Wealth Dynamics and Financial Market Power*

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Abstract

We propose a dynamic theory of financial market concentration in settings where some investors trade strategically because of price impact. The distribution of risk and wealth determines market power, and wealth evolves over time given strategic portfolio choices. In equilibrium, the most well-capitalized investors remain under-diversified to capture rents, generating concentration and volatility in the wealth distribution. Conversely, wealth concentration leads to inflated asset prices, unequal returns to wealth, and poor liquidity that further exacerbates the distortions from market power. We discuss applications of our framework, and derive implications for evaluating welfare using asset pricing data.

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1 Introduction

In many financial markets, there are dominant players with outsize influence on equilibrium outcomes. Because large investors can exploit this influence for private gain, there is concern that financial market concentration might distort asset prices, reduce efficiency and lower welfare, much like market power does in other industries. We are interested in the nature of these effects in financial markets and their persistence over time.

Evaluating the impact of financial market concentration poses several conceptual difficulties that differ from those in other industries. In particular, the extent of financial market power depends on the prevailing distribution of wealth and trading needs, and these evolve over time given asset prices and portfolio choices. Because prices are forward-looking and cash flows are risky, this leads to a stochastic feedback mechanism between current and future market power that transmits through the wealth distribution.

We study this mechanism using a strategic variant of the Lucas (1978) economy in which investors are endowed with heterogeneous real assets whose dividends may be exposed to idiosyncratic and aggregate risk. There is a mass of competitive traders, but also a finite number of strategic agents who have market power because they control discrete shares of real wealth.[1] Depending on prevailing risk exposures and wealth differences, there are gains from trade across states and time. However, these may not be fully realized because large investors take into account their impact on equilibrium prices.

As in the Lucas (1978) model, asset prices can be recovered from the marginal utility process of competitive traders. This allows us to recover the set of price *impact* functions in closed form as well, leading to a tractable no-arbitrage equilibrium in which prices and allocations are invariant to the introduction of redundant assets.

The key object of our analysis is the portfolio choice problem of strategic investors. Because these agents face dividend risk in addition to having price impact, they must weigh standard risk-return considerations against a desire to exploit market power to extract rents for private gain. In general, there is a trade-off between these two objectives. If prices can be pushed up by rationing the supply of certain assets, doing so requires the investor to retain more risk than she otherwise would. Which objective is more important

[1]Such an ownership structure could arise if there are large financial institutions, or private fortunes with large stakes interests in businesses. The baseline model takes the ownership structure of real assets as given, but we later provide an entry game that gives rise to such concentrated ownership.

depends on equilibrium forces, such as prices and price impact, and investor characteristics, such as wealth and dividend risk.

Since equilibrium is invariant in the introduction of redundant securities, it is convenient to work with the set Arrow securities. For competitive traders, it is known that optimal portfolios are pinned down by a condition equalizing the marginal value of consumption in a given state (the investor's *state price*) with the associated security price. Optimal portfolios under price impact follow a distorted version of the rule, whereby state prices differ from asset prices by an optimally chosen state-specific wedge. This wedge turns to be the product of price impact in the Arrow security (i.e. the price change in response to a marginal quantity adjustment) and the quantity traded by the investor.

These wedges have intuitive properties that can be linked to economic primitives. Naturally, sellers reduce supply to raise prices, while buyers lower demand to raise price. Since investors buy a security if and only if they have relatively low income in the associated state, this has the implication that market power begets imperfect risk sharing. Second, wedges depend on the quantity traded. Hence wealthy investors will optimally choose to distort their portfolios more than poorer investors, and remain more exposed to idiosyncratic risk. This is in sharp contrast to other financial frictions, such as limited commitment or imperfect market access, that are more likely to bind at low wealth levels. More broadly, any shock to primitives that raise fundamental gains from trade engenders bigger distortions because they raise trading needs, and therefore quantities.

Although the impact of market concentration on trading volumes and risk sharing is relatively unequivocal, implications for asset prices are more nuanced. A "monopolist" would like to induce price increases for assets she sells, and price decreases prices on asset she buys. But in general equilibrium with many traders, such strategic distortions may be mutually offsetting. Because price impact is nonlinear, there is an illiquidity externality in which the strategic distortions of one large investor impact the incentives to distort of all other large investors. Despite this complication, we show if the top of the wealth distribution is sufficiently symmetric, strategic trading raises prices of *all* traded and non-traded assets if and only if marginal utility is convex. The reason for this uniform increase is investors sell if and only if they have high income, which means that supply curves are necessarily more elastic than demand curves.Away from this symmetric benchmark, strategic trading leads to endogenous risk premia even in the absence of aggregate shocks.

This is because imperfect risk sharing leads to uncertainty about which agents will exert most strategic influence in the future.

Dynamically, the link between market concentration and imperfect risk sharing generates path dependence in market power. In particular, voluntary overexposure to diversifiable risk leads the wealth of some strategic agents grows faster than others ex post. This leads to changes in the distribution of gains from trade that beget worse distortions from market power over time. But inequality is not persistent: because the wealthiest investors are least diversified, there is churn at the top of the wealth distribution. Indeed, precisely because the wealthy have larger trading needs, temporarily poor agents can extract more rents that allow them to grow wealthy again over time. That is, market concentration reduces persistence in the wealth distribution and market power acts as a hedge against future declines in wealth.

Although our model is intentionally abstract, it can be applied to a number of settings. The first relates to financial industry dynamics and intermediary asset pricing. Existing work relating intermediaries to asset prices is built on the idea that shocks to equity capital in the intermediary sector, but this literature typically focus on either a representative intermediary, or competitive markets without price impact. Our work that risk sharing may be impaired even when intermediaries are well-capitalized, and we show that imperfect competition may distort asset prices.

The second application is top wealth inequality. There is empirical evidence the wealthiest investors do not appear to follow fundamental tenets of optimal portfolio theory: their portfolios are highly exposed to idiosyncratic risk, and they often hold concentrated positions in a small number of assets. Fagereng, Guiso, Malacrino, and Pistaferri (2022) and Bach, Calvet, and Sodini (2022) show households' average returns and exposure to idiosyncratic risks are increasing in household wealth, while Hubmer, Krusell, and Smith (2021) argue it is difficult to account for the dynamics of wealth inequality absent such skewed return processes. This is the case although standard financial frictions, such as limited market access or borrowing constraints, are weaker at high wealth levels. Our model can account for these portfolio choice patterns if the wealthy trade in relatively concentrated financial markets, which is the case because a large share of top financial wealth is invested in private businesses, real estate, and illiquid securities. The literature has argued that differences in preferences and portfolio returns (e.g., Gomez

(2017)) or secular declines in long-term real interest rates (e.g., Greenwald, Leombroni, Lustig, and Van Nieuwerburgh (2021)) or capital gains (e.g., Fagereng, Holm, Moll, and Natvik (2021)) are needed to explain how wealth inequality has risen over time. We complement these approaches by exploring the notion that market concentration eventually leads to deviations from price-taking behavior, which can exacerbate wealth inequality via portfolio under-diversification and distorted asset returns.

Conceptually, our paper is related to the literature on endogenous market incompleteness. Kehoe and Levine (1993), Alvarez and Jermann (2000), Hellwig and Lorenzoni (2009), and Ai and Bhandari (2021), for instance, explore how limited commitment impairs risk sharing by imposing endogenous participation constraints. Biais, Hombert, and Weill (2021) illustrate how such constraints give rise to a basis in which the price of a security is below its replicating portfolio of long positions in Arrow securities. In contrast, we examine trading distortions from market power. This not only leads to a *voluntary* misalignment of state prices, a hallmark of exogenously incomplete markets, but can also inflate the prices of all securities when there is sufficient symmetry among strategic agents.[2] Furthermore, such constraints bind for poorer agents because they cannot commit to repay their debts when they have high income, while market power distorts the behavior of wealthy agents because their larger trades move asset prices more. Bocola and Lorenzoni (2020) shows how financial institutions bear too much aggregate risk because complete markets are inefficient at sharing it. In our setting, distortions arise in the sharing of diversifiable risk, which complete markets otherwise effectively facilitate.

Our equilibrium concept is a Cournot-Walras equilibrium (e.g., Gabszewicz and Vial (1972)) with a competitive fringe (e.g., Shitovitz (1973)).[3] In this paradigm, Sahi and Yao (1989) and Amir, Sahi, Shubik, and Yao (1990) focus on the primitive properties of

[2] Roussanov (2007) shows that social status concerns modeled by "keeping up with the Joneses" preferences can also lead to voluntary under-insurance to idiosyncratic risk and higher consumption volatility. In that setting, however, agents earn lower returns, on average, because they are less averse to idiosyncratic risk, whereas in our setting they are compensated with trading rents.

[3] A related approach is the equilibrium-in-demand-schedules concept based on Kyle (1989). Although this concept allows for a richer analysis of strategic interactions among large traders, it often requires strong assumptions on preferences and payoffs (such as the canonical CARA-normal setting) for tractability. In this tradition, Carvajal and Weretka (2012) consider a complete markets model with general preferences but focus on the role of redundant assets in which perceived and actual price impact are linear in asset demands. Similar to us, Malamud and Rostek (2017) emphasize that buyers and sellers shade demand and supply, respectively, but instead examine how decentralized markets can improve welfare relative to centralized exchanges by altering price impact. In Appendix B, we show although how prices are determined differs across the two concepts, the basic strategic forces governing how large agents distort their portfolios are similar. For a more detailed comparison of the two concepts, see also Neuhann and Sockin (2021).

Cournot-Walras equilibria in complete markets endowment economies, such as the competitive limit and whether there is consistent pricing with differentiated goods both with and without money. Basak (1997) examines asset pricing with a monopolistic non-price-taking agent in an Arrow-Debreu economy. Rahi and Zigrand (2009) examines the incentives of large agents to arbitrage across segmented markets. Our focus is instead on the portfolio and asset pricing implications of Cournot-Walras equilibria in the context of wealth accumulation and risk concentration based on the size of agents' trading needs. Our no arbitrage framework has the advantages that prices are uniquely determined by the demands of strategic agents and equilibria are invariant to the introduction of redundant securities, both of which are desirable properties for a general equilibrium analysis of financial market power.

2 Model

We conduct our analysis in two steps. First, we study a two-period model in which the wealth distribution is taken as given. This allows us cleanly derive the key implications of market concentration and wealth inequality for portfolio choice and asset prices. In Section 4, we then extend the model to a fully dynamic framework to study the dynamics of the wealth distribution.

Demographics. There are two classes of agents: a continuum of competitive agents with mass m_f called the *competitive fringe* who takes prices as given, and a discrete number of *strategic agents* who are large relative to the economy and internalize their impact on prices in financial markets. The presence of a competitive fringe is a realistic feature of financial markets given the presence of retail investors. As in many models of oligopolistic competition, it also allows us to recover a unique residual demand curve for each strategic agents in every financial market.

There are N *types* of strategic agents, indexed by $i \in \{1, 2 \ldots, N\}$. Each type receives a stream of state-contingent endowments of the consumption good. To tractably vary market concentration, we assume that, within each type, there exist $1/\mu$ symmetric agents who each have mass μ. Hence μ determines the share of a type's total endowment that is controlled by an individual agent. By measuring the relative size of an agent, μ therefore also determines the extent to which a strategic agent internalizes her market power. This allows us to vary the degree of competition without affecting the aggregate feasibility set. To nest perfect competition as a benchmark, we say that $\mu = 0$ corresponds to a continuum of infinitesimal agents. In the main model, we take μ to be exogenously fixed. Later on, we use an entry game with a fixed cost to determine μ endogenously.

Preferences. Strategic agents share common preferences over consumption at both dates. These are represented by the utility index $u(c)$ that is $\mathcal{C}^2$, strictly increasing, strictly concave, homothetic, and satisfies the Inada condition. Marginal utility $u'(c)$ is further assumed to be strictly convex. Among other preferences, constant relative risk aversion (CRRA) satisfies these restrictions. Homothetic preferences are useful because equilibrium would be invariant to market concentration μ if agents were to trade competitively.

The fringe has quasi-linear preferences: linear in consumption at date 1 and risk-averse at date 2. Its date-2 utility function, $u_f(c)$, satisfies the same properties as that of strategic agents. Although a price-taking fringe is essential for our results, quasi-linearity of its preferences is not. Relaxing this assumption would lead to a more complicated price impact function (the $q'(z)$ in what follows), but would not fundamentally alter the role of market power in distorting agents' portfolios and state prices.[4]

Income and Consumption. Uncertainty is represented by a set of states of the world $\mathcal{Z} \equiv \{1, 2, \dots Z\}$, one of which realizes at date 2. Agents share common beliefs. The probability of generic state $z \in \mathcal{Z}$ is $\pi(z) \in (0, 1)$. The fringe receives initial wealth w_f and state-contingent endowment $y_f(z) > 0$. A strategic agent j of type i receives initial endowment w_j at date 1 per unit of mass, and state-contingent endowment $y_i(z) > 0$ in state z per unit of mass. Let w_i be the total initial endowment of agents of type i, i.e., $w_i = \sum_j w_{j,i}$. The total endowment of type i in state z is $y_i(z) = \sum_{j=1}^{1/\mu} \mu y_i(z)$ and the *aggregate endowment* of all strategic agents is

$$Y(z) = \sum_i y_i(z).$$

Let $c_{1,j,i}$ and $c_{2,j,i}(z)$ denote consumption of agent j of type i at date 1 and in state z, respectively. Aggregating within types gives $c_{1,i} = \sum_{j=1}^{1/\mu} \mu c_{1,j,i}$ and $c_{2,i}(z) =$

[4] We show in the proof of Proposition 2, for instance, that quasi-linearity is not necessary for our result.

$\sum_{j=1}^{1/\mu} \mu c_{2,j,i}(z)$. The aggregate resource constraints are

$$\begin{aligned}\sum_{i=1}^{N} c_{1,i} + m_f c_{2f} &= \sum_{i=1}^{N} w_i + w_f, \\ \sum_{i=1}^{N} c_{2,i}\left(z\right) + m_f c_{2f}\left(z\right) &= Y\left(z\right) + m_f y_f(z).\end{aligned}$$

We restrict all consumption to be nonnegative.

Financial Markets. Financial markets open at date 1. The set of assets is the complete set of Arrow securities. That is, there are Z securities, and security z pays one unit of the numeraire in state z but zero otherwise.

Let $a_{j,i}(z)$ denote the position of agent j of type i in claim z, where $a_{j,i}(z) < 0$ denotes a sale. Aggregating within and across types yields $a_i(z) \equiv \sum_{j=1}^{1/\mu} \mu a_{j,i}(z)$ and $A(z) \equiv \sum_{i=1}^{N} a_i(z)$. The fringe's position in security z is $a_f(z)$. Now define $\mathbf{A}$ to be the $(N+1) \times Z$ matrix summarizing all agents' portfolios. The equilibrium price function of asset z is denoted $Q(\mathbf{A}, z)$. Market clearing in the market for claim z requires:

$$A(z) + m_f a_f(z) = 0 \qquad \text{for all } z. \tag{1}$$

We later show that the equilibrium is invariant in the introduction of redundant securities.

Decision Problems and Equilibrium Concept. We search for a *Cournot-Walras* equilibrium in which the competitive fringe takes asset prices as given while strategic agents place limit orders taking into account the demands of other strategic agents and the residual demand curve of the competitive fringe. A *strategy* $\sigma_{j,i}$ for strategic agent j of type i consists of asset positions and consumption, $\sigma_{j,i} = \{\{a_{j,i}(z)\}_{z\in\mathcal{Z}}, c_{1,j,i}, c_{2,j,i}\}$. The *perceived pricing functional* used by agent j of type i to forecast her influence on the price of security z is $\tilde{Q}_{i,j}(\mathbf{A}, z)$. The decision problem is

$$\begin{aligned} U_{j,i} = \max_{\sigma_{j,i}} \quad & u\left(c_{1,j,i}\right) + \sum_{z\in\mathcal{Z}} \pi\left(z\right) u\left(c_{2,j,i}\left(z\right)\right) \\ \text{s.t.} \quad & \mu c_{1,j,i} = \mu w_i - \sum_{z\in\mathcal{Z}} \tilde{Q}_{i,j}(\mathbf{A}, z) \mu a_{j,i}\left(z\right), \\ & \mu c_{2,j,i}\left(z\right) = \mu y_i\left(z\right) + \mu a_{j,i}\left(z\right). \end{aligned} \tag{2}$$

We define preferences and controls in this manner recognizing the consumption of strategic agent j of type i is actually $\mu c_{1,j,i}$ and $\mu c_{2,j,i}\,(z)$ at dates 1 and 2, respectively, and similarly with optimal asset holdings, $\mu a_{j,i}\,(z)$. Given homothetic utility, however, optimal policies are invariant to defining strategic agent preferences over $\mu c_{t,j,i}$.

A *strategy* σ_f for the competitive fringe consists of asset positions and consumption, $\sigma_f = \{\{a_f(z)\}_{z\in\mathcal{Z}}, c_{1,f}, c_{2,f}\}$. Because the competitive fringe takes prices as given, its perceived pricing functionals depends only on the state, $\tilde{Q}_f(\mathbf{A}, z) = \tilde{Q}_f(z)$. The fringe's decision problem is

$$\begin{aligned} U_f = \max_{\sigma_f} \quad & c_{1f} + \sum_z \pi(z) u(c_{2f}(z)) \\ \text{s.t.} \quad & c_{1f} = w - \sum_z \tilde{Q}_f(z) a_f(z), \\ & c_{2f}\,(z) = y_f(z) + a_f(z). \end{aligned} \tag{3}$$

We can now define our equilibrium concept.[5]

Definition 1 *A Cournot-Walras equilibrium consists of a strategy $\sigma_{j,i}$ for each strategic agent, a strategy σ_f for the competitive fringe, and pricing functions $Q(\mathbf{A}, z)$ for all $z \in \mathcal{Z}$ such that:*

1. *Fringe optimization: σ_f solves decision problem (3) given $\{\tilde{Q}_f(z)\}_{z\in\mathcal{Z}}$*
2. *Strategic agent optimization: For each agent j of type i, $\sigma_{j,i}$ solves decision problem (2) given (i) other agents' strategies $\{\sigma_{-j,i}, \sigma_f\}$ and perceived pricing functions $\{\tilde{Q}_{j,i}(\mathbf{A}, z)\}_{z\in\mathcal{Z}}$.*
3. *Market-clearing: Each market clears with zero excess demand according to (1).*
4. *Consistency: all agents have rational expectations, which requires for strategic agents that $\tilde{Q}_{j,i}(\mathbf{A}, z) = Q(\mathbf{A}, z)$ for all i, j and z.*

We will often contrast Cournot-Walras equilibrium with the competitive benchmark.

Definition 2 (Competitive Equilibrium) *The competitive equilibrium is the Cournot-Walras equilibrium in the special case when $\mu = 0$.*

One may notice that our model of Cournot competition in complete financial markets has technical similarities to one of multi-product Cournot competition in spot exchange markets. This is because Arrow securities enable agents to trade exposures against

[5]The Walras part of the concept stems from the assumption that there is a Walrasian auctioneer in the background who takes the demands of all agents and sets the price vector that clears all asset markets.

specific states, with each state akin to a differentiated product. This analogy, however, quickly breaks down once we consider the pricing of redundant securities and in our dynamic extension where the realization of income shocks introduces path dependence in wealth. In addition, agents sort in financial markets into buyers and sellers based on equilibrium asset prices, whereas product markets have natural consumers and producers. We consider a complete markets setting so that market power is the only friction that impedes risk sharing to make transparent its impact on portfolio choices and asset prices.

In the context of financial markets, our Cournot approach also provides an equilibrium selection mechanism that avoids the typical coordination issues that arise when strategic agents must coordinate on price impact functionals. This is because the competitive fringe's marginal utility pins down asset prices so that large agents face a unique residual demand curve from which to forecast its price impact.[6] This mutes the source of strategic uncertainty that could lead to multiple self-fulfilling price impact functionals (which may not be anonymous) that are consistent with rational expectations. Even if the fringe is infinitesimal in size, its presence selects among all equilibria the rational expectations equilibria that impose no arbitrage and invariance to redundant assets. We view these features as an advantage for a general equilibrium model of many markets.

An alternative equilibrium concept is the Equilibrium-in-Demand-Schedules approach of Kyle (1989). Without a competitive fringe, asset prices solve a system of differential equations that can admit many solutions corresponding to different rational expectations prices that satisfy the Euler Equations of strategic agents and market clearing.[7] These asset prices can admit arbitrage because there is a wedge between asset prices and the state prices of every agent. In Appendix B, we solve an Equilibrium-in-Demand-Schedules version of our model with liquidity traders who take fixed positions in each asset instead of a competitive fringe. This analysis offers two insights. First, conditional on asset prices, how a strategic agent distorts her portfolio because of market power is the same as under Cournot-Walras. Consequently, our analysis is also applicable to Equilibrium-in-Demand-Schedules settings. What differs is how prices are determined. Second, we provide a method for computing such an equilibrium if we restrict our attention to pricing functions in which price impact is the same across all agents. Our method

[6]Models of product markets of oligopolistic Cournot competition often assume price-taking consumers to be able to back out a residual demand curve for strategic agents.

[7]A key insight of Kyle (1989) is in a CARA-normal setting the unique residual demand curve is affine.

reduces characterization to a system of $|\mathcal{Z}|$ first-order differential equations (one for each asset market), with boundary conditions that can be determined by our Cournot-Walras equilibrium in the special case the competitive fringe is infinitesimally small.

3 Equilibrium Characterization

We now characterize equilibrium price impact and explore how market power impacts the portfolio choice. The first step is to derive the equilibrium pricing functional using the decision problem of the competitive fringe. First-order conditions for portfolio optimality require that asset prices are equal to the fringe's marginal utility. By market-clearing, each strategic agent can then infer how much the fringe's consumption will move when the agent demands more or less of a given security, holding other agents' portfolios fixed. Price impact is therefore determined by the change in marginal utility of the competitive fringe. Because each agent's influence on the market-clearing condition scales with her mass, μ, her price impact does as well. Finally, price impact is anonymous because only net risk exposures matter for equilibrium prices.

Proposition 1 (Demand System for Arrow Securities) *The price of the claim on state z is*

$$Q(\mathbf{A}, z) = q(z) \equiv \pi(z) u_f' \left(c_{2f}(z) \right). \tag{4}$$

The price impact of strategic agent i is independent of i and satisfies

$$\frac{\partial \tilde{Q}_{j,i}(\mathbf{A}, z)}{\partial a_i(z)} = \frac{\mu}{m_f} q'(z) \qquad \text{where} \qquad q'(z) \equiv \frac{\partial q(z)}{\partial A(z)} = -\pi(z) u_f'' \left(c_{2f}(z) \right) > 0, \tag{5}$$

and $q'(z)$ is increasing and convex in strategic agent demand.

Focusing on Arrow securities is without loss of generality. We define redundant securities as follows.

Definition 3 (Redundant securities) *Let security ϕ is a promise a set of state-contingent payoffs $\{\phi(z)\}_{z\in\mathcal{Z}}$. The security is redundant if its payoffs can be replicated using Arrow securities.*

Proposition 2 then establishes that prices and allocations in the Cournot-Walras equilibrium are invariant to the introduction of redundant securities

Proposition 2 (Law of One Price) *The Law of One Price holds: the price of any redundant security is equal to the price of its replicating portfolio of Arrow securities. The consumption allocation and all Arrow security prices are invariant to the introduction of redundant securities.*

The invariance to redundant assets is a direct consequence of the Law of One Price that ensures no arbitrage. These properties are not generically guaranteed in models of strategic interaction. They hold in ours because there is a competitive fringe.[8] We view this is a useful feature when studying asset prices because strict arbitrage opportunities do not appear to be pervasive across financial markets.

3.1 Optimal Strategic Portfolios

We now characterize optimal strategic portfolios and discuss how they map into risk sharing arrangements. A *state price* $\Lambda_{j,i}(z)$ for agent j of type i is the marginal rate of substitution between consumption in state z and date 1, that is

$$\Lambda_{j,i}(z) \equiv \frac{\pi(z)\, u'\left(c_{2,j,i}(z)\right)}{u_1'\left(c_{1,j,i}\right)}. \tag{6}$$

The key comparison is the competitive equilibrium benchmark.

Lemma 1 *The competitive benchmark is obtained when $\mu = 0$ In this equilibrium, there is perfect risk sharing across all states and investors. That is,*

$$\Lambda_{j,i}(z) = \Lambda^{CE}(z) = q^{CE}(z) \qquad \text{for all } z \text{ and } j, i,$$

where $\Lambda^{CE}(z)$ is the unique state price in the competitive equilibrium.

The next proposition proves the existence of a Cournot-Walras equilibrium and states necessary conditions for each agent's optimal trading positions. It is straightforward to show that agents must behave symmetrically within types. Hence we suppress all j subscripts going forward.

[8] Carvajal (2018) shows that no arbitrage generically need not hold when strategic agents trade with price impact in financial markets, and that a competitive fringe is one means of enforcing no arbitrage. With complete markets, this enforcement is "off-equilibrium" in our setting in the sense that we do need to impose cross-equation no arbitrage restrictions on Arrow-Debreu assets because they have orthogonal payoffs. Rather, the fringe lets us generalize our model to price any redundant asset.

Proposition 3 *There exists an equilibrium in which the optimal policies for $a_{j,i}(z)$ satisfy the optimality conditions*

$$\Lambda_i(z) = q(z) + \frac{\mu}{m_f} q'(z) a_i(z). \tag{7}$$

The optimality condition (7) reveals the key mechanism of our model. Even though markets are complete, agents *choose* to misalign marginal valuations (state prices) with marginal prices to extract rents. Sellers reduce their supply and have lower state prices than in the competitive equilibrium, while buyers reduce their demand and have higher state prices than in the competitive equilibrium. The extent of these distortions is increasing in gross asset positions and price impact. If the fringe has convex marginal utility, price impact in turn is increasing in prices itself. Since high prices and large asset positions are increasing in the underlying gains of trade (i.e. average marginal valuations of state-contingent consumption), distortions from market power are most severe when gains from trade are large. Note also that we recover the standard Euler equation if $\mu \to 0$.

Wealth and portfolio choice. How do changes in initial wealth affect portfolio choice? Under homothetic preferences, we can express agent $i's$ optimal consumption and investment polices as $c_{2,i}(z) = \hat{c}_{2,i}(z) w_i$ and $a_{2,i}(z) = \hat{a}_{2,i}(z) w_i$ for $z \in \mathcal{Z}$, respectively. This allows us to rewrite the portfolio optimality condition (7) as

$$\pi(z) u'\left(\frac{y_i(z)/w_i + \hat{a}_i(z)}{1 - \sum_{z' \in \mathcal{Z}} q(z') \hat{a}_i(z')}\right) = q(z) + \frac{\mu}{m_f} \bar{q}'(z) \hat{a}_i(z), \tag{8}$$

Wealth impacts portfolio choice through two channels. First, an increase in wealth is akin to reducing the effective endowment, which raises her state price. This makes the agent less of a seller and more of a buyer in each asset market. Second, an increase in wealth increases the agent's trading needs, and raises how much she moves the price for the same relative position $\hat{a}_i(z)$. This erodes her market power by forcing her to trade more for the same share of wealth allocated to asset z. Importantly, these effects are non-linear. Hence, wealth *inequality* affects the severity of risk-sharing distortions. In the dynamic model, imperfect risk sharing will generate endogenous variation in wealth inequality.

Risk-rent trade-off. How do agents employ market power? To better understand strategic considerations, we derive two objects. First, we consider a thought experiment in which we increase one strategic agent's size from μ to $\mu + \Delta\mu$. Taking a first-order approximation of this agent's portfolio holdings around the original equilibrium, we can express her optimal portfolio in terms of a sharp trade-off between rent extraction and risk management. We then also examine how the trade-off between risks and rents impacts the expected return on agent i's total wealth R_i^W

$$R_i^W = \frac{\sum_z \pi(z) c_{2,i}(z)}{w_i + \sum_z q(z) y_i(z) - c_{1,i}}, \tag{9}$$

where total wealth is the sum of initial wealth and the present value of future income less initial consumption, i.e., $W_i = w_i + \sum_z q(z) y_i(z) - c_{1,i}$, and consumption at date 2 is the dividend on total wealth. We then have the following proposition.

Proposition 4 *Agent i 's: 1) optimal holding of the security for state z can be approximated as*

$$\hat{a}_i(z) \approx a_i(z) + -\underbrace{\frac{\frac{\Delta\mu}{m_f}\frac{q'(z)}{q(z)}\hat{a}_i(z)}{\frac{\mu}{m_f}\frac{q'(z)}{q(z)} + \mathbb{E}\left[\gamma\left(g_i(z')\right) u'\left(g_i(z')\right)\frac{1/c_i(z') - \sum_{\tilde{z}=1}^{Z}\frac{q(\tilde{z}) + \frac{\mu}{m_f}q'(\tilde{z})}{c_{n,1}}\frac{\Delta a_i(\tilde{z})}{\Delta a_i(z)}}{q(z)}\delta(z)\right]}}_{\textit{Risk-Rent Ratio}},$$

where $g_i(z) = \frac{c_{2,i}(z)}{c_{1,i}}$ is the consumption growth rate in state z; and 2) expected return on her wealth portfolio R_W^i can be decomposed as

$$R_i^W = \underbrace{E\left[\frac{\pi(z)}{q(z)}\right]}_{\textit{Risk Premium}} + \underbrace{Cov\left(\frac{\pi(z)}{q(z)}, \frac{v_s(z)}{E[v_s(z)]}\right)}_{\textit{Risk-Rent Premium}}, \tag{10}$$

where $v_s(z) = \frac{q(z)}{\pi(z)} u'^{-1}\left(\frac{q(z)}{\pi(z)\alpha_s(z)}\right)$ and $\alpha_s(z) = \left(1 + \frac{q'(z)}{q(z)} a_i(z)\right)^{-1} \geq 0.$

The first part of the proposition reveals that an incremental increase in size enables an agent of type i to extract more rents by reducing its asset position and increasing the gap between her state price (her marginal valuation) and the market price. That is the numerator. Such rent extraction, however, comes at the expense of exposing the agent to additional consumption risk. That along with the shading term because of price impact is the denominator. This is the risk-rent trade-off.

The second part of the proposition illustrates that, in addition to the typical risk premium that each agent (including competitive agents) earn by trading in financial markets $E\left[\frac{\pi(z)}{q(z)}\right]$, a strategic agent earns a risk-rent premium. This premium is related to how her state-by-state distortions to her financial market trading correlate with the inverse of asset prices. Interestingly, this premium can be negative (i.e., a discount) depending on how the market power distortions to her asset trading correlate with asset prices in financial markets.

This risk-rent premium is most transparent in the case of log utility, and the return on the wealth portfolio simplifies to

$$R_i^W = E\left[\frac{\pi(z)}{q(z)}\right] + Cov\left(\frac{\pi(z)}{q(z)}, \frac{\alpha_s(z)}{E[\alpha_s(z)]}\right). \tag{11}$$

When income and substitution effects cancel, a strategic agent earns a higher expected return on her wealth if she is a seller of Arrow securities against states with low state prices (i.e., aggregate low marginal utility states) and a buyer of Arrow securities referencing high marginal utility states. In contrast, she earns a lower expected return if she is a seller of Arrow securities referencing high and a buyer of those referencing low marginal utility states, in which case $Cov\left(\frac{\pi(z)}{q(z)}, \frac{\alpha_s(z)}{E[\alpha_s(z)]}\right) < 0$.

Alternatively, we can measure the surplus strategic agents extract by maintaining a wedge between public (or market-based) and private valuations of their wealth. We measure this "private" surplus as the investor's Excess Wealth $\tilde{W}_i$, which we define as

$$\underbrace{\tilde{W}_i}_{\text{Excess Wealth}} = \underbrace{c_{1,i} - w}_{\text{Excess Expenditure}} + \underbrace{\sum_z \Lambda_i(z)(c_{2,i}(z) - y_i(z))}_{\text{NPV of consumption stream}} = \sum_z q'(z)\, a_i(z)^2. \tag{12}$$

For a competitive agent, $\tilde{W}_i = 0$, but for a strategic agent, it is strictly positive. The more she sells (higher $a_i(z)$ in markets with higher price impact (higher $q'(z)$), the more surplus she extracts, and the more valuable a consumption portfolio she can finance given her private valuation of her wealth.

Interestingly, there can be a disconnect between a strategic agent's excess wealth and the return on her wealth portfolio. This is because she misaligns her marginal valuation of a state (her private state price) with the Arrow asset price to extract inframarginal rents on her trades. As a result, she earns a lower return on her wealth portfolio when she strategically retains exposure to what the market values as high marginal utility states to raise her excess wealth.

Rents versus trading costs. There are two ways to interpret the equilibrium trading behavior of strategic traders. The first might be called an industrial organization perspective. According to this view, state prices represent the marginal cost (or willingness to pay) for state-contingent consumption, and wedges between state prices represent rents earned via markdowns or markups. The second is a finance perspective according to which price impact is a friction that prevents agents from trading towards their preferred portfolio.

It is immediate both views are formally equivalent: agents do not pick efficiently diversified portfolios, but they do pick *optimal portfolios*. That is, they choose portfolios to optimally trade off rents and risks. We now show market power is privately valuable if μ is relatively small, but that excessively large μ can be counterproductive. To do so, we measure the rents earned by some agent n in state z as

$$\Pi_n(z) = (\Lambda_n(z) - q(z))a_n(z) = \frac{\mu}{m_f}q'(z)a_n(z)^2, \tag{13}$$

The total rents earned by agent n are $\Pi_n = \sum_{z\in\mathcal{Z}} \Pi_n(z)$ and the total consumption risk as the variance of consumption $Var(c_n(z))$. The following corollary derives their comparative statics with respect to the μ of one agent type, μ_n.

Corollary 1 *$Var(c_n(z))$ is increasing in μ_n. Π_n is either increasing or hump-shaped in μ_n.*

An increase in size increases trading rents because the agent can exert more market power when trading. This does not, however, imply she is unambiguously better-off for being larger. Although internalizing her price impact given her size is beneficial, having price impact from being relatively larger than other agents impairs her ability to share risk in financial markets. Because the competitive fringe has limited risk-bearing capacity (its size is fixed at m_f), prices scale by more than one-for-one as a strategic agent's trading needs become larger from a higher μ. Consequently, there may be an interior optimal size for large agents that balances the ability to manipulate prices with the hampered risk sharing. This differentiates imperfect competition in financial markets from standard Cournot competition in product markets, which does not have endogenous costs of size. We further explore the implications of market power for how large agents would choose their size μ in Section 3.4 when we allow for ex-ante free-entry into each agent type.

3.2 Equilibrium Prices and Returns

We now investigate the asset pricing implications of market concentration. Because wealthy investors own a sizable share of financial assets, their trading moves asset prices so that prices now reflect rents in addition to risk. In turn, the distortions to asset prices from market power feed back into the portfolio choices of wealthy investors.

Whether and how market concentration affects prices is not obvious. Buyers in a given asset market reduce their demand to lower asset prices while sellers reduce their supply to raise prices. As such, asset prices go up only when sellers distort more. Our central insight is when strategic agents are relatively symmetric in their risk exposures and initial wealth, strategic interactions inflate all asset prices and lower the risk-free rate (which is the inverse of the sum of all Arrow security prices). This is because sellers are better able than buyers to distort their trading behavior to manipulate prices. If wealth is unequal, however, wealthier agents demand more of every asset, which strengthens their strategic motives to push down prices.

To focus on the strategic interaction among large agents, we specialize our model to the case in which the market impact of the competitive fringe is marginal. We refer to this limit as a Strategic Equilibrium.

Definition 4 (Strategic Equilibrium) *A strategic equilibrium is a Cournot-Walras equilibrium where m_f is arbitrarily close to zero, holding μ / m_f fixed.*

To develop intuition, we first focus on a setting in which all agents are type-symmetric in that they have symmetric initial wealth w and income risk.

Definition 5 (Type-Symmetric) *Two agent types are type-symmetric if they have the same initial wealth, i.e., $w_i = w_j$, and symmetric income risks so that they face identical decision problems.*

Our key result, summarized in Proposition 5, in this setting, market concentration raises *all* asset prices, $q(z)$, (state-by-state) and depresses the risk-free rate, r_f, relative to the competitive equilibrium.[9] Incrementally breaking this symmetry by making one agent type wealthier, in contrast, exerts downward pressure on the distortion to asset prices from market power. This is because wealthier agents now are able to push prices in their favor.

[9] This result also holds for CARA utility, which satisfies convex marginal utility but not homotheticity.

Proposition 5 *In a Strategic Equilibrium with type-symmetric agents, asset prices $q(z)$ are higher than in the competitive equilibrium for all z, and the risk-free rate, r_f, is strictly lower. An increase in the initial wealth of one agent type from the type-symmetric case lowers prices in markets in which that type is a buyer and reduces the price inflation from market power in markets in which that type is a seller.*

We briefly sketch the proof for intuition. Summing over the first-order condition for optimal portfolios (7), and imposing market-clearing at $m_f \approx 0$ yields:

$$q(z) = E^*[\Lambda_i(z)],$$

where $E^*[\Lambda_i(z)] = \frac{1}{N}\sum_i \Lambda_i(z)$ is the cross-sectional average of large agents' state prices.

The intuition is as follows. In the competitive equilibrium, marginal rates of substitution are aligned with prices for all agents, $q^{CE}(z) = \Lambda^{CE}(z)$. With market concentration, however, inefficient risk sharing leads to state price dispersion. Under convex marginal utility and symmetry in initial consumption, Jensen's inequality implies that the average state price must rise.[10] As a result, distortions to risk sharing *immediately* map into pricing consequences, irrespective of the particular market structure. Since all $q(z)$ are higher with market power, the riskfree rate is lower.[11] Through this channel, the asset pricing predictions of our model are consistent with the empirical patterns over the 5 decades. In particular, rising wealth concentration leads to a secular decline in risk-free rates and the observed increase in valuations.

An increase in one agent type's initial wealth impacts asset prices through the two channels discussed in the context of equation (8). The first is that having more wealth raises the state prices of agents of that type, all else equal, which raises their demand / lowers their supply in all markets. The second is that having more wealth raises their effective price impact compared to poorer agents, reducing their demand and supply. On net, the wealthier agent type buys more and sells less, which reduces asset prices away from their inflated values in the type-symmetric equilibrium.

[10] This is reminiscent of Weretka (2011) who shows that prices increase relative to a Walras equilibrium in a spot exchange economy without uncertainty when agents have quasi-linear preferences and convex marginal utility. His result obtains when buyers and sellers have fixed types (i.e., producer or consumer).

[11] Such a mechanism for inflating prices and depressing risk-free rates is distinct from that in complete markets models with limited commitment. In those models, state prices are aligned state-by-state and equal to the marginal rate of substitution of unconstrained agents. Because such agents have higher growth rates in consumption than (short-sale) constrained agents, asset prices are higher.

Expected Excess Returns. We now investigate how market concentration impacts expected excess returns. Expected excess returns are determined by a risk premium that is based on the covariance between the asset's payoff and the marginal utility of a representative agent with "average" preferences. As a result of market power, however, state prices are dispersed and "average" preferences reflect disparate marginal valuations of consumption. The distribution of aggregate risk in the type-symmetric case determines the overall impact of market concentration on expected excess returns.[12] Let $\gamma(x) = -\frac{xu''(x)}{u'(x)}$ and $P(z) = -\frac{xu'''(x)}{u''(x)}$ be the coefficients of relative risk aversion and prudence associated with utility index $u(x)$, respectively, and z_L and z_H be the states with the smallest and largest aggregate endowments, respectively.

Proposition 6 *The expected excess return of the Arrow-Debreu security for state z is*

$$\mathbb{E}\left[r(z) - r_f\right] = -Cov\left(\frac{E^*\left[\Lambda_i(z)\right]}{E\left[E^*\left[\Lambda_i(z)\right]\right]}, \delta(z)\right),$$

and can be approximated to second-order around the competitive equilibrium as

$$\begin{aligned}\mathbb{E}\left[r(z) - r_f\right] \approx\ & \mathbb{E}\left[r^{CE}(z) - r_f^{CE}\right] + \underbrace{-\left(r_f^{CE}\right)^2 \sum_{z' \in Z} q^{CE}(z')\,\gamma(z')\,P(z')\,\mathbb{E}^*\left[\left(\frac{\Delta c_{2,i}(z')}{Y(z)/N}\right)^2\right]}_{\text{Risk-free Rate Distortion}} \\ & + \underbrace{\frac{\gamma(z)\,P(z)}{q^{CE}(z)}\mathbb{E}^*\left[\left(\frac{\Delta c_{2,i}(z)}{Y(z)/N}\right)^2\right]}_{\text{State Price Distortion}}.\end{aligned}$$

Suppose agents are type-symmetric and $x^2u'''(x)/u'(x)$ is increasing in x,[13] *then if $Y(z_H)$ is sufficiently larger than $Y(z_L)$, market concentration raises expected excess returns more for states with low than for high aggregate endowments.*

The first part of the proposition shows that risk premia are indeed driven by the covariance of payoffs with the average SDF. The second part of Proposition 6 illustrates that expected excess returns are distorted away from the competitive equilibrium through two channels: 1) a risk-free rate distortion that impacts all expected returns; and 2) a state price distortion that impacts expected excess returns in state z. In the type symmetric case,

[12] More generally, outside of the type-symmetric case, states with higher income dispersion will also have higher expected excess returns because of the impaired risk sharing from market concentration.

[13] This assumption is satisfied, for instance, for CRRA preferences.

all prices rise and the risk-free rate falls (Proposition 5), the risk-free rate distortion raises all asset's expected returns while the state price distortion lowers all expected returns.

If there is sufficient aggregate risk (i.e., high dispersion in aggregate incomes), then market concentration in the type-symmetric case raises expected excess returns more for states with low than for high aggregate endowments. This can lead to risk compression in that risk premia, including the market risk premium, can actually fall although there is poorer risk sharing with market power. Away from the type-symmetric case, expected excess returns can rise if asymmetry in the wealth distribution lowers asset prices. This is because market power then has the reverse effect on expected excess returns because buyers now distort more than sellers.

Feedback through Market Illiquidity. Distortions to asset prices from market power feed back to the portfolio choice of wealthy investors through market illiquidity. When wealthy agents have convex marginal utility and are relatively symmetric, asset prices $q(z)$ (and consequently price impact $q'(z)$) are inflated. From the first-order conditions for a strategic agent's optimal portfolio choice (equation (7)), an increase in market illiquidity forces a larger wedge between asset prices and a wealthy investor's state prices. This reduces both her asset positions and her realized gains from trade. Such a reduction in trade further under-diversifies a wealthy investor by exacerbating her exposure to the idiosyncratic risk of her endowment. As a consequently, the wealth distribution becomes more sensitive to idiosyncratic shocks.

3.3 An Illustration

We now illustrate our theoretical findings using a transparent example in which there are two types and pure idiosyncratic risk. This setting is instructive because it is has a clear competitive benchmark in which there is perfect risk sharing, constant consumption, and all asset returns are the risk-free rate.

In this example, all agents have log preferences, i.e., $u(x) = u_f(x) = \log(x)$. There are two agent types, $i \in \{1, 2\}$. At date 2 are two equally likely states, $z \in \{1, 2\}$ with $\pi(z) = \frac{1}{2}$. Strategic agents face pure idiosyncratic risk: $y_i(i) = \bar{y} + \Delta$ and $y_i(-i) = \bar{y} - \Delta$. That is, in either state one type has a high and the other has a low return. The fringe receives $\bar{y}$ in every state. The fringe receives a fixed endowment $\bar{y}$ at time 1 and in every state at date 2. Strategic agents receive some initial endowments w_1 and w_2 such that $w_1 + w_2 = 2\bar{y}$.

We first provide a conceptual understanding of the case of a monopolistic type, i.e., when one agent type is strategic and the other is price-taking. This setting is similar to that in Basak (1997), and will highlight how a strategic agent manipulates price in their favor. We then provide a conceptual analysis when both agents are strategic in the special case when $w_1 = w_2 = w$. This setting provides two possible sources of gains from trade: across states (insurance), and across time (savings). In the sequel where we explore a dynamic version of our model, these motives will evolve endogenously over time. We then consider the special case when $\Delta = 0$ and to focus on gains from trade across time, and not from risk sharing, when agents differ only in initial wealth. Finally, we explore the role of wealth and income heterogeneity using numerical plots.

The case of a monopolistic type. Suppose agents of Type 1 are the *monopolist type*. In this special case, an agent of type 2 chooses her optimal portfolios until the Arrow price equals her state price state-by-state, i.e., $q(z) = \Lambda_2(z)$ for $z \in \{1,2\}$. With some manipulation of these two conditions, her state-contingent consumption is

$$c_{2,2}(1) = \frac{1}{2}\frac{w_2 + \sum_z q(z)y_2(z)}{2q(1)} \qquad \text{and} \qquad c_{2,2}(2) = \frac{1}{2}\frac{w_C + \sum_z q(z)y_2(z)}{2q(2)}, \tag{14}$$

and her initial consumption is

$$c_{1,2} = \frac{w_2 + \sum_z q(z)y_2(z)}{2}. \tag{15}$$

These two equations imply a return on the wealth of competitive agents of Type 2

$$R_2^W = \frac{1}{4}\left(\frac{1}{q(1)} + \frac{1}{q(2)}\right). \tag{16}$$

in the competitive equilibrium, $q(1) = q(2) = q$, and the return on her wealth portfolio is the risk-free rate $R_2^W = \frac{1}{2q} = r_f$. With market power, however, $q(1) > q(2)$ because Type 1 agents restrict supply of state 1 Arrow assets. By Jensen's Inequality for $1/x$, then, $R_2^W > \frac{1}{q(1)+q(2)} = r_f$, and consequently Type 2 agents earn a risk premium because they are under-insured against state 1 and over-consume in state 2.

A strategic agent of Type 1, in contrast, puts a wedge between her state price and the Arrow price, i.e., $\Lambda_1(z) = q(z) + q'(z)a_1(z)$. With some manipulation of these two conditions, her state-contingent consumption is

$$c_{2,1}(1) = \frac{w_1 + \sum_z q(z) y_1(z) + \frac{\mu}{m_f}\left(q'(2)a_1(2)c_{2,1}(2) - 3q'(1)a_1(1)c_{2,1}(1)\right)}{4q(1)}, \tag{17}$$

$$c_{2,1}(2) = \frac{w_1 + \sum_z q(z) y_1(z) + \frac{\mu}{m_f}\left(q'(1)a_1(1)c_{2,1}(1) - 3q'(2)a_1(2)c_{2,1}(2)\right)}{4q(2)}, \tag{18}$$

and her initial consumption is

$$c_{1,1} = \frac{w_1 + \sum_z q(z) y_1(z) + \frac{\mu}{m_f} q'(z) a_1(z) c_{1,1}(z)}{2}. \tag{19}$$

The return on her wealth is then

$$R_1^W = R_2^W + \frac{1}{4}\frac{\mu}{m_f}\frac{q'(2)a_1(2)c_{2,1}(2) - q'(1)a_1(1)c_{2,1}(1)}{w_1 + \sum_z q(z)y_1(z) - \frac{\mu}{m_f}q'(z)a_1(z)c_{2,1}(z)}\left(\frac{1}{q(1)} - \frac{1}{q(2)}\right). \tag{20}$$

The monopolistic agent earns not only the competitive risk premium she creates by retaining risk $\frac{1}{2}\left(\frac{1}{q_1} + \frac{1}{q_2}\right)$, but also an addition expected excess return on her wealth portfolio. If her wealth share is not too high, then the coefficient on the $\frac{1}{q(1)} - \frac{1}{q(2)}$ term is positive, this additional piece in R_1^W is negative because $q_1 > q_2$, and therefore $\frac{1}{q(1)} - \frac{1}{q(2)} < 0$.[14] Consequently, a Type 1 agent earns a lower return on her wealth portfolio than a Type 2 competitive agent, or $R_1^W \leq R_2^W$.

We can further take a first-order approximation of a Type 2 agent's welfare around the competitive equilibrium, in which $\Lambda_1(z) = q(z) = q$ and $c_{2,1}(2) = c_{2,1}(1) = c_2$, to find

$$U_1 - \approx U_1^{competitive} + \frac{\mu}{m_f}\frac{q'}{4q}\left(a_1(1) + a_1(2)\right),$$

In the competitive equilibrium, $a_1(1) + a_1(2) = 0$. Suppose a Type 1 agent puts a wedge δ such that $a_1(1) + a_1(2) = \delta$, then $U_1 > U_1^{competitive}$. Because such an improvement

[14] Notice when the wealth share is close to symmetric $a_1(2) > 0 > a_1(1)$, i.e., Type 1 agents are buyers of state 2 claims and sellers of state 1. In this case, $q'(2)a_1(2)c_{2,1}(2) - q'(1)a_1(1)c_{2,1}(1)$ is positive. If the wealth share of a Type 1 agent becomes sufficiently small, then both $a_s(1)$ and $a_s(2)$ are negative because she becomes a seller of both claims to agent 2. In this case, her consumption in state 1 $c_{2,1}(1)$ is still higher than that in state 2, and the price (impact) of the state 1 Arrow asset price is higher than that of state 2. Again, this implies $q'(2)a_1(2)c_{2,1}(2) - q'(1)a_1(1)c_{2,1}(1)$ is positive. Finally, if agent 1's wealth share becomes sufficiently large, she may become a buyer of both assets. In this case, by continuity $q'(2)a_1(2)c_{2,1}(2) - q'(1)a_1(1)c_{2,1}(1)$ will fall and still be positive when $a_1(1)$ is in the neighborhood of 0. Consequently, if the wealth share of Type 1 agents is not sufficiently high, $q'(2)a_1(2)c_{2,1}(2) - q'(1)a_1(1)c_{2,1}(1)$ is positive.

is feasible, welfare is weakly higher for the monopolistic type than in the competitive equilibrium.

Our example with a monopolistic type highlights two insights. First, in the absence of strategic externalities, a monopolistic agent type has higher welfare than in the competitive equilibrium. This will be in sharp contract to our findings in the oligopolistic case. Second, the return on wealth of a strategic agent type does not necessarily correspond to her welfare. A monopolistic agent type actually earns a lower return on her wealth because the competitive Type 2 agent determines asset prices, and the strategic Type 1 agents voluntarily remains over-exposed to what Type 2 agents view as the high marginal utility state. This reflects that a Type 1 agent's wealth portfolio is a hedge for a Type 2 agent. Consequently, returns on wealth need not correspond to the value of wealth for large agents.

The case when $w_1 = w_2$. If strategic agents are ex-ante symmetric, then risk sharing is the only motive for trade, and we can search for an equilibrium in which each agent sells a_S units of the claim on the state in which she has high income, and buys a_B units of the claim on the state in which she has low income. Perfect risk sharing would require that $a_S = -\Delta$ and $a_B = \Delta$. To highlight the deviation from perfect risk sharing with market power, we write the optimal security positions as $a_S = -\Delta + \delta_S$ and $a_B = \Delta - \delta_B$, where δ_S and δ_B are optimally chosen shading terms for agents in their state-contingent roles as buyers and sellers. As a result, optimal distortions satisfy

$$\text{Seller distortion:} \qquad \left| \frac{\frac{1}{2}u'(\bar{y} + \delta_S)}{u'(w + q^* \cdot (\delta_S - \delta_B))} - q^* \right| = \frac{\mu}{m_f} q^{*\prime} \Big(\Delta - \delta_S \Big)$$

$$\text{Buyer distortion:} \qquad \left| \frac{\frac{1}{2}u'(\bar{y} - \delta_B)}{u'(w - q^* \cdot (\delta_S - \delta_B))} - q^* \right| = \frac{\mu}{m_f} q^{*\prime} \Big(\Delta - \delta_B \Big),$$

Agents find it optimal to distort portfolio holdings as both a buyer and a seller. Because each agent is a seller in one state and a buyer in the other, both agents are forced to imperfectly insure across both states. If agents also have asymmetric initial wealth, there are further distortions in trade across time.

The case when $\Delta = 0$. In strategic agents differ only in their initial wealth, then there is only a market for a risk-free asset, in which case if $w_1 > w_2$

$$\text{Seller distortion:} \qquad \left| \frac{u'(\bar{y} - a_2)}{u'(w_2 + q^* a_2)} - q^* \right| = -\frac{\mu}{m_f} q^{*\prime} a_2$$

$$\text{Buyer distortion:} \qquad \left| \frac{u'(\bar{y} + a_1)}{u'(w_1 - q^* a_1)} - q^* \right| = \frac{\mu}{m_f} q^{*\prime} a_1.$$

It is immediate the efficient trading quantity is increasing in dispersion in initial wealth, $w_1 - w_2$, because the only gains from trade are from Type 2 agents selling risk-free bonds to Type 1 agents to lower Type 1's intertemporal marginal rate of substitution. To the extent that market power hinders this reallocation of resources, the wealthier Type 1 agents consume less at date 1 and more at date 2. Consequently, market power raises the marginal propensity to consume for agents who are wealthier today.

Interestingly, if instead we considered the case where both agents have the same initial wealth w but Type 1 has higher income $\bar{y} + \epsilon$ at date 2, then Type 1 agents would be sellers of risk-free bonds and sell too little. Consequently, market power lowers the marginal propensity to consume for agents who are wealthier in the future. To the extent the wealthy have long duration wealth, our model predicts wealthier households should have lower marginal propensities to consume, as is observed in the data.

Numerical Example. To illustrate the portfolio and asset pricing implications of market power, we further specialize our example to a Strategic Equilibrium where the competitive fringe's mass goes to zero, holding μ/m_f fixed. Market-clearing then forces strategic types to hold essentially offsetting positions, $a_S = -a_B = -a^*$ for some a^*.

In the Strategic Equilibrium, all states then have the same prices q^* and price impact q'^*. In addition, strategic agents net expenditures at date 1 are zero so that $c_1 = \bar{y}$ for both strategic types. Summing up first-order conditions yields

$$q^* = \frac{\frac{1}{2}u'(\bar{y} + \Delta - a^*) + \frac{1}{2}u'(\bar{y} - \Delta + a^*)}{u'(\bar{y})}.$$

By the convexity of marginal utility, prices are increasing in the distortion to risk sharing. This is reflected in Figure 1, which shows that prices are elevated when wealth is symmetric. However, distortions are asymmetric when some agents are richer than others.

Figure 2 shows the equilibrium consumption allocation in the Cournot-Walras

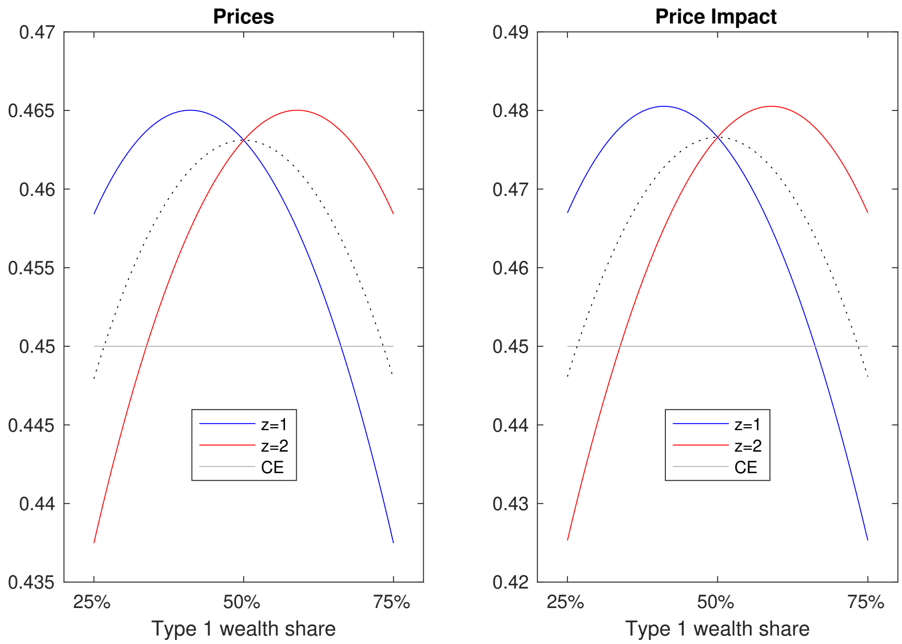

Figure 1: Asset prices (Left) and price impact (Right) as a function of the wealth share $w_1/2\bar{y}$ of Type 1.

equilibrium (CW) and the competitive equilibrium (CE) as a function of Type 1's initial wealth share $w_1/2\bar{y}$. There is noticeably excess volatility in Type 1's consumption at date 2, consistent with strategic agents extracting trading rents at the cost of more consumption volatility. At date 1, because the richer strategic agent internalizes her larger trading needs, she does not save enough, and the poorer agent saves too much.

An advantage of the allocation invariance of our complete markets setting (Proposition 2) is that we can express the Arrow asset exposures of strategic agents in terms of positions in more interpretable assets. Figure 3 clarifies the underlying distortions from market power by decomposing strategic agent portfolios into positions in a risk-free bond with payoffs $[1, 1]$ and in a swap with payoffs $[-1, 1]$. The plot reveals how both margins are distorted away from efficiency in the Cournot-Walras equilibrium. Strategic agents trade two little of both assets, the bond price is inflated for most wealth levels, and the swap price positively correlates with Type 1 agent's wealth share.

Finally, we plot the returns on the total wealth W_i of both agent types. We define total wealth as the sum of initial wealth and the present value of future income less initial consumption, $W_i = w_i + \sum_z q(z)y_i(z) - c_{1,i}$, and the expected return on total wealth R_i^W

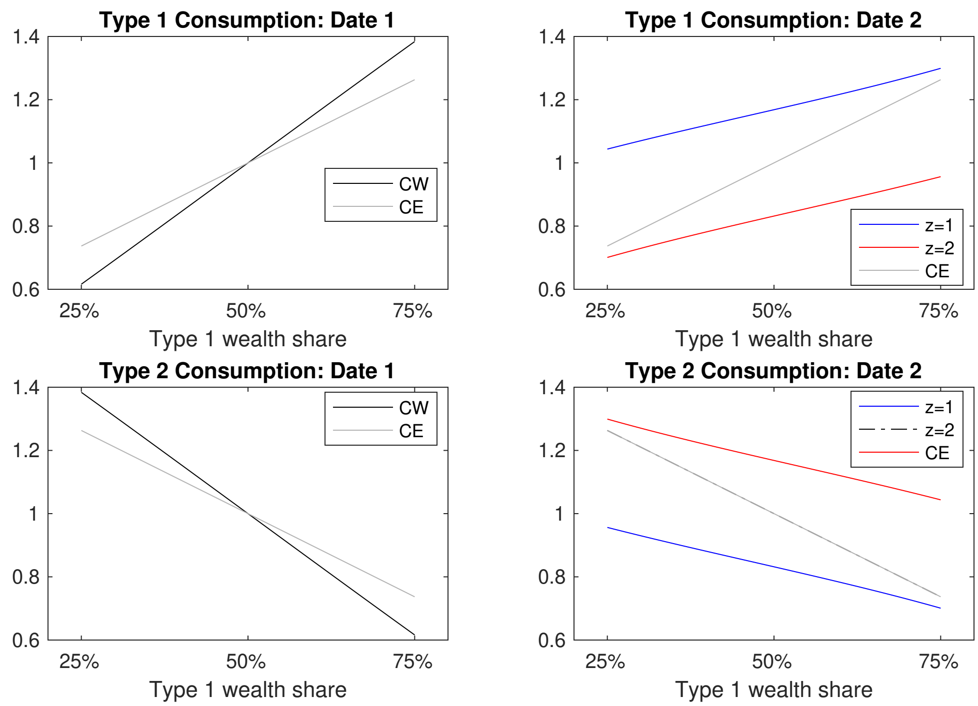

Figure 2: Consumption as a function of the wealth share $w_1/2\bar{y}$ of Type 1.

as in equation . Figure 4 plots the expected return on total wealth for both agent types in both the Cournot-Walras (CW) and the competitive (CE) equilibria, as well as the risk-free rate, for different wealth shares of Type 1 agents. Market power introduces excess volatility into the returns on the total wealth portfolios of both agents, which are constant in the competitive equilibrium. Interestingly, the wealthier agent type earns an expected return in excess of the risk-free rate even though there is no aggregate risk in the economy. In contrast, the poorer agent type earns a lower expected return below the risk-free rate. This reflects that the wealthier agent type under-diversifies more than the poorer agent type, and as a consequence bears more priced idiosyncratic risk in equilibrium.

Finally, we return to the finance versus industrial organization perspectives of market power. Under the finance view, size represents an impediment to risk sharing, and large investors are worse off than small investors because their trades have outsized price impact. We can measure this efficiency loss as the difference in welfare, or ex ante expected utility, for both agents between the Cournot-Walras and Competitive Equilibrium, respectively. Under the industrial organization view, size represents a source of rents, and large investors can extract surplus by maintaining a wedge between public (or market-

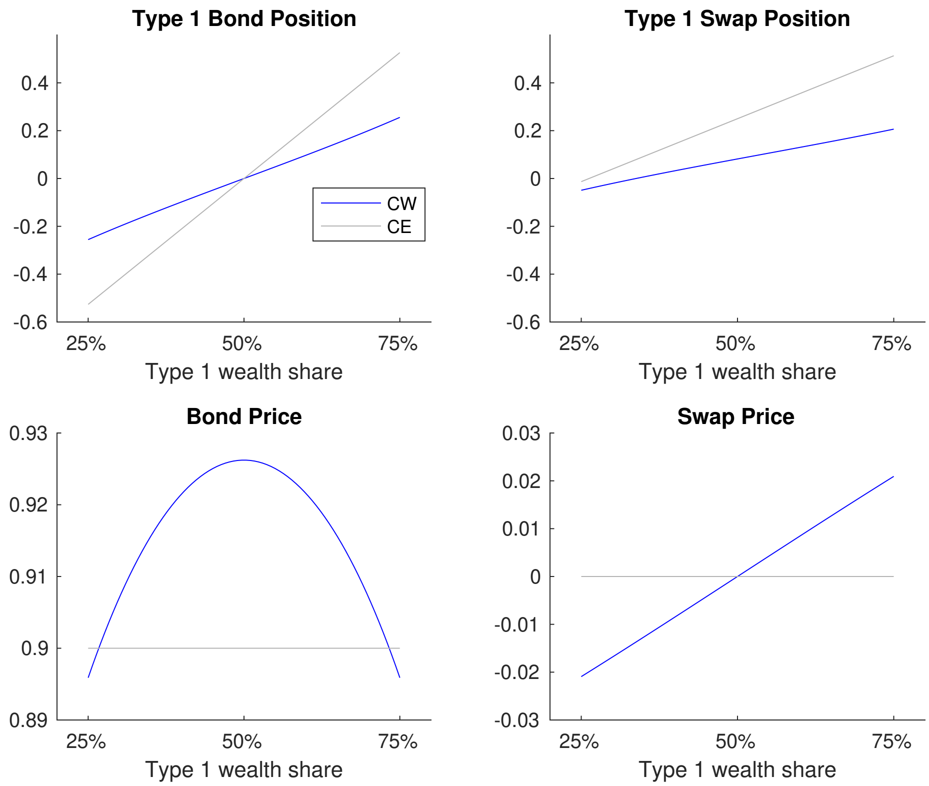

Figure 3: Portfolio of Arrow securities mapped into a risk-free bond with payoffs $[1, 1]$ and a swap with payoffs $[-1, 1]$ as a function of the wealth share $w_1/2\bar{y}$ of Type 1.

based) and private valuations of their wealth. We measure this "private" surplus as the investor's Excess Wealth $\tilde{W}_i$, which we define as

$$\underbrace{\tilde{W}_i}_{\text{Excess Wealth}} = \underbrace{c_{1,i} - w}_{\text{Excess Expenditure}} + \underbrace{\sum_z \Lambda(z)(c_{2,i}(z) - y(iz))}_{\text{NPV of consumption stream}}.$$

For competitive agents, $\tilde{W}_i = 0$, but for strategic agents it is strictly positive.

We plot the efficiency loss and excess wealth of both types of strategic agents in Figure 5. Interestingly, we can harmonize the financial and industrial organization views of market power by recognizing that the two concepts are connected. From the left panel, there are efficiency losses when large agents behave strategically because expected utility is always lower in the Cournot-Walras Equilibrium, with the poorer agent type always being worse off. From the right panel, both agents earn rents as measured by excess

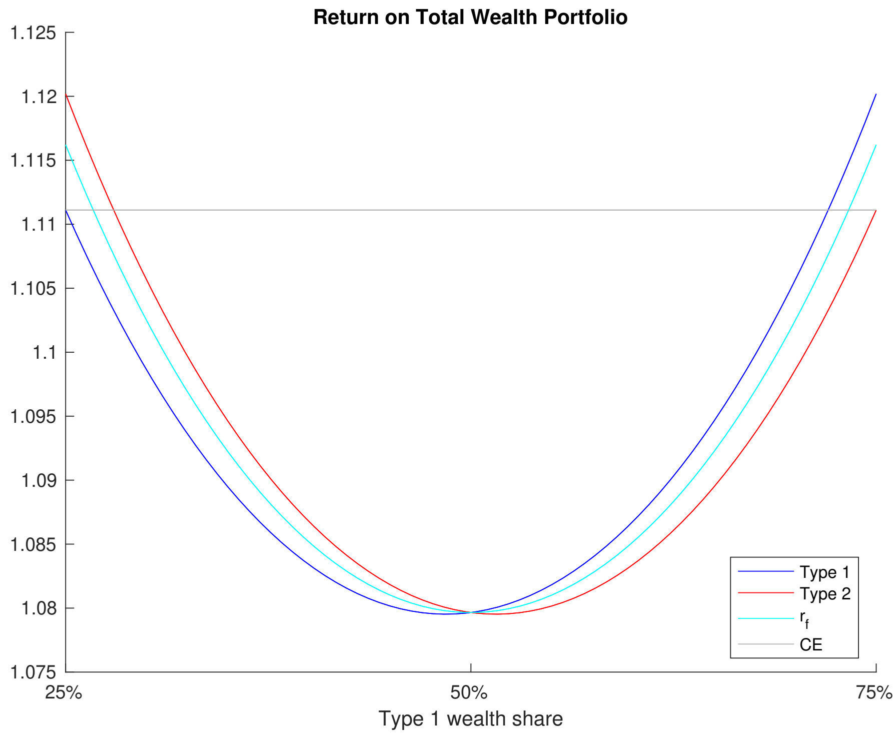

Figure 4: Return on total wealth as a function of the wealth share $w_1/2\bar{y}$ of Type 1.

wealth, which is the same for both types because of symmetry, and these rents are positively related to the efficiency losses in the left panel. Consequently, although privately strategic agents benefit from market power (the industrial organization view), socially they are worse off because of the distortions of price impact (the finance view).

3.4 Endogenizing Real Concentration μ via an Entry Game

In this section, we examine the implications of market power for the size of agents, μ_i, which we now determine and allow to be heterogeneous across types. Notice this size divides up the initial wealth and endowment of agents of type i, w_i and $y_i(z)$, respectively, among $1/\mu$ agents. A larger size corresponds to fewer of agents of that type, and we consider types with fewer agents to be more concentrated industries. Our key insight is financial market power gives rise to returns-to-scale to size that incentives entry into a type, and these returns are endogenous to the risk exposures of other types. Through this channel, there are externalities in the choice of size because financial market concentration

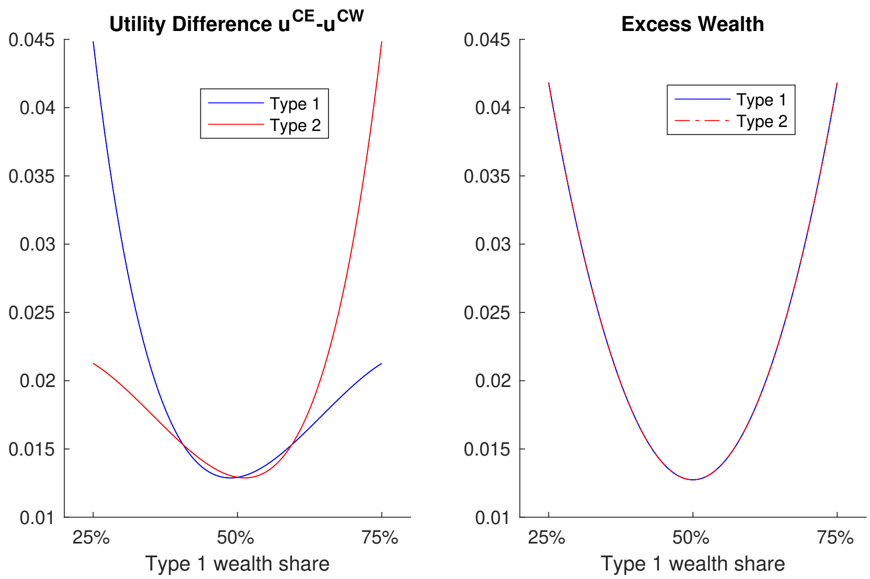

Figure 5: Utility Difference between Cournot-Walras and Competitive Equilibria (Left) and Excess Wealth (Right) as a function of the wealth share $w_1/2\bar{y}$ of Type 1.

within types worsens risk sharing across types, which feeds back into the endogenous benefits of scale.

We now assume there is an initial date 0 before financial market trading occurs. At date 0, an agent can pay a fixed cost $\mu_i f$ in certainty equivalent utility to become an agent of type i with size μ_i, and this cost is the same for all types. As such, there is free-entry into becoming a large agent. Because agent j of type i earns indirect utility $h(\mu_i)\,U_{j,i}$ at date 1 from decision problem (2), where the $h(\mu_i)$ term arises from the homotheticity of agent utility $u(\cdot)$, $1/\mu_i$ agents will enter until

$$\mu_i = \arg\min_{\mu_{i'}\in[0,1]} h(\mu_{i'})\,U_{j,i} \tag{21}$$

$$s.t.\ u\left(U_{j,i}\right)^{-1} \geq f, \tag{22}$$

taking as given the sizes of all other types μ_k for $k \neq i$.

We illustrate the behavior of the entry game in Figure 6 for parameters listed in Section 4.1 when there are two types that are symmetric and have log utility. The left panel plots the utility of agents of type 1 $U_{j,1}$ (normalized by $h(\mu_1)$) across different sizes μ_1 for different sizes of agents of type 2 μ_2.[15] As one can see, the utility of type one agents is

[15]We focus on $U_{j,1}$ to mute the mechanical benefit to agent utility from being larger.

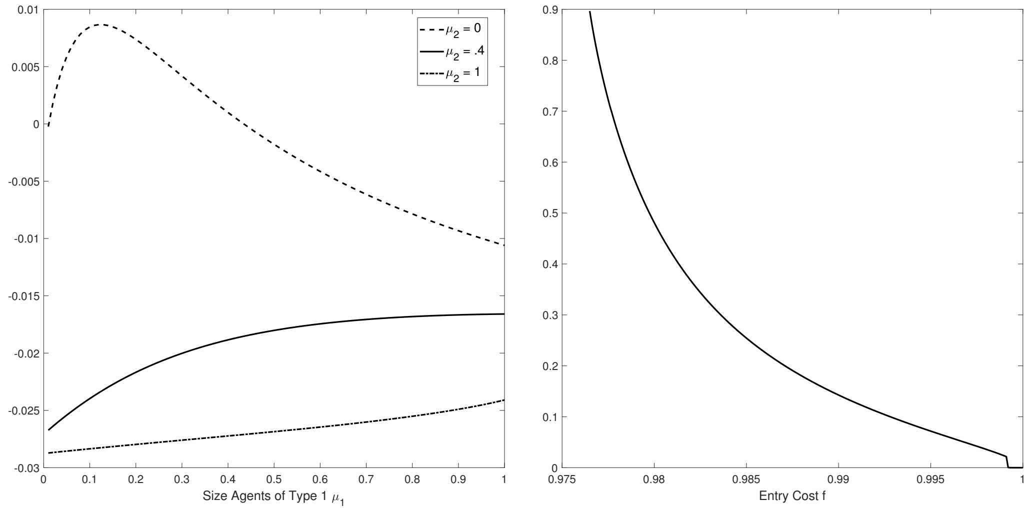

Figure 6: Normalized Utility of Agents of Type 1 $U_{j,1}$ for Different Sizes μ_1 Conditional on the Size of Agents of Type 2 μ_2 for an Entry Cost $f = 0.98$ (Left Panel) and Equilibrium Size of Both Types for Different Entry Costs (Right Panel) for the parameters listed in Section 4.1

increasing in their size when μ_2 is sufficiently large, but is hump-shaped when μ_2 is sufficiently small (small μ_2). This reflects the trade-off between extracting rents, which may be hump-shaped in size, and consumption volatility, which is increasing in size (Corollary 1). When the other type is large, the impact of increasing rents for a larger size dominates type 1 agents' utility, while when the other type is small, for large enough size, the cost of higher consumption volatility drags down type 1 agents' utility. Importantly, there are strategic complementarities in size across agent types and market power incentivizes agents to imperfectly share risks and become large.

The right panel of Figure 6 plots the equilibrium size of both agent types for different fixed costs of entry f. Interestingly, and perhaps surprisingly, the equilibrium size is decreasing in barriers to entry. This is because the size of one agent type imposes an externality on the other type through impaired risk sharing. When agents of one type become larger (i.e., a larger μ_i), this worsens risk sharing for agents of the other type, raising the volatility of their consumption through induced under-diversification. Through this risk-sharing channel, there is complementarity across agent types in entry size, and equilibrium (certainty equivalent) utility for both is actually lower when they are both large compared to when they are both small. This gives rise to a negative relation between fixed costs of entry and the size of agents that enter.

4 Dynamic Model

In this section, we consider a dynamic version of our model to explore the implications of market power for wealth accumulation and inequality. Time is discrete, the horizon infinite, and strategic agents and the competitive fringe now receive income at each date. Specifically, strategic agents of type i receive an income $y_i(z_t)$ and the fringe an income $y_f(z_t)$ that are drawn from bounded and discrete sets. We assume that income processes are time-homogeneous and all income processes can again jointly be summarized by Z possible realizations. Markets are dynamically complete such that there is a full set of Arrow securities at each time t over all possible state realizations at time $t+1$.

Strategic agent of type i has wealth $w_{t,i}$ and subjective discount rate β, and now solves the decision problem

$$\begin{aligned} U_{0,j,i} = \quad & \sup_{\sigma_{j,i}} \quad \sum_{t=0}^{\infty} \sum_{z_t \in \mathcal{Z}} \pi(z_t) \beta^t u\left(c_{t,j,i}(z_t)\right) \\ & \text{s.t.} \quad \mu w_{t,i} \geq \mu c_{t,j,i} + \sum_{z_{t+1} \in \mathcal{Z}} \tilde{Q}_{t,i,j}(\mathbf{A}, z_{t+1}) \mu a_{t,j,i}(z_{t+1}), \\ & \qquad \mu w_{t+1,i}(z_{t+1}) = \mu y_i(z_{t+1}) + \mu a_{t,j,i}(z_{t+1}), \end{aligned} \tag{23}$$

where the two constraints are the budget constraint of agent j of type i and the law of motion of her wealth, respectively. We assume that strategic agents lack commitment and focus on a Markov Perfect Cournot-Walras Equilibrium to avoid issues of reputation.

For simplicity, we assume an overlapping generations structure for the competitive fringe, such that each generation continues to solve (3). As such, the price system at each date is still characterized by (1), with prices at date t given by $q_t(z)$. As in the static model, pricing by the fringe guarantees that there is no arbitrage at each date, and also resolves any strategic uncertainty for strategic agents about equilibrium price impact.

To characterize optimal strategic portfolios in this dynamic setting, we now define the state price $\Lambda_{t,j,i}(z_{t+1})$ for agent j of type i at date t as the marginal rate of substitution between consumption in state z_{t+1} at date $t+1$ and date t, that is

$$\Lambda_{t,j,i}(z) \equiv \frac{\pi(z)\,\beta u'\left(c_{t+1,j,i}(z_{t+1})\right)}{u_1'\left(c_{t,j,i}\right)}. \tag{24}$$

We then have the following proposition.

Proposition 7 *There exists an equilibrium in which the optimal policies for $a_{t,j,i}(z_{t+1})$ satisfy*

$$\Lambda_{t,j,i}(z_{t+1}) = q_t(z_{t+1}) + \frac{\mu}{m_f} q_t'(z_{t+1})\, a_{t,j,i}(z_{t+1}). \tag{25}$$

From Proposition 7, the optimal portfolio choice of agent j of type i balances similar trade-offs as in (7) in our static setting. The right hand side reflects the marginal cost of buying security z_{t+1}, which is the cost of the security, $q_t(z_{t+1})$, plus the marginal impact that the agent has on the price, $\frac{\mu}{m_f} q_t'(z_{t+1})\, a_{t,j,i}(z_{t+1})$. The left-hand side is the marginal benefit of buying the security, her state price $\Lambda_{t,j,i}(z_{t+1})$. This marginal benefit, however, now has a dynamic dimension because the strategic agent is forward-looking and chooses consumption at each date to maximize her expected discounted continuation utility.

To gain further insight into the dynamic impact of market power, we iterate forward on strategic agent i's budget constraint from (23) to arrive at[16]

$$w_{t,j,i} + \sum_{s=t}^{\infty} \Lambda_{t,j,i}(z_s)\, \Pi_{s,j,i} = c_{t,j,i} + \sum_{s=t+1}^{\infty} \sum_{z_s \in \mathcal{Z}} \Lambda_{t,j,i}(z_s) \left(c_{s,j,i}(z_s) - y_i(z_s)\right), \tag{26}$$

with the understanding that $\Lambda_{t,j,i}(z_t) = 1$. In the above, $\Pi_{s,j,i} = \sum_{z_{s+1} \in \mathcal{Z}} \Pi_{s,j,i}(z_{s+1})$ is i's period s total rents and the rent in market z_{s+1} given by (13). Strategic agent i trades such that the present-value of their net expenditures (consumption minus her income), the right-hand side of (26), exceeds the value of her type's current wealth w_{ti}, compared to competitive agents who trade such that the present-value equals their wealth. Intuitively, strategic agent i manipulates prices to earn a surplus on all her trades in financial markets, the present value of which is the second term on the left-hand side of (26). As

[16]Implicitly, we impose the transversality condition:

$$\lim_{T \longrightarrow \infty} \sum_{z_T \in \mathcal{Z}} \Lambda_{it,T}(z_T)\, w_T(z_T) = 0.$$

This is satisfied with bounded aggregate income, $Y(z) + m_f y_f(z)$, because, by market clearing in asset markets, it is equal to total agent wealth.

a result, there is a gap between a strategic agents' private valuation of her wealth and the public valuation of it using market prices. This is because state prices are dispersed across strategic agents with market power even though markets are complete.[17] As the wealth distribution evolves over time, the present value of these rents changes with relative market power.

A key insight from the static model is that there is more upward pressure on asset prices when agents are more symmetric. This result also holds in our dynamic setting.

Corollary 2 *Suppose strategic agents are type-symmetric and have the same wealth at date t. In a Strategic Equilibrium, all asset prices $q_t(z_{t+1})$ are higher, and the risk-free rate is lower, than in the competitive equilibrium.*

Symmetry, however, is not a stable outcome in the dynamic model. Since there is imperfect risk sharing, some agents must be wealthier than others ex-post. Their increased size then raises their market power, leading to a transition to a more monopolistic structure. Market concentration consequently gives rise to a rich set of predictions for asset prices based on the dispersion in the wealth distribution among strategic agents. We illustrate these effects using the following setting.

4.1 Numerical Illustration

We now illustrate the dynamic impact of market power through a numerical example. To emphasize the role of the wealth distribution, we shut down all sources of heterogeneity except for wealth. There are two equally likely states of the world at each date and two type-symmetric strategic agent types that receive i.i.d. income shocks. Agents of type 1 receive $\bar{y}+\Delta$ while type 2 receive $\bar{y}-\Delta$ in state 1, and the reverse in state 2. Each generation of the competitive fringe receives $\bar{y}$ at every date. We set $\bar{y}=1$ and $\Delta=0.25$. Strategic agents and the fringe at date $t+1$ have log utility, $u(x)=u_f(x)=\log(x)$.

To ensure that the fringe has no effect on wealth dynamics, we focus on the *strategic limit* where $m_f \to 0$, holding μ/mu_f constant. In this limit, the fringe determines the residual demand curve, but markets essentially clear among strategic agents. The state variable then is the distribution of wealth (w_1, w_2), which we initialize at $(w_{10}, w_{20}) = (1,1)$. This means that agents are ex-ante symmetric at time 0, and symmetric conditional

[17] A similar phenomenon occurs with competitive agents in incomplete markets, but there differences in valuations arise because certain risks cannot (rather than will not) be traded.

on the state in all other periods. We can consequently write a single value function given an agent's own wealth w and the other agent type's wealth $\bar{w}$. This value function satisfies

$$V(w,\bar{w}) = \max_{a(h),a(l)} \quad u(c) + \beta \sum_z \pi(z) V(w'(z), \bar{w}'(z)) \tag{27}$$

$$\text{s.t.} \qquad c = w_1 - \sum_z q(z)a(z). \tag{28}$$

$$w'(z) = y'(z) + a(z) \qquad \text{and} \qquad \bar{w}'(z) = \bar{y}'(z) + \bar{a}(z). \tag{29}$$

This setting features a stark competitive benchmark: because of perfect risk sharing, the wealth distribution is constant after any sequence of shocks.

Proposition 8 (Wealth Dynamics under Perfect Competition) *If markets are perfectly competitive, there is perfect risk sharing among agents in every period. The wealth distribution therefore remains constant in all periods and after any sequence of shocks, and there is no variation over time in prices, consumption, or portfolios.*

This provides a clear contrast to the wealth dynamics that obtain in imperfectly competitive markets. To illustrate these dynamics, we simulate the model for 20 periods. The first 19 shocks are favorable to Type 1 (i.e. state 1 is realized), while the last shock is favorable to Type 2 (i.e. state 2 is realized). The left panel in Figure 7 shows the resulting *realized* wealth levels. The other panels show the degree of risk management by plotting state-contingent possible future realizations of wealth for Type 1 agents (middle panel) and Type 2 agent (right panel). Blue lines show wealth after a good shock; red lines after a bad shock, and black lines the expectation. The dashed line depict the counterfactual of perfect competition. We construct this counterfactual taking as given the wealth distribution at the beginning of the period.

The left panel reveals that even after a long series of positive shocks, Type 1's wealth remains vulnerable to a negative shock. In fact, equilibrium risk sharing is such that the wealth distribution reverses after a single negative shock, with Type 1 having less wealth in period 20 than in period 1. This is because of the risk-rent trade-off: as investors become wealthier, market power makes it more difficult for agents to manage risk. As a result, agents trade less over time, particularly relative to their wealth. Since agents with high income also become wealthier, market power consequently amplifies underlying income inequality.

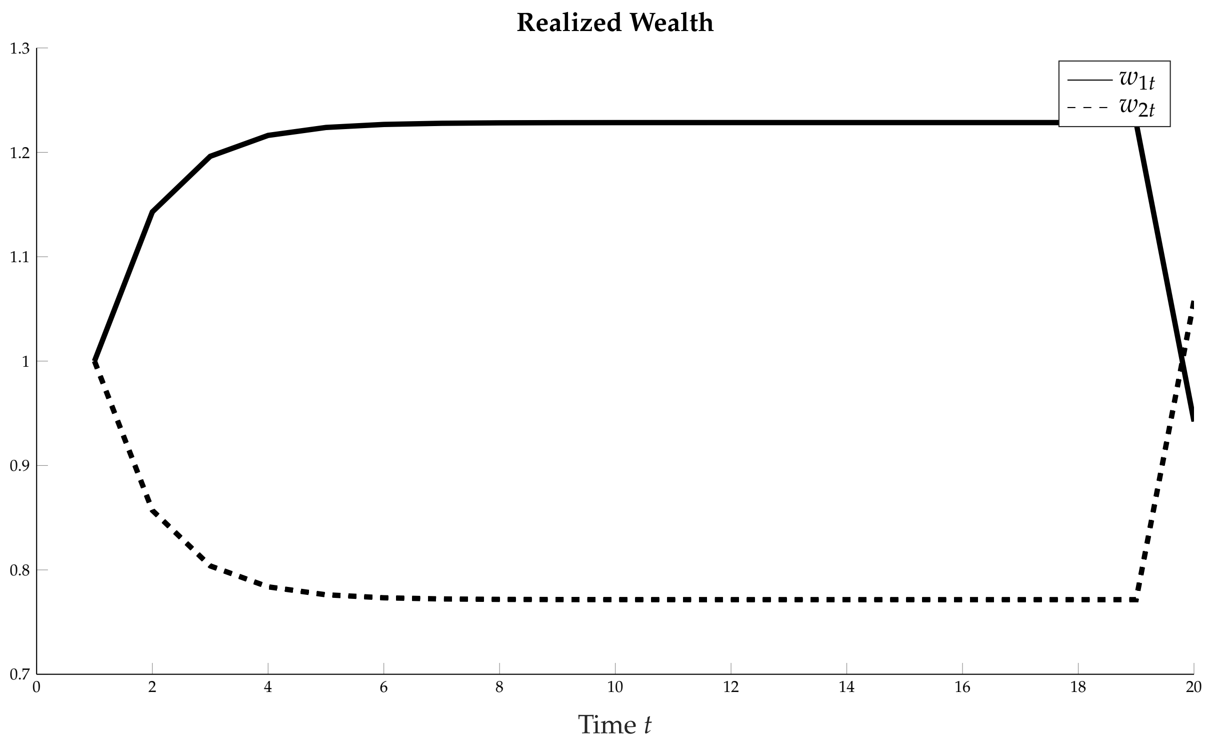

Figure 7: Simulated wealth distribution and exposure to risk over time. The market equilibrium is displayed with solid lines, while the competitive benchmark is displayed with dashed lines.

The comparison to the competitive benchmark further reveals that asymmetries in the wealth distribution also lower agents' savings rates. This reflects that part of agents' returns to their wealth portfolios are accrued in trading rents.[18] It also reflects that agents save less and consume more in the present because price impact acts as a tax on financial assets. The difference from the competitive benchmark increases as the wealth distribution becomes more asymmetric. Market concentration consequently presents a mecha-

[18]From equation (26), the present discounted value of an agent's net consumption (consumption minus income) stream is equal to her wealth plus the present value of her trading rents. Intuitively, a large agent can afford a more expensive wealth portfolio than her wealth supports because of trading rents.

nism that also *raises* the marginal propensity to consume for those at the very top of the wealth distribution.

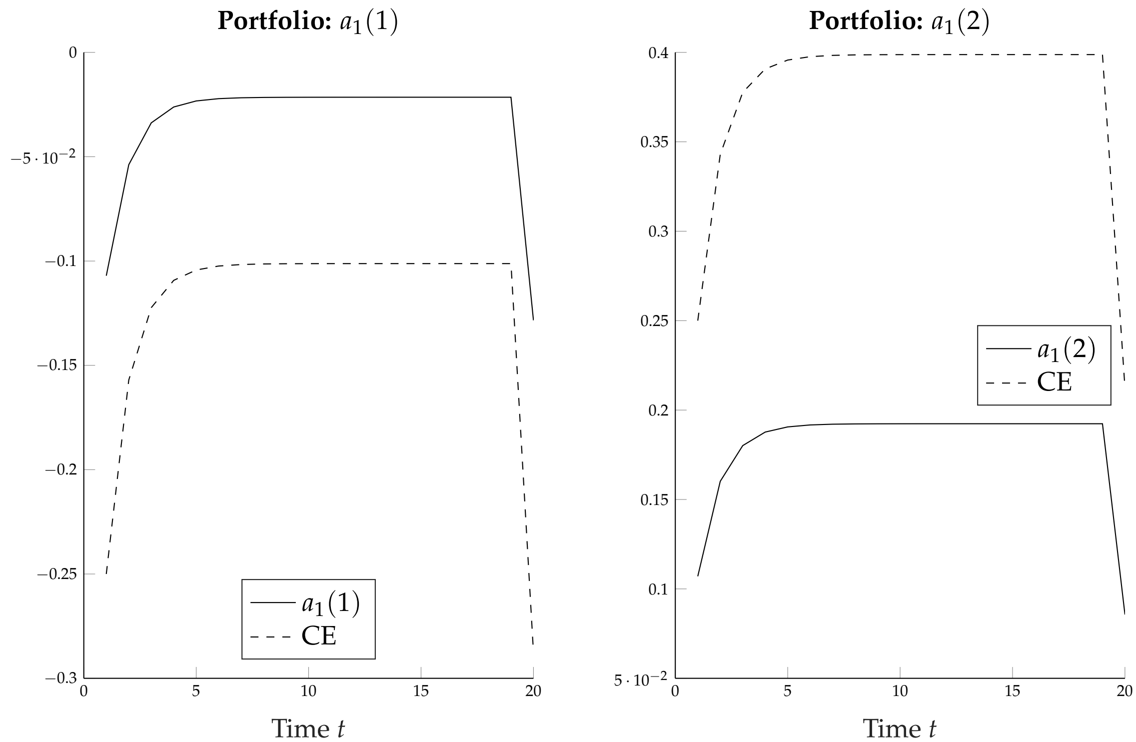

Figure 8: Portfolio choice over time by Type 1 agents (Left Panel) and Type 2 agents (Right Panel). The market equilibrium is displayed with solid lines, while the competitive benchmark is displayed with dashed lines.

Figure 8 shows the portfolios chosen by both agent types.[19] As is apparent, market power leads to quantity shading in both securities. Type 1 agents consequently buy much less insurance against state 2 than they would under perfect competition. Indeed, this gap grows as the agent becomes wealthier because larger positions have higher price impact.

Figure 9 depicts how asset prices evolve over time. At time 0, agents are symmetric and all prices are above the competitive benchmark, consistent with Corollary 2. This no longer holds, however, as the wealth distribution becomes more asymmetric. The price of asset 1 falls as Type 1 becomes relatively wealthier, while the price of asset 2 initially rises. The latter effect is driven purely by market power because prices in the competitive equilibrium are the same in both states and have different behavior over time. Market power, in addition, leads to much sharper reactions of prices to the wealth distribution;

[19]Type 2's portfolio is pinned down By market clearing, i.e., $a_2(z) = -a_1(z)$.

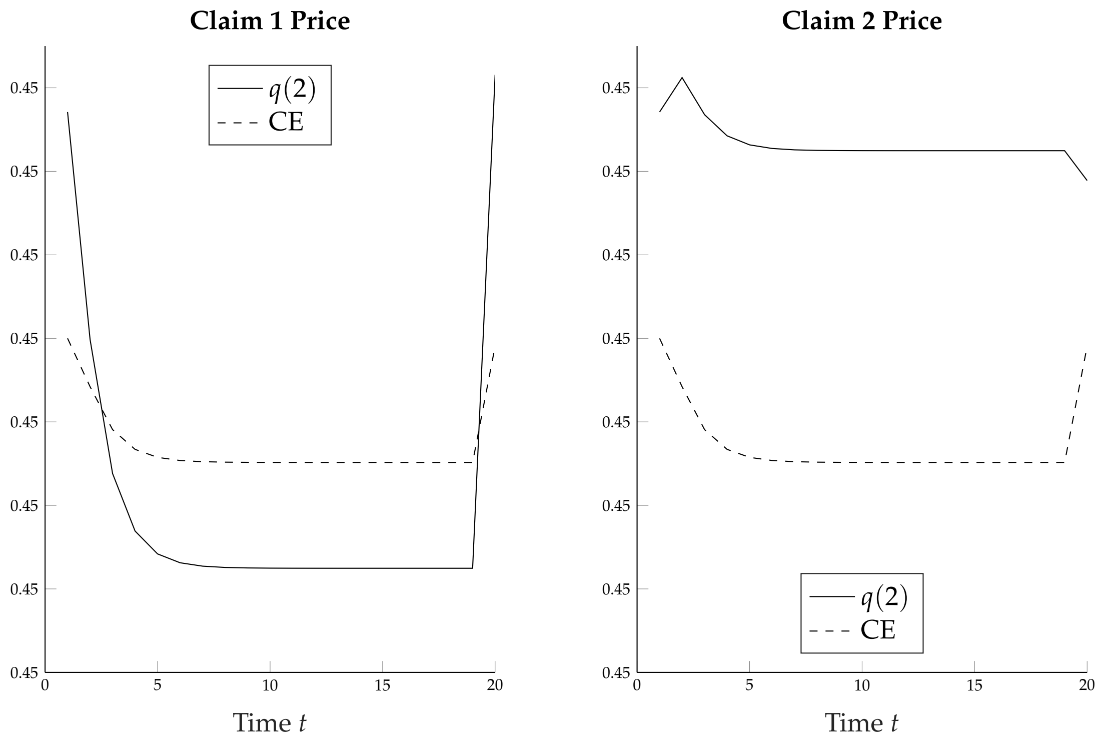

Figure 9: Asset prices of the claim to state 1 (Left Panel) and state 2 (Right Panel) over time. The market equilibrium is displayed with solid lines, while the competitive benchmark is displayed with dashed lines.

that is, prices display more volatility.

The dynamics of asset returns are shown in Figure 10. The left panel reveals that, because risk sharing is impaired, the risk-free rate us lower than in the competitive equilibrium. The risk-free rate rises as the wealth distribution becomes more unequal because wealthier agents have a lower willingness to pay for insurance. Because wealthier Type 1 agents also have a disproportionate impact on the equilibrium, this effect dominates the higher willingness to pay for insurance of the poorer Type 2 agents.

The right panel of Figure 10 shows that the excess returns to claims 1 and 2 have opposite dynamics. As Type 1 agents become wealthier, asset prices reflect more their marginal willingness to pay for both assets. Since Type 1 agents become more highly exposed to their own income state (state 1) as they accumulate wealth, the excess return of claim 1 rises to reflect this amplified risk exposure. In contrast, since Type 1 agents are under-exposed to state 2, the excess return to claim 2 falls to reflect its role as insurance for those agents.

Our analysis can consequently help rationalize several empirical facts about wealth

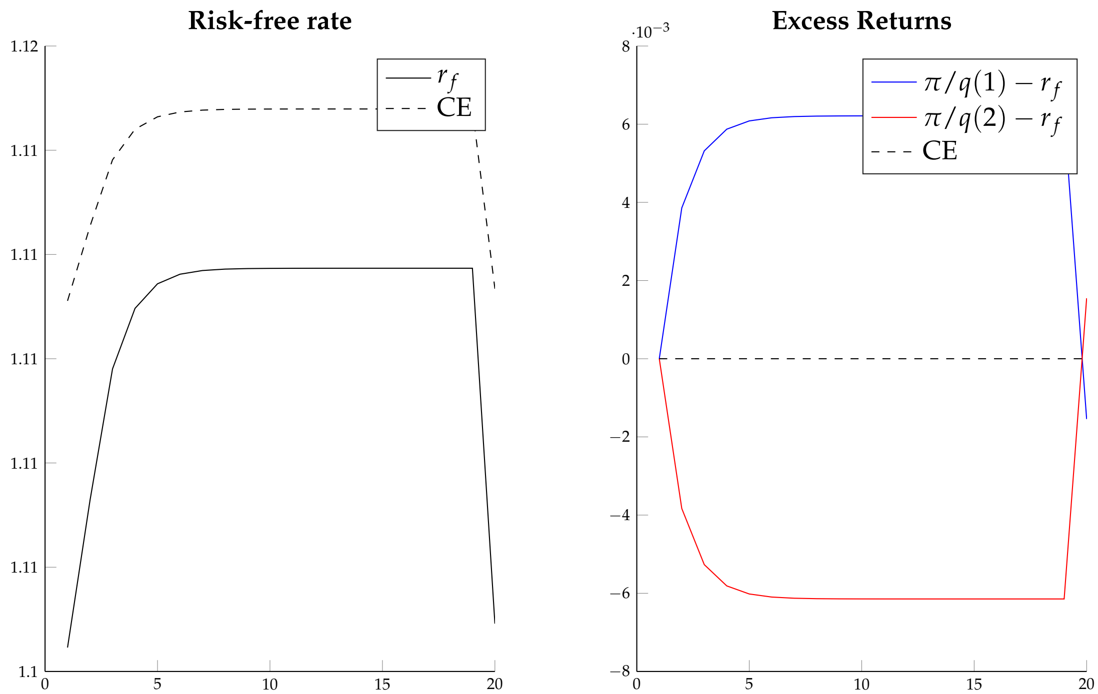

Figure 10: The risk-free rate (Left Panel) and asset excess returns (Right Panel) over time. The market equilibrium is displayed with solid lines, while the competitive benchmark is displayed with dashed lines.

inequality. First, market power gives rise to endogenous illiquidity in financial markets that induces wealthier individuals to remain under-diversified and maintain a large share of their holdings in illiquid paper wealth. Second, because of market power, wealthier earn higher returns on their wealth portfolios than poorer investors, perpetuating inequality. Third, that low risk-free rates may be a symptom rather than a cause of wealth inequality (e.g., Greenwald, Leombroni, Lustig, and Van Nieuwerburgh (2021),). Ultimately, our analysis emphasizes that exposure to idiosyncratic risk is important for understanding the right tail of the wealth distribution, and that even wealth individuals are endogenously highly exposed to idiosyncratic risk.

4.2 Empirical Implications

In this subsection, we discuss the implications of our dynamic model for measuring the distortions from market power in financial markets. Our key insight is that empirical exercises that analyze the behavior of wealthy agents need to take into account the (endogenous) illiquidity of their portfolios.

Our dynamic model emphasizes the rich feedback between market power, asset valuations, and inequality. When strategic agents are relatively symmetric, market power inflates all asset prices relative to perfect competition. As agents become more unequal in wealth, however, market power lowers prices from these elevated values and may even push prices below their competitive benchmark values. Market power consequently distorts the wealth of large agents and their incentives to trade in relatively illiquid markets. It is therefore essential to account for market power when assessing empirically how the wealthy, who disproportionately hold their wealth in illiquid assets, allocate their portfolios. Fagereng, Gomez, Gouin-Bonenfant, Holm, Moll, and Natvik (2022), for instance, measure the benefits of rising asset valuations for the wealthy as the present-discounted value of the relative price gains realized from asset sales. Our analysis suggests such calculations may understate the true value of rising asset prices because of rents wealthy agents garner through strategic trading. This is, in part, because wealthy agents' private valuations differ from public valuations of their wealth using market prices.

Our analysis also provides guidance on how to modify measurement of the wealth of the ultra-wealthy to account for private valuations. Because strategic agents trade until the gap between an asset price $q\left(z\right)$ and her state price $\Lambda_i\left(z\right)$ is $q'\left(z\right)a_i\left(z\right)$, this implies we can recover her private value according to $q\left(z\right)+q'\left(z\right)a_i\left(z\right)$. Consequently, her private valuation of her wealth $\tilde{W}_{t,i}$ is

$$\tilde{W}_{t,i}=w_{t,i}+\sum_{z_{t+1}}\Lambda_{t,i}(z_{t+1})y_i(z_{t+1})=w_{t,i}+\sum_{z_{t+1}}\left(1+\frac{q'\left(z_{t+1}\right)}{q\left(z_{t+1}\right)}a_i\left(z_{t+1}\right)\right)q(z_{t+1})y_i(z_{t+1}). \tag{30}$$

Consequently, observing how ultra-wealthy investors trade, and the price impact in the financial markets they trade, is sufficient to recover their private valuations of their wealth.

5 Conclusion

We construct a dynamic model of concentrated financial markets in which large, risk-averse agents internalize their price impact when trading state-contingent claims. We show that large agents must accept more consumption risk to distort asset prices in their favor. This imperfect risk sharing gives rise to ex-post wealth inequality that worsens market liquidity, reducing trade and further amplifying portfolio under-diversification.

As a result, even wealthy individuals remain highly exposed to idiosyncratic risk and vulnerable to negative income shocks. The distribution of wealth further determines how market power impacts asset prices: valuations are higher than in the competitive equilibrium when the wealth distribution is symmetric, but tilted in favor of wealthier agents when it is asymmetric. Our analysis can consequently explain why wealthy individuals are under-diversified, earn higher returns on their wealth than poorer households because of market power, and have substantial paper wealth that is difficult to trade.

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A Proofs of Propositions

A.1 Proof of Proposition 1:

Step 1: The Problem of the Fringe:

From the first-order condition for $a_f(z)$ from the competitive fringe's problem (3), we can recover the pricing equation of the Arrow-Debreu claim to security z

$$\tilde{q}(z) = \pi(z)u_f'\left(c_{2f}(z)\right) = \Lambda_f(z),$$

where $\Lambda_f(z)$ is the competitive fringe's state price. Since $c_{2f}(z) = y_f(z) + a_f(z)$, imposing the market-clearing condition, (1), reveals that

$$\tilde{q}(z) = \pi(z)u_f'\left(y_f(z) - A(z)\right).$$

In equilibrium, this must be the realized price of the claim, $Q(\mathbf{A}, z)$. Consequently, the competitive fringe's Euler Equation pins down asset prices in the economy. As this price is a function of state variables from the perspective of the fringe, we designate the realized price more concisely as:

$$q(z) = Q(\mathbf{A}, z).$$

Step 2: Equilibrium Price Impact:

We next impose a consequence of our Cournot-Walras equilibrium concept. Since agents of type i take the demands of other agents (even within their type) as given. As a consequence, because $u_f(z)$ is twice continuously differentiable and each agent's position size scales by its mass μ, we can derive each agent's perceived price impact:

$$\frac{\partial \tilde{Q}_{j,i}(\mathbf{A}, z)}{\partial a_i(z)} = -\frac{\mu}{m_f}\pi(z)u_f''\left(c_{2f}(z)\right) = -\frac{\mu}{m_f}\frac{\partial q(z)}{\partial A(z)},$$

which also implies that price impact is symmetric across all strategic agents. Defining $q'(z) = \frac{\partial q(z)}{\partial A(z)}$ yields the expression in the statement of the proposition.

Finally, we recognize that price impact $q'(z)$ is convex as a consequence of the convex marginal utility of the fringe. It is straightforward to see that:

$$q''(z) = \frac{\mu}{m_f}\pi(z)u_f'''\left(c_{2f}(z)\right) > 0,$$

$$q'''(z) = -\left(\frac{\mu}{m_f}\right)^2 \pi(z)u_f''''\left(c_{2f}(z)\right) > 0.$$

As such, price impact is convex in the net demand of strategic agents.

A.2 Proof of Proposition 2:

As a preliminary, suppose that we have some arbitrary asset span indexed by the $|\mathcal{Z}| \times |\mathcal{Z}|$ matrix X that is of full rank. In the special case of Arrow-Debreu assets, $X = I_{|\mathcal{Z}|}$, i.e., the identity matrix of rank $|\mathcal{Z}|$. Let x_k index the k^{th} row vector of X, and $x_k(z)$ be the dividend asset k pays in state z.

If the competitive fringe trades assets with asset span X, then it is immediate from the first-order conditions of the competitive fringe's optimization problem that the vector of asset prices $\vec{q}_X$ satisfies:

$$\vec{q}_X = X\vec{\Lambda}_f = X\vec{q}, \tag{31}$$

where $\vec{\Lambda}_f$ is the vector of the fringe's state prices and $\vec{q}$ the vector of Arrow asset prices.

Since the quasi-linear competitive fringe now maximizes $u_f\left(y_f(z) - \sum_{k=1}^{|\mathcal{Z}|} x(z)\, x_k(z)\, A_{x_k}(z)\right) + \sum_{k=1}^{|\mathcal{Z}|} x(z)\, q_{x_k} A_{x_k}(z)$, where $A_{x_k}(z)$ is the total demand for asset k of the strategic agents, it follows that the price impact function can be summarized by the matrix Γ:

$$\Gamma = XUX', \tag{32}$$

where U is the diagonal matrix with diagonal entries $-\frac{\mu}{m_f}\pi(z)u_f''\left(c_{2f}(z)\right)$.

Step 1: The Law of One Price:

That the Law of One Price holds for redundant assets in our complete markets economy with Arrow-Debreu securities follows immediately from equation (31). Arrow-Debreu prices in the economy therefore satisfy martingale pricing with $\Lambda_f(z)$ as the appropriate state price deflator.

Consequently, what is essential is that the competitive fringe takes prices as given, which ensures no arbitrage across traded assets by the Law of One Price.

Step 2: Trade in Redundant Assets:

We next show that, if a redundant asset $x_k(z)$ is introduced into the Arrow-Debreu complete markets economy, there must trade in that asset. Notice that the first-order condition for strategic agent i's optimal asset position a_{i,x_k} in the redundant asset is:

$$\sum_{z\in\mathcal{Z}} x_k(z)\,\Lambda_i(z) = q_{x_k} + \frac{\partial q_{x_k}}{\partial a_{i,x_k}} a_{i,x_k} \tag{33}$$

Similarly, aggregating the first-order conditions of strategic agent i (see Proposition 3), we also have that:

$$\sum_{z\in\mathcal{Z}} x_k(z)\,\Lambda_i(z) = \sum_{z\in\mathcal{Z}} x_k(z)\,q(z) + \sum_{z\in\mathcal{Z}} \frac{\partial q'(z)}{\partial a_i(z)} a_i(z)\,. \tag{34}$$

Equation (33) and (34), and invoking that $q_{x_k} = \sum_{z\in\mathcal{Z}} x_k(z)\,q(z)$ by no arbitrage, it follows that

$$\sum_{z\in\mathcal{Z}} \frac{\partial q'(z)}{\partial a_i(z)} a_i(z) = \frac{\partial q_{x_k}}{\partial a_{i,x_k}} a_{i,x_k}. \tag{35}$$

Since the left-hand side of equation (35) is nonzero, it follows that the right-hand side must be as well. Consequently, there must be trade in the redundant asset if there is trade in the replicating assets.

Step 3: Market Structure Invariance:

We now establish that whether the complete markets span is $I_{|\mathcal{Z}|}$ or X has no real effects on allocations when X has full rank. Our arguments are similar in spirit to those in (Carvajal (2018)), but applied to our setting and do not impose quasi-linearity of strategic agents. If there are no real effects, then the consumption allocations of the fringe, c_{f1} and $c_{2f}(z)$, and its state prices, $\Lambda_f(z)$, must be the same in both economies.

Notice that we can stack the first-order conditions for strategic agent i with asset span $I_{|\mathcal{Z}|}$ from equation (39) as:

$$\vec{\Lambda}_i = \vec{\Lambda}_f + U\vec{a}_i, \tag{36}$$

where $\vec{\Lambda}_i$ are the stacked state prices of agent i, $\vec{a}_i$ is the vector of her asset positions, and we have substituted for Arrow-Debreu prices $\vec{q}$ with $\vec{\Lambda}_f$.

Let $\vec{a_{i,x}}$ be the vector of asset positions of agent i when she instead trades with the asset span X. Imposing invariance of the consumption allocations of strategic agent i requires that:

$$\vec{a}_i = X'\vec{a_{i,x}}. \tag{37}$$

Substituting with equation (37), we can manipulate equation (36) to arrive at:

$$X\vec{\Lambda}_i = X\vec{\Lambda}_f + XUX'\vec{a_{i,x}} = X\vec{\Lambda}_f + \Gamma\vec{a_{i,x}}, \tag{38}$$

where we have also substituted with equation (32). This is the identical stacked first-order conditions if strategic agent instead traded asset span X.

Consequently, if the competitive fringe's consumption allocations are unchanged between asset spans, then so are the optimal portfolios of each strategic agent. If all strategic agents have the same asset demands, then their aggregate demand for asset exposures in each state z are the same. By market clearing, then, the state-specific asset exposures of the competitive fringe are the same in both asset spans, and consequently so are their consumption allocations, confirming our conjecture.

What remains to show is that the budget sets of strategic agents are unchanged across asset spans. This, however, is trivial because no arbitrage makes invariant the cost of state-specific asset exposures. Consequently, financing the same portfolio of state-specific asset exposures costs the same with asset span $I_{|\mathcal{Z}|}$ as with asset span X.

As such, real allocations in our complete markets economy are invariant to the span of assets that can be traded. As such, studying Arrow-Debreu security markets is without loss of generality. It is straightforward to extend our analysis to allow for a $I_{|\mathcal{Z}|} + n \times I_{|\mathcal{Z}|}$ matrix X of rank $|\mathcal{Z}|$ with n redundant assets.

A.3 Proof of Proposition 3

Step 1: The Problem of Strategic Agents:

We first consider the optimization problem of strategic agent j of type i, (2). In what follows, we attach the Lagrange multiplier φ_i to the budget constraint. The FONCs for $c_{i,j,1}$ and $\{a_{i,j}(z)\}_{z\in\mathcal{Z}}$ are then given by:

$$\begin{aligned} c_{1,j,i} &: u'\left(c_{1,j,i}\right) - \varphi_{j,i} \leq 0 \ \left(= \ if \ c_{1,j,i} > 0\right), \\ a_{j,i}(z) &: -\pi(z)\, u_2^{i\prime}\left(c_{2,j,i}(z)\right) + \varphi_{j,i}\left(\tilde{Q}_{j,i}(\mathbf{A},z) + \frac{\partial \tilde{Q}_{j,i}(\mathbf{A},z)}{\partial a_{j,i}(z)} a_{i,j}(z)\right) = 0. \end{aligned}$$

The above represents the FONCs for agent $i's$ problem. Because $u(\cdot)$ satisfies the Inada condition, $c_{1,j,i} > 0$ and the first FOC binds with equality.

Now that we have derived the FONCs for agent i's optimal asset demands, we can impose the consistency required of a Cournot-Walras equilibrium with the competitive fringe. Because strategic agent i has rational expectations, her perceived price impact must coincide with her actual price impact from (5) in Proposition (1). Consequently, these FONCs reduce to:

$$a_{j,i}(z) : \Lambda_{j,i}(z) = q(z) + \frac{\mu}{m_f} q'(z) a_{j,i}(z)\, \forall\, z \in \mathcal{Z}. \tag{39}$$

We next establish that the correspondence for admissible controls from the constraint set of strategic agent j of type i is compact-valued. This must be true because all strategic agents will have nonnegative consumption at both dates, $c_{1,j',i'}, c_{2,j',i'}(z)\, \forall i', j', z$. Because endowments are bounded, all strategic agents will have a maximum amount of each security they will sell. Similarly, because the fringe has nonnegative consumption at date 2, it is similarly limited in its asset sales. Since every asset is in finite supply, all agent consumptions are bounded at both dates and $\mathbf{A}$ is bounded element-by-element.

Consequently, we can bound all controls of strategic agent j, i's problem, $\{c_{1,j,i}, \{a_{j,i}(z)\}_{z\in\mathcal{Z}}\}$, in a closed and bounded set. By the Heine-Borel Theorem, this set is compact.

We now recall from Proposition (1) that the pricing functional $Q_{j,i}(\mathbf{A}, z)$ is continuously differentiable in $\mathbf{A}$ because it is the marginal utility of the competitive fringe in state z, $\pi(z)\, u_f'\left(c_{f2}(z)\right)$. Since the state prices of the strategic agents and the price impact functional are continuous because all utility functions are $\mathcal{C}^2$, strategic agent j, i's choice correspondence set is also continuous in the optimization problem's primitives (i.e., income processes and initial endowments). As such, the choice correspondence of strategic agent j, i's problem is continuous and compact-valued.

It then follows because the objective function of strategic agent j, i is continuous (in fact, differentiable), and the choice correspondence is continuous and compact-valued, that by Berges' Theory of the Maximum a solution to the decision problem of strategic agent j, i exists. As the choice of j, i was arbitrary, this holds for all agents j of type i and all types $i \in \{1, ..., N\}$.

Step 2: Existence:

As a result of Berge's Theory of the Maximum, the optimal policies of each strategic agent are upper-hemicontinuous correspondences. We can then construct a mapping from a conjectured set of initial consumption and asset decisions for all strategic agents to an optimal set of initial consumption and asset decisions using the market-clearing conditions (1) and the optimal policy correspondences as an equilibrium correspondence whose image is a compact space. Since the budget constraints of strategic agents are not necessarily convex because of market power, we allow for randomization of consumption bundles to ensure that the compact space is also convex. We can then apply Kakutani's Fixed Point Theorem to conclude that an equilibrium exists.

A.4 Proof of Proposition 4

Step 1: Approximating Agent i's Trading Portfolio:

Suppose all strategic agents have size μ and we increase the size of agent n to $\mu_n = \mu + \Delta\mu$ close to μ. Then we can rewrite the FOC for the optimal position in the Arrow-Debreu security in state z from Proposition 3 as:

$$\mathbb{E}\left[\frac{u'\left(\hat{c}_n\left(z'\right)\right)}{u'\left(\hat{c}_{n,1}\right)}\delta\left(z\right)\right] - q\left(z\right) - \frac{\mu_n}{m_f}q'\left(z\right)\hat{a}_n\left(z\right) = 0.$$

Let $g_n\left(z\right) = \frac{c_{2,n}(z)}{c_{1,n}}$ be the consumption growth of agent n in the equilibrium where all strategic agents have size μ. We can take a first-order approximation around this equilibrium to find that:

$$\mathbb{E}\left[u'\left(g_n\left(z'\right)\right)\delta\left(z\right)\right]-q\left(z\right)+\mathbb{E}\left[g_n\left(z'\right)u''\left(g_n\left(z'\right)\right)\left(\frac{\Delta c_n\left(z'\right)}{c_n\left(z\right)}-\frac{\Delta c_{n,1}}{c_{n,1}}\right)\delta\left(z\right)\right]-\frac{\mu_n}{m_f}q'\left(z\right)\hat{a}_n\left(z\right)\approx 0.$$

where $c_{n,1}$ and $c_n\left(z\right)$ are consumption at dates 1 and 2 in the original equilibrium. Substituting for the consumption growth around the original equilibrium:

$$\begin{aligned} 0 &\approx \mathbb{E}\left[g_n\left(z'\right)u''\left(g_n\left(z'\right)\right)\left(\left(\hat{a}_n\left(z\right)-a_n\left(z\right)\right)\frac{\delta\left(z\right)}{c_n\left(z'\right)}-\sum_{\tilde{z}=1}^{Z}\frac{q\left(\tilde{z}\right)+\frac{\mu}{m_f}q'\left(\tilde{z}\right)}{c_{n,1}}\Delta a_n\left(\tilde{z}\right)\right)\delta\left(z\right)\right] \\ &\quad +\mathbb{E}\left[u'\left(g_n\left(z'\right)\right)\delta\left(z\right)\right]-q\left(z\right)-\frac{\mu_n}{m_f}q'\left(z\right)\hat{a}_n\left(z\right), \end{aligned}$$

where $\Delta a_n\left(z'\right)=\hat{a}_n\left(z'\right)-a_n\left(z'\right)$, which reduces since $\delta\left(z\right)$ is the indicator for state z to:

$$\begin{aligned} 0 &\approx \mathbb{E}\left[u'\left(g_n\left(z'\right)\right)\frac{\delta\left(z\right)}{q\left(z\right)}\right]-1-\frac{\mu_n}{m_f}\frac{q'\left(z\right)}{q\left(z\right)}\hat{a}_n\left(z\right) \\ &\quad +\mathbb{E}\left[g_n\left(z'\right)u''\left(g_n\left(z'\right)\right)\frac{1/c_n\left(z'\right)-\sum_{\tilde{z}=1}^{Z}\frac{q(\tilde{z})+\frac{\mu}{m_f}q'(\tilde{z})}{c_{n,1}}\frac{\Delta a_n(\tilde{z})}{\Delta a_n(z)}}{q\left(z\right)}\delta\left(z\right)\right]\left(\hat{a}_n\left(z\right)-a_n\left(z\right)\right). \end{aligned}$$

Define $\gamma\left(x\right)=-xu''\left(x\right)/u'\left(x\right)$ to be the agent's coefficient of absolute risk aversion. Then, the above reduces to:

$$\hat{a}_n\left(z\right)\approx a_n\left(z\right)+\frac{\mathbb{E}\left[u'\left(g_n\left(z'\right)\right)\frac{\delta(z)}{q(z)}\right]-1-\frac{\mu_n}{m_f}\frac{q'(z)}{q(z)}\hat{a}_n\left(z\right)}{\mathbb{E}\left[\gamma\left(g_n\left(z'\right)\right)u'\left(g_n\left(z'\right)\right)\frac{1/c_n(z')-\sum_{\tilde{z}=1}^{Z}\frac{q(\tilde{z})+\frac{\mu}{m_f}q'(\tilde{z})}{c_{n,1}}\frac{\Delta a_n(\tilde{z})}{\Delta a_n(z)}}{q(z)}\delta\left(z\right)\right]}.$$

Finally, substituting the first term of the numerator with the FOC from Proposition 3:

$$\hat{a}_n\left(z\right)\approx a_n\left(z\right)+\frac{\frac{q'(z)}{q(z)}\left(-\frac{\mu}{m_f}\left(\hat{a}_n\left(z\right)-a_n\left(z\right)\right)+\left(\frac{\mu}{m_f}-\frac{\mu_n}{m_f}\right)\hat{a}_n\left(z\right)\right)}{\mathbb{E}\left[\gamma\left(g_n\left(z'\right)\right)u'\left(g_n\left(z'\right)\right)\frac{1/c_n(z')-\sum_{\tilde{z}=1}^{Z}\frac{q(\tilde{z})+\frac{\mu}{m_f}q'(\tilde{z})}{c_{n,1}}\frac{\Delta a_n(\tilde{z})}{\Delta a_n(z)}}{q(z)}\delta\left(z\right)\right]},$$

which we can rewrite as:

$$\hat{a}_n(z) \approx a_n(z) - \frac{\frac{\Delta\mu}{m_f}\frac{q'(z)}{q(z)}\hat{a}_n(z)}{\frac{\mu}{m_f}\frac{q'(z)}{q(z)} + \mathbb{E}\left[\gamma\left(g_n(z')\right)u'\left(g_n(z')\right)\frac{1/c_n(z') - \sum_{\tilde{z}=1}^{Z}\frac{q(\tilde{z})+\frac{\mu}{m_f}q'(\tilde{z})}{c_{n,1}}\frac{\Delta a_n(\tilde{z})}{\Delta a_n(z)}}{q(z)}\delta(z)\right]}.$$

Step 1: Return on Agent i's Wealth Portfolio:

We define on return on the wealth portfolio

$$R_i^W = \frac{\sum_{z\in Z}\pi(z)\,c_{2,i}(z)}{w_i + \sum_{z\in Z}q(z)\,y_i(z) - c_{1,i}} = \frac{\sum_{z\in Z}\pi(z)\,c_{2,i}(z)}{\sum_{z\in Z}q(z)\,c_{2,i}(z)}, \tag{40}$$

where we substitute for w_i with strategic agent i's budget constraint.

We begin by rewriting the first-order condition for agent i's optimal asset demand in state z

$$\Lambda_i(z) = \pi(z)\,u'\left(\frac{c_{2,i}(z)}{c_{1,i}}\right) = q(z)\left(1 + \frac{q'(z)}{q(z)}a_i(z)\right). \tag{41}$$

Define $\alpha_s(z) = \left(1 + \frac{q'(z)}{q(z)}a_i(z)\right)^{-1} \geq 0$, it then follows

$$c_{2,i}(z) = u'^{-1}\left(\frac{q(z)}{\pi(z)\,\alpha_s(z)}\right)c_{1,i} \tag{42}$$

In addition, define $v_s(z) = \frac{q(z)}{\pi(z)}u'^{-1}\left(\frac{q(z)}{\pi(z)\alpha_s(z)}\right)$. We can then write the return on strategic agent i's wealth portfolio

$$R_i^W = \frac{E\left[\frac{\pi(z)}{q(z)}v_s(z)\right]}{E\left[v_s(z)\right]} = E\left[\frac{\pi(z)}{q(z)}\right] + Cov\left(\frac{\pi(z)}{q(z)}, \frac{v_s(z)}{E\left[v_s(z)\right]}\right). \tag{43}$$

A.5 Proof of Corollary 1

Step 1: Comparative static for $Var\left[c_n(z)\right]$:

Suppose all strategic agents have size μ and we increase the size of agent n to $\mu_n = \mu + \Delta\mu$ close to μ. It is immediate that:

$$Var\left[\hat{c}_n(z)\right] = Var\left[c_n(z)\right] + Var\left[\Delta\hat{a}_n(z)\right] + 2Cov\left[c_n(z), \Delta\hat{a}_n(z)\right].$$

With incremental market power (i.e., $\mu \to \mu + \Delta\mu$), strategic agent n shades down its purchases ($\Delta\hat{a}_n(z) < 0$) to lower prices in states where its consumption $c_n(z)$ is low (i.e., agent n is a buyer because its consumption in that state is low). Similarly, it reduces its sales ($\Delta\hat{a}_n(z) > 0$) to raise prices for states in which its consumption is high. As a result, $Cov\left[c_n(z), \Delta\hat{a}_n(z)\right] \geq 0$ because the agent is raising consumption in states in which it is already high and lowering it in states in which it is already low.

As a result:

$$Var\left[\hat{c}_n(z)\right] \geq Var\left[c_n(z)\right] + Var\left[\Delta\hat{a}_n(z)\right] > Var\left[c_n(z)\right],$$

Step 2: Comparative static for Π_n:

Notice next that the return on agent n's tradable wealth portfolio from substituting with the FOC for agent n's optimal holdings:

$$\Pi_n^{\hat{a}} = \mathbb{E}\left[\hat{\Lambda}_n(z)\,\hat{a}_n(z)\right] - \sum_{z=1}^{Z} q(z)\,\hat{a}_n(z) = \frac{\mu_n}{m_f}\sum_{z=1}^{Z} q'(z)\,\hat{a}_n^2(z).$$

The return on the tradeable wealth portfolio is the total rents that agent n extracts from financial markets. Two forces drive the comparative static, $\frac{d\Pi_n^{\hat{a}}}{d\mu_n}$: 1) $\sum_{z=1}^{Z} \frac{d}{d\mu_n}\left(\frac{\mu_n}{m_f} q'(z)\right)\hat{a}_n^2(z) > 0$, as an increase in market power increases price impact and therefore rent extraction from market illiquidity; and 2) $\sum_{z=1}^{Z} \frac{\mu_n}{m_f} q'(z)\frac{d\hat{a}_n^2(z)}{d\mu_n} < 0$ because trading positions become smaller as the agent exerts market power.

When $\mu_n = 0$, the agent behaves competitively and $\Pi_n^{\hat{a}} = 0$. Locally around $\mu_n = 0$, the first force dominates as an infinitesimal amount of market power raises profits. As a thought experiment, at the other extreme μ_n arbitrarily large (i.e., $\mu_n \to \infty$), profits are also zero because the trading needs of the agent are so large that it is forced into autarky. Locally around $\mu_n = \infty$, the second force dominates and the burden of size dominates as the large agent is forced into autarky because it moves prices too much to trade even small quantities. As such, $\frac{d\Pi_n^{\hat{a}}}{d\mu_n} > 0$ around $\mu_n = 0$ and $\frac{d\Pi_n^{\hat{a}}}{d\mu_n} < 0$ near $\mu_n = \infty$, and trading rents are strictly positive, or $\Pi_n^{\hat{a}} > 0$, on the interior for μ_n.

As these are the only two trade-offs for the agent, and μ_n in actually bounded between 0 and 1, it follows that trading rents are either increasing or hump-shaped in μ_n.

A.6 Proof of Proposition 5

Step 1: Type-Symmetric Case:

Summing over condition (7) and imposing market-clearing in the Strategic Equilibrium (in which $m_f \approx 0$) yields:

$$q(z) = E^*[\Lambda_i(z)]. \tag{44}$$

In the competitive equilibrium, in contrast, $q^{CE}(z) = E^*\left[\Lambda_i(z)\right] = \Lambda^{CE}(z)$.

The following Lemma characterizes state prices in the competitive equilibrium when all agents have the same initial wealth w.

Lemma 1: State prices in the competitive equilibrium, $\Lambda^{CE}(z)$ satisfy:

$$\Lambda^{CE}(z) = \pi(z)\, u'\left(\frac{Y(z) + A^{CE}(z)}{\sum_{i=1}^{N} w_i + m_f\left(w_f - c_{f1}^{CE}\right)}\right). \tag{45}$$

In the Strategic Equilibrium in which all strategic agents are type-symmetric and $m_f \approx 0$, and consequently $A^{CE}(z) \approx 0$, (45) from Lemma 1 reduces to:

$$\Lambda^{CE}(z) = \pi(z)\, u'\left(\frac{\frac{1}{N}Y(z)}{w}\right).$$

In the special case in which all agents are type-symmetric, then $\sum_{z\in\mathcal{Z}} q(z)\, a_i(z) = 0$ and $c_{1,i} = w$ for all i. We can then apply Jensen's Inequality to (44) and invoke Lemma 1 to conclude that:

$$E^*[\Lambda_i(z)] \geq u'\left(\frac{\frac{1}{N}\sum_{n=1}^{N} c_{2,i}(z)}{w}\right) = u'\left(\frac{\frac{1}{N}Y(z)}{w}\right) = \Lambda^{CE}(z),$$

by market-clearing (1). This holds for all $\mu > 0$.

Since all $q(z)$ are higher with market power, it follows r_f (the inverse of the sum of state prices) is also lower.

Step 1: Asymmetric Wealth Case:

We start from the type-symmetric case in which all strategic agents have the same initial wealth w. In this case, from Proposition (5), asset prices are inflated state-by-state.

We first establish that, *conditional* on the asset price $q(z)$ and an agent's effective income $\tilde{y}_i(z) = y_i(z)/w_i$ (i.e., holding the effect of wealth on the normalized income process constant), her asset demands are linear in her wealth (i.e., $a_i(z) = \hat{a}_i(z)$).[20]

Step a: Conditional homogeneity of optimal policies in wealth:

Suppose that the optimal policies of a strategic agent of type i are linear in wealth. We then rewrite the FONCs (7) for the strategic agent of type i, given the homotheticity of strategic agent preferences as:

$$\hat{a}_i(z) \quad : \quad \pi(z)\frac{u'(\hat{c}_{2,i}(z))}{u'(\hat{c}_{1,i})} - q(z) - \mu\hat{q}'(z)\hat{a}_i(z) = 0, \tag{46}$$

where we recognize that $\hat{q}'(z) = \frac{1}{w_i}q'(z)$, where $\hat{q}'(z) = \frac{\partial\tilde{Q}_i(\mathbf{A},z)}{\partial\hat{a}_i(z)}$. It then follows that, conditional on prices $q(z)$ and $\tilde{y}_i(z)$, the optimal policies of the strategic agent of type i are indeed homogeneous of degree 1 in w_i.

Step b: A perturbation in a strategic agent's wealth:

Now suppose agents of type i have total initial wealth $w' > w$ compared to other agents. There are two relevant forces based on equation (8).

The first force is that an increase in wealth reduces her effective income to $y_i(z)/w'_i$. This raises her state price because she wants to consume more at date 2 because of her higher wealth but is limited to the same income that she has to trade. As a result of the increase in state price, she sells less and buys more, potentially becoming buyers of all securities for large enough w'.

The second force is an indirect effect also explored in Neuhann and Sockin (2021). An increase in her initial wealth raises how much the same normalized asset position, $\hat{a}_i(z)$, moves the asset price $q(z)$. As such, it raises prices when she is a buyer and lowers them when she is a seller for the same $\hat{a}_i(z)$.

[20] If this is the case, then so are her consumption processes by definition (i.e., $c_{1,i} = \hat{c}_{1,i}w_i$ and $c_{2,j,i}(z) = \hat{c}_{2,j,i}(z)\,w_i$).

It is unambiguous that both forces reduce the upward pressure on prices from market power from the symmetric case in markets in which type i is a seller. This is because she extracts less rents from each unit of asset traded and is forced to trade more. In markets in which she is a buyer, she extracts more rent per unit of asset traded but is also forced to trade more.

On net, because she overall buys more and not less when trading needs go up, increasing her distortion from market power that lowers asset prices.

A.7 Proof of Proposition 6:

Step 1: Approximating Expected Returns:

Define the expected return $\mathbb{E}[r(z)] = \frac{\pi(z)}{q(z)}$. Notice in the Strategic Equilibrium that:

$$q(z) = E[E^*[\Lambda_i(z)]\delta(z)] = \frac{\pi(z)}{r_f} + Cov(E^*[\Lambda_i(z)], \delta(z)),$$

where $\delta(z)$ is the indicator that state z realizes. Standard manipulation establishes that the expected excess return, $\mathbb{E}[r(z) - r_f]$, satisfies:

$$\mathbb{E}[r(z) - r_f] = -Cov\left(\frac{E^*[\Lambda_i(z)]}{E[E^*[\Lambda_i(z)]]}, \delta(z)\right)$$

We now consider a small perturbation in market concentration around the competitive equilibrium. Notice the gross return on the Arrow security referencing state z, $\mathbb{E}[r(z)] = \frac{\pi(z)}{q(z)}$, to first-order because $\Delta q(z) > 0$ from Proposition 5:

$$\mathbb{E}[r(z)] - \mathbb{E}\left[r^{CE}(z)\right] \approx -\frac{\pi(z)}{q^{CE}(z)} \frac{\Delta q(z)}{q^{CE}(z)} < 0.$$

In addition, because all state prices rise:

$$r_f - r_f^{CE} \approx -\left(r_f^{CE}\right)^2 \sum_{z' \in Z} \Delta q(z') < 0,$$

It then follows that:

$$\mathbb{E}[r(z) - r_f] - \mathbb{E}\left[r^{CE}(z) - r_f^{CE}\right] \approx \left(r_f^{CE}\right)^2 \sum_{z' \in Z} \Delta q(z') - \frac{\pi(z)}{q^{CE}(z)} \frac{\Delta q(z)}{q^{CE}(z)}. \quad (47)$$

Step 2: Risk Premia in the Type-Symmetric Case:

In a type-symmetric equilibrium, we recognize:

$$\begin{aligned} q(z) &= \mathbb{E}^*\left[\Lambda_i(z)\right] = \pi(z)\,\mathbb{E}^*\left[u'\left(\frac{c_{2,i}(z)}{w}\right)\right], \\ q^{CE}(z) &= \Lambda^*(z) = \pi(z)\,u'\left(\frac{Y(z)}{Nw}\right), \end{aligned}$$

because all agents have common beliefs and homothetic preferences. Furthermore, under a second-order approximation:

$$\begin{aligned} \Delta q(z) &= \pi(z)\,u''\left(\frac{Y(z)}{Nw}\right)\mathbb{E}^*\left[\frac{\Delta c_{2,i}(z)}{w}\right] + \pi(z)\,u'''\left(\frac{Y(z)}{Nw}\right)\mathbb{E}^*\left[\frac{\left(\Delta c_{2,i}(z)\right)^2}{w^2}\right] \\ &= \pi(z)\,u'''\left(\frac{Y(z)}{Nw}\right)\mathbb{E}^*\left[\frac{\left(\Delta c_{2,i}(z)\right)^2}{w^2}\right], \end{aligned}$$

because in a type-symmetric equilibrium all strategic agents consume w, regardless of μ, and by market-clearing in consumption markets (57):

$$\mathbb{E}^*\left[\frac{\Delta c_{2,i}(z)}{w}\right] = 0.$$

Define $P(z) = -\frac{Y(z)}{Nw}u'''\left(\frac{Y(z)}{Nw}\right)/u''\left(\frac{Y(z)}{Nw}\right)$ to be the coefficient of relative prudence and $\gamma(z) = -\frac{Y(z)}{Nw}u''\left(\frac{Y(z)}{Nw}\right)/u'\left(\frac{Y(z)}{Nw}\right)$ to be the coefficient of relative risk aversion. It then follows from (47) that

$$\begin{aligned} \mathbb{E}\left[r(z) - r_f\right] \approx\ & \mathbb{E}\left[r^{CE}(z) - r_f^{CE}\right] - \left(r_f^{CE}\right)^2 \sum_{z' \in Z} q^{CE}(z')\,\gamma(z')\,P(z')\,\mathbb{E}^*\left[\left(\frac{\Delta c_{2,i}(z')}{Y(z)/N}\right)^2\right] \\ & + \frac{\gamma(z)\,P(z)}{q^{CE}(z)}\mathbb{E}^*\left[\left(\frac{\Delta c_{2,i}(z)}{Y(z)/N}\right)^2\right]. \end{aligned} \tag{48}$$

Since strategic agent utility is concave and marginal utility is convex ($u'''(\cdot) > 0$), $P(z) > 0$, and therefore the second term on the right-hand side of (48) is negative (i.e., $\Delta q(z) > 0$ for every state z). The third term on the right-hand side is the state-specific fall in expected returns because market power inflates each state price.

In a type-symmetric setting, market concentration raises all asset prices and lowers the risk-free rate from Proposition 5. The first effect lowers expected excess returns state-by-state while the second raises expected excess returns for all states.

Notice that the last term on the right-hand side of (48) can be expressed as:

$$\frac{\gamma(z)\,P(z)}{q^{CE}(z)}\mathbb{E}^{*}\left[\left(\frac{\Delta c_{2,i}(z)}{Y(z)/N}\right)^{2}\right]=\frac{\left(\frac{Y(z)}{Nw}\right)^{2}u'''\left(\frac{Y(z)}{Nw}\right)}{\pi(z)\,u'\left(\frac{Y(z)}{Nw}\right)}\mathbb{E}^{*}\left[\left(\frac{\Delta c_{2,i}(z)}{Y(z)/N}\right)^{2}\right].$$

Suppose $x^2u'''(x)/u'(x)$ is increasing in x, which is satisfied, for instance, with CRRA preferences. Notice that the ratio $\mathbb{E}^{*}\left[\left(\frac{\Delta c_{2,i}(z)}{Y(z)/N}\right)^{2}\right]$ mutes differences in aggregate endowment growth across states, although it does not mute how aggregate growth interacts with the dispersion in endowments.

It follows that if there is a sufficiently large difference in aggregate endowments at date 2, $Y(z)$, in the high versus low aggregate endowment states, then the $x^2u'''(x)/u'(x)$ force dominates that of differences in $\mathbb{E}^{*}\left[\left(\frac{\Delta c_{2,i}(z)}{Y(z)/N}\right)^{2}\right]$ across states. In this case, expected excess returns increase more for high than low aggregate endowment growth $(Y(z)/Nw)$ states, and there is then risk compression in state prices.

A.8 Proof of Proposition 7:

Step 1: Primal to Dynamic Problem:

We write the primal problem (23) as the dynamic programming problem:

$$V_{t,j,i}(w_{t,j,i})=\sup_{c_{t,j,i},a_{t,j,i}(z_{t+1})}u(c_{t,j,i})+\beta\sum_{z_{t+1}\in\mathcal{Z}}\pi(z_{t+1})\,V_{t+1,j,i}(w_{t+1,j,i}(z_{t+1})),\tag{49}$$

with associated transversality condition:

$$\lim_{T\to\infty}\sum_{z_T\in\mathcal{Z}}\pi(z_T)\,V_{T,j,i}(w_{T,j,i}(z_T))=0.$$

This transversality condition is satisfied because aggregate consumption, $\sum_{j,i}c_{t,j,i}$, is bounded by the aggregate incomes of all strategic agents and the fringe, $Y(z)+m_f y_f(z)$, which is also bounded.

Standard arguments then establish that a solution to the dynamic problem is also a solution to the primal problem. For instance, iterating forward and imposing transversality, we find that:

$$V_{0,j,i} = \sup_{c_{j,i},a_{j,i}} \sum_{t=0}^{\infty} \sum_{z_t \in \mathcal{Z}} \beta^t \pi(z_t) u(c_{t,j,i}(z)).$$

If a solution to the primal problem exists, then a solution to the dynamic problem also exists.

Notice that because Arrow securities reference the state only one period ahead, there is no scope for strategic agents to violate the expectations of other agents ex post, which would give rise to dynamic inconsistency. Since strategic agents lack commitment and we focus on Markov Perfect Equilibrium, we do not need to keep track of promise-keeping auxiliary state variables for strategic agents' policies to be time-consistent.

Step 2: Optimal Policies:

Let $\lambda_{t,j,i}$ be the Lagrange multiplier on the budget constraint from (23). Assuming the value function $V_{t,j,i}(w_{t,j,i})$ is $\mathcal{C}^1$, the first-order condition for the optimal consumption and portfolio choices are:

$$c_{t,j,i} : u'(c_{t,j,i}) \leq \lambda_{t,j,i}, \ (= if c_{t,j,i} > 0), \tag{50}$$

$$a_{t,j,i}(z_{t+1}) : \beta\pi(z_{t+1}) V'_{t+1,j,i}(w_{t+1,j,i}(z_{t+1})) = \lambda_{t,j,i}\left(q_t(z_t) + \frac{\mu}{m_f} q'_t(z_{t+1}) a_{t,j,i}(z_{t+1})\right). \tag{51}$$

In addition, the Envelope Condition further imposes:

$$V'_{t,j,i}(w_{t,j,i}) = \lambda_{t,j,i}. \tag{52}$$

Since $u(\cdot)$ satisfies the Inada condition, (51) holds with equality. Substituting (51) and the (52) into (51), the optimal portfolio condition for security z_{t+1} can be written as:

$$\Lambda_{t,j,i}(z_{t+1}) = q_t(z_{t+1}) + \frac{\mu}{m_f} q'_t(z_{t+1}) a_{t,j,i}(z_{t+1}), \tag{53}$$

where $\Lambda_{t,j,i}(z_{t+1})$ is the state price of agent j of type i at date t for state z_{t+1} given by (24).

Step 3: Existence:

We establish existence in two steps. In the first step, fix some arbitrary final date T. The total value of all endowments across all dates is then bounded. By feasibility, the consumption (which must be nonnegative) and asset positions of all agents must also be bounded (and consequently lie in a compact set). Allowing for randomization, this strategy space for all agents can be made convex.

It then follows because the objective function of strategic agent j, i in the primal problem (23) is continuous (in fact, differentiable), and the choice correspondence is again continuous and compact-valued (maps to a compact set), that by Berges' Theory of the Maximum a solution to the decision problem of strategic agent j, i exists. As the choice of j, i was arbitrary, this holds for all agents j of type i and all types $i \in \{1, ..., N\}$. The optimal policies of strategic agents are also upper hemicontinuous.

We can then apply Kakutani's Fixed Point Theorem to the market clearing conditions for asset positions to conclude an equilibrium exists for finite T.

In the second step, we take the limit as $T \to \infty$. Since agent consumption and asset positions continue to remain bounded at each date, and strategic agent transversality conditions are satisfied, we can pass through the limit to conclude an equilibrium exists in the infinite horizon economy.

A.9 Proof of Corollary 2:

We can aggregate (25) across strategic agents and impose the strategic equilibrium to arrive at:

$$q_t\left(z_{t+1}\right) = \mathbb{E}^*\left[\Lambda_{t,j,i}\left(z_{t+1}\right)\right].$$

Given the definition of state prices from (24), we can apply similar arguments in the type-symmetric case to establish that

$$q_t\left(z_{t+1}\right) \geq q_t^{CE}\left(z_{t+1}\right). \tag{54}$$

Consequently, Arrow prices are higher state-by-state at date t. Since the risk-free rate is the inverse the sum of Arrow prices, the risk-free rate is depressed.

A.10 Proof of Proposition 8:

Consider the limit $\mu \to 0$ in which all large agents behave competitively. Since markets are competitive, all agents align their state prices in equilibrium state-by-state. This can only happen if they each consumed a fixed fraction of the aggregate endowment, i.e., perfect risk sharing, with this fraction increasing in an agent's wealth.

If agents' consumption shares are fixed, then so are the ratios of their wealth. This is because each agent's wealth is equal to the present discounted value of her future consumption stream. Since all agents have the same state prices state-by-state and consumption is a constant fraction of the aggregate endowment, their wealths are their consumption shares multiplied by the present value of the aggregate endowment.

A.11 Proof of Lemma 1:

In this lemma, we characterize the competitive equilibrium without market power. The standard first-order conditions for optimal consumption and asset holdings align state prices for all agents state-by-state:

$$q(z) = \frac{\pi(z) u'(c_{2,i}(z))}{u'(c_{1,i})} = \pi(z) u'_f(c_f(z)) = \Lambda^{CE}(z), \tag{55}$$

which implies for the N types of agents with homothetic preferences:

$$\frac{c_{2,i}(z)}{c_{1,i}} = \frac{c_{2,j}(z)}{c_{1,j}} = \eta(z), \tag{56}$$

and for the competitive fringe:

$$c_f(z) = \eta_f(z) = u_f^{-1}(u'(\eta(z))).$$

Notice that equation (56) implies:

$$\frac{\sum_{i=1}^{N} c_{2,i}(z)}{\sum_{i=1}^{N} c_{1,i}} = \eta(z). \tag{57}$$

Substituting the market-clearing conditions at both dates into (57), and equating $\eta(z)$ with consumption growth in equation (56) and state prices in equation (55), we arrive at:

$$\Lambda^{CE}(z) = \pi(z)\, u'\left(\frac{Y(z) + A^{CE}(z)}{\sum_{i=1}^{N} w_i + m_f\left(w_f - c_{f1}^{CE}\right)}\right).$$

B Comparing Cournot-Walras and Equilibrium-in-Demand-Schedules

Our model of strategic trading in financial markets uses Cournot-Walras equilibrium as our equilibrium concept. This equilibrium concept differs from a long tradition following Kyle (1989), which focuses on Equilibrium-in-Demand-Schedules (also known as double auctions). Although both equilibrium concepts allow strategic traders to submit price-contingent demand schedules taking into account their impact on equilibrium prices, they have subtle differences that render each particularly suitable for some applications but not for others.

An important observation, however, is the basic forces governing how a strategic agent distorts her portfolio are independent of the equilibrium concept in that, conditional on price impact, her partial equilibrium asset demands are the same. What differs is how this price impact function is determined, which in equilibrium leads to nuanced strategic interactions among the strategic agents. Neuhann and Sockin (2021) formalizes this comparison in the special case in which strategic traders have Constant Absolute Risk Aversion preferences and uncertainty is normally distributed (i.e., the CARA-Normal setting), which is the canonical setting for the Equilibrium-in-Demand-Schedules concept. Our contribution in this appendix is to extend this comparison to a more general complete markets setting, and characterize an Equilibrium-in-Demand-Schedules version of our model.

B.1 Liquidity Traders instead of a Competitive Fringe

As in our static model, suppose there are again two dates 1 and 2 and $z \in \mathcal{Z}$ finite potential states of the world. There N types of strategic agents, each consisting of $1/\mu$ agents of size μ, who choose their asset positions to maximize decision problem 2. However, instead of a competitive fringe there are liquidity traders who take a random position ξ_z

in asset z at date 1. Market clearing for each asset now requires

$$\sum_{i=1}^{N} a_i(z) + \xi(z) = 0, \; \forall\, z \in \mathcal{Z}. \tag{58}$$

An advantage of the Equilibrium-in-Demand Schedules approach is that strategic interaction can be studied in this setting in which only strategic agents make portfolio choices. If we attempted to impose a Cournot-Walras equilibrium, in contrast, we would recover the competitive equilibrium because by taking each other strategic agent's asset demand as given, every strategic agent believes she cannot influence prices.

B.2 Strategic Forces with Equilibrium-in-Demand Schedules

Under the Equilibrium-in-Demand-Schedules equilibrium concept, each strategic agent internalizes she can influence the asset price $Q(\boldsymbol{A}, z)$, where $\boldsymbol{A}$ is the vector of all gents' demands across all assets, by shifting each other strategic agents' demand curves (i.e., she internalizes $\frac{\partial a_j(z)}{\partial a_i(z)} \forall (j, z)$). Assuming a $\mathcal{C}^1$ price function $Q(\boldsymbol{A}, z)$, the first-order necessary condition for a strategic agent of type i's demand for asset z is the analogue of equation 7 from Proposition 3

$$\Lambda_i(z) = Q(\boldsymbol{A}, z) + \mu \frac{\partial Q(\boldsymbol{A}, z)}{\partial a_i(z)} a_i(z), \tag{59}$$

where $\Lambda_i(z)$ is the state price of strategic agent i given in equation 6.

We first analyze the partial equilibrium behavior of a strategic agent of type i, taking as given the equilibrium pricing function $Q(\boldsymbol{A}, z)$ (and consequently the agent's price impact). Similar to the decomposition in equation 8, we can rewrite the asset demand of a strategic agent of type i as $a_i(z) = \hat{a}_i(z)\, w_i$ and the first-order condition 59 as

$$\pi(z)\, u'\left(\frac{y_i(z)/w_i + \hat{a}_i(z)}{1 - \sum_{z' \in \mathcal{Z}} Q(\hat{\boldsymbol{A}}, z')\, \hat{a}_i(z')}\right) = Q(\boldsymbol{A}, z) + \mu \frac{\partial Q(\hat{\boldsymbol{A}}, z)}{\partial \hat{a}_i(z)} \hat{a}_i(z), \tag{60}$$

where by the chain rule $\frac{\partial Q(\hat{\boldsymbol{A}}, z)}{\partial a_i(z)} = \frac{1}{w_i} \frac{\partial Q(\hat{\boldsymbol{A}}, z)}{\partial \hat{a}_i(z)}$ and $\hat{\boldsymbol{A}}$ is now the vector of wealth-normalized asset demands.

It is immediate a strategic agent of type i's asset demand responds to changes in initial wealth w_i, normalized endowment $y_i(z)/w_i$, and prices $Q(\boldsymbol{A}, z)$ as in the Cournot-Walras equilibrium characterized in Section 3.1. Consequently, our analysis for a Cournot-

Walras equilibrium also characterizes wealth and endowment effects in a complete markets Equilibrium-in-Demand-Schedules setting.

As discussed in the introduction to this appendix, what differs is the general equilibrium forces that act through asset prices and price impact. In Cournot-Walras, we select among all pricing functions $Q(\boldsymbol{A}, z)$ that are consistent with rational expectations the unique price function consistent with the competitive fringe's optimization, $q(z)$. This is because strategic agents place a wedge between their state prices $\Lambda_i(z)$ and Arrow asset prices $Q(\boldsymbol{A}, z)$. The competitive fringe, however, does not. As a result, the Arrow asset price is always equal to the competitive fringe's state price. There is no other choice consistent with a Cournot-Walras equilibrium.

Under the Equilibrium-in-Demand-Schedules concept, in contrast, there is no price-taking agent whose state price must, in equilibrium, equal the Arrow asset price. As a result, there are potentially many ways to specify asset prices that provide the appropriate wedges such that all strategic agents' Euler Equations and market-clearing conditions are satisfied. As such, small changes in strategic agents' wealth or endowment processes can lead to vastly different equilibria when multiple exist, and it is unclear how to select a principal equilibrium.

B.3 Solving for an Equilibrium-in-Demand Schedules

To solve for an Equilibrium-in-Demand-Schedules, we recognize in addition to the $N \times |\mathcal{Z}|$ first-order conditions for strategic agents' demands from equation 59, we have the $\mathcal{Z}$ market-clearing conditions 58. Notice there may be many equilibrium consistent with rational expectations, strategic agents' Euler Equations, and market clearing. To make progress, previous research has focused on settings in which all agents are symmetric to reduce the number of Euler Equations to $|\mathcal{Z}|$ instead of $N \times |\mathcal{Z}|$ or on one asset to reduce the number to $N \times |1|$. We will instead consider this general setting but restrict our attention to equilibria with pricing functions that satisfy *anonymity* in which price impact is the same for all strategic agent types, i.e., $\frac{\partial Q(\hat{\boldsymbol{A}}, z)}{\partial \hat{a}_i(z)} = \frac{\partial Q(\hat{\boldsymbol{A}}, z)}{\partial \hat{a}_j(z)} = Q'(\boldsymbol{A}, z) \, \forall (j, z)$. Such equilibria are not only sensible given the symmetry of strategic agents' demands in the market clearing conditions 58, but also are most comparable to our Cournot-Walras equilibrium in which anonymity is an equilibrium outcome.

Imposing *anonymity* on the equilibrium pricing function, the first-order conditions for strategic agents' demands from equation 59 reduce to

$$\Lambda_i(z) = Q(\boldsymbol{A}, z) + \mu Q'(\boldsymbol{A}, z)\, a_i(z), \tag{61}$$

and we can aggregate equations 61 state-by-state and impose the market clearing conditions 58 to arrive at

$$Q(\boldsymbol{A}, z) = \frac{1}{N}\sum_{i=1}^{N}\Lambda_i(z) + \frac{\mu}{N}Q'(\boldsymbol{A}, z)\,\xi(z). \tag{62}$$

These $(N+1)\times|\mathcal{Z}|$ necessary (although not necessarily sufficient) equations identify the Equilibrium-in-Demand-Schedules.

Case 1: All liquidity trader demands are nonzero ($\xi(z) \neq 0\ \forall\, z$)

In this case, we can substitute equation 62 into equation 61 to arrive at

$$\Lambda_i(z) = Q(\boldsymbol{A}, z) + N\frac{Q(\boldsymbol{A}, z) - \frac{1}{N}\sum_{i=1}^{N}\Lambda_i(z)}{\xi(z)}a_i(z), \tag{63}$$

Intuitively, price impact introduces a wedge between strategic agents' state prices and Arrow asset prices, and we can substitute for price impact with the average wedge implied by asset prices.

We now express asset prices in terms of the $|\mathcal{Z}|\times 1$ vector of liquidity trader demands $\boldsymbol{\xi}$. Let $\boldsymbol{Q}$ be the $|\mathcal{Z}|\times 1$ vector of asset prices, and define $\boldsymbol{\lambda}(\boldsymbol{Q}, \boldsymbol{\xi},)$ to be the map from $2|\mathcal{Z}|\times 1$ vectors $\boldsymbol{\xi}$ and $\boldsymbol{Q}$ to $\frac{1}{N}\sum_{i=1}^{N}\Lambda_i(z)$ using the $N\times|\mathcal{Z}|$ equations 63. We then can express the $|\mathcal{Z}|\times 1$ equations 62 as functions of $\boldsymbol{\xi}$

$$\boldsymbol{Q} = \boldsymbol{\lambda}(\boldsymbol{Q}, \boldsymbol{\xi}) + \frac{\mu}{N}\boldsymbol{Q}' \odot \boldsymbol{\xi}, \tag{64}$$

where $\odot$ is the Hadamard product and $\boldsymbol{Q}'$ measures the change in prices from an infinitesimal change in $\xi(z)$ that, by market clearing, is the same as the total price impact of a strategic agent type.

We then have a system of first-order differential equations 64 to solve for prices.

Case 2: Some liquidity trader demands are zero ($\xi(z) = 0$ for some z)

In this case, we cannot rely on equation 62 to substitute for price impact. Instead, we recognize that equation 62 reduces to

$$Q(\boldsymbol{A}, z) = \frac{1}{N} \sum_{i=1}^{N} \Lambda_i(z). \tag{65}$$

Notice, however, this expression resembles the strategic limit of our Cournot-Walras model in which the competitive fringe is arbitrarily small. This observation yields two insights. First, we can solve for states in which $\xi(z) = 0$ numerically assuming an arbitrarily small competitive fringe. Second, the strategic limit of our Cournot-Walras equilibrium recovers an Equilibrium-in-Demand-schedules in which there the pricing function is anonymous and there are no liquidity traders, i.e., $\xi(z) \equiv 0$. This is because the key difference between the Cournot-Walras and Equilibrium-in-Demand-schedules is the price function, and in the strategic equilibrium the price function coincides with equation 65.

In addition to solving for the case in which $\xi(z) = 0$, we can use Case 2 to provide boundary conditions for the first-order ODEs 64 characterized in Case 1.

Consequently, we have shown the strategic forces in the Equilibrium-in-Demand-Schedules are the same as in the Cournot-Walras equilibrium conditional on the price function. Second, we characterize an Equilibrium-in-Demand-Schedules in complete markets in which the pricing function is anonymous.