in the middle panel. These elevated interest rates contributed to the economic downturns visible in the top panel of the figure.

Table 1 presents the parameter values estimated using Maximum Likelihood. The estimated output gap and inflation coefficients β_μ and β_π suggest that nominal interest rates

Parameter	Symbol	Value
Output growth volatility	σ_δ	0.0243***
		(0.0005)
Mean inflation	$\overline{\pi}$	0.0345***
		(0.0013)
Mean nominal interest rate	$\overline{r}_N$	0.0452***
		(0.0009)
Mean-reversion speed of inflation index	ω_π	0.4479***
		(0.0364)
Mean-reversion speed of output growth index	ω_μ	0.4236***
		(0.0455)
Interest rate sensitivity to inflation	β_π	1.3247***
		(0.0310)
Interest rate sensitivity to output growth	β_μ	1.0251***
		(0.0790)
Inflation volatility	σ_π	0.0124***
		(0.0002)
Mean inflation under neutral interest rates	$\breve{\pi}$	0.0322***
		(0.0030)
Mean-reversion speed of inflation	λ_π	0.6295***
		(0.1240)
Volatility of the Fed's ability to control inflation	σ_a	0.8412***
		(0.2763)
Mean-reversion speed of the Fed's ability to control inflation	λ_a	1.3149***
		(0.4890)

Table 1: Parameter values estimated by Maximum Likelihood.
This table reports the parameter values estimated by Maximum Likelihood. The estimation procedure is detailed in Appendix B. The data is at the monthly frequency from January 1955 to December 2021. Output data is in real terms. Standard errors are reported in brackets, and statistical significance at the 10%, 5%, and 1% levels is denoted by *, **, and ***, respectively.

respond more to inflation than to output growth (Clarida et al., 2000; Ang, Boivin, Dong, and Loo-Kung, 2011). The inflation and output growth indexes revert to their means at nearly identical rates, ω_π and ω_μ. As a result, and in line with Section 2, we assume equal mean-reversion speeds: $\omega_\pi = \omega_\mu \equiv \omega = 0.4479$. Throughout our sample period, average inflation stands at 3.45%, and nominal interest rates at 4.52%, yielding an approximate average real interest rate of 1%. Notably, the historical average inflation rate is roughly 72% higher than the Fed's current 2% target, raising questions about the attainability and sustainability of this target.

Figure 2 displays the historical paths of the process $\phi_t = r_{N,t} - \overline{r}_N$ (top panel) and the agent-inferred mean of inflation (bottom panel), denoted as $\widehat{\overline{\pi}}_t \equiv \breve{\pi} - \widehat{a}_t(r_{N,t} - \overline{r}_N)$. These

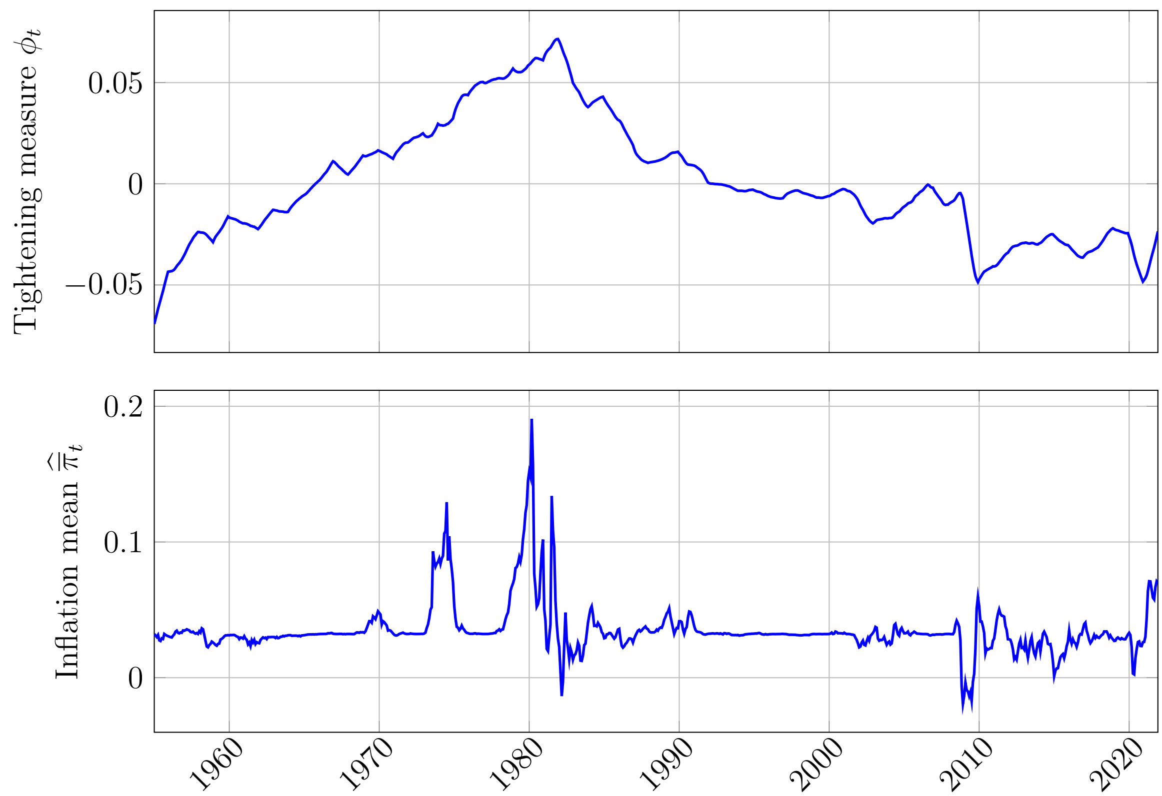

Figure 2: Tightening cycles and inferred mean of inflation.
This figure plots the tightening measure $\phi_t = r_{N,t} - \overline{r}_N$ (top panel) and agent-inferred mean of inflation (bottom panel), denoted as $\widehat{\overline{\pi}}_t \equiv \breve{\pi} - \widehat{a}_t(r_{N,t} - \overline{r}_N)$. These time series are extracted from the Maximum Likelihood estimation.

time series result from the Maximum Likelihood estimation. The process ϕ_t characterizes the Fed's tightening ($\phi_t = r_{N,t} - \overline{r}_N > 0$) and easing ($\phi_t = r_{N,t} - \overline{r}_N < 0$) cycles. The Fed tightened from late 1965 to early 1992 and eased from early 1955 to mid-1965, as well as from mid-1992 to late 2021. The tightening measure ϕ_t exhibits a volatility of around 2.9% and an autocorrelation of approximately 0.996, indicating highly persistent tightening and easing cycles.

In the bottom panel of Figure 2, the inferred mean of inflation, $\widehat{\overline{\pi}}_t \equiv \breve{\pi} - \widehat{a}_t(r_{N,t} - \overline{r}_N)$, is driven by the perceived Fed's ability to control inflation. It has a volatility of 1.9% and an autocorrelation of 0.927, making it a relatively persistent process as well. The inferred mean of inflation hits lows between -1.9% and 0% in early 1982 and late 2008 to mid-2009. The highs range from 7% to 19% in mid-1973 to late 1974, mid-1979 to mid-1981, and mid to late 2021. The 1980–1982 recession lows followed the drastic interest rate increase implemented by the Paul Volcker-led Federal Reserve in mid-1981. The 2009 lows occurred at the end of the Great Recession, spurred by the subprime and financial crises. The highs in the inferred mean of inflation followed the 1973 oil crisis, during which Arab members of

Moment	Data	Model
Real risk-free rate	0.0105	0.0107
Nominal risk-free rate	0.0450	0.0453
Market risk premium	0.0605	0.0818
Market return volatility	0.1431	0.1896
Market Sharpe ratio	0.4228	0.4313

Table 2: Asset-Pricing Moments.
This table presents asset-pricing moments, with the first column displaying empirical moments and the second column showing model-implied counterparts. We calculate empirical moments using the Fed funds rate as the nominal interest rate, the difference between the Fed funds rate and the CPI inflation rate as the real interest rate, and the S&P 500 as the market. Model-implied moments are derived by inputting the state variable time series from the Maximum Likelihood estimation into the model. The data span monthly from January 1955 to December 2021.

the Organization of Petroleum Exporting Countries (OPEC) imposed an oil embargo. The most recent highs resulted from the unprecedented fiscal and monetary stimulus provided during the COVID-19 health crisis.

Consistent with the existing literature, we set the relative risk aversion, the elasticity of intertemporal substitution (EIS), subjective discount rate, dividend leverage on expected consumption growth, and dividend growth volatility to $\gamma = 10$, $\psi = 1.5$, $\rho = 0.0045$, $\alpha = 2.5$, and $\sigma_D = 0.05$, respectively. As discussed later, these chosen parameter values, combined with the estimated parameters in Table 1 yield model-implied real interest rates, nominal interest rates, market risk premium, market return volatility, and market Sharpe ratio that reasonably match the data.

Table 2 presents asset-pricing moments, with the first column displaying empirical moments and the second column showing model-implied moments. We calculate empirical moments using the Fed funds rate as the nominal interest rate, the difference between the Fed funds rate and the CPI inflation rate as the real interest rate, and the S&P 500 as the market. Model-implied moments are derived by inputting the state variable time series from the Maximum Likelihood estimation into the model. The model-implied real and nominal interest rates stand at 1% and 4.5%, respectively, aligning with their empirical counterparts. The model-implied market risk premium, market return volatility, and market Sharpe ratio are 8%, 19%, and 0.43, respectively. These values are reasonably close to their empirical counterparts, suggesting that the model generates realistic asset-pricing moments.

4 Results

In this section, we present the model's predictions and subsequently offer empirical evidence to support them. All the illustrations are derived after solving the model using a numerical algorithm, which relies on the parameters estimated in Section 3. Details of the model solution can be found in Appendix A.

4.1 Model Predictions

Figure 3 illustrates the expected consumption growth $\mu_{\delta,t}$, the real risk-free rate $r_{R,t}$, and the log price-dividend ratio $\Pi(x_t)$ as functions of the model's main state variables. The primary drivers of $\mu_{\delta,t}$, $r_{R,t}$, and $\Pi(x_t)$ are the expected inflation π_t and the tightening variable ϕ_t. As a reminder, high values of ϕ_t indicate tightening, while low values signify easing.

Figure 3 demonstrates that the expected consumption growth rate and the real risk-free rate decline as expected inflation increases. As shown in equation (23), equilibrium expected consumption growth is adversely impacted by inflation. High inflation encourages the agent to consume more today relative to tomorrow, reducing the expected consumption growth. Moreover, the Fisher Equation (7) suggests that when the nominal interest rate remains constant, an increase in expected inflation leads to a decrease in the real risk-free rate. Lastly, an increase in ϕ_t results in monetary tightening and, via the Fisher equation (7), an increase in the real risk-free rate, which in turn leads to higher expected consumption growth as the agent optimally chooses to increase borrowing and delay consumption.

Shifting our attention to the price-dividend ratio (bottom panels), it decreases with expected inflation and the tightening variable ϕ_t and increases with the Fed's perceived ability to control inflation $\widehat{a}_t$. Thus, the inequalities conjectured in Section 2 ($\Pi_\pi < 0$, $\Pi_{\widehat{a}} > 0$, and $\Pi_{\phi_\mu} < 0$) are now verified with our estimated parameter values.

The price-dividend ratio declines with expected inflation through equation (23), which demonstrates that the expected growth rate diminishes as expected inflation rises. This represents the pathway through which inflation introduces long-run risk into the economy. If the inflation process exhibits high persistence, an agent favoring early resolution of uncertainty will be averse to its fluctuations. The price-dividend ratio increases with the Fed's ability to control inflation because when $\widehat{a}_t$ is large and positive, the agent trusts that the Fed will promptly bring inflation back to its target, mitigating the long-run risk that it causes. Conversely, if $\widehat{a}_t$ is large and negative, the Fed will likely lose control of inflation, delaying

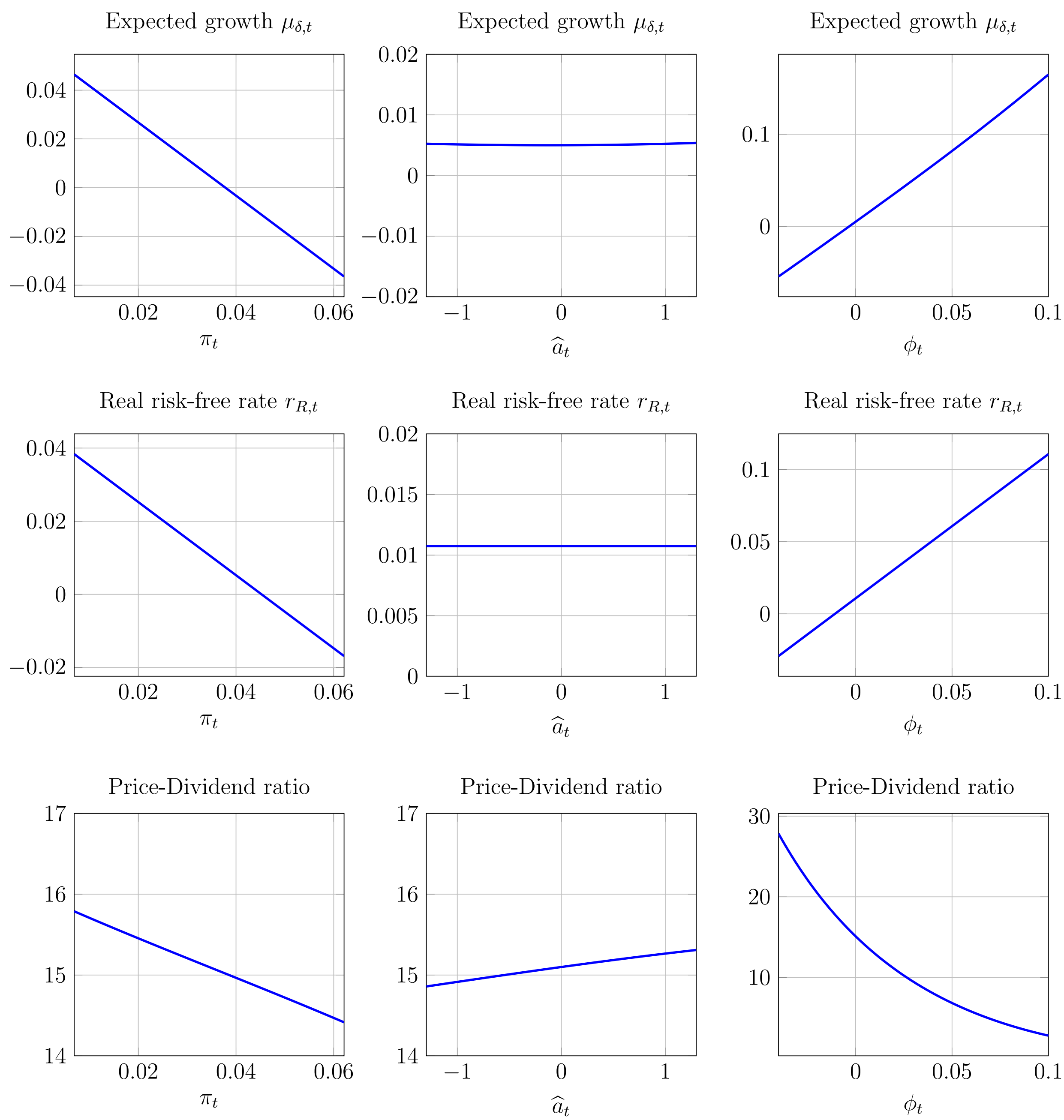

Figure 3: Model Predictions This figure plots the expected consumption growth $\mu_{\delta,t}$, the real risk-free rate $r_{R,t}$, and the price-dividend ratio as functions of the main state variables of the model. For this illustration, we have solved the model numerically (see Appendix A) using the parameters estimated in the Section 3.

its reversion to target and exacerbating long-run risk. Finally, the price-dividend ratio decreases with ϕ_t because a high value for this variable signifies monetary tightening, leading to an increase in the discount rate through a rise in the real risk-free rate. Consequently, the price-dividend ratio falls when the tightening variable ϕ_t increases.

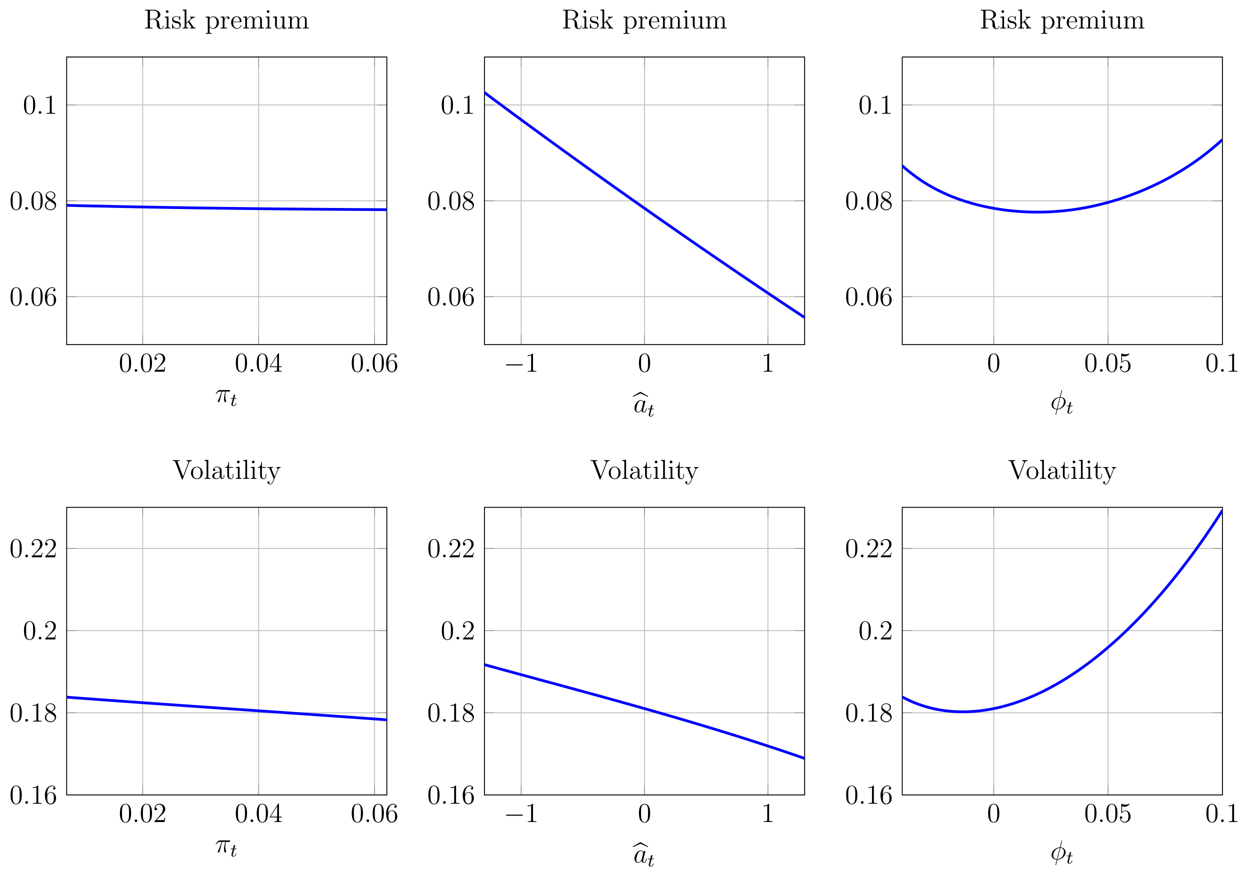

Figure 4: Model Predictions This figure plots the risk premium and the stock market volatility as functions of the main state variables of the model. For this illustration, we have solved the model numerically (see Appendix A) using the parameters estimated in the Section 3.

Figure 4 displays the risk premium and stock market volatility as functions of the model's main state variables. The top three panels reveal that the risk premium is largely unresponsive to expected inflation but decreases significantly with the Fed's perceived ability to control inflation, as denoted by $\widehat{a}_t$. This effect was discussed in relation to equation (28): since inflation is a source of long-run risk in this economy, the Fed's ability to revert it to its target holds value for the agent, resulting in a lower risk premium as $\widehat{a}_t$ increases.

Figure 4 additionally reveals that the risk premium exhibits a U-shaped relationship with the tightening variable ϕ_t. This arises from the uncertainty about the Fed's ability to control inflation. In equation (28), the terms in the second row form a quadratic expression in ϕ_t. Consequently, uncertainty about the Fed's ability to control inflation amplifies the risk premium when the Fed deviates from a neutral monetary policy. Equation (28) also highlights an asymmetry, with the risk premium being higher during tightening; however, this effect is less pronounced with our estimated parameter values.

The bottom panels of Figure 4 depict the volatility of market returns as a function of the state variables, conveying a similar message to that of the risk premium: volatility is mostly unresponsive to expected inflation but decreases as the Fed's ability to control inflation improves and increases with the tightening variable ϕ_t. A notable distinction is the significant surge in volatility during tightening episodes, as shown in the bottom-right panel. This effect directly results from the term linear in ϕ_t in equation (29) and can be interpreted as follows: during a deep tightening cycle, inflation surprises are amplified by the agent's learning process (an increase in inflation is doubly bad news, while a decrease is doubly good news). This intensifies the stock price's sensitivity to inflation news, especially when the Fed embarks on an aggressive tightening cycle.

4.2 Empirical Evidence

Does the data support our model's predictions? To answer this question, we regress both the empirical and model-implied expected output growth rate, real interest rate, market price-dividend ratio, market risk premium, and market return volatility on the state variables. In other words, we verify and confirm that the data support the relationships depicted in Figures 3 and 4.

The empirical expected output growth rate is the fitted value of an ARMA(2,2) model applied to the realized GDP growth rate. The AR(1) and MA(2) coefficients are positive, whereas the AR(2) and MA(1) are negative. The AR(1), AR(2), and MA(2) coefficients are statistically significant at the 1% level, whereas the MA(1) coefficient is statistically significant at the 10% level. The empirical real interest rate is the difference between the Fed funds rate and the CPI inflation rate. The empirical market risk premium is the fitted value obtained by regressing the 1-year-ahead S&P 500 excess return on the current S&P 500 dividend yield (Fama and French, 1989; Cochrane, 2008) and realized S&P 500 return variance (French et al., 1987; Guo, 2006).[2] In the predictive regression, both the dividend yield and realized variance load positively and significantly at the 5% level and 1% level, respectively. The empirical market return volatility is obtained by fitting an Exponential GARCH(1,1) model (Nelson, 1991) on the S&P 500 excess return residual,[3], where the return residual is the difference between the S&P 500 excess return and the empirical risk premium. The ARCH(1) and GARCH(1) coefficients are positive and statistically significant at the 1% level, and the LEVERAGE(1) coefficient is negative and statistically significant at the 1% level. The model-implied moments are obtained by feeding the model with the state variables extracted from the Maximum Likelihood estimation performed in Section 3.

[2]S&P 500 returns, dividend yield, and realized variance are obtained from Amit Goyal's website.

[3]The Exponential GARCH model accounts for the asymmetric response of volatility to return shocks.

	Expected output growth	Real interest rate	Log price-div. ratio	Risk premium	Volatility
$\mu_{\delta,t}$	0.084***				
	(9.864)				
$r_{R,t}$		0.790***			
		(5.649)			
$\log PD_t$			0.485***		
			(4.157)		
RP_t				1.687***	
				(6.332)	
Vol_t					0.318***
					(8.300)
R^2	0.099	0.448	0.404	0.206	0.036
Obs.	804	804	804	804	804

Table 3: Empirical Moments vs. Model-Implied Counterparts.
This table reports the outputs obtained by regressing the empirical moments on their model-implied counterparts. t-statistics are in brackets and are computed using Newey and West (1987)-adjusted standard errors. Statistical significance at the 1%, 5%, and 10% levels are denoted by ***, **, and *, respectively. The data are at the monthly frequency from January 1955 to December 2021.

Table 3 documents the relationships between the empirical moments and their model-implied counterparts. The table shows that the dynamics of the model-implied moments align with the dynamics of the empirical moments. All relations are positive, statistically significant at the 1% level, and feature high R^2s. The explanatory power of the model-implied moments is particularly high for the risk premium, log price-dividend ratio, and real interest rate. Indeed, the model-implied risk premium, log price-dividend ratio, and real interest rate explain respectively 20.6%, 40.4%, and 44.8% of the variation in their empirical counterparts.

We now test the relationships depicted in Figure 3. Table 4 reports the empirical and model-implied relations between the state variables and the expected output growth rate $\mu_{\delta,t}$, real interest rate $r_{R,t}$, and log price-dividend ratio $\Pi(x_t)$. As Figure 3 shows, the main drivers of $\mu_{\delta,t}$, $r_{R,t}$, and $\Pi(x_t)$ are the tightening measure ϕ_t and inflation π_t, which the "Model"-labeled columns in Table 4 confirm. Indeed, the tightening measure and inflation explain more than 99% of the variation in $\mu_{\delta,t}$, $r_{R,t}$, and $\Pi(x_t)$. The "Data"-labeled columns confirm these relations. The expected output growth rate and the real interest rate increase with the tightening measure and decrease with inflation, with statistically significant slopes at the 1% level. This occurs because an increase in ϕ_t leads to monetary tightening, causing

	Expected output growth		Real interest rate		Log price-dividend ratio	
	Model	Data	Model	Data	Model	Data
ϕ_t	1.498^{***}	0.106^{***}	1.000^{***}	0.812^{***}	-15.581^{***}	-7.544^{***}
	(> 100)	(8.125)	(> 100)	(5.253)	$(< -100$	(-2.694)
π_t	-1.489^{***}	-0.175^{***}	-1.000^{***}	-0.732^{***}	-2.787^{***}	-1.198
	(< -100)	(-8.272)	(< -100)	(-9.055)	(-24.163)	(-0.968)
R^2	0.999	0.152	1.000	0.456	0.997	0.391
Obs.	804	804	804	804	804	804

Table 4: Expected Output Growth, Real Interest Rate, and Log Price-Dividend Ratio vs. State Variables.
This table reports the model-implied and empirical relations between the expected real output growth rate, real interest rate, log price-dividend ratio, and their drivers. The drivers are the tightening measure ϕ_t and inflation π_t. t-statistics are in brackets and are computed using Newey and West (1987)-adjusted standard errors. Statistical significance at the 1%, 5%, and 10% levels are denoted by ***, **, and *, respectively. The data are at the monthly frequency from January 1955 to December 2021.

the nominal interest rate to rise through the Fed's Taylor rule (16). The Fisher equation (7) then implies that the real interest rate rises with the tightening measure ϕ_t and decreases with inflation π_t. Furthermore, the equilibrium relation (22) implies that the real interest rate depends linearly on the expected output growth rate. Thus, ϕ_t and π_t drive the expected output growth rate in the same direction they drive the real interest rate.

Table 4 further shows that the price-dividend ratio decreases significantly with the tightening measure ϕ_t, both in the model and in the data. An increase in ϕ_t raises discount rates through tightening. As a result, prices drop as the tightening measure rises. Furthermore, both the model-implied and empirical price-dividend ratios decrease with inflation, although the empirical relation is not statistically significant. A rise in inflation implies a decrease in expected output growth and, therefore, in expected dividend growth, leading to a lower price-dividend ratio.

Table 5 presents the empirical and model-implied relations between the market risk premium, market return volatility, and their primary drivers. As shown in Figure 4, the key drivers include the Fed's ability to control inflation $\widehat{a}_t$, the tightening measure ϕ_t, and the squared tightening measure ϕ_t^2. The "Model"-labeled columns in Table 5 support this observation. These three state variables explain over 86% of the variation in the market risk premium and market return volatility. In both the model and data, an increase in the Fed's ability to control inflation significantly reduces the market risk premium and market return volatility. As the Fed's inflation control ability improves, the likelihood of encountering high

	Risk premium		**Risk premium**	
	Model	Data	Model	Data
$\widehat{a}_t$	-0.012^{***}	-0.017^{***}	-0.011^{***}	-0.016^{***}
	(-9.988)	(-3.581)	(-9.701)	(-3.329)
ϕ_t	0.104^{***}	0.275^{***}	0.070^{***}	0.224^{***}
	(4.508)	(3.964)	(4.728)	(2.724)
ϕ_t^2			2.408^{***}	3.641^{***}
			(6.147)	(2.914)
R^2	0.721	0.205	0.863	0.228
Obs.	804	804	804	804

	Volatility		**Volatility**	
	Model	Data	Model	Data
$\widehat{a}_t$	-0.014^{***}	-0.034^{***}	-0.011^{***}	-0.034^{***}
	(-6.288)	(-6.700)	(-6.406)	(-6.569)
ϕ_t	0.632^{***}	0.151^{***}	0.509^{***}	0.173^{***}
	(10.150)	(5.610)	(13.161)	(7.642)
ϕ_t^2			8.715^{***}	-1.518
			(6.228)	(-1.628)
R^2	0.745	0.101	0.884	0.102
Obs.	804	804	804	804

Table 5: Market Risk Premium and Return Volatility vs. State Variables. This table reports the model-implied and empirical relations between the market risk premium, market return volatility, and their drivers. The drivers are the Fed's ability to control inflation $\widehat{a}_t$, the tightening measure ϕ_t, and the squared tightening measure ϕ_t^2. t-statistics are in brackets and are computed using Newey and West (1987)-adjusted standard errors. Statistical significance at the 1%, 5%, and 10% levels are denoted by ***, **, and *, respectively. The data are at the monthly frequency from January 1955 to December 2021.

future inflation during tightening (or low future inflation during easing) diminishes (refer to equation (6)). In other words, the Fed reduces the persistence of inflation and the associated long-run risk, which consequently leads to a lower market risk premium and decreased market return volatility in equilibrium.

Moreover, both in the model and the data, the market risk premium and market return volatility significantly increase with the tightening measure ϕ_t. As previously mentioned, inflation surprises are amplified during tightening through investor learning. For example, a positive inflation surprise during tightening is doubly bad news because it weakens the agent's confidence in the Fed; in contrast, the same inflation surprise during easing is good news, as it boost the Fed's credibility. This asymmetry contributes to a higher market risk premium and market return volatility during tightening episodes.

Lastly, in the model, the market risk premium and market return volatility increase significantly with the squared tightening measure. This quadratic relationship stems from the last term in equations (28) and (29), where $(r_{N,t} - \overline{r}_N)^2 = \phi_t^2$, and arises due to uncertainty surrounding the Fed's ability to control inflation. The data confirm the positive impact of the squared tightening measure on the market risk premium, with a statistically significant relationship at the 1% level. However, the data reveal no significant empirical correlation between the market return volatility and the squared tightening measure.

5 Conclusion

This paper examines how the market perceives the Fed's ability to control inflation. Investors infer the success of rate hikes from inflation data, which has stock market implications. When the Fed's credibility is high, market risk premiums and volatility decline. Conversely, when investors doubt the Fed's ability to control inflation, these financial measures increase, potentially causing significant market drops. Empirical evidence reinforces these theoretical predictions, highlighting the role of the market's perception of the Fed's inflation-fighting credibility on stock market dynamics.

The Fed has developed effective tools to address inflation by building on experiences from the 1970s' Great Inflation, increased policy autonomy, and a more comprehensive grasp of inflation causes and countermeasures. Among these tools, we argue that credibility in combating inflation may be the Fed's most valuable asset. Our research emphasizes the importance of investors' confidence in the Fed's ability and the need for a solid reputation in effectively managing monetary policy to ensure economic stability.

Additionally, this paper emphasizes the importance of investors' responses to the Fed's actions as a critical economic factor. While our study does not explore the effect of current events on investors' attention, it is plausible that heightened uncertainty, such as during aggressive tightening periods, could lead to increased focus on news, intensifying the observed effects. This insight offers avenues for future research and encourages a deeper understanding of monetary policy's impact on the stock market and the economy.

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Appendix

A Details on Model Resolution in Section 2

Learning: To obtain the agent's posterior mean $\widehat{a}_t \equiv \mathbb{E}[a_t|\mathcal{F}_t^{\pi,r_N}]$ and the posterior variance $\nu_{a,t} \equiv \mathbb{E}[(a_t - \widehat{a}_t)^2|\mathcal{F}_t^{\pi,r_N}]$ as in (4)-(5), apply Theorem 12.7 in Liptser and Shiryaev (2001) with:

$$A_0 = \lambda_\pi(\breve{\pi} - \pi_t), \quad A_1 = -\lambda_\pi(r_{N,t} - \overline{r}_N), \quad B_1 = 0, \quad B_2 = [0 \ \sigma_\pi],$$
$$a_0 = \lambda_a \overline{a}, \quad a_1 = -\lambda_a, \quad b_1 = \sigma_a, \quad b_2 = [0 \ 0].$$

The surprise change in expected inflation according to the agent's information set $\mathcal{F}^\pi$ is

$$d\widehat{B}_{\pi,t} = dB_{\pi,t} + \frac{\lambda_\pi}{\sigma_\pi}(\widehat{a}_t - a_t)(r_{N,t} - \overline{r}_N)dt.$$

HJB equation: The partial differential equation (PDE) that results from (17)-(18) is:

$$\begin{aligned}
0 &= e^{-I} - \rho + \frac{\gamma - 1}{\theta}\left(\frac{\gamma\sigma_\delta^2}{2} - \mu_{\delta,t}\right) + \lambda_\pi\left[\widehat{a}_t(\overline{r}_N - r_{N,t}) + \breve{\pi} - \pi_t\right] I_\pi - \lambda_a \widehat{a}_t I_{\widehat{a}} \\
&+ \omega\left[\beta_\pi(\pi_t - \overline{\pi}) + \beta_\mu(\mu_{\delta,t} - \overline{\mu}_\delta) - \phi_t - \beta_\mu(\gamma - 1)\sigma_\delta^2\right] I_\phi \\
&+ \frac{\sigma_\pi^2}{2} I_{\pi\pi} + \frac{(\overline{r}_N - r_{N,t})^2\lambda_\pi^2\breve{\nu}_a^2}{2\sigma_\pi^2} I_{\widehat{a}\widehat{a}} + \frac{\sigma_\delta^2\beta_\mu^2\omega^2}{2} I_{\phi\phi} + (\overline{r}_N - r_{N,t})\lambda_\pi\breve{\nu}_a I_{\pi\widehat{a}} \\
&+ \frac{\theta\sigma_\pi^2}{2} I_\pi^2 + \frac{\theta(\overline{r}_N - r_{N,t})^2\lambda_\pi^2\breve{\nu}_a^2}{2\sigma_\pi^2} I_{\widehat{a}}^2 + \frac{\theta\sigma_\delta^2\beta_\mu^2\omega^2}{2} I_\phi^2 + \theta(\overline{r}_N - r_{N,t})\lambda_\pi\breve{\nu}_a I_\pi I_{\widehat{a}}.
\end{aligned}$$

To derive this PDE, we set $\nu_{a,t} = \breve{\nu}_a$, which removes one state variable and simplifies the numerical solution process. It is important to note that the theoretical results stated in Section 2 are not affected by this assumption. Moreover, our numerical analysis of the model with a time-varying $\nu_{a,t}$ showed that the price-dividend ratio barely changes in response to $\nu_{a,t}$, although the solution process becomes significantly slower. Consequently, we decided to use a fixed $\nu_{a,t} = \breve{\nu}_a$.

The PDE for $I(\pi_t, \widehat{a}, \phi)$ is solved numerically using the Chebyshev collocation method (Judd, 1998). That is, we approximate the function $I(\pi_t, \widehat{a}, \phi)$ as follows:

$$I(\pi_t, \widehat{a}, \phi) \approx \mathcal{P}(\pi_t, \widehat{a}, \phi) = \sum_{i=0}^{I}\sum_{j=0}^{J}\sum_{k=0}^{K} a_{i,j,k} T_i[\pi] \times T_j[\widehat{a}] \times T_k[\phi],$$

where $T_m[\cdot]$ is the Chebyshev polynomial of order m. The interpolation nodes are obtained by meshing the scaled roots of the Chebyshev polynomials of order $I+1$, $J+1$, and $K+1$. We scale the roots of the Chebyshev polynomials such that they cover approximately 99% of the unconditional distributions of the three state variables (which are all mean-reverting).

The polynomial $\mathcal{P}(\pi_t, \widehat{a}, \phi)$ and its partial derivatives are then substituted into the PDE, and the resulting expression is evaluated at the interpolation nodes. This yields a system of $(I+1) \times (J+1) \times (K+1)$ equations with $(I+1) \times (J+1) \times (K+1)$ unknowns (the coefficients $a_{i,j,k}$). This system of equations is solved numerically.

To verify the solution method's accuracy and address potential concerns about anomalous numerical outcomes, we employed two distinct platforms (Mathematica and Python) and multiple grid dimensions for solving the PDE. In all cases, the results were consistently similar, reinforcing the method's reliability.

Finally, the PDE for the log price dividend ratio Π_t of the asset that is a claim to the dividend process (25) is given by:

$$
\begin{aligned}
0 = {} & e^{-\Pi} - \rho + \frac{\gamma\sigma_\delta^2(\psi+1)}{2\psi} + (1-\alpha)\overline{\mu}_\delta + \left(\alpha - \frac{1}{\psi}\right)\mu_{\delta,t} + \lambda_\pi[\widehat{a}_t(\overline{r}_N - r_{N,t}) + \breve{\pi} - \pi_t]\Pi_\pi \\
& - \lambda_a\widehat{a}_t\Pi_{\widehat{a}} + \omega\left[\beta_\pi(\pi_t - \overline{\pi}) + \beta_\mu(\mu_{\delta,t} - \overline{\mu}_\delta) - \phi_t - \beta_\mu\gamma\sigma_\delta^2\right]\Pi_\phi + \frac{\sigma_\pi^2}{2}\Pi_{\pi\pi} + \frac{(\overline{r}_N - r_{N,t})^2\lambda_\pi^2\breve{\nu}^2}{2\sigma_\pi^2}\Pi_{\widehat{a}\widehat{a}} \\
& + \frac{\sigma_\delta^2\beta_\mu^2\omega^2}{2}\Pi_{\phi\phi} + (\overline{r}_N - r_{N,t})\lambda_\pi\breve{\nu}\Pi_{\pi\widehat{a}} - (\theta-1)\sigma_\delta^2\beta_\mu\omega I_\phi + \frac{\sigma_\pi^2}{2}\Pi_\pi^2 + (\overline{r}_N - r_{N,t})\lambda_\pi\breve{\nu}\Pi_\pi\Pi_{\widehat{a}} \\
& + (\theta-1)\sigma_\pi^2\Pi_\pi I_\pi + (\theta-1)(\overline{r}_N - r_{N,t})\lambda_\pi\breve{\nu}\Pi_\pi I_{\widehat{a}} + (\theta-1)(\overline{r}_N - r_{N,t})\lambda_\pi\breve{\nu}\Pi_{\widehat{a}} I_\pi \\
& + \frac{(\overline{r}_N - r_{N,t})^2\lambda_\pi^2\breve{\nu}^2}{2\sigma_\pi^2}\Pi_{\widehat{a}}^2 + \frac{(\theta-1)(\overline{r}_N - r_{N,t})^2\lambda_\pi^2\breve{\nu}^2}{\sigma_\pi^2}\Pi_{\widehat{a}} I_{\widehat{a}} + \frac{\sigma_\delta^2\beta_\mu^2\omega^2}{2}\Pi_\phi^2 + (\theta-1)\sigma_\delta^2\beta_\mu^2\omega^2\Pi_\phi I_\phi \\
& - \frac{\theta-1}{2}\sigma_\pi^2 I_\pi^2 - (\theta-1)(\overline{r}_N - r_{N,t})\lambda_\pi\breve{\nu} I_\pi I_{\widehat{a}} - \frac{(\theta-1)(\overline{r}_N - r_{N,t})^2\lambda_\pi^2\breve{\nu}^2}{2\sigma_\pi^2} I_{\widehat{a}}^2 - \frac{1}{2}(\theta-1)\sigma_\delta^2\beta_\mu^2\omega^2 I_\phi^2.
\end{aligned}
$$

We replace the solution for the log-wealth consumption ratio I in the above PDE, then solve for the log price-dividend ratio Π using the same numerical procedure.

B Maximum Likelihood Estimation in Section 3

U.S. GDP is from NIPA tables. Real values are used as proxies for the output δ_t and dividend D_t. The Fed funds rate is from FRED, and its annualized continuously compounded value is used as proxy for nominal risk-free rate r_{Nt}. The year-over-year log growth rate of the Consumer Price Index (CPI) is the proxy for π_t. Time series are at the monthly frequency from January 1955 to December 2021.

The GDP growth rate volatility is obtained by maximizing the following log-likelihood function

$$
l_\delta(\Theta_\delta; u_{\delta,\Delta}, \ldots, u_{\delta,J\Delta}) = \sum_{j=1}^{J} \log\left(\frac{1}{(2\pi)^{1/2}\sqrt{\sigma_\delta^2\Delta}}\right) - \frac{1}{2}\left(\sigma_\delta^2\Delta\right)^{-1} u_{\delta,j\Delta}^2,
$$

where $\Delta = 1/12$, $\Theta_\delta \equiv (\sigma_\delta)^\top$, J is the number of observations, $\top$ is the transpose operator, and

$$
u_{\delta,t+\Delta} = \log\left(\delta_{t+\Delta}/\delta_t\right) - \left(\text{avg(GDP growth)} - \frac{1}{2}\sigma_\delta^2\right)\Delta.
$$

avg(GDP growth) stands for the annualized empirical average of the GDP growth rate.

The unconditional mean of inflation is obtained by maximizing the following log-likelihood function

$$
l_p(\Theta_p; u_{p,\Delta}, \ldots, u_{p,J\Delta}) = \sum_{j=1}^{J} \log\left(\frac{1}{(2\pi)^{1/2}\sqrt{\text{var(inflation)}\Delta}}\right) - \frac{1}{2}\left(\text{var(inflation)}\Delta\right)^{-1} u_{p,j\Delta}^2,
$$

where $\Delta = 1/12$, $\Theta_p \equiv (\overline{\pi})^\top$, J is the number of observations, $\top$ is the transpose operator, and

$$
u_{p,t} = \pi_t - \overline{\pi}\Delta.
$$

var(inflation) stands for the annualized empirical variance of inflation.

The parameters driving the Taylor rule are obtained by maximizing the following log-likelihood function

$$l_r(\Theta_r; u_{r,\Delta}, \ldots, u_{r,J\Delta}) = \sum_{j=1}^{J} \log\left(\frac{1}{(2\pi)^{1/2}\sqrt{\sigma_r^2\Delta}}\right) - \frac{1}{2}\left(\sigma_r^2\Delta\right)^{-1} u_{r,j\Delta}^2,$$

where $\Theta_r \equiv (\overline{r}_N, \omega_\pi, \omega_\mu, \beta_\pi, \beta_\mu, \sigma_r)^\top$ and

$$u_{r,t} = r_{Nt} - \left[\overline{r}_N + \beta_\mu\left(\phi_{\mu,t} - \text{avg(GDP growth)}\right) + \beta_\pi\left(\phi_{\pi,t} - \text{avg(Inflation)}\right)\right].$$

The annualized empirical averages of the Fed funds rate, GDP growth rate, and inflation rate are denoted by avg(Fed funds), avg(GDP growth), and avg(Inflation), respectively. The performance indices $\phi_{\mu,t}$ and $\phi_{\pi,t}$ are obtained by discretizing the dynamics in (10) and (9) as follows

$$\phi_{\mu,t} = \omega_\mu \sum_{k=0}^{K} e^{-\omega_\mu k\Delta} \log\left(\delta_{t-k\Delta}/\delta_{t-(k+1)\Delta}\right),$$
$$\phi_{\pi,t} = \omega_\pi \sum_{k=0}^{K} e^{-\omega_\pi k\Delta} \pi_{t-k\Delta}\Delta,$$

where K is the number of observations prior to time t.

To obtain the parameters driving inflation, we discretize the solutions of the stochastic differential equations in (6) and (4) as follows

$$\pi_{t+\Delta} = \pi_t e^{-\lambda_\pi\Delta} + \widehat{\overline{\pi}}_t\left(1 - e^{-\lambda_\pi\Delta}\right) + \sqrt{var_\pi}\,\epsilon_{\pi,t+\Delta},$$
$$\widehat{\overline{\pi}}_t = \breve{\pi} - \widehat{a}_t(r_{Nt} - \overline{r}_N)$$
$$\widehat{a}_{t+\Delta} = \widehat{a}_t e^{-\lambda_a\Delta} - \frac{(r_{Nt} - \overline{r}_N)\lambda_\pi\nu_{a,t}}{\sigma_\pi}\sqrt{\frac{1 - e^{-2\lambda_a\Delta}}{2\lambda_a}}\epsilon_{\pi,t+\Delta}, \tag{B30}$$
$$\nu_{a,t+\Delta} = \nu_{a,t} + \left[\sigma_a^2 - 2\lambda_a\nu_{a,t} - \left(\frac{(r_{Nt} - \overline{r}_N)\lambda_\pi\nu_{a,t}}{\sigma_\pi}\right)^2\right]\Delta, \tag{B31}$$

where $var_\pi = \frac{\sigma_\pi^2}{2\lambda_\pi}\left(1 - e^{-2\lambda_\pi\Delta}\right)$ and $\epsilon_{\pi,t+\Delta}$ is a normally distributed random variable with mean zero and variance one. The parameters driving inflation are obtained by maximizing the following log-likelihood function

$$l_\pi(\Theta_\pi; u_{\pi,\Delta}, \ldots, u_{\pi,J\Delta}) = \sum_{j=1}^{J} \log\left(\frac{1}{(2\pi)^{1/2}\sqrt{var_\pi}}\right) - \frac{1}{2}\left(var_\pi\right)^{-1} u_{\pi,j\Delta}^2,$$

where $\Theta_\pi \equiv (\sigma_\pi, \breve{\pi}, \lambda_\pi, \sigma_a, \lambda_a)^\top$ and

$$u_{\pi,t+\Delta} = \pi_{t+\Delta} - \left[\pi_t e^{-\lambda_\pi\Delta} + \widehat{\overline{\pi}}_t\left(1 - e^{-\lambda_\pi\Delta}\right)\right].$$

The updating rule for $\widehat{a}_t$ and $\nu_{a,t}$ are provided in (B30) and (B31), respectively.