Regulatory Model Secrecy and Bank Reporting Discretion

Shuo Zhao *

Preliminary draft; please do not circulate without permission

Abstract

This paper studies how banking regulators should disclose the models they use to assess banks that have reporting discretion. In my setting, assessments depend on both economic conditions and the fundamental of banks' asset. The regulatory models provide signals about economic conditions and banks report the fundamental of their asset. On the one hand, disclosing the models helps banks to understand how their assets perform under different economic environments. On the other hand, it induces banks with assets that are socially undesirable to manipulate the report and obtain favorable assessments. While the regulator can partially deter manipulation by designing the assessment rule optimally, the disclosure of regulatory models is necessary. The optimal disclosure policy is to disclose the regulatory models when the assessment rule is more likely to induce manipulation and keep them secret otherwise. In this way, disclosure complements the assessment rule by reducing manipulation in cases that harm the regulator more. The analyses speak directly to the supervisory stress test and climate risk stress test.

Keywords: Stress test, Regulatory secrecy, Disclosure, Accounting discretion

*Tilburg University, Department of Finance. E-mail: S.Zhao_3@uvt.nl.
For valuable comments and suggestions, I am grateful to Fabio Castiglionesi, Julio Crego, Qi Chen, Pingyang Gao, Peicong (Keri) Hu, Xu Jiang, Lucas Mahieux, Magdalena Rola-Janicka, Chiara Lattanzio (Discussant), Shuyan Wang (Discussant), Yan Xiong (Discussant) and seminar participants at Duke University, Tilburg University, Dauphine Finance PhD Workshop, Financial Markets and Corporate Governance Conference, Transatlantic Doctoral Conference, and European Banking Center (EBC) Network internal workshop.

1 Introduction

Regulators assess banks on a regular basis to ensure the stability and sustainability of banking industry. In order to do this, regulators rely on models which capture various features of the economy and banks. These models are not always disclosed to banks, making the process of regulatory assessment opaque and its implications unclear. One important reason for not disclosing the models is to prevent banks from gaming the regulatory assessment (Flannery 2019; Clark and Li 2022). A common way for banks to game is to provide uninformative reports that do not represent the underlying risks (Huizinga and Laeven 2012; Bushman and C. D. Williams 2012; Bushman 2016). However, regulatory models contain valuable information which can help banks to understand how their assets perform under different economic conditions. By disclosing the models, regulators enable banks to make more informed decisions. In this paper, I study how regulators should disclose the models they use to assess banks, when banks have reporting discretion.

This study is especially relevant for supervisory stress test and climate risk stress test. The supervisory stress test employs a batch of regulatory models to evaluate the resilience of large banks to adverse macroeconomic shocks. The regulatory models translate the macroeconomic shocks into the risk parameters at banks' level and assign losses to particular positions. While some countries disclose the models transparently, some other countries choose not to disclose them. For example, in the past, the Federal Reserve only provided the broad framework and methodology used in the supervisory stress test. In recent years, it has moved towards more disclosure about the models, including key variables and certain equations. Whereas in Europe, comprehensive disclosure about the stress test methodologies becomes common practice.[1] Disclosing the models helps banks to understand the impact of macroeconomic shocks on their business activities. But it also facilitates banks to manipulate information, which then benefits banks at the expense of the reliability of the stress test results. The severity of such manipulations, which is governed by banks' reporting discretion, crucially affects the tradeoff of disclosing the stress test models.

In the case of climate risk stress tests, the tradeoff of disclosing the regulatory models is even more pertinent. Several countries have conducted climate risk stress tests in recent years.[2] To

[1]For example, the European Banking Authority (EBA) discloses the details of models used in stress test (see https://www.eba.europa.eu/eba-launches-2023-eu-wide-stress-test-0), the Federal Reserve instead discloses high-level information about the models underlying Dodd-Frank Act Stress Test (DFAST) and demonstrates how these models work on hypothetical loan portfolios (See https://www.federalreserve.gov/publications/files/2022-march-supervisory-stress-test-methodology.pdf).

[2]For example, the Bank of England describes the climate risk stress test scenarios in June 2021. See https://www.bankofengland.co.uk/climate-change. The ECB has conducted the climate stress test

fully capture the climate risk factors, the time horizons for climate risk stress tests usually range between 30 and 50 years. Such long time horizons considerably increase the uncertainty about the implication of any business activities.[3] Nevertheless, the regulatory models used in the test are kept confidential. In the meanwhile, banks' report about climate related issue has gained growing attention in recent years. Governments and market watchdogs have proposed several reporting rules for banks to follow.[4] However, the reporting framework is yet to be standardized, leading to substantial reporting discretion.

I develop a tractable model to study the optimal disclosure policy about the regulatory models. The model features one bank and one regulator. The bank has an existing asset whose payoff is increasing in both the economic conditions and the fundamental of the asset. To evaluate the asset, the regulator conducts a stress test. In the stress test, the regulatory models produce a signal about the economic conditions. The regulator discloses the signal to the bank according to the disclosure choice (discussed later). The bank, which has reporting discretion, then reports the fundamental of the asset that can be either high or low. The regulator makes a pass/fail decision based on the signal and the bank's report. If the bank passes the test, the bank can continue to hold the asset. Otherwise, it needs to liquidate the asset.[5]

The bank has large private benefit when retaining the asset such that it prefers to hold the asset regardless of its payoff. But the regulator prefers to keep the asset only when the fundamental is high. This conflict of interests gives bank incentive to manipulate its report. Specifically, manipulation influences the mapping from the asset's fundamental to the report and reduces the informativeness of the report. Manipulation is costly to the bank, and the cost is governed by the prevailing accounting standards which determines the amount of reporting discretion that the bank has.

I first show that the bank's manipulation causes losses to the regulator even when the pass/fail decision is optimal. When making the pass/fail decision, the regulator faces two potential errors: passing a low fundamental asset (i.e., inefficient continuation) and failing a high fundamental asset (i.e., inefficient liquidation). The relative cost of the two inefficiencies, which

since 2021 and published the results. See results for 2022 https://www.bankingsupervision.europa.eu/ecb/pub/pdf/ssm.climate_stress_test_report.20220708~2e3cc0999f.en.pdf.

[3]See https://www.bis.org/fsi/publ/insights34.htm

[4]For example, in Europe, the Corporate Sustainability Reporting Directive (CSRD) entered into force on 5 January 2023, which requires large companies and listed SMEs to disclose social and environmental related information. See https://finance.ec.europa.eu/capital-markets-union-and-financial-markets/company-reporting-and-auditing/company-reporting/corporate-sustainability-reporting_en. The U.S. Securities and Exchange Commission (SEC) also proposed rule changes which require registrants to include certain climate-related disclosures. See https://www.sec.gov/news/press-release/2022-46.

[5]I assume that liquidation is the only possible remedial action after failing the stress test. Further discussions on this point are in Section 2.

depends on both the economic conditions and the asset fundamental, then pins down the regulator's pass/fail decision. Since the regulator observes the economic conditions from the regulatory model and infers the asset fundamental from the bank's report, the two elements also determine how the regulator perceives the relative cost of inefficient continuation and that of inefficient liquidation, which in turn feeds into the pass/fail decision. For given manipulation contained in the bank's report, the expected payoff of the asset is increasing as the regulator receives higher signal from the regulatory models, increasing the relative cost of inefficient liquidation. Accordingly, the regulator's pass/fail decision becomes more lenient in the sense that the bank is more likely to pass the test. The effect of manipulation on the pass/fail decision, however, is more nuanced. Manipulation decreases the informativeness of the bank's report, increasing the similarity between the report of low fundamental asset and that of high fundamental asset. As a result, both inefficient continuation and inefficient liquidation are more likely to occur. Depending on the relative cost of the two, the regulator adjusts the pass/fail decision rule in order to address the error that is more costly. In this way, the pass/fail decision rule optimally trades off inefficient liquidation against inefficient continuation. However, the optimal pass/fail decision is an ex post response to the bank's manipulation. That is, when making the pass/fail decision, the regulator takes the bank's manipulation choice as given. As a result, ex ante, the regulator still bears the cost of passing a low fundamental asset too often, which is the adverse consequence of manipulation on the regulator.

Disclosure of the regulatory models complements the pass/fail decision as an ex ante approach to influence the bank's manipulation incentive. In my model, the disclosure of regulatory models is equivalent to the disclosure of the signal generated by the models. On the one hand, disclosure of the signal enables the bank to learn the economic conditions and their impact on the asset payoff, which then affects the bank's manipulation incentive.[6] On the other hand, since the regulator passes or fails the bank depending on both the regulator's signal and the bank's report, disclosure of the signal also informs the bank how its report affects the regulator's pass/fail decision, which may induce the bank to strategically manipulate its report and self-select the outcome of the stress test.

I show that the disclosure of regulatory models affects the bank's manipulation in two ways. First, it commands the bank's manipulation to vary with the regulator's signal. Since the signal determines the relative cost of inefficient liquidation and inefficient continuation, the disclosure of the signal also commands the bank's manipulation to vary with the tradeoff of the pass/fail decision. I show that, when disclosure incentivizes the bank to learn the asset payoff

[6]In the context of my model, learning affects the bank's manipulation choice and I abstract away any real activity that the bank might take after learning the regulator's signal.

from regulatory models, the bank manipulates less when the asset payoff is low. The regulator benefits from this manipulation choice since the bank manipulates less when the regulator suffers more losses from passing a low fundamental asset. Hence, disclosure of the signal diverts the bank's manipulation from cases where the regulator suffers more losses from manipulation. However, if the bank strategically manipulates to increase the probability of passing the test, then the bank manipulates more particularly in cases where the pass/fail decision is susceptible to manipulation. In this case, disclosure of signal aggravates the regulator's losses caused by the bank's manipulation, confirming the conventional wisdom about the cost of disclosing the regulatory model.

The second effect of disclosure is that it affects the bank's expected amount of manipulation. Disclosing the regulatory models reduces the expected amount of manipulation if bank's manipulation incentive is driven by learning from the regulatory models. If the bank manipulates to exploit the pass/fail decision, then disclosing the regulatory models increases the expected amount of manipulation. This additional effect arises from the interaction between the bank's manipulation response and the regulator's pass/fail decision and it amplifies the first effect of the disclosure of regulatory models.

The optimal disclosure policy is to disclose the regulatory models when the cost of inefficient continuation and that of inefficient liquidation are comparable and keep the models secret otherwise. When the cost of the two inefficiencies is comparable, the pass/fail decision is relatively insensitive to the bank's manipulation. In other words, the bank can increase the probability of passing the test by manipulation without triggering regulatory response in the pass/fail decision. Given that manipulation is effective in increasing the passing probability, the bank's manipulation incentive is then driven by the expected gain after passing the test which is determined by the asset payoff. As a result, disclosing the regulatory models is optimal, since it decreases the expected amount of manipulation and distributes more manipulation to cases where the regulator is less affected by it. However, when the cost of one inefficiency dominates the cost of the other one, the pass/fail decision becomes responsive to the bank's manipulation. The bank then manipulates more when manipulation results in larger increase in the probability of passing the test, which contradicts to the regulator's preference. Hence, the regulator should not disclose the regulatory models.

The optimal disclosure policy crucially depends on the bank's private benefit when passing the test and the bank's reporting discretion. Large reporting discretion and/or large private benefit from passing the test increases the bank's incentive to manipulate, making the bank more likely to exploit pass/fail decision. Hence, the amount of disclosure should decrease in response.

The remainder of the paper is organized as follows. The rest of the introduction discusses the relevant literature. Section 2 presents the model. Section 3 studies the optimal pass/fail decision and the bank's manipulation response. Section 4 analyzes the optimal disclosure policy about the regulatory models. Section 5 conducts comparative statics and demonstrates how the bank's reporting discretion affects the optimal disclosure policy. Section 6 discusses the model assumptions. Section 7 concludes. All proofs are included in Appendix A.

1.1 Related literature

The growing literature on stress test design has focused on disclosure about the results (Goldstein and Sapra 2013; Goldstein and Leitner 2018; Corona, Nan, and Zhang 2019; Quigley and Walther 2020) and scenario design (Parlatore and Philippon 2022). Instead, I focus on, before conducting the stress test, whether the regulator should communicate with the bank about the stress test models. Similar to my paper, Leitner and B. Williams (2023) also study the disclosure policy about the regulatory models. In their paper, revealing the regulatory models induces the bank to always invest in risky asset even when the value is low, but not revealing may lead to underinvestment. While their focus is on the riskiness of bank's investment, I examine the role of bank's information input in the stress test and study how reporting discretion affects the disclosure policy about regulatory models.

Several papers study the impact of stress test assessment on policy design (Agarwal and Goel 2020) and on the bank's opaqueness (Petrella and Resti 2013). In this paper, I show that the disclosure policy about the regulatory models affect the bank's reporting incentive which further influence the accuracy and reliability of stress test results.

The disclosure literature (e.g., Verrecchia (1983) and Dye (1985)) focuses on the disclosure of firms' (in my case, the bank's) information and its impact on the market's expectation. Instead, I focus on the disclosure of the regulator's private information, and I study how it affects the interactions between the regulator and the bank.

Regarding the bank, I study its reporting incentive when reporting discretion exists. The bank's reporting discretion determines how much information the regulator can communicate with the bank. The role of reporting discretion is also analyzed in Gao and Jiang (2018) in the context of bank run. In their paper, the reporting discretion reduces the panic-based runs, but it may also reduce the fundamental-based run. In general, this paper contributes to the literature on the determinants of reporting quality (e.g., Leuz, Nanda, and Wysocki 2003; Barth, Landsman, and Lang 2008; Holthausen 2009; Leuz and Wysocki 2016). The consensus is that the reporting quality depends on various factors. Among others, the regulatory environment and the development of capital market are crucial. This paper shows that the stress test design can affect the reporting quality of banks.

More broadly, this paper contributes to the discussion about the interplay between prudential and accounting regulation. Bertomeu, Mahieux, and Sapra (2020) show that accounting measurement complements capital requirements to affect the level and efficiency of banks' credit decisions. Corona, Nan, and Zhang (2015) examine the impact of accounting information quality on banks' risk-taking incentives, taking into account the interbank competition. This paper shows that the stress test design should be coherent with the prevailing accounting regulation to achieve informative assessment.

2 The model

Consider a risk-neutral economy with no discounting. There is one regulator and one bank. The regulator conducts stress test on the bank. I model the stress test as a four-period game.

At $t = 1$, the stressed scenarios are given exogenously and are observable by everyone. The regulator uses regulatory models to predict the impact of the macroeconomic variables included in stressed scenarios on the banking industry. The output of the regulatory models is summarized in a signal $s \in S = [\underline{s}, \bar{s}]$ with a cumulative distribution function F and density f. The density f has full support. The regulator privately observes s. Throughout the paper, I refer the signal s as the economic condition. The signal s could represent the probability of a liquidity shock in the interbank market, or the aggregate amount of deposit withdrawal by a given industry due to supply chain disruption.

The focus of this paper is to study the optimal disclosure policy about the signal s. At $t = 0$, the regulator commits to a disclosure policy before conducting the stress test. The disclosure policy is defined by the disclosure set $D \subseteq S$ and the no-disclosure sets $N_n \subseteq S$, where $n \in [1, +\infty)$ denotes the number of no-disclosure sets, and, for simplicity, the first no-disclosure set is denoted by $N \equiv N_1$. For any signal $s \in D$, the regulator communicates it truthfully to the bank. For any signal $s \in N_n$, the regulator communicates to the bank that the signal belongs to N_n. I restrict the analyses of no-disclosure sets to monotone disclosure rule in which N_n pools signals from connected intervals.

The continuation value of the bank's asset is $X(s, \omega)$, which depends on a state variable ω and the economic condition s. The variable ω represents the fundamental of the asset. It is either ω_h with probability q_h or ω_l with probability $q_l \equiv 1 - q_h$ and $\omega_h > \omega_l$. The asset's liquidation value is $L(s,\omega)$. Let $x(s,\omega)$ denote the relative gains from continuing the asset. That is,

$$x(s,\omega) = X(s,\omega) - L(s,\omega).$$

In the rest of the paper, I derive solutions in terms of $x(s,\omega)$. To define efficient liquidation and efficient continuation, I assume that $x(s,\omega_l) \leq 0 \leq x(s,\omega_h)$. This suggests that the asset should continue if and only if its fundamental is high. I assume that $x(s,\omega)$ is increasing and weakly concave in s. I also assume that $x(s,\omega_l)$ is weakly log-concave in s to ensure that $x(s,\omega_l)$ is not too concave in s.[7] For example, $x(s,\omega) = s^q + \omega$ for $0 < q \leq 1$ satisfies all the assumptions for $\underline{s} \geq 0$ and appropriate values of ω. I also assume that for high fundamental asset, the relative gain from continuation increases weakly faster and less concave in s. That is, $x''(s,\omega_h) \geq x''(s,\omega_l)$ and $x'(s,\omega_h) \geq x'(s,\omega_l)$. I make the following further assumption about the bank's asset.

Assumption 1. $\mathbb{E}_\omega\left[x(s,\omega)\right] \in [0, q_h x(\bar{s},\omega_h)]$ *for* $s \in [\underline{s},\bar{s}]$.

This assumption states that ex-ante, the bank's asset is worth continuing. Equivalently, this assumption assumes that the bank's asset has higher expected continuation value than liquidation value for any signal s. Consequently, failing high fundamental asset (inefficient liquidation) is more costly than passing low fundamental asset (inefficient continuation). This assumption helps to characterize the optimal disclosure policy, but my main results can be extended to cases where these conditions are violated. I discuss this assumption in Section 6.1.

The fundamental of the bank's asset determines the report distribution. In particular, if the fundamental is ω_i, then the report t is drawn from a distribution with density $g^i(t)$ over $t \in [\underline{t},\bar{t}]$, where $i = \{h,l\}$. The density functions have full support and satisfy monotone likelihood ratio property (MLRP), i.e., $\frac{g^l(t)}{g^h(t)}$ is decreasing in t. Hence, the report t is informative about the asset fundamental. Moreover, I assume that the ratio $\frac{g^l(t)}{g^h(t)}$ is concave in t. Moreover, I impose the regularity condition that the hazard rate $\frac{g^h(t)}{1-G^h(t)}$ and $\frac{g^l(t)}{1-G^l(t)}$ are decreasing on the support of t. This assumption means that the probability that the reported value will be below t conditional on the reported value is already t is decreasing in t. In other words, once the bank gets high value report, it is more likely to get a even higher report.

At $t = 2$, the bank may engage in costly manipulation to affect the report generating process. I follow Gao and Jiang (2020) to model bank's manipulation as ex-ante manipulation. That is, the bank chooses the manipulation level before observing the fundamental of the asset.

[7]Notice that $x(s,\omega_h)$ is non-negative and concave in s, which then implies that $x(s,\omega_h)$ is also weakly log-concave in s. However, given that $x(s,\omega_l)$ is assumed to be non-positive, such implication breaks down for $x(s,\omega_l)$. Hence, I impose the weakly log-concavity assumption on $x(s,\omega_l)$ only.

Specifically, the bank chooses manipulation $m \in [0,1]$ to change the report distribution from $g^i(t)$ to

$$g_m^i(t) = g^i(t) + m\big(g^h(t) - g^i(t)\big). \tag{1}$$

If $m = 0$, the report generating process is not affected by manipulation. If $m = 1$, then the report is always generated from the distribution of high fundamental asset $g^h(t)$. If $m \in (0,1)$, then manipulation improves the distribution in the sense of first-order stochastic dominance. The cost of manipulation m is $kc(m)$ for the bank. Assume that $k \in (0,+\infty)$ and the cost function $c(m)$ is increasing and convex with $c(0) = c'(0) = 0$. I also assume that $\frac{c'(m)}{c''(m)}$ is weakly increasing in m, or, equivalently, that $c'(m)$ is weakly log-concave. The conditions are often used in the literature (see Gao and Jiang (2020)) and are satisfied for common convex functions, e.g. $c(m) = m^q$ for $q \geq 2$.

After observing s and receiving report t from the bank, the regulator makes a pass/fail decision a at $t = 3$. In particular, the regulator passes ($a = 1$) or fails ($a = 0$) the bank to maximize u in the following

$$u \equiv ax(s,\omega). \tag{2}$$

The bank's payoff v is

$$v \equiv a\big(x(s,\omega) + B\big) - kc(m). \tag{3}$$

Where B is the bank's private benefit from continuing the asset. I assume $x(s,\omega)+B > 0$ for all s and ω, meaning that the bank's private benefit B is large enough such that the bank prefers continuation for any value of x. The private benefit then leads to conflict of interest between the regulator and the bank.

The timeline of the model is as follows,

At $t = 0$, the regulator commits to a disclosure policy about the signal s.

At $t = 1$, the regulatory models generate a signal s. The regulator privately observes s and discloses s to the bank according to the disclosure policy.

At $t = 2$, the bank chooses the level of manipulation m to affect the report generating process.

At $t = 3$, state ω is realized, and the bank's report t is generated. Based on the signal s and report t, the regulator passes or fails the bank. And payoffs are realized.

The equilibrium consists of the regulator's disclosure policy about s and the pass/fail decision a, and the bank's manipulation m. I solve the model by backward induction. That is, I first solve for the regulator's pass/fail decision a for given manipulation level m and disclosure policy about s. Anticipating the pass/fail decision rule, the bank then chooses the manipulation m

for given disclosure policy about s. Lastly, the regulator chooses the disclosure policy about s, taking into account its impact on the bank's manipulation choice and in turn on the pass/fail decision.

3 Manipulation and pass/fail decision

In this section, I discuss the bank's manipulation choice and the regulator's pass/fail decision, taking the disclosure policy as given.

At $t = 3$, the regulator forms expectation of the continuation value x based on her own signal s and the bank's report t. The regulator passes the bank if and only if

$$\mathbb{E}_\omega[x(s,\omega)|t,\hat{m}] \geq 0. \tag{4}$$

Where $\hat{m}$ is the regulator's conjecture about the bank's manipulation.[8] Since $g^h(t)$ is a monotone likelihood ratio improvement of $g^l(t)$, the expected continuation value $\mathbb{E}_\omega[x(s,\omega)|t,\hat{m}]$ is increasing in the report t. As a result, the pass/fail decision follows a cutoff rule

Lemma 1. *For given signal s and bank's report t and conjecture about the bank's manipulation $\hat{m}$, the regulator passes the bank if and only if $t \geq t_p(s,\hat{m})$, where the passing threshold $t_p(s,\hat{m})$ solves*

$$\mathbb{E}_\omega[x(s,\omega)|t_p,\hat{m}] = 0.$$

All proofs are included in Appendix A. The passing threshold $t_p(s,\hat{m})$ is defined by the regulator's indifferent condition. That is, the regulator is indifferent between passing and failing the bank when the report is $t_p(s,\hat{m})$. In other words, the passing threshold is chosen to equalize the expected cost of failing high fundamental asset (inefficient liquidation) and the expected cost of passing low fundamental asset (inefficient continuation) *for given signal s and given conjecture about manipulation $\hat{m}$*. The following lemma characterizes the passing threshold $t_p(s,\hat{m})$.

Lemma 2. *For given level of manipulation m, the passing threshold $t_p(s,m)$ is decreasing in s. For given signal s, the passing threshold $t_p(s,m)$ is decreasing in m.*

[8]The conditional expectation is

$$\begin{aligned}\mathbb{E}_\omega[x(s,\omega)|t,\hat{m}] &= x(s,\omega_h)\Pr(\omega=\omega_h|t,\hat{m}) + x(s,\omega_l)\Pr(\omega=\omega_l|t,\hat{m}) \\ &= x(s,\omega_h)\frac{q_h g^h(t)}{q_h g^h(t)+q_l g^l_{\hat{m}}(t)} + x(s,\omega_l)\frac{q_l g^l_{\hat{m}}(t)}{q_h g^h(t)+q_l g^l_{\hat{m}}(t)}.\end{aligned}$$

The intuition of this lemma follows from how the cost of failing high fundamental asset and that of passing low fundamental asset changes with the signal s and the manipulation m. For given manipulation level m, the relative gain from continuing the asset $x(s, \omega)$ is increasing in s which implies that failing high fundamental asset becomes more costly relative to passing low fundamental asset. In response to the increasing relative cost of inefficient liquidation, the regulator is willing to lower the passing threshold and pass the bank more often. The second result captures how manipulation affects the relative cost of failing high fundamental asset and passing low fundamental asset. Assumption 1 assumes that in absence of the report, the regulator's expectation of the relative gain from continuing the asset is non-negative. This implies that inefficient liquidation (failing high fundamental asset) is more costly than inefficient continuation (passing low fundamental asset) in expectation for all signal s. Manipulation makes the report distribution of low fundamental asset and that of high fundamental asset more similar, making it more difficult for the regulator to differentiate the two types of assets. In order to preserve the high fundamental asset, the regulator needs to decrease the passing threshold.

At $t = 2$, the bank anticipates the passing threshold $t_p(s, \hat{m})$ and chooses the manipulation m to maximize the expected payoff. The bank's expected payoff depends on the disclosure of s. If the bank does not observe the regulator's signal s, the expected payoff is

$$V(\hat{m}, m) = \mathbb{E}_s \left[q_h \big(x(s, \omega_h) + B \big) \int_{t \geq t_p(s,\hat{m})} g^h(t) dt + q_l \big(x(s, \omega_l) + B \big) \int_{t \geq t_p(s,\hat{m})} g^l_m(t) dt \,\middle|\, s \in N_n \right] - kc(m).$$

Where N_n is the no disclosure set containing signals s that are not disclosed to the bank. For ease of exposition, I introduce the following definition.

$$\Delta\big(t_p(s, \hat{m})\big) \equiv \int_{t \geq t_p(s,\hat{m})} \big(g^h(t) - g^l(t) \big) dt. \tag{5}$$

This term is the difference in passing probability between high fundamental asset and low fundamental asset. It also measures the increases in passing probability of low fundamental asset after one unit of manipulation. Taking derivative of $V(\hat{m}, m)$ with respect to m, I obtain the following first-order condition of the bank's manipulation m,

$$\mathbb{E}_s \left[q_l \big(x(s, \omega_l) + B \big) \Delta\big(t_p(s, \hat{m})\big) \,\middle|\, s \in N_n \right] - kc'(m) = 0.$$

In equilibrium, the regulator's conjecture about the manipulation $\hat{m}$ is consistent with the bank's choice. The equilibrium manipulation m_{N_n} solves

$$\mathbb{E}_s \left[q_l\big(x(s,\omega_l)+B\big)\Delta\big(t_p(s,m_{N_n})\big) \,\middle|\, s \in N_n \right] - kc'(m_{N_n}) = 0. \tag{6}$$

This condition suggests that no disclosure of s forces the bank's manipulation m_{N_n} to be constant over the regulator's signal s.

If the bank observes the regulator's signal s, the expected payoff is

$$V(s,\hat{m},m) = q_h\big(x(s,\omega_h)+B\big)\int_{t\geq t_p(s,\hat{m})} g^h(t)dt + q_l\big(x(s,\omega_l)+B\big)\int_{t\geq t_p(s,\hat{m})} g^l_m(t)dt - kc(m).$$

The first-order condition of the bank's manipulation response m is as follows,

$$q_l\big(x(s,\omega_l)+B\big)\Delta\big(t_p(s,\hat{m})\big) - kc'(m) = 0.$$

Similar to no disclosure case, the regulator's conjecture about the manipulation is consistent with the bank's choice in equilibrium. The equilibrium manipulation $m_D(s)$ is determined by

$$q_l\big(x(s,\omega_l)+B\big)\Delta\Big(t_p\big(s,m_D(s)\big)\Big) - kc'\big(m_D(s)\big) = 0. \tag{7}$$

I make the following notation for ease of exposition

$$MB_b\big(s,t_p(s,m)\big) \equiv q_l\big(x(s,\omega_l)+B\big)\Delta\big(t_p(s,m)\big). \tag{8}$$

Where "MB" stands for "marginal benefit" and "b" represents "bank". $MB_b\big(s,t_p(s,m)\big)$ is the bank's marginal benefit of manipulation for given regulator's signal s and manipulation level m. It consists of two components. The first component is the expected gain after passing the test with manipulation $q_l\big(x(s,\omega_l)+B\big)$. Given that the relative gain from continuing the asset $x(s,\omega_l)$ is increasing in the signal s, the expected gain after passing the test with manipulation is increasing in s. All else equal, the bank manipulates more when the signal s is high.

The second component $\Delta\big(t_p(s,m)\big)$ represents the increases in the passing probability if the bank changes the report distribution from $g^l(t)$ to $g^h(t)$. This term crucially depends on the passing threshold $t_p(s,m)$. Lemma 2 shows that the passing threshold $t_p(s,m)$ is decreasing in s, since the relative cost of failing the high fundamental asset is rising. As the passing threshold decreases, the test becomes more lenient in the sense that low fundamental asset is more likely to pass the test without manipulation. In other words, the difference in the passing probability between $g^h(t)$ and $g^l(t)$ shrinks. The following lemma summarizes the impact of the signal s on the difference in passing probability $\Delta\big(t_p(s,m)\big)$.

Lemma 3. *For given manipulation level m, $\Delta\big(t_p(s,m)\big)$ is decreasing in s.*

This lemma suggests that all else equal, the bank manipulates less when the signal s is high. When evaluating the bank's manipulation incentive MB_b, the differences in passing probability $\Delta\big(t_p(s,m)\big)$ acts as a counterforce to the expected gain after passing the test with manipulation $q_l\big(x(s,\omega_l)+B\big)$. The magnitude of the two forces then determines how the manipulation $m_D(s)$ responds to the signal s.

Proposition 1. *When s is disclosed, the level of manipulation $m_D(s)$ is unique and it is increasing in s for $s < s_D$ and it is decreasing in s for $s > s_D$, where s_D is the unique solution for $\frac{\partial MB_b\left(s,t_p(s,m_D)\right)}{\partial s} = 0$.*

This result identifies the forces that determines the bank's manipulation $m_D(s)$ when s is disclosed, and it highlights the effect of passing threshold $t_p(s,m)$ on the bank's manipulation $m_D(s)$. When the signal is relatively low, i.e., $s < s_D$, the cost of inefficient liquidation compared to that of inefficient continuation is moderate. Hence, the passing threshold is set at a medium level such that it is effective in preventing both types of error. In this case, the passing probability between $g^h(t)$ and $g^l(t)$ differs substantially, making the manipulation incentive MB_b to be sensitive to the changes in the expected gain after passing the test with manipulation. As a result, the bank's manipulation $m_D(s)$ follows the changes in the expected gain after passing the test with manipulation and it is increasing in s. When $s > s_D$, the inefficient liquidation becomes very costly compared to inefficient continuation. Hence, the passing threshold is set primarily to prevent inefficient liquidation. Consequently, the low fundamental asset is more likely to pass the test even without manipulation, leaving little incremental effect for manipulation, hence, little incentive for the bank to manipulate.

Disclosure of s affects how manipulation changes with s. When s is not disclosed, the bank's manipulation m_{N_n} is constant over the signal s. When s is disclosed, the bank's manipulation incentive changes with both the expected gain after passing the test with manipulation $q_l(x(s,\omega_l)+B)$ and the increases in passing probability of low fundamental asset after manipulation $\Delta\Big(t_p\big(s,m_D(s)\big)\Big)$. Consequently, the manipulation $m_D(s)$ varies with s and such variation further affects the expected level of manipulation. The following proposition compares the expected level of manipulation when s is disclosed with the one when s is not disclosed.

Proposition 2. $\mathbb{E}_s\left[m_D(s)|s \in N\right] \le m_N$ *if* $N \subseteq [\underline{s}, s_D]$ *and* $\mathbb{E}_s\left[m_D(s)|s \in N\right] \ge m_N$ *if* $N \subseteq [s_D, \bar{s}]$.

This result shows the additional effect of disclosing s. When $s \leq s_D$, the bank's manipulation $m_D(s)$ is driven by the expected gain after passing the test with manipulation $q_l\big(x(s,\omega_l)+B\big)$ and it is increasing in the signal s. In response, the regulator decreases the passing threshold $t_p\big(s, m_D(s)\big)$, which makes the bank more likely to pass the test regardless of the fundamental value. Such endogenous response of the regulator's pass/fail decision then decreases the magnitude of passing probability that can be increased by manipulation, leaving manipulation less useful and decreases the bank's manipulation incentive. Such endogenous response is absent if s is not disclosed. Hence, the expected level of manipulation is less if s is disclosed. However, when $s > s_D$, the bank manipulates to increase the passing probability and the manipulation level $m_D(s)$ is decreasing in s. In response, the regulator increases the passing threshold $t_p\big(s, m_D(s)\big)$ to make the test more difficult. Such endogenous response of the regulator's pass/fail decision then widens the difference of passing probability between low and high fundamental asset. More importantly, such response makes the manipulation useful in increasing the passing probability for low fundamental asset, amplifying the bank's manipulation incentive. Hence, the expected level of manipulation when s is disclosed is larger compared to the case when s is not disclosed.

4 Disclosure

In this section, I discuss the optimal disclosure policy about the regulator's signal s, taking into account the bank's manipulation response and its impact on the regulator's pass/fail decision. I show that disclosure and passing threshold are complementary tools for the regulator to minimize the adverse consequence of the bank's manipulation.

For given signal s, the regulator's expected payoff at $t = 1$ is obtained by integrating all reports value that are higher than the passing threshold $t_p(s, m^*)$,

$$\begin{aligned} u(s, m^*) &= \int_{t \geq t_p(s,m^*)} \mathbb{E}_\omega[x(s,\omega)|t, m^*] g_{m^*}(t) dt \\ &= \int_{t \geq t_p(s,m^*)} \big(q_h x(s,\omega_h) g^h(t) + q_l x(s,\omega_l) g^l_{m^*}(t)\big) dt. \end{aligned} \tag{9}$$

Where $m^* = \{m_D(s), m_{N_n}\}$ is the equilibrium manipulation choice of the bank and $g_{m^*}(t)$ is the unconditional distribution of report t when the manipulation is m^*. That is,

$$g_{m^*}(t) = q_h g^h(t) + q_l g^l_{m^*}(t).$$

At $t = 0$, the regulator chooses disclosure policy D and N_n to maximize the ex-ante payoff

$$\begin{aligned} U &= \int_{s\in D} u\big(s, m_D(s)\big) dF(s) + \sum_n \left(\int_{s\in N_n} u(s, m_{N_n}) dF(s) \right) \\ &= \int_{s\in D} \left(\int_{t\geq t_p\left(s, m_D(s)\right)} \big(q_h x(s,\omega_h) g^h(t) + q_l x(s,\omega_l) g^l_{m_D(s)}(t)\big) dt \right) dF(s) \\ &\quad + \sum_n \left(\int_{s\in N_n} \left(\int_{t\geq t_p(s, m_{N_n})} \big(q_h x(s,\omega_h) g^h(t) + q_l x(s,\omega_l) g^l_{m_{N_n}}(t)\big) dt \right) dF(s) \right). \end{aligned} \tag{10}$$

As briefly discussed in Lemma 2, manipulation increases the similarity between the report of low fundamental asset and that of the high fundamental asset, making it more likely that the regulator fails the high fundamental asset (inefficient liquidation) and passes the low fundamental asset (inefficient continuation). The regulator is able to use the pass/fail decision to control this adverse consequence of manipulation, but only partially. Because when choosing the passing threshold $t_p(s, m)$, the regulator trades off the cost of inefficient liquidation against the cost of inefficient continuation *for given level of manipulation* m. However, manipulation increases the likelihood of inefficient continuation ex-ante, which cannot be prevented by using the optimal pass/fail rule. I first define the additional losses caused by manipulation for given signal s. Taking derivative of $u(s, m)$ in (9) with respect to m, I obtain[9]

$$ML_r\big(s, t_p(s, m)\big) \equiv q_l x(s, \omega_l) \Delta\big(t_p(s, m)\big). \tag{11}$$

Where "ML" stands for "marginal loss" and "r" represents "regulator". Given that the asset should be liquidated when fundamental is low, i.e., $x(s, \omega_l) < 0$ for all s, this term is negative. It captures the regulator's marginal losses from continuing the low fundamental asset due to manipulation.

The additional losses caused by manipulation $ML_r\big(s, t_p(s, m)\big)$ consists of two components. The first component is the expected loss of passing the low fundamental asset $q_l x(s, \omega_l)$. The second component $\Delta\big(t_p(s, m)\big)$ is the increases in passing probability after the bank changes the report distribution from $g^l(t)$ from $g^h(t)$. This component captures the regulator's inability to distinguish the low fundamental asset and the high fundamental asset due to the bank's manipulation.

Lemma 4. *For any disclosure set D or no-disclosure set N_n, $ML_r\big(s, t_p(s, m^*)\big)$ is increasing in s for $m^* = \{m_D(s), m_{N_n}\}$.*

[9]Notice that the passing threshold $t_p(s, m)$ is chosen optimally for given signal s and manipulation m, hence, the derivative with respect to $t_p(s, m)$ is zero and does not appear in ML_r, i.e., $\frac{\partial u(s,m)}{\partial t_p(s,m)} \frac{\partial t_p(s,m)}{\partial m} = 0$.

This lemma suggests that, regardless of the disclosure of s, the regulator bears less additional losses from manipulation as the signal s increases. The intuition is as follows. Since the relative gain from continuing the asset $x(s, \omega_l)$ is increasing in s, the regulator's loss from passing the low fundamental asset is ameliorated. In addition, the passing threshold $t_p(s, m^*)$ is decreasing in the signal s, shrinking the difference in passing probability between the low fundamental asset and high fundamental $\Delta\big(t_p(s, m^*)\big)$. This means that when the signal s increases, the increase in passing probability decreases even if the bank shifts the report distribution from $g^l(t)$ to $g^h(t)$, decreasing the chance of passing low fundamental asset due to manipulation. Notice that when the passing threshold is very low, the regulator is more likely to pass the low fundamental asset, but such continuation is not driven by manipulation but rather by the tradeoff of inefficient liquidation and inefficient continuation. Such tradeoff is incorporated in the optimal passing threshold $t_p(s, m^*)$. $ML_r\big(s, t_p(s, m^*)\big)$ does not capture such continuation and it only reflects the regulator's additional mistakes caused by the bank's manipulation.

The regulator needs additional tool to control $ML\big(s, t_p(s, m)\big)$. In what follows, I discuss how disclosure of the regulatory signal s can affect $ML\big(s, t_p(s, m)\big)$. Lemma 4 shows that the regulator's additional loss caused by manipulation $ML_r\big(s, t_p(s, m)\big)$ is increasing in the signal s. To minimize the regulator's expected loss from manipulation, the regulator should distribute more manipulation to cases where the marginal loss $ML_r\big(s, t_p(s, m)\big)$ is small. Recall that Proposition 1 and Proposition 2 state that disclosure not only affect how manipulation distributes across the signal s but also affect the expected amount of manipulation across all signal s. Hence, the regulator can use the disclosure of the regulatory signal s to minimize the expected loss from manipulation.

First, for given expected amount of manipulation, the disclosure of the regulatory signal s affects how manipulation distributes across the regulator's marginal loss ML_r. Disclosing s reveals $\Delta\big(t_p(s, m)\big)$ which is the increases in the passing probability after changing the report distribution from $g^l(t)$ to $g^h(t)$. As captured by $MB_b(s, t_p(s, m))$, all else equal, the bank's gain from manipulation is higher when $\Delta\big(t_p(s, m)\big)$ is large. A large $\Delta\big(t_p(s, m)\big)$ also means that the regulator is more likely to be misled by manipulation and make wrong passing decisions, which in turn increases the regulator's expected loss from bank's manipulation $ML(s, t_p(s, m))$. This means that the bank's and the regulator's interests are not aligned after observing $\Delta\big(t_p(s, m)\big)$. Hence, disclosure of s incurs cost for the regulator because it facilitates the bank to manipulate more when the regulator is more susceptible to manipulation. Disclosing s also gives benefit to the regulator. Since the payoff of the asset $x(s, \omega)$ depends both on the regulator's information s and on the state ω, disclosing s reduces the bank's uncertainty about asset payoff. All else equal, the bank manipulates less when the expected gain after passing the test with manipulation is low, i.e., when $q_l(x(s,\omega_l)+B)$ is low. This manipulation choice is beneficial to the regulator. Because when $x(s,\omega_l)$ is low, passing the bank incurs large loss for the regulator. In other words, the regulator demands more informative report when $x(s,\omega_l)$ is low. Disclosing s then makes the regulator's pass/fail decision more accurate. Given the result in Proposition 1, the benefit dominates the cost of disclosure when the signal s is small.

In addition, Proposition 2 shows that disclosure of the signal s also changes the expected manipulation level. This additional layer strengthens the existing tradeoff of disclosure. As a result, the optimal disclosure policy follows a simple cutoff rule.

Proposition 3. *The optimal disclosure policy follows a cutoff rule where $D = [\underline{s}, s^*)$ and $N = [s^*, \bar{s}]$. That is, the regulator discloses the signal s when $s < s^*$ and does not disclose the signal s when $s > s^*$, where $s^* \in [\underline{s}, s_D]$.*

The intuition for this result is embedded in the tradeoff of disclosure. It is beneficial for the regulator to disclose the signal s when the manipulation is driven by the expected gain after passing the test with manipulation $q_l\big(x(s,\omega_l)+B\big)$. In this case, the bank's manipulation is increasing in s which implies that the bank's manipulation is less (more) when it causes more (less) losses to the regulator as measured by $ML(s, t_p(s,m))$. In addition, the bank manipulates less in expectation when observing the regulator's signal s. Hence, disclosure improves the regulator's ex-ante payoff. However, as the signal s increases, the bank's manipulation is driven by the increases in passing probability after manipulation $\Delta\big(t_p(s,m)\big)$. If the signal is disclosed to the bank, then the bank would manipulate more when the regulator is more susceptible to manipulation. Hence, no disclosure complements the passing threshold to deter the bank's manipulation. No disclosure at all can be optimal if it sufficiently reduces the expected level of manipulation. In sum, the disclosure of s complements the pass/fail decision to minimize the adverse consequence of the bank's manipulation for the regulator.

5 Comparative statics

In this section, I analyze how the optimal disclosure policy changes with the bank's private benefit B when passing the test and the cost of manipulation k.

All else equal, increasing the private benefit B or decreasing the manipulation cost k incentivizes the bank to manipulate more. Such increase in manipulation occurs no matter the signal s is disclosed or not disclosed to the bank. As a result, the implications on the disclosure policy is unclear. However, the following lemma shows that disclosure is more likely to happen when the cost of manipulation increases and/or when the bank's private benefit decreases.

Proposition 4. *The disclosure cutoff point s^* is increasing in k and decreasing in B.*

As the cost of manipulation increases or the private benefit decreases, the disclosure point cutoff point s^* becomes greater, suggesting more disclosure. The intuition is as follows. Recall that disclosure is beneficial to the regulator when the bank's manipulation incentive is driven by the expected gain after passing the test with manipulation. In this case, the disclosure of s commands the bank to manipulate less when the regulator suffers more losses from manipulation, maximizing the regulator's utility. Since the disclosure cutoff point s^* is less than s_D, the manipulation incentive $MB_b\big(s, t_p(s, m_D(s))\big)$ is still driven by the expected gain after passing the test with manipulation, which means that the regulator would gain from increasing the disclosure as long as the manipulation is controlled. Therefore, when the cost of manipulation k increases, the regulator's concern over manipulation is alleviated, supporting more disclosure. On the contrary, when the bank's private benefit B increases, the bank has stronger incentive to manipulate to increase the passing probability, which suggests that the regulator should reduce disclosure.

6 Discussions

6.1 Cost of inefficient liquidation and inefficient continuation

Assumption 1 assumes that the bank's asset is worth continuing ex-ante. This assumption affects how the regulator's choice of passing threshold $t_p(s, m)$ responds to the bank's manipulation m (Lemma 2) and how the bank's manipulation changes with the regulatory signal s when s is disclosed (Lemma 3 and Proposition 1). Nevertheless, the main insight for the disclosure of the regulator's signal s does not depend on this assumption. The regulator's pass/fail decision is still insufficient in restricting the adverse consequence of bank's manipulation. Hence, disclosure of the regulatory signal s is useful. When the signal s is disclosed, the bank's manipulation is determined by two forces: the expected gain after passing the test with manipulation $q_l\big(x(s, \omega_l) + B\big)$ and the increases in passing probability after manipulation $\Delta\big(t_p(s, m)\big)$. And the regulator's losses from manipulation $ML_r(s, t_p(s, m))$ depends on the expected losses of inefficient continuation $q_l x(s, \omega_l)$ and the increases in passing probability after manipulation $\Delta\big(t_p(s, m)\big)$. Disclosure is always beneficial to the regulator when both the bank's manipulation and the regulator's loss from manipulation are driven by the changes in the relative gain from continuing the asset $x(s, \omega_l)$. Instead, no disclosure is preferred when the changes in manipulation is driven by the increases in passing probability after manipulation $\Delta\big(t_p(s, m)\big)$. The former force is more likely to dominate when the tradeoff of the cost of inefficient liquidation and the cost of inefficient continuation is moderate. Because in such case, the regulator's choice of passing threshold leads to large increases in passing probability if the bank manipulates, i.e., $\Delta\big(t_p(s, m)\big)$ is large. This then incentivizes the bank to care about the gain after passing the test when choosing the manipulation. Hence, the bank's manipulation is more likely to be driven by the expected gain after passing the test with manipulation $q_l\big(x(s, \omega_l) + B\big)$. As discussed above, such manipulation choice benefits the regulator. In any case, the disclosure of s still complements the pass/fail decision. In Appendix B, I derive the results formally.

6.2 No commitment to disclosure policy

Suppose that the regulator cannot commit to any disclosure policy about s. Instead, the regulator chooses to disclose or not to disclose the signal s after observing the realization of it. In the following, I show that the only equilibrium is full disclosure.

The intuition is as follows. Suppose that the no disclosure set is $N = [s_1, s_2]$ with $s_1 < s_2$. Denote the bank's manipulation response as m_N. After observing the signal s, the regulator would disclose s with which the bank has manipulation $m_D(s) < m_N$. This implies that the no disclosure set must consist of signals s such that $m_N \leq m_D(s)$, implying that $\mathbb{E}\left[MB(s, t_p(s, m_N))|s \in [s_1, s_2]\right] \leq MB(s, t_p(s, m_D(s))$ for $s \in [s_1, s_2]$. Since $MB(s, t_p(s, m))$ is a continuous function of s, the regulator must be indifferent between disclosing and not disclosing the signals at the boundary of not disclosure set, i.e., $m_N = m_D(s_1) = m_D(s_2)$. Hence, the following condition must hold

$$\mathbb{E}_s\left[MB(s, t_p(s, m_N)|s \in [s_1, s_2]\right] = MB(s_1, t_p(s_1, m_N)) = MB(s_2, t_p(s_2, m_N)).$$

However, given that $MB(s, t_p(s, m))$ is first increasing and then decreasing in s for any given manipulation m, this condition cannot hold if $s_1 < s_2$. Hence, $s_1 = s_2$ and full disclosure is the equilibrium.

6.3 Real activity

In the baseline model, the bank exerts costly effort to manipulate the report generating process. Manipulation improves the report in the sense of first-order stochastic dominance but it does not affect the asset payoff. Hence, the disclosure of the regulator's private information only has informational consequence on the bank. It informs the bank about the gain from manipulation and the probability of obtaining the gain. In Appendix C, I discuss an extension in which the disclosure of the regulator's private information not only affects the bank's reporting choice, but also affects how the bank invests. More specifically, the bank exerts costly effort to improve the payoff of the asset and such effort manifests itself in the report. Such effort still increases the similarity of the report of low fundamental asset and that of high fundamental asset, but such increases in report similarity comes from the actual improvement in asset quality. As a result, the disclosure of the regulator's private information affects the real activities of the bank, i.e., effort choice.

7 Conclusion

This paper presents a tractable model to characterize the optimal disclosure policy about the regulatory assessment models, when facing the manipulation concern. Disclosing the regulatory models helps the bank to learn about its asset, which deters the bank's manipulation incentive. However, disclosing the model also makes it easier for the bank to game the assessment. The main message of the paper is that the disclosure policy about regulatory models complements the assessment rule. I also show that the accounting regulation, which governs the banks' reporting discretion, complements the design and improves the effectiveness of regulatory assessment. This study may provide regulatory implications for supervisory and climate risk stress test design, and it may help us better understand the interactions between performing stress tests and reporting incentives of banks.

A Proofs

For ease of exposition, I define the following ratio

$$r(t) \equiv \frac{g^l(t)}{g^h(t)}. \tag{12}$$

Due to the assumption that the density function of report t satisfies MLRP, the ratio $r(t)$ is decreasing in t.

Proof. Lemma 1

All the necessary steps for the cutoff rule are explained in the text. □

Proof. Lemma 2

The regulator chooses the passing threshold based on the signal s and the conjecture about the bank's manipulation $\hat{m}$. I drop the $\hat{}$ for simplicity.

The passing threshold is determined by

$$\mathbb{E}_\omega[x(s,\omega)|t_p, m] = 0.$$

This condition is equivalent to

$$x(s,\omega_h)\frac{q_h g^h(t_p)}{q_h g^h(t_p) + q_l g^l_m(t_p)} + x(s,\omega_l)\frac{q_l g^l_m(t_p)}{q_h g^h(t_p) + q_l g^l_m(t_p)} = 0,$$

Since the density function $g^l(t)$ and $g^h(t)$ have full support, the condition reduces to

$$x(s,\omega_h)q_h g^h(t_p) + x(s,\omega_l)q_l g^l_m(t_p) = 0.$$

This is equivalent to

$$x(s,\omega_h)q_h + x(s,\omega_l)q_l - x(s,\omega_l)q_l(1-m)\big(1 - r(t_p)\big) = 0. \tag{13}$$

Apply implicit function theorem, I derive the following two partial derivatives.

$$\frac{\partial t_p}{\partial s} = -\frac{q_h\left(x(s,\omega_l)\frac{dx(s,\omega_h)}{ds} - x(s,\omega_h)\frac{dx(s,\omega_l)}{ds}\right)}{(1-m)q_l\big(x(s,\omega_l)\big)^2 r'(t_p)}. \tag{14}$$

Given that the relative gain from continuing the asset $x(s,\omega_l)$ and $x(s,\omega_h)$ are increasing in s and the ratio $r(t)$ is decreasing in t, this derivative is negative.

And the following is the partial derivative of t_p with respect to m,

$$\frac{\partial t_p}{\partial m} = -\frac{q_h x(s,\omega_h) + q_l x(s,\omega_l)}{(1-m)^2 q_l x(s,\omega_l) r'(t_p)}. \tag{15}$$

where $r'(t_p)$ is the derivative of $r(t_p)$ with respect to t_p. Given Assumption 1, the unconditional expected relative gain from continuing the asset is non-negative. Hence, this derivative is non-positive and it equals to zero only when $s = \underline{s}$. □

Proof. Lemma 3

For given passing threshold $t_p(s,m)$, the difference in passing probability between g^l and g^h is $\Delta\big(t_p(s,m)\big)$. I repeat the definition of $\Delta\big(t_p(s,m)\big)$ here

$$\Delta\big(t_p(s,m)\big) \equiv \int_{t \geq t_p(s,m)} \big(g^h(t) - g^l(t)\big) dt.$$

Taking derivative with respect to s, I obtain the following

$$\begin{aligned}\frac{\partial \Delta\big(t_p(s,m)\big)}{\partial s} &= \frac{d\Delta\big(t_p(s,m)\big)}{dt_p(s,m)} \frac{\partial t_p(s,m)}{\partial s} \\ &= \Big(g^l\big(t_p(s,m)\big) - g^h\big(t_p(s,m)\big)\Big) \frac{\partial t_p(s,m)}{\partial s} \\ &\propto \Big(r\big(t_p(s,m)\big) - 1\Big) \frac{\partial t_p(s,m)}{\partial s}.\end{aligned}$$

Recall that equation (13) pins down the passing threshold $t_p(s,m)$, and the ratio $r\big(t_p(s,m)\big)$ solves

$$r\big(t_p(s,m)\big) = \frac{mq_l x(s,\omega_l) + q_h x(s,\omega_h)}{mq_l x(s,\omega_l) - q_l x(s,\omega_l)} \geq \frac{mq_l x(s,\omega_l) - q_l x(s,\omega_l)}{mq_l x(s,\omega_l) - q_l x(s,\omega_l)} = 1. \tag{16}$$

The inequality holds because Assumption 1 implies that $q_h x(s,\omega_h) \geq -q_l x(s,\omega_l)$ for all s and equality holds only when $s = \underline{s}$. As a result, the derivative $\frac{d\Delta\big(t_p(s,m)\big)}{dt_p(s,m)}$ is non-negative. Given the result of Lemma 2 that $\frac{\partial t_p(s,m)}{\partial s} < 0$, the derivative $\frac{\partial \Delta\big(t_p(s,m)\big)}{\partial s} \leq 0$ and equality holds only when $s = \underline{s}$. □

Proof. Proposition 1

When s is disclosed, the manipulation level is determined by the first-order condition in equation (7). I repeat the first-order condition here,

$$q_l\big(x(s,\omega_l) + B\big)\Delta\big(t_p(s,m_D)\big) - kc'(m_D) = 0.$$

The first term of the left-hand side is $MB_b\big(s, t_p(s,m_D)\big)$. Apply implicit function theorem to the first-order condition, I derive the derivative of m_D with respect to s,

$$\frac{\partial m_D}{\partial s} = \frac{\frac{\partial MB_b\left(s,t_p(s,m_D)\right)}{\partial s}}{kc''(m_D) - \frac{\partial MB_b\left(s,t_p(s,m_D)\right)}{\partial m_D}} = \frac{q_l\left(\Delta\left(t_p(s,m_D)\right)\frac{dx(s,\omega_l)}{ds} + \left(x(s,\omega_l)+B\right)\frac{\partial\Delta\left(t_p(s,m_D)\right)}{\partial s}\right)}{kc''(m_D) - q_l\left(x(s,\omega_l)+B\right)\frac{\partial\Delta\left(t_p(s,m_D)\right)}{\partial m_D}}. \tag{17}$$

The derivative $\frac{\partial\Delta\left(t_p(s,m)\right)}{\partial m}$ is given by

$$\frac{\partial\Delta\left(t_p(s,m)\right)}{\partial m} = \frac{d\Delta\left(t_p(s,m)\right)}{dt_p(s,m)}\frac{\partial t_p(s,m)}{\partial m} \propto \left(r\left(t_p(s,m)\right)-1\right)\frac{\partial t_p(s,m)}{\partial m} \leq 0. \tag{18}$$

I omit the proof, since it is similar to the proof of Lemma 3. Consequently, the following holds

$$\frac{\partial m_D}{\partial s} \propto \frac{\partial MB_b\left(s,t_p(s,m_D)\right)}{\partial s} \propto \Delta\left(t_p(s,m_D)\right)\frac{dx(s,\omega_l)}{ds} + \left(x(s,\omega_l)+B\right)\frac{\partial\Delta\left(t_p(s,m_D)\right)}{\partial s}.$$

For ease of exposition, I introduce the following notation

$$F \equiv \Delta\left(t_p(s,m_D)\right)\frac{dx(s,\omega_l)}{ds} + \left(x(s,\omega_l)+B\right)\frac{\partial\Delta\left(t_p(s,m_D)\right)}{\partial s}$$

In the following, I first show that $F=0$ holds at some $s\in(\underline{s},\bar{s})$ and then I prove that $F=0$ is unique at $s=s_D$.

When $s=\underline{s}$, Assumption 1 assumes that $x(\underline{s},\omega_h)q_h + x(\underline{s},\omega_l)q_l = 0$. According to equation (13), the passing threshold satisfies $r\left(t_p(\underline{s},m)\right)=1$ which implies that $\frac{\partial\Delta\left(t_p(s,m)\right)}{\partial s}=0$, hence, the function F is

$$F|_{s=\underline{s}} = \Delta\left(t_p(\underline{s},m_D)\right)\left.\frac{dx(s,\omega_l)}{ds}\right|_{s=\underline{s}} > 0.$$

When $s=\bar{s}$, Assumption 1 implies that $x(\bar{s},\omega_l)=0$. Hence, the passing threshold is $t_p(\bar{s},m_D)=\underline{t}$ and $\Delta(\underline{t})=0$. Hence, the function F is

$$F|_{s=\bar{s}} = B\left.\frac{\partial\Delta\left(t_p(s,m_D)\right)}{\partial s}\right|_{s=\bar{s}} < 0.$$

By the intermediate value theorem, $F=0$ must hold at some value of $s\in(\underline{s},\bar{s})$.

Next, I show that $F=0$ is unique at $s=s_D$. When $F=0$, the following equation holds,

$$\Delta\left(t_p(s,m_D)\right)\frac{\frac{dx(s,\omega_l)}{ds}}{x(s,\omega_l)+B} = -\frac{\partial\Delta\left(t_p(s,m_D)\right)}{\partial s}.$$

I drop the indicator D for the manipulation m_D. Then $F = 0$ is equivalent to

$$\frac{G^l\big(t_p(s,m)\big) - G^h\big(t_p(s,m)\big)}{g^h\big(t_p(s,m)\big) - g^l\big(t_p(s,m)\big)} \frac{\frac{dx(s,\omega_l)}{ds}}{x(s,\omega_l)+B} = \frac{\partial t_p(s,m)}{\partial s}. \tag{19}$$

I first show that the left-hand side is increasing in s. I drop the arguments for t_p when no confusion caused. The left-hand side is equivalent to

$$LHS \equiv -\frac{\Delta(t_p)}{\Delta'(t_p)} \frac{\frac{dx(s,\omega_l)}{ds}}{x(s,\omega_l)+B}.$$

Where $\Delta'(t_p) \equiv \frac{d\Delta(t_p)}{dt_p}$. The derivative of LHS with respect to s is

$$\frac{\partial LHS}{\partial s} = -\frac{d\left(\frac{\Delta(t_p)}{\Delta'(t_p)}\right)}{dt_p} \frac{\partial t_p}{\partial s} \frac{\frac{dx(s,\omega_l)}{ds}}{x(s,\omega_l)+B} - \frac{d\left(\frac{\frac{dx(s,\omega_l)}{ds}}{x(s,\omega_l)+B}\right)}{ds} \frac{\Delta(t_p)}{\Delta'(t_p)}.$$

The derivative $\frac{d\left(\frac{\Delta(t_p)}{\Delta'(t_p)}\right)}{dt_p}$ is

$$\frac{d\left(\frac{\Delta(t_p)}{\Delta'(t_p)}\right)}{dt_p} = \frac{\Delta'(t_p)^2 - \Delta(t_p)\Delta''(t_p)}{\Delta'(t_p)^2}.$$

By assumption, the decreasing hazard rate $\frac{g^i(t)}{1-G^i(t)}$ implies that $g^i(t)$ is decreasing in t. Moreover, the MLRP assumption implies that $r(t)$ is decreasing in t, that is

$$\frac{dr(t)}{dt} = \frac{\frac{dg^l(t)}{dt} g^h(t) - \frac{dg^h(t)}{dt} g^l(t)}{g^h(t)^2} < 0.$$

Recall that at the passing threshold t_p, it holds that $r(t_p) > 1$ which is equivalent to $g^l(t_p) > g^h(t_p)$. Hence, it also holds that $\frac{dg^l(t_p)}{dt_p} < \frac{dg^h(t_p)}{dt_p}$. That is, $\Delta''(t_p) < 0$, which in turn implies that $\frac{d\left(\frac{\Delta(t_p)}{\Delta'(t_p)}\right)}{dt_p} > 0$. The derivative $\frac{d\left(\frac{\frac{dx(s,\omega_l)}{ds}}{x(s,\omega_l)+B}\right)}{ds}$ is

$$\frac{d\left(\frac{\frac{dx(s,\omega_l)}{ds}}{x(s,\omega_l)+B}\right)}{ds} = \frac{-\left(\frac{dx(s,\omega_l)}{ds}\right)^2 + \big(x(s,\omega_l)+B\big)\frac{d^2x(s,\omega_l)}{ds^2}}{\big(x(s,\omega_l)+B\big)^2}.$$

Since $x(s,\omega_l)$ is increasing and concave in s, this derivative is negative. As a result, $\frac{\partial LHS}{\partial s} > 0$.

Now consider the right-hand side of equation (19),

$$RHS = \frac{\partial t_p(s,m)}{\partial s}.$$

And the derivative of the right-hand side is

$$\frac{\partial RHS}{\partial s} = \frac{\partial^2 t_p(s,m)}{\partial s^2}.$$

Recall the derivative $\frac{\partial t_p(s,m)}{\partial s}$ in equation (14). By the chain rule, the second derivative $\frac{\partial^2 t_p}{\partial s^2}$ is,

$$\begin{aligned}
\frac{\partial^2 t_p}{\partial s^2} =& \frac{\partial \frac{\partial t_p}{\partial s}}{\partial s} + \frac{\partial \frac{\partial t_p}{\partial s}}{\partial t_p}\frac{\partial t_p}{\partial s} \\
=& q_h \frac{\mathbb{E}_\omega\left[x(s,\omega)\right] r'(t_p)^2 \frac{dx(s,\omega_l)}{ds}\left(x(s,\omega_l)\frac{dx(s,\omega_h)}{ds} - x(s,\omega_h)\frac{dx(s,\omega_l)}{ds}\right)}{(1-m)^2 q_l^2 x(s,\omega_l)^4 \big(1-r(t_p)\big) r'(t_p)^3} \\
&+ q_h \frac{\mathbb{E}_\omega\left[x(s,\omega)\right] r'(t_p)^2 x(s,\omega_h)\left(x(s,\omega_l)\frac{d^2x(s,\omega_l)}{ds^2} - \left(\frac{dx(s,\omega_l)}{ds}\right)^2\right)}{(1-m)^2 q_l^2 x(s,\omega_l)^4 \big(1-r(t_p)\big) r'(t_p)^3} \\
&+ q_h \frac{\mathbb{E}_\omega\left[x(s,\omega)\right] r'(t_p)^2 x(s,\omega_l)\left(-x(s,\omega_l)\frac{d^2x(s,\omega_h)}{ds^2} + \left(\frac{dx(s,\omega_l)}{ds}\right)\left(\frac{dx(s,\omega_h)}{ds}\right)\right)}{(1-m)^2 q_l^2 x(s,\omega_l)^4 \big(1-r(t_p)\big) r'(t_p)^3} \\
&+ q_h \frac{q_h\big(r(t_p)-1\big) r''(t_p)\left(x(s,\omega_l)\frac{dx(s,\omega_h)}{ds} - x(s,\omega_h)\frac{dx(s,\omega_l)}{ds}\right)^2}{(1-m)^2 q_l^2 x(s,\omega_l)^4 \big(1-r(t_p)\big) r'(t_p)^3}.
\end{aligned} \tag{20}$$

This derivative is negative. Hence, $\frac{\partial RHS}{\partial s} < 0$.

I have shown that when $F = 0$, the left-hand side of equation (19) is increasing in s whereas the right-hand side of equation (19) is decreasing in s, which implies that $F = 0$ has a unique solution s_D. And $F > 0$ for $s < s_D$ and $F < 0$ for $s > s_D$. Recall that $\frac{\partial m_D(s)}{\partial s}$ is proportionate to F, hence, $m_D(s)$ is increasing in s for $s < s_D$ and is decreasing in s for $s > s_D$. Since $\frac{\partial MB_b\left(s,t_p(s,m_D)\right)}{\partial s}$ is proportionate to F, s_D also solves $\frac{\partial MB_b\left(s,t_p(s,m_D)\right)}{\partial s} = 0$. □

Proof. Proposition 2

I prove this proposition by contradiction.

Suppose that $\mathbb{E}_s\left[m_D(s)|s \in [\underline{s}, s_D]\right] > m_N$ for $N = [\underline{s}, s_D]$. Denote $\mathbb{E}_s\left[m_D(s)|s \in [\underline{s}, s_D]\right]$ by $\overline{m_D}$,

$$\begin{aligned}
&\overline{m_D} - m_N \propto kc'(\overline{m_D}) - kc'(m_N) \\
&\leq \mathbb{E}_s\left[\, kc'\big(m_D(s)\big)\,\middle|\, s \in [\underline{s}, s_D]\right] - kc'(m_N) \\
&= \frac{\int_{\underline{s}}^{s_D} kc'\big(m_D(s)\big) dF(s)}{\int_{\underline{s}}^{s_D} dF(s)} - kc'(m_N) \\
&\propto \int_{\underline{s}}^{s_D} \Big(kc'\big(m_D(s)\big) - kc'(m_N)\Big) dF(s).
\end{aligned}$$

The inequality is due to the assumption that $kc'(m)$ is weakly convex in m.

The first-order condition for $m_D(s)$ is

$$MB_b\Big(s, t_p\big(s, m_D(s)\big)\Big) = kc'\big(m_D(s)\big). \tag{21}$$

And the first-order condition for m_N when $N = [\underline{s}, s_D]$ is

$$\mathbb{E}_s\left[MB_b\big(s, t_p(s, m_N)\big)|s \in [\underline{s}, s_D]\right] = kc'(m_N).$$

I can simplify the difference between $\overline{m_D}$ and m_N further.

$$\begin{aligned}
\overline{m_D} - m_N \leq& \int_{\underline{s}}^{s_D} \Big(MB_b\Big(s, t_p\big(s, m_D(s)\big)\Big) - \mathbb{E}\left[MB_b\big(s, t_p(s, m_N)\big)|s \in [\underline{s}, s_D]\right]\Big)\, dF(s)\\
\leq& \int_{\underline{s}}^{s_D} \Big(MB_b\Big(s, t_p\big(s, m_D(s)\big)\Big) - \mathbb{E}_s\left[MB_b\big(s, t_p(s, \overline{m_D})\big)|s \in [\underline{s}, s_D]\right]\Big)\, dF(s)\\
=& \int_{\underline{s}}^{s_D} \Big(MB_b\Big(s, t_p\big(s, m_D(s)\big)\Big)\Big)\, dF(s) - \mathbb{E}\left[MB_b\big(s, t_p(s, \overline{m_D})\big)|s \in [\underline{s}, s_D]\right] \int_{\underline{s}}^{s_D} dF(s)\\
=& \int_{\underline{s}}^{s_D} \Big(MB_b\Big(s, t_p\big(s, m_D(s)\big)\Big) - MB_b\big(s, t_p(s, \overline{m_D})\big)\Big)\, dF(s)\\
=& \int_{\underline{s}}^{s_D} \Big(q_l\big(x(s, \omega_l) + B\big)\Big(\Delta\big(t_p(s, m_D(s))\big) - \Delta\big(t_p(s, \overline{m_D})\big)\Big)\Big)\, dF(s)\\
\leq& 0.
\end{aligned}$$

The first line is obtained by using the first-order condition of m_N and $m_D(s)$. The second line is due to the fact that $MB_b\big(s, t_p(s, m)\big)$ is decreasing in m. This is verified by the following derivative

$$\frac{\partial MB_b\big(s, t_p(s, m)\big)}{\partial m} = q_l\big(x(s, \omega_l) + B\big)\frac{\partial \Delta\big(t_p(s, m)\big)}{\partial m} \leq 0. \tag{22}$$

The derivative $\frac{\partial \Delta\big(t_p(s,m)\big)}{\partial m}$ is non-positive as shown in equation (18). Then the assumption that $m_N < \overline{m_D}$ implies the second line. The third and fourth line follow from the definition of conditional expectation. The last inequality is obtained by applying FKG inequality, which I now explain in details. The manipulation level $m_D(s)$ is increasing in s when $s < s_D$. And equation (18) shows that $\frac{\partial \Delta\big(t_p(s,m)\big)}{\partial m} \leq 0$. This means that the term $\Delta\big(t_p(s, m_D(s))\big)$ is decreasing in s through $m_D(s)$. The term $q_l\big(x(s, \omega_l) + B\big)$ is increasing in s. By FKG inequality, the following holds

$$\mathbb{E}_{s \leq s_D}\left[q_l\big(x(s, \omega_l) + B\big)\Delta\Big(t_p\big(s, m_D(s)\big)\Big)\right] \leq \mathbb{E}_{s \leq s_D}\left[q_l\big(x(s, \omega_l) + B\big)\Delta\Big(t_p\big(s, \mathbb{E}_{s \leq s_D}[m_D(s)]\big)\Big)\right].$$

Where $\mathbb{E}_{s \leq s_D}$ denotes expectation over s conditional on $s \leq s_D$. This implies that the last inequality holds and it contradicts to $m_N < \overline{m_D}$.

Next I prove by contradiction that $\overline{m_D} \geq m_N$ for $N = [s_D, \bar{s}]$. Suppose that the opposite holds, that is, $\overline{m_D} < m_N$ for $N = [s_D, \bar{s}]$. Then the following holds,

$$\begin{aligned}
&\overline{m_D} - m_N \propto \log\big(kc'(\overline{m_D})\big) - \log\big(kc'(m_N)\big) \\
&\geq \mathbb{E}_s\Big[\log\Big(kc'\big(m_D(s)\big)\Big)\Big| s \in [s_D, \bar{s}]\Big] - \log\big(kc'(m_N)\big) \\
&= \frac{\int_{s_D}^{\bar{s}} \log\Big(kc'\big(m_D(s)\big)\Big) dF(s)}{\int_{\underline{s}}^{s_D} dF(s)} - \log\big(kc'(m_N)\big) \\
&\propto \int_{s_D}^{\bar{s}} \Bigg(\log\Big(kc'\big(m_D(s)\big)\Big) - \log\big(kc'(m_N)\big)\Bigg) dF(s) \\
&= \int_{s_D}^{\bar{s}} \frac{kc'\big(m_D(s)\big) - kc'(m_N)}{\frac{1}{kc'(m_s)}} dF(s) \\
&\geq \int_{s_D}^{\bar{s}} \frac{kc'\big(m_D(s)\big) - kc'(m_N)}{\frac{1}{kc'(m_N)}} dF(s) \\
&\propto \int_{s_D}^{\bar{s}} \Big(kc'\big(m_D(s)\big) - kc'(m_N)\Big) dF(s)
\end{aligned}$$

The first inequality holds because $c'(m)$ is weakly log-concave. By the definition of conditional expectation, I obtain the first equality. I derive the second equality by using mean value theorem, where $kc'(m_s) \in \Big(kc'\big(m_D(s)\big), kc'(m_N)\Big)$ or $kc'(m_s) \in \Big(kc'(m_N), kc'\big(m_D(s)\big)\Big)$ depending on the relation between $kc'(m_N)$ and $kc'\big(m_D(s)\big)$. I now explain the second inequality.

- If $kc'\big(m_D(s)\big) < kc'(m_N)$, then $kc'(m_s) \in \Big(kc'\big(m_D(s)\big), kc'(m_N)\Big)$. Hence, the following holds

$$\frac{1}{\frac{1}{kc'(m_N)}} \geq \frac{1}{\frac{1}{kc'(m_s)}} \geq \frac{1}{\frac{1}{kc'(m_D(s))}}.$$

  Which implies that

$$\frac{kc'\big(m_D(s)\big) - kc'(m_N)}{\frac{1}{kc'(m_N)}} \leq \frac{kc'\big(m_D(s)\big) - kc'(m_N)}{\frac{1}{kc'(m_s)}} \leq \frac{kc'\big(m_D(s)\big) - kc'(m_N)}{\frac{1}{kc'(m_D(s))}}.$$

- If $kc'\big(m_D(s)\big) > kc'(m_N)$, then $kc'(m_s) \in \Big(kc'(m_N), kc'\big(m_D(s)\big)\Big)$. Hence, the following holds

$$\frac{1}{\frac{1}{kc'(m_N)}} \leq \frac{1}{\frac{1}{kc'(m_s)}} \leq \frac{1}{\frac{1}{kc'(m_D(s))}}.$$

  Which implies that

$$\frac{kc'\big(m_D(s)\big) - kc'(m_N)}{\frac{1}{kc'(m_N)}} \leq \frac{kc'\big(m_D(s)\big) - kc'(m_N)}{\frac{1}{kc'(m_s)}} \leq \frac{kc'\big(m_D(s)\big) - kc'(m_N)}{\frac{1}{kc'(m_D(s))}}.$$

Hence, regardless of the difference between $kc'\big(m_D(s)\big)$ and $kc'(m_N)$, the second inequality holds.

The first order condition for $m_D(s)$ is the same as in equation (21). And the first-order condition for m_N when $N = [s_D, \bar{s}]$ is

$$\mathbb{E}_s\left[MB_b\big(s, t_p(s, m_N)\big)|s \in [s_D, \bar{s}]\right] = kc'(m_N).$$

I further simplify the difference between $\overline{m_D}$ and m_N,

$$\begin{aligned}
\overline{m_D} - m_N \geq & \int_{s_D}^{\bar{s}} \left(MB_b\Big(s, t_p\big(s, m_D(s)\big)\Big) - \mathbb{E}\left[MB_b\big(s, t_p(s, m_N)\big)|s \in [s_D, \bar{s}]\right]\right) dF(s) \\
\geq & \int_{\underline{s}}^{s_D} \left(MB_b\Big(s, t_p\big(s, m_D(s)\big)\Big) - \mathbb{E}\left[MB_b\big(s, t_p(s, \overline{m_D})\big)|s \in [s_D, \bar{s}]\right]\right) dF(s) \\
= & \int_{s_D}^{\bar{s}} \left(MB_b\Big(s, t_p\big(s, m_D(s)\big)\Big)\right) dF(s) - \mathbb{E}\left[MB_b\big(s, t_p(s, \overline{m_D})\big)|s \in [s_D, \bar{s}]\right] \int_{\underline{s}}^{s_D} dF(s) \\
= & \int_{s_D}^{\bar{s}} \left(MB_b\Big(s, t_p\big(s, m_D(s)\big)\Big) - MB_b\big(s, t_p(s, \overline{m_D})\big)\right) dF(s) \\
= & \int_{s_D}^{\bar{s}} \Big(q_l\big(x(s, \omega_l) + B\big)\Big(\Delta\big(t_p(s, m_D(s))\big) - \Delta\big(t_p(s, \overline{m_D})\big)\Big)\Big) dF(s) \\
\geq & 0.
\end{aligned}$$

The second inequality uses the assumption that $m_N > \overline{m_D}$. The last inequality is derived by using FKG inequality. The manipulation level $m_D(s)$ is decreasing in s when $s > s_D$. Hence $\Delta\big(t_p(s, m_D(s))\big)$ is increasing in s through $m_D(s)$. Given that the term $q_l\big(x(s, \omega_l) + B\big)$ is increasing in s, FKG inequality implies the last inequality. $\square$

Proof. Lemma 4

I first show that $ML_r\big(s, t_p(s, m)\big)$ is increasing in s for any given m.

$$\frac{dML_r\big(s, t_p(s, m)\big)}{ds} = q_l \frac{dx(s, \omega_l)}{ds} \Delta\big(t_p(s, m)\big) + q_l x(s, \omega_l) \frac{d\Delta\big(t_p(s, m)\big)}{ds}.$$

Lemma 3 shows that for any given m, $\Delta\big(t_p(s, m)\big)$ is decreasing in s. Hence, $\frac{d\Delta\left(t_p(s,m)\right)}{ds} < 0$. Since the low fundamental asset has negative value, i.e., $x(s, \omega_l) < 0$, the derivative $\frac{dML_r\left(s, t_p(s,m)\right)}{ds} > 0$. Hence, the derivative $\frac{dML_r\left(s, t_p(s, m_{N_n})\right)}{ds} > 0$ for any no-disclosure set N_n.

Next, consider $ML_r\big(s, t_p(s, m_D(s))\big)$.

$$\begin{aligned}
&\frac{dML_r\big(s, t_p(s, m_D(s))\big)}{ds} = q_l \frac{dx(s, \omega_l)}{ds} \Delta\big(t_p(s, m_D(s))\big) + q_l x(s, \omega_l) \frac{d\Delta\big(t_p(s, m_D(s))\big)}{ds} \\
&= q_l \frac{dx(s, \omega_l)}{ds} \Delta\big(t_p(s, m_D(s))\big) + q_l x(s, \omega_l) \frac{d\Delta\big(t_p(s, m_D(s))\big)}{dt_p} \left(\frac{\partial t_p(s, m_D(s))}{\partial s} + \frac{\partial t_p(s, m_D(s))}{\partial m} \frac{\partial m_D(s)}{\partial s}\right).
\end{aligned}$$

Lemma 2 shows that $\frac{\partial t_p}{\partial m}$ and $\frac{\partial t_p}{\partial s}$ are non-positive. Therefore, when $\frac{\partial m_D(s)}{\partial s}$ is non-negative, the derivative $\frac{dML_r\left(s,t_p(s,m_D(s))\right)}{ds}$ is positive. In the following, I show that the derivative $\frac{dML_r\left(s,t_p(s,m_D(s))\right)}{ds}$ is non-negative even when $\frac{\partial m_D(s)}{\partial s}$ is negative. When $m_D(s)$ is decreasing in s, the marginal benefit $MB_b\big(s,t_p(s,m_D)\big)$ is decreasing in s for given m_D. Taking into account of the changes in $m_D(s)$, the following shows that the total derivative of $\frac{dMB_b\left(s,t_p(s,m_D(s))\right)}{ds}$ is proportionate to $\frac{\partial MB_b\left(s,t_p(s,m_D)\right)}{\partial s}$,

$$
\begin{aligned}
&\frac{dMB\Big(s,t_p\big(s,m_D(s)\big)\Big)}{ds} = \frac{\partial MB}{\partial s} + \frac{\partial MB}{\partial t_p}\left(\frac{\partial t_p(s,m_D)}{\partial s} + \frac{\partial t_p(s,m_D)}{\partial m_D}\frac{\partial m_D(s)}{\partial s}\right) \\
&= \left(\frac{\partial MB}{\partial s} + \frac{\partial MB}{\partial t_p}\frac{\partial t_p(s,m_D)}{\partial s}\right) + \frac{\partial MB}{\partial t_p}\frac{\partial t_p(s,m_D)}{\partial m_D}\frac{\partial m_D(s)}{\partial s} \\
&= \left(\frac{\partial MB}{\partial s} + \frac{\partial MB}{\partial t_p}\frac{\partial t_p(s,m_D)}{\partial s}\right) + \frac{\partial MB}{\partial t_p}\frac{\partial t_p(s,m_D)}{\partial m_D}\left(\frac{\frac{\partial MB}{\partial s} + \frac{\partial MB}{\partial t_p}\frac{\partial t_p(s,m_D)}{\partial s}}{kc''(m_D(s)) - \frac{\partial MB}{\partial t_p}\frac{\partial t_p(s,m_D)}{\partial m_D}}\right) \\
&= \left(\frac{\partial MB}{\partial s} + \frac{\partial MB}{\partial t_p}\frac{\partial t_p(s,m_D)}{\partial s}\right)\left(1 + \frac{\frac{\partial MB}{\partial t_p}\frac{\partial t_p(s,m_D)}{\partial m_D}}{kc''(m_D(s)) - \frac{\partial MB}{\partial t_p}\frac{\partial t_p(s,m_D)}{\partial m_D}}\right) \\
&= \left(\frac{\partial MB}{\partial s} + \frac{\partial MB}{\partial t_p}\frac{\partial t_p(s,m_D)}{\partial s}\right)\frac{kc''(m_D(s))}{kc''(m_D(s)) - \frac{\partial MB}{\partial t_p}\frac{\partial t_p(s,m_D)}{\partial m_D}} \\
&\propto \frac{\partial MB}{\partial s} + \frac{\partial MB}{\partial t_p}\frac{\partial t_p(s,m_D)}{\partial s}.
\end{aligned}
$$

Since $\frac{kc''(m_D(s))}{kc''(m_D(s)) - \frac{\partial MB}{\partial t_p}\frac{\partial t_p(s,m_D)}{\partial m_D}}$ is positive, the total derivative of $MB\Big(s,t_p\big(s,m_D(s)\big)\Big)$ with respect to s is proportionate to the partial derivative of $MB\Big(s,t_p\big(s,m_D(s)\big)\Big)$ with respect to s taking $m_D(s)$ as given.

When $\frac{\partial m_D(s)}{\partial s}$ is negative, the following holds

$$\frac{\partial m_D(s)}{\partial s} \propto \frac{\partial MB\Big(s,t_p\big(s,m_D\big)\Big)}{\partial s} \propto \frac{dMB\Big(s,t_p\big(s,m_D(s)\big)\Big)}{ds} < 0.$$

The total derivative $\frac{dMB\Big(s,t_p\big(s,m_D(s)\big)\Big)}{ds}$ equals to

$$\frac{dMB\Big(s,t_p\big(s,m_D(s)\big)\Big)}{ds} = q_l\frac{dx(s,\omega_l)}{ds}\Delta\big(t_p(s,m_D(s))\big) + q_l\big(x(s,\omega_l)+B\big)\frac{d\Delta\big(t_p(s,m_D(s))\big)}{ds}.$$

Hence, $\frac{\partial m_D(s)}{\partial s} < 0$ implies

$$\frac{d\Delta\big(t_p(s,m_D(s))\big)}{ds} < -\frac{\Delta\big(t_p(s,m_D(s))\big)}{x(s,\omega_l)+B}\frac{dx(s,\omega_l)}{ds}. \tag{23}$$

Then the total derivative $\frac{dML_r\left(s,t_p(s,m_D(s))\right)}{ds}$ is

$$\begin{aligned}
\frac{dML_r\left(s,t_p(s,m_D(s))\right)}{ds} &= q_l\frac{dx(s,\omega_l)}{ds}\Delta\left(t_p(s,m_D(s))\right)+q_l x(s,\omega_l)\frac{d\Delta\left(t_p(s,m_D(s))\right)}{ds}\\
&\propto \frac{dx(s,\omega_l)}{ds}\Delta\left(t_p(s,m_D(s))\right)+x(s,\omega_l)\frac{d\Delta\left(t_p(s,m_D(s))\right)}{ds}\\
&\geq \frac{dx(s,\omega_l)}{ds}\Delta\left(t_p(s,m_D(s))\right)+x(s,\omega_l)\left(-\frac{\Delta\left(t_p(s,m_D(s))\right)}{x(s,\omega_l)+B}\frac{dx(s,\omega_l)}{ds}\right)\\
&= \frac{B}{x(s,\omega_l)+B}\Delta\left(t_p(s,m_D(s))\right)\frac{dx(s,\omega_l)}{ds}\geq 0.
\end{aligned}$$

The first inequality uses the results in equation (23) and the assumption that $x(s,\omega_l)\leq 0$. Hence, the derivative $\frac{dML_r\left(s,t_p(s,m_D(s))\right)}{ds}\geq 0$ always hold. □

Proof. Proposition 3

I complete the proof in three steps. I first show that a cutoff disclosure dominates all other form of disclosures. Next, I solve for the optimal cutoff point s^* and show that $s^* < s_D$. Lastly, I show that $m_D(s^*) = m_N$ where $N = [s^*, \bar{s}]$ holds.

Suppose that $D = [\underline{s}, s_D)$ and $N = [s_D, \bar{s}]$. The regulator's ex-ante expected utility with this disclosure policy is denoted as U

$$U = \int_{\underline{s}}^{s_D} u\left(s, m_D(s)\right)dF(s) + \int_{s_D}^{\bar{s}} u(s, m_N)dF(s).$$

In the following, I show that adding more cutoff points to partition the signal space does not improve the regulator's ex-ante expected utility. First, I show that adding cutoff point in D does not improve the regulator's ex-ante utility. Without loss of generality, consider a disclosure policy which partition the signal space into $N_2 = [\underline{s}, s_1]$, $D = (s_1, s_D)$ and $N_1 \equiv N = [s_D, \bar{s}]$. The regulator's ex-ante expected payoff with such disclosure policy is

$$U' = \int_{\underline{s}}^{s_1} u\left(s, m_{N_2}\right)dF(s) + \int_{s_1}^{s_D} u\left(s, m_D(s)\right)dF(s) + \int_{s_D}^{\bar{s}} u(s, m_N)dF(s).$$

The difference in the regulator's expected utility is

$$
\begin{aligned}
U - U' &= \int_{\underline{s}}^{s_D} u\big(s, m_D(s)\big) dF(s) - \int_{\underline{s}}^{s_1} u\big(s, m_{N_2}\big) dF(s) - \int_{s_1}^{s_D} u\big(s, m_D(s)\big) dF(s) \\
&= \int_{\underline{s}}^{s_1} u\big(s, m_D(s)\big) dF(s) - \int_{\underline{s}}^{s_1} u\big(s, m_{N_2}\big) dF(s) \\
&= \int_{\underline{s}}^{s_1} \big(m_D(s) - m_{N_2}\big) \left.\frac{du(s,m)}{dm}\right|_{m=m(s)} dF(s) \\
&= \int_{\underline{s}}^{s_1} \big(m_D(s) - m_{N_2}\big) \left(\left.\frac{\partial u(s,m)}{\partial m}\right|_{m=m(s)} + \left.\frac{\partial u(s,m)}{\partial t_p(s,m)} \frac{\partial t_p(s,m)}{\partial m}\right|_{m=m(s)} \right) dF(s) \\
&= \int_{\underline{s}}^{s_1} \big(m_D(s) - m_{N_2}\big) \left.\frac{\partial u(s,m)}{\partial m}\right|_{m=m(s)} dF(s) \\
&= \int_{\underline{s}}^{s_1} \big(m_D(s) - m_{N_2}\big) ML_r\big(s, t_p(s, m(s))\big) dF(s) \\
&\geq \int_{\underline{s}}^{s_1} \big(m_D(s) - m_{N_2}\big) ML_r\big(s, t_p(s, m_{N_2})\big) dF(s) \\
&\propto \mathbb{E}_{s \leq s_1} \Big[m_D(s) ML_r\big(s, t_p(s, m_{N_2})\big) \Big] - m_{N_2} \, \mathbb{E}_{s \leq s_1} \Big[ML_r\big(s, t_p(s, m_{N_2})\big) \Big] \\
&\geq \mathbb{E}_{s \leq s_1} \Big[m_D(s) ML_r\big(s, t_p(s, m_{N_2})\big) \Big] - \mathbb{E}_{s \leq s_1} \big[m_D(s) \big] \, \mathbb{E}_{s \leq s_1} \Big[ML_r\big(s, t_p(s, m_{N_2})\big) \Big] \\
&\geq 0.
\end{aligned}
$$

The first two lines are derived from simplifications of the differences in expected utility. Apply mean-value theorem to the second line gives the third line, where $m(s) \in (m_D(s), m_{N_2})$ if $m_D(s) < m_{N_2}$ or $m(s) \in (m_{N_2}, m_D(s))$ if $m_{N_2} < m_D(s)$. The fourth line shows the total derivative of $u(s,m)$ with respect to m, and it reduces to the fifth line because the passing threshold $t_p(s,m)$ maximizes the regulator's utility $u(s,m)$ for given signal s and given manipulation m, hence, $\frac{\partial u(s,m)}{\partial t_p(s,m)} = 0$. Equation (11) defines $ML_r\big(s, t_p(s,m)\big)$. I now explain the first inequality in details. The following derivative shows that $ML_r\big(s, t_p(s,m)\big)$ is increasing in m,

$$
\frac{\partial ML_r\big(s, t_p(s,m)\big)}{\partial m} = q_l x(s, \omega_l) \frac{\partial \Delta\big(t_p(s,m)\big)}{\partial m}.
$$

Equation (18) implies that this derivative is non-negative. The following proves the first inequality

- If $m_D(s) < m_{N_2}$, then $m(s) \in (m_D(s), m_{N_2})$. Hence, the following holds

$$
ML_r\big(s, t_p(s, m_D(s))\big) \leq ML_r\big(s, t_p(s, m(s))\big) \leq ML_r\big(s, t_p(s, m_{N_2})\big),
$$

which implies that

$$\big(m_D(s) - m_{N_2}\big)ML_r\big(s, t_p(s, m_D(s))\big) \geq \big(m_D(s) - m_{N_2}\big)ML_r\big(s, t_p(s, m(s))\big)$$
$$\geq \big(m_D(s) - m_{N_2}\big)ML_r\big(s, t_p(s, m_{N_2})\big).$$

- If $m_D(s) > m_{N_2}$, then $m(s) \in (m_{N_2}, m_D(s))$. Hence, the following holds

$$ML_r\big(s, t_p(s, m_D(s))\big) \geq ML_r\big(s, t_p(s, m(s))\big) \geq ML_r\big(s, t_p(s, m_{N_2})\big),$$

which implies that

$$\big(m_D(s) - m_{N_2}\big)ML_r\big(s, t_p(s, m_D(s))\big) \geq \big(m_D(s) - m_{N_2}\big)ML_r\big(s, t_p(s, m(s))\big)$$
$$\geq \big(m_D(s) - m_{N_2}\big)ML_r\big(s, t_p(s, m_{N_2})\big).$$

Hence, regardless of the difference between $m_D(s)$ and m_{N_2}, the first inequality holds. The second inequality generalize the result from Proposition 2. Since $m_D(s)$ is increasing in s for $s \leq s_1$, and Lemma 4 shows that $ML_r\big(s, t_p(s, m_D(s))\big)$ is increasing in s, hence, the last inequality is obtained by FKG inequality. This proof can be generalized to the cases where more than one cutoff point is added on D.

Apply the same approach, I show that adding cutoff point in N does not improve the regulator's ex-ante utility. The proof is similar and thus omitted.

Given that the disclosure policy with $D = [\underline{s}, s_D]$ and $N = [s_D, \bar{s}]$ dominates all other forms of disclosure, I next solve for the optimal cutoff point. Denote the regulator's ex-ante utility with the optimal disclosure policy by U^*,

$$U^* = \int_{\underline{s}}^{s^*} u\big(s, m_D(s)\big)dF(s) + \int_{s^*}^{\bar{s}} u(s, m_N)dF(s).$$

First, the optimal cutoff point $s^* \leq s_D$ must hold. Otherwise, by the previous proof, the regulator can gain by not disclosing the signals $s \in [s_D, s^*]$. But such disclosure policy features two no-disclosure sets which is dominated by the disclosure policy with signal no-disclosure set $[s_D, \bar{s}]$. Hence, $s^* \leq s_D$ must hold.

Take derivative of U^* with respect to the cutoff point s^*, the first-order condition determines the optimal cutoff point,

$$\Big(u\big(s^*, m_D(s^*)\big) - u(s^*, m_N)\Big)f(s^*) + \frac{\partial m_N}{\partial s^*}\int_{s^*}^{\bar{s}} ML_r\big(s, t_p(s, m_N)\big)dF(s) = 0. \tag{24}$$

The first-order condition for m_N when $N = [s^*, \bar{s}]$ is

$$\mathbb{E}_s \left[MB_b\big(s, t_p(s, m_N)\big) | s \in [s^*, \bar{s}] \right] = kc'(m_N).$$

By implicit function theorem, I derive the derivative $\frac{\partial m_N}{\partial s^*}$,

$$\frac{\partial m_N}{\partial s^*} = \frac{f(s^*)}{\int_{s^*}^{\bar{s}} dF(s)} \frac{\mathbb{E}_{s \geq s^*} \left[MB_b\big(s, t_p(s, m_N)\big) \right] - MB_b\big(s^*, t_p(s^*, m_N)\big)}{kc''(m_N) - \mathbb{E}_{s \geq s^*} \left[\frac{\partial MB_b\left(s, t_p(s, m_N)\right)}{\partial m_N} \right]}.$$

With this derivative, I further reduce the first-order condition in equation (24) to the following

$$\begin{aligned}
&\Big(u\big(s^*, m_D(s^*)\big) - u(s^*, m_N) \Big) \\
&+ \Big(\mathbb{E}_{s \geq s^*} \left[MB_b\big(s, t_p(s, m_N)\big) \right] - MB_b\big(s^*, t_p(s^*, m_N)\big) \Big) \frac{\mathbb{E}_{s \geq s^*} \left[ML_r\big(s, t_p(s, m_N)\big) \right]}{kc''(m_N) - \mathbb{E}_{s \geq s^*} \left[\frac{\partial MB_b\left(s, t_p(s, m_N)\right)}{\partial m_N} \right]} = 0.
\end{aligned} \tag{25}$$

□

B Cost of inefficiencies

In this appendix, I derive the regulator's optimal disclosure policy assuming that the bank's asset is not worth continuing ex-ante. That is, I replace Assumption 1 by the following

Assumption 2. $\mathbb{E}_\omega\left[x(s,\omega)\right] \in [q_l x(\underline{s},\omega_l), 0]$ *for* $s \in [\underline{s}, \bar{s}]$.

For given bank's report t with conjectured manipulation level $\hat{m}$ and the signal s, the regulator's pass/fail decision still follows equation (4). As in Lemma 1, the regulator's pass/fail decision is characterized by a cutoff rule on the bank's report t. That is, the regulator passes the bank if and only if the bank's report t is higher than the threshold $t_p(s,\hat{m})$.

Lemma 5. *For given level of manipulation m, the passing threshold $t_p(s,m)$ is decreasing in s. For given signal s, the passing threshold is increasing in m.*

Proof. The proof follows the proof of Lemma 2. The only difference is that Assumption 2 assumes that $q_h x(s,\omega_h) + q_l x(s,\omega_l) < 0$ which implies that $\frac{\partial t_p}{\partial m}$ in equation (15) is positive. □

Compare to Lemma 2, the effect of the signal s on the regulator's choice of the passing threshold remains the same. However, the effect of manipulation is the opposite. The reason is that with Assumption 2, the inefficient continuation is more costly than the inefficient liquidation. Although manipulation still increases the report similarity between low and high fundamental asset, the regulator is more concerned with the inefficient continuation. Hence, when the regulator is facing a report that is less informative, the regulator would increase the passing threshold in order to avoid inefficient continuation.

The regulator's choice of passing threshold determines the difference in passing probability between low and high fundamental asset. The following lemma shows that this difference in passing probability becomes larger as the signal increases.

Lemma 6. *For given manipulation level m, $\Delta\big(t_p(s,m)\big)$ is increasing in s.*

Proof. The proof is similar to the proof of Lemma 3. The only difference is that Assumption 2 implies that $q_h x(s,\omega_h) \le -q_l x(s,\omega_l)$ for all s. As a result, the ratio $r\big(t_p(s,m)\big)$ is less than 1, which then implies that $\Delta\big(t_p(s,m)\big)$ is increasing in s. □

Anticipate the regulator's pass/fail decision, the bank chooses the manipulation level. The bank's manipulation choice depends on whether regulator discloses the regulatory signal s.

Proposition 5. *When s is disclosed, the level of manipulation $m_D(s)$ is increasing in s for all s. When s is not disclosed, the level of manipulation m_{N_n} is a constant over the no-disclosure set N_n for $n \in [1, +\infty)$.*

Recall that the bank's manipulation incentive is determined by the increases in the passing probability after manipulation $\Delta\big(t_p(s,m)\big)$ and the expected gain after passing the test with manipulation $q_l\big(x(s,\omega_l)+B\big)$. As the signal s increases, both incentives become stronger, leading the bank to manipulate more.

Compare the manipulation level under different disclosure policy

Proposition 6. *$\mathbb{E}_s\left[m_D(s)|s\in N\right]\leq m_N$ for any $N\subseteq S$.*

Proof. The proof follows the proof of Proposition 2 when $m_D(s)$ is increasing in s. □

The disclosure of regulator's private information reduces the expected manipulation level. The reason is rooted in the interaction between the bank's manipulation choice and the regulator's passing threshold choice when s is disclosed. When manipulation increases, the regulator increases the passing threshold $t_p(s,m)$ according to Lemma 5. Such response of the passing threshold reduces the bank's passing probability, more importantly, it reduces the difference in passing probability between high fundamental asset and low fundamental asset, lowering the bank's manipulation incentive. Such interaction between the bank's manipulation choice and the regulator's passing threshold choice is muted, if the signal s is not disclosed. Hence, the expected manipulation level is lower when s is disclosed.

I now analyze the regulator's disclosure policy of signal s. Following Lemma 4, I derive how the regulator's loss caused by the bank's manipulation $ML_r\big(s,t_p(s,m^*)\big)$ changes with the signal s.

Lemma 7. *If the following condition holds,*

$$\frac{d}{ds}\left(\frac{\frac{d\Delta\left(s,t_p(s,m^*)\right)}{ds}}{\Delta\big(s,t_p(s,m^*)\big)}\right)\leq 0, \tag{26}$$

then $ML_r\big(s,t_p(s,m^*)\big)$ is decreasing in s for $s<s_r$ and increasing in s for $s>s_r$, where $m^*=\{m_D(s),m_{N_n}\}$ and s_r is the unique solution for $\frac{dML_r\left(s,t_p(s,m^*)\right)}{ds}=0$.

Proof. The derivative of $ML_r\big(s,t_p(s,m^*)\big)$ with respect to s is given by the following,

$$\frac{dML_r\big(s,t_p(s,m^*)\big)}{ds}=q_l\left(x(s,\omega_l)\frac{d\Delta\big(s,t_p(s,m^*)\big)}{ds}+\Delta\big(s,t_p(s,m^*)\big)\frac{dx(s,\omega_l)}{ds}\right).$$

When $s=\underline{s}$, Assumption 2 assumes that $x(\underline{s},\omega_h)=0$. According to equation (13), the passing threshold is $t_p(\underline{s},m^*)\big)=\bar{t}$ which implies that $\Delta\big(t_p(\underline{s},m^*)\big)=0$, hence, the derivative

$\frac{dML_r\big(s,t_p(s,m^*)\big)}{ds}$ is

$$\left.\frac{dML_r\big(s,t_p(s,m^*)\big)}{ds}\right|_{s=\underline{s}} = q_l x(\underline{s},\omega_l)\left.\frac{d\Delta\big(s,t_p(s,m^*)\big)}{ds}\right|_{s=\underline{s}} < 0.$$

When $s=\bar{s}$, Assumption 2 implies that $x(\bar{s},\omega_h)q_h + x(\bar{s},\omega_l)q_l = 0$. According to equation (13), the passing threshold satisfies $r\big(t_p(\bar{s},m^*\big) = 1$. Hence, the derivative $ML_r\big(\bar{s},t_p(\bar{s},m^*)\big)$ is

$$\left.\frac{dML_r\big(s,t_p(s,m^*)\big)}{ds}\right|_{s=\bar{s}} = q_l\Delta\big(\bar{s},t_p(\bar{s},m^*)\big)\left.\frac{dx(s,\omega_l)}{ds}\right|_{s=\bar{s}} > 0.$$

By the intermediate value theorem, $\frac{dML_r\big(s,t_p(s,m^*)\big)}{ds} = 0$ must hold at some value of $s \in (\underline{s},\bar{s})$.

Next, I show that $\frac{dML_r\big(s,t_p(s,m^*)\big)}{ds} = 0$ is unique at $s=s_r$. When $\frac{dML_r\big(s,t_p(s,m^*)\big)}{ds} = 0$, the following equation holds,

$$x(s,\omega_l)\frac{d\Delta\big(s,t_p(s,m^*)\big)}{ds} + \Delta\big(s,t_p(s,m^*)\big)\frac{dx(s,\omega_l)}{ds} = 0.$$

This is equivalent to

$$\frac{\frac{d\Delta\big(s,t_p(s,m^*)\big)}{ds}}{\Delta\big(s,t_p(s,m^*)\big)} = -\frac{\frac{dx(s,\omega_l)}{ds}}{x(s,\omega_l)}.$$

The left-hand side is positive for $s \in (\underline{s},\bar{s})$. In addition, condition in equation (26) ensures that it is weakly decreasing in s. The right-hand side is also positive for $s \in (\underline{s},\bar{s})$. And the function $x(s,\omega_l)$ is log-concave in s which implies that the right-hand side is weakly increasing in s. Hence, $\frac{dML_r\big(s,t_p(s,m^*)\big)}{ds} = 0$ is unique and the solution is denoted as s_r. □

This result shows that under certain condition, the regulator's marginal loss from manipulation has U-shape. That is, the marginal loss ML_r first decreases in s and then increases in s. The condition in equation (26) means that $\Delta\big(s,t_p(s,m)\big)$ is log-concave in s which ensures that $\Delta\big(s,t_p(s,m)\big)$ is not too convex in s.

Following the intuition of Proposition 3, the optimal disclosure policy should minimize the part of regulator's loss that cannot be controlled by the optimal pass/fail decision. For given level of manipulation, the disclosure policy should allocate less manipulation to cases when the regulator is more susceptible to it. In addition, the optimal disclosure policy should minimize the expected level of manipulation. To see this, I decompose the regulator's ex-ante utility difference between disclosure U_D and no disclosure U_N for a given set of signals S'

$$\begin{aligned}
U_D - U_N &= \int_{S'} \Big(u\big(s, m_D(s)\big) - u(s, m_N)\Big) dF(s) \\
&= \int_{S'} \big(m_D(s) - m_N\big) ML_r\big(s, t_p\big(s, m(s)\big)\big) dF(s) \\
&= \mathbb{E}_{s\in S'} \Big[m_D(s) ML_r\big(s, t_p\big(s, m(s)\big)\big)\Big] - m_N\, \mathbb{E}_{s\in S'} \Big[ML_r\big(s, t_p\big(s, m(s)\big)\big)\Big] \\
&= \mathbb{E}_{s\in S'} \Big[m_D(s) ML_r\big(s, t_p\big(s, m(s)\big)\big)\Big] - \mathbb{E}_{s\in S'} \big[m_D(s)\big]\, \mathbb{E}_{s\in S'} \Big[ML_r\big(s, t_p\big(s, m(s)\big)\big)\Big] \\
&\quad + \mathbb{E}_{s\in S'} \big[m_D(s)\big]\, \mathbb{E}_{s\in S'} \Big[ML_r\big(s, t_p\big(s, m(s)\big)\big)\Big] - m_N\, \mathbb{E}_{s\in S'} \Big[ML_r\big(s, t_p\big(s, m(s)\big)\big)\Big] \\
&= \underbrace{\mathbb{E}_{s\in S'} \Big[m_D(s) ML_r\big(s, t_p\big(s, m(s)\big)\big)\Big] - \mathbb{E}_{s\in S'} \big[m_D(s)\big]\, \mathbb{E}_{s\in S'} \Big[ML_r\big(s, t_p\big(s, m(s)\big)\big)\Big]}_{\text{Distribution effect}} \\
&\quad + \underbrace{\Big(\mathbb{E}_{s\in S'} \big[m_D(s)\big] - m_N\Big)\, \mathbb{E}_{s\in S'} \Big[ML_r\big(s, t_p\big(s, m(s)\big)\big)\Big]}_{\text{Expected level effect}}
\end{aligned}$$

Where m_N is the bank's manipulation response when $N = S'$ and $m(s)$ is the manipulation level that satisfies the mean value theorem. The first two terms capture whether disclosure is able to distribute more manipulation to cases where the regulator suffers less from it. And the last two terms captures the impact of disclosure on the expected level of manipulation. The following proposition characterizes the optimal disclosure policy. It shows that the optimal disclosure policy still follows a single cutoff rule.

Proposition 7. *Suppose that condition (26) holds. The optimal disclosure policy follows a cutoff rule where $D = (s^*, \bar{s}]$ and $N = [\underline{s}, s^*]$. That is, the regulator discloses the signal s when $s > s^*$ and does not disclose the signal s when $s < s^*$, where $s^* \in [s_r, \bar{s}]$.*

Proof. The proof of the optimality of a cutoff disclosure policy is omitted, because it is similar to the proof of Proposition 3. The only difference is that the signal cutoff disclosure policy is optimal because the regulator's marginal loss caused by the bank's manipulation $ML_r\big(s, t_p(s, m)\big)$ has U-shape across s. The regulator's ex-ante utility with the optimal disclosure policy is U^*,

$$U^* = \int_{\underline{s}}^{s^*} u(s, m_N) dF(s) + \int_{s^*}^{\bar{s}} u\big(s, m_D(s)\big) dF(s).$$

Where the optimal cutoff point s^* solves the following,

$$\Big(u(s^*, m_N) - u\big(s^*, m_D(s^*)\big)\Big) f(s^*) + \frac{\partial m_N}{\partial s^*} \int_{\underline{s}}^{s^*} ML_r\big(s, t_p(s, m_N)\big) dF(s) = 0. \tag{27}$$

The first-order condition for m_N when $N = [s^*, \bar{s}]$ is

$$\mathbb{E}_s\left[MB_b\big(s, t_p(s, m_N)\big)|s \in [\underline{s}, s^*]\right] = kc'(m_N).$$

By implicit function theorem, I derive the derivative $\frac{\partial m_N}{\partial s^*}$.

$$\frac{\partial m_N}{\partial s^*} = \frac{f(s^*)}{\int_{\underline{s}}^{s^*} dF(s)} \frac{MB_b\big(s^*, t_p(s^*, m_N)\big) - \mathbb{E}_{s \le s^*}\left[MB_b\big(s, t_p(s, m_N)\big)\right]}{kc''(m_N) - \mathbb{E}_{s \le s^*}\left[\frac{\partial MB_b\left(s, t_p(s, m_N)\right)}{\partial m_N}\right]}.$$

With this derivative, I further reduce the first-order condition to the following

$$\begin{aligned}&\Big(u(s^*, m_N) - u\big(s^*, m_D(s^*)\big)\Big)\\ &+ \Big(MB_b\big(s^*, t_p(s^*, m_N)\big) - \mathbb{E}_{s \le s^*}\left[MB_b\big(s, t_p(s, m_N)\big)\right]\Big) \frac{\mathbb{E}_{s \le s^*}\left[ML_r\big(s, t_p(s, m_N)\big)\right]}{kc''(m_N) - \mathbb{E}_{s \le s^*}\left[\frac{\partial MB_b\left(s, t_p(s, m_N)\right)}{\partial m_N}\right]} = 0.\end{aligned} \tag{28}$$

□

C Real activity

In this appendix, I discuss an extension in which the bank exerts costly effort to improve the payoff of the asset and such effort manifests itself in the report. Although, similar to the baseline model, the bank's effort choice improves the report in the sense of first order stochastic dominance, such improvement in report arises endogenously from the improvement in asset quality. As a result, the disclosure of the regulator's private information affects the real activities of the bank, i.e., effort choice.

Suppose that the bank can exert effort m to improve the quality of the asset. To keep the notation consistent, I still use m to denote the bank's effort. The effort m determines the probability that bank's fundamental value is ω_h. That is, if the bank exerts effort m, then the relative gain from continuing the asset is $x(s, \omega_h)$ with probability m and $x(s, \omega_l)$ with probability $1 - m$ when the regulator's signal is s.

The fundamental of the asset determines the report distribution in the same way as in the baseline model. That is, the report t is drawn from a distribution with density $g^i(t)$ when the fundamental is ω_i, where $i = \{h, l\}$. Hence, the effort manifests itself in the bank's report. With effort m, the bank's report generating process is $g^h(t)$ with probability m and $g^l(t)$ with probability $1 - m$. Notice that Assumption 1 cannot hold since the expected relative gain from continuing the asset for given s is now endogenously determined by the bank's effort. I assume that $x(s, \omega_l) \leq x(\bar{s}, \omega_l) \equiv 0$ and $x(s, \omega_h) \geq x(\underline{s}, \omega_h) \equiv 0$. All other elements of the model are the same as in Section 2. The focus of the following analysis is to show how the regulator should disclose the regulatory signal to the bank to affect the bank's effort choice. I solve the model backwards.

Consider the regulator's pass/fail decision after observing the private information s and the bank's report t. Same as in the baseline model, the regulator passes the bank if and only if the expected gain from passing the bank is greater than failing the bank. That is,

$$\mathbb{E}_\omega[x(s, \omega)|t, \hat{m}] \geq 0.$$

Where $\hat{m}$ is the regulator's conjecture about bank's effort. The conditional expectation is

$$\begin{aligned}\mathbb{E}_\omega[x(s, \omega)|t, \hat{m}] &= x(s, \omega_h) \Pr(\omega = \omega_h|t, \hat{m}) + x_{\hat{m}}(s, \omega_l) \Pr(\omega = \omega_l|t, \hat{m}) \\ &= x(s, \omega_h) \frac{\hat{m} g^h(t)}{\hat{m} g^h(t) + (1 - \hat{m}) g^l(t)} + x(s, \omega_l) \frac{(1 - \hat{m}) g^l(t)}{\hat{m} g^h(t) + (1 - \hat{m}) g^l(t)}.\end{aligned}$$

The pass/fail decision still features a threshold $t_p(s, \hat{m})$ on the bank's report. Specifically, the bank passes the test if and only if the report t satisfies $t \geq t_p(s, \hat{m})$. The passing threshold $t_p(s, \hat{m})$ solves $\mathbb{E}_\omega[x(s,\omega)|t_p, \hat{m}] = 0$, which indicates that the regulator is indifferent between passing and failing the bank when the bank's report is $t_p(s, \hat{m})$.

Lemma 8. *For given level of effort m, the passing threshold $t_p(s,m)$ is decreasing in s. For given signal s, the passing threshold $t_p(s,m)$ is decreasing in m.*

This lemma echoes to Lemma 2. The explanation for the first result is the same as in Lemma 2. However, the effect of effort is different from manipulation. Effort improves the relative gain from continuing the asset $x(s,\omega)$, which endogenously increases relative cost of inefficient liquidation. Consequently, the regulator decreases the passing threshold.

Anticipate the regulator's pass/fail decision, the bank chooses the effort level. Suppose that the bank observes the regulator's private signal s. The bank's payoff is

$$V(s,\hat{m},m) = m\big(x(s,\omega_h)+B\big)\int_{t\geq t_p(s,\hat{m})} g^h(t)dt + (1-m)\big(x(s,\omega_l)+B\big)\int_{t\geq t_p(s,\hat{m})} g^l(t)dt - kc(m).$$

The first-order condition with respect to m determines the bank's effort choice. In equilibrium, the regulator's conjecture about the effort is consistent with the bank's choice. Hence, the equilibrium manipulation $m_D(s)$ is determined by

$$\big(x(s,\omega_h)+B\big)\int_{t\geq t_p\left(s,m_D(s)\right)} g^h(t)dt - \big(x(s,\omega_l)+B\big)\int_{t\geq t_p\left(s,m_D(s)\right)} g^l(t)dt - kc'\big(m_D(s)\big) = 0. \quad (29)$$

The first two terms are the marginal benefit of effort. I modify the definition of MB_b in equation (8) to the following

$$\begin{aligned} MB_b\big(s, t_p(s,m)\big) &\equiv \big(x(s,\omega_h)+B\big)\int_{t\geq t_p(s,m)} g^h(t)dt - \big(x(s,\omega_l)+B\big)\int_{t\geq t_p(s,m)} g^l(t)dt \\ &= \big(x(s,\omega_h) - x(s,\omega_l)\big)\int_{t\geq t_p(s,m)} g^h(t)dt + \big(x(s,\omega_l)+B\big)\Delta\big(t_p(s,m)\big). \end{aligned} \quad (30)$$

Where $\Delta\big(t_p(s,m)\big)$ is defined in equation (5) and it captures the difference in passing probability between low and high fundamental asset. The first term of MB_b is the bank's gain from improving the fundamental from low to high, provided that the bank passes the test. This term captures *the effect of effort on the relative gain from holding the asset.* The second term is identical to equation (8) and it captures the bank's expected gain from having low fundamental asset pass the test. This term captures *the effect of effort on the bank's report.* This effect is identical to the effect of manipulation in the baseline model.

Lemma 9. *For given signal s, if*

$$\big(x(s,\omega_h)-x(s,\omega_l)\big)g_h(\underline{t}) < \big(x(s,\omega_l)+B\big)\big(g_l(\underline{t})-g_h(\underline{t})\big),$$

then $MB_b\big(s,t_p(s,m)\big)$ is increasing in m for $m<m_r(s)$ and it is decreasing in m for $m>m_r(s)$, where $m_r(s)$ solves $\frac{\partial MB_b\left(s,t_p(s,m)\right)}{\partial m}=0$. Otherwise, $MB_b\big(s,t_p(s,m)\big)$ is increasing in m.

This result shows that the bank's marginal benefit of exerting effort is nonmonotonic in the level of effort m. The intuition is as follows. The first component of $MB_b\big(s,t_p(s,m)\big)$ is $\big(x(s,\omega_h)-x(s,\omega_l)\big)\int_{t\geq t_p(s,m)} g^h(t)dt$, and it represents the bank's gain from improving the asset fundamentals. As the effort m increases, the bank is more likely to have high fundamental asset. In response, the regulator is more likely to pass the bank (by lowering the passing threshold). Consequently, the first term $\big(x(s,\omega_h)-x(s,\omega_l)\big)\int_{t\geq t_p(s,m)} g^h(t)dt$ is increasing in the amount of effort, incentivizing the bank to exert more effort. However, the second component $\big(x(s,\omega_l)+B\big)\Delta\big(t_p(s,m)\big)$ may be decreasing in the effort m, depending on how $\Delta\big(t_p(s,m)\big)$ changes with m. Lemma 8 shows that the passing threshold $t_p(s,m)$ is decreasing in effort m, meaning that the bank is more likely to pass the test when exerting more effort. When the level of effort is very low (high), the regulator will set the passing threshold very high (low) which makes the bank less (more) likely to pass the test regardless of the fundamental of the asset. When the level of effort is intermediate, the regulator will also set the passing threshold at an intermediate level which makes the passing probability depend crucially on the fundamental of the asset. Hence, the *difference* in passing probability between low and high fundamental asset $\Delta\big(t_p(s,m)\big)$ is first increasing and then decreasing in the effort level m, which then makes the second component of $MB_b\big(s,t_p(s,m)\big)$ follow the same pattern.

The condition in Lemma 9 guarantees that the effect of first component does not always dominate the effect of the second component. This condition depends on the bank's private benefit of passing the test. When the private benefit B is relatively small, the first effect ("learning the regulator's signal to improve the asset quality") always dominates the second effect in which the bank exerts effort only when it leads to higher chance of passing the test ("learning the regulator's signal to improve the report without improving the asset quality") .

One implications of Lemma 9 is that the first-order condition in equation (29) may be nonmonotonic in m, hence, the interior solution of $m_D(s)$ may not exist and the solution of $m_D(s)$ may not be unique. One sufficient condition for the interior solution of $m_D(s)$ to exist is

$$x(s,\omega_h)-x(s,\omega_l) < kc'(1),\ \forall s. \tag{31}$$

This condition means that the effort is costly such that the bank does not have incentive to improve the fundamental to $x(s,\omega_h)$. To avoid having multiple equilibria for the effort $m_D(s)$, I assume that the bank chooses the highest effort level when the bank is indifferent. The solution $m_D(s)$ must satisfy the second order condition $\frac{\partial FOC}{\partial m} < 0$.

Proposition 8. *When s is disclosed, the level of manipulation $m_D(s)$ is increasing in s for $s < s_D$ and it is decreasing in s for $s > s_D$, where s_D is the unique solution for $\frac{\partial MB_b}{\partial s} = 0$.*

Now consider the effort choice when the bank does not observe the regulator's signal s. The equilibrium effort m_N solves,

$$\mathbb{E}_s \left[MB_b\big(s, t_p(s, m_{N_n})\big) \big| \, s \in N_n \right] - kc'(m_{N_n}) = 0.$$

The effort m_N is unique and it is a constant over the regulator's signal s.

The following proposition compares the effort level when the bank observes the signal with the effort level when the bank does not observes the signal.

Proposition 9. *If the following holds for all s*

$$\left. \frac{\partial MB_b\big(s, t_p(s,m)\big)}{\partial m} \right|_{m=m_D(s)} >= 0, \tag{32}$$

then $\mathbb{E}_s \left[m_D(s) | s \in N \right] \geq m_N$ if $N \subseteq [\underline{s}, s_D]$ and $\mathbb{E}_s \left[m_D(s) | s \in N \right] \leq m_N$ if $N \subseteq [s_D, \bar{s}]$.

The intuition is as follows. When $s \in [\underline{s}, s_D]$, the effort $m_D(s)$ is increasing in s if s is disclosed. In response, the regulator decreases the passing threshold t_p, which makes the test to be more lenient regardless of the asset fundamental. Such endogenous response of the passing threshold has two opposite effects on the bank's incentive to exert effort. One the one hand, an easier test allows the bank to pass the test even without exerting effort, which then decreases the bank's incentive to exert effort. On the other hand, an easier test increases the possibility that the bank's effort is realized, i.e., the bank passes the test after increase the asset quality. This effect increases the bank's incentive to exert effort. (Notice that this second effect is missing in the baseline model.) Depending on the magnitude of the two forces, the bank may increase or decrease effort. When condition (32) holds, the second effect dominates. As a result, the interactions between the regulator's pass/fail decision and the bank's effort choice increases the expected level of effort, comparing to the case when such interactions are absent, i.e. when s is not disclosed. When $s \in [s_D, \bar{s}]$, the manipulation $m_D(s)$ is decreasing in s if s is disclosed. In response, the regulator increases the passing threshold t_p to make the test more difficult. Such endogenous response of the pass/fail decision then increases the magnitude of passing probability that can be increased by exerting effort, incentivizing the bank to exert more effort. However, as the test gets more difficult, it also increases the likelihood that the bank may not be paid off by exerting effort. That is, the bank may still fail the test even after exerting effort. (Again, this second effect is missing in the baseline model.) The second effect dominates the bank's effort choice if condition (32) holds. As a result, the interactions between the regulator's pass/fail decision and the bank's effort choice decreases the expected level of effort when s is disclosed compare to the case when s is not disclosed.

Consider the regulator's disclosure policy. For given signal s and equilibrium effort m^*, the regulator's payoff is

$$\begin{aligned} u(s, m^*) &= \int_{t \geq t_p(s,m^*)} \mathbb{E}_\omega[x(s,\omega)|t, m^*] g_{m^*}(t) dt \\ &= m^* x(s, \omega_h) \int_{t \geq t_p(s,m^*)} g^h(t) dt + (1-m^*) x(s, \omega_l) \int_{t \geq t_p(s,m^*)} g^l(t) dt. \end{aligned}$$

Where $g_{m^*}(t)$ is the unconditional distribution of report t when the bank's effort is m^*. That is,

$$g_{m^*}(t) = m^* g^h(t) + (1-m^*) g^l(t).$$

Taking derivative of $u(s,m)$ with respect to m, I obtain the marginal effect of bank's effort on the regulator. I modify the definition of ML_r in equation (11) to the following,

$$\begin{aligned} ML_r\big(s, t_p(s,m)\big) &\equiv x(s,\omega_h) \int_{t \geq t_p(s,m)} g^h(t) dt - x(s,\omega_l) \int_{t \geq t_p(s,m)} g^l(t) dt \\ &= \big(x(s,\omega_h) - x(s,\omega_l)\big) \int_{t \geq t_p(s,m)} g^h(t) dt + x(s,\omega_l) \Delta\big(t_p(s,m)\big). \end{aligned}$$

Lemma 10. *For any disclosure set D or no-disclosure set N_n, $ML_r\big(s, t_p(s,m^*)\big)$ is increasing in s for $m^* = \{m_D(s), m_{N_n}\}$.*

As argued in the baseline model, disclosure is less likely to be optimal when the changes in manipulation m and changes in the regulator's marginal utility change ML_r are driven by the the difference in passing probability between low and high fundamental asset $\Delta(t_p(s,m))$. This is still the case in this extension with effort choice. However, the effort exertion makes disclosure more likely to occur. Because in addition to inform the bank about the gain after passing the test, the regulator's disclosure about s is also informative about the improvement of such gain by exerting effort. Hence, disclosure is more useful to align the regulator's and the bank's interest regarding when effort is more desirable.

Proposition 10. *When condition (32) holds, the optimal disclosure policy follows a cutoff rule where $D = [\underline{s}, s^*)$ and $N = [s^*, \bar{s}]$. That is, the regulator discloses the signal s when $s < s^*$ and does not disclose the signal s when $s > s^*$, where $s^* \in [\underline{s}, s_D)$.*

References

Agarwal, Isha and Tirupam Goel (2020). “Limits of Stress-Test Based Bank Regulation”. *Available at SSRN 3712632*.

Barth, Mary E, Wayne R Landsman, and Mark H Lang (2008). “International accounting standards and accounting quality”. *Journal of accounting research* 46.3, pp. 467–498.

Bertomeu, Jeremy, Lucas Mahieux, and Haresh Sapra (2020). “Interplay between accounting and prudential regulation”. *Available at SSRN 3266348*.

Bushman, Robert M (2016). “Transparency, accounting discretion, and bank stability”. *Economic Policy Review, Issue Aug*, pp. 129–149.

Bushman, Robert M and Christopher D Williams (2012). “Accounting discretion, loan loss provisioning, and discipline of banks’ risk-taking”. *Journal of accounting and economics* 54.1, pp. 1–18.

Clark, Robert and Shaoteng Li (2022). “The strategic response of banks to macroprudential policies: Evidence from mortgage stress tests in Canada”. *Review of Finance* 26.1, pp. 187–216.

Corona, Carlos, Lin Nan, and Gaoqing Zhang (2015). “Accounting information quality, interbank competition, and bank risk-taking”. *The Accounting Review* 90.3, pp. 967–985.

— (2019). “The Coordination Role of Stress Tests in Bank Risk-Taking”. *Journal of Accounting Research* 57.5, pp. 1161–1200.

Dye, Ronald A (1985). “Disclosure of nonproprietary information”. *Journal of accounting research*, pp. 123–145.

Flannery, Mark J (2019). “Transparency and Model Evolution in Stress Testing”. *Available at SSRN 3431679*.

Gao, Pingyang and Xu Jiang (2018). “Reporting choices in the shadow of bank runs”. *Journal of Accounting and Economics* 65.1, pp. 85–108.

— (2020). “The economic consequences of discrete recognition and continuous measurement”. *Journal of Accounting and Economics* 69.1, p. 101250.

Goldstein, Itay and Yaron Leitner (2018). “Stress tests and information disclosure”. *Journal of Economic Theory* 177, pp. 34–69.

Goldstein, Itay and Haresh Sapra (2013). “Should banks’ stress test results be disclosed? An analysis of the costs and benefits”. *Foundations and Trends in Finance, forthcoming.*

Holthausen, Robert W (2009). “Accounting standards, financial reporting outcomes, and enforcement”. *Journal of accounting research* 47.2, pp. 447–458.

Huizinga, Harry and Luc Laeven (2012). “Bank valuation and accounting discretion during a financial crisis”. *Journal of Financial Economics* 106.3, pp. 614–634.

Leitner, Yaron and Basil Williams (2023). “Model secrecy and stress tests”. *The Journal of Finance* 78.2, pp. 1055–1095.

Leuz, Christian, Dhananjay Nanda, and Peter D Wysocki (2003). “Earnings management and investor protection: an international comparison”. *Journal of financial economics* 69.3, pp. 505–527.

Leuz, Christian and Peter D Wysocki (2016). “The economics of disclosure and financial reporting regulation: Evidence and suggestions for future research”. *Journal of accounting research* 54.2, pp. 525–622.

Parlatore, Cecilia and Thomas Philippon (2022). *Designing stress scenarios.* Tech. rep. National Bureau of Economic Research.

Petrella, Giovanni and Andrea Resti (2013). “Supervisors as information producers: Do stress tests reduce bank opaqueness?” *Journal of Banking & Finance* 37.12, pp. 5406–5420.

Quigley, Daniel and Ansgar Walther (2020). “Inside and outside information: reverse unraveling and stress test design”. *Unpublished Working Paper.*

Verrecchia, Robert E (1983). “Discretionary disclosure”. *Journal of accounting and economics* 5, pp. 179–194.