Value Without Employment*

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Abstract

Young firms' contribution to aggregate employment has been underwhelming. However, we show that a similar trend is not apparent in their contribution to aggregate sales or stock-market capitalization, suggesting that these firms have exhibited a high ratio of average-to-marginal revenue-product-of-labor. We study the implications of the arrival of such firms in a standard model of dynamic firm heterogeneity, and show that their arrival provides a unified explanation for a large number of facts related to the decline in "business dynamism". We provide an analytical framework to gauge the quantitative impact of the decline in business dynamism on aggregate economic activity.

Keywords: Business dynamism, productivity, firm dynamics, entrepreneurship

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1 Introduction

An extensive body of research has documented and analyzed the decline in dynamism of the US economy since the early 1980s. The main findings of this literature are declining gross labor flows, declining firm entry and exit rates, declining contribution of young firms to aggregate employment, and since the early 2000s a decline in high growth firms. Motivated by these facts, a significant body of research has sought to offer an explanation for the growing weakness of young firms. Curiously, the alarming tone of the labor literature appears at odds with the far more optimistic outlook expressed by practitioners in the venture capital industry, which has experienced robust growth over the last 15 years.

In this paper we provide empirical evidence that helps reconcile the seemingly contradictory views expressed in the labor literature on the one hand and the financial industry on the other. Using a variety of data sources, we confirm that while young firms (even the largest ones) over the last couple of decades had an underwhelming performance in terms of creating jobs, their performance in terms of generating sales and creating new wealth was not similarly weak.

From this empirical evidence, we conclude that recent cohorts of firms are not necessarily "weaker" than their predecessors, but rather different in terms of how their revenue and employment relate to each other. In particular, our empirical findings raise the possibility that young firms have exhibited a higher average-to-marginal revenue product of labor (high "ARPL-to-MRPL" ratio) compared to their predecessors. Using a standard model of firm dynamics, we show that *this fact alone*, namely the arrival of high ARPL-to-MRPL firms, can help provide a unified explanation not just for the decline in the labor share, but also for the collection of facts that have been referred to as the "decline in business dynamism" (declining employment by young firms, declines in re-allocation, longer life-span of existing firms, increased TFP dispersion, etc.). We also show that the arrival of high ARPL-to-MRPL firms may help explain why the quite dramatic decline in employment by young firms (around 40-50%) has not led to a dramatic collapse of aggregate output and consumption over the last couple of decades.

We next provide a more detailed summary of the empirical evidence, the model, and our analytical results.

Our empirical analysis starts by showing that the decline in business dynamism (when measured using labor) is not just a phenomenon that pertains to smaller firms. Specifically, Compustat data, which capture the largest firms in the US economy, show a steady and large decline in the fraction of employment that is due to young, Compustat firms. An advantage of Compustat data is that company valuation data and sales data are readily available. We show that despite the underwhelming contribution of recent firms towards job creation, there is no similar clear trend in the contribution of these young cohorts towards increasing aggregate market capitalization or sales.

We confirm the fact that recent cohorts of young firms do not appear weaker in terms of their contribution to wealth creation by using Pitchbook data. Pitchbook considers the broader universe of firms that have been the target of an acquisition, or have gone through an IPO. Data from Pitchbook suggest little difference in the aggregate market value of recent young firm cohorts (expressed as a fraction of the aggregate stock market capitalization at the time of their exit) compared to their predecessors at a similar age.

For the universe of firms where no market valuation exists, we use data from the National Establishment Time Series (NETS). We document a recent trend in the data, which did not exist prior to mid-2000. Specifically, young firms purchasing an establishment with similar characteristics as an establishment purchased by an older firm operate that establishment with fewer workers than their predecessors.

Using minimal assumptions, we argue that all of the above findings point to a changing relation between revenue and labor for recent cohorts of young firms. Specifically, the joint occurrence of weak performance in terms of employment, but rising valuation-to-employment (and sales-to-employment) ratios implies a high ratio of average revenue-product of labor, $\frac{py}{l}$, to marginal revenue-product of labor, $\frac{\partial py}{\partial l}(l_t)$.

To study the economic implications of this high ARPL-to-MRPL ratio for arriving firms, we introduce this feature of the data into a standard model of dynamic firm heterogeneity. The goal of the model is not to provide an explanation for the root cause of a higher ARPL-to-MRPL ratio. Instead, we take the changing relation between revenue and employment for young firms as given, and merely explore the implications of this changing relationship.

In the model, firms arrive with heterogeneous initial productivities, experience idiosyncratic productivity shocks, and eventually perish either due to an exogenous, geometrically distributed death shock, or because of endogenous bankruptcy.[1] We characterize the steady state of this model analytically, and then perform a transition experiment, whereby from some time t_0 onward, the arriving firms exhibit a higher ARPL-to-MRPL ratio compared to the older firms. To keep the exercise transparent, we abstract from the possibility that old firms may also start exhibiting a higher ARPL-to-MRPL ratio compared to their own past. We simply note that extending the model to allow for this possibility (in order to account for the fact that even some older firms have exhibited a reduced responsiveness to idiosyncratic shocks) is straightforward, but immaterial for our results.[2]

We show that from time t_0 onward, this economy exhibits declining wages, a declining aggregate labor share (which is not driven by a decline in the average labor share), declining gross job creation and destruction for both older and younger firms, declining output creation by arriving firms, declining death rates by existing firms, greater cross-sectional dispersion in TFP, and an aging population of firms.

Despite the decline in all these measures of dynamism, aggregate consumption and output don't experience as large a change, by comparison. To better understand the link between young-firm dynamism and aggregate output, we provide a formula that connects a given change in steady-state consumption with the change in the revenue produced by young firms. This formula helps illustrate two points: First, the elasticity between the decline of young firm output and steady state consumption is a small number. To highlight an indicative quantitative result, under our baseline-case assumptions, this elasticity is 8%.

[1] Endogenous bankruptcy is driven by the presence of "operating leverage", i.e., the requirement that in order to remain operative, a firm must pay a fixed labor cost per unit of time.

[2] It is straightforward to enrich our model by introducing an idiosyncratic, firm-level Poisson shock, after which old firms start exhibiting a higher ARPL-to-MRPL ratio compared to their own past. Such a modification would be essentially isomorphic to increasing the birth rate of "new" firms in our current model, and would speed up the convergence to the new steady state. With this modification, our model would be able to match the evidence that even mature firms have shown signs of reduced responsiveness in their hiring and firing decisions, though this decline is smaller than what has been shown for young firms. Allowing for the re-birth of old firms as "young" firms would not have a material impact on our analytical results, since in the new steady state all firms born prior to t_0 will have perished anyway, and it is immaterial whether they went through an episode of "re-birth" at some point. All that is needed for our analytical formulas in sections 4 and 5 is that the new cohort of firms that start arriving after time t_0 have a *comparatively* higher ARPL-to-MRPL ratio (when compared to the firms born prior to t_0).

This means that an observed 40% decline in the contribution of young firms translates into a steady state decline in consumption of 3.2%. The reason for this small elasticity is that the decline of output by young firms reflects largely transitory factor adjustments along the transition path. Second, the magnitude of the decline in entering-firm output caused by a high ARPL-to-MRPL ratio is not a sufficient statistic for the decline in aggregate output and consumption; the root cause behind the rise in the ARPL-to-MRPL ratio also matters.

The model is sufficiently rich to allow for three (non-exclusive) possibilities behind the rise in the ARPL-to-MRPL ratio: a) A rise in the labor intensity in production, b) A rise in (within-industry) markups, c) Sectoral shifts that raise the revenue share of either low labor-intensity and/or high-markup sectors. Depending on the view that one takes behind the likely cause of the higher ARPL-to-MRPL ratio, our formulas allow a mapping to aggregate output and consumption; more importantly, the formulas allow us to provide some rough quantitative bounds and worst-case scenarios on the impact of a rising ARPL-to-MRPL on output and consumption. As it turns out, if the rise in the ARPL-to-MRPL ratio is the result of either a lower labor intensity or changing sectoral composition, then the impact on consumption is likely to be quantitatively quite small, even if the drop in young-firm employment is as large as what we observed in the data. The worst case scenario is that the rise in the ARPL-to-MRPL ratio reflects entirely a rise in markups across all industries; in that case consumption drops by about 15% once the transition to the new steady state is complete (a process that takes around 50-70 years in our calibration).

The paper is organized as follows. After a brief literature review, Section 2 presents the empirical facts concerning the rise in the value-to-employment and the sales-to-employment ratio. Section 3 presents the model, the steady-state analysis and the transition-path analysis. Section 4 presents the formulas connecting the decline in dynamism with steady-state consumption. Section 5 extends results to a multi-sector economy. Section 6 concludes. Proofs and additional results are contained in the appendix.

1.1 Literature Review

An extensive body of research had documented and analyzed the decline in dynamism of the US economy since the early 1980s. The main findings of this literature are declining gross labor flows (Decker et al., 2014), declining firm entry and exit rates (Decker et al., 2016*a*), declining contribution of young firms to aggregate employment (Decker et al., 2016*a*), and since the early 2000s a decline in high growth firms (Decker et al., 2016*b*). Decker et al. (2020) and Ilut, Kehrig and Schneider (2018)[3] present evidence that the decline in dynamism is driven by a decline in the responsiveness of firms to shocks, rather than a decline in the dispersion of shocks. An overview of this literature is presented in Akcigit and Ates (2019*a*).

Motivated by these facts, a significant body of research has sought to offer an explanation for the growing weakness of young firms. This literature has focused on financial frictions (Davis and Haltiwanger, 2019; Decker et al., 2020; Clara, Corhay and Kung, 2019), information frictions (Akcigit and Ates, 2019*b*), barriers to entry (Clara, Corhay and Kung, 2019; De Loecker, Eeckhout and Mongey, 2021), the changing demographic landscape (Karahan, Pugsley and Şahin, 2019; Pugsley and Şahin, 2019; Hopenhayn, Neira and Singhania, 2018), and growing innate weaknesses determined prior to firm birth (Sterk, Sedláček and Pugsley, 2021).

In sharp contrast to the literature on declining dynamism, which finds a growing weakness of young firms, a separate literature on Venture Capital has also studied long run trends since the 1980s and has found a growing strength of young firms. This research is nicely summarized by Guzman and Stern (2020) who argue that we should not be focusing on the number of young firms or even on their employment contribution, but instead on their quality and conclude "Simply put, alternative definitions of entrepreneurship suggest different assessments of the state of American entrepreneurship."

In this paper we provide empirical evidence that helps to reconcile the labor and venture capital views on the strength/weakness of recent cohorts of young firms. From this empirical evidence, we conclude that recent cohorts of firms are not weaker (Labor view) or stronger (VC view) than past cohorts, but instead they appear to exhibit a higher ARPL-to-MRPL ratio. We view this fact as important because a) it affects the long-term implications of the decline in dynamism, and b) it provides a new fact that may help distinguish between alternative theories of the decline in dynamism.

Our empirical evidence and model results further tie together the literature on declining

[3] This results is presented in table 5 of the earlier NBER working paper version dated September 2014.

dynamism with the literature on the decline in the labor share. As is the case in Akcigit and Ates (2019*b*) and De Loecker, Eeckhout and Mongey (2021), our model jointly explains the decline in the labor share and declining dynamism. In addition to generating both features in a single model, the analytical equations we derive tell us that the consequences of declining dynamism may exhibit a close link with the origins of the decline in the labor share.[4]

Our paper is related to a recent paper by De Loecker, Eeckhout and Mongey (2021). Both papers quantitatively relate increases in markups to the declining labor share and declining business dynamism. While the two papers use different classes of models, both manage to generate multiple moments of declining dynamism from rising markups. De Loecker, Eeckhout and Mongey (2021) present a repeated (mostly) static structural model in the style of Atkeson and Burstein (2008) that uses declining labor reallocation as one of several moment that helps quantitatively separate out different theories of rising markups. In addition to matching the targeted moments, they successfully reproduces several dynamism trends. The authors use the model to study the welfare consequences of rising markups. We differ from De Loecker, Eeckhout and Mongey (2021) in three important ways: a) we present empirical evidence that the weakness in employment is not fully mirrored in sales and market value, b) we use a simple dynamic model to show how this one fact alone can generate several aspects of the decline in dynamism and c) we provide detailed analytical formulas to study the connection between the decline in dynamism and the change in steady-state consumption.

The paper is broadly related to the literature that links financial economics with labor economics and production. We don't attempt to summarize this large literature. Indicative examples of papers that have used an operating leverage channel (as we do) include Favilukis and Lin (2016) and Donangelo et al. (2019) among many others. Indicative examples of papers that analyze the firm-valuation implications of firm birth, death, displacement, technological transformation and imperfect competition include Papanikolaou (2011), Gârleanu, Panageas and Yu (2012), Gârleanu, Kogan and Panageas (2012), Kogan et al. (2020), Gar-

[4]For instance, if the decline in the labor share is due to the replacement of workers with machines, (e.g., Karabarbounis and Neiman (2014), Brynjolfsson and McAfee (2014), and Acemoglu and Restrepo (2018) among others) then our model predicts no significant long-run decline in consumption. If, on the other hand, the decline in the labor share is due to declining competition, (for example Barkai (2020), De Loecker, Eeckhout and Unger (2020), Autor et al. (2020), and Philippon (2019)) then our model predicts a larger long-run decline in consumption.

lappi and Song (2017), Knesl (2018), and Loualiche (2021).

The paper also makes a technical contribution. In the appendix we prove a mathematical correspondence between an appropriately distorted planning problem and the market outcome of an imperfectly competitive, dynamic economy with endogenous firm exit and entry. This correspondence is convenient, since it allows us to use the Envelope theorem to obtain the long-run implications of marginal changes in the fundamental parameters of the model.[5]

2 Motivating Facts

Studies referring to the decline in business dynamism typically study the number of jobs in young firms as a fraction of aggregate employment. In this section, we study the contribution of young firms to aggregate wealth (rather than job) creation.

This section is structured as follows. In sections 2.1 and 2.2 we document that while the contribution of young firms to job creation has been underwhelming and below historical levels in the last couple of decades, aggregate firm valuations of recent cohorts (as well as their sales) do not show a similar weakness. Section 2.1 documents this pattern in Compustat data and section 2.2 in Pitchbook data. In section 2.3 we show that the different behavior of market valuations and employment implies that recent firm cohorts had a higher ratio of ARPL-to-MRPL compared to their predecessors. Since only a fraction of firms undergo an IPO or acquisition, in section 2.4 we study all establishments in the National Establishment Time Series (NETS) data set, which covers a much larger set of firms and their establishments. We show that by studying the employment creation in similar establishments purchased by young firms (as compared to older firms), we can draw a similar conclusion about the behavior of the ARPL-to-MRPL ratio.

2.1 Employment vs Market Value Creation (Compustat Data)

In this section we study the employment, sales, and firm value creation of recent firm cohorts in the Compustat dataset. For the formation of cohorts, we use 5-year bins of all the firms

[5]This correspondence is not an immediate implication of results such as Leahy (1993) and Baldursson and Karatzas (1996), since in those papers all firms in the industry are affected by the same shock, in contrast to our economy where the shocks are idiosyncratic.

that went through an initial public offering (IPO) during that period. To avoid measuring the contribution of mature firms that happen to go public late, we use information on each firm's founding year and exclude firms that are founded more than 10 years prior to their IPO. We measure total employment, sales, and firm value of each 5-year IPO cohort and express it as a share, by dividing with the aggregate value of the respective quantity (employment, sales, firm value respectively) of all Computstat firms in the prior year.

To preview and summarize the results that follow, we show that there is a widening gap between the employment share of recent cohorts and their share of market value. More recent cohorts account for a progressively smaller share of employment, their share of of aggregate sales is slightly smaller, and there is no clear trend in their share of aggregate market value. To highlight some numbers, the employment share of the 2010–2014 IPO cohort bin is half the employment share of the 1985–1989 IPO cohort bin, its share of sales is only 10 percent less than the 1985–1989 IPO cohort, and its share of aggregate firm value is similar and even slightly larger than that of the 1985–1989 IPO cohort bin.

We next provide the details of our analysis.

2.1.1. Data and Sample Selection

We use firm level data on public U.S. firms from Compustat covering the period 1985–2014. We construct the sample of non-financial U.S. public firms that are traded on NYSE, AMEX, and NASDAQ. We further remove from the sample utilities, the United States Post Office, and firms that are classified as part of Public Administration. Last, we require that firms have positive employment and market value (market value is the sum of debt and equity). In total, our sample consists of 7,559 firms and 83,189 firm$\times$year observations.

We construct IPO years using the first year that a firm has non-missing information on the number and price of common shares in the Compustat database. The data contain precise IPO dates for about half of the firms in our sample. For the set of firms with precise IPO dates, our constructed measure of IPO year is within one year of the precise date in over 95% of cases. To avoid measuring the contribution of mature firms that go public, we use data on firm founding year and exclude firms that were founded more than 10 years prior to their IPO. As an example, Revlon was founded in the 1930s and went public in 1996 and we therefore exclude Revlon when measuring the contribution of the 1996 IPO cohort.

To ensure near complete coverage, data on firm founding year are taken from a large number of sources. These sources include Loughran and Ritter (2004), Jovanovic and Rousseau (2001), SDC Platinum, Crunchbase, Wikipedia, Bloomberg, Funding Universe, and Google. We adjust the firm founding years to account for spin-offs and mergers.[6] Our founding year data cover 90% of firms in our sample. Importantly, our founding year data cover 96% of the firms that go public over the sample period. At the start of our sample in 1985, the set of firms for which we have a founding year account for 94% of employment, 95% of market value, and 95% of total assets. At the end of our sample, the set of firms for which we have a founding year account for over 99% of employment, market value, and total assets.

We measure the employment, sales, and market value contributions of the year t IPO cohort as

$$\text{Employment Contribution}_t = \frac{\text{Employment of IPO Firms (Excluding Mature Firms)}_t}{\text{Total Employment}_{t-1}}$$

$$\text{Sales Contribution}_t = \frac{\text{Sales of IPO Firms (Excluding Mature Firms)}_t}{\text{Total Sales}_{t-1}}$$

$$\text{Market Value Contribution}_t = \frac{\text{Market Value of IPO Firms (Excluding Mature Firms)}_t}{\text{Total Market Value}_{t-1}}$$

where "Employment of IPO Firms (Excluding Mature Firms)$_t$" is employment of the IPO cohort in the year of the IPO, excluding firms that were founded more than 10 years prior to their IPO, "Total Employment$_{t-1}$" is total employment in the sample in the prior year. Sales contribution and market value (sum of debt and equity) contribution are defined similarly.

We present results by IPO cohort bin. We bin IPO cohorts into the following 5-year bins: 1985–1989, 1990–1994, 1995–1999, 2000–2004, 2005–2009, and 2010-2014. For each IPO cohort bin we construct the cumulative employment, sales, and market value contributions

[6] We use the information from Wikipedia, Bloomberg, Funding Universe, and Google to identify spin-offs and mergers. If firm Y is spun-off from firm X, we assign firm Y the founding year of firm X. In the case of a merger, we attempt to determine the relative size of the two parties at the time of the merger using these same online sources and we then assign the founding year of the larger of the two parties of the merger.

of the IPO cohort bin as follows:

$$\text{Employment Contribution}_{bin} = \sum_{i \in Bin} \text{Employment Contribution}_i$$

$$\text{Sales Contribution}_{bin} = \sum_{i \in Bin} \text{Sales Contribution}_i$$

$$\text{Market Value Contribution}_{bin} = \sum_{i \in Bin} \text{Market Value Contribution}_i.$$

2.1.2. Results

Figure 1 Panel A presents the logarithm of the employment, sales, and market value contributions of each IPO cohort bin since 1985. Panel B presents the normalized (1985–1989 cohort = 0) logarithm. Panel C presents the logarithm of the ratio of the sales (respectively market value) contributions to the employment contributions. Panel D presents the normalized (1985–1989 cohort = 0) logarithm of the ratio of the sales (respectively market value) contributions to the employment contributions.

To understand the units, the log employment contribution of the 1985–1989 cohort bin to is -3.5. This means that the employment contribution of the 1985–1989 cohort bin is 3% and the average employment contribution of each IPO cohort in the 5-year bin is 0.6%.

The figure shows that there is a large decline in the employment contribution of IPO cohorts over time, but a more stable sales and market value contribution of IPO cohorts. This can be most easily seen in the normalized series presented in Panels B and D. Panel B shows that the sales and market value contribution of the most recent IPO cohorts in the sample (2010–2014) is in line with the contributions of the early cohorts in the sample. By contrast, the employment contribution of the most recent IPO cohorts in the sample is half that of the early cohorts in the sample.

As a consequence of the stable contribution to market values and declining contribution to employment, Panel D shows that the ratio of market value contribution to employment contribution has increased steadily over time. The ratio of market-value-contribution to employment-contribution has more than doubled over the sample period. While the 2000–2004 and 2005–2009 IPO cohort bins contributed less to employment, sales, and market value than previous cohorts, the relative drop in employment contribution is much larger than the drop in sales or market value contribution.

In terms of magnitudes, the employment contribution of the 2010–2014 IPO cohort bin is only 1.5%, half the employment contribution of the 1985–1989 IPO cohort bin. By contrast, the sales contribution of the 2010–2014 IPO cohort bin is 2.1%, only 10% below the contribution of the 1985–1989 IPO cohort bin, and the market value contribution of the 2010–2014 IPO cohort bin is 3.7%, slightly above the market value contribution of the 1985–1989 IPO cohort bin.

Appendix F shows that the same patterns that apply at the level of Compustat as a whole also apply when we perform the analysis at the level of individual sectors within Compustat.

2.2 Exit Values by Cohort (Pitchbook Data)

To further examine the finding that market valuations of recent firm cohorts have not behaved very differently than their predecessors, we next turn attention to the broader set of companies contained in Pitchbook. We use Pitchbook to compute the equity market valuations of young firms, which underwent an IPO, or were the target of a merger, or an acquisition. From the perspective of the original shareholders (and the various private equity funds that may have financed these firms at youth), an IPO or an acquisition is commonly referred to as an "exit" event. These exit events allow us to infer the market valuations of these firms. We find that recent cohorts of firms have aggregate exit valuations – expressed as a fraction of U.S. stock market capitalization at the time of exit – that are at least as large as earlier cohorts, controlling for age at exit.

2.2.1. Data and Sample

PitchBook is a financial data provider that covers private capital markets, including venture capital, private equity and M&A transactions. The data contain firm level information on both private and public companies including the line of business, key personnel, founding year, recent news, and detailed financing history. When a firm exits by IPO or M&A, the data provide a post-money valuation which we use to measure exit value.

From the PitchBook platform, we extract aggregate exit values in each year for each founding-year cohort. We limit our sample to U.S. firms and we separately extract exit values for firms exiting by IPO and for firms exiting by M&A. As an example, our data contain the aggregate exit value of all firms founded in 2002 that exited by IPO and M&A in 2016. We aggregate the data across exit types to construct total exit value by year and founding year cohort. We note that because we are interested in exits by both IPOs and M&A, in this section we form cohorts by founding year rather than by IPO year. This also serves as a robustness check that our results don't depend on whether we form cohorts by IPO year (as with Compustat data), or founding year.

There are firms covered by PitchBook for which an exit occurs, but the exit value is missing. We ignore firms with missing values, that is we implicitly assume that the exit values of these firms are zero. The incidence of missing values appears to be more prevalent in recent years, and hence by assigning a value of zero, we are downplaying the market values of recent cohorts, which makes our estimates on the value creation of recent cohorts (compared to earlier cohorts) conservative.[7]

To smooth out year-to-year fluctuations and facilitate the presentation of the results, we bin cohorts into 5-founding-year cohort-bins. Specifically, we present results using the following founding-year cohort-bins: 1990–1994, 1995–1999, 2000–2004, 2005–2009, 2010–2014, 2015–2019. We aggregate the data to construct a year-by-bin panel by summing the exit values in year t of each of the firms belonging in the cohort-bin. For each cohort-bin, we define "age" as year t minus the birth year of the youngest cohort in the cohort-bin. For example, the age of the 2000–2004 cohort-bin in 2005 is 5.

2.2.2. Results

Figure 2 presents the raw data on nominal exit values. Specifically, the figure shows the cumulative sum of nominal exit values of each cohort-bin (s) by age ($t - s$). Nominal exit values are measured in millions of U.S. Dollars. The figure shows that, with the exception of the 1995–1999 cohort, more recent cohorts of firms have nominal exit valuations that are

[7] In addition, firms with missing data are also more likely to be small, so they probably wouldn't affect aggregate valuations. The data provider continuously back-fills missing information and has incentives to provide exit values for all firms with significant market valuations. We therefore consider it likely that firms with missing exit value are indeed small.

larger than earlier cohorts. In nominal terms, by the age of four (the last age for which we have data for the 2015–2019 cohort-bin), the cumulative exit value of the 1990–1994 cohort-bin is \$3bil, the cumulative exit value of the 1995–1999 cohort-bin is \$175bil, the cumulative exit value of the 2000–2004 cohort-bin is \$55bil, the cumulative exit value of the 2005–2009 cohort-bin is \$126bil, the cumulative exit value of the 2010–2014 cohort-bin is \$168bil, and the cumulative exit value of the 2005–2009 cohort-bin is \$229bil. Each successive cohort-bin has created more market value than past cohort-bins.

To deflate the exit values in year t we use aggregate stock market capitalization at the end of year $t-1$. Specifically, we use 2000 as a base year and construct :

$$\text{deflated exit value}_t = \text{nominal exit value}_t \times \frac{\text{market capitalization}_{2000}}{\text{market capitalization}_{\text{end of } t-1}}. \tag{1}$$

As with the definition of firm value contribution in the previous section, the presence of aggregate stock market capitalization in the denominator controls for aggregate fluctuations in stock market values.

Figure 3 shows the cumulative deflated exit values of each cohort-bin by age. The figure clearly shows that, with the exception of the 1995–1999 cohort, more recent cohorts of firms have (deflated) exit valuations similar or larger than those of earlier cohorts.

Figure 4 shows the cumulative deflated exit values of each cohort-bin by age and by exit type, separately for IPO and M&A exits. To keep the figure easy to read, we only plot the four recent cohorts. The results are very similar when we plot all cohorts. The figure shows that exits through M&A are nearly twice as large in combined value as exit values through IPO. This points to the importance of using data that includes both IPO and M&A exits.

A notable outlier in the figure is the 1995-1999 cohort, which exhibits an unusual increase around the large stock market run-up of 1999. Similar to the Compustat data, the firms with birth years 2000-2004 have exhibited an underwhelming performance. One possible explanation is the collapse in venture capital in the aftermath of the stock market crash of 1999, along with the fact that the great recession was unfolding at the time when these companies would be ready for exit. However, firms with birth years since 2010 have not exhibited an alarmingly weak performance, when measured by deflated exit values. Overall, the deflated exit values in Pitchbook paint a very similar picture to Compustat: An exceptionally strong performance for firms that experienced their exits in the late 1990s, weakness in 2000s and a rebound post 2010. Other than these fluctuations that are likely to be cyclical, there is no noticeable time trend in these series.

It is also worth noting that while the number of IPOs has significantly declined in recent years, the total capitalization of IPOs by year has not experienced a similar decline. This shows that it was mostly the smaller market-capitalization IPOs that disappeared over the years.

2.3 Implications of the Empirical Findings

Before proceeding with the empirical analysis of the NETS data, we take stock of the implications of our findings for the behavior of the marginal and average revenue product of labor of recent firm cohorts. Under some assumptions, the observation that the market capitalization of younger firms (as a fraction of aggregate market capitalization) is not changing in any noticeable way, while their employment (as a fraction of aggregate employment) declines, suggests that the ratio of the ARPL to the prevailing wage (which is equal to the MRPL) has gone up for these young firms.

To show this, let s be the time of birth of a firm, let t denote calendar time, and define $\pi^i_{t,s} \equiv p^i_{t,s} y^i_{t,s}(l^i_{t,s}) - w_t l^i_{t,s}$, where $p^i_{t,s} y^i_{t,s}$ is the time -t revenue of a firm i born at time s, $l^i_{t,s}$ is the labor input and w_t the prevailing wage at time t. Let $P^i_{t,s}$ be the value of the firm defined as the present value of $\pi^i_{t,s}$, discounted with the prevailing stochastic discount factor in the economy. We have the following result:

Lemma 1 *Assume that $\pi^i_{t,s}$ can be decomposed in a time-age-cohort fashion, $\log \pi_{i,t,s} = \log \Pi_s + \log \Pi_t + \lambda\,(t-s) + \varepsilon_{i,t}$, where Π_s is a cohort-effect, common to all firms born at time s, Π_t is a time-effect, affecting all firms in existence at time t, $\lambda\,(t-s)$ is an age effect, and $\varepsilon_{i,t}$ is a permanent idiosyncratic shock, which evolves according to $\varepsilon_{i,t+1} = \varepsilon_{i,t} + \tilde{\eta}_{i,t}$, with $\tilde{\eta}_{i,t}$ independent across firms and time.*

Define the ARPL-to-MRPL ratio as $\delta^i_{t,s} \equiv \frac{\frac{p^i_{t,s} y^i_{t,s}}{l^i_{t,s}}}{w_t}$, where we used the first-order condition $w_t = MRPL^i_{t,s}$. Letting Δ denote the first-difference operator, we have

$$\Delta \log\left(\frac{P_{t,t}}{\sum_{s\le t} P_{t,s}}\right) - \Delta \log\left(\frac{l_{t,t}}{\sum_{s\le t} l_{t,s}}\right) \equiv \Delta \log\left(\frac{\sum_{i,t} P^i_{t,t}}{\sum_{i,s\le t} P^i_{t,s}}\right) - \Delta \log\left(\frac{\sum_{i,t} l^i_{t,t}}{\sum_{i,s\le t} l^i_{t,s}}\right)$$
$$= \Delta \log\left(\frac{\sum_{i,t} \omega^i_{t,t}\left(\delta^i_{t,t}-1\right)}{\sum_{i,s\le t} \omega^i_{t,s}\left(\delta^i_{t,s}-1\right)}\right), \quad (2)$$

where $\omega^i_{t,t} \equiv \frac{l^i_{t,t}}{\sum_{i,t} l^i_{t,t}}$ *are the (relative) employment weights of firms born at time* t *and* $\omega^i_{t,s\le t} \equiv \frac{l^i_{t,s}}{\sum_{i,s\le t} l^i_{t,s}}$ *are the employment weights of all firms.*

The term $\left(\frac{l_{t,t}}{\sum_{s\le t} l_{t,s}}\right)$ maps to our definition of the employment share of young firms (i.e., firms born in "period" t). Similarly, the term $\left(\frac{P_{t,t}}{\sum_{s\le t} P_{t,s}}\right)$ is the market value share of young firms. Lemma 1 shows that the discrepancy in the time-series evolution between a market capitalization-based measure of dynamism $\left(\frac{P_{t,t}}{\sum_{s\le t} P_{t,s}}\right)$ and an employment-based measure of dynamism $\left(\frac{l_{t,t}}{\sum_{s\le t} l_{t,s}}\right)$ has implications for the evolution of the ARPL-to-MRPL ratio, $\delta^i_{t,s}$, of young firms vs. older firms: When the logarithmic change in the market capitalization share of young firms exceeds the logarithmic change in the employment share of young firms, then the (employment-weighted) value of $\delta^i_{t,s}$ must have increased for recent cohorts of young firms.

We note that in the proof of Lemma 1, the assumption that $\log \pi^i_{t,s}$ admits a time-age-cohort decomposition is only useful to conclude that the the employment contribution of young firms decreased relative to their revenue contribution, $\Delta \log\left(\frac{p_{t,t}y_{t,t}}{\sum_{s\le t} p_{t,s}y_{t,s}}\right) - \Delta \log\left(\frac{l_{t,t}}{\sum_{s\le t} l_{t,s}}\right) > 0$. In cases where we observe directly that this difference is positive, as is the case for Compustat firms, the conclusion of Lemma 1 holds without any assumption.

2.4 Micro-Level Evidence from Establishments (NETS Data)

A natural concern about the conclusions of section 2.3, is that they may be special to the subset of firms for which some form of equity valuation exists.

In this section we use an alternative set of assumptions to establish the occurrence of a higher ARPL-to-MRPL ratio for young firms in instances where no firm valuation exists. This alternative approach relies on the comparison between the job creation in business establishments purchased by young and old firms.

To explain the key idea, we introduce a set of assumptions. Specifically, letting s denote the date of firm birth and t the current time period, suppose that a company contemplates purchasing a given establishment and expects to reap profits equal to

$$\pi_{t,s}^{e} = \max_{l_{t,s}} \left[p_{t,s} y_{t,s} - w_t l_{t,s} \right] = \max_{l_{t,s}} \left[w_t l_{t,s} \left(\delta_{t,s} - 1 \right) \right], \tag{3}$$

where $l_{t,s}$ is the labor that will be employed at that establishment and $p_{t,s} y_{t,s}$ is the revenue that will be produced at the establishment. We let $\Pi_{t,s}^{e}$ denote the present value of the stream of profits, $\pi_{t,s}^{e}$.

Next, consider two identical establishments "1" and "2", the first purchased by a firm born in period t and the second purchased by a firm born in period $s < t$. Being identical, the two establishments command the same price $P_{1,t}^{est.} = P_{2,t}^{est.}$. Assuming that buyers and sellers of establishments split the rents from a purchase in some fixed proportion, we have

$$\phi \Pi_{t,t}^{e} = P_{1,t}^{est.} = P_{2,t}^{est.} = \phi \Pi_{t,s}^{e}, \tag{4}$$

where ϕ is a parameter that controls the fraction of rents going to sellers and $\Pi_{t,s}^{e}$ is the present value of profits from purchasing the establishment accruing to the firm born at time s. Assuming the same ratio $\frac{\Pi_{t,t}^{e}}{\pi_{t,t}^{e}} = \frac{\Pi_{t,s}^{e}}{\pi_{t,s}^{e}}$ for both firms and using (3) and (4) yields

$$\frac{w_t l_{t,t}}{w_t l_{t,s}} = \frac{\left(\delta_{t,s} - 1 \right)}{\left(\delta_{t,t} - 1 \right)}. \tag{5}$$

Therefore, if we observe in the data that $l_{t,t} < l_{t,s}$, then (5) implies that $\delta_{t,t} > \delta_{t,s}$.

The idea is quite intuitive: if the young and the old firm are willing to pay the same price for an establishment, they must be expecting to obtain similar profits. If the young firm expects to operate the establishment with fewer workers, then it must be the case that the revenue per worker must be higher than for the older firm in order to arrive at the same level of profits. Since the marginal product of labor must be the same for the two firms (and equal to the wage), the ratio ARPL-to-MRPL must be higher for the younger firm.

Using this idea, we study examine situations in the data where an establishment with similar characteristics in the same year and geographical location is purchased by a young vs. an old firm. We find that prior to 2005 there is no difference in the employment creation at that plant in the subsequent year, whether the acquirer is a young or an old firm. However, post 2005 we find that establishments with similar characteristics experience smaller employment growth when purchased by a young firm vs. an old firm. In light of equation (5), this suggests a higher value of $\delta^i_{t,s}$ for young firms post 2005.

2.4.1. Data

We use for our analysis establishment-level data from the National Establishment Time Series (NETS) Database. The NETS data are constructed from annual snapshots of the Dun and Bradstreet (D&B) archival national establishment data and cover the period 1990–2015. Neumark, Zhang and Wall (2007) and Neumark, Wall and Zhang (2011) provide an extensive overview of the data and Haltiwanger, Jarmin and Miranda (2013), Barnatchez, Crane and Decker (2017), and Crane and Decker (2019) provide a comparison of NETS to the Longitudinal Business Database (LBD).

In the NETS data, each establishment is identified by a unique 9-digit DUNS Number. The data report annual establishment-level employment and industry. The data further provide the first address of each establishment and records significant moves.[8] In our analysis, we use the first location of an establishment. D&B collect firm-level sales data, but these are not available in the NETS database. Instead, NETS provide estimates of establishment-level sales that are primarily determined by the estimated industry-level ratio of sales-to-employees. For this reason the data do not allow us to measure sales growth separate from employment growth. Therefore, we exclusively use employment data in our analysis.

All establishments have an identified headquarters in each year and firms are defined as the collection of all establishments with the same headquarters. We construct the founding year of a firm as the first year that the headquarters appear in the data and we adjust the founding year to account for firm reorganizations, spin-offs, and mergers.[9] We identify a change in the ownership of an establishment by a change in its reported headquarters.

[8] A significant move is a change in 5-digit ZIP Code and physical address that isn't later reversed.

[9] See Appendix G for our classification of changes in ownership and adjustments to firm age.

We construct the sample of all private-sector, payroll[10] establishments that change its reported headquarters. We exclude reorganizations and spin-offs from our sample of changes in ownership. The results are robust to including these in the sample. We measure the log difference in employment from the year before the acquisition ($t-1$) to the year after the acquisition ($t+1$). This measure of employment growth implicitly requires that the establishment survives one year after the acquisition. We refer to the establishment that changes headquarters as the target establishment, and we refer to the new headquarters as the acquiring firm.

Data limitations require the following additional filters. First, the headquarter variable in 1990 appears to be unreliable.[11] For this reason we construct our measure of firm founding year using the 1991–2015 data. Second, our method of determining a firm's founding year is unable to determine the precise founding year of a firm that appears in the data at the start of the sample. Put simply, we cannot tell apart firms founded in 1950 from firms founded in 1990. For this reason, we start the analysis in 1998, the first year in which we can determine whether a firm is at least 8 years old. Third, as discussed by Crane and Decker (2019), the data include many imputed values for employment. We remove all acquisitions for which employment of the target establishment is imputed in the year before the acquisition or in the year after the acquisition. Last, as discussed by Neumark, Wall and Zhang (2011), reported employment is sticky. In NETS, less than 20% of establishments report any change in employment over a two-year period. We restrict the sample to establishments that report a change in employment from the year before the acquisition to the year after the acquisition.

Table 1 presents a summary of our sample construction and reports the number of acquisitions in our sample after each data filter. Our analysis sample consists of 213,255 acquisitions over the period 1998–2014.

Despite these limitations, the NETS data have a significant advantage over the confidential census data. While NETS shows a smooth aggregate number of changes in ownership across years, the census data don't often know the year in which an establishment is sold

[10]We require that an establishment has 2 employees, including the owner, prior to the acquisition.

[11]The data show an abnormally high level of changes in ownership in 1991 that are not supported by external data. In addition, some headquarter identifiers in 1990 do not correspond to an establishment that exists in 1990.

and this results in large aggregate spikes in economic census years (years ending 2 and 7).

2.4.2. Results

Table 2 shows that establishments purchased by young firms grow slower than similar establishments purchased by older firms. The dependent variable is the log difference in employment from the year before the acquisition ($t-1$) to the year after the acquisition ($t+1$). The independent variables are an indicator Young Acquirer equal to one if the acquirer is less than 8 years old in the year of the acquisition and fixed effects that vary by column. The first column includes year fixed effects. The second column includes year×industry fixed effects where industry is defined as a 4-digit SIC. The third column further accounts for geographic variation by including year×industry×state fixed effects. Standard errors are clustered by year×industry×state. The tables shows that establishments purchased by young firms grow 2.7 to 3.9 percentage points slower than similar establishments purchased by older firms.

Table 3 reports our main results. In this analysis we split our sample into an early period (1998–2005) and late period (2006–2014). The table shows that, up until 2005, establishment purchase by young firms grew at a similar rate as establishments purchased by older firms. Only starting in 2005 do establishment purchased by young firms grow at a slower rate than similar establishments purchased by older firms. The point estimates show that since 2005 establishments purchased by young firms growth between 12 and 13.4 percentage points slower than similar establishments purchased by older firms.

One potential concern is that (since 2005) young firms systematically acquire establishments that differ in age and size from those acquired by older firms. To address this concern we repeat the analysis with controls for the age and size of the target establishment. To account for establishment age, we bin establishments into three age bins: [1,3], [4,7], and 8+. Table 4 reports the results. All columns include year×industry×state fixed effects. The first column repeats the analysis of Table 3. The second column includes establishment age bin dummies. The third column includes a control for log employment in the year before the acquisition ($t-1$). The fourth column includes both establishment age bin dummies and a control for log employment in the year before the acquisition ($t-1$). The results are very similar when we include establishment age bin dummies or a control for log employment at the target establishment in the year before the acquisition.

3 Model

3.1 Overview

The ARPL-to-MRPL ratio can be expressed as[12]

$$\frac{ARPL_i}{MRPL_i} = \underbrace{\frac{1}{1 + \frac{y_i}{p_i}\frac{\partial p_i}{\partial y_i}}}_{\text{markup}} \times \underbrace{\frac{1}{\frac{l_i}{y_i}\frac{\partial y_i}{\partial l_i}}}_{\text{inverse of labor intensity}} \tag{6}$$

The first term in equation (6) is the markup charged by a firm (we show this more formally shortly). The second term is the (inverse of) the output elasticity of labor, which we refer to as the "labor intensity" of production. The fact that younger firms start exhibiting a higher ARPL-to-MRPL ratio implies that they either charge higher markups, or that they employ less labor-intensive technologies relative to their predecessors.

For the remainder of the paper, we use a "textbook" model of firm dynamics to study the implications of the arrival of firms with high ARPL-to-MRPL ratio (either because of higher markups, or because of a lower labor intensity). The model features firm birth, death, and endogenous continuation decisions. To study the co-existence of firms with different ARPL-to-MRPL ratios, the model allows for heterogeneous markups and labor intensities across firms.

In section 3.2 we lay out the model in detail and in section 3.3 we derive its steady state assuming that all firms charge the same markup and have the same labor intensity. In section 3.4 we study the transition of this economy to a new steady state, when, starting from some time t_0, newly-arriving firms have a higher ARPL-to-MRPL ratio. We show that this one

[12]To derive this equation use the definition $ARPL_i = \frac{p_i y_i}{l_i}$ and then the definition of $MRPL_i$ to obtain

$$\begin{aligned} MRPL_i &= \frac{\partial (p_i y_i)}{\partial y_i}\frac{\partial y_i}{\partial l_i} = \left(y_i \frac{\partial p_i}{\partial y_i} + p_i\right)\frac{\partial y_i}{\partial l_i} = p_i\left(1 + \frac{y_i}{p_i}\frac{\partial p_i}{\partial y_i}\right)\frac{\partial y_i}{\partial l_i} = \\ &= p_i\left(1 + \frac{y_i}{p_i}\frac{\partial p_i}{\partial y_i}\right)\left(\frac{y_i}{l_i}\right)\left(\frac{l_i}{y_i}\frac{\partial y_i}{\partial l_i}\right) = ARPL_i\left(1 + \frac{y_i}{p_i}\frac{\partial p_i}{\partial y_i}\right)\left(\frac{l_i}{y_i}\frac{\partial y_i}{\partial l_i}\right) \end{aligned}$$

feature alone can account for a number of facts that have been collectively referred to as a decline in "business dynamism"(fewer jobs and output created by young firms, smaller gross job flows, larger fraction of the population employed by mature firms, larger dispersion in total factor productivity).

One of the surprising features of the model in section 3.4 is that while there is a dramatic drop in employment contribution of young firms, the declines in long-run aggregate output and consumption are quite small.

Motivated by this finding, sections 4 and 5 provide analytical, quantifiable equations that connect a given drop in young-firm output with an eventual drop in aggregate output and consumption. These equations can help explain (both qualitatively and quantitatively) why consumption may not drop a lot, despite very large drops in new-firm employment. We also consider a multi-sector extension of the model, which helps underscore that same conclusion.

3.2 Setup

3.2.1. Preliminaries

The setup is one of monopolistic competition with heterogeneous intermediate-goods producers. Time is continuous and indexed by t. Setting up the model in continuous time allows us to provide simple, closed-form solutions for several quantities.

3.2.2. Final-good producer

There is a representative, competitive, final-good producing firm, which assembles intermediate inputs y_{it} to supply the final good quantity Y_t. Since we want to allow for the possibility of heterogeneous markups we proceed as in Kimball (1995) and assume that the quantity of the final good Y_t is given implicitly by the equation

$$1 = \int_{i \in I} \mathcal{Y}\left(\frac{y_{it}}{Y_t}\right) di. \tag{7}$$

where y_{it} is the quantity of intermediate good i at time t and $\mathcal{Y}(\cdot)$ is a function satisfying $\mathcal{Y}(1) = 1, \mathcal{Y}' > 0$ and $\mathcal{Y}'' < 0$.

The appealing feature of (7) is that the production function exhibits constant returns to scale, since doubling all inputs y_{it} necessarily doubles the final good output Y_t.

The widely used Dixit-Stiglitz specification is a special case of (7). Indeed, choosing $\mathcal{Y}(x) = x^{\xi}$, $\xi \in (0,1)$ equation (7) can be equivalently written (after multiplying both sides by Y_t^{ξ} and raising both sides to the power $\frac{1}{\xi}$) as

$$Y_t = \left(\int_{i \in I} y_{it}^{\xi} di \right)^{\frac{1}{\xi}}. \tag{8}$$

In this special case, all firms charge the same markup $\frac{1}{\xi} - 1$.

Throughout, we use the final good as the numeraire and fix its price to one. Letting p_{it} denote the relative price of intermediate good i at time t, the optimization problem of the final-goods firm can be formulated as

$$\max_{y_{it}} Y_t - \int_{i \in I} p_{it} y_{it} di, \tag{9}$$

subject to the production equation (7). The first order condition for intermediate good i is

$$p_{it} = \frac{\partial Y_t}{\partial y_{it}} = \frac{\mathcal{Y}'\left(\frac{y_{it}}{Y_t}\right)}{\int_{i \in I} \frac{y_{it}}{Y_t} \mathcal{Y}'\left(\frac{y_{it}}{Y_t}\right) di}, \tag{10}$$

where the second equality in (10) follows from the implicit function theorem. In the special case where $\mathcal{Y}(x) = x^{\xi}$, equation (10) specializes to

$$p_{it} = \left(\frac{y_{it}}{Y_t} \right)^{\xi - 1}, \tag{11}$$

the well known Dixit-Stiglitz demand function for intermediate good i. Throughout, we use the final good as the numeraire and normalize its price to one.

3.2.3. Intermediate-goods producers

At each point in time a continuum of mass $\phi > 0$ of new firms arrives. Firms produce output utilizing the production function

$$y_{it} = Z_{it} k_{it}^{1-\alpha_i} l_{it}^{\alpha_i}, \tag{12}$$

where l_{it} is the labor employed by firm i, k_{it} is capital, $\alpha_i \in (0,1)$ controls the relative importance of labor in production, and Z_{it} is a time-varying, firm-specific productivity shock that evolves according to

$$dZ_{it} = \mu Z_{it} dt + \sigma Z_{it} dW_{it}, \tag{13}$$

where μ and σ are parameters common to all firms. W_{it} are firm-specific Brownian motions, independent across firms. The assumption that Z_{it} follows a geometric Brownian motion is not essential for our results, but facilitates some calculations.

The initial value of $\log(Z_{it})$ for each newly-born firm is drawn from some distribution $m(\cdot)$.

To operate, firms have to employ $\bar{l}$ units of labor per unit of time. This labor cost should be viewed as "overhead". It does not impact the output of the firm, but it is a requirement for the firm to be able to exist. If a firm decides to shut down, then it terminates permanently.

A firm is also subject to an exogenous, idiosyncratic, exponentially-distributed death shock that arrives with intensity $\lambda > 0$. By the Law of Large Numbers, a fraction λdt of firm perishes for exogenous reasons per unit of time dt.

Since there are no aggregate shocks in this model, all aggregate quantities are deterministic, and there is no risk premium. Therefore, each firm chooses its labor demand l_{it} and its optimal stopping time of termination, τ, so as to maximize the expected present value of its profits

$$V_{i,t} = \max_{\tau, l_{iu}, k_{iu}} E_t \int_t^{\tau} e^{-\int_t^u (r_s+\lambda) ds} \left[p_{iu} y_{iu} - w_u \left(l_{iu} + \bar{l} \right) - r_u^K k_{iu} \right] du, \tag{14}$$

where r_t is the interest rate and r_t^K is the rental rate of capital at time t. We next define the demand elasticity, ε_i, as $\varepsilon_i \equiv -\frac{\mathcal{Y}'\left(\frac{y_{it}}{Y_t}\right)}{\frac{y_{it}}{Y_t}\mathcal{Y}''\left(\frac{y_{it}}{Y_t}\right)}$. Maximizing over labor and capital inputs leads to the following two first order conditions for capital and labor:

$$
\begin{aligned}
(1-\alpha_i)\left(1-\frac{1}{\varepsilon_i}\right)p_{it}Z_{it}k_{it}^{-\alpha_i}l_{it}^{\alpha_i} &= r_t^K, \qquad (15)\\
\alpha_i\left(1-\frac{1}{\varepsilon_i}\right)p_{it}Z_{it}k_{it}^{1-\alpha_i}l_{it}^{\alpha_i-1} &= w_t. \qquad (16)
\end{aligned}
$$

These conditions are the familiar requirements that the marginal revenue products of capital and labor equal their respective marginal cost. Equations (15) and (16) together with the definition of y_{it} imply that the markup for firm i is $\frac{p_{it}y_{it}}{w_t l_{it}+r_t^K k_{it}} = \left(1-\frac{1}{\varepsilon_i}\right)^{-1}$.

In the important special case, where $\mathcal{Y}(x) = x^{\xi_i}$, and ξ_i differs across i, we have that $\xi_i = 1-\frac{1}{\epsilon_i}$. Therefore, $1-\xi_i$ can be interpreted as the fraction of output that takes the form of rents.[13]

3.2.4. Labor and capital market clearing

Letting $g_t(Z)$ denote the measure of firms with productivity Z that are active at time t, labor market clearing requires that

$$
\int_0^\infty g_t(Z)\left[l_{it}(Z)+\bar{l}\right]dZ = L, \qquad (17)
$$

where L is the labor supply in this economy. In the body of the paper we assume a constant, inelastic labor supply equal to L, for simplicity. Appendix D introduces elastic labor supply and derives the implications of this extension for the main analytical results of the paper.

Similarly, capital market clearing requires that $\int_0^\infty g_t(Z)\,k_{it}(Z)\,dZ = K_t$.

3.2.5. Representative household and goods market clearing

The assumptions we make about the representative household and capital accumulation are standard, so we keep the presentation brief. The household has constant relative risk aversion preferences with an intertemporal elasticity of substitution (IES) equal to $\frac{1}{\gamma}$. The representative household collects all wages and profits, which equal to aggregate output.

[13] Using $\frac{p_{it}y_{it}}{w_t l_{it}+r_t^K k_{it}} = \left(1-\frac{1}{\varepsilon_i}\right)^{-1}$ and $\xi_i = 1-\frac{1}{\epsilon_i}$ implies that $\xi p_{it}y_{it} = w_t l_{it} + r_t^K k_{it}$. Accordingly, $1-\xi_i$ is the fraction of output that takes the form of economic rents, and ξ_i is the non-rent share of output.

Since there are no aggregate shocks, the Euler equation implies that the interest rate (in terms of the final good) is given by $r_t = \rho + \gamma \left(\frac{\dot{C}_t}{C_t} \right)$.

The final good can be used either for investment or consumption. Goods market clearing requires that aggregate consumption plus investment equal aggregate output, $C_t + I_t = Y_t$.

Finally, the household can save in either zero-net supply bonds yielding the interest rate r_t or capital. There are no adjustment costs and hence capital evolves according to $\dot{K}_t = -\delta K_t + I_t$.

3.3 Analysis: Steady State

In this section we study the steady state of an economy, where all firms charge the same markup ($\mathcal{Y}(x) = x^{\xi}$) and use the same technology ($\alpha_i = \alpha$). The characterization of this steady state is available analytically. We present the detailed propositions in the appendix (Section A) and here we only summarize the main results.

First, the steady-state cross-sectional distribution of the log-productivity $z = \log(Z)$ is given by

$$g(z; z^*) = \frac{2}{\sigma^2} \frac{\phi}{\eta_2 - \eta_1} \left[\begin{array}{c} \left(\int_{z^*}^{z} e^{\eta_1 (z-s)} m(s)\, ds + \int_{z}^{\infty} e^{\eta_2 (z-s)} m(s)\, ds \right) \\ - \left(\int_{z^*}^{\infty} e^{\eta_2 (z^*-s)} m(s)\, ds \right) e^{\eta_1 (z-z^*)} \end{array} \right], \tag{18}$$

where η_1, η_2 are appropriate constants, and z^* is the cutoff value of z that triggers firm exit. (Explicit analytical expressions for η_1, η_2, z^* are provided in the appendix).

The equilibrium rate of return on capital is $r^K = \rho + \delta$, and the equilibrium wage is

$$w = \left(\alpha\xi \left(\frac{\alpha}{1-\alpha} (\rho + \delta) \right)^{\alpha - 1} \left(\int_{z^*}^{\infty} g(z; z^*)\, e^{\frac{\xi}{1-\xi} z} dz \right)^{\frac{1-\xi}{\xi}} \right)^{\frac{1}{\alpha}}, \tag{19}$$

while aggregate output is

$$Y = \frac{w}{\alpha\xi} \times \left(L - \bar{l} \int_{z^*}^{\infty} g(z; z^*)\, dz \right). \tag{20}$$

In the special case of a labor-only economy ($\alpha = 1$), $Y = \left(L - \bar{l} \int_{z^*}^{\infty} g(z; z^*)\, dz \right) \times$

$\left(\int_{z^*}^{\infty} g\left(z; z^*\right) e^{\frac{\xi}{1-\xi} z} dz\right)^{\frac{1-\xi}{\xi}}$, which is a product of two terms, the labor utilized in production, $L - \bar{l} \int_{z^*}^{\infty} g\left(z; z^*\right) dz$, and an aggregator of productivities $\left(\int_{z^*}^{\infty} g\left(z; z^*\right) e^{\frac{\xi}{1-\xi} z} dz\right)^{\frac{1-\xi}{\xi}}$. As ξ approaches one (perfect competition limit), this aggregator puts progressively higher weight on only the highest productivities.

A direct implication of (A.10) is that the labor share is given by

$$\frac{wL}{Y_t} = \alpha\xi \times \left(\frac{L}{L - \bar{l} \int_{z^*}^{\infty} g\left(z; z^*\right) dz}\right). \tag{21}$$

Equation (21) shows that in this economy the labor share is a product of i) the labor intensity, α, ii) the non-rent share of output, ξ, and iii) $\frac{L}{L - \bar{l} \int_{z^*}^{\infty} g(z; z^*) dz}$, which is the inverse of the share of the labor force that is employed in the production of output (rather than "overhead" activities.)

3.4 Transition to a New Steady State

In section 2 we documented that the ratio $\delta_{it} = \frac{p_{it} y_{it}}{w_t l_{it}}$ appears to have increased for recent cohorts of entering firms. Equations (6) and (16) imply two (non-exclusive) possibilities: either markups, $(1 - \frac{1}{\varepsilon_i})^{-1}$, increased, or the labor intensity, α_i, decreased for these firms.

In this section, we focus on the first possibility and study the transition path of an economy, whereby from some time t_0 onward the economic rents for the arriving cohorts of firms is higher than for their predecessors (markups increase). The goal of this section is not to provide a full-blown calibration, but to illustrate that an increase in the share of economic rents for incoming firms can explain not just a drop in the labor share of output, but also a number of other facts related to the decline in business dynamism. We also illustrate how a quantitatively large decline in the employment share of young firms can be still consistent with only small drops in output and consumption, which is a result that we analyze in detail in the next section.

The reason why we focus on a rise in economic rents, rather than a decline in labor intensity is the following. Equation (A.4) in the appendix shows that (in steady state) the labor demand of a firm is proportional to $Z_{i,t}^{\varepsilon-1}$.[14] Taking logarithms on both sides of this equation shows that the standard deviation of log-employment changes , $\sigma(d\log l_{it})$, is equal to $(\varepsilon-1)\sigma$, which depends on ε but is invariant to α. As is well understood in the literature, one key aspect for explaining the decline in business dynamism is that firms became less responsive to their idiosyncratic shocks, i.e., $\sigma(d\log l_{it})$ declined. Therefore, by assuming a decline in the elasticity, ε, for young firms we can explain the decline in the labor share, the rise in the ARPL-to-MRPL ratio for young firms, and the decline in business dynamism as *joint phenomena* emanating from a single source. Of course, this doesn't preclude that both ξ and α have decreased, and therefore in the next section where we provide our main analytic result, we allow both ξ and α to change.

Since the goal of this section is illustrative, in the interest of parsimony we simplify matters even further, and consider a labor-only economy ($\alpha = 1$), whereby from some time t_0 onward the function $\mathcal{Y}\left(\frac{y_{it}}{Y_t}\right)$ *for arriving firms* becomes $\mathcal{Y}\left(\frac{y_{it}}{Y_t}\right) = \left(\frac{y_{it}}{Y_t}\right)^{\xi^*}$ with $\xi^* < \xi$, which implies that their demand elasticity, $\varepsilon^* = \frac{1}{1-\xi^*} < \frac{1}{1-\xi} = \varepsilon$. We also allow for the possibility that the incoming cohorts of firms (from t_0 onward) employ the technology $Z^c Z_{it} l_{it}$, where Z^c is a common TFP shifter. By considering a joint change in Z^c and ξ^*, we can control separately the level of the ARPL and the ratio of ARPL-to-MRPL. In the interest of transparency, in this section we set $Z^c = 1$, but we allow Z^c to differ from one in the next section, when we derive our analytical results. Finally, while it would be straightforward to modify the model to allow older firms to face a different demand function (and hence charge higher markups and be less responsive to their idiosyncratic shocks), we do not pursue this possibility, for simplicity.[15]

Providing an analytical solution for the evolution of all equilibrium quantities on the way to the new steady state is infeasible. The key difficulty is that wages and interest rates become time-dependent as we transit between steady states. This makes the bankruptcy

[14] To see this, combine (A.4) with $\varepsilon = \frac{1}{1-\xi}$.

[15] Research on declining dynamism has found that mature firms show signs of reduced responsiveness in their hiring and firing decisions, though this decline is smaller than what has been shown for young firms. The declining responsiveness of mature firms can be easily incorporated into the model by introducing a firm-specific Poisson process that moves mature firms from the old technology to the new technology. Incorporating this additional feature speeds up the transition of the economy to the new steady state, but does not have a material impact on the (analytical) steady state analysis in sections 4 and 5. See also the discussion in footnote 2 for more details on this point.

thresholds both time-, and productivity-dependent, requiring a numerical algorithm to derive the bankruptcy threshold. We provide a brief description of the numerical algorithm in appendix C.

The parameter values for our numerical example are provided in table 5. Since this is a labor-only economy (for simplicity we choose $\alpha = 1$), we assume that $\xi = 0.66$, to approximately match a labor share of $\alpha\xi \approx \frac{2}{3}$. (We use the word "approximately" because the presence of fixed labor costs actually implies a slightly higher labor share.) We choose $\mu = -0.01$ and $\sigma = 0.14$ so that the model's implications are in line with figures 1.3A and 1.3B of Haltiwanger et al. (2017). Specifically these figures in Haltiwanger et al. (2017) report a median employment (and output) growth around zero and an inter-decile range that declines with age (staring from about 1.3 and declining to 0.5). By choosing $\sigma = 0.14$, the standard deviation of employment growth (excluding overhead) is approximately $\xi\sigma/(1-\xi) \approx 0.28$, which results in an inter-decile range approximately equal to 0.72, which is in line with the typical values for firms between 5-15 years. [16] For λ we choose a value of 0.08, which implies a 8% death rate for exogenous reasons. This death rate is mostly relevant for the mature firms, since in the early years firms mostly terminate due to endogenous bankruptcy in the model. For the discount rate we choose $\rho = 0.02$, to account for the low interest rates during the sample period. The choice of the intertemporal elasticity of substitution, $\frac{1}{\gamma}$, is largely irrelevant for the purposes of our exercise, since we are considering a pure labor economy without capital accumulation. Therefore, we assume a high intertemporal elasticity of substitution (formally, $\gamma = \infty$,) so that the interest rate remains constant and equal to $\rho = 0.02$ along the transition path.[17] We choose the number for $\bar{l}/L$ to match a bankruptcy cutoff z^* approximately equal to the lowest possible value of the (log) productivities of entering firms, which we normalize to zero. With this tight bankruptcy cutoff we can capture the strong selection effects (several exits, high growth rates conditional on survival, etc.) that are typical of firms that enter the economy. We choose the initial distribution of productivities

[16]Firm growth for very young firms (0-5 years) is substantially skewed both in the data and the model, so it makes more sense to match the cross-sectional standard deviation of firms past 5 years of age.

[17]A smaller intertemporal elasticity of substitution would have introduced some time-variation in the interest rate and therefore in firms' optimal termination decisions. However, for our parameters the fluctuations in aggregate output turn out to be very small on a yearly basis along the transition path. Thus the interest rate variability would be negligible for plausible values of the IES. Therefore we simply set $\gamma = \infty$ and keep the interest rate constant along the transition path.

$m(z) \propto 2.5e^{-2.5z}$ with $z \in [0, 3]$. With these choices the stationary distribution of firm size (in the original steady state) can approximate the very large dispersion of employment across the firm size distribution that we observe in the data.[18]

For the firms that arrive after the onset of the transition, we choose $\xi^* = 0.58$ and $Z^c = 1$ to match a drop in the labor share of approximately 8%. [19]

Figure 5 plots the equilibrium wage, labor share and output for our example. The x-axis depicts years, with year $t_0 = 20$ being the year of transition from the old to the new steady state. We divide the wage and the output number by their values in the original steady state, so that these numbers can be interpreted as a fraction of their original steady state value. Not surprisingly, the labor share declines essentially monotonically (up to numerical approximation error). Wages decline by about 14% between the steady states, while the level of aggregate output drops by about 6% over a course of approximately 80 years.

While the decline of the labor share is not our primary focus, we point out that the model is consistent with the fact that while the aggregate labor share has declined in the last decades, the "average" labor share has increased.[20] Put differently, the decline in the labor share has been primarily driven by compositional forces. To expedite the presentation of our main results, we relegate a more detailed discussion of this issue to appendix E.

Turning to measures of business dynamism, the left plot of figure 6 depicts $l_t^{\text{new}} = \int_{z^*}^{\infty} m(z)\, l_t^{\text{new}}(z)\, dz$, the employment by entering firms at time t. The right plot of the same figure depicts $Y_t^{\text{new}} = \int_{z^*}^{\infty} m(z)\, p_t^{\text{new}}(z)\, y_t^{\text{new}}(z)\, dz$, the output produced by young firms at time t . Both output and employment of young firms decline and the magnitudes are substantial. For instance, comparing the employment of arriving firms before and after the onset of the transition, we see a drop around 35%, which is comparable to the very large drops observed in the data. Remarkably, the model generates this large drop with only a small

[18]For instance, in 1995 the ratio of workers at a firm at the top 10% of the employment distribution is about 5% of the number of employees for a firm at the top 0.3% of the employment distribution. By comparison, in the model the ratio of employment in a firm at the top 10% of the employment distribution is approximately 0.07 times the employment of a firm at the top 0.3% of the employment distribution.

[19]In results that we don't report in the interest of brevity, we have also experimented with values of $Z^c > 1$ in order to capture the fact that in the data the output drop of young firms seems more muted than the employment drop. None of the conclusions we draw in this section are affected, except that the percentage decline in steady state output is smaller by $d\log(Z^C)$.

[20]See, e.g., Hartman-Glaser, Lustig and Xiaolan (2019).

decline in ξ for young firms with other plausible channels entirely absent (e.g., decline in the demographic measure of entrepreneurs, or a rise in the barriers to entry for young firms.)

Figure 7 presents results for additional measures of business dynamism. The top left plot presents the sum of gross job creation and destruction defined in a similar manner to Davis and Haltiwanger (1991). These measures are computed as the employment-weighted average of $\frac{|l_{i,t+1}-l_{i,t}|}{\frac{1}{2}(l_{i,t+1}+l_{i,t})}1_{\{l_{i,t+1}>l_{i,t}\}}$ (gross job creation) and $\frac{|l_{i,t+1}-l_{i,t}|}{\frac{1}{2}(l_{i,t+1}+l_{i,t})}1_{\{l_{i,t+1}<l_{i,t}\}}$ (gross job destruction). Both measures show a decline along the transition path (and in the new steady state). This decline is driven by three distinct forces inside the model. First, there is less job creation by entering firms, as noted above. Second, there is less gross job reallocation among existing firms, since the firms arriving after the onset of the transition have lower values of ξ^* and hence the same variance of idiosyncratic shocks results in less variance of employment demands. This decline in the responsiveness of firms to shocks is consistent with the empirical evidence presented in Decker et al. (2020). Third, there is less job destruction due to exit, since the lower wages decrease the fixed cost $w_t\bar{l}$ that firms have to pay to remain alive. The bottom left plot of Figure 7 illustrates the lower death rates. These lower death rates are also responsible for the greater dispersion of TFP in the new steady state (bottom right plot). The combination of reduced exit and weaker job creation by entering firms naturally implies that an increasing share of the workforce is employed at older firms.

We conclude with two remarks:

First, the goal of this section was mostly to provide an illustration rather than a large-scale calibration of the model. The goal was merely to show that a single change (namely the shift to an economy with firms exhibiting a higher share of rents in revenue) can result in a significant decline in young-firm employment, while also accounting for a wide host of other facts related to the decline in business dynamism.

Second, an intriguing observation is that the (percentage) drop in total output is much smaller than the (percentage) drop in young-firm output and employment. The next two sections analyze the relationship between the drop in young-firm output and the drop in steady-state output and consumption in detail. In analyzing this relationship we consider a more general model.

4 Young-firm output and aggregate consumption

In this section, we provide a formula connecting the drop in the output of arriving firms with the drop in steady-state consumption. The goal of this formula is two-fold. First, it helps explain (theoretically) why large declines in the output of young firms may still be consistent with modest drops in output. Second, it allows us to easily gauge the quantitative magnitude of the decline in steady state output as a function of the possible root causes behind the rise in the ARPL-to-MRPL ratio.

The setup is similar to the transition experiment of the previous section. From some time t_0 onward, the parameter ξ increases by $\xi^* = \xi + d\xi$ for arriving firms. In addition we allow capital to be an additional factor of production and assume that the technological labor share of arriving firms is $\alpha^* = \alpha + d\alpha$, and the total factor productivity of the arriving firms is multiplied by a common constant e^{dz^c}. In short, we allow that new firms differ from their predecessors in terms of their markups (ξ^*), their technological labor share α^*, and the total factor productivity e^{dz^c}. Our goal is to provide a formula linking a given, empirically observed drop in young-firm output to a decline in steady-state consumption.

The following proposition contains the main result of this section:

Proposition 1 *Define the distribution of revenue among an entering cohort of firms,* $\widetilde{m}(z) \equiv \frac{m(z)p(z)y(z)}{\int_{z^*}^{\infty} m(z)p(z)y(z)}$, *and the stationary distribution of revenue among all firms,* $\widetilde{g}(z) \equiv \frac{g(z)p(z)y(z)}{\int_{z^*}^{\infty} g(z)p(z)y(z)}$, *(evaluated at the old steady state). Define the revenue produced by an incoming cohort of firms as* $Y^{new} = \int_{z^*}^{\infty} m(z)\, p(z)\, y(z)\, dz$, *where* $p(z)$, $y(z)$ *correspond to the price and output of an entering firm with log-productivity* z. *Finally, let* C^{SS} *denote consumption in the new steady state. Then as the discount rate* ρ *approaches zero, the percentage change in steady state consumption (compared to the old steady state),* $\frac{dC^{SS}}{C^{SS}}$, *is given by*

$$\frac{dC^{SS}}{C^{SS}} = \frac{1}{\alpha}\frac{1-\xi}{\xi}\frac{dY^{new}}{Y^{new}} - \left(\frac{1-\xi}{1-\xi(1-\alpha)}\right)\frac{d\alpha}{\alpha} + \frac{1}{\alpha}(G-D)\frac{d\xi}{\xi}, \tag{22}$$

where $D \equiv \left(1+\frac{1}{\xi}\right) + \alpha\frac{\xi(1-\alpha)}{1-\xi(1-\alpha)} - (1-\alpha)$ *and* $G \equiv \int_{z^*}^{\infty} (\widetilde{g}(z) - \widetilde{m}(z)) \log y(z)\, dz$.

The proposition shows a connection between the percentage change of the output of new firms ($\frac{dY^{\text{new}}}{Y^{\text{new}}}$) and the percentage change in steady state consumption ($\frac{dC^{SS}}{C^{SS}}$).

A first observation about equation (22) is that only a fraction $\frac{1}{\alpha}\frac{1-\xi}{\xi}$ of $\frac{dY^{\text{new}}}{Y^{\text{new}}}$ "passes through" to steady-state consumption $\frac{dC^{SS}}{C^{SS}}$. As an illustration, if rents are 5% of revenue (at the old steady state), so that $\xi = 0.95$, and $\alpha = 0.65$, the decline in long-term consumption is at most $\frac{1-\xi}{\alpha\xi} \approx 8\%$ of the decline in $\frac{dY^{\text{new}}}{Y^{\text{new}}}$.

A second observation about (22) is that in order to connect the decline in new-firm revenue to an eventual decline in steady-state consumption, one needs to make two adjustments, which are captured by the second and the third term in equation (22). These two terms show that it makes a difference whether a rise in the average revenue product of labor of young firms is driven by a rise in economic rents $(d\xi < 0)$, or a decline in the technological importance of labor $(d\alpha < 0)$, since – as we argue below – the second and the third term in equation (22) have opposite signs when evaluated with empirical data.

To better understand the observations we made in the above two paragraphs, it is useful to start by providing a heuristic derivation of equation (22). In the appendix we show that the revenue of a young firm is proportional to $\xi\left(\frac{y_{it}^{\text{new}}}{Y_t}\right)^{\xi-1} y_{it}^{\text{new}}$. Totally differentiating the revenue of a young firm with respect to ξ, α, z^c yields

$$\begin{aligned} d\log\left(p_{it}^{\text{new}} y_{it}^{\text{new}}\right) \quad = \quad & \left(\frac{1}{\xi} + \log\left(\frac{y_{it}^{\text{new}}}{Y}\right)\right) d\xi + \xi\left(dz^c + \log\left(\frac{l_{it}^{\text{new}}}{k_{it}^{\text{new}}}\right) d\alpha\right) \\ & + \xi\left(\alpha d\log l_{it}^{\text{new}} + (1-\alpha)\, d\log k_{it}^{\text{new}}\right) \end{aligned} \tag{23}$$

where $y_{it}^{\text{new}}, l_{it}^{\text{new}}, k_{it}^{\text{new}}$ are the output, labor (used in production), and capital employed by the representative entering firm. Since there are no within-cohort differences in the labor-to-capital ratio, we can write $\frac{l_{it}^{\text{new}}}{k_{it}^{\text{new}}} = \frac{l^{\text{new}}}{k^{\text{new}}}$. The first two terms on the right hand side of (23) capture the change in revenue resulting from the changes in $d\xi, dz^c, d\alpha$, while keeping the labor and capital inputs of arriving firms unchanged. The last term captures that the changes in $d\xi, dz^c, d\alpha$ will also trigger a change in the inputs $d\log l_{it}^{\text{new}}$, $d\log k_{it}^{\text{new}}$ chosen by arriving firms.

At the onset of the transition, the measure of the new firms is zero, and therefore the wage and capital rental rate do not experience a marginal change $(d\log(w_t) = d\log(r_t^K) = 0)$.

Using this observation, the first order conditions for labor and capital (16) and (15) imply

$$d\log l_{it}^{\text{new}} = d\log(w_t) + d\log l_{it}^{\text{new}} = \frac{d\alpha}{\alpha} + \frac{d\xi}{\xi} + d\log\left(p_{it}^{\text{new}} y_{it}^{\text{new}}\right), \tag{24}$$

$$d\log k_{it}^{\text{new}} = d\log(r_t^K) + d\log k_{it}^{\text{new}} = -\frac{d\alpha}{1-\alpha} + \frac{d\xi}{\xi} + d\log\left(p_{it}^{\text{new}} y_{it}^{\text{new}}\right). \tag{25}$$

Substituting (24) and (25) into (23) and aggregating across the cohort of arriving firms implies that the change in the revenue, Y^{new}, of the incoming cohort is

$$\begin{aligned}\frac{dY^{\text{new}}}{Y^{\text{new}}} = & \left[1 + \frac{1}{\xi} + \int_{z^*}^{\infty} \widetilde{m}(z) \log\left(\frac{y^{\text{new}}(z)}{Y}\right) dz\right] d\xi \\ & + \xi\left[dz^c + \log\left(\frac{l^{\text{new}}}{k^{\text{new}}}\right) d\alpha\right] + \xi \frac{dY^{\text{new}}}{Y^{\text{new}}},\end{aligned}$$

where $\widetilde{m}(z)$ are revenue weights. Solving for $\frac{dY^{\text{new}}}{Y^{\text{new}}}$ and re-arranging leads to

$$\frac{dY^{\text{new}}}{Y^{\text{new}}} = \left(\frac{\xi}{1-\xi}\right) \times \left\{dz^c + \log\left(\frac{l^{\text{new}}}{k^{\text{new}}}\right) d\alpha + \left[1 + \frac{1}{\xi} + \int_{z^*}^{\infty} \widetilde{m}(z) \log\left(\frac{y^{\text{new}}(z)}{Y}\right) dz\right] \frac{d\xi}{\xi}\right\}. \tag{26}$$

Equation (26) shows how the output of arriving firms changes as a function of $dz^c, d\alpha, d\xi$. Note that the term inside angular brackets is multiplied by the term $\frac{\xi}{1-\xi}$. The presence of the rent-share of output, $1-\xi$, in the denominator of $\frac{\xi}{1-\xi}$ captures that any direct impact on output resulting from $dz^c, d\xi, d\alpha$ will be amplified by the firm's endogenous factor choices. This follows from equations (24) and (25), which show that the change in the factor inputs depends on the change in revenue, which in turn depends on the factor-input changes (equation (23)), thus creating a feedback loop of amplification.

To evaluate how the marginal changes in dz^c, $d\alpha$ and $d\xi$ affect long-term output and consumption we start with some preliminary observations. In the new steady state output will be given by (8), evaluated at $\xi = \xi^*$ (since all the firms of the old type will eventually perish). Taking logarithms on both sides of (8) and totally differentiating the resulting expression with respect to dz^c, $d\alpha$ and $d\xi$, leads to[21]

$$d\log Y^{SS} = dz^c + \left(\int_{z^*}^{\infty} \widetilde{g}(z)\left(\log\frac{y(z)}{Y}\right)\right)\frac{d\xi}{\xi} + \left(\log\left(\frac{l}{k}\right)\right)d\alpha + (1-\alpha)\,d\log K^{SS}. \quad (27)$$

The first three terms on the right-hand side of Equation (27) capture the impact of the changes in $dz^c, d\xi, d\alpha$ on output, while $(1-\alpha)\,d\log K^{SS}$, captures the impact of the (endogenous) reaction of the capital stock to these changes. To compute $d\log K^{SS}$ we aggregate firms' first-order conditions across firms to obtain $\xi^*(1-\alpha)Y^{SS} = (\rho+\delta)K^{SS}$, and therefore[22] $d\log K^{SS} = \frac{d\xi}{\xi} - \frac{1}{1-\alpha}d\alpha + d\log Y^{SS}$. Substituting this expression into (27), solving for $d\log Y^{SS}$ and then combining with (26) leads to[23]

$$\frac{dY^{SS}}{Y^{SS}} = \frac{1-\xi}{\alpha\xi}\frac{dY^{\text{new}}}{Y^{\text{new}}} - \frac{d\alpha}{\alpha} + \left[(1-\alpha) - \left(1+\frac{1}{\xi}\right) + \int_{z^*}^{\infty}\left(\widetilde{g}(z) - \widetilde{m}(z)\right)\left(\log y(z)\right)\right]\frac{d\xi}{\alpha\xi}. \quad (28)$$

The reason that the percentage change in the revenue of young firms, $\frac{dY^{\text{new}}}{Y^{\text{new}}}$, is multiplied by the factor $\frac{1-\xi}{\alpha\xi}$, which is quantitatively small in the data, reflects the observation we made earlier that a substantial part of $\frac{dY^{\text{new}}}{Y^{\text{new}}}$ captures transient labor and capital choices by the new type of firms, which are faced with a factor-price vector that has not yet adjusted to its new steady-state values.

The second term, $-\frac{d\alpha}{\alpha}$, in (28) reflects that a reduction in α raises the marginal product of capital in production and boosts capital accumulation, which leads to higher output.

The third term, contained in the square brackets of (28), captures the impact of a change in rents. There are three separate components inside the square brackets. First, $1-\alpha$

[21]In deriving equation (27) we ignored the endogenous reaction of firms' employment and shutdown decisions to changes in dz^c, $d\alpha$ and $d\xi$. This step is justified in the appendix, where we show that as ρ approaches zero, the market outcome coincides with an (appropriately distorted) planning problem, and therefore the validity of (27) is a consequence of the envelope theorem.

[22]Recall that we evaluate all changes around $\xi = \xi^*$.

[23]Using (27) to solve for $d\log Y^{SS}$ gives

$$d\log Y^{SS} = \frac{dz^c}{\alpha} + \left[\int_{z^*}^{\infty}\widetilde{g}(z)\left(\log\frac{y(z)}{Y}\right) + (1-\alpha)\right]\frac{d\xi}{\alpha\xi} + \left(\log\frac{l}{k}\right)\frac{d\alpha}{\alpha} - \frac{d\alpha}{\alpha}.$$

Solving for dz^c from (26), substituting the resulting expression into the equation for $d\log Y^{SS}$ and noting that we are considering a marginal change evaluated around the old steady state (so that $\frac{l_i}{k_i} = \frac{l^{\text{new}}}{k^{\text{new}}}$) leads to (28).

captures the impact of a change in ξ on capital accumulation. A rise in rents (a lower ξ) reduces the capital share in revenue, $\xi(1-\alpha)$, and leads to smaller capital accumulation and output. The second component reflects that if $d\xi < 0$, then part of the drop in $\frac{dY^{\text{new}}}{Y^{\text{new}}}$ is due to the change in $d\xi$, and thus it is there to avoid double-counting. The third component is the most interesting one. A lower value of ξ in our economy has similar effects to decreasing the returns to scale.[24] With diverse productivities, a lower value of ξ impedes labor and capital from getting allocated to the comparatively more productive firms. Among young firms the dispersion of revenue is $\widetilde{m}(z)$, while among all firms it is $\widetilde{g}(z)$. Thus, if $\widetilde{g}(z)$ is more disperse than $\widetilde{m}(z)$, the third term captures the notion that as the young firms start growing and becoming the dominant firm type, the misallocation effect associated with a lower ξ will spread across the economy.

Proposition 1 uses equation (28) to arrive at an expression for the change in steady-state consumption.[25]

The key implication of (22) is that it is not so much the magnitude of the drop in $\frac{dY^{\text{new}}}{Y^{\text{new}}}$ that is disconcerting for long-term output and consumption, but rather its source. For example, if one took the extreme view that the rise in the ARPL-to-MRPL ratio for entering firms is exclusively driven by a change in the labor intensity, $d\alpha < 0$, while $dz = d\xi = 0$, then $\frac{dC^{SS}}{C^{SS}}$ would be bounded below by $\frac{1-\xi}{\alpha\xi}\frac{dY^{\text{new}}}{Y^{\text{new}}}$.[26] Under the assumption that $\frac{1-\xi}{\alpha\xi}$ is about 8%, this would imply that even a 40% drop in new firm output, would lead to a consumption drop no larger than 3.2%.

Matters change somewhat if the observed change in $\frac{dY^{\text{new}}}{Y^{\text{new}}}$ results from a decline in ξ (a rise in the economic rents of young firms). In the data, the constant G is a number around 2.85.[27] The constant D is approximately equal to 2 for values of ξ close to one. As a result

[24] In a previous version of this paper, we obtained very similar conclusions in a version of the model that featured decreasing returns to scale and perfect competition.

[25] To arrive from the change in steady-state output (28) to the steady-state change in consumption (22), it suffices to note that as ρ approaches zero, $C^{SS} = (1-\xi(1-\alpha))Y^{SS}$. Taking logarithms on both sides and totally differentiating gives $\frac{dC^{SS}}{C^{SS}} = \frac{\xi d\alpha-(1-\alpha)d\xi}{1-\xi(1-\alpha)} + \frac{dY^{SS}}{Y^{SS}}$. Using this equation and (28) leads to (22).

[26] Note that $\frac{1-\xi}{1-\xi(1-\alpha)} > 0$.

[27] Assuming that overhead costs are small, in our model one can evaluate G with employment rather than revenue data: Since we are considering a marginal change in the parameters around the old steady state, equation (16) implies that $\log(w)+\log(l_i) = \log(\alpha\xi)+\log(p_i y_i)$ for all firms (young and old). In turn, since $\log(p_i y_i)$ is equal to $\xi\log(y_i)$ plus a constant that is the same for all firms (since we are evaluating around $\xi = \xi^*$), it follows that $G = \int_{z^*}^{\infty}(\widetilde{g}(z)-\widetilde{m}(z))\log y(z)\,dz \approx \frac{1}{\xi}\int_{z^*}^{\infty}(\widetilde{g}(z)-\widetilde{m}(z))\log l(z)\,dz$. Moreover,

$\frac{1}{\alpha}(G-D)$ is likely to be a number close to one. Therefore, the term $\frac{1}{\alpha}(G-D)\frac{d\xi}{\xi}$ in (22) is approximately equal to $\frac{d\xi}{\xi}$. For instance if one took the view that ξ decreased from 0.95 to 0.85,[28] the third term would be equal to a reduction of $\frac{1}{\alpha}(G-D)\frac{d\xi}{\xi}\approx -13\%$ in steady-state consumption.

5 Sectoral shifts

So far we have assumed that the firms within a given cohort utilize the same technology and charge the same markup. In this section we relax this assumption by introducing different "sectors" that firms belong to. Specifically, we assume that before and after the transition there are N^S sectors, where N^S is a natural number. Aggregate output is given as the solution to the equation $1=\sum_{S\in N^S}\int_{i\in S}\left(\frac{y_i^S}{Y}\right)^{\xi^S}di$, where $\xi^S\in(0,1)$ is a sector-specific constant and $y_i^S=\exp(z_{i,t})\,k_{it}^{1-\alpha^S}l_{it}^{\alpha^S}$ are sector-specific production functions. For simplicity, we assume that the overhead labor cost $\bar{l}$ is the same across sectors. The next proposition generalizes Proposition 1.

Proposition 2 *Let* $\omega^S=\frac{\int_{z^{S,*}}^{\infty}g^S(z)p^S(z)y^S(z)dz}{Y}$ *denote the revenue weight of sector* S *and also define* $\widehat{\alpha}\equiv\sum_S\omega^S\alpha^S$, $\widehat{\sigma}\equiv\sum_S\omega^S(1-\alpha^S)\xi^S$, *and* $G_S\equiv\int_{z^{S,*}}^{\infty}\left(\widetilde{g}^S(z)-\widetilde{m}^S(z)\right)\ln\left(y_i^S(z)\right)dz$, *where* $\widetilde{g}^S(z)=\frac{g^S(z)p^S(z)y^S(z)}{\int_{z^{S,*}}^{\infty}g^S(z)p^S(z)y^S(z)dz}$ *is the within-sector stationary distribution of revenue and* $\widetilde{m}^S(z)=\frac{m^S(z)p^S(z)y^S(z)}{\int_{z^{S,*}}^{\infty}m^S(z)p^S(z)y^S(z)dz}$ *is the within-sector distribution of revenue for newly-arriving firms in sector* S*. Then the change in steady-state consumption is*

$$\begin{aligned}\frac{dC^{SS}}{C^{SS}} &= \frac{1}{\widehat{\alpha}}\sum_S\omega^S\left(\frac{1-\xi^S}{\xi^S}\right)\frac{dY^{S,new}}{Y^{S,new}}+\sum_S\omega^S\left[G_S-\left(1+\frac{1}{\xi^S}\right)\right]\frac{d\xi^S}{\widehat{\alpha}\xi^S}+\frac{1}{\widehat{\alpha}}\frac{1-\widehat{\alpha}-\widehat{\sigma}}{1-\widehat{\sigma}}\frac{d\widehat{\sigma}}{\widehat{\sigma}}\\ &\quad+\frac{1}{\widehat{\alpha}}\sum_S\left\{\frac{wL^S}{\xi^SY}\frac{dL^S}{L^S}+\omega^S\left(1-\alpha^S\right)\left(\frac{dK^S}{K^S}-\frac{dK}{K}\right)\right\},\end{aligned}\tag{29}$$

where $Y^{S,new}$ *is revenue produced by newly arriving firms in sector* S*.*

maintaining the assumption that overhead costs are small, we can replace the revenue weights $\widetilde{g}$ and $\widetilde{m}$ with the respective employment weights, since the wage bill is the same fraction of revenue for all firms (young and old) and all firms face the same wage. The implications is that G can be approximated exclusively with employment data, which are readily available. Defining young firms as firms 5 years or younger, and using $\xi=0.95$ for the old steady state, we compute a value of G around $\frac{1}{0.95}\times 2.7=2.85$ in the data.

[28] Barkai (2020).

The next corollary confirms that (29) equals (22) in a one-sector economy.

Corollary 1 *If $S = 1$, then (29) equals (22).*

The change in steady-state consumption in equation (29) is comprised of four terms. The first three terms can be viewed as sector-weighted averages of the corresponding terms in the one-sector economy, as the proof of corollary 1 (in the appendix) illustrates. The fourth term in (29) has no corresponding term in (22), since it captures the effect of *relative* sectoral expansions or contractions. Indeed, if all sectors grow in the same proportion ($\frac{dK^S}{K^S} = \frac{dK}{K}$ and $\frac{dL^S}{L^S} = \frac{dL}{L} = 0$), then the fourth term disappears.

To better understand this fourth term, it is most useful to consider a situation where any sectoral shifts $\left(d\omega^S\right)$ are purely the result of productivity changes ($dz^c \neq 0$, but $d\alpha^S = d\xi^S = 0$). To make matters interesting, let $\widehat{\phi} \equiv \sum_S \omega^S \alpha^S \xi^S$ and assume that $d\widehat{\sigma} \leq 0$ and $d\widehat{\phi} \leq 0$. The assumptions $d\widehat{\sigma} \leq 0$ and $d\widehat{\phi} \leq 0$ capture the idea that the productivity shifts dz^c are such that sectors with low value of ξ^S (high markup sectors) grow in relative importance,[29] and the increase in the ARPL-to-MRPL ratio of arriving firms is purely the result of sectoral shifts in productivity. Assuming that the fixed component of labor utilization is small $\left(\bar{l} \approx 0\right)$, we have that

Lemma 2 *Under the assumptions of the previous paragraph, $\frac{dC^{SS}}{C^{SS}}$ is bounded below by $\frac{1}{\widehat{\alpha}} \sum_S \omega^S \left(\frac{1-\xi^S}{\xi^S}\right) \frac{dY^{S,new}}{Y^{S,new}}$.*

In short, if the increase in economic rents is purely the result of sectoral shifts – caused by shifts in sector-specific productivities – then the drop in steady-state consumption is bounded by a sector-weighted average of $\left(\frac{1-\xi^S}{\widehat{\alpha}\xi^S}\right) \frac{dY^{S,\text{new}}}{Y^{S,\text{new}}}$. As in the previous section, supposing that $1 - \xi^S$ is a small number for all sectors S, even large declines in $\frac{dY^{\text{S,new}}}{Y^{\text{S,new}}}$ have a muted effect on steady-state consumption.

As a practical matter, this means that it makes a material difference whether the increase in economy-wide rents is the result of a within-sector rise in markups ("intensive margin"), or

[29] The easiest way to see this is to consider the case where $\alpha^S = \alpha$ across all sectors. In that case $d\widehat{\sigma}$ and $d\widehat{\phi}$ are proportional to each other and their sign is negative whenever $\frac{1}{S} \sum_S \xi^S \left(d\omega^S\right) = E\xi^S \left(d\omega^S\right) = cov\left(\xi^S d\omega^S\right) \leq 0$, where E is an equal-weighted average across sectors and the second equality follows from $\frac{1}{S} \sum_S d\omega^S = 0$.

an across-sector shift in the economic importance of the relatively more rent-intensive sectors ("extensive margin"). In the latter case, the decline in young-firm output and employment is likely to have a small impact on long-term consumption, as opposed to the opposite extreme where the rise in rents is a within-sector phenomenon.

6 Conclusion

In the last decades, young firms have created fewer jobs than their predecessors at similar ages. In this paper we use Compustat and Pitchbook data to document that despite the underwhelming performance of young firms in terms of creating new jobs, their contribution to wealth and revenue creation was not similarly weak. This implies a rise in the ratio of average-to-marginal revenue product of labor for these firms (ARPL-to-MRPL ratio). NETS data corroborates this explanation: young firms purchasing similar establishments to older firms create fewer jobs in the purchased establishments starting around 2005.

Taking these observation as given, we consider their implications within an intentionally stylized model of dynamic firm heterogeneity. In particular, we consider an economy where entering firms start exhibiting a higher ARPL-to-MRPL ratio. We show that this fact alone can provide a unified explanation for a large number of facts that have been collectively referred to as a decline in business dynamism. This suggests that the decline in the labor share (and likely rise in the rent share) and the decline in business dynamism may be two facets of the same phenomenon.

A surprising implication of the model is that even when the model is asked to reproduce a very large percentage decline in the revenue produced by young firms, the decline of aggregate consumption and output are both quite muted.

Our theoretical results provide some analytical insights on the relationship between young-firm output and steady-state consumption. These mathematical equations show that it is not the decline in young-firm output –per se– that is a disconcerting sign for long term consumption. The elasticity of the decline in long-term consumption to new-firm-output is quite small, since most of the reaction of the output of young firms may just reflect transient factor adjustments. The decline in dynamism becomes more ominous if the decline in dynamism is driven by a within- (as opposed to across-) sector rise in economic rents.

In short, the paper makes three contributions: First, it documents a rise in the ARPL-to-MRPL ratio for young firms. Second, it shows that the rise in the ARPL-to-MRPL ratio of entering firms, the decline in the labor share, and the decline in business dynamism are three facets of the same phenomenon. Third, it provides analytical insights as to why dramatic changes in business dynamism can have muted effects on aggregate output and consumption.

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Figure 1: **Employment, Sales, and Market Value Contributions of IPO Cohorts**
Data on employment, sales, and market values of US public firms are taken from Compustat. Data on firm founding years are described in the text. We exclude from IPO cohorts all firms that were founded more than 10 years prior to their IPO. We measure the employment, sales, and market value contribution of an IPO cohort as a share of the total employment, sales, and market value of public firms in the prior year. We then measure the contribution of an IPO cohort bin as the sum of the contributions of the different IPO cohorts in the bin. Panel A presents the logarithm of the employment, sales, and market value contributions of each IPO cohort bin since 1985. Panel B presents the normalized (1985–1989 cohort = 0) logarithm of the employment, sales, and market value contributions. Panel C presents the logarithm of the ratio of the sales and market value contributions to the employment contributions. Panel D presents the normalized (1985–1989 cohort = 0) logarithm of the ratio of the sales and market value contributions to the employment contributions. See Section 2.1 for further details.

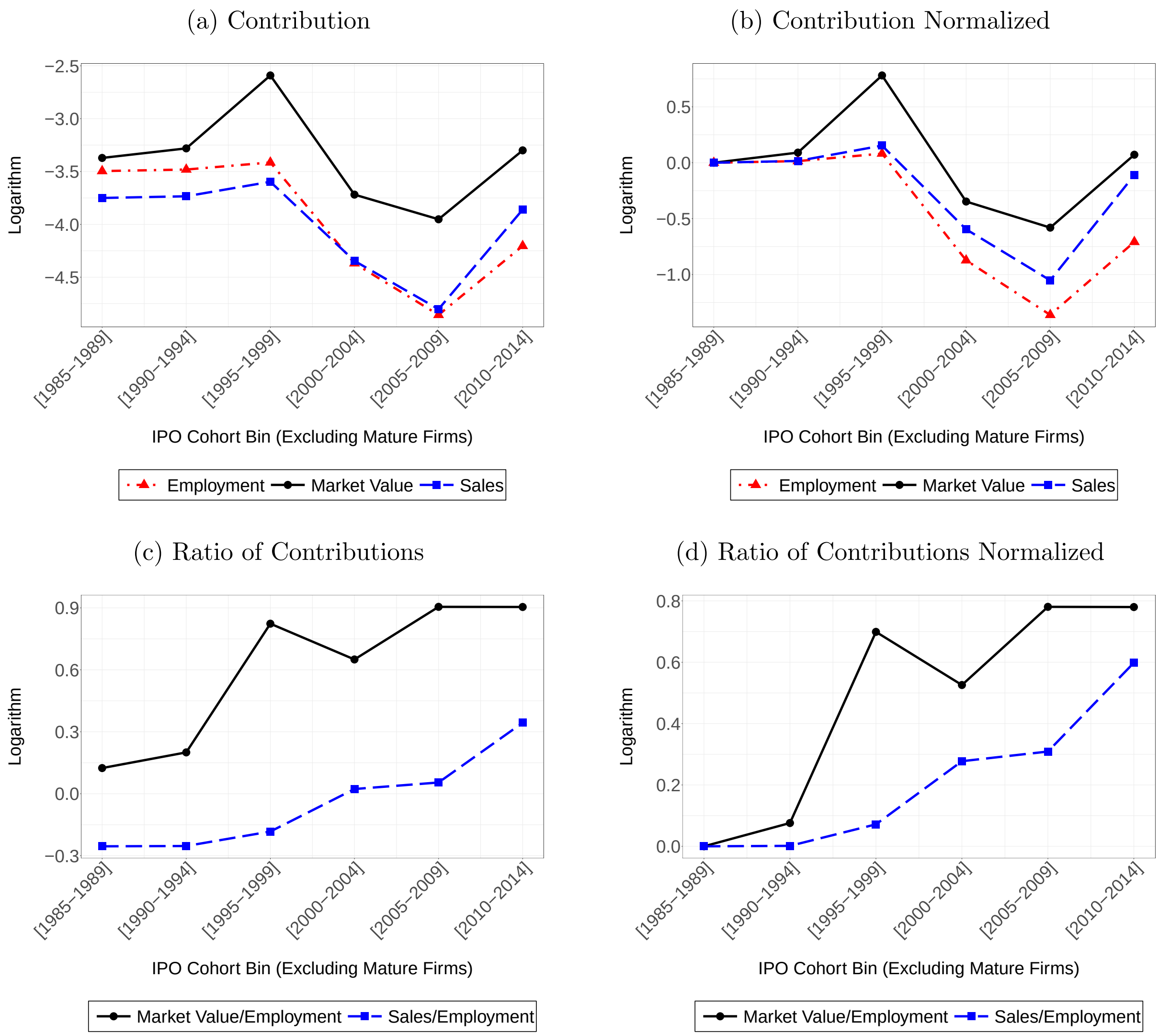
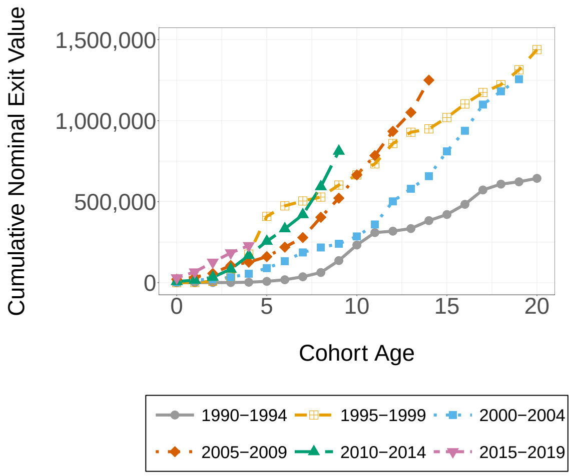


Figure 2: **Nominal Exit Values by Cohort**
The figure shows the cumulative nominal exit values of each cohort-bin by age. Exit values are measured in millions of U.S. Dollars. We bin cohorts into 5-year cohort-bins. Age is defined as year less youngest cohort in the cohort-bin. Exit value is post-money valuation. We include both IPO and M&A exits. Data on exit values are taken from PitchBook. See Section 2.2 for further details.

(a) All Cohorts



(b) Excluding 1995–1999 Cohort

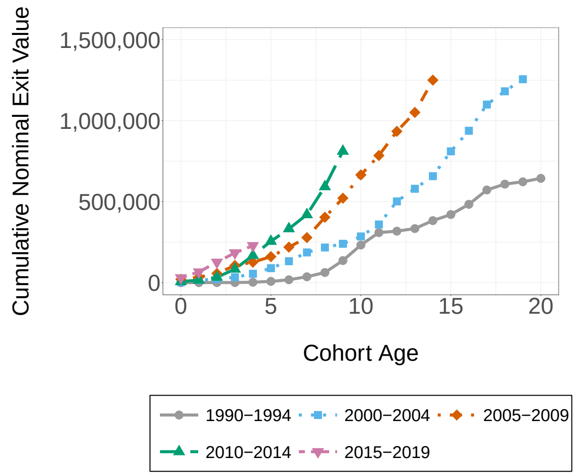
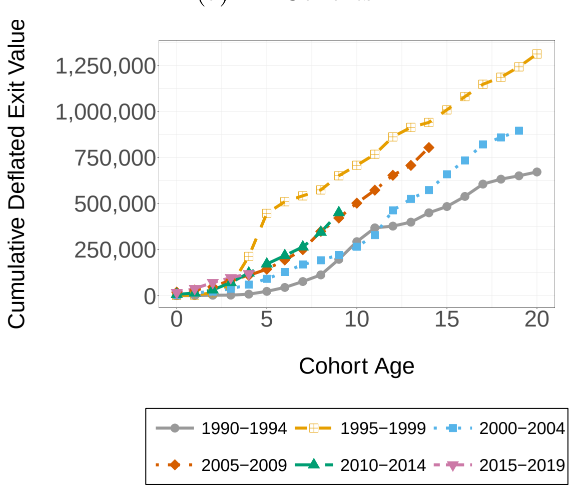


Figure 3: **Deflated Exit Values by Cohort**
The figure shows the cumulative deflated exit values of each cohort-bin by age. Nominal exit values are measured in millions of U.S. Dollars. Deflated exit values are nominal exit values deflated by the U.S. stock market capitalization, where the deflator is normalized to one at the start of 2000. We bin cohorts into 5-year cohort-bins. Age is defined as year less youngest cohort in the cohort-bin. Exit value is post-money valuation. We include both IPO and M&A exits. Data on exit values are taken from PitchBook. See Section 2.2 for further details.

(a) All Cohorts



(b) Excluding 1995–1999 Cohort

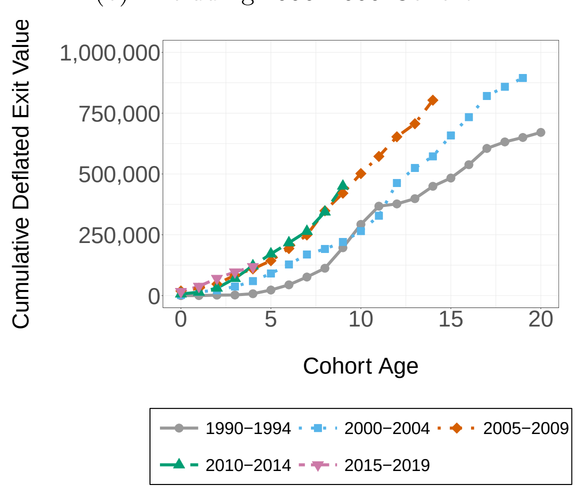


Figure 4: **Deflated Exit Values by Cohort and Exit Type**
The figure shows the cumulative deflated exit values of each cohort-bin by age. The figure presents deflated exit values for IPO exits, M&A exits and Total exits. Data on exit values are taken from PitchBook. See Section 2.2 and figure 3 for further details.

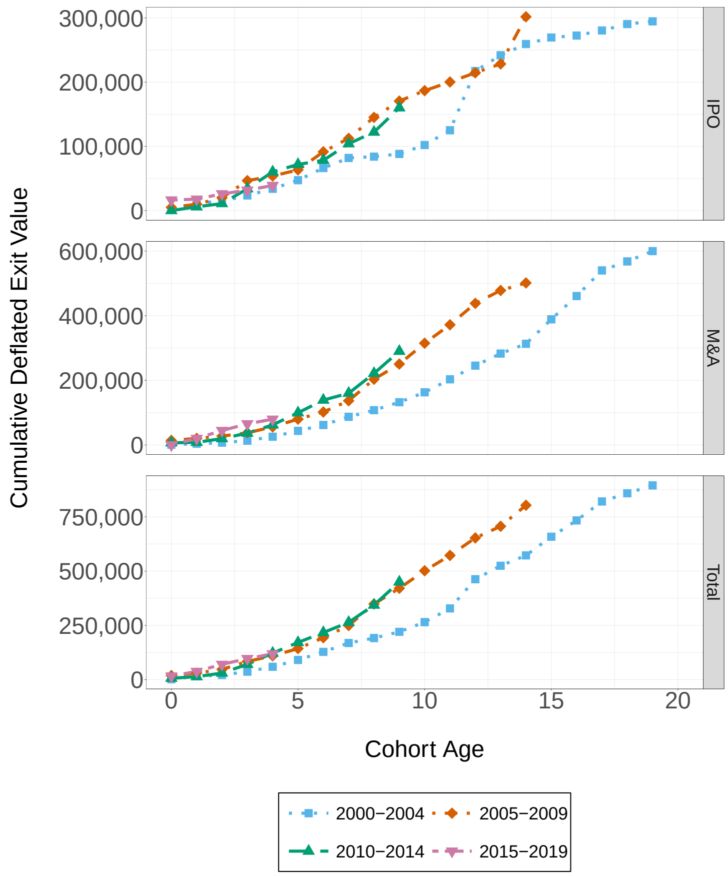


Figure 5: **Model-Implied Equilibrium Wage, Output, and Profit Share**. The vertical dashed line denotes the onset of the transition. Output and the wage are normalized to one at the onset of the transition. The "noise" in the figures is due to numerical approximation.

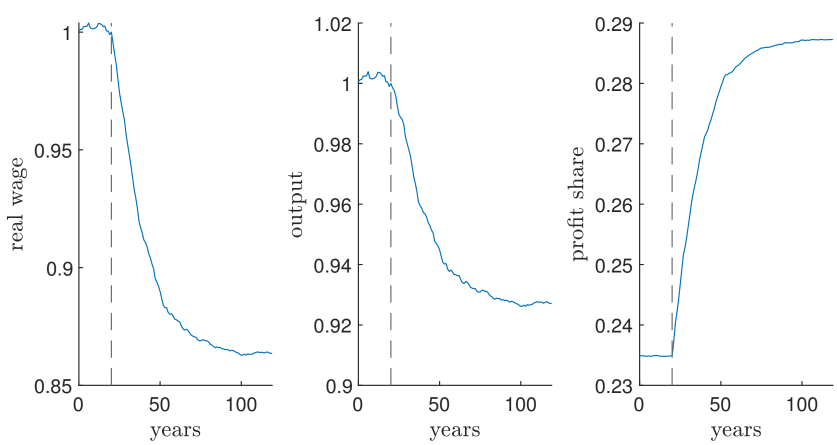


Figure 6: **Model-Implied Employment Share and Output of Entering Firms**. The left plot depicts the employment share of firms entering in year t. The right plot depicts output of these firms (normalized to one in year 20). The vertical dashed line denotes the onset of the transition. The “noise” in the figures is due to numerical approximation.

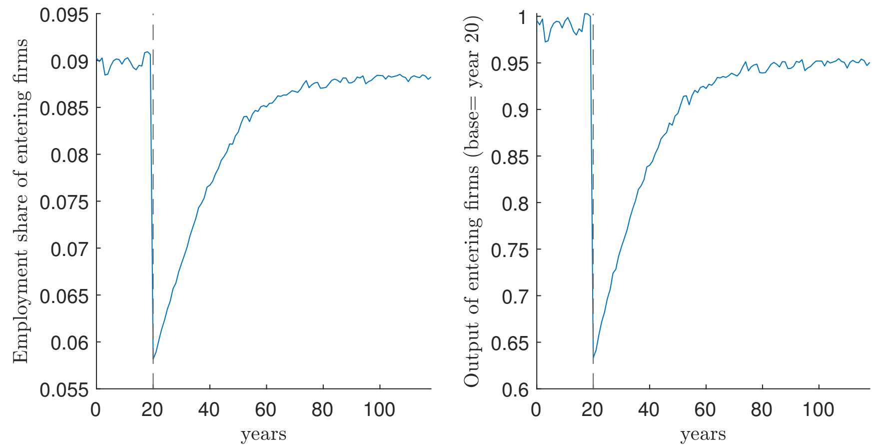


Figure 7: **Model-Implied Gross Job Creation and Destruction and TFP Dispersion.** The top left plot depicts gross job creation and destruction (see the text for definitions). The top right plot isolates job creation. The bottom left plot depicts the fraction of firms that exit every year. The bottom right plot depicts the cross-sectional standard deviation of the logarithm of total factor productivity. ($log(Z)$). The vertical dashed line denotes the onset of the transition. The “noise” in the figures is due to numerical approximation.

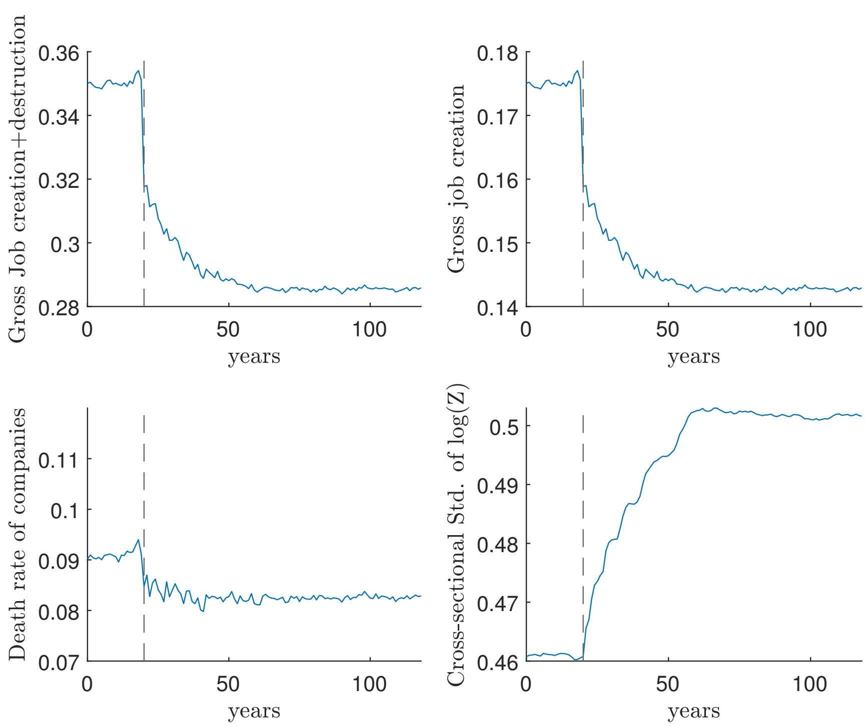


Table 1: **Sample of Switchers in NETS**
This table presents the number of NETS establishments that report a change in ownership over the years 1998–2014. Row 1 consists of all private payroll establishments. An establishment is private if it has a four digit SIC code less than 9000. Payroll establishments are those with at least 2 reported employees (including the owner) in the year prior to the acquisition. Row 2 removes establishments that exit the sample at the end of the year of the acquisition. Row 3 further removes transactions that we classify as a reorganization or spin-off. Row 4 further removes all observations where employment is imputed in the year before the acquisition or in the year after the acquisition. Row 5 (our analysis sample) further removes all establishments that report no change in employment from the year before the acquisition to the year after the acquisition. See Section 2.4 and Appendix G for further details.

Sample	N
Changes in ownership, all private payroll establishments	1,728,088
After removing exiting establishments	1,618,286
After further removing reorganizations and spin-offs	1,546,055
After further removing imputed employment	982,131
After further removing sticky employment	213,255

Table 2: **Employment Growth of Switchers in NETS** (First Specification)
This table reports results of regressions of changes in log-employment on an indicator equal to one if the acquiring firm is young. Data on establishment level employment and changes in ownership are taken from the National Establishment Time Series (NETS). The unit of observation is an establishment. The change in log-employment is measured from the year before the acquisition ($t-1$) to the year after the acquisition ($t+1$). The independent variables are an indicator Young Acquirer equal to one if the acquirer is less than 8 years old in the year of the acquisition and fixed effects that vary by column. The first column includes year fixed effects. The second column includes year×industry fixed effects where industry is defined as a 4-digit SIC industry. The third column further accounts for geographic variation by including year×industry×state fixed effects. All standard errors are clustered by year×industry×state. See Section 2.4 for further details.

	Dependent variable:		
	$\log L_{t+1} - \log L_{t-1}$		
	(1)	(2)	(3)
Young Acquirer	-0.039^{***}	-0.035^{***}	-0.027^{**}
	(0.010)	(0.010)	(0.012)
Fixed Effects	Year	Year×SIC4	Year×SIC4×State
S.E. Cluster	Year×SIC4×State	Year×SIC4×State	Year×SIC4×State
Sample Period	1998–2014	1998–2014	1998–2014
Observations	213,255	213,255	213,255
R^2	0.015	0.119	0.504
Note:			*p<0.1; **p<0.05; ***p<0.01

Table 3: **Employment Growth of Switchers in NETS** (Main Specification)
This table reports results of regressions of changes in log-employment on an indicator equal to one if the acquiring firm is young and an interaction term equal to one if the acquiring firm is young and the acquisition occurs after 2005. Data on establishment level employment and changes in ownership are taken from the National Establishment Time Series (NETS). The unit of observation is an establishment. The change in log-employment is measured from the year before the acquisition ($t-1$) to the year after the acquisition ($t+1$). The independent variables are an indicator Young Acquirer equal to one if the acquirer is less than 8 years old in the year of the acquisition, an indicator Young Acquirer×Post-2005 equal to one if the acquirer is less than 8 years old in the year of the acquisition and the acquisition occurs after 2005, and fixed effects that vary by column. The first column includes year fixed effects. The second column includes year×industry fixed effects where industry is defined as a 4-digit SIC industry. The third column further accounts for geographic variation by including year×industry×state fixed effects. All standard errors are clustered by year×industry×state. See Section 2.4 for further details.

	Dependent variable:		
	$\log L_{t+1} - \log L_{t-1}$		
	(1)	(2)	(3)
Young Acquirer	-0.018^{*}	-0.018^{*}	-0.017
	(0.009)	(0.010)	(0.013)
Young Acquirer × Post-2005	-0.134^{***}	-0.127^{***}	-0.120^{**}
	(0.035)	(0.039)	(0.055)
Fixed Effects	Year	Year×SIC4	Year×SIC4×State
S.E. Cluster	Year×SIC4×State	Year×SIC4×State	Year×SIC4×State
Sample Period	1998–2014	1998–2014	1998–2014
Observations	213,255	213,255	213,255
R^2	0.015	0.119	0.504
Note:			*p<0.1; **p<0.05; ***p<0.01

Table 5: **Parameter values for the model**

μ	-0.01	ξ	0.66	$\bar{l}/L$	0.115
σ	0.14	ξ^*	0.58	ρ	0.02
λ	0.08	$\log(Z^c)$	0.0	$m(z)$	$\propto e^{-2.5z}1_{(z\in[0,3])}$

Table 4: **Employment Growth of Switchers in NETS** (Target Age and Size Controls)
This table presents results of regressions of changes in log-employment on an indicator equal to one if the acquiring firm is young, an interaction term equal to one if the acquiring firm is young and the acquisition occurs after 2005, and additional controls for the age and size of the target establishment. The first column repeats the specification presented in Table 3. The second column includes establishment age bin dummies, using the age bins [1,3], [4,7], and 8+. The third column includes a control for log employment of the target establishment in the year before the acquisition ($t-1$). The fourth column includes both establishment age bin dummies and a control for log employment of the target establishment in the year before the acquisition ($t-1$). See Section 2.4 and Table 3 for further details.

	Dependent variable:			
	$\log L_{t+1} - \log L_{t-1}$			
	(1)	(2)	(3)	(4)
Young Acquirer	−0.017	−0.018	−0.029**	−0.023**
	(0.013)	(0.013)	(0.012)	(0.012)
Young Acquirer × Post-2005	−0.120**	−0.123**	−0.116**	−0.110**
	(0.055)	(0.055)	(0.050)	(0.050)
Fixed Effects	Year×SIC4×State	Year×SIC4×State	Year×SIC4×State	Year×SIC4×State
Establishment Age Bins	No	Yes	No	Yes
Control for $\log L_{t-1}$	No	No	Yes	Yes
S.E. Cluster	Year×SIC4×State	Year×SIC4×State	Year×SIC4×State	Year×SIC4×State
Sample Period	1998–2014	1998–2014	1998–2014	1998–2014
Observations	213,255	213,255	213,255	213,255
R^2	0.504	0.504	0.589	0.591
Note:				*p<0.1; **p<0.05; ***p<0.01

All Appendices are Intended for Online Publication

Appendix A Steady-State Analysis

A.1 Preliminary results: Capital and labor demands

To be able to analyze the steady state of this model in closed-form, for the remainder of this section and until section 3.4, we adopt the Dixit-Stiglitz specification $\mathcal{Y}(x) = x^{\xi}$ for all i, in which case $\varepsilon_i = \varepsilon = \frac{1}{1-\xi}$ and the price p_{it} is given by (11). Moreover, we assume that $\alpha_i = \alpha$.

Dividing (15) by (16) gives the labor-to-capital ratio

$$\frac{l_{it}}{k_{it}} = \frac{\alpha}{1-\alpha} \frac{r_t^K}{w_t}. \tag{A.1}$$

Combining (15) with (11) and (A.1) gives

$$k_{it}(Z_{it}) = \varphi_t Z_{it}^{\frac{\xi}{1-\xi}}, \tag{A.2}$$

where φ_t is defined as

$$\varphi_t \equiv Y_t \left[\frac{1}{\xi} \left(\frac{r_t^K}{1-\alpha} \right)^{1-\alpha\xi} \left(\frac{w_t}{\alpha} \right)^{\alpha\xi} \right]^{\frac{1}{\xi-1}}. \tag{A.3}$$

The quantity φ_t is not firm specific. It only depends on aggregate quantities. Using (A.1) implies the labor demand

$$l_{it}(Z_{it}) = \left(\varphi_t \frac{\alpha}{1-\alpha} \frac{r_t^K}{w_t} \right) Z_{it}^{\frac{\xi}{1-\xi}}. \tag{A.4}$$

A.2 Main Proposisitions

Since we have abstracted from trend-growth, in the steady state both C_t and K_t are constant, and the interest rate is equal to $r_t = \rho$, while the return on capital is $r^K = \rho + \delta$.

Furthermore, we define the steady state wage as $w_t = w$ and use the lowercase $z_{i,t} = \log(Z_{i,t})$ to refer to the logarithm of a firm's productivity. Ito's Lemma implies that $dz_{i,t} = \left(\mu - \frac{\sigma^2}{2}\right) dt + \sigma dW_{i,t}$.

The next proposition provides the cut-off level of log-productivity z^* below which a firm decides to terminate.

Proposition 3 *Assume that* $\mathcal{Y}(x) = x^{\xi}$. *Let* w *denote the steady-stage wage of the economy. Define* ω_1 *as the negative and* ω_2 *as the positive root of the quadratic equation* $\omega^2 \frac{\sigma^2}{2} + \omega\left(\mu - \frac{\sigma^2}{2}\right) - (\rho + \lambda) = 0$, *which are given by* $\omega_{1,2} = \frac{-\left(\mu - \frac{\sigma^2}{2}\right) \pm \sqrt{\left(\mu - \frac{\sigma^2}{2}\right)^2 + 2\sigma^2(\rho+\lambda)}}{\sigma^2}$. *Assume that* $\omega_2 > \frac{\xi}{1-\xi}$ *and define* $\Xi \equiv \frac{1-\xi}{\xi} \frac{1}{1-\alpha} (\rho + \delta) \left(\frac{\varphi}{w}\right) > 0$, *where* φ *is the steady-state value of* φ_t . *The value-maximizing termination decision for a firm is to stop operating the first time that* z_{it} *becomes smaller than*

$$z^* \equiv \frac{1-\xi}{\xi} \log\left(\frac{\omega_2 - \frac{\xi}{1-\xi}}{\omega_2} \frac{\bar{l}}{\Xi}\right). \tag{A.5}$$

Equation (A.5) provides the optimal termination threshold up to the constant Ξ, which in turn depends on the equilibrium quantity $\frac{\varphi}{w}$. To determine $\frac{\varphi}{w}$, we next examine the steady state distribution of firm productivity.

Proposition 4 *Define* η_1 *as the negative and* η_2 *as the positive root of the quadratic equation* $\frac{\sigma^2}{2}\eta^2 - \left(\mu - \frac{\sigma^2}{2}\right)\eta - \lambda = 0$, *which are given by* $\eta_{1,2} = \frac{\left(\mu - \frac{\sigma^2}{2}\right) \pm \sqrt{\left(\mu - \frac{\sigma^2}{2}\right)^2 + 2\sigma^2\lambda}}{\sigma^2}$. *The steady-state distribution of log-productivity is given by*

$$g(z; z^*) = \frac{2}{\sigma^2} \frac{\phi}{\eta_2 - \eta_1} \left[\begin{array}{c} \left(\int_{z^*}^{z} e^{\eta_1(z-s)} m(s)\, ds + \int_{z}^{\infty} e^{\eta_2(z-s)} m(s)\, ds\right) \\ -\left(\int_{z^*}^{\infty} e^{\eta_2(z^*-s)} m(s)\, ds\right) e^{\eta_1(z-z^*)} \end{array} \right], \tag{A.6}$$

where $m(\cdot)$ *is the (log) productivity distribution of the incoming firm cohort.*

To complete the construction of equilibrium, evaluate the labor market clearing condition (17) at the steady state to arrive at

$$\frac{\alpha}{1-\alpha} (\rho + \delta) \left(\frac{\varphi}{w}\right) = \frac{L - \bar{l} \int_{z^*}^{\infty} g(z; z^*)\, dz}{\int_{z^*}^{\infty} g(z; z^*)\, e^{\frac{\xi}{1-\xi} z} dz}. \tag{A.7}$$

To ensure that the denominator of (A.7) is integrable, we assume that

$$-\eta_1 > \frac{\xi}{1-\xi}. \tag{A.8}$$

Remark 1 *Assumption (A.8) implies the condition* $\omega_2 > \frac{\xi}{1-\xi}$ *of Proposition 3, since* $\omega_2 > -\eta_1$.

By equations (A.5) the cutoff z^* is a linear function of $\log\left(\frac{\varphi}{w}\right)$, so equation (A.7) is a non-linear equation in $\frac{\varphi}{w}$. As the next Lemma asserts, it has a unique, positive solution $\frac{\varphi}{w}$.

Lemma 3 *There is a unique* $\frac{\varphi}{w} > 0$, *for which equation (A.7) holds.*

With this unique $\frac{\varphi}{w}$, the cut-off value z^* is uniquely determined and the equilibrium wage and output can be computed as[30]

$$w = \left(\alpha\xi\left(\frac{\alpha}{1-\alpha}(\rho+\delta)\right)^{\alpha-1}\left(\int_{z^*}^{\infty} g(z;z^*)\, e^{\frac{\xi}{1-\xi}z}dz\right)^{\frac{1-\xi}{\xi}}\right)^{\frac{1}{\alpha}}, \tag{A.9}$$

and

$$Y = \frac{w}{\alpha\xi} \times \left(L - \bar{l}\int_{z^*}^{\infty} g(z;z^*)\, dz\right). \tag{A.10}$$

Appendix B Proofs

Proof of Lemma 1. Letting H_t be the stochastic discount factor in this economy, we have

$$\begin{aligned} \frac{P_{i,t,s}}{\pi_{i,t,s}} &= E_t\left\{\sum_{u\geq t}\frac{H_u}{H_t}\frac{\pi_{i,u,s}}{\pi_{i,t,s}}\right\} = \\ &= \sum_{u\geq t} E_t\left\{\frac{H_u\Pi_u}{H_t\Pi_t}\exp\left[\lambda(u-t)\right]\right\}\times E_t\left\{e^{\sum_{u\geq t}\eta_{i,u}}\right\} \equiv A_t. \end{aligned} \tag{B.1}$$

Using (B.1) we obtain

$$\begin{aligned} \frac{\sum_{i,t} P^i_{t,t}}{\sum_{i,s\leq t} P^i_{t,s}} &= \frac{\sum_{i,t}\pi^i_{t,t}}{\sum_{i,s\leq t}\pi^i_{t,s}} = \frac{\sum_{i,t} p^i_{t,t}y^i_{t,t} - w_t l^i_{t,t}}{\sum_{i,s\leq t}\left(p^i_{t,s}y^i_{t,s} - w_t l^i_{t,s}\right)} \\ &= \frac{\sum_{i,t} l^i_{t,t}\left(\frac{p^i_{t,t}y^i_{t,t}}{l^i_{t,t}} - w_t\right)}{\sum_{i,s\leq t} l^i_{t,s}\left(\frac{p^i_{t,s}y^i_{t,s}}{l^i_{t,s}} - w_t\right)} = \frac{w_t}{w_t}\times\frac{\sum_{i,t} l^i_{t,t}\left(\delta^i_{t,t}-1\right)}{\sum_{i,s\leq t} l^i_{t,s}\left(\delta^i_{t,s}-1\right)} \\ &= \frac{\sum_{i,t} l^i_{t,t}}{\sum_{i,s\leq t} l^i_{t,s}}\times\frac{\sum_{i,t}\left(\frac{l^i_{t,t}}{\sum_{i,t} l^i_{t,t}}\right)\times\left(\delta^i_{t,t}-1\right)}{\sum_{i,s\leq t}\left(\frac{l^i_{t,s}}{\sum_{i,s\leq t} l^i_{t,s}}\right)\left(\delta^i_{t,s}-1\right)} \\ &= \frac{\sum_{i,t} l^i_{t,t}}{\sum_{i,s\leq t} l^i_{t,s}}\times\frac{\sum_{i,t}\omega^i_{t,t}\left(\delta^i_{t,t}-1\right)}{\sum_{i,s\leq t}\omega^i_{t,s}\left(\delta^i_{t,s}-1\right)}. \end{aligned}$$

[30]Equations (8), (A.1) and (A.2) imply that $Y_t = w_t\frac{\varphi_t}{w_t}\left(\frac{\alpha}{1-\alpha}\frac{r^K_t}{w_t}\right)^{\alpha}\left(\int_{z^*}^{\infty} g(z;z^*)\, e^{\frac{\xi}{1-\xi}z}dz\right)^{\frac{1}{\xi}}$. Using (A.7) to substitute for $\frac{\varphi_t}{w_t}$ and using the fact that $w_t\left(L-\bar{l}\int_{z^*}^{\infty} g(z;z^*)\, dz\right) = \xi\alpha Y_t$ leads to (A.9) and (A.10).

Taking logarithms on both sides and then first differences leads to (2). ■

Proof of Proposition 3. Multiplying (15) by k_{it}, (16) by l_{it} and adding the resulting equations implies that

$$\pi\left(Z_{i,t}\right) \equiv p_{it}y_{it} - w_t\left(l_{i,t} + \bar{l}\right) - r_t^K k_{it} = (1-\xi)\, p_{i,t}y_{i,t} - w_t\bar{l}.$$

In turn, (16) implies

$$\pi\left(Z_{i,t}\right) = (1-\xi)\, p_{u,t}y_{u,t} - w_t\bar{l} = \frac{1-\xi}{\xi}\frac{w_t l_{it}}{\alpha} - w_t\bar{l}.$$

In steady state $\varphi_t = \varphi,\quad w_t = w$ and $r_t^K = r^K = \rho + \delta$. Using these constants, equation (A.4) and the definition of Ξ gives

$$\pi\left(z_{i,t}\right) = \frac{1-\xi}{\xi}\frac{w l_{it}}{\alpha} - w\bar{l} = w\left(\Xi e^{\frac{\xi}{1-\xi}z} - \bar{l}.\right) \tag{B.2}$$

Before its termination, the value function of the firm solves the following differential equation:

$$V_{zz}\frac{\sigma^2}{2} + V_z\left(\mu - \frac{\sigma^2}{2}\right) - (r+\lambda)\, V + \pi\left(z_{i,t}\right) = 0. \tag{B.3}$$

A particular solution V^P of this differential equation is

$$V^P(z) = \frac{2}{\sigma^2}\frac{1}{\omega_2 - \omega_1}\left(\int_{z^*}^{z} e^{\omega_1(z-s)}\pi(s)\, ds + \int_{z}^{\infty} e^{\omega_2(z-s)}\pi(s)\, ds\right), \tag{B.4}$$

which can be verified by substituting (B.4) into (B.3). As a result, the general solution of (B.3) is

$$V(z) = D_1 e^{\omega_1 z} + D_2 e^{\omega_2 z} + V^P(z). \tag{B.5}$$

By standard arguments (value matching, smooth pasting, no bubble condition) we have that

$$V(z^*) = 0 \tag{B.6}$$

$$V_z(z^*) = 0 \tag{B.7}$$

$$\lim_{z\to\infty} V(z) = V^P(z) \tag{B.8}$$

Condition (B.8) implies that $D_2 = 0$ and condition (B.6) implies that

$$D_1 = -e^{-\omega_1 z^*} \frac{2}{\sigma^2} \frac{1}{\omega_2 - \omega_1} \left(\int_{z^*}^{\infty} e^{\omega_2 (z^* - s)} \pi(s)\, ds \right). \tag{B.9}$$

Differentiating (B.5) with respect to z, evaluating the resulting expression at z^* and using (B.7) gives

$$\omega_1 D_1 e^{\omega_1 z^*} + \frac{2}{\sigma^2} \frac{\omega_2}{\omega_2 - \omega_1} \int_{z^*}^{\infty} e^{\omega_2 (z-s)} \pi(s)\, ds = 0. \tag{B.10}$$

Using (B.9) inside (B.10) and re-arranging implies that

$$\int_{z^*}^{\infty} e^{-\omega_2 s} \pi(s)\, ds = 0. \tag{B.11}$$

Substituting (B.2) into (B.11) and integrating leads after some simplifications to (A.5).

■

Proof of Proposition 4. Letting $g(z)$ denote the mass of firms with log-productivity z in the steady state, the forward Kolmogorov equation implies that the density $g(z)$ obeys the differential equation

$$\frac{\sigma^2}{2} g_{zz} - \left(\mu - \frac{\sigma^2}{2} \right) g_z - \lambda g + \phi m(z) = 0 \tag{B.12}$$

subject to the boundary condition $g(z^*) = 0$ and $\lim_{z \to \infty} g(z) = 0$. Similar to the proof of Proposition 3, a particular solution of (B.12) is

$$g^P(z) = \frac{2}{\sigma^2} \frac{\phi}{\eta_2 - \eta_1} \left(\int_{z^*}^{z} e^{\eta_1 (z-s)} m(s)\, ds + \int_{z}^{\infty} e^{\eta_2 (z-s)} m(s)\, ds \right). \tag{B.13}$$

The general solution is therefore

$$g(z) = K_1 e^{\eta_1 z} + K_2 e^{\eta_2 z} + g^P(z) \tag{B.14}$$

The two boundary conditions $g(z^*) = 0$ and $\lim_{z \to \infty} g(z) = 0$ imply that $K_2 = 0$ and

$$K_1 = \frac{-g^P(z^*)}{e^{\eta_1 z^*}}. \tag{B.15}$$

Substituting (B.15) and $K_2 = 0$ into (B.14) leads to

$$g(z; z^*) = g^P(z) - g^P(z^*)\, e^{\eta_1 (z - z^*)}, \tag{B.16}$$

which leads to (A.6). ■

Proof of Lemma 3. The left hand side of (A.7) is an increasing function of $\frac{\varphi}{w}$. The right hand side is decreasing in $\frac{\varphi}{w}$, which can be shown by differentiating the right hand side of (A.7) with respect to z^* and then using the fact that z^* is equal to $\log\left(\frac{\varphi}{w}\right)$ plus an additive constant, by (A.5). Specifically, using the fact that $g(z^*) = 0$, we have that

$$\frac{d}{dz^*}\int_{z^*}^{\infty} g(z; z^*)\, dz = \int_{z^*}^{\infty} \frac{\partial g(z; z^*)}{\partial z^*} dz =$$

$$= -\left(\int_{z^*}^{\infty} e^{\eta_2 (z^* - s)} m(s)\, ds\right)\left(\int_{z^*}^{\infty} e^{\eta_1 (z - z^*)} dx\right)\frac{2\phi}{\sigma^2} < 0 \tag{B.17}$$

and

$$\frac{d}{dz^*}\int_{z^*}^{\infty} \exp\left(\frac{\xi}{1-\xi} z\right) g(z; z^*)\, dz = \int_{z^*}^{\infty} \exp\left(\frac{\xi}{1-\xi} z\right)\frac{\partial g(z; z^*)}{\partial z^*} dz =$$

$$= -\frac{2\phi}{\sigma^2}\left(\int_{z^*}^{\infty} e^{\eta_2 (z^* - s)} m(s)\, ds\right)\int_{z^*}^{\infty} e^{\eta_1 (z - z^*) + \frac{\xi}{1-\xi} z} dz < 0 \tag{B.18}$$

Using (B.17) and (B.18) implies that the right hand side of (A.7) is increasing in z^*. Since z^* is decreasing in $\log\left(\frac{\varphi}{w}\right)$, the right-hand side of (A.7) is decreasing in $\frac{\varphi}{w}$. By inspection, the right hand side becomes strictly positive as $\frac{\varphi}{w} \to 0$. Combining all the above facts implies that the difference between the right and the left hand side of (A.7) is decreasing in $\frac{\varphi}{w}$, becomes positive as $\frac{\varphi}{w} \to 0$, and tends to negative infinity as $\frac{\varphi}{w} \to \infty$. By continuity, we conclude that there is a unique positive $\frac{\varphi}{w}$, for which equation (A.7) holds. ■

Proof of Proposition 1. To prove proposition 1, we start by proving the following Lemma, which shows the correspondence of the decentralized equilibrium with an (appropriately distorted) planning problem.

Lemma 4 *Assume that $\rho = 0$ and consider the optimization problem of maximizing H, where*

$$H \equiv \max_{l(z), k(z), x^*} Y - \frac{\delta}{\xi} K \tag{B.19}$$

subject to the constraint

$$\int_{x^*}^{\infty} g(z,x^*)\left(\bar{l}+l(z)\right)dz = L, \tag{B.20}$$

where

$$Y \equiv \left(\int_{x^*}^{\infty} g(z,x^*)\, y_i(z)^{\xi} dz\right)^{\frac{1}{\xi}}, \tag{B.21}$$

$y_i(z) = \exp(z)\, k(z)^{1-\alpha}\, l(z)^{\alpha}$ *and*

$$K \equiv \int_{x^*}^{\infty} g(z,x^*)\, k(z)\, dz$$

The optimization problem (B.19) has the same solution as the market equilibrium, namely $x^* = z^*, k^*(z) = k^{market}(z)$ *and* $l^*(z) = l^{market}(z)$*, where* $k^{market}(z)$ *and* $l^{market}(z)$ *are the capital and labor values in the decentralized equilibrium, and stars indicate the optimal solution to the planning problem. Accordingly,* $Y^* = Y^{market}$ *and* $C^* = C^{market}$*. The maximized objective* H *is related to* Y^* *and* C^* *as follows*

$$H = \alpha Y^* \text{ and } C^* = \frac{1-\xi(1-\alpha)}{\alpha} H. \tag{B.22}$$

Proof of Lemma 4. Maximizing (B.19) over $k(z)$ gives

$$(1-\alpha)\, Y^{1-\xi} y(z)^{\xi-1} \exp(z) \left(\frac{k(z)}{l(z)}\right)^{-\alpha} = \frac{\delta}{\xi}, \tag{B.23}$$

while maximizing over $l(z)$ gives

$$\alpha Y^{1-\xi} y(z)^{\xi-1} \exp(z) \left(\frac{k(z)}{l(z)}\right)^{1-\alpha} = \zeta, \tag{B.24}$$

where ζ is a Lagrange multiplier associated with the constraint (B.20). Maximizing over x^* leads to[31]

$$\int_{x^*}^{\infty} \frac{\partial}{\partial x^*} g(z,x^*) \left[\frac{1}{\xi} Y^{1-\xi} (y(z))^{\xi} dz - \frac{\delta}{\xi} k(z) - \zeta\left(l(z)+\bar{l}\right)\right] dz = 0 \tag{B.25}$$

Using (A.6) and differentiating implies

$$\frac{\partial}{\partial x^*} g(z;x^*) = -\left(\int_{x^*}^{\infty} e^{\eta_2(x^*-s)} m(s)\, ds\right) \frac{2\phi}{\sigma^2} \left[e^{\eta_1(z-x^*)}\right]. \tag{B.26}$$

[31] Note that $g(x^*, x^*) = 0$.

Using (B.26) inside (B.25) and noting that $g(x^*; x^*) = 0$, leads after some simplifications to

$$\int_{x^*}^{\infty} e^{\eta_1 z} \left[\frac{1}{\xi} Y^{1-\xi} y(z)^{\xi} \, dz - \frac{\delta}{\xi} k(z) - \zeta \left(l(z) + \bar{l} \right) \right] dz = 0. \tag{B.27}$$

We next argue that $Y = Y^{\text{market}}, l(z) = l^{\text{market}}(z), k(z) = k^{\text{market}}(z)$, $x^* = z^*$ and $\zeta = \frac{w}{\xi}$ satisfies (B.23), (B.24) and (B.27). Indeed, using equation (11), equation (B.23) coincides with (15) when $\rho = 0$ (since $r_K = \delta$). Similarly, (B.24) coincides with (16) since $\zeta = \frac{w}{\xi}$. The expression inside square brackets in (B.27) is equal to the profits, $\frac{1}{\xi}\pi(z)$, in the decentralized market. When $\rho = 0$, we have that $\eta_1 = -\omega_2$ and hence (B.27) coincides with $\int_{x^*}^{\infty} e^{\eta_1 z} \pi(z) \, dz = 0$, i.e., the optimality condition for z^* (equation (B.11)). Moreover, since the market allocation is feasible (it clears the labor market), $\zeta = \frac{w}{\xi}$ is the Lagrange multiplier associated with the constraint (B.20).

Multiplying both sides of (B.23) with $k(z)$, and aggregating across firms gives

$$\frac{\delta}{\xi} K = (1 - \alpha) Y^*, \tag{B.28}$$

which implies $H = \alpha Y^*$ and $C = Y^* - \delta K = Y^* - \xi \frac{\delta}{\xi} K = [1 - \xi(1 - \alpha)] Y^* = \frac{1 - \xi(1-\alpha)}{\alpha} H.$ ■

Having established the equivalence between the market equilibrium and the planning problem (B.19), we use the simpler notation $Y = Y^*, C = C^*$, etc. to refer to the output, consumption, etc. that arise in market equilibrium (equivalently, at an optimum of (B.19)).

We start by determining the partial derivatives of the objective H, which captures the steady-state value of $Y - \frac{\delta}{\xi} K$ (equation (B.19)) with respect to various parameters.. Letting $z^c = \log(Z^c)$, $y(z) = \exp(z^c + z) k^{1-\alpha}(z) l(z)$, and applying the envelope theorem (around $z^c = 0$) to (B.19) gives

$$\frac{\partial H}{\partial z^c} = Y.$$

Similarly, using the definition

$$\widetilde{g}(z) \equiv \frac{g(z, x^*) e^{\xi \log y(z)}}{\int_{x^*}^{\infty} g(z, x^*) e^{\xi \log y(z)} dz},$$

and equation (B.28) yields

$$\frac{\partial H}{\partial \xi} = \frac{1}{\xi} Y \left[\int_{x^*}^{\infty} \widetilde{g}(z) \log y(z) \, dz - \log(Y) + (1-\alpha) \right]. \tag{B.29}$$

Similarly,

$$\frac{\partial H}{\partial \alpha} = Y \log \left(\frac{l(z)}{k(z)} \right). \tag{B.30}$$

With these partial derivatives and using (B.22) and $H = \alpha Y$ we next proceed to compute dC. To that end we compute $\frac{\partial \left(\frac{1-\xi(1-\alpha)}{\alpha} \right)}{\partial \xi} = -\frac{1-\alpha}{\alpha}$, and $\frac{\partial \left(\frac{1-\xi(1-\alpha)}{\alpha} \right)}{\partial \alpha} = -\frac{1-\xi}{\alpha^2}$ and hence

$$\begin{aligned} \frac{dC}{C} &= \left[-\frac{1-\xi}{\alpha} \frac{1}{1-\xi(1-\alpha)} + \frac{1}{\alpha} \log \left(\frac{l(z)}{k(z)} \right) \right] d\alpha + \frac{1}{\alpha} dz^c \\ &\quad + \left\{ -\frac{1-\alpha}{1-\xi(1-\alpha)} + \frac{1}{\alpha\xi} \left[\int_{x^*}^{\infty} \widetilde{g}(z) \log y(z) \, dz - \log(Y) + (1-\alpha) \right] \right\} d\xi. \end{aligned} \tag{B.31}$$

We next show how to relate the steady-state change in $\frac{dC}{C}$ to the change in the revenue produced by the incoming cohort of firms at the beginning of the transition to the new steady state. We start by assuming that arriving firms have a coefficient $\xi = \xi^{\text{old}} + d\xi$, where ξ^{old} pertains to the old firms and $d\xi$ is a marginal change. Similarly, we assume that the new firms' productivity is multiplied by e^{dz^c} for a marginal change in dz^c, and their labor share is $\alpha = \alpha^{\text{old}} + d\alpha$.

The production function from the onset of the transition onward is given by

$$1 = \int_{i \in I_{\text{old}}} \left(\frac{y_i^{\text{old}}}{Y} \right)^{\xi^{\text{old}}} di + \int_{i \in I_{\text{new}}} \left(\frac{y_i^{\text{new}}}{Y} \right)^{\xi} di.$$

Using the implicit function theorem and (10) leads to

$$p_{it}^{\text{old}} = \frac{\xi^{\text{old}} \left(\frac{y_{it}^{\text{old}}}{Y_t} \right)^{\xi^{\text{old}}-1}}{\int_{i \in I_{\text{old}}} \xi^{\text{old}} \left(\frac{y_{it}}{Y_t} \right)^{\xi_{\text{old}}} di + \int_{i \in I_{\text{new}}} \xi \left(\frac{y_{it}}{Y_t} \right)^{\xi} di}, \tag{B.32}$$

$$p_{it}^{\text{new}} = \frac{\xi \left(\frac{y_{it}^{\text{new}}}{Y_t} \right)^{\xi-1}}{\int_{i \in I_{\text{old}}} \xi^{\text{old}} \left(\frac{y_{it}}{Y_t} \right)^{\xi_{\text{old}}} di + \int_{i \in I_{\text{new}}} \xi \left(\frac{y_{it}}{Y_t} \right)^{\xi} di}. \tag{B.33}$$

Note that at the onset of the transition the second integral in the denominator of (B.32) and (B.33) is zero, since the total measure of arriving firms with coefficient $\xi = \xi^{\text{old}} + d\xi$

are of zero measure. In the long run, the first integral becomes measure zero, and (B.32) becomes identical to (11).

While of measure zero at the beginning of the transition, the percentage change in the revenue of incoming firms normalized by the measure of these firms is well defined and given by

$$Y^{\text{new}} \equiv \int_{z^*}^{\infty} m(z)\, p^{\text{new}}(z)\, y^{\text{new}}(z)\, dz,$$

where p_{it}^{new} is shorthand notation for $p_{it}^{\text{new}}\left(z; z+z^c, \xi^{\text{old}}+d\xi, \alpha^{\text{old}}+d\alpha\right)$ and similarly for $y^{\text{new}}(z)$. Multiplying both sides of (B.33) by $y^{\text{new}}(z)$, aggregating across entering firms (and noting that at the onset of the transition the measure of firms employing the new technologies is of measure zero) gives

$$\begin{aligned} d\log Y^{\text{new}} \quad = \quad & \left(\frac{1}{\xi} - \log Y + \left(\int_{z^*}^{\infty} \widetilde{m}(z) \log y^{\text{new}}(z)\, dz\right)\right) d\xi \\ & + \xi\left(dz^c + \log\left(\frac{l^{\text{new}}}{k^{\text{new}}}\right) d\alpha\right) \\ & + \xi \int_{z^*}^{\infty} \widetilde{m}(z)\left(\alpha d\log l^{\text{new}}(z) + (1-\alpha)\, d\log k^{\text{new}}(z)\right) dz \end{aligned} \tag{B.34}$$

where $\widetilde{m}(z)$ is defined as

$$\widetilde{m}(z) = \frac{m(z)\, e^{\xi \log y^{\text{new}}}}{\int_{z^*}^{\infty} m(z)\, e^{\xi \log y^{\text{new}}} dz} = \frac{m(z)\, p^{\text{new}}(z)\, y^{\text{new}}(z)}{\int_{z^*}^{\infty} m(z)\, p^{\text{new}}(z)\, y^{\text{new}}(z)\, dz}, \tag{B.35}$$

where the second equality follows from equation (B.33).

The first order conditions for labor and capital (16) and (15) give $w_t l^{\text{new}}(z) = \alpha\xi p^{\text{new}}(z)\, y^{\text{new}}(z)$ and $r_t^K k^{\text{new}}(z) = (1-\alpha)\,\xi p^{\text{new}}(z)\, y^{\text{new}}(z)$ and therefore

$$d\log l^{\text{new}}(z) = \frac{d\alpha}{\alpha} + \frac{d\xi}{\xi} + \frac{d\left[p^{\text{new}}(z)\, y^{\text{new}}(z)\right]}{p^{\text{new}}(z)\, y^{\text{new}}(z)}, \tag{B.36}$$

and

$$d\log k^{\text{new}}(z) = -\frac{d\alpha}{(1-\alpha)} + \frac{d\xi}{\xi} + \frac{d\left[p^{\text{new}}(z)\, y^{\text{new}}(z)\right]}{p^{\text{new}}(z)\, y^{\text{new}}(z)}. \tag{B.37}$$

Combining (B.36) and (B.37) gives

$$\alpha d\log l^{\text{new}} + (1-\alpha)\, d\log k^{\text{new}} = \frac{d\xi}{\xi} + \frac{d\left[p^{\text{new}}(z)\, y^{\text{new}}(z)\right]}{p^{\text{new}}(z)\, y^{\text{new}}(z)}. \tag{B.38}$$

Combining (B.34) with (B.38) gives

$$\frac{dY^{\text{new}}}{Y^{\text{new}}} = \left(1+\frac{1}{\xi}-\log Y+\left(\int_{z^*}^{\infty}\widetilde{m}(z)\log y^{\text{new}}(z)\,dz\right)\right)d\xi + \xi\left(dz^c+\log\left(\frac{l^{\text{new}}}{k^{\text{new}}}\right)d\alpha\right)+\xi\frac{dY^{\text{new}}}{Y^{\text{new}}}, \tag{B.39}$$

or after re-arranging

$$\left(\frac{1-\xi}{\xi}\right)\frac{dY^{\text{new}}}{Y^{\text{new}}} = \left(1+\frac{1}{\xi}-\log Y+\int_{z^*}^{\infty}\widetilde{m}(z)\log y^{\text{new}}dz\right)\frac{d\xi}{\xi}+dz^c+\log\left(\frac{l^{\text{new}}}{k^{\text{new}}}\right)d\alpha. \tag{B.40}$$

Solving for dz^c in (B.40), substituting the result inside (B.31) and evaluating around $z^c = 0, \xi = \xi^*, \alpha = \alpha^*$, and noting that the labor-capital ratio is equalized across all firms when $z^c = 0, \xi = \xi^*, \alpha = \alpha^*$ leads to

$$\frac{dC}{C} = \frac{1}{\alpha}\frac{1-\xi}{\xi}\frac{dY^{\text{new}}}{Y^{\text{new}}}-\left(\frac{1-\xi}{1-\xi(1-\alpha)}\right)\frac{d\alpha}{\alpha} \tag{B.41}$$

$$+\frac{1}{\alpha}\left(\int_{x^*}^{\infty}\widetilde{g}(z)\log y(z)\,dz-\int_{x^*}^{\infty}\widetilde{m}(z)\log y(z)\,dz-D\right)\frac{d\xi}{\xi} \tag{B.42}$$

where $D \equiv \left(1+\frac{1}{\xi}\right)+\alpha\frac{\xi(1-\alpha)}{1-\xi(1-\alpha)}-(1-\alpha)$. ■

Proof of Proposition 2. To start, we prove the next Lemma, which extends Lemma 4 to a multi-sector setup.

Lemma 5 *Assume that $\rho = 0$ and consider the optimization problem of maximizing H, where*

$$\widehat{H} \equiv \max_{l^S(z),k^S(z),x^{S,*}} Y-\delta\sum_S\frac{1}{\xi^S}\int_{x^{S,*}}^{\infty}g^S\left(z,x^{S,*}\right)k^S(z)\,dz, \tag{B.43}$$

subject to the constraints

$$\int_{x^*}^{\infty}g\left(z^S,x^{S,*}\right)\left(\bar{l}+l^S(z)\right)dz = L^{S,market}, \tag{B.44}$$

where $L^{S,market}$ is the amount of labor employed in sector S in the market equilibrium. The production function for Y is given by

$$1=\sum_S\int_{x^*}^{\infty}g\left(z^S,x^{S,*}\right)\left(\frac{y_i^S(z)}{Y}\right)^{\xi^S}dz, \tag{B.45}$$

with $y_i^S(z) = \exp(z)\, k^S(z)^{1-\alpha^S}\, l^S(z)^{\alpha^S}$. *The optimization problem (B.43) has the same solution as the market equilibrium, namely* $x^{S,*} = z^{S,*}$, $k^{S,*}(z) = k^S(z)$ *and* $l^{S,*}(z) = l^S(z)$, *where* $k^S(z)$ *and* $l^S(z)$ *are the capital and labor values in the decentralized equilibrium, and stars indicate the optimal solution to the planning problem. Accordingly,* $Y^* = Y$ *and* $C^* = C$.

Proof of Lemma 5. The proof is essentially the same as the proof of Lemma 4, so we only provide a sketch. The first-order conditions for capital by firm i in sector S is

$$\left(1-\alpha^S\right)\frac{\partial Y}{\partial y_i^S}\exp(z)\left(\frac{k^S(z)}{l^S(z)}\right)^{-\alpha} = \frac{\delta}{\xi^S}. \tag{B.46}$$

Letting ζ^S denote the Lagrange multiplier on the constraint (B.44), the respective first-order condition for labor is

$$\alpha^S\frac{\partial Y}{\partial y_i^S}\exp(z)\left(\frac{k^S(z)}{l^S(z)}\right)^{-\alpha} = \zeta^S. \tag{B.47}$$

Using the implicit function theorem in equation (B.45), the first-order condition for the termination cut-off, $x^{*,S}$, in sector S is

$$\frac{\partial Y}{\partial x^{*,S}} = \int_{x^{S,*}}^{\infty}\frac{\partial g^S\left(z,x^{S,*}\right)}{\partial x^{*,S}}\left[\frac{Y^{1-\xi^S}\left(y_i^S(z)\right)^{\xi^S}}{\sum_S \xi^S Y^{-\xi^S}\int_{i\in S}^{\infty}\left(y_i^S(z)\right)^{\xi^S} g^S\left(z,x^{S,*}\right)dz} - \frac{\delta}{\xi^S}k^S(z) + \zeta^S\left(\bar{l}+l^S(z)\right)\right]dz = 0, \tag{B.48}$$

In addition, implicit differentiation of equation (B.45) gives

$$\frac{\partial Y}{\partial y_i^S} = \frac{\xi^S Y^{1-\xi^S}\left(y_i^S(z)\right)^{\xi^S-1}}{\sum_S \xi^S Y^{-\xi^S}\int_{i\in S}^{\infty}\left(y_i^S(z)\right)^{\xi^S}di},$$

and since $p_i^S = \frac{\partial Y}{\partial y_i^S}$, we can re-write (B.48)

$$\int_{x^{S,*}}^{\infty}\frac{\partial g^S\left(z,x^{S,*}\right)}{\partial x^{*,S}}\left[\frac{p_i(z)\,y_i(z)}{\xi^S} - \frac{\delta}{\xi^S}k^S(z) + \zeta_S\left(\bar{l}+l^S(z)\right)\right]dz = 0. \tag{B.49}$$

From this point onward, the argument is exactly the same as in the proof of Lemma 4. Multiplying both sides of (B.49) by ξ^S and setting $\zeta_S = \frac{w}{\xi^S}$, we recognize that the term inside square brackets in expression (B.49) captures profits, and the equations (B.46) and (B.47) are just the first order conditions for capital and labor in a market equilibrium. ■

Let w^S denote the revenue weight of sector S, defined as

$$\omega^S \equiv \frac{\int_{i\in S} p_i y_i di}{Y} = \frac{\xi^S Y^{-\xi^S} \int_{i\in S} \left(y_i^S(z)\right)^{\xi^S} di}{\sum_S \xi^S Y^{-\xi^S} \int_{i\in S}^{\infty} \left(y_i^S(z)\right)^{\xi^S} di}.$$

Using implicit differentiation on (B.45) and the envelope theorem (fixing a given labor allocation across sectors) gives

$$\begin{aligned} \frac{\partial \widehat{H}}{\partial z^{c,S}} &= \omega^S Y,\ \frac{\partial \widehat{H}}{\partial \alpha^S} = \ln\left(\frac{l^S}{k^S}\right)\omega^S Y, \text{ and} \\ \frac{\partial \widehat{H}}{\partial \xi^S} &= \frac{1}{\xi^S}\omega^S Y\left[\int_{x^{S,*}}^{\infty} \widetilde{g}^S\left(z, x^{S,*}\right)\ln y_i^S(z)dz - \ln Y - \left(1-\alpha^S\right)\right] \end{aligned} \tag{B.50}$$

where $\widetilde{g}^S\left(z, x^{S,*}\right)$ are within-sector revenue weights defined as

$$\widetilde{g}^S\left(z, x^{S,*}\right) \equiv \frac{p_i y_i}{\int_{i\in S} p_i y_i di} = \frac{g^S\left(z, x^{S,*}\right)\left(y_i^S(z)\right)^{\xi^S}}{\int_{x^{S,*}}^{\infty} g^S\left(z, x^{S,*}\right)\left(y_i^S(z)\right)^{\xi^S} dz}.$$

Note that the partial derivatives of $\widehat{H}$ with respect to the various parameters are analogous to their counterparts in the single-sector economy, except for the presence of the sector weights ω^S. Using the relation $C = \widehat{H} - \delta\sum_S\left(1-\frac{1}{\xi^S}\right)K^S$, totally differentiating C and using the Envelope theorem gives[32]

$$\begin{aligned} dC &= Y\sum_S \omega^S\left\{dz^{c,S} + \ln\left(\frac{l^S}{k^S}\right)d\alpha^S + \frac{1}{\xi^S}\left(\int_{x^{S,*}}^{\infty}\widetilde{g}^S\left(z, x^{S,*}\right)\ln\left(\frac{y_i^S(z)}{Y}\right)dz + \left(1-\alpha^S\right)\right)d\xi^S\right\} \\ &\quad + \zeta^S\sum_S dL^S - \delta\sum_S\left(1-\frac{1}{\xi^S}\right)dK^S - \sum_S\frac{\delta K^S}{\xi^S}\frac{d\xi^S}{\xi^S}. \end{aligned} \tag{B.51}$$

The proof of Lemma 5 shows that the Lagrange multipliers, ζ^S, associated with the constraints (B.44) obey the relation $\zeta_S = \frac{w}{\xi^S}$. Moreover, aggregating the first-order conditions for capital within a sector implies that $\xi^S\left(1-\alpha^S\right)\int_{i\in S} p_i y_i di = \delta K^S$. Accordingly, (B.51)

[32]In applying the envelope theorem, we used the fact that the Lagrange multipliers are the solution to the min-max problem

$$\min_{\zeta^S}\left\{\begin{array}{c} \max_{l^S(z),k^S(z),x^{S,*}} Y - \delta\sum_S\frac{1}{\xi^S}\int_{x^{S,*}}^{\infty} g^S\left(z, x^{S,*}\right)k^S(z)\,dz \\ -\sum_S\left(\int_{x^*}^{\infty} g\left(z^S, x^{S,*}\right)\left(\bar{l}+l^S(z)\right)dz - L^S\right). \end{array}\right\}$$

simplifies to

$$
\begin{aligned}
dC &= Y\sum_S \omega^S \left\{dz^{c,S} + \ln\left(\frac{l^S}{k^S}\right)d\alpha^S + \left(\int_{x^{S,*}}^{\infty} \widetilde{g}^S\left(z, x^{S,*}\right)\ln\left(\frac{y_i^S(z)}{Y}\right)dz\right)\frac{d\xi^S}{\xi^S}\right\} \\
&\quad + \sum_S \frac{1}{\xi^S}\left(wdL^S + \delta dK^S\right) - \delta K\left(\frac{dK}{K}\right).
\end{aligned}
$$

Let $\widehat{\sigma} \equiv \sum_S \omega^S\left(1-\alpha^S\right)\xi^S$. Aggregating the first-order conditions for capital across sectors gives $\delta K = \widehat{\sigma}Y$ and by implication $C = Y - \delta K = (1-\widehat{\sigma})\,Y$. Therefore,

$$
\begin{aligned}
dC &= Y\sum_S \omega^S \left\{dz^{c,S} + \ln\left(\frac{l^S}{k^S}\right)d\alpha^S + \left(\int_{x^{S,*}}^{\infty} \widetilde{g}^S\left(z, x^{S,*}\right)\ln\left(\frac{y_i^S(z)}{Y}\right)dz\right)\frac{d\xi^S}{\xi^S}\right\} \\
&\quad + Y\sum_S \left(\frac{wL^S}{\xi^S Y}\frac{dL^S}{L^S} + \omega^S\left(1-\alpha^S\right)\frac{dK^S}{K^S}\right) - \widehat{\sigma}Y\left(\frac{dK}{K}\right),
\end{aligned}
$$

which implies

$$
\begin{aligned}
dC &= Y\sum_S \omega^S \left\{dz^{c,S} + \ln\left(\frac{l^S}{k^S}\right)d\alpha^S + \left(\int_{x^{S,*}}^{\infty} \widetilde{g}^S\left(z, x^{S,*}\right)\ln\left(\frac{y_i^S(z)}{Y}\right)dz\right)\frac{d\xi^S}{\xi^S}\right\} \\
&\quad + Y\sum_S \left\{\frac{wL^S}{\xi^S Y}\frac{dL^S}{L^S} + \omega^S\left(1-\alpha^S\right)\left(\frac{dK^S}{K^S} - \frac{dK}{K}\right)\right\} + Y\left(1-\widehat{\alpha}-\widehat{\sigma}\right)\left(\frac{dK}{K}\right),
\end{aligned}
$$

where $K = \sum_S K^S$, $\widehat{\alpha} \equiv \sum_S \omega^S \alpha^S$. Using $C = (1-\widehat{\sigma})\,Y$ and $\delta K = \widehat{\sigma}Y$ implies $\frac{C}{\delta K} = \frac{1-\widehat{\sigma}}{\widehat{\sigma}}$ and hence $\frac{dC}{C} - \frac{dK}{K} = -\frac{1}{1-\widehat{\sigma}}\frac{d\widehat{\sigma}}{\widehat{\sigma}}$ and therefore we obtain after some re-arranging

$$
\begin{aligned}
(1-\widehat{\sigma})\frac{dC}{C} &= \sum_S \omega^S \left\{dz^{c,S} + \ln\left(\frac{l^S}{k^S}\right)d\alpha^S + \left(\int_{x^{S,*}}^{\infty} \widetilde{g}^S\left(z, x^{S,*}\right)\ln\left(\frac{y_i^S(z)}{Y}\right)dz\right)\frac{d\xi^S}{\xi^S}\right\} \\
&\quad + \sum_S \left\{\frac{wL^S}{\xi^S Y}\frac{dL^S}{L^S} + \omega^S\left(1-\alpha^S\right)\left(\frac{dK^S}{K^S} - \frac{dK}{K}\right)\right\} + \left(1-\widehat{\alpha}-\widehat{\sigma}\right)\left(\frac{dC}{C} + \frac{1}{(1-\widehat{\sigma})}\frac{d\widehat{\sigma}}{\widehat{\sigma}}\right)
\end{aligned}
$$

Solving for $\frac{dC}{C}$ gives

$$
\begin{aligned}
\frac{dC}{C} &= \sum_S \omega^S \left\{\frac{dz^{c,S}}{\widehat{\alpha}} + \log\left(\frac{l^S}{k^S}\right)\frac{d\alpha^S}{\widehat{\alpha}} + \left(\int_{x^{S,*}}^{\infty} \widetilde{g}^S\left(z, x^{S,*}\right)\ln\left(\frac{y_i^S(z)}{Y}\right)dz\right)\frac{d\xi^S}{\widehat{\alpha}\xi^S}\right\} \quad \text{(B.52)} \\
&\quad + \frac{1}{\widehat{\alpha}}\sum_S \left\{\frac{wL^S}{\xi^S Y}\frac{dL^S}{L^S} + \omega^S\left(1-\alpha^S\right)\left(\frac{dK^S}{K^S} - \frac{dK}{K}\right)\right\} + \frac{1}{\widehat{\alpha}}\frac{1-\widehat{\alpha}-\widehat{\sigma}}{1-\widehat{\sigma}}\frac{d\widehat{\sigma}}{\widehat{\sigma}}.
\end{aligned}
$$

The remainder of the proof follows exactly the same steps as the proof of Proposition 22.

Specifically, the following identity continues to be true

$$\left(\frac{1-\xi^S}{\xi^S}\right)\frac{dY^{S,\text{new}}}{Y^{S,\text{new}}} = \left(1+\frac{1}{\xi^S}+\int_{z^{*,s}}^{\infty}\widetilde{m}^S(z)\log\left(\frac{y^{S,\text{new}}(z)}{Y}\right)dz\right)\frac{d\xi^S}{\xi^S}+dz^{c,S}+\log\left(\frac{l^{S,\text{new}}}{k^{S,\text{new}}}\right)d\alpha^S. \tag{B.53}$$

Combining (B.53) with (B.52) leads to (29). ■

Proof of Corollary 1. If $S=1$ then $\omega^S=1, \alpha^S=\alpha$ and $\xi^S=\xi$. This implies that that the term on the second line of (29) is zero since $\sum_S dL_S = dL = 0$ and $\frac{dK^S}{K^S}=\frac{dK}{K}$. Therefore to prove that (22) and (29) are identical, it suffices to show that

$$\frac{1}{\alpha}\frac{1-\alpha-\sigma}{1-\sigma}\frac{d\sigma}{\sigma} = -\left(\frac{1-\xi}{1-\xi(1-\alpha)}\right)\frac{d\alpha}{\alpha}-\left(\alpha\frac{\xi(1-\alpha)}{1-\xi(1-\alpha)}-(1-\alpha)\right)\frac{d\xi}{\alpha\xi} \tag{B.54}$$

To prove (B.54), note that when $S=1$, $\sigma=(1-\alpha)\xi$. Therefore

$$\frac{d\sigma}{\sigma} = -\frac{\alpha\xi}{(1-\alpha)\xi}\frac{d\alpha}{\alpha}+\frac{d\xi}{\xi}. \tag{B.55}$$

Substituting $\sigma=(1-\alpha)\xi$ and (B.55) into the left-hand side (B.54) and re-arranging gives the right-hand side of (B.54). ■

Proof of Lemma 2. The assumption that the fixed labor component is small implies that[33] $\frac{\alpha^S\xi^S\omega^S}{\widehat{\phi}} \approx \frac{L^S}{L}$ and therefore $\frac{dL^S}{L^S}-\frac{dL}{L}=\frac{dL^S}{L^S}\approx\frac{d\omega^S}{\omega^S}-\frac{d\widehat{\phi}}{\widehat{\phi}}$. Similarly we have that $\frac{K^S}{K}=\frac{\omega^S(1-\alpha^S)\xi^S}{\widehat{\sigma}}$ and accordingly $\frac{dK^S}{K^S}-\frac{dK}{K}=\frac{d\omega^S}{\omega^S}-\frac{d\widehat{\sigma}}{\widehat{\sigma}}$. Therefore, using the approximation[34] $\frac{wL^S}{\xi^S Y}\approx\omega^S\alpha^S$, the third and fourth term on the right-hand side of (29) can be combined to

$$\frac{1}{\widehat{\alpha}}\sum_S\omega^S\left\{\alpha^S\left(\frac{d\omega^S}{\omega^S}-\frac{d\widehat{\phi}}{\widehat{\phi}}\right)+\left(1-\alpha^S\right)\left(\frac{d\omega^S}{\omega^S}-\frac{d\widehat{\sigma}}{\widehat{\sigma}}\right)\right\}+\frac{1}{\widehat{\alpha}}\frac{1-\widehat{\alpha}-\widehat{\sigma}}{1-\widehat{\sigma}}\frac{d\widehat{\sigma}}{\widehat{\sigma}}. \tag{B.56}$$

Since $\sum_S\omega^S=1$, we have $\sum_S d\omega^S=0$, and therefore the expression (B.56) becomes $\left(-\frac{d\widehat{\phi}}{\widehat{\phi}}\right)+\left(-\frac{d\widehat{\sigma}}{\widehat{\sigma}}\right)\frac{\widehat{\sigma}}{1-\widehat{\sigma}}>0$. Since the sum of the third and the fourth term in (29) are positive,

[33] Aggregating the first-order condition for a firm in sector S, $\alpha^S\xi^S p^S(z)\,y^S(z)=wl^S(z)$, to the sector level and dividing both sides by Y gives $\alpha^S\xi^S\omega^S=\frac{w}{Y}\left(L^S(z)-\overline{L}^S\right)$, where $\overline{L}^S$ is labor employed in overhead activities in sector S. Using the approximation $\overline{L}^S\approx 0$, and aggregating across sectors gives $\widehat{\phi}=\frac{w}{Y}\left(L-\overline{L}\right)$, and hence $\frac{\alpha^S\xi^S\omega^S}{\widehat{\phi}}\approx\frac{L^S}{L}$.

[34] As in footnote 33, aggregating the first-order condition for a firm in sector S, $\alpha^S\xi^S p^S(z)\,y^S(z)=wl^S(z)$, to the sector level and dividing both sides by Y gives $\alpha^S\xi^S\omega^S\approx\frac{wL^S(z)}{Y}$, where we used the approximation $\overline{L}^S\approx 0$.

and the second term is zero (by assumption), the change in $\frac{dC^{SS}}{C^{SS}}$ is bounded below by $\frac{1}{\bar{\alpha}}\sum_S \omega^S \left(\frac{1-\xi^S}{\xi^S}\right)\frac{dY^{S,\text{new}}}{Y^{S,\text{new}}}$. ■

Appendix C Numerical algorithm for computing the transition dynamics

In this section we provide a brief description of our numerical algorithm to solve for the transition path. The key difficulty preventing a closed from solution along the transition path is that both the wage, the interest rate and the threshold level of productivity that leads to endogenous bankruptcy (for the two different kinds of firms) are now functions of time rather than constants. To solve for the transition dynamics, we start with an initial guess for the productivity thresholds that trigger bankruptcy. With that guess in hand, we simulate an economy whereby new firms arrive each year, with idiosyncratic shocks that follow the dynamics (13). The number of new firms is in principle irrelevant for the model, with a higher number reducing simulation error at the expense of computational power. We verified the validity of Monte Carlo by checking (analytically) that for our chosen parameters the numerically integrated quantities posses finite moments.[35,36]

Using the cross-section of simulated productivities we determine the market clearing prices, wages and output. Fixing these time-series of prices, wages and output, we then use a binomial tree with 200 years and time increment $dt = 0.1$ to solve for the optimal termination thresholds for the pre- and post-transition firms separately. Using these optimal termination policies, we repeat the wage and output calculation and iterate to convergence (which typically takes two-three iterations of the algorithm).

Appendix D Elastic Labor supply

Throughout the paper we maintained the assumption of inelastic labor supply for simplicity. Here we show how to extend the key results of the paper to the case where labor supply is

[35]For our simulations we set the number of incoming firms to 10,000 per year and per computer processor We use parallel computing, repeat our calculations on a 12-processor parallel cluster (resulting in 120,000 draws in total) and report the average value.

[36]An alternative to Monte Carlo would be to numerically solve the forward Kolmogorov equations.

elastic. Specifically, suppose that workers have a utility of the form

$$E_t \int_t^{\infty} e^{-\rho(s-t)} \left[u\left(c_s\right) + h\left(L^e - L_s\right)\right] ds,$$

where L^e is the household's endowment of hours, L denotes the hours supplied by the representative household, and $h(\cdot)$ is an increasing and concave function. The first-order condition for labor supply is

$$\frac{h'\left(L^e - L_t\right)}{u'\left(C_t\right)} = w_t. \tag{D.1}$$

To determine the steady-state labor, L – which equations (A.7) - (21) took as fixed – we start by noting that Lemma 3 implies that $\frac{\varphi}{w}$ is a function of L; by implication, equations (A.9) and (A.10) imply that $w = w(L)$ and $Y = Y(L)$ are functions of L. In a steady state we have that $C = Y - \delta K = Y\left(1 - \delta \frac{K}{Y}\right)$, where the capital-to-output ratio, $\frac{K}{Y} = \frac{\xi(1-\alpha)}{\rho+\delta}$, is a constant independent of L. Therefore, consumption, C, is proportional to output, and therefore $C = C(L)$. With these observations, the equilibrium L amounts to solving the labor-supply equation

$$\frac{h'\left(L^e - L\right)}{u'(Y(L)\left(1 - \delta \frac{K}{Y}\right))} = w\left(L\right)$$

for L.

We next prove the following generalization of Lemma 4.

Lemma 6 *Assume that $\rho = 0$ and consider the optimization problem of maximizing H, where*

$$H \equiv \max_{L, l(z), k(z), x^*} u\left(Y - \frac{\delta}{\xi} K\right) + bh(L^e - L) \tag{D.2}$$

subject to the constraints (B.20), and the definition (B.21), where

$$b \equiv \frac{u'\left(\frac{\alpha}{1-\xi(1-\alpha)}\right)}{\xi}.$$

The optimization problem (D.2) has the same solution as the market equilibrium, namely $L = L^*$ $x^* = z^*, k^*(z) = k^{market}(z)$ and $l^*(z) = l^{market}(z)$, where $k^{market}(z)$ and $l^{market}(z)$ are the capital and labor values in the decentralized equilibrium, and stars indicate the optimal solution to the planning problem. Accordingly, $Y^* = Y^{market}$ and $C^* = C^{market}$.

Proof of Lemma 6. We provide a sketch since the proof is essentially identical to the proof of Lemma 4. Equation (B.23) remains unchanged, while maximizing over $l(z)$ gives

$$\alpha Y^{1-\xi} y(z)^{\xi-1} \exp(z) \left(\frac{k(z)}{l(z)}\right)^{-\alpha} = \frac{\zeta}{u'\left(Y - \frac{\delta}{\xi}K\right)}, \tag{D.3}$$

where ζ is a Lagrange multiplier associated with the constraint (B.20). Maximizing over L gives

$$bh'(L - L^e) = \zeta. \tag{D.4}$$

Maximizing over x^*leads to[37]

$$\int_{x^*}^{\infty} \frac{\partial}{\partial x^*} g(z, x^*) \left[\frac{1}{\xi} Y^{1-\xi} (y(z))^{\xi} dz - \frac{\delta}{\xi} k(z) - \frac{\zeta}{u'\left(Y - \frac{\delta}{\xi}K\right)} \left(l(z) + \bar{l}\right)\right] dz = 0 \tag{D.5}$$

From this point on one can repeat the same steps as in Lemma 4 in order to confirm that the market equilibrium corresponds to a planning optimum. The arguments are identical to Lemma 4, except that the relation between the Lagrange multiplier, ζ, and the wage is now $\frac{\zeta}{U'\left(Y - \frac{\delta}{\xi}K\right)} = \frac{w}{\xi}$. The only new step is to confirm that the market equilibrium L and the planner-chosen L coincide. To confirm this, use the relation $\frac{\zeta}{U'\left(Y - \frac{\delta}{\xi}K\right)} = \frac{w}{\xi}$ together with $\frac{\delta}{\xi}K = (1-\alpha)Y$, $C = Y - \delta K = Y - \xi\frac{\delta}{\xi}K = [1 - \xi(1-\alpha)]Y$ and (D.4) to obtain

$$\frac{w}{\xi} = \frac{\zeta}{u'\left(Y - \frac{\delta}{\xi}K\right)} = \frac{bh'(L^e - L)}{u'(C)\, u'\left(\frac{\alpha}{1-\xi(1-\alpha)}\right)} = \frac{h'(L^e - L)}{\xi u'(C)}, \tag{D.6}$$

which is equation (D.1), the equation that determines labor supply in the market equilibrium. Therefore the optimal labor L in the planning problem and the free market equilibrium coincide. ■

To generalize proposition 1, let $z^c = \log(Z^c)$, $y(z) = \exp(z^c + z)\, k^{1-\alpha}(z)\, l(z)$, and apply the envelope theorem (around $z^c = 0$) to (D.2) to obtain

$$\frac{\partial H}{\partial z^c} = u'\left(Y - \frac{\delta}{\xi}K\right) Y.$$

[37]Note that $g(x^*, x^*) = 0$.

Similarly,

$$\frac{\partial H}{\partial \xi} = u'\left(Y - \frac{\delta}{\xi}K\right) \times \frac{1}{\xi}Y\left[\int_{x^*}^{\infty} \widetilde{g}(z)\log y(z)\,dz - \log(Y) + (1-\alpha)\right] + \frac{\partial b}{\partial \xi}h(L^e - L), \quad \text{(D.7)}$$

Similarly,

$$\frac{\partial H}{\partial \alpha} = u'\left(Y - \frac{\delta}{\xi}K\right) \times Y\log\left(\frac{l(z)}{k(z)}\right) + \frac{\partial b}{\partial \alpha}h(L^e - L). \quad \text{(D.8)}$$

Since $U(\cdot)$ is monotone, its inverse exists and therefore

$$u^{-1}(H - bh(L)) = Y - \frac{\delta}{\xi}K = \alpha Y, \quad \text{(D.9)}$$

where the last equality follows from $\frac{\delta}{\xi}K = (1-\alpha)Y$. Total differentiation of (D.9) gives

$$\begin{aligned} d\alpha Y + \alpha dY &= u^{-1\prime}(H - bh(L^e - L))(dH + h'(L^e - L)dL) \\ &= \frac{1}{u'\left(Y - \frac{\delta}{\xi}K\right)}(dH + bh'(L^e - L)dL) \\ &= Y\left\{dz^c + \log\left(\frac{l(z)}{k(z)}\right)d\alpha + \frac{1}{\xi}\left(\int_{x^*}^{\infty}\widetilde{g}(z)\log\left(\frac{y(z)}{Y}\right)dz + (1-\alpha)\right)d\xi\right\} \\ &+ \frac{bh'(L^e - L)}{u'\left(Y - \frac{\delta}{\xi}K\right)}dL, \end{aligned}$$

or equivalently

$$\begin{aligned} \frac{dY}{Y} &= \frac{1}{\alpha}\left\{dz^c + \left(\log\left(\frac{l(z)}{k(z)}\right) - 1\right)d\alpha + \frac{1}{\xi}\left(\int_{x^*}^{\infty}\widetilde{g}(z)\log\left(\frac{y(z)}{Y}\right)dz + (1-\alpha)\right)d\xi\right\} \\ &+ \frac{bh'(L^e - L)}{\alpha Y u'\left(Y - \frac{\delta}{\xi}K\right)}dL. \end{aligned}$$

Note that $\frac{dY}{Y}$ is the same as in equation (28), except for the presence of the term

$$\frac{bh'(L^e - L)}{\alpha Y u'\left(Y - \frac{\delta}{\xi}K\right)}dL = \frac{\zeta}{\alpha Y u'\left(Y - \frac{\delta}{\xi}K\right)}dL = \frac{wL}{\alpha\xi Y}\frac{dL}{L},$$

where we used (D.6). The term $\frac{wL}{\alpha\xi Y}\frac{dL}{L}$, which encapsulates the effects of elastic labor, is comprised of the percentage change in labor $\frac{dL}{L}$and the term $\frac{wL}{\alpha\xi Y} \approx 1$, where the approximation is accurate as long as the fraction of labor that is due to overhead, $\bar{l}$, is close to zero. To determine $\frac{dL}{L}$, let $\Phi \equiv \int_{z^*}^{\infty} g(z, z^*)\, dz$ denote the measure of firms in the stationary distribution and note that equations (21) and (D.1) imply

$$log\xi\alpha + logY = logw + log\left(L - \bar{l}\Phi\right), \tag{D.10}$$

$$logh'(L^e - L) - logU'(C) = logw. \tag{D.11}$$

Because of our additively separable specification for labor utility, we assume for the remainder of this section that $u(c) = log(c)$, in order to ensure balanced growth. This implies that

$$b = \frac{\alpha\xi}{1 - \xi(1-\alpha)}.$$

Combining (D.10) with (D.11) gives

$$log\xi\alpha = logh'(L^e - L) + log(1 - \xi(1-\alpha)) + logL + log\left(\frac{L - \bar{l}\Phi}{L}\right). \tag{D.12}$$

Letting $\frac{1}{f} \equiv -\frac{h''(L^e-L)L}{h'(L^e-L)}$ denote the inverse of the Frisch elasticity of labor supply (f), totally differentiating both sides of (D.12), using the definition of b and simplifying gives

$$\begin{aligned} \frac{dL}{L} &= -\frac{1}{1+\frac{1}{f}}\left\{dlogb + dlog\left(\frac{L - \bar{l}\Phi}{L}\right)\right\} \\ &= -\frac{1}{1+\frac{1}{f}}\left\{dlogb + \frac{\bar{l}\Phi}{L}\left(1 - \frac{\bar{l}\Phi}{L}\right)^{-1}\left(\frac{dL}{L} - \frac{d\Phi}{\Phi}\right)\right\} \end{aligned}$$

Assuming that the fraction of overhead in production is small ($\frac{\bar{l}\Phi}{L} \approx 0$) we obtain the simple formula

$$\frac{dL}{L} \approx -\frac{1}{1+\frac{1}{f}} dlogb,$$

where

$$dlogb = \frac{(1-\xi)}{1-\xi(1-\alpha)}\frac{d\alpha}{\alpha} - \frac{1}{1-\xi(1-\alpha)}\frac{d\xi}{\xi}.$$

A decline in $log\xi$ and a decline in $log\alpha$ have opposite effects. In particular a decline in ξ (higher rent share) results in lower labor supply. Moreover, $\frac{1}{1-\xi(1-\alpha)} > \frac{(1-\xi)}{1-\xi(1-\alpha)}$, so that the magnitude of the elasticity of labor supply with respect to a change in ξ is higher than with respect to a change in α.

The quantitative magnitude of $\frac{dL}{L}$ depends on the assumed elasticity of labor supply, f. As $f \to 0$, $\frac{dL}{L} \to 0$. If one were to use an elasticity around 0.5, then a decline in $\frac{d\xi}{\xi}$ of approximately 0.1 would result in a drop in $\frac{dL}{L}$ of approximately 0.05 for $\xi \approx 1$ and $\alpha \approx \frac{2}{3}$.

Appendix E The decomposition of the labor share

To start, we observe that the aggregate labor share can be expressed as

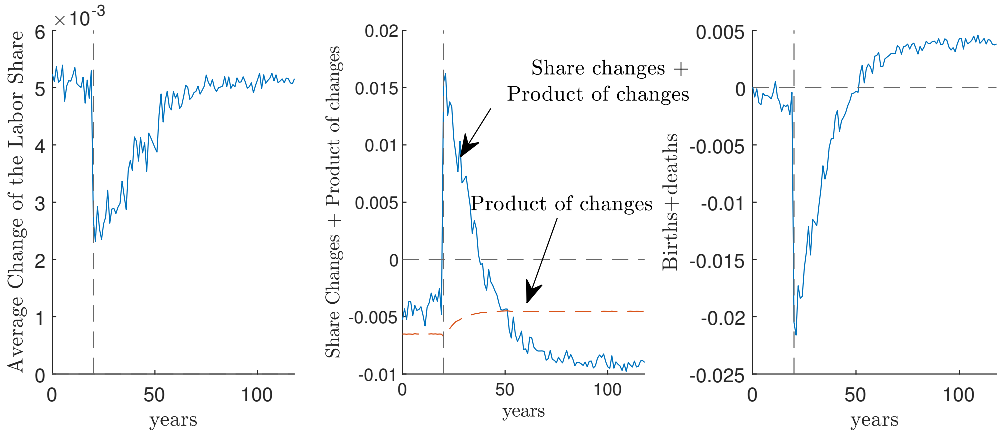
$$\frac{w_t L}{Y_t} = \frac{w_t \int_i l_{it} di}{Y_t} = \int_i \left(\frac{p_{it} y_{it}}{Y_t}\right)\left(\frac{w_t l_{it}}{p_{it} y_{it}}\right) di = \int_i \omega_{it} a_{it} di, \tag{E.1}$$

where $\omega_{it} \equiv \frac{y_{it}}{Y_t}$ is the output weight of firm i and $a_{it} \equiv \frac{w_t l_{it}}{p_{it} y_{it}}$ is firm $i's$ labor share. In light of (E.1), we obtain the "time-share" decomposition

$$\begin{aligned} \frac{w_{t+1} L}{Y_{t+1}} - \frac{w_t L}{Y_t} &= \int_i \omega_{it+1} a_{it+1} di - \int_i \omega_{it} a_{it} di \\ &= \underbrace{\int_i \omega_{it} \left(a_{it+1} - a_{it}\right) di}_{\text{Average change of the labor share}} + \\ &\quad \underbrace{\int_i \left(\omega_{it+1} - \omega_{it}\right) a_{it} di}_{\text{Share changes}} + \underbrace{\int_i \left(\omega_{it+1} - \omega_{it}\right)\left(a_{it+1} - a_{it}\right) di.}_{\text{Product of changes}} \end{aligned} \tag{E.2}$$

The above equation shows that the change in the labor share can be decomposed into three distinct terms. The first term is the output-share-weighted change in individual labor shares. The second term captures the effect of changing shares and the third term is a term that resembles a covariance term. Figure E.1 shows that the first term is always positive both in the old steady state and in the transition phase. This is driven by the fact that in our model $\mu - \frac{\sigma^2}{2} < 0$ and hence for the "median" firm the log productivity declines slightly. Due to the presence of a fixed labor cost, the labor share for the average firm increases. The decline in the labor share is driven mostly by the sum of the second and the third components

Figure E.1: **Model-Implied Decomposition of the Labor share decline.** The left plot depicts the evolution of the term labeled "Average change of the labor share" in equation (E.2). The middle plot depicts the sum of "Share changes" and "Product of changes" (solid line). The dashed line depicts the term "Product of changes". The last plot depicts the effect of deaths and births, i.e., the difference between the change in the labor share and the sum of the three components in (E.2). The vertical dashed line in all three plots depicts the onset of the transition. The "noise" in the figures is due to numerical approximation.



of equation (E.2) (middle plot), which capture the effects of changing firm weights as the output weight of the more productive firms (which have the smaller labor shares) increases. In equation (E.2) we aggregate only over firms that are alive both at time t and $t+1$. The impact of births and deaths on labor share changes is depicted in the third plot of the figure.

Appendix F Additional Compustat Results

F.1 Compustat by Sector

Our measures of employment, sales, and market value contribution measures in Section 2.1 show that the ratio of the ratio of market value (or sales) contribution to employment contribution has increased over time. In this section we show that the same patterns hold when we perform the analysis at the level of individual sectors.

We construct two additional measures of employment, sales, and market value contribution that account for firms' sector. Our first alternative measure, presented in equation (F.1), separately measures the contribution of young firms from each sector relative to the universe of public firms. Our second alternative measure, presented in equation (F.2), separately measures the contribution of young firms from each sector relative to the mature public firms in the same sector.

In equation form, letting X denote either employment, sales, or market value, we define the contribution of the year t IPO cohort from sector s as:

$$\text{X Contribution in Total}_{s,t} = \frac{\text{X of IPO Firms (Excluding Mature Firms)}_{s,t}}{\text{Total X}_{t-1}} \tag{F.1}$$

$$\text{X Contribution in Sector}_{s,t} = \frac{\text{X of IPO Firms (Excluding Mature Firms)}_{s,t}}{\text{Sector X}_{s,t-1}} \tag{F.2}$$

Continuing with the format of our main results, for each of the two measures of sector specific contributions, we construct the cumulative employment, sales, and market value contributions of 5-year IPO cohort bin as follows:

$$\text{X Contribution in Total}_{s,bin} = \sum_{i \in Bin} \text{X Contribution in Total}_{s,i} \tag{F.3}$$

$$\text{X Contribution in Sector}_{s,bin} = \sum_{i \in Bin} \text{X Contribution in Sector}_{s,i} \tag{F.4}$$

Figure F.2 presents the ratio of the sales and market value contributions to the employment contributions for each sector. In Panels A and B, we measure the contribution of young firms from each sector of the economy relative to the universe of public firms. Panel A presents the logarithm of the ratio of the sales and market value contributions to the employment contributions. Panel B presents the normalized (1985–1989 cohort = 0) logarithm of the ratio of the sales and market value contributions to the employment contributions. In Panels C and D, we measure the contribution of young firms from each sector of the economy relative to mature public firms in the same sector. Panel C presents the logarithm of the ratio of the sales and market value contributions to the employment contributions. Panel D presents the normalized (1985–1989 cohort = 0) logarithm of the ratio of the sales and market value contributions to the employment contributions. Panel E presents the number of firms going public in each sector and IPO cohort bin, after excluding firms that were founded more than 10 years prior to their IPO.

F.2 Operating Income

Figure F.3 presents a slightly modified version of the analysis of Section 2.1, in which we present results for employment, operating income, and market value. Operating income is Compustat variable OIBDP.

We measure the employment, operating income, and market value contribution of an IPO cohort as a share of the total market value and employment of public firms in the prior year. We then measure the contribution of an IPO cohort bin as the sum of the contributions of the different IPO cohorts in the bin.

Figure F.2: **Contribution of IPO Cohorts, By Sector**
Data on employment, sales, and market values of US public firms are taken from Compustat. Data on firm founding years are described in the text. We exclude from IPO cohorts all firms that were founded more than 10 years prior to their IPO. In Panels A and B, we measure the contribution of young firms from each sector of the economy relative to the universe of public firms. Panel A presents the logarithm of the ratio of the sales and market value contributions to the employment contributions. Panel B presents the normalized (1985–1989 cohort = 0) logarithm of the ratio of the sales and market value contributions to the employment contributions. In Panels C and D, we measure the contribution of young firms from each sector of the economy relative to mature public firms in the same sector. Panel C presents the logarithm of the ratio of the sales and market value contributions to the employment contributions. Panel D presents the normalized (1985–1989 cohort = 0) logarithm of the ratio of the sales and market value contributions to the employment contributions. Panel E presents the number of firms going public in each sector and IPO cohort bin, after excluding firms that were founded more than 10 years prior to their IPO. See Section F.1 for further details. [Images are on the next five pages.]

Figure F.2: **Contribution of IPO Cohorts, By Sector (Continued from Previous Page)**

(a) Ratio of Contributions, Relative to All of Public Firms

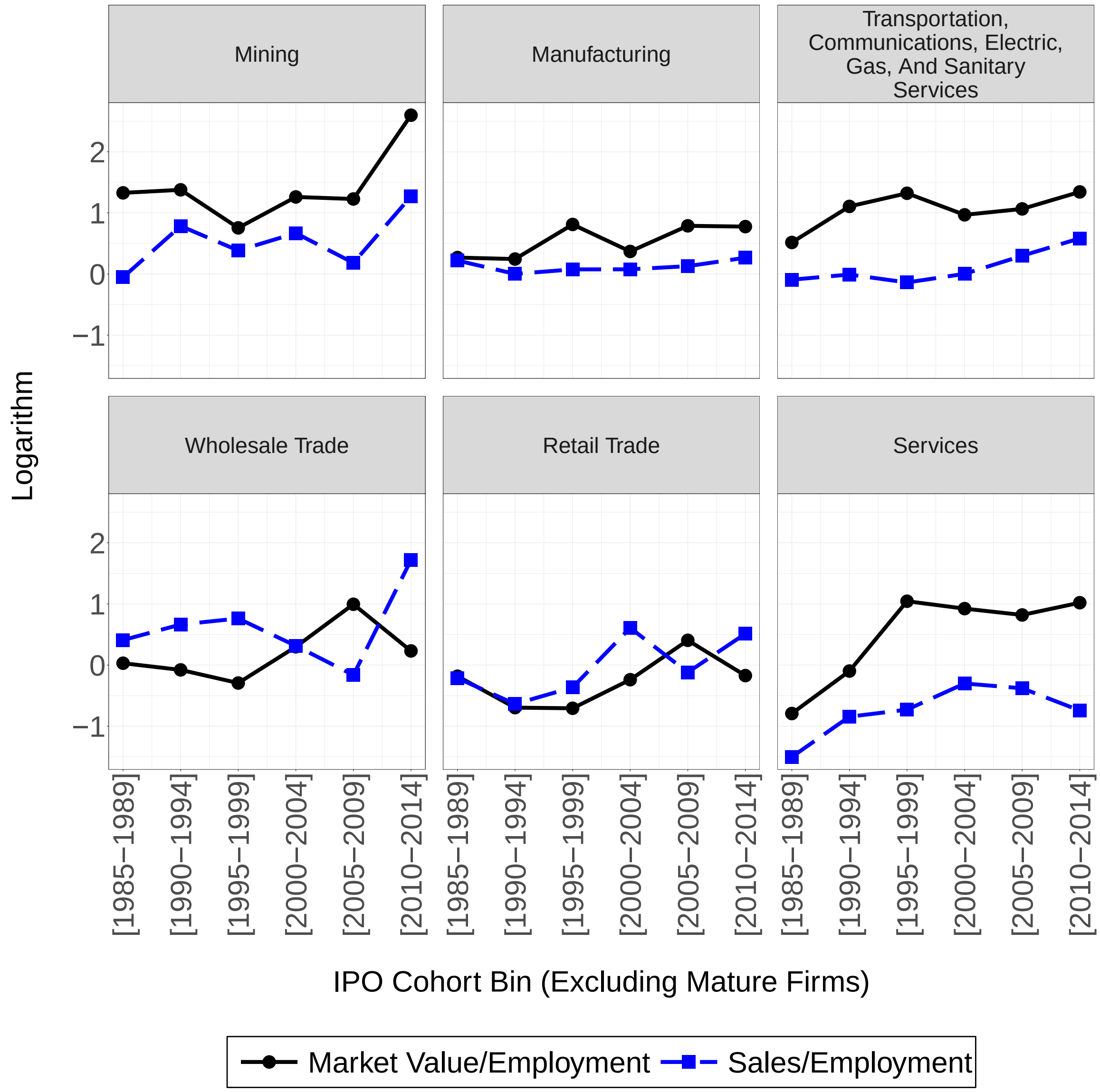


Figure F.2: **Contribution of IPO Cohorts, By Sector (Continued from Previous Page)**

(b) Ratio of Contributions Normalized, Relative to All of Public Firms

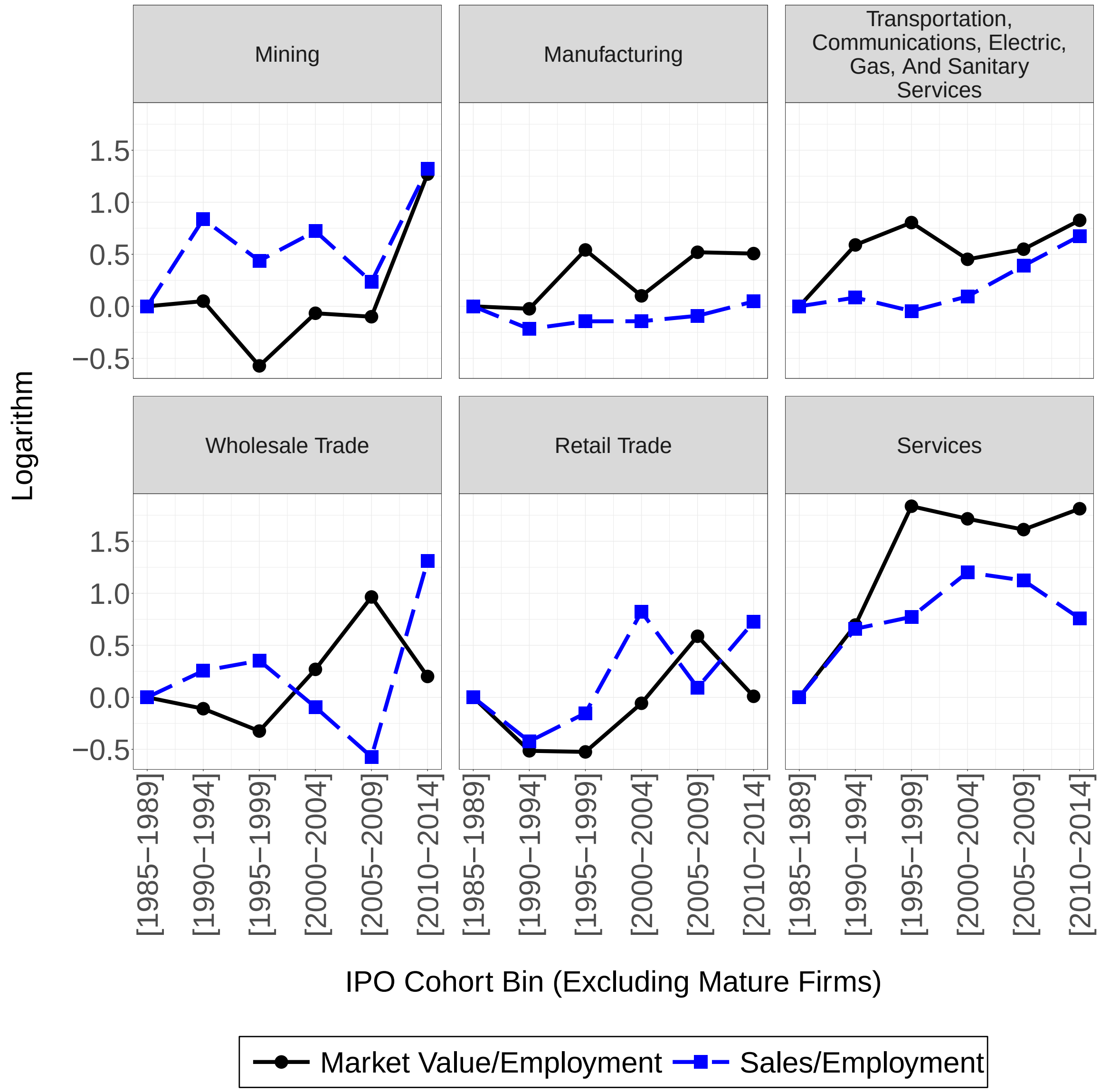


Figure F.2: **Contribution of IPO Cohorts, By Sector (Continued from Previous Page)**

(c) Ratio of Contributions, Relative to Public Firms in Sector

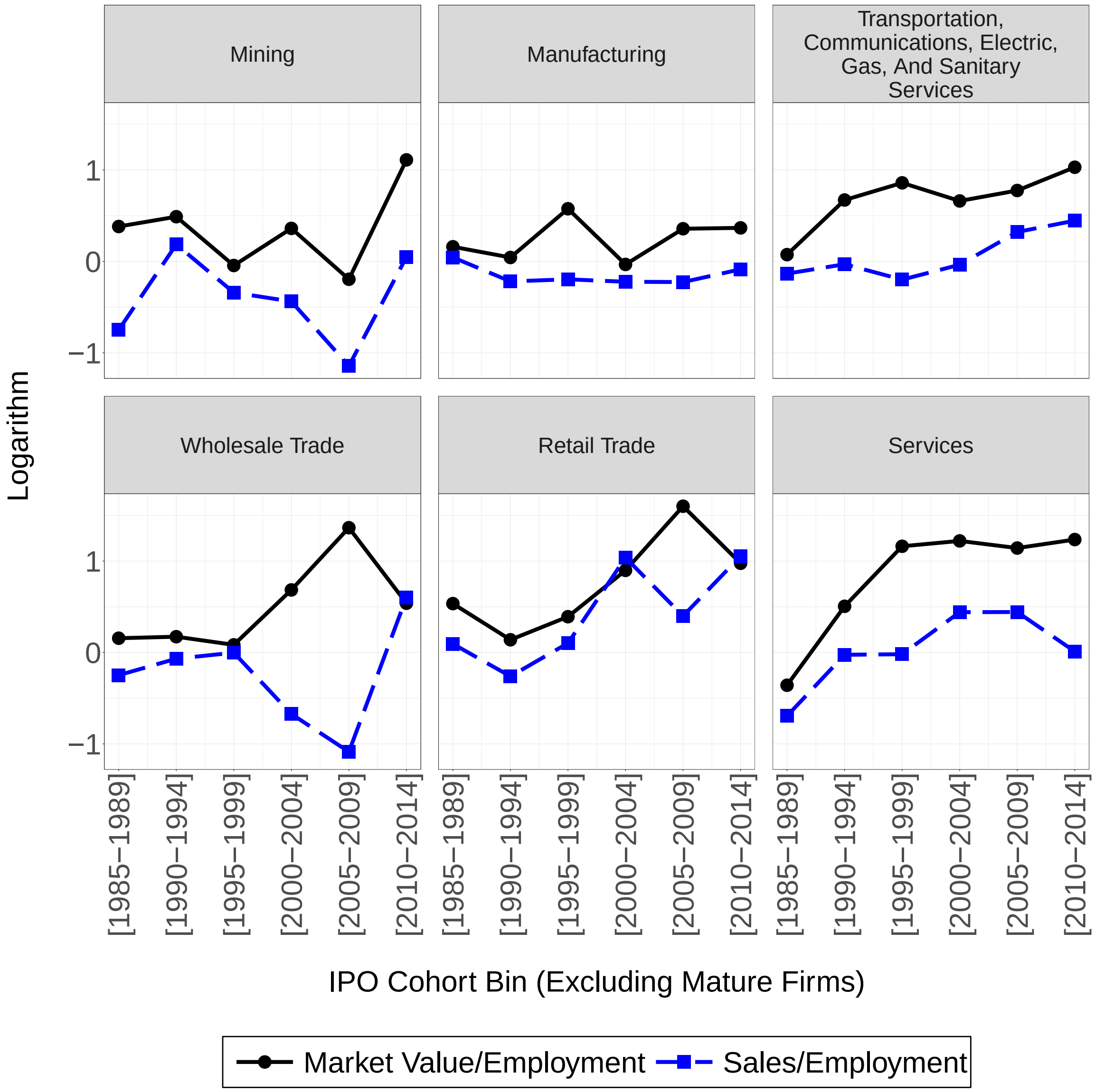


Figure F.2: **Contribution of IPO Cohorts, By Sector (Continued from Previous Page)**

(d) Ratio of Contributions Normalized, Relative to Public Firms in Sector

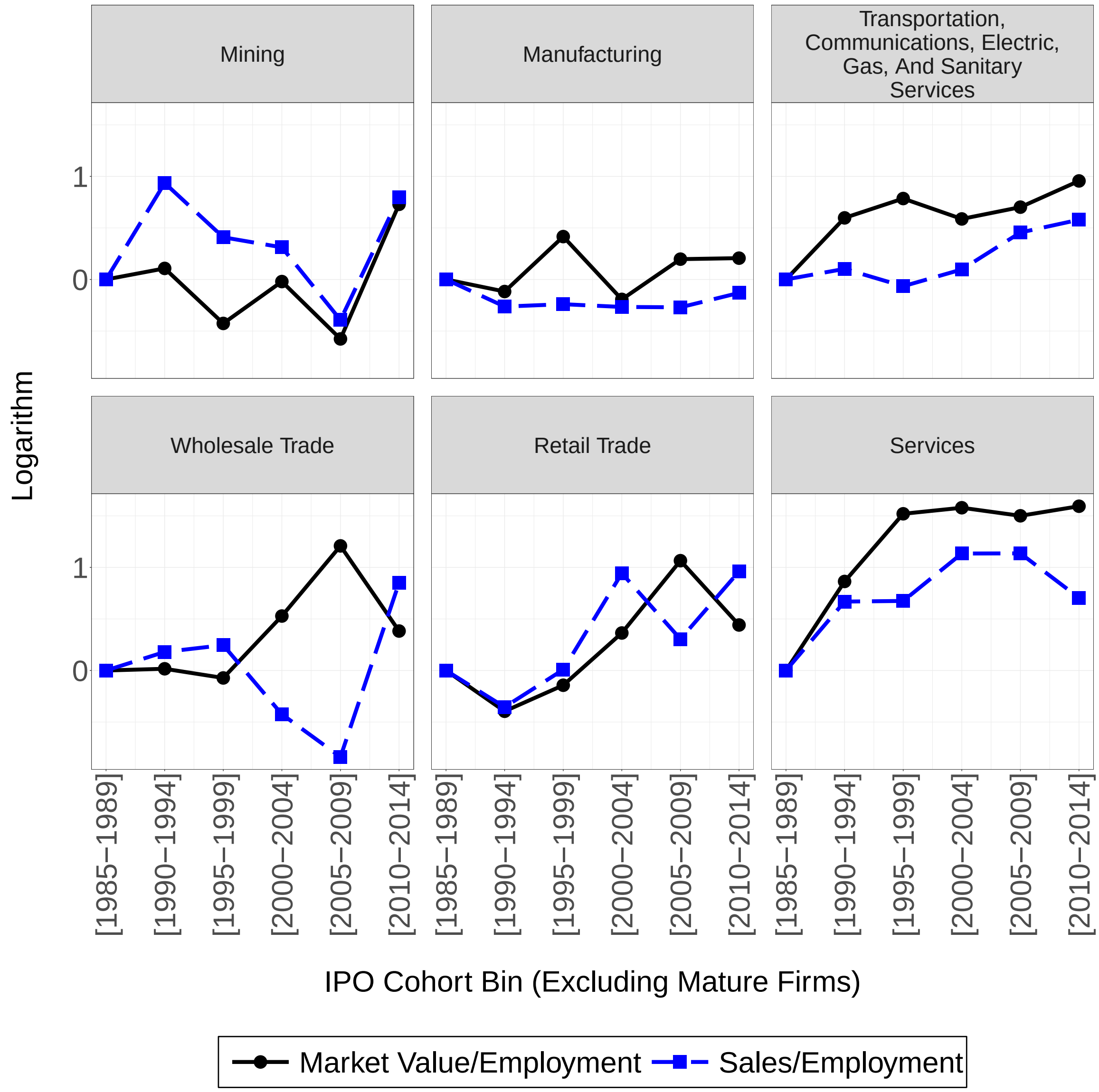


Figure F.2: **Contribution of IPO Cohorts, By Sector (Continued from Previous Page)**

(e) Number of Firms in each IPO Cohort, Excluding Mature Firms

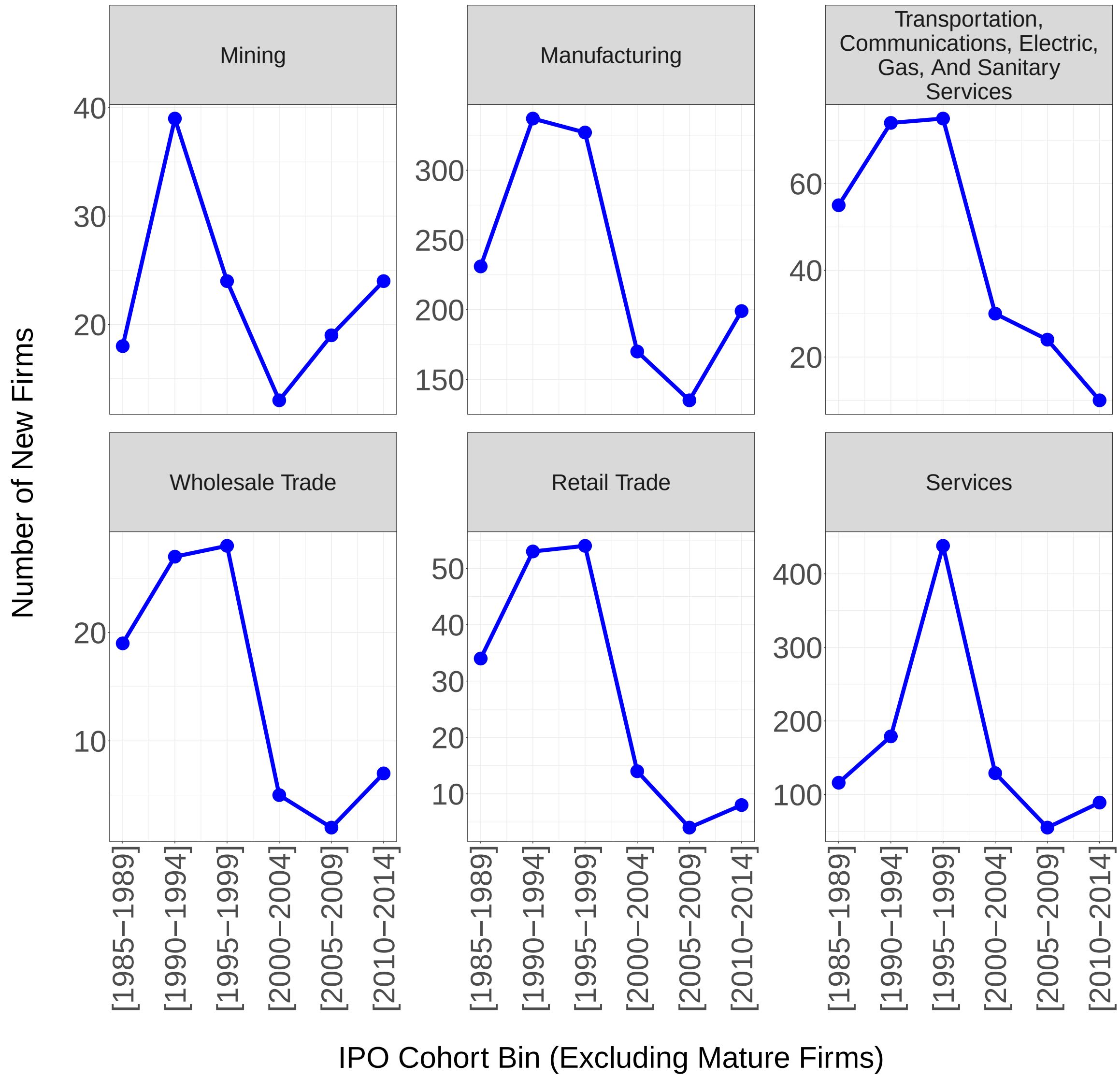
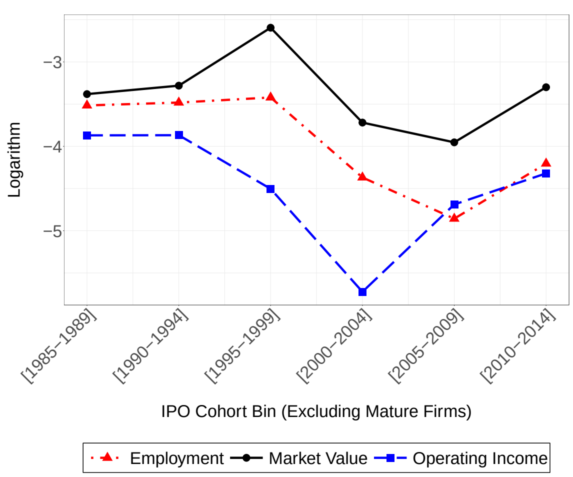


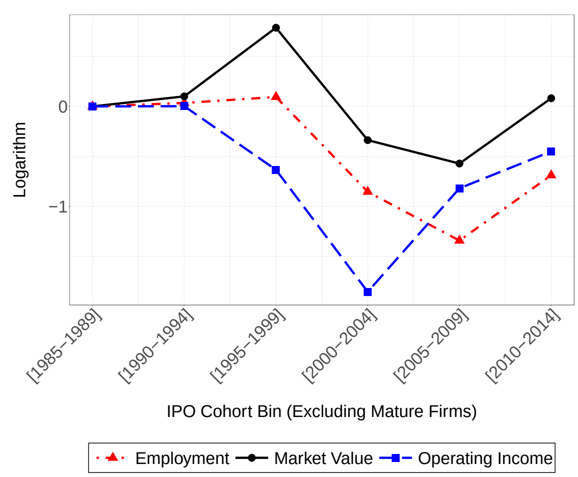
Figure F.3: **Employment, Operating Income, and Market Value Contributions of IPO Cohorts**

Data on employment, operating income, and market values of US public firms are taken from Compustat. Operating income is Compustat variable OIBDP. Data on firm founding years are described in the text. We exclude from IPO cohorts all firms that were founded more than 10 years prior to their IPO. We measure the employment, operating income, and market value contribution of an IPO cohort as a share of the total employment, operating income, and market value of public firms in the prior year. We then measure the contribution of an IPO cohort bin as the sum of the contributions of the different IPO cohorts in the bin. Panel A presents the logarithm of the employment, operating income, and market value contributions of each IPO cohort bin since 1985. Panel B presents the normalized (1985–1989 cohort = 0) logarithm of the employment, operating income, and market value contributions. Panel C presents the logarithm of the ratio of the operating income and market value contributions to the employment contributions. Panel D present the normalized (1985–1989 cohort = 0) logarithm of the ratio of the operating income and market value contributions to the employment contributions. See Section 2.1 for further details.

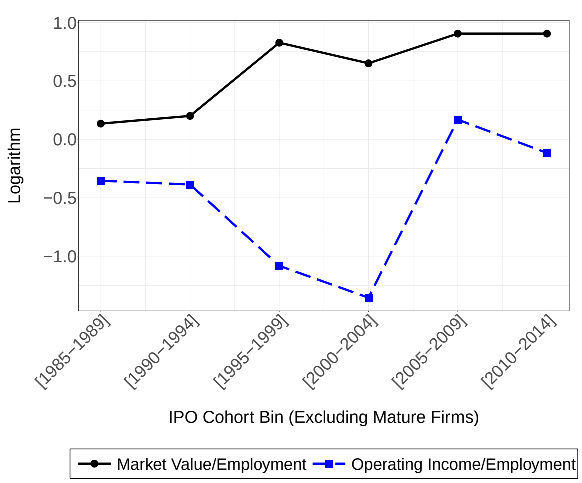
(a) Contribution



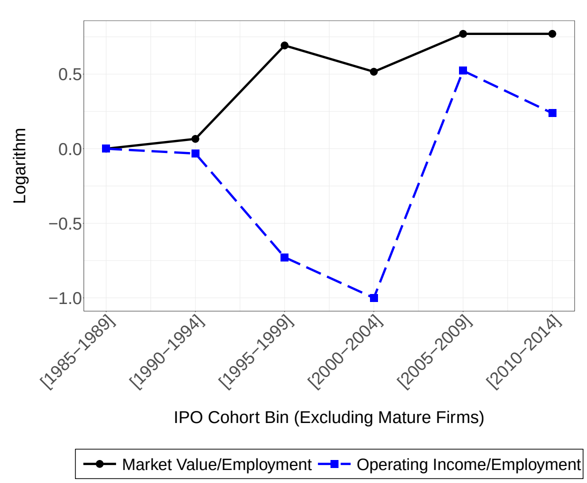
(b) Contribution Normalized



(c) Ratio of Contributions



(d) Ratio of Contributions Normalized



Appendix G Constructing Founding Year in NETS

This appendix describes our classification of changes in ownership and our adjustments to firm founding year that account for firm reorganizations, spin-offs, and mergers.

G.1 Classifying Changes in Ownership

We construct the set of all firms in year t that are destination of a switcher (destination) and all firms in year t-1 that are home of a switcher (home). Each home-destination pair is classified as one of the following mutually exclusive transactions.

1. **Reorganization** A home-destination pair is defined as a reorganization if all of the following are true:
   (a) The destination is a firm that had no establishments in year t-1.
   (b) The establishments of the destination firm are precisely the continuing establishments of the home firm.
2. **Spin-Off** A home-destination pair is defined as a spin-off if all of the following are true:
   (a) The destination is a firm that had no establishments in year t-1.
   (b) The establishments of the destination firm are a strict subset of the continuing establishments of the home firm.
3. **Merger** A home-destination pair is defined as a merger if all of the following are true:
   (a) The destination is a firm that had no establishments in year t-1.
   (b) The destination acquired establishments from more than one firm.
4. **Acquisition** A home-destination pair is defined as part of an acquisition if it is not a reorganization, spin-off, or merger. These are cases in which the destination is not a new firm.

G.2 Adjusting Firm Founding Year

We repeat the following process sequentially from the start to the end of the sample.

1. **Reorganization** In the case of a reorganization the destination firm is assigned the founding year of home firm.

2. **Spin-Off** In the case of a spin-off we distinguish between two possibilities. (1) If the spun-off destination is a new firm we assign the founding year of home firm. (2) If the spun-off destination had existed in the past we assign the minimum of the founding year of the home firm and the founding year of the previously existed firm. This second possibility arises in cases where a firm is purchased and then spun-off several years later.

3. **Merger** In the case of a merger the destination firm is assigned the founding year of largest of the home firms (measured by employment in year t-1).

G.3 Sample of Changes in Ownership

We exclude reorganizations from our sample of changes in ownership. The results are robust to including these in the sample.