Breaking the Correlation between Corporate Bonds and Stocks: The Role of Asset Variance *

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ABSTRACT

We show that firm default risk is the primary predictor of the correlation between corporate bond and stock returns, both in the cross-section and over time. Bonds of less creditworthy firms behave more like the issuing firms' stocks, resulting in higher future comovement. As a direct implication, investing in bonds and stocks of the most creditworthy firms significantly enhances diversification benefits and Sharpe ratios out-of-sample. We develop a structural model with stochastic asset variance that rationalizes these findings, whereby time-variation in asset variance plays a critical for breaking down the perfect stock-bond correlation implied by the Merton model.

JEL Classification Numbers: G12, G13

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Stocks and bonds are both claims on a firm's underlying assets, implying that their prices should co-move positively with each other. Despite this basic intuition, and the fact that stock-bond correlation plays a central role for portfolio and risk management decisions, we know surprisingly little about the comovement between stock and corporate bond returns (see Bali, Goyal, Huang, Jiang and Wen 2022 for a review). For example, we still have limited knowledge on its magnitude, how it varies across firms and over time, or about its underlying economic forces.[1] A primary reason for this limited understanding is that the no-arbitrage model of Merton (1974), which has become the de facto workhorse framework for pricing corporate securities, implies a perfect correlation between a firm's stock and bond returns. In the data, however, the correlation between corporate bonds and stocks is neither constant nor perfect, as illustrated by Figure 1, suggesting that the connection between these asset classes is far from trivial.

In this paper, we show that time-variation in asset variance plays a critical role for breaking down the perfect stock-bond correlation implied by the Merton model. The reason is that a firm's bond and equity prices are typically exposed to changes in its asset variance in opposite directions. On one hand, higher variance of assets increases a firm's equity value, as equity can be viewed as a call option on the firm's assets. On the other hand, it also increases the value of the short put option embedded in the firm's bond, thereby decreasing bond valuation.

Despite its importance for enabling a less than perfect correlation between stocks and bonds, asset variance is not the driver of the stock-bond comovement variation. But default risk is. Intuitively, a bond's sensitivity to firm fundamentals, and thus to equity, should increase when creditworthiness deteriorates. To see this, consider a risk-free corporate bond, whose valuation is independent on the firm's fundamentals. In this case, the correlation with equity is nil. The bond of a distressed firm, however, behaves more like its equity, thereby generating a high correlation between the two. So we can expect stock-bond correlation to increase with the level of a firm's default risk, especially when firm asset variance strongly varies over time.

Using these insights, we provide novel evidence on the comovement between corporate bonds and stocks

[1] Stocks and corporate bonds have long been treated as separate asset classes and studied independently. To a large extent, this is due to a view that markets are segmented and different investors operate in debt and equity markets, respectively. Boyarchenko and Mueller (2019), for example, show that a large proportion of corporate bonds are held by pension funds and insurance companies. The Flow of Funds report released by the Federal Reserve Board also shows that from 1986 to 2017, 78% of corporate bonds are held by institutional investors, as opposed to equity markets, where 43% of the market is held by the household sector, 33% by mutual funds and 15% by pension funds (see Bai et al., 2021).

issued by the same firm and make three main contributions. First, we show that firm default risk is the primary predictor of the future comovement between corporate bond and stock returns, both in the time series and in the cross-section. This predictive relation arises directly from the time variation in firm asset variance, as the relation is strongest for firms exhibiting highest fluctuations in asset variance but disappears for firms with constant asset variance. Second, we develop a model that rationalizes these stylized facts by introducing stochastic idiosyncratic and systematic asset variances into a standard credit risk framework. Third, our findings have relevant implications for practitioners as we show that the comovement predictability can be exploited to generate tangible out-of-sample economic gains for investors adopting an active portfolio strategy.

Our empirical analysis first shows that stock-bond covariance and correlation varies strongly across firms and over time, and that firm default risk is the primary driver of the one-year-ahead stock-bond comovement.[2] Such predictability is robust to using various firm-level default risk proxies such as market leverage, bond yield spreads, credit default swap (CDS) spreads, distance-to-default, or a composite default risk measure. A panel regression analysis demonstrates that the predictive ability of default risk is robust in the cross-section and with firm fixed effects, which indicates that the predictability is not caused by unobservable changes in aggregate conditions or by time-invariant differences across firms. We then verify that the predictive ability of default risk holds after controlling for a wide range of variables that are informative about firm and bond liquidity, shocks to the capital of financial intermediaries, changes in Treasury yields and expected inflation, or macroeconomic conditions. Relative to these measures, default risk is by far the most relevant variable for predicting stock-bond comovement; it accounts for more than 70% of the total variation in comovement explained by our regression models. Importantly, the impact of default risk on future stock-bond comovement is large and economically meaningful. Unconditionally, the stock-bond correlation increases from 0.07 to 0.33, when moving from AAA/AA rated firms to firms rated BB/C.

We rationalize these findings with a structural credit risk model which extends Du et al. (2019) along two key dimensions. First, we allow for stochastic asset variance to have a systematic and an idiosyn-

[2]Our main sample uses TRACE data combined with stock and firm-level data from CRSP/COMPUSTAT and consists of 1,290 firms with a total of 9,103 bonds over the period from August 2002 to August 2020. For robustness, we merge various corporate bond data sets (i.e., Lehman Brothers Fixed Income Database, TRACE, and Datastream) to obtain an extended sample consisting of 12,756 corporate bonds issued by 1,652 firms spanning the period April 1987 to August 2020. Results remain similar across the two samples.

cratic component. Second, the model introduces a factor structure in asset returns, such that firms differ in their exposure to aggregate asset returns and variance via their asset beta. Both extensions are non-trivial, yet useful in generating reasonable predictions. On the one hand, the presence of priced systematic variance is key to obtaining reasonable credit spreads, along with the level of systematic risk in bond and stock markets we observe empirically. Systematic variation in the level and the variance of a firm's assets also helps generate meaningful time-series fluctuations in aggregate stock-bond comovement. On the other hand, firm heterogeneity in asset betas and idiosyncratic variances contributes to generating the observed cross-sectional variation in stock-bond correlation and covariance.

To grasp the fundamental role of stochastic asset variance in shaping stock-bond comovement, consider a Merton-type model with constant asset variance. When the variation in the firm's assets is the unique source of uncertainty, bond and equity valuations perfectly comove with each other (i.e., good news for equity is always good news for the bonds). Within our model, heterogeneity in equity and bond exposures to asset variance risk is key to break down this perfect correlation. The reason is that equity can be viewed as a call option on the firm's assets and, hence, equity valuation increases with total asset variance. In contrast, higher asset variance increases the firm's default probability and reduces the value of its bonds. Shocks to asset variance thus affect the pricing of stocks and bonds in opposite directions in our model, consistent with what we observe empirically.

We compare the model predictions with the data by simulating a large cross-section of firms with one bond and one stock each over 10 years. This allows us to calculate various bond and stock pricing moments and structurally estimate the model parameters to match a set of empirical moments in the bond and stock markets. The main finding is that we can reproduce the predictive relation between future stock-bond comovement and current firm default risk, both over time and in the cross-section. For robustness, we augment our model with stochastic interest rates and find that default risk remains a strong driver of future stock-bond covariance and correlation.[3]

Our work complements the literature studying the stock-bond return relation with the hedge ratio, which measures the sensitivity of corporate bond returns to changes in the value of equity. We have two novel

[3]Note that stochastic variation in interest rates helps to decrease the level of stock-bond comovement. A rise in the risk-free rate effectively reduces the discounted value of future coupons, thereby decreasing the value of the bond. This in turn increases the value of the firm's stock for a given level of assets. So bond and stock valuations move in the opposite direction upon a change in interest rates.

insights.[4] First, we decompose the empirical hedge ratio across rating categories into a stock-bond correlation term and the Merton component, which is purely impacted by leverage. We document that the correlation increases by a factor of about 6 from the safest to the riskiest ratings, whereas there is virtually no cross-sectional variation in the Merton component. Thus, the relation between the hedge ratio and default risk documented in Schaefer and Strebulaev (2008), among others, is almost uniquely driven by fluctuations in the stock-bond correlation, consistent with our model. Second, we show that the Merton model, which assumes constant asset volatility, can only fit the data once we compute the model-implied hedge ratios with time-varying asset volatility. This finding highlights the importance of accounting for stochastic asset volatility in credit risk models to generate meaningful variation in stock-bond correlation and, thus, hedge ratios.

The findings documented in this paper also have relevant implications for investors and managers of funds with a joint exposure to stocks and bonds. Investing in more creditworthy firms can generate tangible out-of-sample economic gains through enhanced diversification benefits. To show this, we first sort firms into quintiles based on default risk. We then evaluate, for each quintile, the diversification benefits from investing in an equal-weighted allocation of stocks and bonds issued by the same firm, compared to investing in the two asset classes separately. We find that the diversification benefits are significantly higher for firms with less default risk.[5] Notably, selecting the most creditworthy firms reduces portfolio volatility more than it decreases excess returns, resulting in an improved risk-return tradeoff. For example, the Sharpe ratio doubles when investing in stocks and bonds issued by firms with the lowest instead of highest default risk. This finding suggests that reaching for yield, by selecting bonds of firms with high default risk, can be particularly detrimental for balanced (and multi-asset) mutual funds in terms of risk-adjusted performance. Exploiting the strong diversification between stocks and bonds of the most creditworthy firms can add significant value to managers of such funds, which tend to spread their overall investment across various rating categories.[6]

[4] See, for example, Kwan (1996), Schaefer and Strebulaev (2008), Bao and Hou (2017), Choi and Kim (2018), Augustin et al. (2020), Bali et al. (2022), and Kelly et al. (2022).

[5] A portfolio combining stocks and bonds issued by firms with the lowest (highest) default risk can eliminate about 47% (26%) of the average stock and bond return volatility. A central driver of this result is the fall in the correlation between stock and bond portfolio returns, which decreases from 0.71 to 0.11 when moving from the highest to the lowest-default-risk quintile portfolio.

[6] For example, consider BlackRock's "Multi-Asset Income Fund", which invests in corporate bonds (60%) and stocks (40%), with about $15.6 billion under management. The fund's bond allocation as of April 29, 2022 is as follows: 2.8% in AAA, 3.7% in AA, 7.1% in A, 14.5% in BBB, 28.3% in BB, 27% in B, 7.2% in CCC, and 1.5% in CC and below.

This paper builds on a growing literature analyzing the relation between stocks and corporate bonds. The seminal work of Collin-Dufresne et al. (2001) suggests that the relation between stock returns and changes in credit spreads is weak compared to what structural models would predict. In contrast, Schaefer and Strebulaev (2008) find that the sensitivity of corporate bond returns to stock returns (hedge ratio) is consistent with this class of models, while Bao and Hou (2017) document that such sensitivity increases with a bond's seniority. Bali et al. (2022) find that the return predictability of corporate bonds, using stock and bond characteristics with machine learning, improves when imposing the economic structure from the Merton model. Kelly et al. (2022) show that debt and equity markets become more integrated than previous estimates suggest when computing the hedge ratio with the systematic components of bond and stock returns. Based on portfolio sorts, Friewald et al. (2014) find that firms' stock returns strongly increase with credit risk premia. In contrast, Chordia et al. (2017) and Choi and Kim (2018) suggest that stock characteristics do not help predict the cross-section of corporate bond returns, thus questioning evidence regarding the existence of a tight link between stock and corporate bond pricing. Consistent with this view, Bali, Subrahmanyam and Wen (2021) point out that the correlation between stock and corporate bond returns is low on average.[7] Relative to these studies, our contribution is to investigate how default risk predicts stock-bond correlation and to rationalize these findings through the lens of a new structural model.[8]

The results of our paper also complement the literature studying the degree of integration between the stock and bond markets. Having different types of investors in either markets naturally implies some degree of segmentation, as evidenced in Kapadia and Pu (2012), Chordia et al. (2017), Choi and Kim (2018), Augustin et al. (2020), Sandulescu (2021), or Collin-Dufresne et al. (2021), among others. Market segmentation is thus an important feature of the data to potentially explain the low average correlation *level* between stocks and bonds. However, market segmentation is unlikely to drive the *variation* in correlation across firms, especially regarding the role of default risk. Market segmentation indeed tends to be more severe for speculative-grade firms (e.g., see Chernenko and Sunderam, 2012) and, therefore, could not explain our finding that stock-bond comovement is higher for firms with more default risk. In addition, a key contribution of the paper is to explore to what extent one can rationalize

[7] They document an average correlation of about 0.18, consistent with previously reported evidence (e.g., Kapadia and Pu, 2012; Chordia et al., 2017).

[8] Table A.1 in the Online Appendix provides a selective summary of this literature.

stylized facts about stock-bond comovement with a model assuming fully integrated markets.

On the theoretical front, our paper relates to recent structural credit risk models allowing for time variation in asset variance and risk premia (e.g., see Cremers et al., 2008; Chen et al., 2009; Bhamra et al., 2010). These papers aim to match empirical features of the equity and bond markets, such as the level of credit spreads and the equity risk premium. Du et al. (2019) show that accounting for priced asset variance risk helps generate the levels of credit spreads together with equity and bond total volatilities we observe in the data.[9] Collin-Dufresne et al. (2021) introduce a jump-diffusion process with idiosyncratic and systematic risk to explain the level of implied volatilities in equity and credit derivative markets. We contribute to this literature with novel insights on the dependence between stock and bond returns while extending our understanding of their respective pricing moments. Specifically, we show that a model with stochastic idiosyncratic and systematic asset variance can replicate the predictability of the stock-bond comovement with default risk consistent with the data, and helps understand its underlying economic forces. This makes the model particularly useful not only to study the pricing of stocks and bonds but also their joint dynamics.

While the credit spread puzzle has been a central focus of the credit risk literature over the last two decades (e.g., Huang and Huang, 2012), we uncover a novel dimension that structural models struggle to reproduce: the low average correlation level between stock and bond returns. We find that a no-arbitrage model with stochastic asset variance is successful in matching key individual equity and bond pricing moments (e.g., firm-level and aggregate stock/bond volatilities) and default risk (e.g., CDS spreads, financial leverage, and default probability), but implies a level of stock-bond correlation that is too high relative to the data: around 0.79 (0.64) in our model with constant (stochastic) interest rates, while it is around 0.20 in the data. We label this finding the 'stock-bond correlation puzzle.' One potential way to reduce the unconditional correlation is to consider financial frictions, thereby preventing stock and bond markets from being fully integrated.[10] Given our focus on the *conditional variation* of the stock-bond comovement across firms and over time, rather than on its *unconditional level*, we intentionally do not introduce any friction in the model. The resolution of this correlation

[9] The presence of the variance risk premium creates a wedge between physical and risk-neutral asset variance, which implies a high credit spread along with reasonable firm leverage.

[10] The introduction of financial constraints, stock/bond-specific shocks, asset-specific transaction costs, or asymmetric information would mechanically weaken the tight link between corporate bonds and stocks that existing rational pricing models imply.

puzzle constitutes, however, a promising agenda for future research on credit models.

The remainder of the paper is organized as follows. Section 1 describes the data and presents the results regarding the predictive relation between stock-bond comovement and default risk. Section 2 discusses the out-of-sample diversification gain by default risk. Section 3 outlines a credit risk model with stochastic variance that rationalizes our empirical findings and sheds new light on the comovement between corporate bonds and stocks. Section 4 concludes. The Online Appendix contains technical details and presents additional results not included in the main body of the paper.

1 Predictability of stock-bond comovement with default risk

In this section, we empirically explore how default risk predicts stock-bond comovement at the firm level, both in the cross-section and over time. We document a positive and asymmetric predictive relation, that is, an increase in default risk has a stronger impact on the one-year-ahead stock-bond comovement than a decrease in default risk. We verify that such predictability is robust to controlling for alternative explanations, bond characteristics, and changes in financial/economic conditions. We first present the data and then discuss the empirical results.

1.1 Data

We start by describing our data and introducing the measures of stock-bond comovement and default risk.

Sample construction Our main analysis uses a comprehensive dataset of transaction-based bond prices from the cleaned Enhanced TRACE dataset provided by WRDS. We extract corporate bond characteristics data from Mergent FISD, which we then match with equity and accounting data from CRSP and COMPUSTAT.[11] Online Appendix A contains details on the construction, merging and filtering rules applied to the datasets, which closely follow the literature. The final sample comprises 9,103 corporate bonds issued by 1,290 firms spanning the period from August 2002 to August 2020.

[11]We merge corporate bond data to firm-level data using the WRDS Bond Linker for the TRACE data and historical Committee on Uniform Securities Identification Procedures (NCUSIP) identifiers at both the firm and issue levels for the other databases. The remaining corporate bonds are manually matched using Bloomberg's data point (BDP) function.

Stock-bond comovement The estimation of stock-bond comovement requires firm-level measures of stock and corporate bond returns. We compute stock returns as $r_{S,i,t} = \frac{S_{i,t}+D_{i,t}}{S_{i,t-1}} - 1$, where $S_{i,t}$ is the stock price of firm i and $D_{i,t}$ is the dividend paid during month t (if any). For bond returns, since there are multiple bonds issued by the same firm, we first compute monthly returns of individual bonds and then aggregate the returns at the firm level. Individual corporate bond returns are computed as follows:

$$r_{B,i,j,t} = \frac{B_{i,j,t} + AI_{i,j,t} + Coupon_{i,j,t}}{B_{i,j,t-1} + AI_{i,j,t-1}} - 1, \tag{1}$$

where $B_{i,j,t}$ is the price of bond j of firm i in month t, $AI_{i,j,t}$ is the accrued interest, and $Coupon_{i,j,t}$ is the coupon payment, if any. The aggregate corporate bond return for firm i in month t, denoted by $r_{B,i,t}$, is the equal-weighted average of the firm's outstanding corporate bond returns $r_{B,i,j,t}$.[12] Our main measures of stock-bond comovement for firm i in month t are the covariance, $\sigma_{S,B,i,t}$, and correlation, $\rho_{S,B,i,t}$, between stock and bond returns estimated using 12 monthly observations from $t-11$ to t.[13]

Default risk measures We consider four firm-month default risk measures, which we describe in detail in Online Appendix B. First, we use market leverage, $L_{i,t}$, defined as the ratio between total book debt and the sum of total book debt and the market value of equity. Second, we use the distance-to-default (DD), given by (the log of) the distance between a firm's assets and the face value of its debt scaled by asset volatility. We follow Choi and Richardson (2016) and compute asset volatility as the standard deviation of monthly asset returns measured over the previous year. The asset return for firm i in month t, denoted by $r_{A,i,t}$, is constructed as a weighted-average of bond and stock returns: $r_{A,i,t} = L_{i,t} \times r_{B,i,t} + (1 - L_{i,t}) \times r_{S,i,t}$. A lower distance-to-default indicates a higher probability of default and is thus commonly used as a negative default predictor in reduced-form models (e.g., Duffie et al., 2007; Bharath and Shumway, 2008; Campbell et al., 2008). Third, we consider the equal-weighted credit spreads of a firm's outstanding bonds, which determines the compensation for bearing default risk. The credit spread of an individual bond is computed as the difference between the yield of the bond and the associated yield of the Treasury curve for the same maturity. We use the Benchmark Treasury

[12] Results remain quantitatively similar when using weights based on the outstanding amount in each of the firm's bonds.

[13] For our regression analysis, we apply the Fisher transformation to the correlation, which is defined by $\rho^F_{S,B,i,t} = \frac{1}{2} ln(\frac{1+\rho_{S,B,i,t}}{1-\rho_{S,B,i,t}})$. The Fisher transformation follows a standard normal distribution asymptotically and allows for correct inference on the regression coefficients. We also consider longer estimation windows (up to 60 months) for robustness.

rates from Datastream for maturities of 3, 5, 7, 10, and 30 years, and then use a linear interpolation scheme to estimate the entire yield curve, following Duffee (1998) and Collin-Dufresne et al. (2001), among others. Finally, we construct an aggregate firm-level default risk variable, labeled 'Default Risk', as the sign-corrected average of the three standardized default risk measures.[14] As we use one year of data to compute the distance-to-default, the sample we consider for the remainder of our analysis spans the period August 2003 to August 2020.

Table 1 summarizes the construction and the sources of the variables used in the paper. Panel A of Table 2 presents summary statistics for the different measures of stock-bond comovement and default risk, while Panel B presents the pairwise correlations between the variables. Once the sign for distance-to-default is flipped, all measures are positively correlated and exhibit a positive correlation with both measures of stock-bond comovement.

Table 1 and Table 2 [about here]

1.2 Default risk as the main driver of stock-bond comovement

In this section, we establish our main result that default risk predicts future comovement between corporate bond and stock returns by running the following regression:

$$Comovement_{i,t+12} = a + \delta DR_{i,t} + \mathbf{Y}'_{i,t}\delta_C + b_i + \epsilon_{i,t+12}, \quad (2)$$

where $Comovement_{i,t+12}$ is either the one-year-ahead stock-bond covariance, $\sigma_{S,B,i,t+12}$, or Fisher correlation, $\rho^F_{S,B,i,t+12}$, of firm i computed between months $t+1$ and $t+12$, $DR_{i,t}$ denotes firm-level default risk observed in month t, and $\mathbf{Y}_{i,t}$ is a vector of firm characteristics and aggregate variables observed in month t that we use as controls. We include firm fixed effects b_i to account for unobserved time-invariant differences across firms. Standard errors are clustered at the firm and month level to account for potential correlation across residuals, $\epsilon_{i,t}$, across firms and time.

Table 3 reports the baseline predictive regression results using individual default risk measures. To

[14]Specifically, 'Default Risk' is computed as $(Lev - DD + CS)/3$, where Lev, DD, and CS are standardized measures of leverage, distance-to-default, and credit spread, respectively. This variable is also standardized to ensure a mean of zero and a standard deviation of one.

make the coefficient estimates comparable, all measures are standardized.[15] While Panel A reports results for covariance, the results for correlation are presented in Panel B. In all cases the coefficients are positive and highly statistically significant, i.e., default risk positively predicts the one-year-ahead stock-bond covariance as well as the correlation. The predictive relation is not only statistically strong but also economically meaningful. Using leverage as an example, a one-standard deviation increase in firm default risk is associated with an increase in the correlation of about 28% over the following year.[16]

Table 3 [about here]

Given that the results are qualitatively similar across the individual default risk measures, we use the aggregate default risk measure for the remainder of the empirical analysis. In Columns (1) and (3) of Panel A in Table 4, we repeat the baseline regressions from Table 3 with firm fixed effects but using 'Default Risk' as the regressor. Additionally, Columns (2) and (4) present results using the Fama-MacBeth approach to explore the predictability across firms. Throughout, the impact of default risk remains highly statistically significant and further strengthens in the cross-section. The results with firm fixed effects suggest that stock-bond comovement doesn't merely capture time-invariant differences across firms.[17] At the same time, the cross-sectional results suggest that the predictability is not driven by unobservable changes in aggregate conditions. Furthermore, the results are similar whether we compute stock-bond comovement using raw or excess returns, hence, we focus on raw returns throughout the paper.[18]

Table 4 [about here]

To complement our regression analysis we study conditional portfolio sorts based on S&P bond ratings. To this end, each month t, we form five groups of firms according to the following rating ranges: AAA

[15] In addition, the sign on distance-to-default is flipped so that the interpretation of the coefficient estimates remains consistent across columns.

[16] Recall that our regressions use the Fisher correlation, such that the dependent variable is unbounded. Inverting the Fisher transformation yields $\rho_{S,B,i,t} = \frac{e^{2\rho^F_{S,B,i,t}}-1}{e^{2\rho^F_{S,B,i,t}}+1}$. A one-standard-deviation increase in leverage increases the Fisher correlation ($\rho^F_{S,B,i,t}$) from 0.24 (unconditional mean) to 0.31 (as $\hat{\delta} = 0.07$), which implies that the stock-bond correlation ($\rho_{S,B,i,t}$) effectively changes from 0.235 to 0.300, i.e., an increase of 28%.

[17] In Online Appendix C, we consider additional panel specifications using different combinations of time and firm fixed effects, as well as different dimensions of standard error clustering. Regardless of the specification, the predictive relationship between default risk and measures of stock-bond comovement remains robust.

[18] Table A.3 in the Online Appendix reproduces the results of Table 4 using two different measures of excess returns: stock and bond returns are computed in excess of i) the one-month T-Bill return and ii) the maturity-matched Treasury Bond return. Results are robust across all return specifications.

to AA-, A+ to A-, BBB+ to BBB-, BB+ to BB-, and B+ to C. For each group, we create value-weighted portfolios, using market capitalization for the weights, and compute the respective stock-bond covariances and correlations over the following year. Confirming our regression analysis, Panels A and B of Figure 2 show that the one-year-ahead stock-bond comovement is monotonically decreasing in the average level of credit rating. Specifically, the stock-bond correlation increases from 0.07 to 0.33, when moving from AAA/AA rated firms to BB/C rated ones. This difference is economically sizable.

Figure 2 [about here]

The bar plots also suggest that the predictive relation between stock-bond comovement and default risk is asymmetric. For example, when moving from A/BBB to BB/C rated firms, the increase in correlation is four times larger than the increase measured when moving from AAA/AA to A/BBB rated firms. To verify the robustness of this finding, we further sort firms into default risk quintiles using the aggregate default risk measure. Panels C and D of Figure 2 again confirm the predictive relationship between default risk and stock-bond comovement. Furthermore, the relationship continues to be strongly convex, i.e., the impact of a decrease in default risk (e.g., from Q3 to Q1) is not the mirror image of an increase in default risk (e.g., from Q3 to Q5). The asymmetry implies that the predictive relation between stock-bond comovement and default risk strengthens for less creditworthy firms.

In summary, the level of default risk dictates the fundamental relation between corporate bond and stock valuations. When a firm has higher default risk, corporate bond prices become more sensitive to changes in firm conditions and start behaving more like the firm's equity. Consistent with this intuition, we provide evidence that default risk is a strong predictor of the one-year-ahead covariance and correlation between stocks and bonds. In addition, this predictability holds conditionally and is particularly acute among firms with higher default risk.

These results have important implications for portfolio allocation. Investing in a balanced portfolio of stocks and bonds of firms with low default risk (e.g., AAA-AA to A ratings) should offer a high level of diversification, as the correlation of stocks and bonds issued by these firms is relatively low. In contrast, a balanced allocation strategy that would invest in stocks and bonds of firms with high default risk (e.g., B+ to C ratings) should offer lower diversification benefits as the correlation of stocks and bonds issued by these firms is relatively high. We quantify the role of default risk on the diversification gain of investing in stocks and bonds in Section 2.

1.3 Controlling for alternative explanations

We now verify that default risk remains a key predictor of stock-bond comovement after accounting for various alternative channels. Although the literature on the determinants of the stock-corporate bond covariance and correlation remains limited to date, several strands of studies are useful to identify potential alternative factors that could impact the comovement between corporate bonds and stocks. We now reestimate the benchmark specification controlling for such alternative channels. We present the results in Table 5, where each column includes a different set of controls.

Table 5 [about here]

Bond and equity characteristics As a first robustness check, we control for various corporate bond characteristics in Column (1). Bao and Hou (2017) find that the maturity of a bond shapes its relation with the firm's stock. To account for this channel, we directly control for time-to-maturity as well as the debt coupon, which we average at the firm level. Other bond features such as the possibility to call the bond (callability) could also affect its yield, return, and eventually its comovement with stock returns. Bond callability is a crucial feature of most corporate bonds, as over 70% of the full sample of corporate bonds have an embedded call option. Hence, we also control for a dummy variable equal to one if the bond is callable and zero otherwise, following Chordia et al. (2017).[19] In addition, we control for the total size of a firm's bonds with (the log of) the sum of all bonds outstanding for a given firm. Finally, we account for firm-level bond illiquidity, which we compute as the value-weighted bond bid-ask spread. Complementing these bond features, in Column (2), we account for firm-level stock characteristics that are potentially related to default risk. Specifically, we control for stock illiquidity using Amihud (2002)'s ratio, equity size using the log of stock market capitalization, and equity valuation using the market-to-book ratio.

[19] We also verify that our findings remain qualitatively similar if we exclude all callable bonds and estimate our main results using the reduced sample of option-free bonds. The results are reported in the Online Appendix C

Aggregate liquidity conditions The literature on the comovement between stocks and Treasury bonds suggests that aggregate liquidity conditions are central drivers of the stock-treasury bond covariance (Baele et al., 2010). As such, aggregate liquidity conditions likely affect investment-grade bonds, as they tend to fluctuate in line with risk-free securities. We thus control in Column (3) for the potential impact of aggregate illiquidity using Pastor and Stambaugh (2003)'s liquidity risk-factor and the 'noise' illiquidity proxy of Hu et al. (2013).[20] The noise measure captures episodes of liquidity crises of different origins, thereby providing information beyond traditional liquidity proxies.

Intermediary capital risk We then check that the impact of default risk on the stock-bond covariance and correlation is not capturing an intermediary asset pricing story. To this end, we control in Column (4) for the intermediary capital risk factor of He et al. (2017), which is known to explain the cross-sectional pricing of various asset classes, including stocks and bonds.[21]

Interest rates and expected inflation Another body of literature provides evidence that the term structure of interest rates and expected inflation drive the stock-treasury bond covariance (see, e.g., David and Veronesi, 2013; Campbell et al., 2017, 2020). We thus control in Column (5) for these effects using the 10-year U.S. Treasury rate, the 3-month U.S. Treasury Bill secondary market rate, and their difference (slope) to proxy for the term structure. We separately control in Column (6) for the one-quarter-ahead mean inflation forecast from the Survey of Professional Forecasters, obtained from the Philadelphia FED, to proxy for expected inflation.

Macroeconomic conditions The comovement between stocks and corporate bonds tends to vary with macroeconomic conditions, as depicted in Figure 1. In particular, the covariance and correlation are expected to increase in times of financial stress, when default risk increases. At the same time, Laarits (2022) finds that the covariance between stocks and risk-free bonds becomes more negative in bad times, when the precautionary savings motive increases. So the effect may also vary with a firm's creditworthiness. To account for the role of global financial and economic conditions, we control in Column (7) for the macro uncertainty index of Jurado et al. (2015) and the business conditions index

[20]We obtain the noise measure from Jun Pan's website.

[21]We obtain the series from Asaf Manela's website. The sample period is shorter in this analysis, as the intermediary capital risk factor ends in December 2018.

of Aruoba et al. (2009).

In all cases, default risk continues to positively predict the one-year-ahead stock-bond comovement, with statistical significance robust at the 1% level. The same finding holds when including all the controls jointly, as presented in Column (8).[22] Notably, the inclusion of the various controls does not weaken the economic magnitude of the predictive ability of default risk: the impact of default risk on the stock-bond correlation in fact doubles (0.08 vs. 0.04) when adding controls. Overall, these results show that, even after controlling for a multitude of stock-bond comovement proxies identified in the market segmentation and asset pricing literature, the positive predictability coefficient of default risk remains highly statistically significant and economically meaningful.

As a complementary exercise, we explore the relative importance of default risk vs. the different sets of controls in explaining future fluctuations in stock-bond comovement. To do this, Figure 3 decomposes the total R-squared of our baseline specification presented in Equation (2), excluding fixed effects, into two parts: i) the part that is attributed to default risk; and ii) the part that is captured by a given set of controls. The procedure is known as the Shapley-Owen marginal R^2 decomposition, following the methodology outlined in Shorrocks (1982). The results indicate that the role of default risk largely dominates any alternative channel in explaining the one-year-ahead variation in stock-bond comovement.

Figure 3 [about here]

1.4 Additional robustness checks

We consider additional tests to assess the robustness of our findings, which we discuss in detail in Online Appendix C. The results are reported in Table 6, where the first column reproduces the specification with all controls (see Column 8 of Table 5) for comparison purposes.

Table 6 [about here]

[22]For conciseness, Table 5 does not present the coefficients of the controls. Table A.4 in the Online Appendix presents the regression estimates of all controls used in Column (8). Stock-bond comovement appears to increase with various bond characteristics (amount outstanding, and bid-ask spread) and decrease with equity size and illiquidity. In terms of the aggregate controls, stock-bond comovement increases with the short-term risk-free rate, but decreases with the long-term risk-free rate, expected inflation, macroeconomic uncertainty, and business conditions. Default risk is, however, clearly the most relevant explanatory variable in terms of the t-statistic.

Data and sample Our baseline panel regressions exploit firm-level data, but a firm can have several distinct bonds outstanding. Column (2) verifies that the results continue to hold at the corporate bond level.[23] Given that a large number of bonds in the sample contain an embedded call option, Column (3) reproduces the main results at the bond level excluding bonds with call options. Column (4) shows that the results remain similar after excluding financial and insurance-related firms. Furthermore, Column (5) reports similar comovement predictability with default risk using an extended data sample, which combines several corporate bond databases starting in April 1987. Hence, our results are not limited to the TRACE dataset or the post-2000 period.

Comovement measure Our inferences are also robust to the way we measure the stock-bond covariance and correlation. For example, Column (6) shows that the firm-level results remain similar when using covariance and correlation implied from an asymmetric DCC-GARCH model, rather than using rolling estimates.

Default risk measure As an alternative measure of default risk, we consider firm-level credit default swap (CDS) spreads using Markit data. Column (7) shows that the predictability of stock-bond comovement with the (standardized) 10-year CDS spread is of similar economic magnitude and statistical significance as for the other default risk measures.[24] This robustness analysis also verifies that the baseline results are not driven by the relatively small firms, which typically do not have outstanding CDS contracts.

Persistence and overlapping observations We then verify that the comovement predictability does not arise mechanically from the persistence in stock-bond comovement or from overlapping observations. Column (8) controls for the lagged stock-bond covariance/correlation (computed between month $t-12$ and $t-1$), while Column (9) uses yearly non-overlapping observations. In both cases, the estimated coefficients and R-squared values are similar in magnitude to those reported with the baseline specification.

[23] In this analysis, the covariance and correlation are computed between individual bond's returns and firm-level stock returns.

[24] Results are similar when using 5-year CDS spreads.

Forecast horizon Our analysis thus far focused on the one-year-ahead comovement. Table A.5 presents results when stock-bond comovement is computed over various horizons, ranging between 12 and 60 months. The predictive ability of default risk decreases with the forecast horizon, but remains highly significant in all cases.

Overall, the role of default risk continues to be robust after controlling for a wide array of competing explanations, exploiting alternative data and samples, as well as using different econometric specifications. Our empirical results therefore strongly support the idea that default risk is a primary predictor of stock-bond comovement, both over time and in the cross-section.

1.5 Understanding the hedge ratio

An extant literature studies the stock-bond return relation using the hedge ratio, which measures the sensitivity of corporate bond returns to changes in the value of equity.[25] For instance, Schaefer and Strebulaev (2008) and Bao and Hou (2017) find that the hedge ratio increases with default risk and that this result is consistent with the Merton model. We complement and extend this literature with two novel insights. First, we empirically document the first-order impact of stock-bond correlation on the relation between the hedge ratio and default risk. Second, we show that the Merton model, which assumes constant asset volatility, can only fit the data once we compute the model-implied hedge ratios with time-varying asset volatility.

Building on Schaefer and Strebulaev (2008), we first regress firm-level excess corporate bond returns, $r^e_{B,i,t}$, on the associated excess stock returns, $r^e_{S,i,t}$, and the 10-year constant maturity U.S. Treasury bond returns, $r_{f,t}$:

$$r^e_{B,i,t} = \alpha_0 + \alpha_S r^e_{S,i,t} + \alpha_{r_f} r_{f,t} + \epsilon_{i,t}, \tag{3}$$

where α_S is the sensitivity of corporate bond returns for firm i at time t to the corresponding stock returns, or an empirical estimate of the hedge ratio, while α_{r_f} reflects the sensitivity of corporate bond returns to interest rates.[26] Panel A of Table 7 reports the regression results by rating categories. Con-

[25] See, for example, Kwan (1996), Schaefer and Strebulaev (2008), Bao and Hou (2017), Choi and Kim (2018), Augustin et al. (2020), and Bali et al. (2022).

[26] Note that we use the 1-month T-bill to compute excess returns for both bonds and stocks.

sistent with previous studies, the hedge ratio increases as the creditworthiness of the firm deteriorates.

Table 7 [about here]

We now investigate the relation between the empirical hedge ratio and default risk in more detail. By definition, the hedge ratio h_S is

$$h_S \equiv \mathbb{COV}\left(r_S, r_B\right)/\sigma_S^2 = \rho_{B,S} \times \sigma_B/\sigma_S, \tag{4}$$

where $\rho_{B,S}$ is the stock-bond correlation, while σ_B and σ_S denote bond and stock return volatility, respectively. Recall that the Merton model features a unique source of risk (firm assets), which in turn implies a perfect correlation ($\rho_{B,S} = 1$) and $h_S = \sigma_B/\sigma_S$. Using equation (4) we can decompose the empirical hedge ratio estimate α_S into a correlation term ($\rho_{B,S}$) and the Merton component (σ_B/σ_S) that is purely impacted by leverage. This allows then to explore the relative contribution of the two components in determining fluctuations in the hedge ratio across ratings. From Panel B, we see that the correlation increases by a factor of 6.45 from the safest to the riskiest ratings, whereas there is virtually no cross-sectional variation in the Merton component. Thus, the variation in the hedge ratio is almost uniquely driven by fluctuations in the stock-bond correlation.

To test whether the Merton model correctly estimates the sensitivity of corporate bond returns to changes in the value of equity, Schaefer and Strebulaev (2008) run the following regression:

$$r^e_{B,i,t} = \alpha_0 + \beta_S h_{S,i,t}(\widehat{\sigma_A}) r^e_{S,i,t} + \alpha_{rf} r_{f,t} + \epsilon_{i,t}, \tag{5}$$

where $h_{S,i,t}(\widehat{\sigma_A})$ is the hedge ratio calculated with the Merton model for which σ_A is re-estimated every month based on conditional measures of stock-bond volatilities and correlation. Under the null hypothesis of no model misspecification, the slope coefficient β_S should be unity. We revisit their empirical analysis and adopt a similar regression specification using our sample (see Appendix E for the details). Panel A of Table 8 reports the results. In line with, e.g., Schaefer and Strebulaev (2008) and Kelly et al. (2022), we find that β_S is not statistically different from one.

Table 8 [about here]

However, as the Merton model assumes constant asset volatility, we reproduce the regression specification (5) with a hedge ratio $h_{S,i,t}(\widehat{\sigma_A})$ computed with an asset volatility σ_A that remains constant over time (rather than re-estimating every month). Panel B of Table 8 contains the results. Unlike the estimates in Panel A, β_S is now significantly different from unity. That is, when the assumption of constant volatility is strictly imposed, the Merton model is not well suited to estimate the hedge ratio. This is illustrated in Figure 4 where we plot the time variation in the hedge ratios with both conditional as well as constant asset volatility.[27] Shutting down the asset volatility channel substantially reduces the time variation in the hedge ratio. The discrepancy between the two measures is particularly pronounced during market downturns, when the dependence between bond and stock returns tends to increase. Confirming this intuition, a variance decomposition reveals that fluctuations in asset volatility explain about 90% of the time variation in the hedge ratio, while only 10% is attributed to fluctuations in the Merton component (or leverage), on average.

Figure 4 [about here]

Thus, while the Merton model can generate hedge ratios increasing in default risk through the ratio of stock-bond volatility, our results show fluctuations on stock-bond correlations largely drive the variation in hedge ratios. Overall, the finding documented in this section highlight the importance of allowing for stochastic asset volatility in credit risk models to break down the perfect link between stock and bond returns. Developing such a model and studying its implications for stock-bond correlation is precisely the subject of Section 3.

2 The economic value of stock-bond diversification

In this section, we show that our empirical findings have important implications for investors and managers of funds with a joint exposure to stocks and bonds. Specifically, we assess the economic value of an active asset allocation strategy that exploits the predictability of stock-bond comovement with default risk. Our analysis thus far suggests that the diversification benefit of combining stocks and bonds should be highest when investing in the most creditworthy firms. We quantify the gain investors can attain using this information and find that sorting firms based on default risk strongly increases the out-of-sample Sharpe ratio of a balanced investment strategy. We first describe the methodology and

[27] In the constant volatility case, time-variation in hedge ratios is purely induced by fluctuations in firm leverage.

then discuss the results.

2.1 Portfolio construction and diversification

In each month t we sort firms in quintiles based on their level of default risk and form conditional portfolios for stocks and corporate bonds separately.[28] Then, we compute the value-weighted average excess return over the following month, $t+1$, for each stock (S) and bond (B) portfolio. We denote the time series of the excess returns of security $j = \{S, B\}$ and quintile $q = \{1, .., 5\}$ by $r_{j,q,t+1}$. For each quintile we compute the portfolio variance of security j's excess returns $r_{j,q,t+1}$, denoted by $\sigma^2_{j,q}$. For a given quintile, the portfolio variance of next month's excess returns is then $\sigma^2_{S,q}$ when investing in stocks only, while it is $\sigma^2_{B,q}$ when investing in bonds only.

We also construct another set of quintile portfolios that combine both stocks and bonds, and thus account for potential diversification across the two asset classes. The variance of these portfolios now depend on the covariance between stock and bond excess returns, $\sigma_{S,B,q}$. For simplicity, we focus on an equal allocation of stocks and bonds, which we hereafter refer to as a 'balanced' portfolio, but consider alternative allocations for robustness in the Online Appendix.

To assess the degree of stock-bond diversification, we compute Goetzmann et al. (2005)'s variance ratio, VR_q, as follows:

$$VR_q = \frac{var\left(\frac{1}{2}\sum r_{j,q}\right)}{\frac{1}{2}\sum var\left(r_{j,q}\right)} = \frac{\frac{1}{4}\sigma^2_{S,q} + \frac{1}{4}\sigma^2_{B,q} + \frac{1}{2}\sigma_{S,B,q}}{\frac{1}{2}\sigma^2_{S,q} + \frac{1}{2}\sigma^2_{B,q}} \tag{6}$$

$$= \frac{1}{2} + \frac{\sigma_{S,B,q}}{\sigma^2_{S,q} + \sigma^2_{B,q}}. \tag{7}$$

The numerator of Equation (6) is the balanced portfolio variance, whereas the denominator equals the average of the stock and bond portfolio variances, $\sigma^2_{S,q}$ and $\sigma^2_{B,q}$. A lower stock-bond covariance increases the diversification benefit of pooling stocks and bonds in a balanced portfolio, compared to investing in bonds and stocks separately, and thus reduces the variance ratio.

Based on this insight, we label 'diversification benefit' the fraction of the average stock and bond

[28] We measure a firm's default risk with its average bond yield (or credit spread) because it is directly available to investors and does not require any estimation. The results are quantitatively similar when sorting firms based on alternative default risk proxies, such as the composite default risk measure, credit rating, or leverage.

variance that can be diversified away by combining stocks and bonds in a balanced portfolio. More precisely, diversification benefit is defined as $1 - VR_q$ and is expressed in percentage terms. We expect higher diversification benefits for balanced portfolios invested in more creditworthy firms. We now turn to an empirical assessment of this prediction and discuss the implications for the risk-return tradeoff.

2.2 Out-of-sample diversification and risk-return tradeoff

Panel A of Table 9 presents the average excess returns, volatilities, and Sharpe ratios of the default risk-sorted balanced portfolios. While excess returns moderately decrease with default risk, the reduction in return volatility is much more pronounced. The balanced portfolio with the least creditworthy firms (Q5) displays a return volatility of 18.6%, which drops to 6.9% for the portfolio with the most creditworthy firms (Q1).

As a direct consequence, the level of default risk strongly affects the risk-return tradeoff of the balanced portfolios. The Sharpe ratio is monotonically increasing in firm creditworthiness, ranging from 0.49 for the most default risky firms (Q5) to 0.95 for the least default risky firms. The difference (i.e., Q5-Q1) is economically sizable and statistically significant at the 5% level, based on HAC standard errors (Ledoit and Wolf, 2008). Panel A shows that portfolios of firms with less default risk also have a higher Sortino ratio, lower kurtosis, and lower Value at Risk (VaR), indicating that the higher Sharpe ratio does not come at the cost of higher left-tail risk. Hence, the risk-return trade-off of investing in stocks and bonds issued by firms with lower default risk is substantially more attractive than for firms with higher default risk.[29] Note that all portfolio moments are out-of-sample as they are based on excess returns measured at $t + 1$ while the sorting is based on information available at time t.

Table 9 [about here]

To better grasp the implications of these results, Panel B of Table 9 reports the diversification benefits of pooling stocks and bonds by default risk quintile. The diversification benefit for Q1 is 46.68% compared to 25.57% for Q5, which means that a balanced portfolio can eliminate about 47% and 26% of the average risk of stocks and bonds, respectively. A primary driver of this improvement in diversification benefit is the correlation between stock and bond returns, which falls from 0.71 for Q5 to 0.11 for Q1.

[29] The results are similar using alternative allocations of corporate bonds and stocks (e.g., 60%/40% and 40%/60%), as reported in Table A.7 in the Online Appendix.

Lower default risk, therefore, implies weaker comovement between stocks and corporate bonds in the following month, resulting in higher diversification benefits and Sharpe ratios.

These results are particularly relevant for investors and managers of balanced (or multi-asset) funds. We find that investing exclusively in firms in the lowest default risk segment can substantially improve the risk-return tradeoff, compared to spreading the bond part of the portfolio across various rating categories. The reason is that reaching for yields by selecting bonds with higher default risk, a common practice among portfolio managers, would substantially reduce the diversification benefits of combining stocks and bonds in a balanced allocation strategy.

2.3 Mechanism

We now investigate the channels through which stock-bond diversification decreases with default risk. To disentangle the impact of default risk on stocks vs. bonds, we explore two alternative portfolio strategies. First, we consider an allocation in which one invests in a fixed bond market index and a default risk-sorted stock portfolio, with equal allocation. Results reported in Panel C show that there is no material difference in diversification benefits across portfolios with different default risk levels. For the second strategy, we construct portfolios that invest in a fixed stock market index and a default risk-sorted bond portfolio, with equal allocation. As shown in Panel D, we now recover the monotonically decreasing diversification benefit from Q1 to Q5. These results indicate that the impact of default risk on diversification is induced by a change in the behaviour of corporate bonds rather than stocks. This analysis provides direct support to the main hypothesis studied in the paper, which is that corporate bonds of firms with higher default risk start behaving more like their stocks.

Next, we analyze how the difference in diversification between portfolios of low and high default risk varies over time. To this end, we construct balanced portfolios sorted on default risk as before and then estimate our measure of diversification benefits using a 12-month rolling window. In Figure 5 we plot the resulting series for portfolios Q1 and Q5. The difference in diversification benefit between portfolios Q1 and Q5 is large on average and positive during 185 out of 193 months (i.e., more than 95% of the time). This further demonstrates that the results reported in Table 9 are not simply driven by a few specific subperiods and, thus, can be consistently exploited by managers of balanced funds.

Figure 5 [about here]

In sum, we show that the future degree of diversification between stocks and corporate bonds depends critically on the current creditworthiness of the invested firms. While default risk is typically associated with reaching for yields, our analysis shows that investing primarily in the most creditworthy firms can generate tangible economic value through enhanced diversification across stocks and bonds. The level of default risk shapes the out-of-sample diversification gains of a balanced investment strategy and, eventually, its risk-return tradeoff.

3 A stochastic volatility model of the firm

In this section, we develop a credit risk model embedded in a multiple-firm economy to understand how stock-bond comovement varies with default risk. We first present our framework, which extends Du et al. (2019) to allow for a factor structure in asset returns and a decomposition of stochastic asset variance into a systematic and an idiosyncratic component. We show that this environment can generate a rich cross-section of firms, thereby inducing substantial variation in the comovement between corporate bond and stock returns. Also, accounting for stochastic asset variance is key to break down the perfect stock-bond correlation implied in the Merton model, as equity and bond prices have a positive exposure to asset return but an opposite exposure to asset variance (see Figure 6).

Figure 6 [about here]

We then estimate the model and discuss its ability to capture the empirical facts documented in Section 1, while matching key asset pricing and credit risk moments in the data. We find that a simulated economy can replicate the positive predictability of the stock-bond covariance and correlation with default risk, both over time and across firms. Finally, we use the model to better understand the economic forces driving the level of stock-bond comovement and its relation with default risk.

3.1 Environment and firm asset dynamics

We consider an economy with multiple firms whose assets are exposed to a common and an idiosyncratic source of shocks, as well as to stochastic volatility.[30] The common factor driving the level of firm assets, Y_t, which reflects changes in aggregate economic conditions, follows the dynamics

$$\frac{dY_t}{Y_t} = \mu_{Y,t}dt + \sigma_{Y,t}dW_t^Y, \tag{8}$$

where $\mu_{Y,t}$ is the expected aggregate asset growth, dW_t^Y is a Brownian motion capturing systematic risk under the physical probability measure $\mathbb{P}$, and $\sigma_{Y,t}$ denotes aggregate asset volatility. The latter follows a square root process:

$$d\sigma_{Y,t}^2 = \kappa_Y(\theta_Y - \sigma_{Y,t}^2)dt + \delta_Y\sigma_{Y,t}dW_t^{\sigma_Y}, \tag{9}$$

where κ_Y captures the speed of mean reversion of aggregate asset variance $\sigma_{Y,t}^2$ toward its long-run mean θ_Y, δ_Y is the volatility of aggregate asset variance, and $dW_t^{\sigma_Y}$ captures the variance innovations.

To account for the dependence between aggregate assets Y_t and its variance $\sigma_{Y,t}^2$, we assume that $dW_t^Y = \rho_Y dW_t^{\sigma_Y} + \sqrt{1-\rho_Y^2}dW_t^{Y\perp\sigma_Y}$, where ρ_Y captures the correlation between aggregate asset shocks and aggregate asset variance shocks, while $dW_t^{\sigma_Y}$ and $dW_t^{Y\perp\sigma_Y}$ are two mutually independent Brownian motions. When $\rho_Y < 0$, aggregate asset variance is high when the level of aggregate assets is low, that is, in bad economic times. This case implies a negative skewness in the distribution of aggregate shocks, consistent with the evidence in Berger et al. (2020), among others.

The dynamics of firm i's total assets, denoted by $X_{i,t}$, and its idiosyncratic asset variance, $\sigma_{X,i,t}^2$, jointly follow

$$\frac{dX_{i,t}}{X_{i,t}} = (r-q)\,dt + \beta_i\left(\frac{dY_t}{Y_t} - rdt\right) + \sigma_{X,i,t}dW_{i,t}^X \tag{10}$$

$$d\sigma_{X,i,t}^2 = \kappa_X\left(\theta_X - \sigma_{X,i,t}^2\right)dt + \delta_X\sigma_{X,i,t}dW_{i,t}^{\sigma_X}, \tag{11}$$

[30] We assume that information is complete and that financial assets are continuously traded without frictions in arbitrage-free markets.

under the physical probability measure $\mathbb{P}$, where r is the risk-free rate and q is the total payout rate to security holders. Turning to the idiosyncratic asset variance dynamics, κ_X denotes the speed of mean reversion, θ_X the unconditional mean, and δ_X the volatility of variance. These parameters are identical across firms.[31] For parsimony, we assume independence between idiosyncratic asset shocks and variance shocks (i.e., $dW_{i,t}^X dW_{i,t}^{\sigma_X} = 0$).

A specific feature of our model is that firms differ in their β_i, which captures the exposure of firm i to aggregate asset fluctuations, $\frac{dY_t}{Y_t}$. That is, Equation (10) implies a factor structure in firm i's total asset variance, which is given by $\sigma_{i,t}^2 = \beta_i^2 \sigma_{Y,t}^2 + \sigma_{X,i,t}^2$. Cross-sectional variation in firms' total variance thus arises in this framework from cross-sectional variation in both firm betas and conditional idiosyncratic variances.

We consider a stochastic discount factor (SDF), denoted by ϕ_t, that depends linearly on aggregate asset and variance risk. Its dynamics is given by:

$$\frac{d\phi_t}{\phi_t} = -r dt - \sigma_{Y,t}\left(\lambda_{Y\perp\sigma_Y} dW_t^{Y\perp\sigma_Y} + \lambda_{\sigma_Y} dW_t^{\sigma_Y}\right), \tag{12}$$

where $\sigma_{Y,t}\lambda_{Y\perp\sigma_Y}$ and $\sigma_{Y,t}\lambda_{\sigma_Y}$ reflect the price of aggregate asset risk ($dW_t^{Y\perp\sigma_Y}$) and aggregate variance risk ($dW_t^{\sigma_Y}$), respectively. Our SDF builds on the long-run risk and variance risk literature, which provides theoretical and empirical support for priced variance risk.[32] We use this SDF to derive the model's risk premia and the dynamics (8)-(11) under the risk-neutral measure (see Online Appendix F).

3.2 Bond and stock returns

We now turn to the pricing of bonds and stocks, which are contingent claims on a firm's assets, and express their return dynamics. Firms issue a perpetual consol bond with a coupon rate c per unit of time. A firm defaults when its asset value hits an exogenously-specified default boundary X_B.[33] At

[31] Despite this simplification, it is worth noting that firms' asset values and conditional idiosyncratic variances vary in the cross-section due to firm-specific idiosyncratic return and variance shocks, $dW_{i,t}^X$ and $dW_{i,t}^{\sigma_X}$.

[32] See, for example, Bansal and Yaron (2004), Bollerslev et al. (2009), and Koijen et al. (2010). Closely related to our study, Du et al. (2019) also show the importance of priced variance risk to help resolve the credit spread puzzle.

[33] Note that we explicitly depart from the case of endogenous financing and default policies. The reason is that we estimate the debt coupon and default boundary, amongst other parameters, to match the level of default risk, leverage, and various asset pricing moments empirically. Our goal is to explore stock-bond comovement in a model that closely fits the data, rather than analyzing a firm's capital structure decisions.

default, debtholders recover a fraction of the after-tax unlevered asset value of the firm, whereas the remaining fraction is lost due to liquidation costs. We denote the constant liquidation cost by α and the tax rate by ζ. Shareholders are entitled to the firm's assets net of debt servicing as long as the firm does not default. When the firm is in default, shareholders recover nothing and lose their rights to any future claim. We assume that c, α, X_B, and ζ are identical across firms.

The bond price of firm i, denoted by $B_{i,t}$, equals the sum of the present value of the coupon payments before default and the recovered value of the firm at default, which is given by

$$B_{i,t} = \frac{c}{r} + \left[(1-\alpha)X_B - \frac{c}{r}\right] p_D\left(X_{i,t}, \sigma_{Y,t}, \sigma_{X,i,t}\right), \tag{13}$$

while the firm's stock price is

$$S_{i,t} = X_{i,t} - \frac{(1-\zeta)c}{r} + \left[(1-\zeta)\frac{c}{r} - X_B\right] p_D\left(X_{i,t}, \sigma_{Y,t}, \sigma_{X,i,t}\right), \tag{14}$$

where p_D is the price of an Arrow-Debreu default claim, which reflects the risk-neutral present value of \$1 at default. More precisely, the Arrow-Debreu default claim is defined by $p_D\left(X_{i,t}, \sigma_{Y,t}, \sigma_{X,i,t}\right) \equiv \mathbb{E}_t^{\mathbb{Q}}[e^{-r(\tau_{D,i}-t)}]$, where $\tau_{D,i} = \inf\{s \geqslant t \mid X_{i,s} \leq X_B\}$ denotes firm i's random default time and $\mathbb{E}^{\mathbb{Q}}$ the expectation under the risk-neutral measure $\mathbb{Q}$. In absence of arbitrage, the levered firm value is the sum of $B_{i,t}$ and $S_{i,t}$.

Applying Itô's lemma to Equations (13) and (14) allows us to express the dynamics of stock and bond returns, which jointly satisfy

$$\frac{dS_{i,t}}{S_{i,t}} = \mu_{S,i,t}dt + \Delta_X^S\left[\beta_i\sigma_{Y,t}dW_t^Y + \sigma_{X,i,t}dW_{i,t}^X\right] + \Delta_{\sigma_Y}^S\delta_Y\sigma_{Y,t}dW_t^{\sigma_Y} + \Delta_{\sigma_X}^S\delta_X\sigma_{X,i,t}dW_t^{\sigma_X} \tag{15}$$

$$\frac{dB_{i,t}}{B_{i,t}} = \mu_{B,i,t}dt + \Delta_X^B\left[\beta_i\sigma_{Y,t}dW_t^Y + \sigma_{X,i,t}dW_{i,t}^X\right] + \Delta_{\sigma_Y}^B\delta_Y\sigma_{Y,t}dW_t^{\sigma_Y} + \Delta_{\sigma_X}^B\delta_X\sigma_{X,i,t}dW_t^{\sigma_X} \tag{16}$$

where $\mu_{S,i,t}$ and $\mu_{B,i,t}$ are the instantaneous expected stock and bond returns, respectively. The sensitivities of stock and bond prices to a change in firm assets satisfy $\Delta_X^S = \frac{X_{i,t}}{S_{i,t}}\frac{\partial S_{i,t}}{\partial X_{i,t}}$ and $\Delta_X^B = \frac{X_{i,t}}{B_{i,t}}\frac{\partial B_{i,t}}{\partial X_{i,t}}$. The sensitivities of stock and bond prices to a change in aggregate and idiosyncratic asset variance are

$\Delta^S_{\sigma_Y} = \frac{1}{S_{i,t}}\frac{\partial S_{i,t}}{\partial \sigma^2_{Y,t}}$, $\Delta^S_{\sigma_X} = \frac{1}{S_{i,t}}\frac{\partial S_{i,t}}{\partial \sigma^2_{X,i,t}}$ $\Delta^B_{\sigma_Y} = \frac{1}{B_{i,t}}\frac{\partial B_{i,t}}{\partial \sigma^2_{Y,t}}$, and $\Delta^B_{\sigma_X} = \frac{1}{B_{i,t}}\frac{\partial B_{i,t}}{\partial \sigma^2_{X,i,t}}$.[34] We can then study the comovement between bond and stock returns, which is the main focus of our theoretical analysis.

3.3 Stock-bond covariance

Given the joint dynamics of stock and bond returns, it is relatively straightforward to derive the stock-bond covariance (Proposition 1) and correlation (Proposition 2) implied by our model.

Proposition 1: *From the dynamics of stock and bond returns in Equations (15) and (16), the instantaneous stock-bond covariance for firm i at time t, denoted by $\sigma_{S,B,i,t} \equiv cov\left(\frac{dS_{i,t}}{S_{i,t}}, \frac{dB_{i,t}}{B_{i,t}}\right)$, satisfies*

$$\begin{aligned}
\sigma_{S,B,i,t} &= \underbrace{\Delta^S_X \Delta^B_X var\left(\frac{dX_{i,t}}{X_{i,t}}\right)}_{Asset\, risk} + \underbrace{\Delta^S_{\sigma_Y}\Delta^B_{\sigma_Y} var\left(d\sigma^2_{Y,t}\right) + \Delta^S_{\sigma_X}\Delta^B_{\sigma_X} var\left(d\sigma^2_{X,i,t}\right)}_{Variance\, risk} \\
&\quad + \underbrace{\left[\Delta^S_X \Delta^B_{\sigma_Y} + \Delta^B_X \Delta^S_{\sigma_Y}\right] cov\left(\frac{dX_{i,t}}{X_{i,t}}, d\sigma^2_{Y,t}\right)}_{Co-skewness\, risk} \qquad (17) \\
&= \underbrace{\Delta^S_X \Delta^B_X \left[\beta_i^2 \sigma^2_{Y,t} + \sigma^2_{X,i,t}\right]dt}_{Asset\, risk} + \underbrace{\Delta^S_{\sigma_Y}\Delta^B_{\sigma_Y}\delta^2_Y \sigma^2_{Y,t} dt + \Delta^S_{\sigma_X}\Delta^B_{\sigma_X}\delta^2_X \sigma^2_{X,i,t} dt}_{Variance\, risk} \\
&\quad + \underbrace{\left[\Delta^S_X \Delta^B_{\sigma_Y} + \Delta^B_X \Delta^S_{\sigma_Y}\right] \rho_Y \beta_i \delta_Y \sigma^2_{Y,t}}_{Co-skewness\, risk} dt. \qquad (18)
\end{aligned}$$

Proof. Multiplying Equations (15) and (16) gives the required result. □

Proposition 1 is insightful to understand how the various sources of shocks drive the stock-bond covariance, which can be decomposed into three components: asset risk (first term), variance risk (second term), and co-skewness risk (third term). We now discuss each of these components.

The first component in Equation (17) and (18) reflects the role of asset risk, which is the product of bond and stock return sensitivities to changes in firm assets, Δ^S_X and Δ^B_X, and total asset variance, $var\left(\frac{dX_{i,t}}{X_{i,t}}\right)$. Several key predictions arise from this component. First, since bond and stock prices increase with the value of firm assets, stock and bond return sensitivities are positive, i.e., $\Delta^S_X > 0$ and $\Delta^B_X > 0$. Asset variance is also strictly positive, so the asset risk component of stock-bond covariance is positive. Second, stock and bond returns become more sensitive to fluctuations in firm assets when

[34] We purposely omit the dependence of equity and bond sensitivities to firm i and time t for ease of exposition.

leverage is high, which implies that Δ_X^S and Δ_X^B increase with a firm's default risk and so does the covariance between its stock and bond returns. Third, the strength of the link between the stock-bond covariance and default risk is intrinsically determined by the level of a firm's total asset variance, $var\left(\frac{dX_{i,t}}{X_{i,t}}\right) = \beta_i^2\sigma_{Y,t}^2 + \sigma_{X,i,t}^2$.[35] Observe that stock-bond covariance is expected to vary in the cross-section due to firm heterogeneity in asset risk, induced by different bond and stock return exposures (Δ_X^S and Δ_X^B), asset betas (β_i), and idiosyncratic asset variances ($\sigma_{X,i,t}^2$) across firms.

Variance risk, the second term in Equations (17) and (18), captures the role of fluctuations in aggregate and idiosyncratic asset variance on the stock-bond covariance. An increase in (aggregate or idiosyncratic) asset variance decreases the stock-bond covariance, and the intuition for this result is as follows. Equity can be interpreted as a call option on the firm's assets, such that equity valuation increases with a firm's asset variance (i.e., $\Delta_{\sigma_Y}^S > 0$ and $\Delta_{\sigma_X}^S > 0$). In contrast, an increase in asset variance increases the firm's default probability and reduces the value of its bonds, so $\Delta_{\sigma_Y}^B < 0$ and $\Delta_{\sigma_X}^B < 0$. As a result, a change in asset variance affects stock and bond valuation in opposite directions, as we have $\Delta_{\sigma_Y}^S\Delta_{\sigma_Y}^B < 0$ and $\Delta_{\sigma_X}^S\Delta_{\sigma_X}^B < 0$. The variance risk component of stock-bond covariance is thus negative.

The third term in Equations (17) and (18) corresponds to the impact of co-skewness risk on the stock-bond covariance, which arises when aggregate variance co-moves with the level of aggregate assets (i.e., $\rho_Y \neq 0$).[36] Although the sign of this term is a priori unclear, we find it to be positive: Based on our model calibration (see Section 3.5), we find that $\Delta_X^S\Delta_{\sigma_Y}^B < 0$ quantitatively dominates $\Delta_X^B\Delta_{\sigma_Y}^S > 0$ because of the negligible economic magnitude of $\Delta_{\sigma_Y}^S$, so $\left[\Delta_X^S\Delta_{\sigma_Y}^B + \Delta_X^B\Delta_{\sigma_Y}^S\right] < 0$. In addition, we have $\rho_Y < 0$ in the data given the negative skewness in the aggregate risk distribution (i.e., asset variance goes up as aggregate assets drop in value). Overall, the impact of co-skewness risk on the stock-bond covariance is positive and strengthens when firms are closer to default, as both Δ_X^S and $\Delta_{\sigma_Y}^B$ increase in absolute value. By showing that co-skewness risk can amplify the positive comovement between stock and bond returns, our model extends the existing literature, which has thus far primarily focused on the pricing of co-skewness for stock returns.

[35] Note that the linear factor structure in asset return and variance translates into a non-linear factor structure in the stock-bond covariance. This result arises from the non-linear exposures of bond and equity to asset value and stochastic volatilities.

[36] Although the exact definition of co-skewness risk varies in the literature, the essence of co-skewness is to capture the magnitude of the covariance of a financial asset return with squared-factor returns, or the factor variance.

In summary, the stock-bond covariance can be decomposed into three distinct components. The first term, which we call asset risk, generates a positive stock-bond covariance because bond and stock prices jointly vary with changes in firm assets. The second and third terms account for additional impacts of higher-order risk on the stock-bond covariance, which corresponds to the sum of variance risk and co-skewness risk. Variance risk dampens the positive stock-bond comovement because fluctuations in asset volatility affect bond and stock prices in opposite directions. In contrast, co-skewness risk increases the stock-bond covariance because higher asset variance increases the level of the risk premium, which jointly reduces stock and bond prices. This decomposition of the stock-bond covariance is useful to understand the sources of the cross-sectional and time-series fluctuations, which are ultimately linked to default risk.

3.4 Stock-bond correlation

Following our empirical analysis, we consider correlation as a second measure of comovement between stock and bond returns. Proposition 2 presents the implications of the model for the stock-bond correlation.

Proposition 2: *Given the dynamics of firm i's stock and bond returns in Equations (15) and (16) and firm i's covariance (17), stock and bond instantaneous variance and correlation are given by*

$$\begin{aligned}\sigma^2_{S,i,t} =& \left(\Delta^S_X\right)^2 var\left(\frac{dX_{i,t}}{X_{i,t}}\right) + \left(\Delta^S_{\sigma_Y}\right)^2 var\left(d\sigma^2_{Y,t}\right) + \left(\Delta^S_{\sigma_X}\right)^2 var\left(d\sigma^2_{X,i,t}\right) \\ &+ 2\left[\Delta^S_X \Delta^S_{\sigma_Y}\right] cov\left(\frac{dX_{i,t}}{X_{i,t}}, d\sigma^2_{Y,t}\right)\end{aligned} \tag{19}$$

$$\begin{aligned}\sigma^2_{B,i,t} =& \left(\Delta^B_X\right)^2 var\left(\frac{dX_{i,t}}{X_{i,t}}\right) + \left(\Delta^B_{\sigma_Y}\right)^2 var\left(d\sigma^2_{Y,t}\right) + \left(\Delta^B_{\sigma_X}\right)^2 var\left(d\sigma^2_{X,i,t}\right) \\ &+ 2\left[\Delta^B_X \Delta^B_{\sigma_Y}\right] cov\left(\frac{dX_{i,t}}{X_{i,t}}, d\sigma^2_{Y,t}\right)\end{aligned} \tag{20}$$

$$\rho_{S,B,i,t} = \frac{\sigma_{S,B,i,t}}{\sigma_{S,i,t}\sigma_{B,i,t}}, \tag{21}$$

where $\sigma^2_{S,i,t} \equiv var\left(\frac{dS_{i,t}}{S_{i,t}}\right)$ is the stock return variance, $\sigma^2_{B,i,t} \equiv var\left(\frac{dB_{i,t}}{B_{i,t}}\right)$ is the bond return variance, $\sigma_{S,B,i,t}$ is the stock-bond covariance, and $\rho_{S,B,i,t}$ denotes the stock-bond correlation.

Proof. Taking the quadratic variation of Equations (15) and (16) gives Equations (19) and (20), while Equation (21) directly follows from the definition of correlation. □

Proposition 2 is particularly useful to understand the role of stochastic variance in shaping the comovement between stock and bond returns. In traditional capital structure models based on Merton (1974), Leland (1994), or Goldstein et al. (2001), which assume constant asset volatility, the stock-bond covariance boils down to the first term in Equations (17) and (18). That is, asset risk becomes the only driver of the stock-bond covariance, as the second and third terms vanish when there is no uncertainty about aggregate and idiosyncratic asset variance (i.e., $\delta_Y = \delta_X = 0$). In such a case, stocks and bonds perfectly comove together. To see this, Proposition 2 shows that stock and bond return volatilities are equal to $\sigma_{S,i} = \Delta_X^S \sqrt{\beta_i^2 \sigma_Y^2 + \sigma_{X,i}^2}$ and $\sigma_{B,i} = \Delta_X^B \sqrt{\beta_i^2 \sigma_Y^2 + \sigma_{X,i}^2}$ in the case of constant asset volatility, while the stock-bond covariance becomes $\sigma_{S,B,i} = \Delta_X^S \Delta_X^B (\beta_i^2 \sigma_Y^2 + \sigma_{X,i}^2)$. Stock and bond returns are then perfectly correlated, as we have $\rho_{S,B,i} = \sigma_{S,B,i} / (\sigma_{S,i} \sigma_{B,i}) = 1$. This analysis highlights the importance of introducing stochastic asset variance for breaking down the perfect correlation between stocks and bonds implied by standard credit risk models.

Based on the model, we explore how default risk drives the stock-bond covariance and correlation, both across firms and over time. We first present our estimation strategy and discuss the model fit. We then show that the predictions of the model based on simulations replicate the empirical findings documented in Section 1. Finally, we use the estimated model to quantify the relative importance of the different covariance components, which will provide valuable insights on the economic forces driving the comovement between stocks and bonds.

3.5 Simulation-based calibration

Here, we describe the calibration of the model, which features 17 structural parameters and 3 latent variables. The starting point of our calibration strategy consists of reducing the dimensionality of the problem by fixing the values of some parameters. To this end, we follow Du et al. (2019), Feldhütter and Schaefer (2018), and Bai et al. (2020), among others, and set the parameters driving firm distress costs, corporate taxes, and asset payout ratio to $\alpha = 40\%$, $\zeta = 20\%$, and $q = 0\%$, respectively. We normalize the initial asset value $X_{i,0}$ to \$1 and set the risk-free rate $r = 1.01\%$, which corresponds to the long-run mean of a square-root process fitted to the 3-month Treasury Bill rate between August 2003 to August 2020. This leaves us with 11 structural parameters to estimate, $\Theta \equiv \{\Psi_Y, \Psi_X, X_B, c\}$ with $\Psi_Y \equiv \{\kappa_Y, \theta_Y, \delta_Y, \rho_Y, \lambda_{Y\perp\sigma_Y}, \lambda_{\sigma_Y}\}$ and $\Psi_X \equiv \{\kappa_X, \theta_X, \delta_X\}$.

We estimate the parameter vector Θ using 9 moment conditions capturing various dimensions of a firm's credit, equity, and bond risk. More precisely, we consider market leverage (1 moment), stock and bond total volatilities (2 moments), proportions of stock and bond systematic variance to total variance (2 moments), volatilities of aggregate bond and stock indices (2 moments), credit default swap (CDS) spread (1 moment), and physical default probability (1 moment). We purposely do not consider moments related to the dependence between stocks and bonds, such as the covariance or correlation. Our goal is to explore whether a rich credit risk model fitted to the 9 moments described above is able to match the joint dynamics of stocks and bonds out-of-sample.

We compute the empirical target moments as pooled sample averages from the merged corporate bond and equity dataset used in Section 1. There are two exceptions: i) the physical default probability, which corresponds to the 10-year historical default rate of BB-rated firms for the 1981-2020 period from Standard and Poor's (2021); and ii) the 10-year CDS spread, which we extract from Markit for BB-rated firms over the same period as the other variables.

For the model-implied moments, we rely on the following simulation strategy. We first create an economy of firms that differ in their factor structure. For parsimony, we assume that firms can take on 5 values (0.6, 0.8, 1, 1.2, 1.4) of asset beta β_i with equal probability. We then simulate an economy of 1,250 firms over 10 years, which results in 250 firms on average by asset beta. We repeat this exercise 10 times to reduce the impact of a particular simulated path of the aggregate asset dynamics on the results. For each simulated path of the state variables, we compute firms' leverage, CDS spread, stock (bond) total volatility, proportion of stock (bond) systematic variance, and aggregate stock (bond) index volatility implied by the model. We then calculate the pooled average of each model-implied moment m across firms, time, and simulations. See the Online Appendix G for details about the simulation procedure and the computation of model-implied moments.

Armed with the set of moment conditions, we estimate the vector of parameters Θ by solving the following optimization problem:

$$\hat{\Theta} = \arg\min \sum_{m=1}^{9} \left(Data^m - Model(\Theta)^m \right)^2 , \tag{22}$$

where $Data^m$ and $Model(\Theta)^m$ denote the empirical and model-implied moment condition m, respectively. Table 10 summarizes the estimated values of the model parameters $\hat{\Theta}$.

Table 10 [about here]

3.6 Assessment of the model fit

We now discuss the model's goodness-of-fit. Table 11 compares the empirical and model-implied moments and reports their descriptive statistics. Panel A presents the set of moments used in the estimation, which are referred to as in-sample moments, while Panel B presents a set of out-of-sample moments, which have not been used in the estimation. It is worth noting that the model parameters are only estimated based on the moment conditions identified in bold in Panel A, i.e., not their standard deviations which can be viewed as additional out-of-sample moments. Comparing the first and second columns of Panel A in Table 11, we can see that the simulated economy provides a good fit to what we observed for U.S. firms over the last two decades.

Table 11 [about here]

Specifically, the model generates leverage and default risk levels that closely match their empirical counterparts. Leverage is 50.8% in the model while it is 49.9% in the data. The 10-year CDS spread is 251 bps in the simulated economy and 236 bps in the data. In terms of physical default risk, the 10-year cumulative default rate is 14.1% in the data and 12.7% in the simulated economy. The model addresses the well-known credit spread puzzle, which arises from the difficulty in generating high credit spreads with reasonable levels of physical default probability and leverage. In addition, the model-implied total volatilities, proportions of systematic variances, and index volatilities for bonds and stocks are also close to their empirical counterparts. For example, the average stock and bond total volatility is 32.9% and 9.2% in the data while the simulation generates values of 32.6% and 9.1%, respectively. Historically, the proportion of systematic variance is 28.5% for stocks and 38.1% for bonds, which is of the same order of magnitude as in the simulations (27.9% and 36.8%).

The last three columns of Table 11 provide further support for the model fit. Not only is the simulated economy able to match the unconditional moments observed in the data, it is also able to generate reasonable cross-sectional and time-series variation in these moments. Except for leverage and bond return volatility, the (pooled) standard deviations of the simulated moments are generally close to those of the data. Recall that these standard deviations are out-of-sample as they are not part of the model calibration. The cross-sectional variation in idiosyncratic asset variance plays a key role in generating the cross-sectional variability in the moments, whereas the aggregate (asset and variance) shocks help generate sizable time-series variation.

Panel B of Table 11 reports the results for additional out-of-sample moments: asset volatility, distance-to-default, and the measures of stock-bond comovement. Our choice to study asset volatility and distance-to-default as additional moments is guided by the difficulty of standard credit risk and asset pricing models in matching these dimensions, while predicting reasonable levels of equity volatility, firm leverage, and CDS spread. The model calibration generates a level of asset volatility and distance-to-default (14.9% and 5.9) that is similar to the data (15.5% and 6.5). These results confirm that the high CDS spreads and stock and bond volatilities reported in Panel A do not come at the cost of unreasonably high asset volatility or low distance-to-default.

Stock-bond correlation puzzle

There is one dimension, however, that the model fails in replicating: the *level* of comovement between stock and bond returns. From Panel B of Table 11 we see that the level of stock-bond covariance and correlation is much higher in the simulations (about 2.5 times higher) than in the data. The size of the discrepancy is particularly striking given that the model is able to fit a large set of asset pricing moments and firm-level characteristics. The difference in covariance estimates is likely due to the model's difficulty in generating a sufficiently low level of correlation between stock and bond returns, given its ability to closely match empirical stock and bond volatilities. Confirming this intuition, the level of correlation is 0.20 in the data while it is 0.79 in the simulations. Surprisingly and despite this shortcoming, the model can generate sizable fluctuations in stock-bond covariance and correlation, as indicated by the last three columns of Panel B.

One potential avenue to reduce the stock-bond correlation is to introduce stochastic interest rates in the model. An increase in the risk-free rate reduces the discounted value of future coupons, which decreases the value of the bond. This in turn increases the value of the firm's stock for a given level of assets.[37] As a result, bond and stock valuations react in opposite directions to a change in the risk-free rate. While economically intuitive, it remains an open question as to how much this mechanism can help quantitatively reduce the level of correlation. To address this question, we augment the model with stochastic interest rates. This alternative specification and its calibration are discussed in detail in Online Appendix H. Panel A of Table A.6 shows that accounting for variation in interest rates reduces the average stock-bond correlation from 0.79 to 0.64. Thus, even with stochastic interest rates, the model-implied correlation remains much higher than in the data.

Overall, our model with stochastic systematic and idiosyncratic volatilities can generate substantial variation in stock-bond comovement across firms and time, while addressing the credit spread puzzle and matching a large set of empirical moments. However, it fails in fully breaking down the tight relation between stocks and bonds. This result indicates a fundamental tension in the existing theory between matching key dimensions of credit, equity, and bond risk and simultaneously generating a reasonable level of stock-bond correlation. We refer to this finding as to the *stock-bond correlation puzzle*. Various model extensions could be considered to reduce the strong link between stocks and bonds. For instance, allowing for frictions that would introduce asset-specific shocks in the model may help decrease stock-bond correlation.[38] Given our focus on studying how stock-bond comovement varies with default risk, solving the *stock-bond correlation puzzle* is beyond the scope of this paper.

3.7 Rationalizing the predictability of stock-bond comovement with default risk

We turn to the central part of our exercise, which is exploring the model's ability to reproduce the predictive positive relation between stock-bond comovement and default risk uncovered empirically in Section 1. For a given simulation, we construct the model-implied composite default risk at the firm level as the average of the following (standardized) variables: leverage, sign-corrected distance-to-default, and CDS spread. Using simulated data, we then regress stock-bond covariance and correlation against

[37] We can abstract away from the negligible effect of risk-free rate on the discounted tax shield and bankruptcy costs.

[38] Additional extensions include incorporating bond- and equity-specific liquidity shocks, demand/supply effects, or measurement error in returns through the modeling of bid-ask spreads.

this composite default risk variable.[39] The specifications we consider consist of the baseline panel regression with firm fixed effects (2), which allows us to examine the time-series predictability, and the Fama-MacBeth approach to analyze the cross-section. Panel B of Table 4 presents the results, which can be easily compared with the empirical estimates (Panel A).

Various results emerge from our analysis. First, the predictive regression coefficients for default risk in the simulations have the correct sign and are highly statistically significant. The predictability is robust across specifications, as in Panel A. That is, the model can rationalize the main empirical finding that the one-year-ahead stock-bond comovement increases with default risk, both over time and across firms. In addition, the impact of default risk on the stock-bond comovement in the simulations is of the same order of magnitude, albeit a bit lower than in the data. Note that the model exhibits more difficulty in matching the relation between default risk and stock-bond comovement in the cross-section than in the time-series.[40] This is apparent from the regression coefficients and R^2, which are both of smaller magnitude in Columns (2) and (4) of Panel B compared to in Panel A. Unsurprisingly, this pattern is more pronounced for the correlation than the covariance. Despite this shortcoming, we can conclude that the model is successful in achieving one of the key objectives of our simulation exercise: to show that default risk positively predicts stock-bond comovement, both in the cross-section and over time. Furthermore, this result continues to hold when we account for stochastic interest rates in the model simulations (see Panel B of Table A.6).

3.8 Understanding stock-bond comovement by default risk

Our model is also useful for studying how the different sources of risk drive the relation between stock-bond comovement and default risk. To this end, we exploit the simulations to compute the model-implied covariance and its components for each firm and month, using the covariance decomposition in Proposition 1. We first sort firms into quintiles based on their (composite) default risk and then calculate the value-weighted average covariance for each quintile.[41] Quintile 1 (Q1) is the portfolio

[39] For the simulated economy, we compute statistics as the averages across 10 simulations. The composite default risk variable is based on 10-year CDS spreads, matching the maturity of bond credit spreads which is about 10 years in our sample.

[40] Extending the model to allow some of the idiosyncratic variance parameters or the default barrier, for example, to vary across firms may help generating additional cross-sectional variation in default risk and stock-bond comovement.

[41] The results are qualitatively similar using any of the individual default risk variables.

with the lowest default risk, while Quintile 5 (Q5) is the portfolio with the highest default risk. Panel A of Table 12 presents results for the levels of covariance and its components, while Panel B presents the percentage contributions of asset risk vs. higher-order risk (obtained by combining variance risk and co-skewness risk).

Table 12 [about here]

First note that the level of comovement between corporate bond and stock returns monotonically increases with default risk, consistent with the regression results. Also, the stock-bond covariance is more sensitive to a change in default risk for firms closer to financial distress, as the increase in the covariance is greater from Q4 to Q5 than from Q1 to Q2. These results illustrate the ability of our model to capture the positive and asymmetric relation between stock-bond covariance and default risk uncovered empirically (see Figure 2).

We now provide new insights on the economic forces shaping the link between stock-bond comovement and default risk. We find that stock-bond covariance is primarily driven by asset risk across default risk quintiles and explains at least 93.8% of the total level. Variance risk and co-skewness risk jointly explain at most 6.2% of the covariance level. Recall that the variance risk component is negative, given that a change in asset variance affects equity and bond valuation in opposite directions. The positive contribution of higher-order risk is thus mainly driven by co-skewness risk, which captures how firm assets comove with aggregate variance. The high level of stock-bond comovement implied by the model is therefore driven primarily by asset risk and then by co-skewness risk. While the variance risk component is critical for breaking the perfect correlation between stocks and bonds, its contribution to the covariance remains modest, on average.

The role of the asset and higher-order risk in the stock-bond comovement displays pronounced variation across default risk portfolios. In Panel A of Table 12, both components of the stock-bond covariance increase in absolute terms with default risk. That is, both sources of risk help generating the positive relation between stock-bond comovement and default risk. In relative terms, however, Panel B shows that the percentage contributions of asset and higher-order risk behave in opposite directions: the lower the default risk, the higher the relative contribution of higher-order risk in explaining the level of stock-bond comovement.[42]

To summarize, introducing stochastic asset variance in a credit risk model allows for a rich decomposition of the stock-bond comovement. We find that asset risk constitutes the central economic force driving stock-bond covariance, although higher-order risk becomes a non-negligible contributor for highly creditworthy firms. That is, the lower a firm's default risk, the more it becomes relevant to depart from a Merton (1974)-type model with constant asset variance for understanding stock-bond comovement. This result echoes the finding that stochastic asset variance plays a critical role in resolving the credit spread puzzle, which is predominant among investment-grade firms (Du et al., 2019).

4 Conclusion

The paper shows that default risk is a critical driver of the comovement between stock and corporate bond returns, both empirically and theoretically. We start by documenting a series of new empirical stylized facts. First, we show that default risk robustly predicts the one-year-ahead stock-bond covariance as well as the correlation. The intuition is that corporate bonds issued by firms with a greater level of default risk behave more like the issuing firms' stock, thereby increasing their future comovement. The predictive power of default risk is both statistically and economically meaningful, and survives an extensive list of control variables informative about firm characteristics, economic and financial conditions, intermediary capital risk, and liquidity. We then show that these results have important asset pricing implications, as illustrated by a conditional portfolio analysis. When investing in a balanced portfolio of stocks and corporate bonds, selecting the most creditworthy firms significantly enhances the out-of-sample diversification gain and Sharpe ratio. This result is directly relevant to mutual and hedge funds pursuing a multi-asset investment strategy.

We propose a model that helps rationalize these empirical findings. Specifically, we extend existing Merton-type models to have the dynamics of firms' assets impacted by a common factor and firm-specific risk, both of which have stochastic variance. Simulating a large cross-section of firms, we first show that the model is able to fit various stock and bond pricing moments, as well as financial

[42] To understand this result, recall that higher-order risk is almost entirely driven by co-skewness risk. The model predicts that the negative skewness of systematic risk impacts firms that are further away from default than firms closer to their default boundary.

leverage, default risk, and CDS spreads. Moreover, the model reproduces the empirical predictability of stock-bond comovement with default risk, both across firms and over time. We then use the model to decompose the economic forces driving stock-bond comovement by isolating the roles of asset, variance, and co-skewness risks. We find that asset risk is by far the primary driver of stock-bond comovement, although the role of the variance and co-skewness risks increases with firm creditworthiness.

While the model matches key asset pricing moments and addresses the credit spread puzzle, it faces difficulty in generating the low level of unconditional stock-bond correlation observed in the data. We label this model shortcoming the 'stock-bond correlation puzzle', which refers to the fact that a no-arbitrage contingent claim model with multiple sources of risk is unable to produce a sufficiently low level of stock-bond correlation. Potential extensions that could help address this puzzle include the introduction of alternative asset-specific shocks, financial constraints, information asymmetry, or transaction costs. Arguably, such extensions could help reduce the tight link between corporate bond and stock valuation but they are beyond the scope of this paper. Exploring such mechanisms constitutes, however, a fruitful avenue for future research.

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Table 1 : Definitions of variables and data sources This table defines the variables used in this study, describes their calculations, and identifies their sources. Additional definitions of the variables and their data sources are presented in Section 1.1 and in Online Appendix A and B.

Variable	Definition	Source
Panel A: Firm-level variables		
Leverage	Market leverage computed as the ratio between total book debt (LT) and the market value of equity (monthly closing values of stock prices (PRC) multiplied by the stocks shares outstanding (SHROUT)) plus total book debt (LT).	CRSP/COMPUSTAT
Distance-to-Default	Log of the distance between firm assets and the default threshold (equal to one-half of long-term debt value plus short-term debt value from COMPUSTAT) divided by asset volatility, following Campbell and Thompson (2008). As in Choi and Richardson (2016), asset volatility is the rolling 12-month standard deviation of asset returns, given by $r_{A,i,t} = L_{i,t} \times r_{B,i,t} + (1 - L_{i,t}) \times r_{S,i,t}$, where $r_{B,i,t}$ and $r_{S,i,t}$ are stock and bond returns for firm i in month t, while $L_{i,t}$ denotes the firm's leverage.	CRSP/COMPUSTAT
Default Risk	Sign-corrected average of the three standardized individual default risk variables, which are Leverage, Distance-to-Default (sign-corrected), and Credit Spread (defined in Panel B).	CRSP/COMPUSTAT/Bond databases
Amihud Ratio	Amihud (2002)'s ratio computed as the the sum of the daily absolute returns divided by the sum of dollar trading volume, averaged across all days in a month.	CRSP
Market Capitalization	Stock price (PRC) multiplied by the amount of shares outstanding (SHROUT).	CRSP/COMPUSTAT
Market-to-Book	Ratio of Market Capitalization to Book Value of Equity.	CRSP/COMPUSTAT
CDS Spread	Spread of Credit Default Swap (CDS) contracts with 10-year maturity, expressed in basis points.	Markit
Panel B: Bond-level variables		
Bond Coupon	Corporate bond coupon rate.	Mergeant FISD
Bond Callability	Dummy variable equal to 1 if a corporate bond has an embedded call option, zero otherwise.	
Time-to-Maturity	Corporate bond time-to-maturity, expressed in months.	Mergeant FISD
Amount Outstanding	Initial offering amount of a corporate bond, adjusted for units of the offering which have been called.	Mergeant FISD
Bond Liquidity	Average corporate bond bid-ask spread.	Bond databases
Bond Credit Ratings	Standard & Poor's corporate bond-level rating.	Mergeant FISD
Credit Spread	Difference between the yield of a corporate bond and the associated yield of the Treasury curve at the same maturity. Using the Benchmark Treasury rates for maturities of 3, 5, 7, 10, and 30 years, we use linear interpolation to estimate the entire yield curve, following Duffee (1998) and Collin-Dufresne et al. (2001). Expressed in basis points.	CRSP/Bond databases, Datastream
Panel C: Stock-bond comovement variables		
Covariance/Correlation	Rolling 12-month covariance/correlation computed between firm-level corporate bond returns (a firm-level bond return is the equally-weighted average of all of the firm's outstanding corporate bond returns) and the corresponding issuing firm's stock returns, using monthly observations. Covariances are annualized, except in the regression tables, and multiplied by 1,000.	Bond databases
aDCC Covariance/Correlation	Covariance/correlation between corporate bond returns and the corresponding issuing firm's stock returns estimated with the asymmetric dynamic conditional correlation (aDCC) model, using monthly observations. Covariances are annualized, except in the regression tables, and multiplied by 1,000.	Bond databases
Panel D: Aggregate control variables		
Liquidity Risk Factor	Traded-version of the Pastor and Stambaugh (2003) liquidity risk factor.	WRDS
Aggregate Liquidity Factor	Aggregate liquidity factor of Hu et al. (2013).	Jun Pan's website
Intermediary Capital Risk Factor	Intermediary capital risk factor of He et al. (2017).	Asaf Manela's website
Term Structure	10-year U.S. Treasury rate, 3-month U.S. Treasury Bill secondary market rate, and their difference (slope).	WRDS
Macroeconomic Risk	Macro Uncertainty Index of Jurado et al. (2015) and Business Conditions Index of Aruoba et al. (2009).	Sydney Ludvigson's website, Philadelphia FED
Expected Inflation	One-quarter-ahead mean inflation forecast from the Survey of Professional Forecasters.	Philadelphia FED

Table 2 : Descriptive statistics This table presents statistics for the stock-bond comovement and default risk variables. Panel A reports their key descriptive statistics, while Panel B reports their pairwise correlations. The covariance (scaled by 1,000) and correlation between corporate bond and stock returns are computed over the following 12 months. The Fisher Correlation is given by $\rho^F_{S,B,i,t+12} = \frac{1}{2}ln(\frac{1+\rho_{S,B,i,t+12}}{1-\rho_{S,B,i,t+12}})$, where $\rho_{S,B,i,t+12}$ is the stock-bond correlation for firm i computed over the period $t+1$ to $t+12$. Leverage is defined as the ratio between total book debt and the sum of total book debt and the market value of equity. Distance-to-default (DD) is the log of the distance between a firm's assets and the default threshold (equal to one-half of long-term debt value plus short-term debt value) divided by asset volatility. Bond credit spreads are computed as the difference between maturity-matched corporate bond yields and the associated U.S. Treasury Bond yield. Default Risk is the sign-corrected average of the above three standardized default risk variables. The sample period spans August 2003 - August 2020. All variables are annualized when applicable. The definitions of the variables and their data sources are presented in Section 1.1 and in Online Appendix A and B.

Panel A: Descriptive statistics

					Percentiles					
	Obs.	Mean	Median	Std	1	5	25	75	95	99
Covariance	117,822	10.78	1.88	28.97	-11.02	-4.58	-0.37	7.84	60.60	162.05
Correlation	117,822	0.20	0.22	0.36	-0.64	-0.43	-0.05	0.49	0.76	0.87
Fisher Correlation	117,822	0.24	0.23	0.44	-0.76	-0.46	-0.05	0.53	1.00	1.33
Leverage	117,822	49.90	48.07	21.53	10.79	17.94	32.73	65.46	88.45	96.97
DD	117,822	14.75	12.57	10.29	0.89	2.69	7.70	19.09	34.64	54.00
Credit Spread [bps]	117,822	287.69	193.61	296.16	38.14	59.84	112.28	359.34	773.59	1625.59
Default Risk	117,822	0.00	-0.11	1.00	-1.90	-1.40	-0.66	0.53	1.74	3.16

Panel B: Pairwise correlations

	Covariance	Correlation	Leverage	DD	Credit Spread	Default Risk
Covariance	1	0.50	0.24	-0.27	0.46	0.43
Correlation		1	0.14	-0.26	0.24	0.27
Leverage			1	-0.18	0.43	0.50
DD				1	-0.47	-0.81
Credit Spread					1	0.82
Default Risk						1

Table 3 : Predictability of stock-bond comovement with individual default risk measures This table presents results on the predictive relation between stock-bond comovement and various proxies of firm-level default risk, based on panel regressions. The dependent variable is the one-year-ahead covariance (Panel A) and correlation (Panel B) between stock and corporate bond returns. All regressors are mean-variance standardized. We include firm fixed effects and report t-statistics in parentheses, using standard errors (SE) double clustered at the month and firm levels. The sample period spans August 2003 - August 2020. The definitions of the variables and their data sources are presented in Section 1.1 and in Online Appendix A and B. Significance at the 10%, 5%, 1% level is indicated by *, **, ***, respectively.

	Panel A: Covariance			*Panel B: Correlation*		
	(1)	(2)	(3)	(4)	(5)	(6)
Leverage	0.96***			0.07***		
t-stat	(10.03)			(5.72)		
Distance-to-default		0.31***			0.03***	
t-stat		(7.12)			(3.37)	
Credit Spread			0.70***			0.04***
t-stat			(8.21)			(4.88)
$R^2_{Adj.}$	0.359	0.336	0.370	0.280	0.287	0.279
Obs.	117,822	117,822	117,822	117,822	117,822	117,822
SE	Month & Firm	Month & Firm	Month & Firm	Month & Firm	Month & Firm	Month & Firm
Fixed Effects	Firm	Firm	Firm	Firm	Firm	Firm
Controls	None	None	None	None	None	None

Table 4 : Predictability of stock-bond comovement with default risk – data vs. model This table presents results on the predictive relation between stock-bond comovement and a composite measure of default risk. Columns (1) and (3) report the baseline results with firm fixed effects. Columns (2) and (4) present cross-sectional results using the Fama-MacBeth approach. The dependent variable is the one-year-ahead covariance (Columns 1 and 2) and correlation (Columns 3 and 4) between stock and corporate bond returns. Panel A presents results for the data. The 'Default Risk' variable is constructed as the sign-corrected average of the three standardized default risk proxies, namely Leverage, Distance-to-Default, and Credit Spread. Panel B presents results using simulated economies based on the model. Model-based 'Default Risk' is constructed in a similar way to the empirical measure, but with the CDS spread replacing the bond credit spread. Regressions are performed on each simulated economy comprising 1,250 firms over a 10-year period. The results correspond to the average of each statistic across 10 simulated economies. We report t-statistics in parentheses, using standard errors (SE) double clustered at the month and firm levels for the baseline panel regressions. Newey-West corrected standard errors are used for the Fama-MacBeth procedure with 12 lags. The sample period spans August 2003 - August 2020. The definitions of the variables and their data sources are presented in Section 1.1 and in Online Appendix A and B. Significance at the 10%, 5%, 1% level is indicated by *, **, ***, respectively.

Panel A: Data

	Covariance		**Correlation**	
	Baseline (1)	Fama-MacBeth (2)	Baseline (3)	Fama-MacBeth (4)
Default Risk	0.77***	1.08***	0.04***	0.16***
t-stat	(9.86)	(8.44)	(4.88)	(10.64)
$R^2_{Adj.}$	0.240	0.311	0.279	0.126
Obs.	117,822	117,822	117,822	117,822
SE	Month & Firm	Newey- & West	Month & Firm	Newey- & West
Fixed Effects	Firm	None	Firm	None
Controls	None	None	None	None

Panel B: Model

	Covariance		**Correlation**	
	Baseline (1)	Fama-MacBeth (2)	Baseline (3)	Fama-MacBeth (4)
Default Risk	0.30***	0.58***	0.03***	0.05***
t-stat	(8.90)	(25.61)	(6.55)	(4.66)
$R^2_{Adj.}$	0.311	0.087	0.363	0.029
Obs.	133,255	133,255	133,255	133,255
SE	Month & Firm	Newey- & West	Month & Firm	Newey- & West
Fixed Effects	Firm	None	Firm	None
Controls	None	None	None	None

Table 5 : Predictability of stock-bond comovement with default risk – including controls This table presents results on the predictive relation between stock-bond comovement and default risk, controlling for alternative explanations. The dependent variable is the one-year-ahead covariance (Panel A) and correlation (Panel B) of stock and corporate bond returns. Column (1) controls for corporate bond characteristics, which include a Callability dummy, Coupon, Time-to-Maturity, Bond Size (log of amount outstanding), and Bond Illiquidity (bid-ask spread). Column (2) controls for stock characteristics, which include Amihud (2002)'s Illiquidity Ratio, Market-to-Book, and Equity Size (log of market capitalization). Column (3) controls for global liquidity factors, which include the Pastor and Stambaugh (2003)'s Liquidity Risk Factor and Hu et al. (2013)'s Aggregate Liquidity Factor. Column (4) controls for the Intermediary Capital Risk Factor of He et al. (2017). Column (5) controls for risk-free interest rates, which include the 10-year U.S. Treasury rate, 3-month U.S. Treasury Bill secondary market rate, and their difference (term structure slope). Column (6) controls for Expected Inflation, measured as the one-quarter-ahead mean inflation forecast from the Survey of Professional Forecasters. Column (7) controls for the Macroeconomic Uncertainty Index of Jurado et al. (2015) and the Business Conditions Index of Aruoba et al. (2009). Column (8) controls for all bond/stock features and global variables from Columns (1)-(7). The 'Default Risk' variable is constructed as the sign-corrected average of the three standardized default risk proxies, namely Leverage, Distance-to-Default, and Credit Spread. We include firm fixed effects and report t-statistics in parentheses, using standard errors (SE) double clustered at the month and firm levels. The sample period spans August 2003 - August 2020. The definitions of the variables and their data sources are presented in Section 1.1 and in Online Appendix A and B. Significance at the 10%, 5%, 1% level is indicated by *, **, ***, respectively.

Panel A: Covariance

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Default Risk	0.77***	0.80***	0.68***	0.79***	0.94***	0.72***	0.60***	0.67***
t-stat	(9.74)	(9.08)	(7.76)	(9.59)	(11.61)	(7.97)	(7.23)	(9.62)
$R^2_{Adj.}$	0.373	0.367	0.371	0.366	0.379	0.367	0.380	0.414
Obs.	114,825	117,748	117,822	117,822	117,822	117,822	117,822	114,755
SE	Month & Firm	Month & Firm	Month & Firm	Month & Firm	Month & Firm	Month & Firm	Month & Firm	Month & Firm
Fixed Effects	Firm	Firm	Firm	Firm	Firm	Firm	Firm	Firm
Controls	Bond Charact.	Stock Charact.	Liquidity Risk	Intermediary Capital Risk	Interest Rates	Expected Inflation	Macro Conditions	All

Panel B: Correlation

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Default Risk	0.05***	0.09***	0.05***	0.05***	0.07***	0.05***	0.04***	0.08***
t-stat	(5.48)	(6.93)	(5.14)	(5.84)	(7.21)	(4.88)	(4.18)	(6.52)
$R^2_{Adj.}$	0.290	0.284	0.281	0.280	0.294	0.279	0.285	0.308
Obs.	114,825	117,748	117,822	117,822	117,822	117,822	117,822	114,755
SE	Month & Firm	Month & Firm	Month & Firm	Month & Firm	Month & Firm	Month & Firm	Month & Firm	Month & Firm
Fixed Effects	Firm	Firm	Firm	Firm	Firm	Firm	Firm	Firm
Controls	Bond Charact.	Stock Charact.	Liquidity Risk	Intermediary Capital Risk	Interest Rates	Expected Inflation	Macro Conditions	All

Table 6 : Predictability of stock-bond comovement with default risk – robustness checks This table presents robustness results on the predictive relation between stock-bond comovement and default risk. The dependent variable is the one-year-ahead covariance (Panel A) and correlation (Panel B) of stock and corporate bond returns. Column (1) reproduces the specification with all controls (Column 8 of Table 5). Column (2) presents results at the corporate bond level, where covariance and correlation are computed between individual bond's returns and firm-level stock returns. Column (3) replicates Column (2) whilst excluding bonds with an embedded call option. Column (4) replicates Column (1) whilst excluding all financial and regulated utility firms (those with a SIC code between 6000–6999 and 4900–4949, respectively). Column (5) uses an extended data sample, combining various databases (Lehman Brothers/TRACE/Datastream), starting April 1987. Column (6) uses stock-bond covariance and correlation computed with an asymmetric DCC-GARCH model at the firm level. Column (7) uses the 10-year CDS spread (mean-variance standardized) as the proxy for default risk, using Markit CDS data. Column (8) controls for persistence in stock-bond comovement by introducing lagged stock-bond covariance/correlation (computed over the previous 12 months). Finally, Column (9) uses yearly non-overlapping observations. The 'Default Risk' variable is constructed, in all columns except Column (7), as the sign-corrected average of the three standardized default risk proxies, namely Leverage, Distance-to-Default, and Credit Spread. Regressions include all control variables and firm fixed effects. We report t-statistics in parentheses, using standard errors (SE) double clustered at the month and firm levels for Columns (1)-(8). In Column (9), we cluster the standard errors at the year and firm level. The sample period spans August 2003 - August 2020 for all columns except Column (5), which spans April 1987 - August 2020. The definitions of the variables and their data sources are presented in Section 1.1 and in Online Appendix A and B. Significance at the 10%, 5%, 1% level is indicated by *, **, ***, respectively.

Panel A: Covariance

	Baseline (1)	Bond level (2)	Exclude call options (3)	Exclude fin./util. (4)	Extended sample (5)	Alternative comovement (6)	CDS spread (7)	Control for persistence (8)	No overlapping (9)
Default Risk	0.67***	1.25***	1.19***	0.79***	0.49***	0.35***	0.62***	0.79***	0.71***
t-stat	(9.62)	(9.14)	(5.23)	(8.73)	(10.38)	(7.44)	(7.15)	(10.49)	(3.52)
$R^2_{Adj.}$	0.414	0.322	0.336	0.423	0.410	0.336	0.388	0.418	0.460
Obs.	114,755	514,017	104,743	81,619	188,111	91,863	63,778	114,432	7,427
SE	Month & Firm	Month & Firm	Month & Firm	Month & Firm	Month & Firm	Month & Firm	Month & Firm	Month & Firm	Year & Firm
Fixed Effects	Firm	Firm	Firm	Firm	Firm	Firm	Firm	Firm	Firm
Controls	All	All	All	All	All	All	All	All	All

Panel B: Correlation

	Baseline (1)	Bond level (2)	Exclude call options (3)	Exclude fin./util. (4)	Extended sample (5)	Alternative comovement (6)	CDS spread (7)	Control for persistence (8)	No overlapping (9)
Default Risk	0.08***	0.09***	0.08***	0.07***	0.06***	0.02***	0.04***	0.09***	0.09***
t-stat	(6.52)	(8.07)	(4.04)	(5.02)	(5.57)	(5.87)	(3.25)	(7.23)	(3.10)
$R^2_{Adj.}$	0.308	0.225	0.192	0.321	0.274	0.746	0.287	0.309	0.399
Obs.	114,755	514,017	104,743	81,619	188,111	91,863	63,778	114,432	7,427
SE	Month & Firm	Month & Firm	Month & Firm	Month & Firm	Month & Firm	Month & Firm	Month & Firm	Month & Firm	Year & Firm
Fixed Effects	Firm	Firm	Firm	Firm	Firm	Firm	Firm	Firm	Firm
Controls	All	All	All	All	All	All	All	All	All

Table 7 : Hedge ratio decomposition by default risk This table presents results on the hedge ratio, extending the analysis of Schaefer and Strebulaev (2008). Panel A reports the results of regressing firm-level excess corporate bond returns, $r^e_{B,i,t}$, on the associated excess stock returns, $r^e_{S,i,t}$, and the 10-year constant maturity U.S. Treasury bond returns, $r_{f,t}$. For each firm i, we estimate the following regression model: $r^e_{B,i,t} = \alpha_0 + \alpha_S r^e_{S,i,t} + \alpha_{rf} r_{f,t} + \epsilon_{i,t}$, where α_S is the hedge ratio, while α_{rf} reflects the corporate bond sensitivity to the risk-free bond. Panel A replicates Table 4 (p.6) of Schaefer and Strebulaev (2008), using our updated sample. The hedge ratio is the regression coefficient of excess corporate bond returns on excess stock returns, which is equal to $\mathbb{COV}(r_S, r_B)/\sigma^2_S = \rho_{B,S} \times \sigma_B/\sigma_S$, where $\rho_{B,S}$ is the stock-bond correlation while σ_B and σ_S denote the bond and stock volatility, respectively. In Panel B, we report the hedge ratio obtained in Panel A and the unconditional stock-bond correlation $\rho_{B,S}$ calculated by rating. The ratio between the two gives the Merton component (σ_B/σ_S), as $\rho_{B,S} = 1$ in the Merton model. Column (2) contains the most creditworthy firms whilst Column (8) contains the least creditworthy (speculative grade) firms. We report t-statistics in parentheses, using standard errors (SE) double clustered at the month and firm levels. The sample period spans August 2003 - December 2020. The definitions of the variables and their data sources are presented in Section 1.1 and in Online Appendix A and B. Significance at the 10%, 5%, 1% level is indicated by *, **, ***, respectively.

Panel A: Estimated Hedge Ratio

	All (1)	AAA (2)	AA (3)	A (4)	BBB (5)	BB (6)	B (7)	CCC to C (8)
β_S	15.27***	3.45***	8.35***	9.27***	10.33***	13.49***	16.57***	24.43***
t-stat	(9.31)	(3.04)	(4.34)	(5.44)	(4.97)	(6.86)	(9.88)	(11.30)
α_{rf}	30.78***	62.31***	45.96***	52.27***	37.37***	7.66	-4.33	-68.35***
t-stat	(4.43)	(9.00)	(8.02)	(9.49)	(5.50)	(0.91)	(-0.35)	(-2.63)
$R^2_{Adj.}$	0.206	0.521	0.304	0.264	0.191	0.252	0.279	0.329
Obs.	106,338	913	5,384	26,956	39,066	17,833	12,901	3,240

Panel B: Hedge Ratio Decomposition

	All (1)	AAA (2)	AA (3)	A (4)	BBB (5)	BB (6)	B (7)	CCC to C (8)	(8)/(2)
(A) Hedge Ratio ($\rho_{B,S} \times \sigma_B/\sigma_S$)	15.27	3.45	8.35	9.27	10.33	13.49	16.57	24.43	7.081
(B) Stock-bond Correlation ($\rho_{B,S}$)	0.44	0.11	0.24	0.27	0.30	0.39	0.48	0.71	6.455
(A)/(B) Merton Component (σ_B/σ_S)	34.70	31.36	34.79	34.33	34.43	34.59	34.52	34.41	1.097

Table 8 : Hedge ratio regressions – conditional vs. constant asset volatility This table presents results from pooled OLS regressions of excess bond returns on hedged equity returns computed with conditional asset volatility in Panel A and constant asset volatility in Panel B. Columns (1) and (4) reports the results using a sample that is representative of that used by Schaefer and Strebulaev (2008) which uses the Intercontinental Exchange (ICE) corporate bond data. Columns (2) and (5) report the results using all bonds in the TRACE sample. Columns (3) and (6) report the results for all bonds from the extended sample which includes TRACE, Datastream and the Lehman Brothers database. The details of the computation of the hedge ratio and it's inputs are detailed in Online Appendix E. We include firm fixed effects and report t-statistics in parentheses, using standard errors (SE) double clustered at the month and firm levels. The t-statistics for β_S are with respect to the difference from unity, while the t-statistics for α_{rf} are with respect to zero. The α_{rf} coefficient is reported in basis points. The sample period spans December 1996 - December 2003 for the Schaefer and Strebulaev (2008) matched sample, August 2003 - December 2020 for the TRACE data, and April 1987 - December 2020 for the extended sample. The definitions of the variables and their data sources are presented in Section 1.1 and in Online Appendix A and B. Significance at the 10%, 5%, 1% level is indicated by *, **, ***, respectively.

	Panel A: Conditional asset volatility			Panel B: Constant asset volatility		
	SS 2008 Sample (1)	TRACE Sample (2)	Extended Sample (3)	SS 2008 Sample (4)	TRACE Sample (5)	Extended Sample (6)
β_S	1.120	1.084	1.026	1.735***	1.794***	1.721***
t-stat	(0.932)	(0.671)	(0.264)	(3.777)	(4.836)	(4.49)
α_{rf}	55.41***	34.55***	49.69***	57.25***	37.02***	49.82***
t-stat	(10.66)	(4.91)	(12.87)	(12.08)	(5.16)	(12.35)
$R^2_{Adj.}$	0.354	0.226	0.214	0.345	0.214	0.1922
Obs.	41,982	534,459	1,189,856	41,982	534,459	1,189,856
Firm FE	YES	YES	YES	YES	YES	YES
SE	Month & Firm	Month & Firm	Month & Firm	Month & Firm	Month & Firm	Month & Firm
Sample start	1996-12	2003-07	1987-04	1996-12	2003-07	1987-04
Sample end	2003-12	2020-12	2020-12	2003-12	2020-12	2020-12

Table 9 : Stock-bond diversification by default risk and out-of-sample performance This table presents results on the diversification between stocks and bonds by default risk and the implications for the performance of a balanced allocation strategy. Panel A reports performance statistics for default-risk-sorted portfolios with equal allocation in stocks and corporate bonds. Panel B reports, for each portfolio, the corresponding 'Diversification Benefit' and the correlation between each stock and bond portfolio. Diversification Benefit represents the percentage of the average stock/bond variance that can be diversified away by combining stocks and bonds in a single portfolio. Panels C and D report the results for two special cases: i) an investment in a bond market index and default risk-sorted stocks; ii) an investment in a stock market index and default risk-sorted corporate bonds. Portfolios are presented by default risk quintile and are value-weighted based on market capitalization. Quintile portfolios are formed every month by sorting firms based on their default risk, measured with the Credit Spread. Quintile 1 (Q1) is the portfolio with lowest default risk, while Quintile 5 (Q5) is the portfolio with highest default risk. The Sortino ratio is computed as a portfolio's excess return divided by downside volatility, defined as the standard deviation of negative returns. The one-month value-at-risk (VaR) is the historical 95% quantile of each portfolio. The t-statistics for the individual joint portfolio Sharpe ratios are computed using heteroskedasticity and autocorrelation consistent (HAC) errors, as in Lo (2002). The t-statistic for the difference in the portfolio Sharpe Ratio between Q5 and Q1 is computed using HAC standard errors, as in Ledoit and Wolf (2008). The stock (bond) market index returns are the value-weighted (using market capitalization) firm-level stock (bond) returns. The data sample contains stocks and bonds spanning August 2003 - August 2020. The definitions of the variables and their data sources are presented in Section 1.1 and in Online Appendix A and B.

Panel A: Characteristics of balanced stock-bond portfolios

	Q1	Q2	Q3	Q4	Q5	Q5-Q1
Excess Return (%)	6.568	6.813	6.772	8.056	9.162	2.594
Volatility (%)	6.884	8.191	10.607	13.783	18.634	11.75
Sharpe Ratio	0.954***	0.832***	0.638***	0.584***	0.492***	-0.462**
t-stat	[3.871]	[3.406]	[2.629]	[2.409]	[2.031]	[-2.478]
Sortino Ratio	1.172	0.974	0.688	0.661	0.664	-0.506
Skew	-0.607	-0.831	-1.019	-0.522	0.235	0.842
Kurtosis	3.094	4.472	6.171	7.887	5.418	2.324
VaR-95 (%)	-8.044	-11.09	-14.86	-17.48	-21.39	-13.35

Panel B: Portfolio diversification between the bond and stock portfolios

Diversification Benefit (%)	46.68	40.49	36.65	30.52	25.57	-21.11
Correlation	0.109	0.299	0.486	0.692	0.706	0.597

Panel C: Portfolio diversification between a bond index and individual stocks

Diversification Benefit (%)	41.93	41.92	42.81	43.66	45.57	3.638
Correlation	0.254	0.278	0.305	0.325	0.284	0.031

Panel D: Portfolio diversification between a stock index and individual bonds

Diversification Benefit (%)	46.78	40.33	34.96	26.90	21.55	-25.22
Correlation	0.114	0.312	0.445	0.580	0.585	0.471

Table 10 : Model parameters This table presents the estimates of the model parameters. Panel A reports the set of calibrated parameters, including distress costs, corporate tax rate, and asset payout ratio which are set exogenously. The risk-free rate is equal to the long-run mean of a square-root process fitted to the 3-month Treasury Bill rate. Panel B reports the parameter values obtained from a structural estimation over the period August 2003 - August 2020. Panel C presents the goodness-of-fit of the estimation, measured with the sum of squared errors. The estimation methodology and the set of target moments are described in Section 3.5 and in Online Appendix G. All variables are annualized.

Variable	**Symbol**	**Value**
Panel A: Exogenous parameters		
Distress costs	α	0.4
Corporate tax rate	ζ	0.2
Asset payout ratio	q	0
Risk-free rate	r	0.0101
Panel B: Structurally-estimated parameters		
Speed of mean reversion of aggregate asset variance	κ_Y	0.9703
Unconditional mean of aggregate asset variance	Θ_Y	0.0060
Volatility of aggregate asset variance	δ_Y	0.1076
Correlation between the Brownian motions dW_t^Y and $dW_t^{\sigma_Y}$	ρ_Y	-0.2698
Price of variance risk	λ_{σ_Y}	-7.7231
Price of asset risk	$\lambda_{Y\perp\sigma_Y}$	1.6261
Speed of mean reversion of idiosyncratic asset variance	κ_X	0.8245
Unconditional mean of idiosyncratic asset variance	Θ_X	0.0211
Volatility of idiosyncratic asset variance	δ_X	0.1861
Debt coupon	c	0.0097
Exogenous barrier	X_B	0.5596
Panel C: Goodness of fit		
Sum of squared errors		0.0011

Table 11 : Empirical vs. model-implied moments This table compares empirical moments to those obtained in the simulated economy. Panel A presents the moments that are used in the structural estimation of the model, where the mean values highlighted in bold are the target empirical moments. The remaining values (standard deviations) are out-of-sample. Panel B presents a set of additional out-of-sample moments. The empirical moments are constructed as follows: Leverage is the ratio of total book debt to the sum of total book debt and market value of equity; Credit Default Swap (CDS) Spread is the 10-year spread for BB-rated firms from Markit. Default Probability is the 10-year historical default rate for BB-rated firms over the 1981 - 2020 period from Standard and Poor's (2021); Stock (Bond) Return Volatility is the standard deviation of stock (bond) returns estimated with a 12-month rolling window; Stock (Bond) Systematic Variance Ratio is the ratio of stock (bond) systematic variance to total variance. This ratio is estimated as the R-squared of a regression of stock (bond) returns on the returns of a stock (bond) value-weighted index estimated with a 12-month rolling window; Aggregate Stock (Bond) Return Volatility is equal to sample standard deviation of value-weighted stock (bond) index returns; Asset volatility is computed as the 12-month rolling standard deviation of asset returns, given by $r_{A,i,t} = L_{i,t} \times r_{B,i,t} + (1 - L_{i,t}) \times r_{S,i,t}$, where $r_{B,i,t}$ and $r_{S,i,t}$ are stock and bond returns for firm i in month t, while $L_{i,t}$ denotes the firm's leverage; Distance-to-default is the log of the distance between firm assets and the default threshold (equal to total debt value) divided by asset volatility; Stock-Bond Correlation/Covariance are computed using stock and bond returns with a 12-month rolling window. The model-implied moments are obtained as averages of spot moments across firms and months, using a simulated economy of 1,250 firms over 10 years. The empirical data sample period spans August 2003 - August 2020. The moments are annualized when applicable. The definitions of the variables/moments and their data sources are presented in Section 1.1 and Online Appendix A and B. The simulation procedure and the calculation of model-implied and empirical moments are detailed in Section 3.5 and in Online Appendix G.

	Mean			**Standard Deviation**		
	Data (A)	Model (B)	Ratio (A/B)	Data (C)	Model (D)	Ratio (C/D)
Panel A: In-sample moments						
Leverage [%]	**49.90**	50.78	0.98	21.53	9.95	2.16
CDS Spread [10y, %bps]	**236.22**	250.78	0.94	181.17	164.40	1.10
Physical Default Probability [10y, %]	**14.13**	12.70	0.90	-	-	-
Stock Return Volatility [%]	**32.87**	32.59	1.01	21.57	16.05	1.34
Bond Return Volatility [%]	**9.19**	9.11	1.01	8.84	3.25	2.72
Stock Systematic Variance Ratio [%]	**28.51**	27.94	1.02	18.76	25.62	0.73
Bond Systematic Variance Ratio [%]	**38.12**	36.80	1.04	22.30	29.09	0.77
Aggregate Stock Return Volatility [%]	**13.05**	14.02	0.93	6.08	7.29	0.83
Aggregate Bond Return Volatility [%]	**4.71**	4.88	0.96	2.51	2.38	1.05
Panel B: Out-of-sample moments						
Asset Volatility [%]	15.50	14.94	1.04	8.48	5.94	1.43
Distance-to-default	6.52	5.92	1.09	4.02	4.17	0.97
Stock-Bond Covariance	10.78	27.16	0.40	28.97	22.69	1.28
Stock-Bond Correlation	0.20	0.79	0.25	0.36	0.16	2.28

Table 12 : Decomposition of stock-bond comovement by default risk This table presents a decomposition of the model-implied stock-bond covariance by default risk, following Proposition 1 of Section 3.3. The stock-bond covariance has three components: Asset Risk reflects the impact of a firm's asset variance and leverage; Variance Risk is the impact of the variance of aggregate and idiosyncratic asset variances; Co-skewness Risk corresponds to the comovement between aggregate asset variance and the level of aggregate assets. Higher-order risk corresponds to the sum of Variance Risk and Co-skewness Risk. Quintile portfolios are formed every month by sorting firms based on the Default Risk composite variable, defined as the sign-corrected average of the three standardized default risk proxies, namely Leverage, Distance-to-Default, and CDS spread. Quintile 1 (Q1) is the portfolio with the lowest default risk and Quintile 5 (Q5) is the portfolio with the highest default risk. The quintiles are value-weighted by model implied equity value computed over the previous month. Panel A reports the annualized total stock-bond covariance and its components for each default risk portfolio. Panel B reports the percentage contribution of these components to the total stock-bond covariance. The results are based on a simulated economy of 1,250 firms over 10 years. Results reported are averaged over the 10 economies. The simulation procedure is detailed in Section 3.5 and in Online Appendix G.

Panel A: Stock-bond covariance and its components

	Q1	Q2	Q3	Q4	Q5
Stock-bond covariance	9.744	17.97	25.58	34.94	52.33
Asset Risk	9.144	17.36	25.02	34.45	51.89
Higher-Order Risk (i+ii)	0.600	0.615	0.568	0.490	0.440
(i) Variance Risk	-0.395	-0.689	-0.940	-1.180	-1.390
(ii) Co-skewness Risk	0.995	1.304	1.508	1.670	1.830

Panel B: Stock-bond covariance decomposition

Asset Risk (%)	0.938	0.966	0.978	0.986	0.992
Higher-Order Risk (%)	0.062	0.034	0.022	0.014	0.008

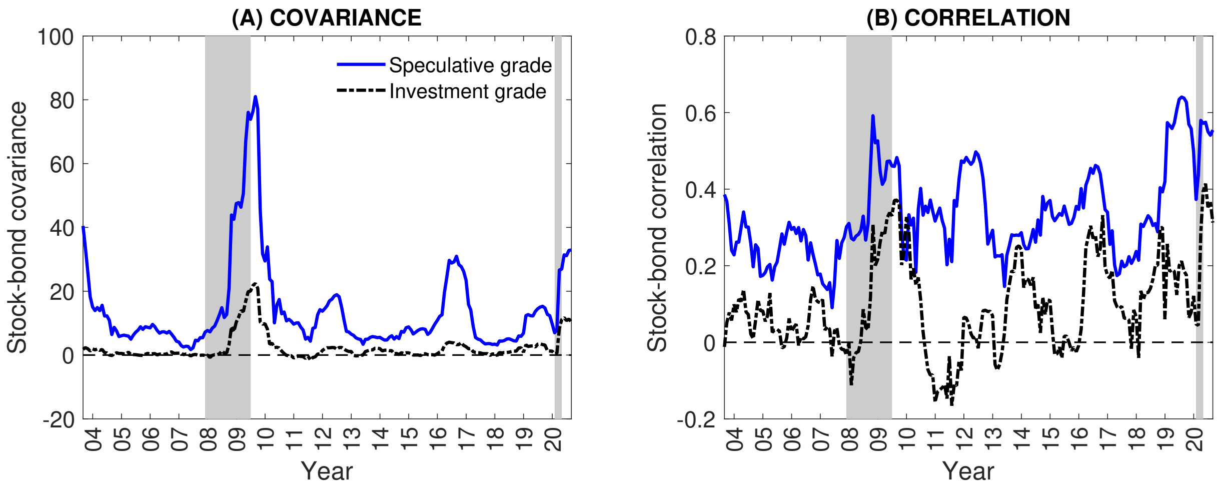

Figure 1: Conditional comovement between stock and corporate bond returns. This figure plots the monthly (annualized) covariance in Panel A and the correlation in Panel B between stock returns of U.S. firms and their respective corporate bond returns. The stock-bond covariance and correlation are computed at the firm level using 12-month rolling windows and then averaged across firms. Firms are split into two classes based on their rating: investment grade firms (rated AAA to BBB-) and junk firms (rated BB+ and below). The Shaded area represents the NBER recession period. The sample spans August 2003 - August 2020.

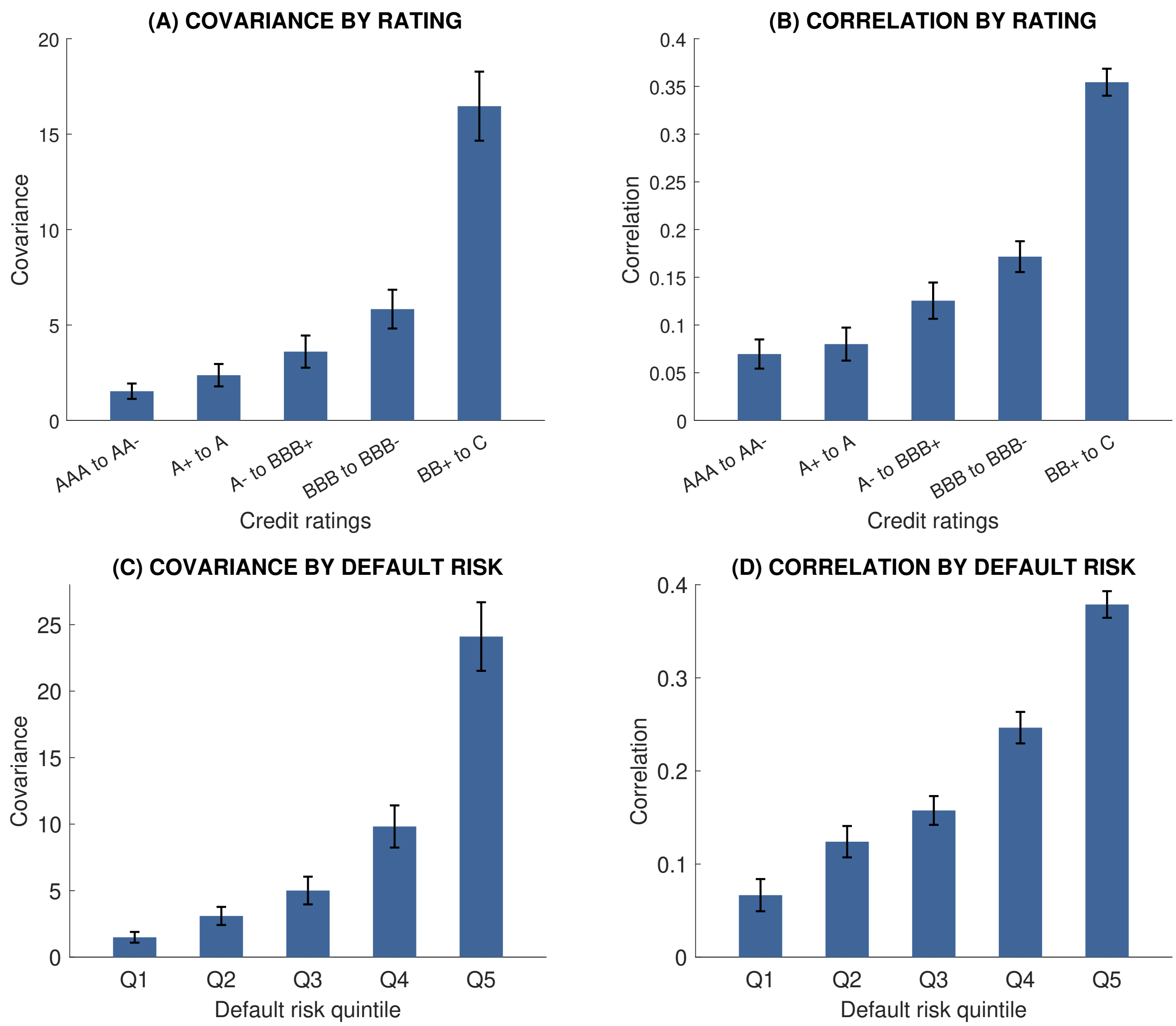

Figure 2: Portfolio sorts of stock-bond comovement This figure plots the average stock-bond covariance and correlation by default risk. Each month, we separate firms into five portfolios based on either their S&P credit ratings (Panels A and B) or the aggregate default risk variable (Panels C and D). We then compute the covariance (Panels A and C) and the correlation (Panels B and D) between stock and corporate bond returns over the following year with monthly observations. Within each portfolio, firms are value weighted based on their market capitalization. The reported standard error intervals correspond to the 90% confidence level. The sample period spans August 2003 - August 2020. The definitions of the variables and their data sources are presented in Section 1.1 and in Online Appendix A and B.

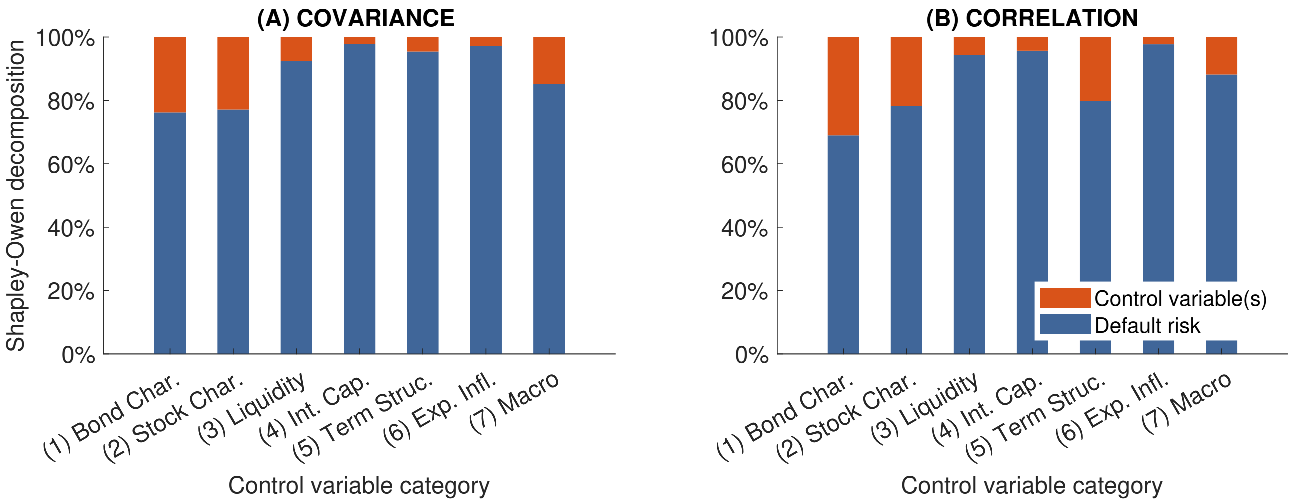

Figure 3: Predictive power of default risk vs. alternative channels This figure plots the relative contribution of default risk in predicting variation in stock-bond covariance (Panel A) and correlation (Panel B). The blue bars capture the fraction of the total variation (R-squared) in the one-year-ahead stock-bond comovement explained by default risk, in a pooled OLS regression, while the red bars capture the fraction of the total variation attributed to a specific group of controls. In each panel, the first bar accounts for corporate bond characteristics, which include a Callability dummy, Coupon, Time-to-Maturity, Bond Size (log of amount outstanding), and Bond Illiquidity (bid-ask spread). Bar (2) accounts for stock characteristics, which include Amihud (2002)'s Illiquidity Ratio, Market-to-Book, and Equity Size (log of market capitalization). Bar (3) accounts for global liquidity factors, which include the Pastor and Stambaugh (2003)'s Liquidity Risk Factor and Hu et al. (2013)'s Aggregate Liquidity Factor. Bar (4) accounts for the Intermediary Capital Risk Factor of He et al. (2017). Bar (5) accounts for the risk-free term structure, which includes the 10-year U.S. Treasury rate, 3-month U.S. Treasury Bill secondary market rate, and their difference (slope). Bar (6) accounts for Expected Inflation, measured as the one-quarter-ahead mean inflation forecast from the Survey of Professional Forecasters. Bar (7) accounts for the Macroeconomic Uncertainty Index of Jurado et al. (2015) and the Business Conditions Index of Aruoba et al. (2009). The Shapley-Owen marginal R-squared decomposition follows the methodology outlined in Shorrocks (1982). The sample period spans August 2003 - August 2020. The definitions of the variables and their data sources are presented in Section 1.1 and in Online Appendix A and B.

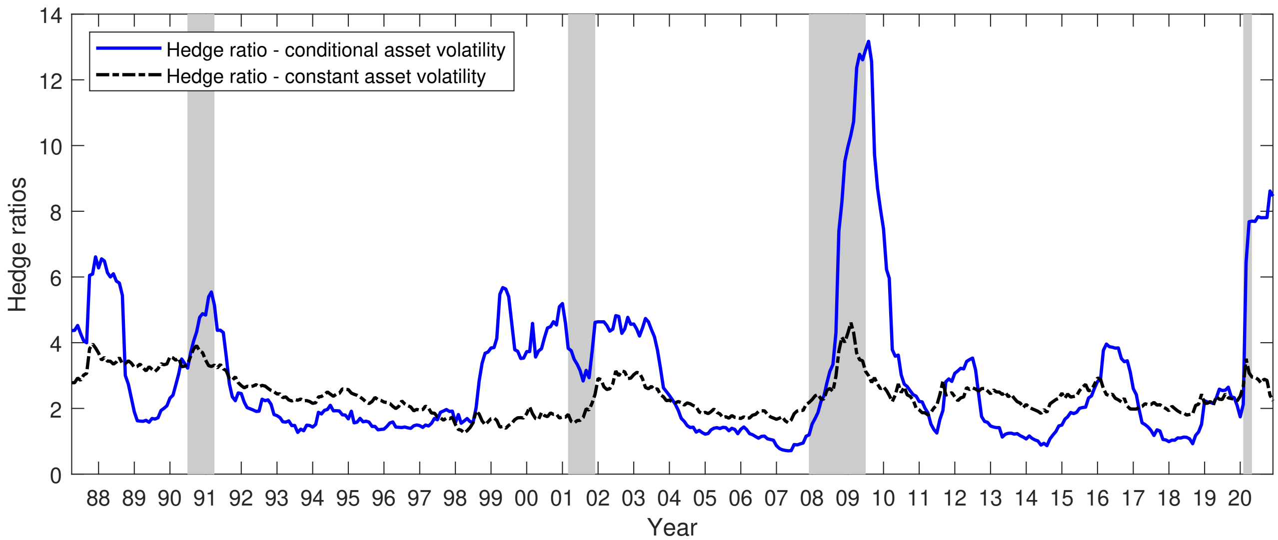

Figure 4: Aggregate hedge ratios This figure plots the monthly aggregate hedge ratios computed with conditional asset volatility in Panel A and constant asset volatility in Panel B. The details of the computation of the hedge ratio and it's inputs are detailed in Online Appendix E. The Shaded area represents the NBER recession period. The sample spans April 1987 - December 2020.

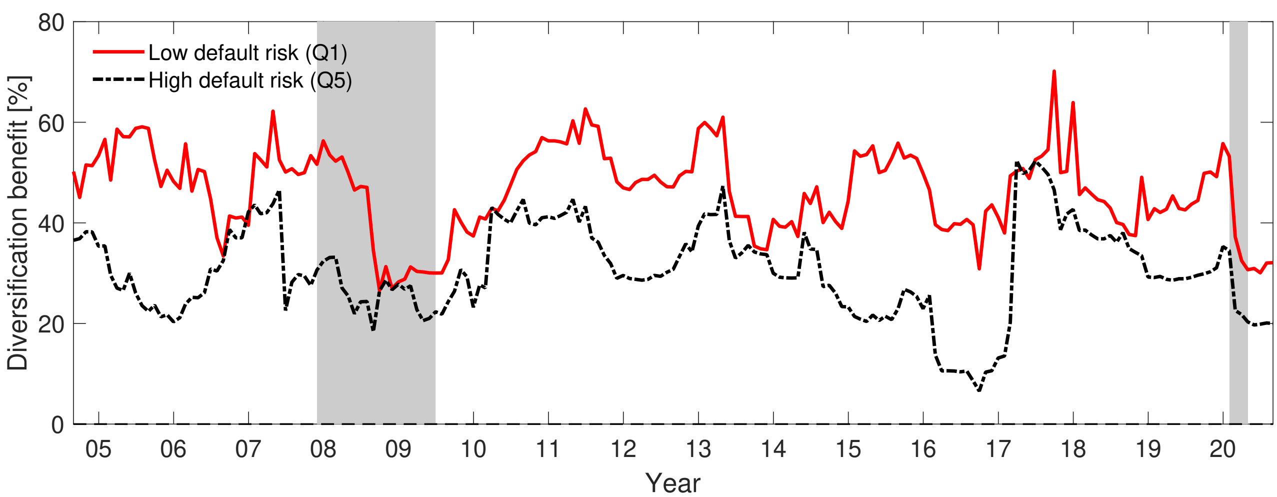

Figure 5: Conditional out-of-sample diversification benefit by default risk This figure plots the conditional out-of-sample diversification benefit of an equal allocation between corporate bonds and stocks by default risk. Diversification Benefit represents the percentage of the average stock/bond variance that can be diversified away of the following year by combining stocks and bonds in a single portfolio. The two time-series show the results for portfolios of firms sorted in the lowest (Q1) and highest (Q5) default risk quantiles. Shaded areas represent NBER recession periods. We estimate the conditional series of Diversification Benefit with a 12-month rolling window. The sample period spans August 2004 - August 2020. The definition of Diversification Benefit is presented in Section 2.

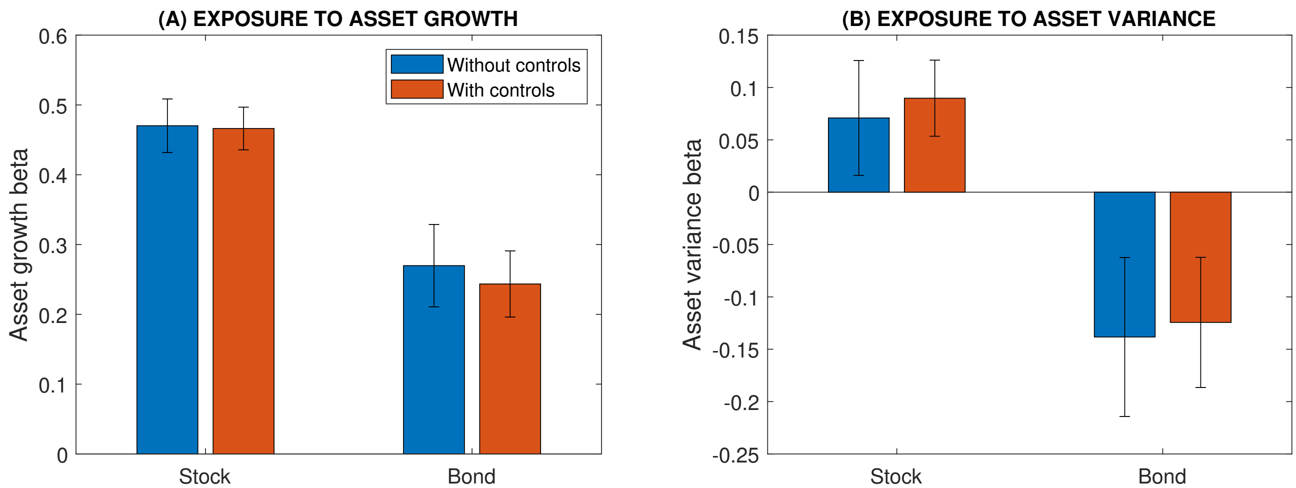

Figure 6: Stock and bond exposures to asset return and asset variance This figure illustrates the exposure of stock and corporate bond returns to asset return and asset variance. The exposures are estimated jointly as the slope coefficients (beta) from a pooled regression of contemporaneous stock and bond excess returns on aggregate asset return (Panel A) and aggregate asset variance (Panel B), with and without controls. The control variables include bond characteristics (Callability dummy, Coupon, Time-to-Maturity, Bond Size, Bond Illiquidity), stock characteristics (Amihud (2002)'s Illiquidity Ratio, Market-to-Book, and Equity Size), and global market conditions (the Intermediary Capital Risk Factor of He et al. (2017), Slope and Term Structure of interest rates, Expected Inflation, the Macroeconomic Uncertainty Index of Jurado et al. (2015), and the Business Conditions Index of Aruoba et al. (2009)). Aggregate asset return is a leverage-weighted average of the aggregate excess stock market return and aggregate excess bond market return. Aggregate asset variance is the factor mimicking portfolio for aggregate asset variance, following Choi and Richardson (2016). Full details of the construction of these measures are detailed in Section D of the Online Appendix. All variables are mean-variance standardized. The error bar denotes 95% confidence intervals. The sample period spans August 2003 - August 2020. The definitions of the variables and their data sources are presented in Section 1.1 and in Online Appendix A and B.

Online Appendix to

"Understanding the Comovement between Corporate Bonds and Stocks: The Role of Default Risk"

(not for publication)

Abstract

This Online Appendix presents supplementary material and results not included in the main body of the paper.

A Corporate bond data

This Appendix describes the procedure related to sourcing, cleaning, and preparing the corporate bond database. The main results of the paper rely on the cleaned version of the TRACE dataset which spans the period August 2002 - August 2020 made available from WRDS. In a robustness analysis (see Online Appendix C), we consider an extended dataset that we obtain by merging various corporate bond databases. For completeness, we present all sources below as well as the details for constructing the comprehensive dataset. We largely follow Chordia et al. (2017), Bai et al. (2019) and Elkamhi et al. (2022) in cleaning and merging the respective corporate bond databases.

A.1 Bond databases

The following describes the filtering rules we apply to each corporate bond database:

1. The Lehman Brothers Fixed Income (LB) database contains monthly corporate bond price quotes, yields, ratings, amount outstanding, accrued interest and returns. The sample spans the period January 1973 to March 1998. The bond prices are quote based, but are considered to be reliable according to Warga (1992), among others. In some cases, the price observations are based off matrix pricing. Matrix pricing infers a bond price which lacks its own quote based on quoted prices of similar bonds. We follow Feldhütter and Schaefer (2018) and Avramov et al. (2013) and start the sample in April 1987, because there are very few non-callable or junk bonds before this date. Furthermore, Standard & Poor's (S&P) firm ratings do not exist before 1986.
2. The cleaned Enhanced TRACE database, which is provided by the WRDS Bond Database, provides cleaned monthly transactions data. The processed data sample spans the period July 2002 to August 2020 and we use end-of-the-month prices to compute monthly returns.
3. The Datastream (DS) database includes quote based price data, yields, and returns. The sample spans the period January 1990 to December 2019.

A.2 Data cleaning

We only retain bonds from all aforementioned databases if they link to the Mergent FISD bond characteristics database. We use the following static Mergent FISD variables to filter all bonds in the sample:

1. Bond Type: We only include corporate bonds which are classified as US Corporate Debentures ('CDEB'), US Corporate MTN ('CMTN') or US Corporate MTN Zero ('CMTZ').
2. Public Firm: We exclude bonds that are not listed, or traded in the US public market. This includes bonds issued via private placement, bonds issued under the 144A rule, and bond issuers not in the jurisdiction of the United States.

3. Bond Coupon: We exclude bonds with a variable coupon ('V'), i.e., we only include bonds with a fixed ('F') or zero coupon ('Z').
4. Convertible: We exclude all convertible bonds.
5. Asset-Backed: We exclude all asset-backed bonds.
6. Yankee Bonds: We exclude all Yankee bonds (a debt obligation issued by a foreign entity which is traded in the United States and denominated in US dollars).
7. Foreign Currency: We only include US denominated bonds
8. Embedded options: We exclude putable bonds, but include callable bonds.
9. Security Level: We exclude all junior bonds, that is the 'Junior', 'Junior Subordinate', and 'Subordinate' bonds.
10. Rating: We exclude bonds which are 'Unrated'.

In quote-based databases, corporate bonds may trade infrequently. To address the issue of stale prices, we follow Chordia et al. (2017) and apply these additional filters:

1. Prices that bounce back in an extreme manner relative to preceding days (bid-ask bounce) are excluded. Specifically, denoting by $r_{B,i,j,t}$ the month-t return for bond j of firm i, we delete a month-t observation if $r_{B,i,j,t} \times r_{B,i,j,t-k} < -0.02$ for $k = 1, \ldots, 12$.
2. Prices that do not change for more than 3 months are excluded.

Finally, we remove observations if a corporate bond's monthly price is less than $1 or above $1000 and if the bonds time to maturity is less than 12 months, as in Bai et al. (2019).

A.3 Final dataset

The individual corporate bond datasets are combined with the following order of precedence when overlaps are present: i) Lehman Brothers Fixed Income Database, ii) the cleaned Enhanced TRACE data, and iii) Datastream.

We then merge with stock price and accounting data from CRSP and COMPUSTAT. We follow the literature and only include stocks listed on one of the NYSE, AMEX, or NASDAQ stock exchanges. We exclude stocks with share prices below $5 and only include stocks with a share code of 10 or 11.

Once a bond is in 'default', the bond is removed from the sample - we do not not use an imputed return as a proxy. This is to align the empirical results with the model. In the simulation, upon breaching the default threshold, the respective simulated firm ceases to exist. However, we verify that all reported empirical results are practically unchanged when including defaulted firms and their respective proxied returns upon default. We define 'default' as the firm having a numerical S&P Bond credit rating of 22 (D) or 'In Default' - the lowest possible rating.

We use the WRDS Bond CRSP Link to merge all corporate bonds from the TRACE dataset to their respective CRSP permanent company number (PERMNO). The remaining two databases are merged to their respective PERMNOs using the Committee on Uniform Securities Identification Procedures (CUSIP) identifiers at both the firm and issue levels. Due to CUSIP identifiers changing over time, we also use the historical CUSIP (NCUSIP) from CRSP. Remaining unmatched bonds are merged using Capital IQ from Compustat and manually matched using the ticker information provided by Bloomberg's data point (BDP) function.

The final 'full' sample consists of 12,756 corporate bonds issued by 1,652 firms spanning the period April 1987 to August 2020. Our main analysis uses TRACE data only, which yields a sample of 1,290 firms and 9,103 bonds over the period August 2002 to August 2020. Our regressions effectively use the period starting in August 2003 because one year of bond return data is required to compute some of the default risk measures.

B Description of default risk variables

This Appendix describes the firm-level variables used to measure default risk in our empirical analysis.

Market leverage

Market leverage is the ratio between total book debt (LT) and the market value of equity (monthly closing values of stock prices (PRC) multiplied by the number of shares outstanding (SHROUT)) plus total book debt (LT). We denote market leverage by $L_{i,t}$.

Distance-to-default

We compute a firm's distance-to-default ($DD_{i,t}$) as follows:

$$DD_{i,t} = \frac{-log(\frac{D_{i,t}}{X_{i,t}}) + (0.06 + r_{f,t} - \frac{1}{2}\sigma^2_{X,i,t})(T-t)}{\sigma_{X,i,t}\sqrt{T-t}}, \tag{A.1}$$

where $X_{i,t}$ is the level of firm i's total assets at time t, defined as the value of market equity plus short-term book debt plus one half long-term book debt, $D_{i,t}$ is short-term book debt plus one half long-term book debt. 6% is the proxy for the equity premium used by Campbell et al. (2008) and $r_{f,t}$ is the one-month US Treasury-Bill rate. Finally, $\sigma_{X,i,t}$ is the firm's asset volatility, computed as the rolling 12-month standard deviation of asset returns, given by $r_{A,i,t} = L_{i,t} \times r_{B,i,t} + (1 - L_{i,t}) \times r_{S,i,t}$, as in Choi and Richardson (2016), where $r_{B,i,t}$ and $r_{S,i,t}$ are stock and bond returns for firm i in month t. We follow Campbell and Thompson (2008) and set $T - t = 1$.

The distance-to-default can be interpreted as the number of standard deviations of asset growth by which a firm's market value of assets exceeds a given debt (liability) threshold.

Credit spread

We first compute the credit spread of an individual bond as the difference between the yield of the bond and the associated yield of the Treasury curve at the same maturity. We use the Benchmark Treasury rates from Datastream for maturities of 3, 5, 7, 10, and 30 years, and then use a linear interpolation scheme to estimate the entire yield curve, following Duffee (1998) and Collin-Dufresne et al. (2001), among others. We then compute the credit spread at the firm level by equal-weighting each bond's credit spread for a given firm (at each time t).

Ratings

We use bond level ratings as a measure of default risk. Bond-level ratings are obtained from S&P (S&P Global Ratings). When an S&P rating is unavailable, we use the one provided by Moody's.

CDS spread

Credit Default Swap (CDS) spread data is obtained from Markit over the period January 2002 to December 2020. We follow the cleaning methodology of Kelly et al. (2019). We keep a firm only if it has a valid CDS spread with maturities of 1, 3, 5, 7, and 10 years. We only retain USD-denominated contracts written on senior debt with the modified restructuring credit event clause. We exclude firms with a Markit implied rating of 'D' and trim extreme values of CDS spreads by imposing a cap on any spread value at 2%. We construct monthly observations based on the last available daily data each month for each company for each maturity of CDS. We then merge the Markit CDS data to CRSP/COMPUSTAT by manually matching the Markit REDCODE/CUSIP pairs to the NCUSIP codes from CRSP. Thereafter, we sample the data between August 2003 and August 2020 to align the CDS data to the main sample used in the paper. To compute the statistics for the empirical CDS spreads used in Table 11 we use firms with a Markit implied rating of 'BB' and a CDS maturity of 10-years. This results in a pooled mean and standard deviation of 236.22 bps and 181.17 bps with 23,249 observations. Once merged with the main dataset (which includes the TRACE data), the pooled mean and standard deviation is 231.26 bps and 172.36 bps with 12,854 observations.[43]

C Additional empirical results

This Appendix reports various robustness checks on the predictive relation between stock-bond co-movement and default risk. The results are reported in Table 6, where the first column reproduces

[43] The number of total observations for the merged Markit/CRSP/COMPUSTAT database is 117,865. This drops to 68,693 when merged to the full data panel used in the main results of the paper.

the specification with all controls (Column 8 of Table 5) for comparison purposes. All results use the TRACE sample, except Column (5) that uses the extended sample comprising all databases.

Analysis at the corporate bond level

The results in the main body of the paper use aggregated corporate bond returns (using equal weights) to create a 'firm-level' corporate bond return. We verify that the results also hold at the corporate bond level. To do so, we now estimate, for each firm, the rolling 12-month covariance and correlation between a firm's stock return and the return on its individual corporate bonds. We then estimate a pooled panel regression in the form of Equation (2) using corporate bond observations. Column (2) presents the results using all controls. The impact of default risk on the 12-month ahead stock-bond covariance and correlation remains of the same economic magnitude and statistical significance as in the baseline results with firm-level observations (Column 1). This analysis confirms that our findings do not arise because of the way we aggregate individual bond returns at the firm level.

Excluding bonds with embedded call options

To address concerns that the results are driven by corporate bonds with embedded call options, we estimate the baseline panel regression excluding embedded call options. Column (2) presents the results, which indicate that the impact of default risk on the 12-month ahead stock-bond covariance and correlation is similar using bonds with and without call features.

Excluding financial and regulated firms

Most empirical studies in the corporate finance literature exclude financial and regulated firms. Firms operating in these sectors are subject to specific regulations that influence their leverage policies and thus their default risk. We now test whether the relationship between stock-bond comovement and default risk is sensitive to the inclusion/exclusion of financial institutions and regulated utilities. To do so, we replicate Column (1) whilst excluding all financial and regulated utility firms (those with a SIC codes between 6000–6999 and 4900–4949, respectively). This exclusion removes about 20% of the observations. As evidenced in Column (4), the exclusion of financial institutions and regulated utilities in our sample has no discernible impact on the magnitude, or the statistical significance, of our baseline results.

Extended dataset

We replicate our baseline specification using a comprehensive dataset that combines TRACE data with the Lehman and Datastream databases. Column (5) presents the results. When using data spanning the period April 1987 to August 2020, the results are similar to those with the TRACE sample (August 2003 to August 2020), in terms of the magnitude of the predictability coefficients and explanatory power, as measured with the R-squared.[44] In particular, the impact of default risk on the 12-month ahead stock-bond covariance and correlation remains statistically significant at the 1% level. Our findings are thus not limited to the last two decades.

Alternative measure of stock-bond comovement

We verify here that the results are robust the computation of stock-bond comovement. To do so we estimate stock-bond covariance and correlation estimated from an asymmetric dynamic conditional correlation (DCC) GARCH model as an alternative proxy of comovement to the rolling 12-month estimates. Column (6) confirms that the impact of default risk on the 12-month ahead stock-bond covariance and correlation is not specific to the the way we measure stock-bond comovement.

Alternative measure of default risk

As an alternative measure of default risk, we consider credit default swap (CDS) spreads using Markit data. We use the (standardized) 10-year CDS spread at the firm level instead of the 'Default Risk' variable. Column (7) shows that predictability of stock-bond comovement with the CDS spread is of similar economic magnitude and statistical significance as for the baseline default risk measure. Note that, after merging the main sample to Markit, we recover only about 50% of the observations. This robustness analysis thus also verifies that our results are not driven by relatively small firms without traded CDS contracts.

Controlling for persistence in stock-bond comovement

We also verify that our predictability results do not arise from the persistence in stock-bond comovement. To do so, we augment our predictive regression model (2) as follows:

$$Comovement_{i,t+12} = a + \delta DR_{i,t} + \mathbf{Y}'_{i,t}\delta_C + \delta_L Comovement_{i,t} + b_i + \epsilon_{i,t+12}, \qquad \text{(A.2)}$$

where $Comovement_{i,t+12}$ is the one-year-ahead stock-bond comovement measure of firm i computed between months $t+1$ and $t+12$, $DR_{i,t}$ is a firm-level default risk measure observable in month t, $\mathbf{Y}_{i,t}$ is the vector of controls, and $Comovement_{i,t}$ accounts for the lagged stock-bond comovement, computed between months $t-11$ and t. Column (7) confirms that our results are not driven by some persistence in the dependent variable.

Non-overlapping observations

In our core analysis, we consider predictive regressions with overlapping stock-bond covariance/correlation, which may be a source of concern. Here we provide an additional exercise that complements the baseline

[44] All control variables are the same as in the main results except for bond bid-ask spread which is replaced with the Bao et al. (2011) illiquidity proxy because bond bid-ask spreads are not available for the quote based databases.

specification by running predictive regressions with non-overlapping covariance/correlation. Specifically, we convert monthly data into yearly observations by sampling every July and predict stock-bond comovement between August and July of the following year. Default risk and the set of controls are all observed in July of each year. We report our empirical evidence in Column (8) and find that the slope coefficient estimates associated with default risk continue to be statistically significant. This exercise provides evidence that our findings on the stock-bond comovement predictability with default risk cannot be attributed to the use of overlapping observations.

Different panel specifications

Our final robustness check involves considering various panel specifications. Table A.2 presents the results using bond-level data. Columns (1)-(3) report results from panel regressions including firm fixed effects with different combinations of standard error clustering (bond, firm, industry, and month). Columns (4)-(6) report results from panel regressions including firm and time (month) fixed effects standard errors clustered at the month and firm level for various fixed effect models. In all cases, the impact of default risk on the stock-bond covariance and correlation is statistically significant at the 1% level. We can conclude that our results are robust to the consideration of different standard error corrections and fixed effect dimensions.

Table A.2 [about here]

D Construction of aggregate asset return and asset variance

This Appendix describes the construction of our empirical proxies for aggregate asset return and asset variance, used to generate Figure 6.

Aggregate asset return We compute aggregate asset return as the weighted average of stock and bond market returns. For stocks, we use the excess return market risk factor denoted as MKT_t^S from Kenneth French's data library. For bonds, we use the ICE Bank of America (BofA) US Corporate Bond Total Return Index in excess of the one-month U.S. T-Bill rate of return, which we denote MKT_t^B. Then, using the merged CRSP/COMPUSTAT database, we first compute market leverage for each firm i in each month t as $\frac{D}{(D+E)}$, where D is total book debt from COMPUSTAT and E is market equity. The aggregate market leverage (L_t) is the value-weighted average of firm-level market leverage using total firm value ($V_{i,t} = D_{i,t} + E_{i,t}$) as weights. Finally, we compute aggregate asset return as $AR_t = MKT_t^B \times L_t + MKT_t^S \times (1 - L_t)$. We thus obtain, by construction, a 'traded' risk-factor since it is a time-series of excess aggregate asset returns.

Aggregate asset variance To estimate aggregate asset variance, we follow Choi and Richardson (2016) and fit an exponential-GARCH(1,1) model to aggregate asset return, as described above. We then compute the conditional asset variance using the estimated parameters from the fitted exponential-GARCH model. We use the first difference in the conditional asset variance as the proxy for the 'non-traded' version of aggregate variance risk, which we denote VR*. To compute the 'traded' version of aggregate asset variance, we form a factor mimicking portfolio (FMP). To generate a set of basis assets for the FMP, we sort firms into 20 portfolios based on their firm-level credit spread for each month in the sample. Thereafter, in the following month, we compute the value-weighted (using firm-level weights as defined above) average excess stock and bond returns. This yields a total of 40 portfolios of stock and bond excess portfolio returns. The aggregate asset variance risk FMP is constructed from a time-series regression of the VR* factor onto the set of stock and bond excess portfolio returns constructed above such that the resultant mimicking portfolio is maximally correlated with the VR* factor. To do this, VR* is projected onto the space of excess asset (stock and bond) excess returns to obtain a vector of portfolios weights (ω_t) which are normalized to sum to one (100% invested). The VR* mimicking portfolio weights are given by,

$$w_T = -\frac{\hat{\beta}_T}{|\hat{\beta}_T' \iota|}, \;\; [\hat{\alpha}_T, \hat{\beta}_T'] = \underset{\hat{\alpha_T}, \hat{\beta_T}}{\text{arg min}} \frac{1}{T} \sum_{t=1}^{T} (VR_T - \alpha_T - \beta_T \mathbf{R}_t^e)^2,$$

where ι is a vector of conformable ones, w_T is the vector of normalized factor mimicking portfolio weights, $\hat{\beta}_T$ is the vector of estimated stock and bond loadings on the VR* factor and $\mathbf{R}_t^e$ are the 40 stock and bond portfolios of excess returns. The traded version of aggregate asset variance is then obtained as VR $= w_T' \mathbf{R}_t^e$.[45]

E Hedge ratios

This Appendix describes the methodology which underlies the construction of the firm-level hedge ratios.

F Asset risk premium and risk-neutral dynamics

This Appendix describes additional parts of the model that are unreported in the body of the paper. Specifically, we here discuss the asset risk premium and risk-neutral dynamics of a firm's assets, which are used to evaluate stocks and bonds.

We first present and discuss the asset risk premium in the economy. Combining the dynamics of

[45] The correlation between the non-traded (VR*) and the traded (VR) versions is 81%, which is in-line with the equity VIX factor mimicking portfolio constructed in Ang et al. (2006).

aggregate assets (8) and the SDF (12), the level of asset risk premium is given by:

$$\mu_{Y,t} - r = \sigma^2_{Y,t}\left(\sqrt{1-\rho_Y^2}\lambda_{Y\perp\sigma_Y} + \rho_Y\lambda_{\sigma_Y}\right) = \sigma^2_{Y,t}\lambda_Y, \tag{A.3}$$

where ρ_Y is the correlation between aggregate asset shocks and aggregate asset variance shocks, and $\lambda_Y \equiv \sqrt{1-\rho_Y^2}\lambda_{Y\perp\sigma_Y} + \rho_Y\lambda_{\sigma_Y}$.

The intuition is as follows. When the level and the variance of aggregate assets co-move with each other (i.e., $\rho_Y \neq 0$), the asset risk premium reflects the representative investor's aversion to fluctuations in aggregate asset and its variance.[46] Note that the risk premium increases with aggregate variance when $\lambda_Y > 0$. The factor structure in firm asset returns in Equation (10), combined with Equation (A.3), implies that firm i's asset risk premium is the product of β_i and the asset risk premium, i.e., $\mu_{X,i,t} - r = \beta_i\left(\mu_{Y,t} - r\right)$.[47]

From the physical dynamics (8)-(11) and the SDF (12), we can determine the risk-neutral dynamics, which are given by:

$$\frac{dY_t}{Y_t} = rdt + \sigma_{Y,t}d\widetilde{W}_t^Y \tag{A.4}$$

$$d\sigma^2_{Y,t} = \widetilde{\kappa}_Y(\widetilde{\theta}_Y - \sigma^2_{Y,t})dt + \delta_Y\sigma_{Y,t}d\widetilde{W}_t^{\sigma_Y} \tag{A.5}$$

$$\frac{dX_{i,t}}{X_{i,t}} = (r-q)\,dt + \beta_i\left(\frac{dY_t}{Y_t} - rdt\right) + \sigma_{X,i,t}d\widetilde{W}_{i,t}^X \tag{A.6}$$

$$d\sigma^2_{X,i,t} = \kappa_X\left(\theta_X - \sigma^2_{X,i,t}\right)dt + \delta_X\sigma_{X,i,t}d\widetilde{W}_{i,t}^{\sigma_X}, \tag{A.7}$$

where $\widetilde{\kappa}_Y = (\kappa_Y + \delta_Y\lambda_{\sigma_Y})$ and $\widetilde{\theta}_Y = \frac{\kappa_Y}{\widetilde{\kappa}_Y}\theta_Y$ are the risk-adjusted speed of mean reversion and unconditional aggregate variance. The Brownian motions under the risk-neutral measure $\mathbb{Q}$ are defined by $d\widetilde{W}_t^Y = \rho_Y d\widetilde{W}_t^{\sigma_Y} + \sqrt{1-\rho_Y^2}d\widetilde{W}_t^{Y\perp\sigma_Y}$ with $d\widetilde{W}_t^{\sigma_Y} = dW_t^{\sigma_Y} + \sigma_{Y,t}\lambda_{\sigma_Y}dt$, $d\widetilde{W}_t^{Y\perp\sigma_Y} = dW_t^{Y\perp\sigma_Y} + \sigma_{Y,t}\lambda_{Y\perp\sigma_Y}dt$, , and $d\widetilde{W}_{i,t}^X = dW_{i,t}^X$ and $d\widetilde{W}_{i,t}^{\sigma_X} = dW_{i,t}^{\sigma_X}$, respectively.[48]

Observe that a negative price of variance risk, $\lambda_{\sigma_Y} < 0$, implies a higher persistence in the volatility process under the risk-neutral measure than under the physical measure ($\widetilde{\kappa}_Y < \kappa_Y$) and, therefore, a higher unconditional systematic variance ($\theta_Y < \widetilde{\theta}_Y$). The unconditional total asset variance under the risk-neutral measure, which is given by $\widetilde{\theta}_i = \beta_i^2\widetilde{\theta}_Y + \theta_X$, also increases, i.e., $\theta_i < \widetilde{\theta}_i$. The model thus features a negative variance risk premium, which plays a critical role in generating reasonable levels of

[46] Instantaneously, the asset risk premium, $E_t^{\mathbb{P}}\left[dY_t/Y_t\right] - E_t^{\mathbb{Q}}\left[dY_t/Y_t\right]$, solves $-cov\left(d\phi_t/\phi_t, dY_t/Y_t\right)$.

[47] Similarly to aggregate asset risk, the risk premium of firm i's asset risk is $E_t^{\mathbb{P}}\left[dX_{i,t}/X_{i,t}\right] - E_t^{\mathbb{Q}}\left[dX_{i,t}/X_{i,t}\right]$ and solves $-cov\left(d\phi_t/\phi_t, dX_{i,t}/X_{i,t}\right)$ in absence of arbitrage opportunities.

[48] Note that absence of arbitrage opportunities implies $E_t^{\mathbb{Q}}\left[dY_t/Y_t\right] = rdt$ given that, without frictions, aggregate asset risk can be replicated by large diversified portfolios of bonds and/or stocks which also must earn the risk-free rate under the risk-neutral measure.

risk premia and asset pricing moments.[49]

G Model calibration and simulation

This Appendix provides details about the calibration of the model and the simulation methodology. We first present our simulation strategy. We then describe the estimation of model-implied measures that are used to compute moment conditions. Finally, we discuss the estimated parameters and provide some insights about their economic implications.

G.1 Simulation methodology

We estimate model-implied moments using simulation methods. Firms in the economy can take on 5 values of asset beta (0.6, 0.8, 1, 1.2, 1.4) with equal probability. We denote by $\Pi_k \equiv \{\Theta, \beta_k\}$ the vector of parameters used to simulate a firm with a given asset beta β_k, where $k = \{1, .., 5\}$. From Equations (13) and (14), we can see that knowledge of the functional form of p_D, which is the price of an Arrow-Debreu default claim, is key for computing model-implied stock and bond prices, $S_{i,t}$ and $B_{i,t}$ respectively. We also compute the model-implied spread of a credit default swap (CDS) contract, which is a critical moment condition we consider. We estimate the Arrow-Debreu default price $\hat{p}_D(\cdot, \cdot, \cdot, \Pi_k)$ and the spread of a CDS with 10-year maturity, denoted by $\widehat{CS}(\cdot, \cdot, \cdot, 10, \Pi_k)$, as a function of the state variables $\left\{X_{i,t}, \sigma^2_{Y,t}, \sigma^2_{X,i,t}\right\}$ using non-parametric methods (see Section G.2 and Section G.3). Once we know $\hat{p}_D(\cdot, \cdot, \cdot, \Pi_k)$ and $\widehat{CS}(\cdot, \cdot, \cdot, 10, \Pi_k)$ for all k, we can generate an entire cross-section of firm-level variables, as it only requires simulating $\left\{X_{i,t}, \sigma^2_{Y,t}, \sigma^2_{X,i,t}\right\}$ and computing $\hat{p}_D(X_{i,t}, \sigma^2_{Y,t}, \sigma^2_{X,i,t}, \Pi_k)$, $\widehat{CS}(X_{i,t}, \sigma^2_{Y,t}, \sigma^2_{X,i,t}, 10, \Pi_k)$, and the remaining measures such as bond and stock total volatility. It is important to note that all remaining moments we consider only depend on $\hat{p}_D(X_{i,t}, \sigma^2_{Y,t}, \sigma^2_{X,i,t}, \Pi_k)$ and its partial derivatives. We postpone specific discussions about the estimation of p_D, CS, and other relevant firm-level measures for now, but provide details about their computation in Section G.2, Section G.3, and Section G.4.

To initialize the economy, we start with a cross-section of 250 firms for each asset beta. Each firm starts with the same initial unlevered asset value $X_{i,0} = 1$ and with $\sigma^2_{X,i,0} = \theta_X$. Initial aggregate spot variance is set to $\sigma^2_{Y,0} = \theta_Y$. Using these initial values, we then simulate firms and aggregate asset risk under the physical probability measure over 10 years. Our simulation of the state variables exploits a Euler discretization of the dynamics (8)-(11) with daily time steps. Although all firms have $X_{i,0} = 1$ and $\sigma^2_{X,i,0} = \theta_X$ initially, the levels of their assets and their conditional idiosyncratic variances diverge over time due to different idiosyncratic asset and variance shocks. We thus obtain a cross-section of

[49] A large literature suggests that the variance risk premium has important pricing implications. For example, Du et al. (2019) find that the asset variance risk premium helps address the credit spread puzzle. For discussions about the equity variance risk premium, see Bollerslev et al. (2009), Carr and Wu (2009), and Todorov (2010), among others.

about 1,250 firms, with one bond and one stock each.

In the simulations, we replace a firm by a new one in any of the following cases: (i) when its market leverage (computed as $B_{i,t}/\left(B_{i,t}+S_{i,t}\right)$) is below 0.20 or above 0.80 (i.e., the upper bound implicitly filters out firms close to default); (ii) when its 10-year CDS spread is below 0.20% or above 25%; (iii) when its annualized total equity volatility is above 150%; or (iv) when its annualized total bond volatility is above 50%. In any of these cases, we replace a firm by a new firm with the same beta, whose simulated path starts one year prior to the one exiting the sample. When initializing the simulation of a new firm on day t, we set $X_{i,t}=1$, $\sigma^2_{X,i,t}=\theta_X$, and aggregate variance to $\sigma^2_{Y,t}$. This simulation procedure helps generate a stationary economy, where firm leverage does not vanish over time, and prevents that the economy becomes dominated by a few very large or extremely risky firms.

We then adopt the following strategy to estimate the model-implied moments from the simulations. For all measures, except the physical default probability, we use the spot values implied by our model, which we compute based on the simulated state variables (see the following sections for details). Every month, we sample these values for each firm and thus obtain monthly firm-level measures of market leverage, credit spread, stock and bond return volatilities, and asset volatility. The physical default probability corresponds to the proportion of firms defaulting during the 10-year simulation horizon among the initial set of 1,250 firms (i.e., that is before replacing any firm given the aforementioned filters). For a given simulation path of the state variables, we repeat the steps above for each firm and every month to get the monthly estimates of all moments. We repeat this entire exercise 10 times to have 10 different paths of aggregate asset and variance risk. We then set $Model(\Theta)^m$ to the (pooled) average of moment m across all firms, months, and simulations.

G.2 Estimating the price of an Arrow-Debreu default claim and its partial derivatives

We now describe the methodology to estimate $\hat{p}_D(\cdot,\cdot,\cdot,\Pi_k)$ and its partial derivatives for a given set of structural parameters Π_k. First, we discretize the state space of the three state variables, $X_{i,t}$, $\sigma^2_{Y,t}$, and $\sigma^2_{X,i,t}$.[50] A given combination of the discretized state variables is then used as initial values for our simulation exercise. Specifically, we adopt a daily discretization of the risk-neutral dynamics (A.4)-(A.7) using an Euler approximation scheme.[51] Using the discretized dynamics, the vector of parameters Π_k, and a given combination of the initial state variables $\left\{X_{i,0},\sigma^2_{Y,0},\sigma^2_{X,i,0}\right\}$, we simulate firm assets under the risk-neutral measure over 10 years. Using the simulated paths of firm assets, we then estimate $\hat{p}_D(X_{i,0},\sigma^2_{Y,0},\sigma^2_{X,i,0},\Pi_k)$ as $\hat{E}^{\mathbb{Q}}_0\left[e^{-r\tau_{D,i}}\right]=\frac{1}{MC}\sum_{n=1}^{MC}e^{-r\tau^n_{D,i}}$, where $\tau^n_{D,i}=\inf\left\{s\geq 0: X^n_{i,s}\leq X_B\right\}$

[50] More precisely, we adopt a grid composed of 5 nodes for asset value and 4 nodes for spot variances. The lower and upper bounds for asset values are $1.1\cdot X_B$ and 2, respectively. The lower and upper bounds for spot systematic (idiosyncratic) variances correspond to $\theta_Y/100$ ($\theta_X/100$) and $\theta_Y\cdot 8$ ($\theta_X\cdot 8$), respectively. These bounds combined with our Chebychev polynomial approach define the nodes.

[51] Unlike the simulation of the economy, which is done under the physical probability measure, the estimation of the Arrow-Debreu price of default and asset prices requires simulating the state variables $\left\{X_{i,0},\sigma^2_{Y,0},\sigma^2_{X,i,0}\right\}$ under the risk-neutral measure.

and $X^n_{i,s}$ denotes the firm i's asset value at time s for the simulated path n, X_B represents the default boundary, and MC is the number of Monte-Carlo simulations.[52]

We repeat this simulation exercise for each combination of the initial values of the state variables. This gives us an entire cross-section of p_D as a function of the initial states (i.e., the initial combination of state variables used to simulate the firm forward). Using the cross-section of p_D and the combination of states, we then estimate the loadings of Chebychev polynomials for a given vector of parameters Π_k by projecting the p_D on the state variables. The estimated Chebychev loadings provide us with the required (smooth) mapping between p_D and the state variables for a given Π_k, that is, $\hat{p}_D(\cdot,\cdot,\cdot,\Pi_k)$. Because Chebychev polynomials guarantee a smooth mapping between $\hat{p}_D(\cdot,\cdot,\cdot,\Pi_k)$ and $\left\{X_{i,t},\sigma^2_{Y,t},\sigma^2_{X,i,t}\right\}$, the estimates of the partial derivatives of $\hat{p}_D(\cdot,\cdot,\cdot,\Pi_k)$ with respect to $X_{i,t}$, $\sigma^2_{Y,t}$, and $\sigma^2_{X,i,t}$ are then relatively straightforward to compute.

G.3 Estimating the CDS spread

We begin this section by presenting the pricing of the CDS contract. We then explain the way we estimate the CDS spread implied by our model for a given set of state variables and vector of parameters.

A CDS contract involves two parties: the protection buyer and the protection seller. The protection buyer makes quarterly premium payments to the protection seller until the maturity of the contract or until the firm's default. When the running spread paid by the buyer is 1 basis point (bp) per annum, the present value of future premiums (or premium leg) is given by:

$$\mathrm{LEG}_{\mathrm{prem,t}} = 0.0001 \times \sum_{u=1}^{4T} e^{-r(t_u-t)}\left[1-\hat{G}(X_{i,t},\sigma^2_{Y,t},\sigma^2_{X,i,t},t_u,\Pi_k)\right]/4, \tag{A.8}$$

where T is the maturity of the given CDS contract which we set to 10, $\{t_1,t_2,\cdots t_{4T}\}$ denote quarterly premium payment dates, and $\hat{G}(X_{i,t},\sigma^2_{Y,t},\sigma^2_{X,i,t},T,\Pi_k)$ captures the cumulative risk-neutral default probability, which is estimated as $\hat{E}^{\mathbb{Q}}_0\left[1_{(\tau_{D,i}\leq T)}\right] = \frac{1}{MC}\sum_{n=1}^{MC} 1_{(\tau^n_{D,i}\leq T)}$.

In exchange for paying insurance premiums, the protection buyer acquires a contingent claim. In the event of the firm's default, the protection seller has an obligation to buy the defaulted bond at par from the protection buyer, making up the loss from the default. The present value of a contingent protection payment (or protection leg) is computed as:

$$\mathrm{LEG}_{\mathrm{prot,t}} = (1-R)\sum_{u=1}^{4T} e^{-r(t_u-t)}\left[\hat{G}(X_{i,t},\sigma^2_{Y,t},\sigma^2_{X,i,t},t_u,\Pi_k)-\hat{G}(X_{i,t},\sigma^2_{Y,t},\sigma^2_{X,i,t},t_{u-1},\Pi_k)\right], \tag{A.9}$$

[52]We exploit both antithetic and control variable techniques in the simulation of the model. We use $MC = 2000$ for computational efficiency.

where R represents the recovery rate which we set to 40%, measured as a fraction of the CDS notional value. In the case of zero upfront fee, the CDS spread refers to the fair market spread that equates the premium leg ($\mathrm{LEG}_{\mathrm{prem}}$) with the protection leg ($\mathrm{LEG}_{\mathrm{prot}}$). That is, the CDS spread for a contract with 10-year maturity is given by

$$CS(X_{i,t}, \sigma^2_{Y,t}, \sigma^2_{X,i,t}, 10, \Pi_k) = \frac{\mathrm{LEG}_{\mathrm{prot,t}}(X_{i,t}, \sigma^2_{Y,t}, \sigma^2_{X,i,t}, 10, \Pi_k)}{\mathrm{LEG}_{\mathrm{prem,t}}(X_{i,t}, \sigma^2_{Y,t}, \sigma^2_{X,i,t}, 10, \Pi_k)}. \tag{A.10}$$

We now describe the methodology to estimate $\widehat{CS}(\cdot,\cdot,\cdot,10,\Pi_k)$ for a given set of structural parameters Π_k, closely following the estimation of p_D.[53] First, we discretize the state space of $X_{i,t}$, $\sigma^2_{Y,t}$, and $\sigma^2_{X,i,t}$ and use a given combination of the discretized state variables as initial values for a subsequent simulation. Using the discretized dynamics of (A.4)-(A.7), the vector of parameters Π_k, and a given combination of the initial state variables $\left\{X_{i,0}, \sigma^2_{Y,0}, \sigma^2_{X,i,0}\right\}$, we simulate firm assets under the risk-neutral measure $\mathbb{Q}$ over 10 years. Using these simulated paths, we then estimate $CS(X_{i,0}, \sigma^2_{Y,0}, \sigma^2_{X,i,0}, 10, \Pi_k)$ given the term-structure of risk-neutral default probability implied by the simulations.

We repeat this procedure for each combination of initial values of the state variables. This gives us an entire cross-section of CS as a function of the initial states. Using the cross-section of CS and the combination of initial states, we then estimate the loadings of Chebychev polynomials for a given vector of parameters Π_k by projecting the CS on the state variables. The estimated Chebychev loadings provide us with the required (smooth) mapping between CS and the state variables for a given Π_k, that is, $\widehat{CS}(\cdot,\cdot,\cdot,10,\Pi_k)$.

G.4 Estimating stock and bond moments

Model-implied moments We now present the construction of other measures that are relevant to compute model-implied stock and bond moments. For a given set of simulated state variables $\left\{X_{i,t}, \sigma^2_{Y,t}, \sigma^2_{X,i,t}\right\}$ at time t and a given set of parameters (e.g., firm beta), we obtain $\hat{p}_D$. The estimate of $\hat{p}_D$ combined with the semi-closed form formulae for stock and bond prices allows us to obtain $S_{i,t}$ and $B_{i,t}$, from which firm leverage can also be inferred as follows: $B_{i,t}/\left(B_{i,t}+S_{i,t}\right)$. Using the partial derivatives of $\hat{p}_D$ and the stock and bond price semi-closed form formulae, we can then estimate $\Delta^S_X = \frac{X_{i,t}}{S_{i,t}}\frac{\partial S_{i,t}}{\partial X_{i,t}}$, $\Delta^B_X = \frac{X_{i,t}}{B_{i,t}}\frac{\partial B_{i,t}}{\partial X_{i,t}}$, $\Delta^S_{\sigma_Y} = \frac{1}{S_{i,t}}\frac{\partial S_{i,t}}{\partial \sigma^2_{Y,t}}$, $\Delta^S_{\sigma_X} = \frac{1}{S_{i,t}}\frac{\partial S_{i,t}}{\partial \sigma^2_{X,i,t}}$, $\Delta^B_{\sigma_Y} = \frac{1}{B_{i,t}}\frac{\partial B_{i,t}}{\partial \sigma^2_{Y,t}}$, and $\Delta^B_{\sigma_X} = \frac{1}{B_{i,t}}\frac{\partial B_{i,t}}{\partial \sigma^2_{X,i,t}}$ at any point of time. Finally, using the results in Propositions 1 and 2 allows us to compute model-implied stock-bond spot covariance and correlation, as well as stock and bond spot volatilities.

[53] Although we present the estimation of p_D and CS separately in this Appendix, it is worth noting that their estimation is done simultaneously and relies on the same simulations.

Because idiosyncratic risk can be diversified away, it can be shown that the returns of an equally-weighted portfolio of stocks and bonds follow[54]

$$\frac{1}{N}\sum_{i=1}^{N}\frac{dS_{i,t}}{S_{i,t}} = \frac{1}{N}\sum_{i=1}^{N}\left[\mu_{S,i,t}dt + \Delta_X^S\beta_i\sigma_{Y,t}dW_t^Y + \Delta_{\sigma_Y}^S\delta_Y\sigma_{Y,t}dW_t^{\sigma_Y}\right] \tag{A.11}$$

$$\frac{1}{N}\sum_{i=1}^{N}\frac{dB_{i,t}}{B_{i,t}} = \frac{1}{N}\sum_{i=1}^{N}\left[\mu_{B,i,t}dt + \Delta_X^B\beta_i\sigma_{Y,t}dW_t^Y + \Delta_{\sigma_Y}^B\delta_Y\sigma_{Y,t}dW_t^{\sigma_Y}\right]. \tag{A.12}$$

Building on these equations, we approximate aggregate stock and bond return variance, denoted by $\sigma^2_{S,t}$ and $\sigma^2_{B,t}$, as

$$\sigma^2_{S,t} = \frac{1}{N}\sum_{i=1}^{N}\left[\left(\Delta_X^S\right)^2\beta_i^2 + \left(\Delta_{\sigma_Y}^S\right)^2\delta_Y^2 + 2\left[\Delta_X^S\Delta_{\sigma_Y}^S\right]\left(\rho_Y\beta_i\delta_Y\right)\right]\sigma^2_{Y,t} \tag{A.13}$$

$$\sigma^2_{B,t} = \frac{1}{N}\sum_{i=1}^{N}\left[\left(\Delta_X^B\right)^2\beta_i^2 + \left(\Delta_{\sigma_Y}^B\right)^2\delta_Y^2 + 2\left[\Delta_X^B\Delta_{\sigma_Y}^B\right]\left(\rho_Y\beta_i\delta_Y\right)\right]\sigma^2_{Y,t}. \tag{A.14}$$

Finally, we estimate the stock and bond systematic variance ratios, denoted by $Syst_{S,i,t}$ and $Syst_{B,i,t}$, as

$$Syst_{S,i,t} = \frac{\left[\left(\Delta_X^S\right)^2\beta_i^2 + \left(\Delta_{\sigma_Y}^S\right)^2\delta_Y^2 + 2\left[\Delta_X^S\Delta_{\sigma_Y}^S\right]\left(\rho_Y\beta_i\delta_Y\right)\right]\sigma^2_{Y,t}}{\sigma^2_{S,i,t}} \tag{A.15}$$

$$Syst_{B,i,t} = \frac{\left[\left(\Delta_X^B\right)^2\beta_i^2 + \left(\Delta_{\sigma_Y}^B\right)^2\delta_Y^2 + 2\left[\Delta_X^B\Delta_{\sigma_Y}^B\right]\left(\rho_Y\beta_i\delta_Y\right)\right]\sigma^2_{Y,t}}{\sigma^2_{B,i,t}}, \tag{A.16}$$

where $\sigma^2_{S,i,t}$ and $\sigma^2_{B,i,t}$ are the stock and bond total variances, and which are presented in Proposition 2.

Empirical moments The following describes the construction of the empirical moments, used in the calibration and reported in Table 11. The empirical data sample period spans August 2003 - August 2020.

1. The construction/collection of Leverage and the CDS spread (from Markit) is detailed in Table 1 and Appendix B.
2. Default Probability is the 10-year historical default rate for BB-rated firms over the 1981-2020 period from Standard and Poor's (2021).

[54] Note that in Equations (A.11), (A.12), (A.13), (A.14), (A.15), and (A.16), the dependence of bond and stock sensitivities to i (and t) is omitted for ease of notation.

3. Stock (Bond) Return Volatility is the standard deviation of stock (bond) returns estimated with a rolling window, whereby we require a minimum of 12 monthly observations.[55]

4. Stock (Bond) Systematic Variance Ratio is the ratio of stock (bond) systematic variance to total variance. This ratio is estimated as the R-squared of a regression of stock (bond) returns on a stock (bond) value-weighted index estimated with a 12-month rolling window.

5. Aggregate Stock (Bond) Return Volatility is equal to the 12-month rolling standard deviation of a value-weighted stock (bond) index's returns. The index is constructed using market capitalisation as weights and uses stock and bond returns from firms that are contained within our sample.

6. Asset Volatility is computed as the 12-month rolling window standard deviation of asset returns, given by $r_{A,i,t} = L_{i,t} \times r_{B,i,t} + (1 - L_{i,t}) \times r_{S,i,t}$, where $r_{B,i,t}$ and $r_{S,i,t}$ are stock and bond returns for firm i in month t, while $L_{i,t}$ denotes the firm's leverage.

7. Stock-bond Correlation/Covariance are computed using firm-level stock and bond returns with a 12-month rolling window.

G.5 Parameter estimates

Using the set of moment conditions described above, we estimate the vector of parameters Θ by solving the optimization problem (22). We obtain the following parameter estimates. The debt coupon is $c = 0.0097$ and the default barrier is $X_B = 0.5596$, which generates leverage and default probability that closely match their empirical counterparts. Regarding the aggregate asset dynamics and the corresponding price of risk, we have $\kappa_Y = 0.9703$, $\theta_Y = 0.60\%$, $\delta_Y = 0.1076$, $\rho_Y = -0.2698$, and an unconditional growth of asset of $\mu_{Y,\infty} = \lambda_Y \cdot \theta_Y = 2.1959\%$, where $\lambda_Y = \sqrt{1-\rho_Y^2}\lambda_{Y\perp\sigma_Y} + \rho_Y \lambda_{\sigma_Y} = 3.6495$ (see Equation A.3). We discuss these parameter values in light of the literature below.

Because the pricing of stocks and bonds is obtained under the risk-neutral measure $\mathbb{Q}$, it is important to study the implications of the parameter estimated for risk-neutral aggregate variance dynamics. The calibrated risk-neutral unconditional variance of aggregate risk is $\theta_Y^* = 4.19\%$. Thus, the level of risk-neutral variance increases substantially from $\mathbb{P}$ to $\mathbb{Q}$. This negative variance risk premium is important to generate reasonable levels of risk premia and, thus, CDS spreads. This is consistent with Du et al. (2019) who also find that CDS spread data requires a large and negative asset variance risk premia.

Regarding the firm idiosyncratic variance parameters, we have $\kappa_X = 0.8245$, $\theta_X = 2.11\%$, and $\delta_X = 0.1861$. Note that idiosyncratic asset variance is more persistent than aggregate asset variance, since $\kappa_X \leqslant \kappa_Y$. This parametrization is broadly consistent with existing evidence indicating that idiosyncratic variance for the average stock is more persistent than for the stock market index variance (see, e.g., Christoffersen et al., 2018). Furthermore, a level of idiosyncratic variance of 2.11% implies that a firm with unit exposure to aggregate risk has an unconditional proportion of asset systemic risk equal to

[55] The rolling-window expands up from 12 to 36-months to align the window with the rolling period used in Bai et al. (2021).

22.22%, i.e., $\theta_Y/(\theta_Y+\theta_X)$. Finally, a firm with unit exposure to aggregate risk has an unconditional asset Sharpe ratio, $\lambda_Y\theta_Y/\sqrt{(\theta_Y+\theta_X)}$, of 0.1335. The estimated parameters imply the model-implied moment conditions presented in Table 11. Overall the model fit is good, as indicated by the fact that the data and model in-sample moment conditions are close to each other, on average.

These parameter values are comparable to those reported in the equity and credit risk literature. For instance, the parameter estimated implies that the total variance of assets of a unit-beta firm is estimated to be about 2.76% (i.e., $\theta_Y+\theta_X$). This estimate is comparable to the median total asset variance estimate of 3% reported in Du et al. (2019). The authors calibrate an asset Sharpe ratio of 22% which is somewhat close to the 13% implied by our calibration.[56]

H Model extension – stochastic interest rates

This Appendix presents an extended version of the model with stochastic interest rates. First, we present the dynamics of the short rate and of the modified stochastic discount factor (SDF). We then highlight the impact of stochastic interest rates on equity and corporate bond pricing. Finally, we describe the simulation methodology.

H.1 Dynamics of the short rate and the SDF

We assume that the instantaneous risk-free rate (also called the short rate) is stochastic and can be described by the following square-root process:

$$dr_t \quad = \quad \kappa_r(\theta_r - r_t)dt + \delta_r\sqrt{r_t}dW_t^r, \tag{A.17}$$

where κ_r is the mean reversion speed of the short rate, θ_r its long-run mean, and δ_r its volatility parameter. The new Brownian dW_t^r is the source of interest rate risk, which we assume to be independent of $dW_t^{\sigma_Y}$ and $dW_t^{Y\perp\sigma_Y}$ for parsimony. We replace the constant interest rate with r_t in individual asset and aggregate risk dynamics.

Consistent with our previous specification, the SDF depends linearly on systematic risks as follows:

$$\frac{d\phi_t}{\phi_t} \quad = \quad -r_t dt - \sigma_{Y,t}\lambda_{Y\perp\sigma_Y}dW_t^{Y\perp\sigma_Y} - \sigma_{Y,t}\lambda_{\sigma_Y^2}dW_t^{\sigma_Y} - \sqrt{r_t}\lambda_r dW_t^r, \tag{A.18}$$

where $\lambda_{Y\perp\sigma_Y}$, $\lambda_{\sigma_Y^2}$, and λ_r denote the risk premium parameters on the three Brownian motions driving systematic unlevered asset value and variance risk, as well as interest rate risk. Note that firm specific risk, $dB_{i,t}^X$ and $dB_{i,t}^{\sigma_X}$, are deliberately assumed not to be priced, as in our baseline model.

[56]Note that the cross-section of firms we consider is much larger than the one studied in Du et al. (2019) and is composed of riskier firms, which may explain our slightly lower Sharpe ratio.

H.2 Pricing of corporate bond and equity

We now discuss the valuation of corporate bond and equity. We follow the same debt structure as our benchmark model with constant interest rate. Similarly to Leland (1994), the firm issues a consol bond and equity. Default happens when the firm's asset value reaches the exogenous default boundary. The value of the firm's bond can then be written as

$$B_{i,t} \quad = \quad cE_t^{\mathbb{Q}}\left[\int_t^{\infty} e^{-\int_t^s r_s ds} \cdot 1_{(\tau>s)} ds\right] + (1-\alpha)X_B E_t^{\mathbb{Q}}\left[\int_t^{\infty} e^{-\int_t^s r_s ds} \cdot 1_{(\tau=s)} ds\right], \tag{A.19}$$

where τ is the date at which default occurs and $1_{(\tau>s)}$ is an indicator function that equals 1 if the borrower survives beyond date s, and zero otherwise; $1_{(\tau=s)}$ takes the value 1 if the default time occurs at time s.

Given that interest rate shocks are independent from other sources of risk, we can rewrite (A.19) as

$$B_{i,t} \quad = \quad c\int_t^{\infty} p_t(r_t; s) \cdot E_t^{\mathbb{Q}}\left[1_{(\tau>s)}\right] ds + (1-\alpha)X_B \int_t^{\infty} p_t(r_t; s) \cdot E_t^{\mathbb{Q}}\left[1_{(\tau=s)}\right] ds, \tag{A.20}$$

where $p_t(r_t; s) \equiv E_t^{\mathbb{Q}}\left[e^{-\int_t^s r_s ds}\right]$ denotes the time-t price of a default-free discount bond with maturity s, which is known in closed form. Similarly, the discounted values of the tax shield (TS) and distress costs (DC) can be written as

$$TS_{i,t} = \zeta c \int_t^{\infty} p_t(r_t; s) \cdot E_t^{\mathbb{Q}}\left[1_{(\tau>s)}\right] ds \tag{A.21}$$

$$DC_{i,t} = \alpha \int_t^{\infty} p_t(r_t; s) \cdot E_t^{\mathbb{Q}}\left[1_{(\tau=s)}\right] ds, \tag{A.22}$$

where ζ and α denote the effective corporate tax rate and proportional distress costs, respectively. Finally, the value of equity is the difference between the levered firm value $v_{i,t}^L$ and the bond $B_{i,t}$:

$$E_{i,t} \quad = \quad v_{i,t}^L - B_{i,t} = X_{i,t} + TS_{i,t} - DC_{i,t} - B_{i,t}. \tag{A.23}$$

H.3 Simulation methodology

In this section we briefly elaborate on the simulation methodology, which closely follows Section G.1 and Section G.2. The key difference is that we now need to simulate 4 state variables $\left\{X_{i,t}, \sigma_{Y,t}^2, \sigma_{X,i,t}^2, r_t\right\}$

instead of 3. Additionally, we must determine the following functional forms:

$$
\begin{aligned}
z1\left(X_{i,t},\sigma_{Y,t}^2,\sigma_{X,i,t}^2,r_t,\Pi_k\right) &= \int_t^\infty p_t(r_t;s)\cdot E_t^{\mathbb{Q}}\left[1_{(\tau>s)}\right]ds \\
z2\left(X_{i,t},\sigma_{Y,t}^2,\sigma_{X,i,t}^2,r_t,\Pi_k\right) &= \int_t^\infty p_t(r_t;s)\cdot E_t^{\mathbb{Q}}\left[1_{(\tau=s)}\right]ds,
\end{aligned}
$$

as well as $CS\left(X_{i,t},\sigma_{Y,t}^2,\sigma_{X,i,t}^2,r_t,10,\Pi_k\right)$ for a given set of structural parameters $\Pi_k\equiv\{\Theta,\beta_k\}$ where $k=\{1,..,5\}$. To this end, we follow closely the steps outlined in Section G.2 and Section G.3. The only difference between this framework and our benchmark set-up with constant interest rate is that we need to estimate the functional form of 3 functions $z1(\cdot,\cdot,\cdot,\cdot,\Pi_k)$, $z2(\cdot,\cdot,\cdot,\cdot,\Pi_k)$, and $CS(\cdot,\cdot,\cdot,\cdot,10,\Pi_k)$ instead of 2 (i.e., p_D and CS). As before, we estimate these functions using non-parametric methods following the method outlined in Section G.2 and Section G.3.[57]

Similarly to our benchmark model, we obtain model-implied moments using simulation methods. Again, firms in the economy can take on 5 values of asset beta (0.6, 0.8, 1, 1.2, 1.4) with equal probability. To initialize the economy, we start with a cross-section of 250 firms for each asset beta. Each firm starts with the same initial unlevered asset value $X_{i,0}=1$ and with $\sigma_{X,i,0}^2=\theta_X$. Initial aggregate spot variance is set to $\sigma_{Y,0}^2=\theta_Y$ and the short rate to $r_0=\theta_r$. Using these initial values, we then simulate firms and aggregate asset risk under the physical probability measure over 10 years. Our simulation of the state variables exploits a Euler discretization of the state dynamics with daily time steps. Note that we impose the same filters as for our benchmark approach in Section G.1 to replace firms in the simulations.

We then adopt the same strategy to estimate the model-implied moments from the simulations. For all measures, except the physical default probability, we use the spot values implied by our model, which we compute based on the simulated state variables.

H.4 Covariance, correlation, and other model implications

Given the joint dynamics of stock and bond returns, it is relatively straightforward to extend the benchmark results for the stock-bond covariance (Proposition F.1) and correlation (Proposition F.2) implied by the model in the presence of interest risk. We have:

[57] Relative to the constant interest rate set-up, we adopt here the following grid to estimate the Chebychev polynomial loadings of each function. We consider a grid composed of 5 nodes for asset value and 4 nodes for the short rate and spot variances. The lower and upper bounds for asset values are $1.1\cdot X_B$ and 2, respectively. The lower and upper bounds for spot systematic (idiosyncratic) variances correspond to $\theta_Y/100$ ($\theta_X/100$) and $\theta_Y\cdot 8$ ($\theta_X\cdot 8$), respectively. Finally, the lower and upper bounds for short rate are $\theta_r/10$ and $\theta_r\cdot 4$, respectively. These bounds combined with our Chebychev polynomial approach define the nodes.

Proposition F.1: *From the dynamics of stock and bond returns, the stock-bond covariance for firm i at date t, denoted by $\sigma_{S,B,i,t}$, satisfies*

$$\begin{aligned}\sigma_{S,B,i,t} &= \underbrace{\Delta_X^S\Delta_X^B var\left(\frac{dX_{i,t}}{X_{i,t}}\right)}_{Asset\,risk} + \underbrace{\Delta_{\sigma_Y}^S\Delta_{\sigma_Y}^B var\left(d\sigma_{Y,t}^2\right) + \Delta_{\sigma_X}^S\Delta_{\sigma_X}^B var\left(d\sigma_{X,i,t}^2\right)}_{Variance\,risk} \\ &\quad + \underbrace{\left[\Delta_X^S\Delta_{\sigma_Y}^B + \Delta_X^B\Delta_{\sigma_Y}^S\right] cov\left(\frac{dX_{i,t}}{X_{i,t}}, d\sigma_{Y,t}^2\right)}_{Co-skewness\,risk} + \underbrace{\Delta_r^S\Delta_r^B var\left(dr_t\right)}_{Interest\,rate\,risk} \qquad \text{(A.24)} \\ &= \underbrace{\Delta_X^S\Delta_X^B\left[\beta_i^2\sigma_{Y,t}^2 + \sigma_{X,i,t}^2\right]}_{Asset\,risk} dt + \underbrace{\Delta_{\sigma_Y}^S\Delta_{\sigma_Y}^B\delta_Y^2\sigma_{Y,t}^2 dt + \Delta_{\sigma_X}^S\Delta_{\sigma_X}^B\delta_X^2\sigma_{X,i,t}^2}_{Variance\,risk} dt \\ &\quad + \underbrace{\left[\Delta_X^S\Delta_{\sigma_Y}^B + \Delta_X^B\Delta_{\sigma_Y}^S\right]\rho_Y\beta_i\delta_Y\sigma_{Y,t}^2}_{Co-skewness\,risk} dt + \underbrace{\Delta_r^S\Delta_r^B\delta_r^2 r_t dt}_{Interest\,rate\,risk}. \qquad \text{(A.25)}\end{aligned}$$

in the presence of interest risk where $\Delta_r^S = \frac{1}{S_{i,t}}\frac{\partial S_{i,t}}{\partial r_t}$ and $\Delta_r^B = \frac{1}{B_{i,t}}\frac{\partial B_{i,t}}{\partial r_t}$ are the sensitivities of stock and bond to interest rate risk.

Proposition F.2: *With stochastic interest rates, stock and bond instantaneous variance and correlation are given by*

$$\sigma_{S,i,t}^2 = \left(\Delta_X^S\right)^2 var\left(\frac{dX_{i,t}}{X_{i,t}}\right) + \left(\Delta_{\sigma_Y}^S\right)^2 var\left(d\sigma_{Y,t}^2\right) + \left(\Delta_{\sigma_X}^S\right)^2 var\left(d\sigma_{X,i,t}^2\right) \qquad \text{(A.26)}$$

$$+ 2\left[\Delta_X^S\Delta_{\sigma_Y}^S\right] cov\left(\frac{dX_{i,t}}{X_{i,t}}, d\sigma_{Y,t}^2\right) + \left(\Delta_r^S\right)^2 var\left(dr_t\right)$$

$$\sigma_{B,i,t}^2 = \left(\Delta_X^B\right)^2 var\left(\frac{dX_{i,t}}{X_{i,t}}\right) + \left(\Delta_{\sigma_Y}^B\right)^2 var\left(d\sigma_{Y,t}^2\right) + \left(\Delta_{\sigma_X}^B\right)^2 var\left(d\sigma_{X,i,t}^2\right) \qquad \text{(A.27)}$$

$$+ 2\left[\Delta_X^B\Delta_{\sigma_Y}^B\right] cov\left(\frac{dX_{i,t}}{X_{i,t}}, d\sigma_{Y,t}^2\right) + \left(\Delta_r^B\right)^2 var\left(dr_t\right)$$

$$\rho_{S,B,i,t} = \frac{\sigma_{S,B,i,t}}{\sigma_{S,i,t}\sigma_{B,i,t}}, \qquad \text{(A.28)}$$

respectively.

Because idiosyncratic risk can be diversified away, it can be shown that the returns of an equally-weighted portfolio of stocks and bonds follow

$$\frac{1}{N}\sum_{i=1}^{N}\frac{dS_{i,t}}{S_{i,t}} = \frac{1}{N}\sum_{i=1}^{N}\left[\mu_{S,i,t}dt + \Delta_X^S\beta_i\sigma_{Y,t}dW_t^Y + \Delta_{\sigma_Y}^S\delta_Y\sigma_{Y,t}dW_t^{\sigma_Y} + \Delta_r^S\delta_r\sqrt{r_t}dW_t^r\right] \quad (A.29)$$

$$\frac{1}{N}\sum_{i=1}^{N}\frac{dB_{i,t}}{B_{i,t}} = \frac{1}{N}\sum_{i=1}^{N}\left[\mu_{B,i,t}dt + \Delta_X^B\beta_i\sigma_{Y,t}dW_t^Y + \Delta_{\sigma_Y}^B\delta_Y\sigma_{Y,t}dW_t^{\sigma_Y} + \Delta_r^B\delta_r\sqrt{r_t}dW_t^r\right] \quad (A.30)$$

Building on these equations, we can approximate aggregate stock and bond return variance, denoted by $\sigma_{S,t}^2$ and $\sigma_{B,t}^2$, as[58]

$$\sigma_{S,t}^2 = \frac{1}{N}\sum_{i=1}^{N}\left[\left[\left(\Delta_X^S\right)^2\beta_i^2 + \left(\Delta_{\sigma_Y}^S\right)^2\delta_Y^2 + 2\left[\Delta_X^S\Delta_{\sigma_Y}^S\right](\rho_Y\beta_i\delta_Y)\right]\sigma_{Y,t}^2 + \left(\Delta_r^S\right)^2 r_t\right] \quad (A.31)$$

$$\sigma_{B,t}^2 = \frac{1}{N}\sum_{i=1}^{N}\left[\left[\left(\Delta_X^B\right)^2\beta_i^2 + \left(\Delta_{\sigma_Y}^B\right)^2\delta_Y^2 + 2\left[\Delta_X^B\Delta_{\sigma_Y}^B\right](\rho_Y\beta_i\delta_Y)\right]\sigma_{Y,t}^2 + \left(\Delta_r^B\right)^2 r_t\right]. \quad (A.32)$$

Finally, we can estimate the stock and bond systematic variance ratios, denoted by $Syst_{S,i,t}$ and $Syst_{B,i,t}$, as

$$Syst_{S,i,t} = \frac{\left[\left(\Delta_X^S\right)^2\beta_i^2 + \left(\Delta_{\sigma_Y}^S\right)^2\delta_Y^2 + 2\left[\Delta_X^S\Delta_{\sigma_Y}^S\right](\rho_Y\beta_i\delta_Y)\right]\sigma_{Y,t}^2 + \left(\Delta_r^S\right)^2 r_t}{\sigma_{S,i,t}^2} \quad (A.33)$$

$$Syst_{B,i,t} = \frac{\left[\left(\Delta_X^B\right)^2\beta_i^2 + \left(\Delta_{\sigma_Y}^B\right)^2\delta_Y^2 + 2\left[\Delta_X^B\Delta_{\sigma_Y}^B\right](\rho_Y\beta_i\delta_Y)\right]\sigma_{Y,t}^2 + \left(\Delta_r^B\right)^2 r_t}{\sigma_{B,i,t}^2}, \quad (A.34)$$

where $\sigma_{S,i,t}^2$ and $\sigma_{B,i,t}^2$ are the total stock and bond variances, and presented in Proposition F.2.

Using the above results, we can estimate model-implied moments following closely the methodology outlined in Section G.4.

H.5 Parameter estimates

Our first objective is to obtain the structural parameters driving the dynamics of the short rate. To this end, we obtain monthly data on the 3-month T-bill between August 1, 2003 and July 1, 2020. Setting r_t to the observed rate, we estimate the physical parameters of the short rate by Maximum Likelihood. At this stage, the conditional likelihood of the square-root process, when r_t is assumed to be observed without measurement errors, is given by the Bessel function. We get $\kappa_r = 0.1896$, $\theta_r = 0.0101$, and $\delta_r = 0.0619$. We then estimate the λ_r in a second step by maximum likelihood where we assume that

[58] Note that in Equations (A.31), (A.32), (A.33), and (A.34), the dependence of bond and stock sensitivities to i (and t) is omitted for ease of notation.

the errors between the observed yield of the 10-year T-Bond and the model prediction are normally distributed. We get $\lambda_r = -2.2534$ which implies that the risk-neutral unconditional level of the short rate is higher than its physical counterpart. This is consistent with an upward-sloping term structure of interest rates during normal market conditions.

To make the results as comparable as possible between the stochastic interest rate model and the benchmark constant interest rate model without performing a formal optimization, we adopt the following strategy. First, we set the remaining parameters to that of the benchmark model estimates, except the default barrier X_B and the debt coupon c. The reason is that the constraints that need to be imposed to ensure the right signs of equity/debt exposures to the state variables are not the same between the two versions of the models, simply because the pricing formulas are different. To ensure the right economic responses of equity/debt to changes in the state values, while matching the other moments relatively well, we construct a grid of X_B and c. We then estimate for each value the model-implied moments using 10 simulations of 10 years, as in the baseline case. We then select the default barrier and coupon values that imply the smallest errors when using the same target moments as the ones used to estimate the parameters of the benchmark model. We get $X_B = 0.4920$ and $c = 0.0248$. Finally, we conduct simulations based on these values and the remaining structural parameters.

Table A.1 : Studies on the determinants of the relation between stock and bonds In this table, we summarize the key literature on the determinants of the relation between stock and corporate bonds. The literature is characterized by four broad areas which we break down as 'Focus', 'Explanation', 'Econometrics' and 'Theory'. Within each subject area there is a further sub-category unique to each area. The final column of the table emphasizes the scope of this research paper relative to the other papers in the literature with bold tick marks.

		Kwan (1996)	Collin-Dufresne et al. (2001)	Schaefer & Strebulaev (2008)	Kapadia & Pu (2012)	Friewald & et al. (2014)	Bao & Hou (2017)	Chordia et al.(2017)	Choi & Kim (2018)	Augustin et al. (2020)	Collin-Dufresne et al. (2020)	Du et al. (2020)	Bali et al. (2021)	**Present study**
Focus	Stock-bond relation	✓	✓	✓	✓	✓	✓		✓	✓	✓			**✓**
	Individual moments							✓			✓	✓	✓	**✓**
Explanation	Expected inflation													
	Aggregate risk premia								✓		✓	✓		
	Market segmentation/integration				✓				✓	✓	✓			
	Firm/bond characteristics						✓	✓					✓	
	Default risk	✓	✓	✓			✓					✓	✓	**✓**
Econometrics	Cross section				✓	✓	✓		✓	✓			✓	**✓**
	Time series		✓				✓			✓	✓		✓	**✓**
	Hedge ratio	✓		✓			✓		✓	✓				**✓**
	Covariance/correlation				✓					✓				**✓**
	Predictability					✓		✓					✓	**✓**
	Investment strategy													**✓**
Theory	Complete markets			✓		✓	✓			✓	✓	✓		**✓**
	Default risk			✓		✓	✓			✓	✓	✓		**✓**
	Jumps										✓			
	Stochastic variance									✓		✓		**✓**
	Idiosyncratic variance risk													**✓**

Table A.2 : Predictability of stock-bond comovement with default risk – alternative specifications This table presents results on the predictive relation between stock-bond comovement and default risk under alternative econometric specifications. Columns (1)-(3) report results from panel regressions with firm fixed effects and different combinations of standard error (SE) clustering (bond, firm, industry, and month), including all control variables. Columns (4)-(6) report results from panel regressions with firm and time (month) fixed effects and different combinations of standard error clustering (bond, firm, industry, and month). Due to the inclusion of the time fixed effect, Columns (4)-(6) only include controls that vary in the cross-section. The 'Default Risk' variable is constructed as the sign-corrected average of the three standardized default risk proxies, namely Leverage, Distance-to-Default, and Credit spread. The dependent variable is the 12-month ahead rolling covariance (Panel A) and correlation (Panel B) of stock and corporate bond returns. Clustering at the industry level uses the Fama and French 17 industry classification. Observations are at the corporate bond level. The sample period spans August 2003 - August 2020. The definitions of the variables and their data sources are presented in Section 1.1 and in Online Appendix A and B. Significance at the 10%, 5%, 1% level is indicated by *, **, ***, respectively.

Panel A: Covariance

	Firm Fixed Effects			*Firm & Time Fixed Effects*		
	(1)	(2)	(3)	(4)	(5)	(6)
Default Risk	1.25***	1.25***	1.25***	1.24***	1.24***	1.24***
t-stat	(9.14)	(6.22)	(12.78)	(8.59)	(6.50)	(12.37)
$R^2_{Adj.}$	0.322	0.322	0.322	0.382	0.382	0.382
Obs.	514,017	514,017	514,017	514,017	514,017	514,017
SE	Month & Firm	Month & Industry	Month & Bond	Month & Firm	Month & Industry	Month & Bond
Fixed Effects	Firm	Firm	Firm	Month & Firm	Month & Firm	Month & Firm
Controls	All	All	All	Stock & Bond Characteristics	Stock & Bond Characteristics	Stock & Bond Characteristics

Panel B: Correlation

	Firm Fixed Effects			*Firm & Time Fixed Effects*		
	(1)	(2)	(3)	(4)	(5)	(6)
Default Risk	0.09***	0.09***	0.09***	0.07***	0.07***	0.07***
t-stat	(8.07)	(7.01)	(10.74)	(7.27)	(7.08)	(10.22)
$R^2_{Adj.}$	0.225	0.225	0.225	0.281	0.281	0.281
Obs.	514,017	514,017	514,017	514,017	514,017	514,017
SE	Month & Firm	Month & Industry	Month & Bond	Month & Firm	Month & Industry	Month & Bond
Fixed Effects	Firm	Firm	Firm	Month & Firm	Month & Firm	Month & Firm
Controls	All	All	All	Stock & Bond Characteristics	Stock & Bond Characteristics	Stock & Bond Characteristics

Table A.3 : Predictability of stock-bond comovement with default risk – Raw vs. excess returns This table presents results on the predictive relation between stock-bond comovement and default risk across different return specifications. Columns (1) and (3) use raw stock and bond returns. Columns (2) and (4) consider stock and bond returns computed in excess of the one-month U.S. T-Bill return. Columns (3) and (6) consider stock and bond returns computed in excess of the maturity-matched Treasury bond return. Corporate bond excess returns are computed at the bond level and then aggregated (using equal-weights) at the firm level. Columns (1)-(3) in Panels A and B report results from panel regressions with firm fixed effects. Columns (4)-(6) in Panel A and B report results using the Fama-MacBeth approach. The sample period spans August 2003 - August 2020. The definitions of the variables and their data sources are presented in Section 1.1 and in Online Appendix A and B. Significance at the 10%, 5%, 1% level is indicated by *, **, ***, respectively.

Panel A: Covariance

	Baseline			Fama-MacBeth		
	(1)	(2)	(3)	(4)	(5)	(6)
	Raw returns	Excess of 1M T-Bill return	Excess of maturity-matched Treasury return	Raw return	Excess of 1M T-Bill return	Excess of maturity maturity-matched Treasury return
Default Risk	0.772***	0.769***	0.896***	1.078***	1.075***	1.195***
t-stat	(9.857)	(9.828)	(10.240)	(8.418)	(8.398)	(8.441)
$R^2_{Adj.}$	0.366	0.365	0.365	0.240	0.239	0.241
Obs.	117,822	117,822	117,822	117,822	117,822	117,822
SE	Month & Firm	Month & Firm	Month & Firm	Newey- & West	Newey- & West	Newey- & West
Fixed Effects	Firm	Firm	Firm	None	None	None
Controls	None	None	None	None	None	None

Panel B: Correlation

	Baseline			Fama-MacBeth		
	(1)	(2)	(3)	(4)	(5)	(6)
	Raw returns	Excess of 1M T-Bill return	Excess of maturity-matched Treasury return	Raw return	Excess of 1M T-Bill return	Excess of maturity maturity-matched Treasury return
Default Risk	0.043***	0.043***	0.038***	0.160***	0.160***	0.073***
t-stat	(4.879)	(4.850)	(3.144)	(10.614)	(10.588)	(11.019)
$R^2_{Adj.}$	0.279	0.279	0.210	0.126	0.126	0.035
Obs.	117,822	117,822	117,822	117,822	117,822	117,822
SE	Month & Firm	Month & Firm	Month & Firm	Newey- & West	Newey- & West	Newey- & West
Fixed Effects	Firm	Firm	Firm	None	None	None
Controls	None	None	None	None	None	None

Table A.4 : Predictability of stock-bond comovement with default risk – role of the control variables This table presents results on the predictive relation between stock-bond comovement and default risk after controlling for alternative explanations. The dependent variable is the one-year-ahead covariance and correlation at the firm level in Panel A and at the bond level in Panel B. All specifications include firm fixed effects. We report t-statistics in parentheses, using standard errors (SE) double clustered at the month and firm levels. The sample period spans August 2003 - August 2020. The definitions of the variables and their data sources are presented in Section 1.1 and in Online Appendix A and B. Significance at the 10%, 5%, 1% level is indicated by *, **, ***, respectively.

	Panel A: Firm Level		*Panel B: Bond Level*	
	Covariance	Correlation	Covariance	Correlation
	(1)	(2)	(3)	(4)
Default Risk	0.67***	0.08***	1.25***	0.09***
t-stat	(9.62)	(6.52)	(9.14)	(8.07)
Bond Coupon	-0.21***	-0.01	-0.18***	-0.01***
t-stat	(-3.88)	(-1.02)	(-3.75)	(-3.30)
Bond Maturity	0.01	0.00	0.10***	0.01**
t-stat	(0.61)	(0.05)	(4.57)	(2.11)
Bond Size (Amount Outstanding)	0.39***	0.07***	0.13***	0.03***
t-stat	(5.14)	(4.91)	(3.10)	(4.60)
Callable Dummy	-0.31***	-0.05**	-0.15*	-0.02**
t-stat	(-2.86)	(-2.04)	(-1.73)	(-2.10)
Bond Liquidity (Bid-Ask Spread)	0.03	0.00	0.11***	-0.00
t-stat	(1.33)	(0.29)	(2.69)	(-0.85)
Equity Size (Market Capitalization)	-0.28***	0.03**	-0.29*	0.04**
t-stat	(-3.64)	(2.09)	(-1.89)	(2.56)
Equity Illiquidity (Amihud, 2002)	-0.11***	-0.00	-0.17**	-0.01**
t-stat	(-2.98)	(-0.55)	(-2.45)	(-2.23)
Market-to-Book	-0.01	0.01	-0.00	0.01*
t-stat	(-0.70)	(1.43)	(-0.09)	(1.73)
Aggregate Illiquidity (Hu et al., 2013)	-0.14	-0.02	0.03	-0.03**
t-stat	(-0.89)	(-1.41)	(0.17)	(-2.07)
Aggregate Illiquidity (Pastor and Stambaugh, 2003)	-0.13	-0.02	-0.10	-0.01
t-stat	(-1.49)	(-1.49)	(-1.05)	(-1.35)
Intermediary Capital Risk Factor	-0.08	0.02	-0.14	0.01
t-stat	(-0.87)	(1.57)	(-1.19)	(0.91)
3m T-Bill Rate	1.47***	0.14**	2.03***	0.08
t-stat	(3.42)	(2.25)	(4.30)	(1.42)
10y T-Bond Rate	-0.68**	-0.10**	-1.07***	-0.07
t-stat	(-2.11)	(-2.11)	(-2.99)	(-1.60)
Slope (10Y-3m Rate)	0.60**	0.04	0.78**	0.01
t-stat	(2.25)	(1.05)	(2.60)	(0.20)
Expected Inflation	-0.38**	-0.04**	-0.46**	-0.03**
t-stat	(-2.13)	(-2.26)	(-2.08)	(-2.05)
Macro Uncertainty Index	-0.14	-0.01	-0.23	0.01
t-stat	(-1.16)	(-0.72)	(-1.59)	(1.05)
Business Conditions Index	-0.32***	-0.03**	-0.35***	-0.02**
t-stat	(-3.25)	(-2.57)	(-3.06)	(-2.27)
$R^2_{Adj.}$	0.414	0.308	0.322	0.225
Obs.	114,755	114,755	514,017	514,017
SE	Month & Firm	Month & Firm	Month & Firm	Month & Firm
Fixed Effects	Firm	Firm	Firm	Firm
Controls	All	All	All	All

Table A.5 : Predictability of stock-bond comovement with default risk – different horizons This table presents results on the predictive ability of default risk for stock-bond comovement computed over different horizons. The dependent variable is the covariance (Panel A) and correlation (Panel B) between stock and corporate bond returns over the following 12 to 60 months. The 'Default Risk' variable is constructed as the sign-corrected average of the three standardized default risk proxies, namely Leverage, Distance-to-Default, and Credit Spread. Regressions include all controls and firm fixed effects. We report t-statistics in parentheses, using standard errors (SE) double clustered at the month and firm levels. The sample period spans August 2003 - August 2020. The definitions of the variables and their data sources are presented in Section 1.1 and in Online Appendix A and B. Significance at the 10%, 5%, 1% level is indicated by *, **, ***, respectively.

	Panel A: Covariance					*Panel B: Correlation*				
	12m (1)	24m (2)	36m (3)	48m (4)	60m (5)	12m (6)	24m (7)	36m (8)	48m (9)	60m (10)
Default Risk	0.67***	0.45***	0.34***	0.26***	0.13***	0.08***	0.08***	0.07***	0.06***	0.03***
t-stat	(9.62)	(7.92)	(6.54)	(5.29)	(2.98)	(6.52)	(7.80)	(7.20)	(6.49)	(3.18)
$R^2_{Adj.}$	0.414	0.538	0.637	0.712	0.752	0.308	0.467	0.560	0.632	0.677
Obs.	114,755	96,024	81,267	68,641	58,125	114,755	96,024	81,267	68,641	58,125
SE	Month & Firm	Month & Firm	Month & Firm	Month & Firm	Month & Firm	Month & Firm	Month & Firm	Month & Firm	Month & Firm	Month & Firm
Fixed Effects	Firm	Firm	Firm	Firm	Firm	Firm	Firm	Firm	Firm	Firm
Controls	All	All	All	All	All	All	All	All	All	All

Table A.6 : Additional model results – Constant vs. stochastic interest rates This table compares results from a model with constant vs. stochastic interest rates. Panel A presents descriptive statistics for covariance and correlation, while Panel B presents results on the predictive relation between stock-bond comovement and a composite measure of default risk. Columns (1) and (3) report the baseline results with firm fixed effects. Columns (2) and (4) present cross-sectional results using the Fama-MacBeth approach. The dependent variable is the one-year-ahead covariance (Columns 1 and 2) and correlation (Columns 3 and 4) between stock and corporate bond returns. The 'Default Risk' variable is constructed as the sign-corrected average of the three standardized default risk proxies, namely leverage, distance-to-default, and credit (CDS) spread. Both panels present results using simulated economies based on each version of the model. Regressions are performed on each simulated economy comprising 1,250 firms over a 10-year period - reported results are the average over 10 economies. We report t-statistics in parentheses, using standard errors (SE) double clustered at the month and firm levels for the baseline panel regressions. Newey-West corrected standard errors are used for the Fama-MacBeth procedure with 12 lags. The definitions of the variables and their data sources are presented in Section 1.1 and in Online Appendix A and B. The simulation procedure and the calculation of model-implied and empirical moments are detailed in Section 3.5 and in Online Appendix G. The model extension with stochastic interest rates is described in Online Appendix H. Significance at the 10%, 5%, 1% level is indicated by *, **, ***, respectively.

Panel A: Comovement descriptive statistics

	Mean	StdDev	25%	50%	75%
Covariance					
Constant interest rate	27.16	22.69	11.32	21.86	37.17
Stochastic interest rates	27.30	27.15	9.08	20.06	36.97
Correlation					
Constant interest rate	0.787	0.160	0.758	0.814	0.854
Stochastic interest rates	0.638	0.231	0.559	0.709	0.790

Panel B: Regressions with simulated data

	Covariance		**Correlation**	
	Baseline	Fama-MacBeth	Baseline	Fama-MacBeth
	(1)	(2)	(3)	(4)
	Constant interest rate			
Default Risk	0.30***	0.58***	0.03***	0.05***
t-stat	(8.90)	(25.61)	(6.55)	(4.66)
$R^2_{Adj.}$	0.311	0.087	0.363	0.029
	Stochastic interest rates			
Default Risk	0.28***	0.80***	0.08***	0.10***
t-stat	(7.04)	(10.63)	(5.369)	(7.681)
$R^2_{Adj.}$	0.403	0.103	0.396	0.088
SE	Month & Firm	Newey & West	Month & Firm	Newey & West
Fixed Effects	Firm	None	Firm	None
Controls	None	None	None	None

Table A.7 : Out-of-sample performance by default risk for alternative stock-bond allocations This table presents the out-of-sample performance of portfolios combining stocks and bonds by default risk using different allocations. Panel A reports performance statistics for default-risk-sorted portfolios with a 60% allocation in corporate bonds and 40% in stocks. Panel B reports performance statistics for default-risk-sorted portfolios with a 40% allocation in corporate bonds and 60% in stocks. Portfolios are presented by default risk quintile and are value-weighted based on market capitalization. Quintile portfolios are formed every month by sorting firms based on their default risk, measured with the Credit Spread. Quintile 1 (Q1) is the portfolio with lowest default risk, while Quintile 5 (Q5) is the portfolio with highest default risk. The Sortino ratio is computed as a portfolio's excess return divided by downside volatility, defined as the standard deviation of negative returns. The one-month value-at-risk (VaR) is the historical 95% quantile of each portfolio. The t-statistics for the individual joint portfolio Sharpe ratios are computed using heteroskedasticity and autocorrelation consistent (HAC) errors, as in Lo (2002). The t-statistic for the difference in the portfolio Sharpe Ratio between Q5 and Q1 is computed using HAC standard errors, as in Ledoit and Wolf (2008). The data sample contains stocks and bonds spanning August 2003 - August 2020. The definitions of the variables and their data sources are presented in Section 1.1 and in Online Appendix A and B.

Panel A: Characteristics of portfolios with 60% in bonds and 40% in stocks

	Q1	Q2	Q3	Q4	Q5	Q5-Q1
Excess Return (%)	6.023	6.457	6.604	7.986	9.66	3.637
Volatility (%)	5.913	7.178	9.243	12.17	16.87	10.957
Sharpe Ratio	1.019***	0.900***	0.714***	0.656***	0.573**	-0.446**
t-stat	[4.119]	[3.677]	[2.938]	[2.702]	[2.362]	[-2.240]
Sortino Ratio	1.242	1.027	0.749	0.725	0.774	-0.468
Skew	-0.574	-0.881	-1.166	-0.558	0.335	0.909
Kurtosis	3.568	5.332	7.117	8.494	5.731	2.163
VaR-95 (%)	-7.191	-10.083	-13.46	-15.896	-19.26	-12.069

Panel B: Characteristics of portfolios with 40% in bonds and 60% in stocks

	Q1	Q2	Q3	Q4	Q5	Q5-Q1
Excess Return (%)	7.113	7.169	6.94	8.125	8.664	1.551
Volatility (%)	7.946	9.291	12.033	15.443	20.478	12.532
Sharpe Ratio	0.895***	0.772***	0.577**	0.526**	0.423*	-0.472**
t-stat	[3.648]	[3.157]	[2.373]	[2.167]	[1.746]	[-2.636]
Sortino Ratio	1.114	0.923	0.64	0.605	0.572	-0.542
Skew	-0.612	-0.777	-0.908	-0.494	0.159	0.771
Kurtosis	2.687	3.741	5.446	7.369	5.135	2.448
VaR-95 (%)	-8.897	-12.095	-16.267	-19.657	-23.537	-14.64