CBDC, Monetary Policy Implementation, and The Interbank Market*

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Abstract

We study the effect of a central bank digital currency (CBDC) on the money market. A CBDC is equivalent to a 100% reserve requirement to fund those transactions that require CBDC, contrary to transactions that require bank deposits that only need partial reserve backing. We find that a higher fraction of transactions conducted with CBDC will drain reserves and tend to increase the interbank rate. The effect of CBDC remuneration is however ambiguous. A higher CBDC rate increases its value as a payment instrument. This leads to lower funding costs and larger investment, decreasing or increasing the demand for reserves and the interbank market rate, depending on which effect dominates. We show that a cap on CBDC will reduce the interbank rate and the deposit rate, as banks need less deposits to buy reserves. A CBDC design with tiered remuneration does not bring additional benefits relative to a single (lower) remuneration rate.

Keywords: Central bank digital currency (CBDC), monetary policy, interbank markets, payments

JEL-Codes: E42, E58, G21

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1 Introduction

Central bank digital currencies are high on the agenda of central banks. Some central banks already launched pilots to understand the implications of the CBDC design.[1] While the effects on the economy and the financial systems have been extensively studied, fewer studies seek to understand the effects of CBDC on the market for bank reserves and the implementation of monetary policy.[2] This paper seeks to fill that gap by analyzing the effects of CBDC on the demand for reserves in a model of the interbank market featuring uncertainty in the form of a Poole (1968) shock.

Specifically, we embed in a Poole (1968) model the environment of Chiu et al. (2022), but without banks having market power. In an initial round of investment, banks finance some entrepreneurs with deposits. Banks are subject to a reserve requirement as a function of the amount of deposits they issue. Banks can borrow reserves by issuing deposits and from each other on the interbank market. After the market for reserves closes, banks learn if the entrepreneurs they fund need to purchase more inputs, either with deposits or with CBDC. Hence, this is a refinancing shock for banks. The fraction of entrepreneurs who need to refinance is stochastic and is the Poole (1968) shock in our model. At this stage, banks only have access to the central bank's deposit and lending facilities. We assume that CBDC can be bought with reserves directly at the central bank. Since CBDC has a 100% reserve requirement in our model, a bank may have to borrow an amount of reserves at the lending facility which is equal to the full amount of CBDC needed to purchase more input goods.

In this context, we study the effect of increasing the market share of CBDC (the likelihood that entrepreneurs need CBDC), as well as the effect of the remuneration rate of CBDC on the interbank market rate and the level of banks' investment. We also study whether CBDC can serve as another policy tool and how it should be used, as well as the effects of limits on CBDC holdings.

Several key findings arise. First, as the market share of CBDC increases, it will clearly drain reserves. As a result, the interbank market rate will tend to increase and banks will tend to more often access the lending facility. Since reserves become more expensive, everything else constant, there will be disintermediation in the sense of lower investment. However, there will not necessarily be disintermediation in the sense of a reduction of banks' liabilities because banks will seek to "attract" deposits

[1] These include for example the Bank of China's digital Renminbi or the Sand dollar issued by the Central Bank of The Bahamas.

[2] A good reference is the special issue of the Journal of Economic Dynamics and Control (2022), as well as the CEPR report edited by Niepelt (2022).

in order to increase their reserve holdings by raising their deposit rates. Of course, the central bank could limit the drain in reserves and, therefore, the impact of CBDC on the money markets by supplying more reserves.

Second, we find that the effect of a higher CBDC remuneration rate on the money market is ambiguous, as it relies on two effects. By paying a higher interest rate on CBDC, the central bank makes it a better payment instrument that can buy more (investment) goods. Therefore, the same level of activity can be sustained with less CBDC, which plays to reduce the demand for reserves. This is the funding effect. However there is a counteracting investment effect: Since it is cheaper to fund entrepreneurs, banks invest more, which in turn puts pressure on the demand for reserves — the investment effect. Which effect dominates depends on the model parameters; specifically on how costly it is for banks to find more entrepreneurs with investment projects.

In a next step, we investigate the effect of quantitative limitations on the amount of CBDC holdings — another CBDC design feature that is actively discussed at central banks. Specifically, we assume that when entrepreneurs need more CBDC at the refinancing stage as the amount defined by the CBDC cap, they need to liquidate some of their initial investment. We find that for a given market share of CBDC or a given CBDC rate, a cap will decrease the demand for reserves and, therefore, the interbank market rate. By decreasing the effective productivity level, a cap could also decrease banks' investment and deposits. However, this decrease can be reversed when a tighter cap is accompanied by a higher CBDC rate.

Finally, we add to our model a two-tiered CBDC remuneration, as this is another design feature in current policy discussions (see, e.g. Bindseil (2020)). Specifically, CBDC holdings up to a certain threshold are remunerated at a higher rate than holdings above this threshold. We find that a tiered remuneration of CBDC is equivalent to a decrease in the CBDC rate when there is no two-tiered system in place.

The literature on CBDC is growing rapidly (a structured overview is provided by Ahnert et al. (2022a)). Major studies include Andolfatto (2020), Chiu et al. (2022), Keister and Sanches (2022), as well as Williamson (2022). How CBDC can affect the fragility of the financial system, see, for example, Keister and Monnet (2022) featuring bank runs, or Assenmacher et al. (2022) who describe a channel through which CBDC could stabilize the economy. On the use of CBDC for online transactions see Garratt and van Oordt (2021) and Ahnert et al. (2022b).

There are only few studies that focus on the demand for bank reserves in the presence of CBDC. Malloy et al. (2022) illustrate through stylized balance sheet analyses that CBDC could decrease aggregate reserves, putting upward pressure on the policy rate. Fegatelli (2021) points out that CBDC could improve banks' profitability by reducing the amount of potentially expensive excess reserves. We add to this literature by developing a dynamic general equilibrium with uncertainty regarding the amount of reserves needed. Through the lens of our model, we can analyze the effects of an increasing adoption of CBDC, as well as its remuneration rate, on the money market, bank deposits, and investment. Furthermore, we implement in our model different CBDC design features that are currently discussed at central banks, such as a holding limit or a tiered CBDC remuneration.

2 Environment

Our model combines elements of the money market model of Berentsen and Monnet (2009) and the CBDC model of Chiu et al. (2022) . Time $t = 1, 2, ...$ is discrete and continues forever. The discount factor is β. There are four types of agents: a measure one of buyers, sellers, and bankers, and a large measure (greater than one) of entrepreneurs. In addition, there is a central bank that manages the supply of reserves and offers a lending and a borrowing facility. In each period, two (goods) markets open sequentially: Goods market 1 (DM) and goods market 2 (CM). Both markets are Walrasian. An interbank market for central bank reserves opens at the same time as goods market 1.

Buyers and sellers Buyers and sellers are infinitely lived. Buyers have utility $u(y)$ from consuming y units of market 1 goods. $u(.)$ is increasing, strictly concave, and $u(0) = 0$. Sellers have a linear disutility $-y$ of producing y units of market 1 goods. As a result, the efficient level of market 1 good is y^* such that $u'(y^*) = 1$. Buyers cannot commit and promises cannot be enforced. Therefore, buyers need a means of payment to pay sellers on market 1. As will be clear below, they will use digital cash issued by the central bank and/or bank deposits.

In market 2 (CM), buyers and sellers work and consume x units of the consumption good of market 2. Their labor h is transformed into market 2 goods, one-for-one. The utility of consumption is $U(x)$, increasing, and strictly concave. All in all, buyers' and sellers' instant utility function for any period, given an allocation (y, x, h), is

$$\begin{aligned} U^B(y, x, h) &= u(y) + U(x) - h \\ U^S(y, x, h) &= -y + U(x) - h. \end{aligned}$$

Entrepreneurs Entrepreneurs are born in market 2 of period t, become old and die in market 2 of period $t+1$. So they live for 1 and 1/2 period. In the first market 2 of their life, entrepreneurs are "young" and then they become "old" when they enter the second market 2 of their life. Entrepreneurs cannot work in market 2 and they consume only when old. However, young entrepreneurs are endowed with a technology $F(x,y):\{0,1\}^2 \to \mathbb{R}_+$. The technology has two different returns depending on the state of nature,

$$F(1,y) = \begin{cases} Ax & \text{state 1} \\ Axy & \text{state 2} \end{cases},$$

where $x, y \in \{0,1\}$. In words, young entrepreneurs make an initial investment $x \in \{0,1\}$ of market 2 goods. In state 1 they produce Ax in the next market 2 when they are old. In state 2, young entrepreneurs have to purchase $y = 1$ of market 1 goods in order to bring their project to fruition, as long as they made an initial investment of $x = 1$. We interpret state 2 as a re-financing state. It will become clear that the probability of each state is only important for bankers and we will specify it later. In state 2, young entrepreneurs will purchase goods from sellers in market 1. However, there is no credit arrangement possible between entrepreneurs and sellers, and the former need a means to pay the latter. As will be clear below, they will use cash, CBDC and/or bank deposits.

Bankers Bankers have the same life span as entrepreneurs: they are born in market 2 of period t, become old in period $t+1$, and die in market 2 of period $t+1$. Bankers only consume market 2 goods when old. They have a commitment technology to repay their liabilities, therefore, the latter can be used as a means to pay. Also, bankers have a technology to enforce repayment from entrepreneurs. Therefore, they are willing to lend to entrepreneurs. There is free entry into the business of lending to entrepreneurs. A banker who lends to n entrepreneurs will suffer a utility cost $c(n)$ that can be interpreted as a search cost for n entrepreneurs. Bankers face uncertainty regarding the refinancing shock of entrepreneurs: the fraction of state 2 entrepreneurs in a banker's portfolio is $\gamma \in [0,1]$ and γ is distributed according to a distribution function with cdf $G(\cdot)$. A banker does not know his own γ when choosing n and only learns it in market 1.

A banker finances its loans in market 2 at date t and market 1 at date $t+1$ by issuing liquid checkable deposits to buyers and entrepreneurs, and possibly IOUs to other banks and the central bank. A banker's checkable deposits can be used as a medium of exchange between buyers, entrepreneurs who need reinvestment, and sellers in market 1. Finally, bankers are subject to a reserve requirement: they must hold in reserves at least a fraction $\chi \in [0, 1]$ of the amount of checkable deposits they issued. Required reserves are remunerated at rate i_r, which can be negative. Banks can purchase CBDC from the central bank by spending reserves.

Before learning their shock γ, bankers can trade reserves on an interbank market. The interbank market rate is R_m. As in Poole, there is no reason why banks would trade reserves on this market (since they are identical and they have the same information), but there is an equilibrium interest rate R_m that will leave them indifferent between borrowing or lending reserves.

Government/Central bank The central bank issues three forms of liabilities: cash, central bank reserves, and CBDC. Only bankers have access to reserves. CBDC is a digital entry on the central bank's balance sheet that can be used for retail payments, and it pays a net rate i_e, which can be negative. CBDC are like bank deposits that are fully backed by reserves at the central bank. Therefore, CBDC carries a 100% reserve requirement. Being ear-marked for CBDC, these reserves are remunerated at rate i_e (and whoever holds the CBDC can redeem it at the central bank and get the interest).

The central bank operates standing facilities for reserves: It offers a lending facility and stands ready to lend any amount of reserves bankers require at a rate $i_{\ell f}$. Similarly, it offers a deposit facility and remunerates at rate i_{df} any amount of reserves that bankers would deposit. The central bank also manages the supply of cash in the economy by making lump-sum transfers to buyers. As a result, cash grows at a constant rate $\mu \geq \beta$. Finally, seignorage goes to the government that rebates it through lump-sum transfers to buyers.

Market 1 and payment instruments Sellers are differentiated by the type of payment instruments they accept and they know their types. A fraction $\omega_1 \in [0, 1]$ of sellers only accept deposits, while a fraction $1 - \omega_1$ of sellers only accept CBDC. No sellers accept cash in our economy, and given the declining use of cash in modern economies, we do not see this assumption as being especially strong.[3] To keep things simple, we assume those sellers are located on two separate trading venues, trading venue 1 and 2 (tv-1 and tv-2) and both trading venues are Walrasian markets (with possibly different prices). Buyers and entrepreneurs cannot choose their trading venues and we assume that with probability ω_1 they enter tv-1 and with probability

[3] In Appendix ____ we analyze the case where sellers accept both cash and deposits indifferently.

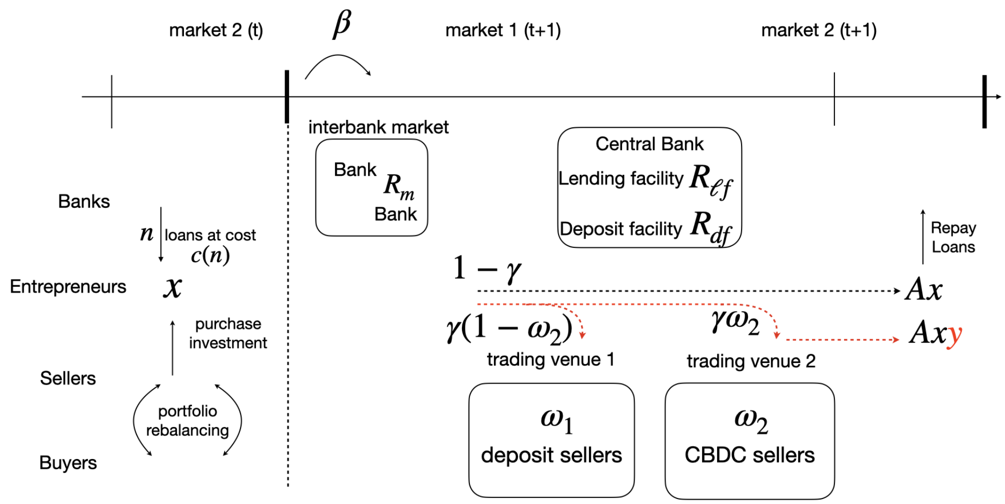

Figure 1: Timeline

$\omega_2 = 1 - \omega_1$ they enter tv-2.[4] Since buyers have no use for cash, they will not bring any into market 1. This does not mean, however, that cash will have no value, but it will only be used in market 2, by banks as reserves and to pay the interest on deposits previously issued, and by buyers who can purchase deposits with it.

Timing Figure 1 shows the timing of the economy.

We proceed to solve this economy by first looking at the buyers' and sellers' decisions, which are standard. Then we consider the banks' problem. Since entrepreneurs have no bargaining power, the banks' problem is the most interesting to analyze.

3 Agents' problems and market clearing

Buyers' and sellers' problems

The buyers' and sellers' problems are rather standard and follow the same structure as in the basic Lagos and Wright (2005) literature. First, it is clear that a seller will

[4]We could have assumed that buyers and entrepreneurs meet randomly sellers, at the cost of having to introduce tools from bargaining. We chose to keep market 1 as simple as possible while using some ideas from models with random matching.

carry no means of payment from market 2 to market 1 since they only produce and they have no needs to consume in market 1. Therefore, a seller of type m (which refers to the payment type they accept) solves

$$W_S^m(\mathbf{a}) = \max U(x) - h + \beta V_S^m$$

subject to

$$x = h + \mathbf{R}.\mathbf{a},$$

with

$$V_S^m = \max_y -y + p_m y + W_m^S(0),$$

where the price in trading venue $m = 1, 2$ in terms of market 1 goods is p_m. Hence, the only equilibrium has for both trading venues,

$$p_m = 1.$$

We now turn to buyers. In market 2, a buyer solves

$$W_B(\mathbf{a}) = \max U(x) - h + \beta V_B(\mathbf{a}')$$

subject to

$$x + \mathbf{1}.\mathbf{a}' = T + h + \mathbf{R}.\mathbf{a},$$

where $\mathbf{a} = (e, d)$ is the vector of real CBDC (e) and bank deposits (d), while $\mathbf{R}$ is the vector of gross real return $\mathbf{R} = (R_e, R_d)$ where $R_s = (1 + i_s)/\mu$. At this stage, it is important to notice that inflation μ will directly negatively affect R_e but not necessarily R_d since i_e is a policy variable, while the effect on R_d will be determined in equilibrium.

Given the price is $p_m = 1$ in both trading venues $m = 1, 2$, the buyer's value function when entering market 1 is

$$V_B(\mathbf{a}) = \sum_{m=1,2} \omega_m \left[u(Y_m(\mathcal{L}_m)) - Y_m(\mathcal{L}_m) + W_B(\mathbf{a}) \right],$$

where

$$\begin{aligned} \mathcal{L}_1 &= R_d d \\ \mathcal{L}_2 &= R_e e \end{aligned}$$

is the usable liquidity in trading venue $m = 1, 2$, including the expected return of the asset. The buyers' budget constraint in both trading venues is

$$Y_m \leq \mathcal{L}_m \qquad (\lambda(\mathcal{L}_m)).$$

Hence, the first order condition of buyers in trading venue m of market 1 gives

$$u'(Y(\mathcal{L}_m)) = 1 + \lambda(\mathcal{L}_m),$$

with the envelope condition (with some abuse of the vectorial notation)

$$V_B'(\mathbf{a}) \;=\; \sum_{m=1}^{2} \omega_m \left[u'(Y(\mathcal{L}_m)) - 1\right] \mathbf{R}\mathbb{I}_{\{\text{a used in venue } m\}} + \mathbf{R}.$$

The first order condition of buyers in market 2 is

$$\beta V_B'(\mathbf{a}') \leq 1 \quad (= \text{ if } \mathbf{a}' > 0).$$

Combining the last two equations, we obtain the Euler conditions of buyers,

$$\begin{aligned} \omega_1 \left[u'(Y(\mathcal{L}_1)) - 1\right] R_d + R_d &\leq \beta^{-1}, \\ \omega_2 \left[u'(Y(\mathcal{L}_2)) - 1\right] R_e + R_e &\leq \beta^{-1}. \end{aligned}$$

Since buyers do not bring any cash, $R_z < 1/\beta$, and

$$(\beta R_d)^{-1} = \begin{cases} 1 & \text{if } Y^* \leq R_d d \\ \omega_1 u'\left(R_d d\right) + (1 - \omega_1) & \text{otherwise} \end{cases}$$

and

$$(\beta R_e)^{-1} = \begin{cases} 1 & \text{if } Y^* \leq R_e e \\ \omega_2 u'\left(R_e e\right) + (1 - \omega_2) & \text{otherwise} \end{cases}.$$

Finally, notice that since increasing inflation μ will decrease R_e, it affects the amount of CBDC that buyers choose to carry over from market 2 at t to market 1 at $t + 1$.

Bankers' problem

There is a measure one of bankers. Banker j chooses to invest with n_j entrepreneurs in market 2 when they are born. Bankers fund entrepreneurs by issuing deposits. Bankers are perfectly competitive in lending and in issuing deposits. In the following market 1, a banker learns that it will have to refinance a fraction γ of these loans, where γ is i.i.d. across bankers. We assume that, of those entrepreneurs who need to invest an additional unit, a fraction ω_1 do it in the first trading venue and the rest in the second trading venue. Therefore, a bank's problem in the CM is to choose how many entrepreneurs n_j to fund (they will each need deposits to purchase 1 unit of the investment good in the CM), how many deposits d_j to issue and how much reserves r_j to hold, to maximize

$$\max_{n_j, r_j, d_j} \quad V\left(n_j, r_j - \chi d_j\right) + \underbrace{R_r \chi d_j}_{\text{int on req. res.}} - R_d d_j \tag{1}$$

subject to

$$\begin{aligned} n_j + \underbrace{r_j}_{\text{cash reserves}} &= d_j, \\ r_j &\geq \chi d_j. \end{aligned}$$

The banker issues d_j deposits to fund n_j entrepreneurs in the CM, and also to purchase cash reserves r_j from buyers and /or sellers. For each deposit it issues, the banker has to pay the equilibrium deposits (real) rate R_d, and it has to set aside a fraction $\chi \in [0, 1]$ of required reserves remunerated at the (real) rate R_r. $V(n, e)$ is the banker's value of holding a portfolio of n loans and excess reserves $e = r - \chi d$.

Next, in market 1 of period $t + 1$, bankers learn the extent to which they have to refinance their loans to entrepreneurs. A banker draws the fraction γ of entrepreneurs it has to refinance from a distribution $G(\gamma)$. Given γ, a banker has to refinance γn entrepreneurs. A fraction ω_1 of those will have to purchase the (investment) good on the first trading venue with deposits, and a fraction ω_2 will need CBDC to purchase the (investment) good on the second trading venue. We assume the banker refinances all entrepreneurs who need refinancing.[5]

Therefore, the bank issues $\omega_1 \gamma n / R_d$ new deposits to $\omega_1 \gamma n$ entrepreneurs so that they can each purchase 1 unit of the investment good with deposits on tv-1. The bank also acquires $\omega_2 \gamma n / R_e$ new CBDC from the central bank so that the $\omega_2 \gamma n$ entrepreneurs on tv-2 can purchase 1 unit of the investment good with CBDC. The bank acquires CBDC by using its excess reserves or borrowing reserves from the interbank market or the central bank (issuing IOUs).

When new deposits are used as means to pay sellers, the reserve requirement is still $\chi < 1$, while the banker needs the full amount of reserves to purchase CBDC. Therefore, at the refinancing stage, the banker has $r_j - \chi d_j$ reserves that it carried over from the last period's market 2, plus the amount of reserves y_j the banker borrowed on the interbank market, and it needs $\frac{\omega_1 \gamma n_j}{R_d} \chi + \frac{\omega_2 \gamma n_j}{R_e}$. Hence, the banker

[5] A priori, the bank could choose to refinance some but not all entrepreneurs. We assume $A > A^*$ (defined in Appendix XYZ) so that the banker chooses to refinances all entrepreneurs on both trading venues.

has a reserve shortfall if

$$\frac{\omega_1 \gamma n_j}{R_d}\chi + \frac{\omega_2 \gamma n_j}{R_e} - (r_j - \chi d_j + y_j) \quad \equiv \quad \sum_{s=1,2} \chi_s \omega_s \gamma n_j - (r_j - \chi d_j + y_j) > 0$$

with

$$\chi_1 \quad \equiv \quad \chi / R_d \quad \text{and } \chi_2 \equiv 1/R_e.$$

If the inequality is reversed, the banker has a reserve surplus. This expression is positive (a reserve shortfall) whenever the refinancing shock is large enough, that is if $\gamma > \bar{\gamma}$, and a surplus otherwise, where $\bar{\gamma}$ is

$$\bar{\gamma}(d_j, n_j, y_j) = \frac{(r_j - \chi d_j + y_j)}{\sum_s \omega_s \chi_s n_j}.$$

Since the refinancing shock γ is stochastic, we obtain an expression for $V(n, e)$ that is close to the one in the famous contribution by Poole (1968) on banks' reserve management:

$$V(n_j, \underbrace{r_j - \chi d_j}_{e}) = A n_j - c(n_j)/\beta \tag{2}$$

$$+ \max_{y_j} \left[\begin{array}{c} \int_{\gamma < \bar{\gamma}(d_j, n_j, y_j)} R_{df} \underbrace{\left((r_j - \chi d_j + y_j) - \sum_s \omega_s \chi_s \gamma n_j \right)}_{\text{long in reserves - can lend}} dG(\gamma) \\ - \int_{\gamma \geq \bar{\gamma}(d_j, n_j, y_j)} R_{\ell f} \underbrace{\left(\sum_s \omega_s \chi_s \gamma n_j - (r_j - \chi d_j + y_j) \right)}_{\text{short in reserves - needs to borrow}} dG(\gamma) - R_m y_j \end{array} \right]$$

$$- \int \omega_1 \gamma n_j dG(\gamma) + \int \underbrace{\frac{\omega_1 \gamma n_j}{R_d} \chi R_r}_{\text{interest on required reserves}} dG(\gamma)$$

The first line of (2) shows the gain from investing with n_j entrepreneurs, net of the investment (search) cost which occured at the end of the previous period. The second line of (2) shows the problem of the banker on the interbank market: When the banker faces a low enough refinancing shock ($\gamma < \bar{\gamma}$) it has a reserve surplus after refinancing all entrepreneurs and can deposit that surplus at the deposit facility. If the banker faces a high refinancing shock, it will have to cover the reserve deficit at the lending facility. The first term on the last line refers to the interest payment that the bank has to make when it issued $\omega_1 \gamma n / R_d$ new deposits: then she will have to pay the interest rate cost $R_d (\omega_1 \gamma n / R_d)$ on it. Finally, the last term of (2) shows the interest rate that the bank obtains on the required reserves backing newly issued deposits.

The solution of the problem of the banker on the interbank market is standard and the first order condition gives y_j as the solution to

$$R_m = R_{df} G(\bar{\gamma}(d_j, n_j, y_j)) + R_{\ell f} \left[1 - G(\bar{\gamma}(d_j, n_j, y_j))\right]. \tag{3}$$

The marginal cost of borrowing more reserves on the interbank market R_m has to equal the marginal gain of having more reserves, which is an average of the standing facility rates, weighted by the probability to access those facilities. Accordingly, the effect of a lower R_m is to increase the demand for reserves in the interbank market. Since the marginal cost of borrowing is lower, the marginal gain must also be lower. But, everything else constant, increasing y_j makes $\bar{\gamma}(d_j, n_j, y_j)$ higher, since a larger shock is needed for the bank to run short of reserves. This implies that the bank is more likely to have to use the central bank deposit facility than its lending facility, which lowers the gain of holding more reserves, that is the right hand side of (3).

Figure 2 shows a typical demand for reserves y_j for different levels of ω_2 and R_e. For a given level of the interbank market rate R_m, an increase in the market share of CBDC ω_2 will increase the demand for reserves, and the demand becomes much less elastic. While the effect of ω_2 is more similar to a higher demand in the standard Poole model (a change in the mean of the shock distribution), the effect of a higher CBDC remuneration rate R_e is similar to a lower variance (more certainty) in the distribution of the liquidity shock in the Poole model: Banks are more certain they will be less likely to be short in reserves. Since they are more sure of their liquidity need, at the same rate, they borrow/lend less reserves.

From there, we derive the marginal payoff of lending to more entrepreneurs, $\frac{\partial V}{\partial n_j}$, as well as the marginal payoff of holding more excess reserves, $\frac{\partial V}{\partial e}$, ahead of the refinancing shock,

$$\begin{aligned} \frac{\partial V}{\partial n_j} &= A - c'(n_j)/\beta - \int \omega_1 \gamma dG(\gamma) + \int \frac{\omega_1 \gamma}{R_d} \chi R_r dG(\gamma) \qquad (4) \\ &\quad - \left[\int_{\gamma < \bar{\gamma}(d_j, n_j, y_j)} R_{df} \left(\sum_s \omega_s \chi_s \gamma \right) dG(\gamma) + \int_{\gamma \geq \bar{\gamma}(d_j, n_j, y_j)} R_{\ell f} \left(\sum_s \omega_s \chi_s \gamma \right) dG(\gamma) \right] \\ \frac{\partial V}{\partial e_j} &= \int_{\gamma < \bar{\gamma}(d_j, n_j, y_j)} R_{df} dG(\gamma) + \int_{\gamma \geq \bar{\gamma}(d_j, n_j, y_j)} R_{\ell f} dG(\gamma) = R_m \qquad (5) \end{aligned}$$

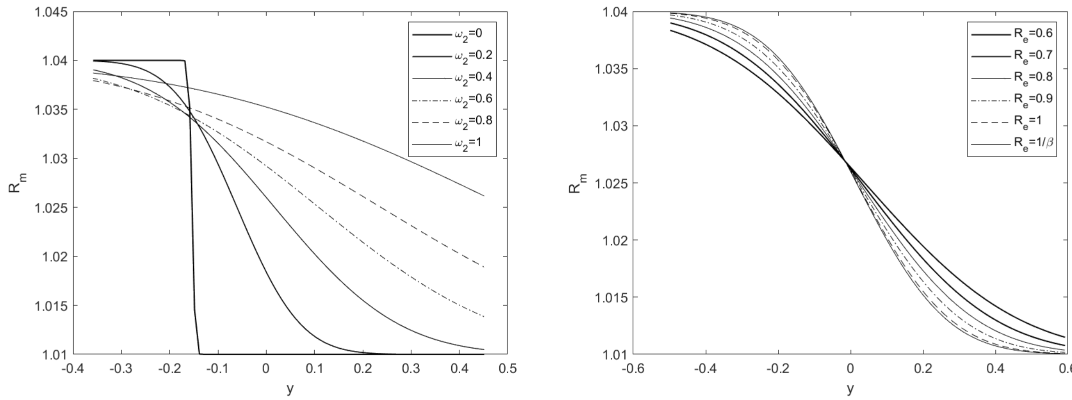

Figure 2: Demand for reserves y_j.

Notice that we can disregard the effect of n_j and e_j on $\bar{\gamma}$ because the integrated function is zero at $\bar{\gamma}$. The marginal benefit of lending to entrepreneurs (4) include the net marginal costs of investment $A - c'(n_j)/\beta$, as well as the marginal interest rate margin $(R_d - \chi R_r)$ on the additional ω_1/R_d deposits, minus the expected refinancing costs. That refinancing cost is a weighted average of the standing facility rates since the liquidity shocks happen after the interbank market closes and have to be covered by accessing the central bank facilities.

(5) shows the benefit of holding more excess reserves when exiting the CM and has to equal the interbank market rate because reserves can always be borrowed there.

Replacing $r_j = d_j - n_j$ in (1), we then obtain the first order conditions of the banker,

$$n_j \quad : \qquad \frac{\partial V}{\partial n_j} - \left(\frac{\partial V}{\partial e} + \lambda \right) = 0, \tag{6}$$

$$d_j \quad : \qquad R_r \chi - R_d + (1 - \chi) \left(\frac{\partial V}{\partial e} + \lambda \right) = 0, \tag{7}$$

where λ is the Lagrange multiplier on the reserve requirement constraint. In a symmetric equilibrium, all bankers finance the same amount of entrepreneurs so that (6) gives the overall level of intermediation in this economy by setting $n_j = N$, and (7) will give us the total amount of private liabilities, $d_j = D$.

Interbank market clearing

The interbank market clearing condition is

$$\int y_j dj = 0.$$

Since all banks are the same, it must be that $y_j = 0$ for all j. Since $n_j = N$ and $d_j = D$, we obtain the interbank market rate as

$$R_m = R_{df} G(\bar{\gamma}(D, N, 0)) + R_{\ell f} \left[1 - G(\bar{\gamma}(D, N, 0))\right]. \tag{8}$$

4 Equilibrium

In this section we analyze two equilibrium regimes. One where the reserve constraint binds (so $\lambda > 0$) and one where it does not (and $\lambda = 0$). For brevity, we write $\bar{\gamma}(D, N) \equiv \bar{\gamma}(D, N, 0)$.

Suppose the reserve constraint does not bind, $\lambda = 0$. Then combining (7) and (8) we obtain

$$R_{df} G(\bar{\gamma}(D, N)) + R_{\ell f} \left[1 - G(\bar{\gamma}(D, N))\right] = \frac{R_d - R_r \chi}{1 - \chi}, \tag{9}$$

while combining (6) together with (8) gives

$$\begin{aligned} A - c'(N)/\beta - \int \omega_1 \gamma dG(\gamma) + \int \frac{\omega_1 \gamma}{R_d} \chi R_r dG(\gamma) &= \\ \left[\int_{\gamma < \bar{\gamma}(D,N)} R_{df} \Omega(\gamma) dG(\gamma) + \int_{\gamma \geq \bar{\gamma}(D,N)} R_{\ell f} \Omega(\gamma) dG(\gamma)\right] &, \end{aligned} \tag{10}$$

where

$$\Omega(\gamma) \equiv 1 + \sum_s \omega_s \chi_s \gamma.$$

Finally, the deposit rate R_d is given by

$$(\beta R_d)^{-1} = \begin{cases} 1 & \text{if } Y^* \leq R_d D \\ \omega_1 u'(R_d D) + (1 - \omega_1) & \text{otherwise} \end{cases}$$

These are three equations in three unknowns R_d, D, and N. If the solution (R_d, D, N) to this system of equations satisfies the non-binding reserves constraint, $(1 - \chi)D \geq N$, then this is the solution.

Suppose the reserve constraint binds ($\lambda > 0$). Then

$$(1-\chi)D = N \tag{11}$$

and combining (6) and (7), we obtain

$$A - c'(N)/\beta - \int \omega_1 \gamma dG(\gamma) + \int \frac{\omega_1 \gamma}{R_d} \chi R_r dG(\gamma) \tag{12}$$

$$-\left[\int_{\gamma<\bar{\gamma}(D,N)} R_{df}\Omega(\gamma)dG(\gamma) + \int_{\gamma\geq\bar{\gamma}(D,N)} R_{\ell f}\Omega(\gamma)dG(\gamma)\right] = \frac{R_d - R_r\chi}{1-\chi} - R_m.$$

and the interbank market is given by

$$R_m = R_{df}G(\bar{\gamma}(D,N)) + R_{\ell f}\left[1 - G(\bar{\gamma}(D,N))\right]. \tag{13}$$

The deposit rate R_d still satisfies

$$(\beta R_d)^{-1} = \begin{cases} 1 & \text{if } Y^* \leq R_d D \\ \omega_1 u'(R_d D) + (1-\omega_1) & \text{otherwise} \end{cases}$$

This is an equilibrium whenever $\lambda > 0$ or

$$\frac{R_d - R_r\chi}{1-\chi} > R_m.$$

5 Simulations

In this section we parametrize the model and derive some simulations for this economy. We are mainly interested in answering two questions: First, what is the impact of increasing the share of CBDC use (ω_2) on investment and the interbank market rate? Second, what are the effects of different CBDC remuneration rates?

Parametrization

In all our simulations, we choose the following functional forms and parameter values (unless otherwise stated). The utility function in market 1 is CRRA, $u(x) = x^{(1-\rho)}/(1-\rho)$, while the search cost for entrepreneurs is $c(N) = 0.5cN^2$ where c is a constant. It will turn out that this cost function is very important to determine the impact of CBDC on interbank markets. We use parameter values reported in Table 1. As will become clear below, we fix these parameter values so that we can analyze the effect of CBDC on the two equilibrium regimes of interest. We assume that γ is uniformly distributed in [0,1] with $E(\gamma) = 0.5$.

Parameter	Value	Description
β	0.96	discount factor
ω_2	0.4	share of tv-2 sellers (CBDC)
ω_1	$1 - \omega_2$	share of tv-1 sellers (deposits)
χ	0.01	reserve requirement
R_r	1.01	interest rate on required reserves (tier 1)
R_e	1.00	interest rate on CBDC
R_{df}	1.01	deposit facility rate (tier 2)
R_{lf}	1.04	lending facility rate
ρ	0.3	CRRA parameter in $u'(X) = X^{-\rho}$
A	2.62	Output per unit of investment
c_L/β	1.5	cost parameter in $c'(N) = c_L N$
c_H/β	2.15	cost parameter in $c'(N) = c_H N^2$

Table 1: Parameter values

CBDC and the demand for reserves

CBDC market share To understand the effects of introducing CBDC on our economy, we first simulate the effect of an increase in the share of CBDC use by sellers, ω_2. This captures whether the central bank makes CBDC more or less attractive for users, with a higher ω_2 being associated with a more attractive CBDC, be it through the absence of fees, or non-pecuniary benefits such as ease of use or the preservation of privacy. Then we concentrate more precisely on the effect of remunerating CBDC: We fix ω_2 and increase R_e. In each simulation, we look at the level of bank intermediation N and D, and we analyze the effect on the deposit rate R_d and the interbank market rate R_m, as well as access to the central bank's standing facilities, $DF$and LF. To understand the effects we also report the aggregate amount of reserves in the economy and $\bar{\gamma}$.

There are three zones in the each graph of Figure 3, that represent different usage of CBDC.

Consider an increase in ω_2 from zero but below around 0.2 so that CBDC has a market share below 20% (of course this value depends on our parametrization) – this is zone 1. Figure 3 shows that there is no effect of increasing ω_2 on $\bar{\gamma}$ in that zone: All banks have enough reserves to satisfy even the highest refinancing shock. As a result, no bank needs to access the central bank's lending facility, while some banks access the deposit facility. In zone 1, both the interest rate on deposits and the interbank market rate equal the deposit facility rate R_{df}. Increasing ω_2 in zone 1

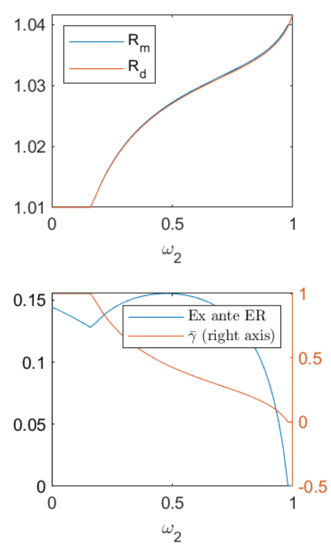

Figure 3: CBDC drains reserves.

does not affect bank's investment N, but there is a decreasing demand for deposits form households since some sellers no longer accept deposits as means of payment but only CBDC. As a result, the level of reserves held by banks ahead of the interbank market, $ER = (1-\chi)D - N$ (ER in Figure 3), decreases.

When ω_2 lies further above 0.2 but below a number close to 1, zone 2 is entered. There, the increasing need for reserves (due to higher CBDC usage) implies that now some banks will be short in reserves ($\bar{\gamma}$ falls below 1). Since banks put a positive probability on having to access the lending facility, the interbank market rate increase from R_{df}. This increases the value of reserves, and banks seek to acquire additional reserves by increasing the deposit rate R_d above R_{df}. While the market share of CBDC increases, the increase in R_d leads, at first, households to demand more (and not less!) deposits. In other words, the effect from an increase in R_d dominates the effect from a higher ω_2 on the demand for deposits, as long as ω_2 stays relatively low. For higher values of ω_2, the direct effect of the CBDC market share eventually dominates and D falls. However, reserves are now more expensive, which is reflected in lower investment N. Accordingly, the level of reserves held by banks ahead of the interbank market first increases and then falls as ω_2 increases.

It is maybe surprising that the interbank market rate increases although banks hold more excess reserves for some small to intermediate values of ω_2. To understand the mechanism, it is useful to look at the threshold $\bar{\gamma}$. In equilibrium, it is

$$\bar{\gamma}(D, N, 0) = \frac{(1-\chi)D - N}{\sum_s \omega_s \chi_s N}.$$

So the interbank market rate will increase with ω_2 ($\bar{\gamma}$ will decline) whenever the change in reserve requirements (here $(1-\chi)D$) offsets the increase in reserves.

For even higher values of ω_2 closer to 1, the economy is in zone 3 where, given policy rates, the demand for CBDC is so high that banks will always access the lending facility. Then the reserve requirement binds ($\lambda > 0$). In that case, the deposit rate R_d increases above the interbank rate which equals the lending facility rate $R_{\ell f}$. Banks hold no reserves when entering the interbank market.

From this narrative of the evolution of the economy as CBDC becomes more accepted by sellers, it is clear that banks will need more reserves (if only because they need 1 unit of reserves to purchase 1 unit of CBDC, while they would need only $\chi < 1$ reserves to issue 1 unit of deposits). As a result of this draining of reserves, the interbank market rate will tend to increase and banks will access more often the lending facility (zones 2-3). There is disintermediation in the sense of lower investment (in zone 2 and 3, N declines with ω_2). However, there is not necessarily disintermediation in the sense of a reduction of banks' liabilities (in zone 2, deposits D can increase with ω_2).

CBDC remuneration rate We have shown above that, everything else constant, an increase in CBDC usage induces an upward pressure on money market rates. Is there any countervailing force to this, in particular, can the central bank reduce the attractiveness of CBDC by lowering its remuneration rate? We answer this question next by analyzing the effect of changing R_e given a fixed market share of CBDC, $\omega_2 = 0.4$, as shown in Figure 4. Notice that we have values of $R_e < 1$, which implies a rate penalizing CBDC usage (i.e. $i_e < 0$).[6]

As the CBDC rate R_e increases, CBDC gains purchasing power and becomes more attractive as a means of payment. Since $\rho < 1$, households demand more of it. The increase in R_e also leads to a decline in $\Omega(\gamma)$ for all γ: This is the "funding effect"; a higher purchasing power reduces the cost of funding one entrepreneur. This funding effect plays to lower the demand for reserves. However, the funding effect gives rise to a subsequent "investment effect"; banks can afford to do more intermediation and N increases. This investment effect plays to increase the demand for reserves

[6] One may wonder if $R_e < 1$ would be possible in a world with cash. We answer yes, as long as we see CBDC being used for online transactions.

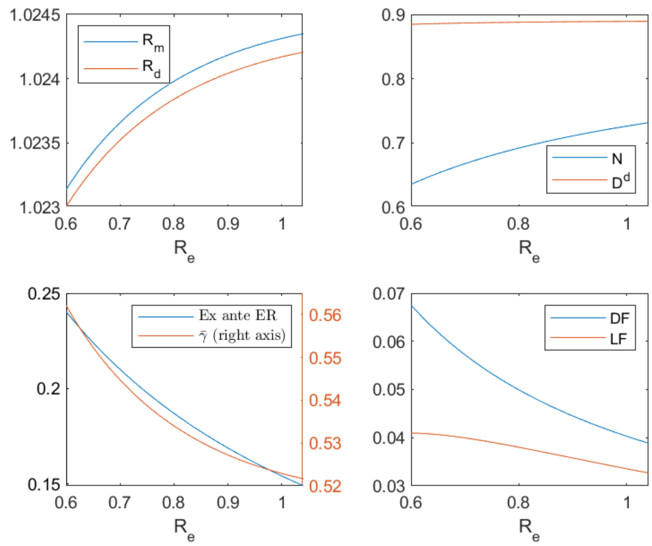

Figure 4: The demand for reserves and R_m are increasing in the CBDC rate.

in the money market and, hence, to increase the money market rate R_m, as shown in Figure 4. If banks expect to need more reserves in market 1, they increase their reserve holdings by increasing the deposit rate R_d. As a result, deposits increase somewhat. However, since N increases by more, excess reserves $(1-\chi)D - N$ held by banks in market 1 ahead of the refinancing shock decline, even if the investment effect dominates the funding effect.

Ambiguous effect of CBDC remuneration rate on R_m We may conclude that the effect of a larger market share of CBDC on the money market can be undone by decreasing its remuneration rate R_e, so as to make this payment instrument less attractive. However, the intuition that a lower R_e always reduces the pressure on the demand for reserves, and in turn the money market rate R_m, is deceptive. Indeed, as the previous paragraph explains, demand for reserves in the money market increases with R_e whenever the investment effect dominates the funding effect. This last condition may, however, not always be satisfied and R_m could decrease with R_e if $c''(N)$ is large, as Figure 5 shows. The intuition is simple: While banks find it cheaper to fund entrepreneurs when R_e is larger, they still incur a search cost $c(N)$. If $c''(N)$ is too large, the increase in intermediation N may not be enough to offset the lower need for reserves. In this case, R_m will drop. Since banks need less reserves, R_d is

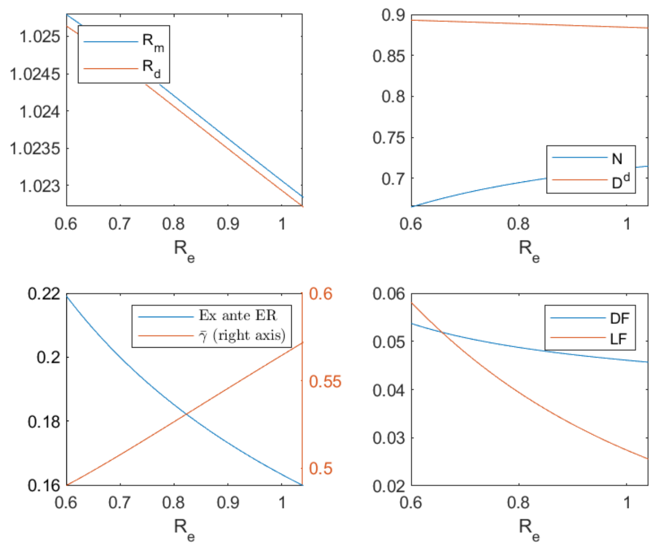

Figure 5: Large $c''(N)$: the demand for reserves and R_m are decreasing in the CBDC rate.

also decreasing in R_e, as is the demand for deposits D. As a consequence, the level of excess reserves held by banks ahead of the money market ($ER = (1-\chi)D - N$) is lower. This may be the case, for example, in downturns when it is allegedly more difficult for banks to find projects with net positive present value.

For completeness, let us stress that the effects of ω_2 that we analyzed in the previous section are not substantially different when $c''(\cdot)$ is higher.

6 A cap on CBDC holdings

Since quantitative limitations on the amount of CBDC holdings are a central part of central banks' discussions on CBDC, in this section, we investigate the role of such caps in our model. We model the cap as a limit $\bar{E}$ on CBDC holdings: no one can hold more CBDC than $\bar{E}$. We first describe the bankers' problem in the presence of a CBDC cap and derive the new equilibrium. In simulations, we illustrate the effects of quantitative limitations on CBDC.

The banker's problem in the presence of a CBDC cap

We assume that entrepreneurs who need CBDC to purchase capital at the refinancing stage have to liquidate some of their initial investment if they need more than the cap on CBDC, $\bar{E}$.[7] Therefore, the bank acquires $\omega_2\gamma n \min\{1/R_e, \bar{E}\}$ new CBDC from the central bank such that the $\omega_2\gamma n$ entrepreneurs on tv-2 can purchase one unit of the investment good with CBDC if $1/R_e < \bar{E}$, or $R_e\bar{E}$ units of the investment good if $\bar{E} < 1/R_e$. In the latter case, the entrepreneur can only produce $A(R_e\bar{E})$ and has to scrap $1 - R_e\bar{E}$ which yields no value. Now, the bank needs $\frac{\omega_1\gamma n_j}{R_d}\chi + \omega_2\gamma n_j \min\{1/R_e, \bar{E}\}$ reserves at the refinancing stage. Hence, the banker has a reserve shortfall if

$$\frac{\omega_1\gamma n_j}{R_d}\chi + \omega_2\gamma n_j \min\{1/R_e, \bar{E}\} - (r_j - \chi d_j d_j + y_j) \quad \equiv \quad \sum_{s=1,2} \chi_s\omega_s\gamma n_j - (r_j - \chi d_j d_j + y_j) > 0$$

with

$$\chi_1 \quad \equiv \quad \chi/R_d \quad \text{and } \chi_2 \equiv \min\{1/R_e, \bar{E}\}.$$

It will be useful to define the effective productivity of entrepreneurs as

$$\mathcal{A}(\bar{E}, R_e) = A \int \left[(1-\gamma) + \gamma(\omega_1 + \omega_2 \left(R_e \min\{1/R_e, \bar{E}\}\right)\right] dG(\gamma)$$

Now, the equations characterizing the equilibrium when the reserve constraint does not bind, hence, $(1-\chi)D \geq N$ with $\lambda = 0$, read as:

$$R_{df}G(\bar{\gamma}(D,N)) + R_{\ell f}\left[1 - G(\bar{\gamma}(D,N))\right] = \frac{R_d - R_r\chi}{1-\chi},$$

$$\begin{aligned} \mathcal{A}(\bar{E}, R_e) - c'(N)/\beta - \int \omega_1\gamma dG(\gamma) + \int \frac{\omega_1\gamma}{R_d}\chi R_r dG(\gamma) \quad &= \\ \left[\int_{\gamma<\bar{\gamma}(D,N)} R_{df}\Omega(\gamma)dG(\gamma) + \int_{\gamma\geq\bar{\gamma}(D,N)} R_{\ell f}\Omega(\gamma)dG(\gamma)\right] \quad &, \end{aligned}$$

where

$$\Omega(\gamma) \equiv 1 + \sum_s \omega_s\chi_s\gamma \text{ with } \chi_2 \equiv \min\{1/R_e, \bar{E}\}.$$

[7] One justification is that entrepreneurs have to refinance from only one seller who cannot hold more than $\bar{E}$ units of CBDC.

Finally, the deposit rate R_d is given by

$$(\beta R_d)^{-1} = \begin{cases} 1 & \text{if } Y^* \leq R_d D \\ \omega_1 u'(R_d D) + (1-\omega_1) & \text{otherwise.} \end{cases}$$

When the reserve constraint binds and ($\lambda > 0$), these equations become:

$$(1-\chi)D = N$$

$$\mathcal{A}(\bar{E}, R_e) - c'(N)/\beta - \int \omega_1 \gamma dG(\gamma) + \int \frac{\omega_1 \gamma}{R_d} \chi R_r dG(\gamma)$$
$$- \left[\int_{\gamma < \bar{\gamma}(D,N)} R_{df} \Omega(\gamma) dG(\gamma) + \int_{\gamma \geq \bar{\gamma}(D,N)} R_{\ell f} \Omega(\gamma) dG(\gamma) \right] = \frac{R_d - R_r \chi}{1-\chi} - R_m.$$

$$R_m = R_{df} G(\bar{\gamma}(D,N)) + R_{\ell f} \left[1 - G(\bar{\gamma}(D,N)) \right].$$

where, again,

$$\Omega(\gamma) \equiv 1 + \sum_s \omega_s \chi_s \gamma \text{ with } \chi_2 \equiv \min\{1/R_e, \bar{E}\}.$$

$$(\beta R_d)^{-1} = \begin{cases} 1 & \text{if } Y^* \leq R_d D \\ \omega_1 u'(R_d D) + (1-\omega_1) & \text{otherwise.} \end{cases}$$

Simulations

In the following simulations, we compare two different caps on CBDC: a large one with $\bar{E} = 1.3$ and a low one with $\bar{E} = 0.95$. In the simulations in Figure 6 in which we increase the share of CBDC meetings, ω_2, the CBDC rate is kept constant at $R_e = 1.00$, implying that the low cap (dashed line) is always binding, while the high one (solid line) is never binding.

When the cap binds, less CBDC can be used for refinancing the investment projects, leading to the liquidation of some of the initial investment and a lower effective productivity. Therefore, overall investment, N, is lower in the presence of a tight cap. Intuitively, the higher the share of CBDC meetings, the larger is the decrease in investment due to the cap. Also, the demand for reserves is lower, leading to lower money market and deposit rates and a lower demand for deposits.

Figure 7 shows the effects of the two caps when the CBDC rate is increased. When R_e is low, both caps are binding. As R_e increases, the funding costs decrease and

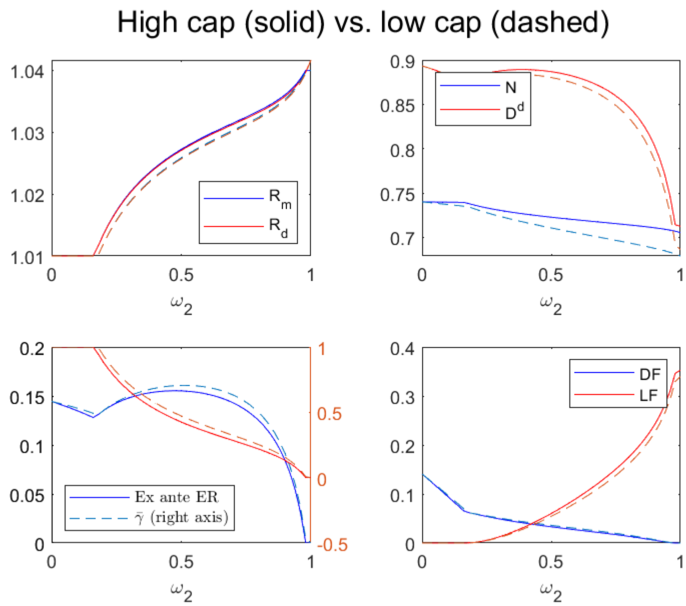

Figure 6: The effects of CBDC caps when the share of CBDC meetings increases.

less CBDC is needed for reinvestment. Therefore, the cap becomes "less binding". At the same time, the amount of excess reserves decreases and demand for reserves and, hence, the money market rate, increase with a high slope. At some point, the high cap (solid line) is not binding anymore. Now, the money market rate increases in R_e with a smaller slope because now the surplus of reserves due to a binding CBDC cap has vanished and the demand for reserves is only driven by the funding and investment effects that we describe above (with the latter dominating). As R_e becomes very large, also the low cap does not bind anymore and both the solid and dotted lines become the same. Note that the central bank can compensate the decrease in investment due to a cap on CBDC by increasing the CBDC rate.

7 Two-tier remuneration system for CBDC

Another central element of the policy discussions on the design features of CBDC is the possibility of a tiered remuneration (see, e.g., Bindseil (2020)). Specifically, central banks could set two different interest rates: up to a certain limit, CBDC could be remunerated at a higher rate, and amounts above this threshold would be

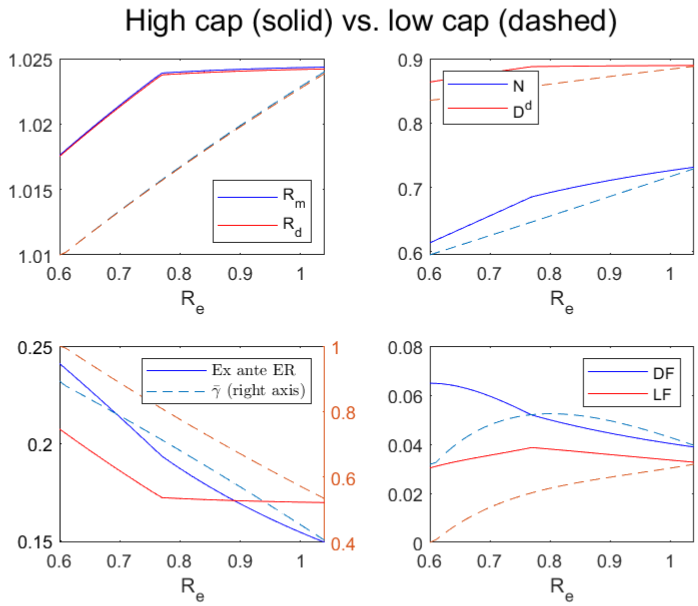

Figure 7: The effects of CBDC caps when the CBDC rate increases.

remunerated at a lower rate. The goal of this design feature is to avoid a large shift from bank deposits into CBDC holdings.

The banker's problem in the presence of a tiered remuneration

Suppose the remuneration of CBDC is tiered. So CBDC holdings c are remunerated at the following contingent rate,

$$R_e(c) = \begin{cases} R_e^1 & \text{if } c \leq \bar{c} \\ R_e^2 & \text{if } c > \bar{c} \end{cases},$$

where $\bar{c}$ is the tiering threshold.

In order for tiering to have a bite, we assume that the bank is investing in k firms with a net discounted payoff of $Ak - c(k)/\beta$, where $c(k)$ is the cost of investing in k firms (e.g. for selecting or monitoring firms). However, it is the bank that has to refinance a fraction γ of its balance sheet, and it can only do it from one seller – so the bank cannot make sure to always be below the remuneration threshold by purchasing a tiny bit from a very large number of sellers. All in all, given γ, the bank has to purchase $\omega_2\gamma k$ with CBDC from the same seller.

The seller charges a price $p(k)$ to sell k units of capital, such that

$$k = R_e^1 \min\left[p(k)k; \bar{c}\right] + R_e^2 \max\left[p(k)k - \bar{c}; 0\right].$$

Therefore

$$p(k) = \begin{cases} \frac{1}{R_e^1} & \text{if } k \leq R_e^1\bar{c} \\ \frac{1}{R_e^2} + \left(R_e^2 - R_e^1\right)\frac{\bar{c}}{R_e^2 k} & \text{if } k > R_e^1\bar{c} \end{cases}.$$

Hence, on the γk units of capital needed, the bank pays $p(\gamma k)$. So the bank needs $\omega_2\gamma k p(\gamma k)$ in CBDC which gives

$$\omega_2\gamma k p(\gamma k) = \begin{cases} \omega_2\frac{\gamma k}{R_e^1} & \text{if } \gamma \leq \frac{R_e^1\bar{c}}{k} \\ \omega_2\left[\frac{\gamma k}{R_e^2} + \left(R_e^2 - R_e^1\right)\frac{\bar{c}}{R_e^2}\right] & \text{if } \gamma > \frac{R_e^1\bar{c}}{k} \end{cases}.$$

The bank is short in reserves whenever

$$\mathcal{R}(\gamma, k) \equiv \chi\omega_1\frac{\gamma k}{R_d} + \omega_2\gamma k p(\gamma k) - (r - \chi d + y) > 0$$

Notice that at this stage, k is fixed. Therefore, we can define different cases depending on γ.

If $\gamma \leq \frac{R_e^1\bar{c}}{k} \equiv \hat{\gamma}$, then

$$\mathcal{R}(\gamma, k) = \mathcal{R}^<(\gamma, k) = \chi\omega_1\frac{\gamma k}{R_d} + \omega_2\frac{\gamma k}{R_e^1} - (r - \chi d + y)$$

while if $\gamma > \hat{\gamma}$ then

$$\mathcal{R}(\gamma, k) = \mathcal{R}^>(\gamma, k) = \chi\omega_1\frac{\gamma k}{R_d} + \omega_2\frac{\gamma k}{R_e^2} + \omega_2\left(R_e^2 - R_e^1\right)\frac{\bar{c}}{R_e^2} - (r - \chi d + y)$$

Then $\mathcal{R}^<(\gamma, k) \geq 0$ iff

$$\gamma \geq \frac{(r - \chi d + y)}{k(\omega_1\chi\frac{1}{R_d} + \omega_2\frac{1}{R_e^1})} \equiv \bar{\gamma}_1$$

while $\mathcal{R}^>(\gamma, k) \geq 0$ iff

$$\gamma \geq \frac{(r - \chi d + y) + \omega_2\left(R_e^2 - R_e^1\right)\frac{\bar{c}}{R_e^2}}{k\left(\omega_1\chi\frac{1}{R_d} + \omega_2\frac{1}{R_e^2}\right)} \equiv \bar{\gamma}_2$$

Given these thresholds it is straightforward to write down the value function of the bank and to compute the first order conditions with respect to y, k, and excess reserves.

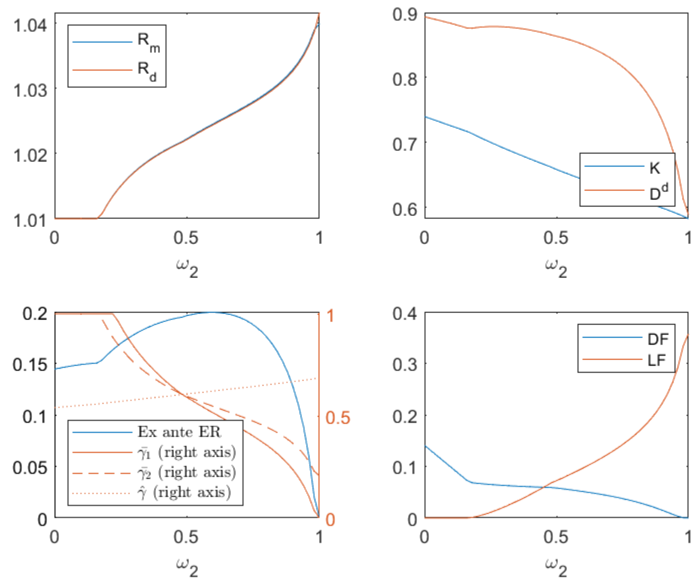

Figure 8: Increasing the share of CBDC meetings in the presence of a two-tiered CBDC remuneration

Simulations

In the following simulations, we set $R_e^1 = 1$ and $R_e^2 = 0.7$ and the tiering threshold $\bar{c} = 0.4$. In Figure 8, we show three different thresholds of γ: $\hat{\gamma}$ is the threshold above which the capital needed for reinvestment becomes so large that the second remuneration rate applies. Since (aggregate) investment K is decreasing in ω_2, this threshold is increasing. As before, $\bar{\gamma}$ is the threshold above which a bank receives such a large refinancing shock, that it is short in reserves. $\bar{\gamma}_1$ denotes this threshold when the tiering threshold is not reached and $\bar{\gamma}_2$ when it is reached.

As Figure 8 shows, the main effects of an increase in the share of CBDC meetings, ω_2, are the same as before. However, now for banks who are hit by a large refinancing shock and, therefore, need a large amount of CBDC, the second and lower remuneration rate applies. This leads to CBDC loosing value relatively and, hence, increases the bank's refinancing costs. Therefore, overall investment is lower as in the case without a tiered remuneration. The larger ω_2 becomes, the larger is this effect and the lower is investment K.

To simulate the effects of an increase in the remuneration of CBDC, we set $R_e^1 = R_e$ and $R_e^2 = R_e^1 - 0.3$. As Figure 9 shows, an increase in both remuneration rates of

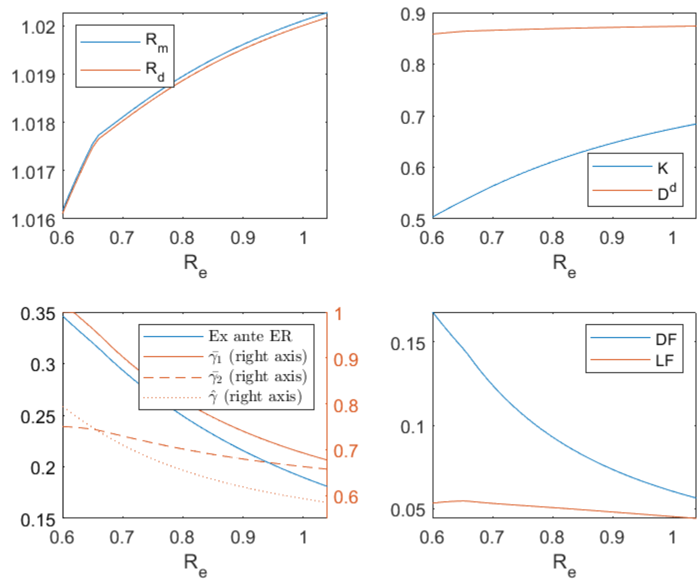

Figure 9: Increasing the remuneration rates of CBDC in the presence of a two-tiered CBDC remuneration

CBDC has the same effects as before. However, the average effective remuneration rate of CBDC is lower now, leading to larger refinancing costs for banks as CBDC is worth less, and, therefore, lower investment. This decrease in investment leads to a smaller demand for reserves and, hence, to a lower money market rate.

In total, we find that a tiered remuneration of CBDC with a lower CBDC rate that is applied once a certain tiering threshold is crossed, is equivalent to a decrease in the CBDC rate when there is no two-tiered system in place.

8 Conclusion and future research

We developed a model of the interbank market featuring uncertainty in the form of a Poole (1968) shock, in which CBDC is introduced in a framework similar to Chiu et al. (2022). In the model, there are two types of payments: bank deposits that only need partial reserve backing, and CDBC which is equivalent to a 100% reserve requirement. We find that, as the market share of CBDC increases, the demand for reserves increases, leading eventually to a higher interbank market rate. While this result is intuitive, the effects from an increase in the CBDC rate are ambiguous.

Since a higher remuneration leads to CBDC increasing in value, the costs to fund the same investment level decrease and less reserves are needed. However, now the banks can fund more entrepreneurs, increasing their investment level and therefore, the demand for reserves. Depending on the model parameters, either the investment or the funding effect dominates, pushing the demand for reserves and, hence, the interbank rate up or down.

Furthermore, we show that quantitative limitations on the amount of CBDC that can be held decrease the reserve demand and the interbank rate and reduces the investment level. Another policy design feature that is actively discussed at central banks, e.g. in Bindseil (2020), is a tiered remuneration of CBDC. We introduce such a tiering system in our model and find that introducing a second, lower CBDC rate for large amounts of CBDC holdings is equivalent to a decrease in the CBDC rate when there is no two-tiered system in place.

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