Decoupling Voting and Cash Flow Rights

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Abstract

The equity lending and option markets both allow investors to decouple voting and cash flow rights of common shares. We provide a theory of this decoupling. While either market enables investors to acquire voting rights without cash flow exposure, empirical studies demonstrate a substantial difference in implied vote prices. Our model explains this surprising difference by showing that vote prices in the equity lending market are endogenously lower than those implied by the option market. Nonetheless, we show that even though votes are cheaper in the equity lending market, activists endogenously choose to decouple using both markets.

Keywords: decoupling, empty voting, shareholder activism, vote trading, empty creditor

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1 Introduction

Modern financial markets allow shareholders to separate voting and cash flow rights of common shares. Thereby, shareholders can influence corporate decision-making while limiting or eliminating their economic exposure to the firm. Understanding this *decoupling* is crucial because acquiring voting rights without economic exposure has become a common practice among activist investors, as documented by the seminal papers Hu & Black (2007), and Lubben (2007). More than 40 cases have been document in Hu & Black (2008*a*), Hu & Black (2008*b*), Bolton & Oehmke (2011), Danis (2017), and Danis & Gamba (2023). Recent media coverage of decoupling includes the fight for control over Premier Foods (2018)[1] and Elliott's involvement in Telecom Italia (2019).[2]

There is a wide range of ways to achieve decoupling but, as we show in the appendix, the most common ones are outcome equivalent to one of two archetypes:

- Using *equity lending*, an activist who wants to acquire voting rights without economic exposure can borrow shares just before the record date and return them soon after. Because a borrower is eligible to vote and does not incur any price risk, this allows her to acquire voting rights without any economic exposure. The price of a voting right is given by the lending fee.

- *Option trading*, on the other hand, involves creating a synthetic stock using a long call, a short put, and a long cash position. The investor then sells this synthetic stock and buys the actual stock, effectively acquiring only the voting right without any net cash flow exposure. The difference between the actual and synthetic stock prices represents the price of a voting right.

To determine the value of a voting right, the empirical literature has analyzed both techniques with surprising results: Contrary to the intuition that voting rights should be worth the same irrespective of how they are acquired, prices observed in the equity lending market are much lower than those inferred from options. In the equity lending market, Christoffersen et al. (2007) and Aggarwal et al. (2015) report meager annualized vote prices of 0 and 2 basis points (bps), respectively. Christoffersen et al. (2007) also find no price effect in the equity lending market around shareholder meetings. By contrast, analyzing the option market, Kalay et al. (2014) find a vote price of 16 bps for a maturity of 38 days (or 153 bps annualized) with US data and a doubling of vote prices in months with special shareholder meetings. Also using options, Kind & Poltera (2013) find an annualized vote price of 37 bps in Europe and 182 bps in months with shareholder meetings.

[1] Financial Times, July 15, 2018, "Market reverberates with accusations of empty voting," https://www.ft.com/content/0e28929e-85dd-11e8-a29d-73e3d454535d.

[2] Financial Times, February 21, 2019, "Vivendi locks horns with Elliott over equity derivatives," https://www.ft.com/content/95e64656-344f-11e9-bb0c-42459962a812.

In this paper, we develop a theory of decoupling. We find that equity lending and option trading can be used by activist investors to decouple voting and cash flow rights and change the outcome of the vote. Still, we show that both ways are very different in terms of incentives they create for shareholders. These differences in incentives translate into different implied vote prices. Our theory thus provides a theoretical underpinning for the empirical findings and gives guidance for their interpretation.

We focus on activist investors whose private agenda runs contrary to a firm value-increasing reform and, thus, benefit most from obtaining voting power without appropriate economic exposure.[3] Examples include activist investors opposing a firm value-increasing merger that would reduce their private profits, e.g., due to cross-holdings in the merging firms (Matvos & Ostrovsky 2008), or ESG investors who attempt to increase stakeholder value while lowering shareholder value. There is ample anecdotal evidence for these firm value-reducing actions, which has caused concern with the Securities and Exchange Commission (SEC) and other regulatory agencies worldwide:

"[It] is a source of some concern that [...] important corporate actions [...] might be decided by persons who could have the incentive to [...] block actions that are in the interests of the shareholders as a whole." — SEC (2010) [4]

Our analysis begins with a simple model featuring a continuum of risk-neutral shareholders who vote on a reform that would increase firm value. We say the firm value is "high" with the reform and "low" otherwise. An activist investor seeks to block the reform because she derives a private benefit from the status quo. To do so, the activist needs sufficiently many voting rights. There is symmetric information among all investors.

First, we find that the implied vote price in the equity lending market (the lending fee) is zero. When shareholders lend out their shares, they retain the economic exposure and only sell their voting rights. Therefore, their reservation value is solely determined by the impact of their lending decision on the vote outcome and its effect on firm value. Since there is a continuum of shareholders, no individual shareholder significantly influences the aggregate vote outcome. Consequently, the vote price is zero, and the activist can block the reform in the unique equilibrium at no cost.

Next, we demonstrate how decoupling with options can lead to positive implied vote prices. The activist utilizes options to solve a commitment problem: after acquiring enough shares to prevent the reform, the economic exposure associated with these shares might otherwise make pursuing the reform more profitable. In this case, the activist cannot succeed and makes zero profits. By hedging with options first, however, the

[3] Investors seeking to improve firm value would like to maximize their economic exposure.

[4] See https://www.sec.gov/rules/concept/2010/34-62495.pdf p. 139. For further discussion, compare "SEC Staff Roundtable on the Proxy Process" (July 2018), https://www.sec.gov/news/public-statement/statement-announcing-sec-staff-roundtable-proxy-process, or ESMA (September 2011), https://www.esma.europa.eu/press-news/consultations/call-evidence-empty-voting.

activist is committed to block the reform, resulting in a low firm value. If shareholders believe that sufficiently many other shareholders will also sell, they expect the firm value to be low. Then, they are willing to sell their shares at prices as low as the low firm value.

Decoupling with options can lead to positive implied vote prices, determined by the difference between the share price (real share) and the expected cash flow entitlement (value of synthetic share). Positive vote prices emerge when the activist blocks the reform, but the share price she pays is higher than the low firm value after blocking the reform. A spread can arise because shareholders may reject low offers they believe others will reject as well. In this case, the reform passes and the firm value appreciates, rendering such a rejection a best response to any offer below the high post-reform firm value. Depending on shareholders' beliefs, this can force the activist to pay substantially more than the low share value to obtain sufficient voting power.

To summarize, implied vote prices are lower in the equity lending market because equity lending allows the activist to directly acquire only the voting rights from shareholders. As a result, shareholders' reservation value is only determined by their expected impact on the collective vote outcome, which is negligible. On the other hand, when an activist decouples using options, shareholders forfeit their cash flow rights upon selling, for which the activist needs to compensate them with at least the expected ex-post firm value. Therefore, shareholders' reservation value depends not only on their marginal impact on the vote outcome but also on the outcome of the shareholder vote as a whole. Depending on shareholders' beliefs about this outcome, multiple equilibria can arise. Except for the boundary case, in all these equilibria, the activist must pay more than the low firm value after blocking the reform, resulting in a positive vote price that is higher than the zero vote price in the lending market.

In practice, many shareholders, such as institutional investors, own large stakes and have a material impact on the outcome of a shareholder vote. To capture this, we consider a variant of the model with a finite number of shareholders. Because shareholders now consider their impact on the vote outcome, one may conjecture that the price ordering between the two forms of decoupling breaks down. Indeed, we show that either method can give rise to positive vote prices as lenders in the equity lending market now need to be compensated for their expected impact on the vote outcome. Nonetheless, *the price-ordering holds*, with lower vote prices when using equity lending than options. Consistent with the empirical evidence (Porras Prado et al. 2016), we further show that a higher dispersion of ownership decreases the equity lending fee. In particular, we show that the total cost[5] of decoupling via equity lending converges to zero as the firm's ownership becomes more dispersed. In contrast, the range of equilibrium total costs of decoupling

[5] For varying ownership concentrations, individual shares have a smaller claim to the overall cash flows of the firm and, thus, the total cost is the only "fair" comparison across different ownership structures.

with options is unaffected by the ownership structure. Hence, the vote price difference between equity lending and option trading is maximized in dispersedly held firms.

Activist investors usually have free choice between decoupling using equity lending or options. Given the price difference, the question arises why the activist would ever use options to decouple. In our final result, we show that it can be optimal for the activist to engage in both forms of decoupling, even when prices in the equity lending market are lower and supply in the lending market is slack. By acquiring voting rights via option trading, the activist can limit the likelihood that a shareholder's lending decision is pivotal for the vote outcome. This reduces the price for all (inframarginal) voting rights bought in the equity lending market. Hence, the activist may be willing to pay a premium for votes acquired with options. As a result, splitting the order optimally between the two forms of decoupling lowers the average vote price for the activist and, thus, her total cost of blocking the reform.

Another contribution of our paper is to clarify what empirical measures of vote prices mean. Our analysis shows that the way voting rights are acquired affects the implied vote price, reconciling the different findings from the equity lending and option markets. Additionally, even within the same decoupling method, there can be a wide range of prices consistent with equilibrium. This means that even after controlling for firm characteristics such as firm size, age, ownership structure, and board composition, there can still be a lot of variation in vote prices. Therefore, it might be difficult to interpret vote prices as the "value of a voting right."

In the appendix, we examine two extensions to the model, which provide new testable predictions. First, we find that the presence of initial stakes owned by the activist investor leads to a decrease in the implied vote prices she is willing to pay when decoupling with options. Second, when firms possess a dual-class share structure, the activist is willing to pay higher implied vote prices. Therefore, vote prices, as implied by the option market, tend to be systematically higher in firms with a dual-class structure than in firms with a single class of shares. This also increases the price gap between equity lending and option trading.

We contribute to the current policy discussion on decoupling. Our results show that decoupling techniques can be used by activist investors to reduce firm value if it suits their private agenda, raising welfare concerns.[6] The analysis provides valuable insights into how firm value-reducing decoupling can be prevented. For example, policy measures that aim at increasing transparency may be insufficient. We show that activists can profit from decoupling even under symmetric information. Moreover, decoupling emerges as a byproduct of modern financial derivatives and the equity lending markets, so banning

[6]Notice that the overall welfare effect of decoupling is not unambiguously negative: An ESG investor may use decoupling to raise aggregate welfare while reducing firm value. Still, our model provides valuable insights into how a policy preventing decoupling ought to be designed if this was a policy goal.

transactions that may be used for decoupling is costly. We propose regulating voting entities instead of the securities used for decoupling. According to our analysis, regulating the eligibility to vote seems the most promising avenue. In particular, we argue that any entity that acquires any voting rights with decoupling should not be eligible to vote at all.

2 Literature

We discuss the relation to the empirical literature in detail in Section 7. In the theoretical literature, Grossman & Hart (1988) as well as Harris & Raviv (1988) provide conditions under which a "one-share-one-vote" policy is optimal. This policy implies that securities should be designed in a way that voting and cash flow rights are equally distributed across all outstanding shares. Deviations from said policy can be optimal to facilitate takeovers as shown by Burkart et al. (1998), At et al. (2011), Burkart & Lee (2015) and Ferreira et al. (2015).[7] An extensive overview of the theoretical and empirical literature on dual-class share structures can be found in Burkart & Lee (2008) and Adams & Ferreira (2008), respectively.

A separation of voting and cash flow rights can, however, not only be obtained by initial design but also by trading in financial markets. The literature has mainly studied models in which decoupling takes the form of direct trading of voting rights, equivalent to equity lending. While Neeman (1999), Bó (2007), and Casella et al. (2012) establish downsides of vote trading, Brav & Mathews (2011) and Eso et al. (2015) show that direct trade of voting rights may be valuable to overcome problems of asymmetric information in corporate decision making. Bar-Isaac & Shapiro (2020) show how abstention in voting can improve corporate decision making. Moreover, Blair et al. (1989) consider the effect of vote trading on control contests. While the theoretical literature has acknowledged different decoupling techniques, it focuses on models in which traders can directly trade voting rights (see e.g., Dekel & Wolinsky (2012)). This turns out to be the appropriate model for some (e.g., equity lending) but not all decoupling techniques (e.g., option trading). Kalay & Pant (2009) show how shareholders use the option market as a commitment device to improve their bargaining position in a subsequent control contest. Levit et al. (2022) and Levit et al. (2021) analyze models of trading and voting. In Levit et al. (2022), trading opportunities render the shareholder base endogenous, introducing a feedback loop and self-fulfilling equilibria. Levit et al. (2021) analyze the "voting premium" in a model where a blockholder trades to influence the vote by changing the median voter. In an extension, they also consider the effect of vote trading (in a continuum model) on vote prices but do not compare it with decoupling through the option market. In general,

[7] Voss & Kulms (2022) show that separating ownership and control can be optimal for takeovers.

none of the above papers compares vote prices that arise from decoupling through different markets and none studies the interaction of option and equity lending market for decoupling.

While this paper focuses on shareholder voting, creditor voting in bankruptcy may also be affected by decoupling techniques. Most relevant, bondholders can acquire CDS contracts to hedge their economic exposure. The effect of these empty creditors on debt restructuring has been analyzed theoretically and empirically by, among others, Bolton & Oehmke (2011), Danis (2017), Danis & Gamba (2018), and Colonnello et al. (2019). Our paper contributes to this literature because one can equally interpret the option contract in our model as a CDS contract. Thus, our paper can be thought of as a theoretical foundation for Feldhütter et al. (2016), who derive the price of a voting right from CDS data.

3 Dispersed shareholder model

We start with a model with a continuum of shareholders to convey the basic intuition. A generalized model with a finite number of strategic shareholders is analyzed in Section 4.

Investors. Consider a public firm owned by a continuum of shareholders with mass 1. Every shareholder owns one share, consisting of a cash flow and a voting right. Further, there is an activist investor (A) who owns no shares. All investors are risk neutral.

Shareholder meeting. The firm has an upcoming shareholder meeting with an exogenously given reform proposal on the agenda. The reform passes if at least $1-\beta \in (0,1)$ votes are cast in favor and is blocked if a fraction β of shareholders vote against the proposal. In the vote, we focus on equilibria in which voters do not play weakly dominated strategies, as is standard in the literature. Thus, voters always vote in favor of their preferred alternative as if they were pivotal in the vote.

Payoffs. If the firm sticks with the status quo, firm value remains unchanged at $v > 0$. Conversely, if the reform passes, the firm value increases by $\Delta > 0$ to $v + \Delta$. As a result, any dispersed shareholder eligible to vote will vote in favor of the reform. In spite of its positive effect on firm value, the activist opposes the reform as she gains a private benefit of $b \in (0, \Delta)$ if the status quo remains. This private benefit may, for instance, stem from other assets of her portfolio: cross-ownership leading to different preferences in a merger[8] or debt in the same company reducing the risk appetite. The status quo may also allow the activist to (continue to) extract b at a cost to the firm of Δ. Alternatively, the activist can be an ESG fund that obtains private benefits (e.g., due to increased investor flows) of b from blocking some shareholder value-increasing but environmentally harmful reform. In any case, we take b as exogenously given. Whether

[8]Matvos & Ostrovsky (2008) provide evidence that cross-ownership of institutional owners leads to different preferences and voting decisions in mergers.

the activist votes for or against the reform depends on her portfolio of shares and hedges. Without decoupling, an activist can only block the reform if she assumes the economic exposure of the blocking fraction of β shares. Thus, she will only acquire the necessary β shares to block the reform if $b \geq \beta\Delta$. By contrast, for $b < \beta\Delta$, decoupling is necessary for A to profitably block the reform. We focus our analysis on the interesting case and assume that $b < \beta\Delta$.

3.1 Decoupling through option trading

"*Elliott last month stepped up its stake in Telecom Italia to 9.4 per cent after renegotiating a hedging facility called an equity collar with JPMorgan [...].*"
— Financial Times (2019)[9]

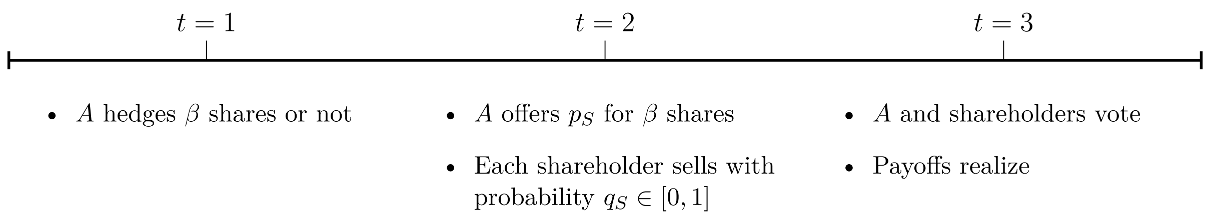


Figure 1: Sequence of events – decoupling with options

We start with the model in which the activist has access to hedging via the option market. The activist first acquires a fairly priced hedge guaranteeing her a share value of $v + \Delta$ for β shares, or not.[10] Afterward, she makes a public take-it-or-leave-it (tioli) offer for shares at price $p_S \in \mathbb{R}_+$ per share. In Appendix B.1, we show that this is A's preferred order of moves: If she bought the shares first, she would not have an incentive to acquire a hedge and block the proposal. A can restrict the number of shares she is willing to buy. If more shareholders decide to sell, they are rationed. It is without loss to assume that the activist makes an offer for up to β shares, the minimal amount of shares she needs to block the reform. The activist conditions her offer price on whether she acquired a hedge, so that her strategy becomes $p_S : \{0, \beta\} \to \mathbb{R}_+$. Shareholders observe whether the activist hedged her position as well as the offer p_S and decide whether they want to sell their share. To capture the predominant anonymity among shareholders, we consider symmetric strategies, denoted by their mixing probability $q_S : \{0, \beta\} \times \mathbb{R}_+ \to [0, 1]$ and consider subgame perfect equilibria.

Share buying. Solving the model backward, we first look at the share-buying stage. The activist can only block the reform when she offers a price p_S such that shareholders

[9]Financial Times, February 21, 2019, "Vivendi locks horns with Elliott over equity derivatives," https://www.ft.com/content/95e64656-344f-11e9-bb0c-42459962a812.
[10]The assumption of a fairly priced hedge stacks the deck against the activist. Still, we show that she can profit from decoupling. The assumption is standard in related models; see Bolton & Oehmke (2011).

sell with probability $q_S^*(p_S) \geq \beta$. Shareholders sell their whole cash-flow claim, for which they demand the (expected) ex-post firm value. This is v if shareholders expect the reform to fail, and $v + \Delta$ otherwise. Shareholders expect the reform to fail if i) they expect that at least $q_S^*(p_S) \geq \beta$ other shareholders sell their share and ii) the activist benefits from blocking the reform. Note that because $b < \beta\Delta$, the activist with β shares only benefits from blocking the reform if she owns a hedge, making it a commitment device. Without a hedge, the activist with β shares would not block the reform, yielding a firm value of $v + \Delta$. As shareholders anticipate this, they demand $v + \Delta$ per share and A's profits will be zero.

Assuming that A owns a hedge and buys enough shares to block the reform, $q_S^*(p_S) \geq \beta$, her payoff is given by

$$\Pi_A^O(p_S; q_S^*) = \underbrace{b}_{\text{private benefit}} + \overbrace{\beta v}^{\text{share value w/o reform}} + \underbrace{\beta\Delta}_{\text{payout hedge}} - \overbrace{\beta p_S}^{\text{payment to shareholders}}. \tag{1}$$

We exclude the upfront cost of the hedge because it is sunk at this point. As Equation (1) shows, at the share buying stage, A's payoff is non-negative if the price is $p_S \leq v + \Delta + \frac{b}{\beta}$ and $q_S^*(p_S) \geq \beta$. By contrast, if $q_S^*(p_S) < \beta$, the activist cannot prevent the reform and her payoff is at most zero. Note that any $p_S > v + \Delta$ guarantees that shareholders sell, so that in equilibrium $p_S^* \leq v + \Delta$ and $q_S^*(p_S^*) \geq \beta$.[11]

Together, this gives rise to a continuum of equilibria in the subgame after the hedge has been bought with $p_S^* \in [v, v + \Delta]$ in which shareholders sell with probability $q_S^*(p_S^*) \geq \beta$ and $q_S^*(p_S) < \beta$ for all $p_S < p_S^*$. Which equilibrium of the subgame prevails, i.e., at which price A is able to acquire the shares depends entirely on the coordination among shareholders and is, thus, self-fulling. For example, it could be that each shareholder expects other shareholders to sell if $p_S \geq v + \Delta/2$ and not to sell otherwise. If A chose $p_S = v$, no shareholder would sell, and A would receive a payoff of 0.

Substituting $p_S^* \in [v, v + \Delta]$ into Equation (1), we find that A's payoff when owning a hedge is $\Pi_A^O(p_S^*; q_S^*) \in [b, b + \beta\Delta]$. If she did not acquire a hedge, her payoff is zero.

Hedging. If the activist decides to buy a hedge and blocks the reform, the hedge pays out $\beta\Delta$. Thereby, she has to pay the fair price of $\beta\Delta$ for the hedge. As a result, it only pays for the activist to buy a hedge and block the reform if the value from owning a hedge is $\Pi_A^O(p_S^*; q_S^*) \geq \beta\Delta$. Since $b < \beta\Delta$, this means that there are two types of equilibria: when $\Pi_A^O(p_S^*; q_S^*) > \beta\Delta$, the activist acquires the hedge, blocks the reform, and $p_S^* \in [v, v + {}^{b}/_{\beta}]$. By contrast, if $\Pi_A^O(p_S^*; q_S^*) < \beta\Delta$, A does not buy the hedge and the reform passes. If A does not buy a hedge, she does not block the reform, and her payoff

[11] When owning a hedge, preventing the reform by offering a price marginally above $v + \Delta$ is strictly profitable and any $p_S > v + \Delta$ induces a $q_S^*(\beta, p_S) = 1$, meaning that the activist is strictly better off lowering her offer to $p' = \frac{p_S + v + \Delta}{2}$.

is zero.

Proposition 1. *When the activist can use the option market for decoupling, there are two types of equilibria:*

1. *A buys the hedge for $\beta\Delta$, acquires β shares, and blocks the reform. In this case, the firm value remains at v. Shares trade at $p_S^* \in [v, v + {}^b/_\beta]$ so that implied vote price is in $[0, {}^b/_\beta]$.*

2. *A does not buy the hedge, the reform passes and firm value increases to $v + \Delta$. The share price is $p_S^* = v + \Delta$ and the implied vote price is 0.*

Proof. Follows from the text. □

Since the option market anticipates the activist's actions and charges the fair value for the hedge, the activist does not benefit *directly* from decoupling through the option market. Nevertheless, acquiring a hedge before the shares can be beneficial for the activist *indirectly* because it ensures that she never holds a long position. Thereby, the preemptive hedge commits her to block the reform.[12] Being committed to block the reform, the activist can potentially exploit coordination failures among the shareholders in that she buys shares below $v + \Delta$. Given A's hedge, selling at these low prices can be optimal for shareholders if they believe other shareholders to sell.

The vote price is given by the difference between the share price and the cash flow entitlement. In the equilibria in which A blocks the reform, Proposition 1 establishes that there is a continuum of implied vote prices given by $p_S^V \equiv (p_S^* - v) \in [0, \frac{b}{\beta}]$. Hence, without any difference in fundamentals, there is a wide variety of implied vote prices consistent with equilibrium.

Kalay et al. (2014) compute vote prices as the difference of the stock price and the synthetic stock price (as given by the put-call parity). In our model, this difference is given by

$$p_S^V = p_S - p_{call} + p_{put} - PV(cash), \tag{2}$$

where p_{call} and p_{put} are the per share prices of a call and put option with the common strike price of $v + \Delta$, respectively. $PV(cash)$ is the present value of cash in the amount of the common strike price. In an equilibrium in which A blocks the reform, the call option is redundant, $p_{call} = 0$, and the put option is priced fairly, $p_{put} = \Delta$. Consequently, the implied vote price(2) is

$$p_S^V = p_S^* - 0 + \Delta - (v + \Delta) = (p_S^* - v) \in [0, {}^b/_\beta]. \tag{3}$$

[12] In Section B.1 we show that if A could only acquire a hedge after buying shares she can never block the reform.

3.2 Decoupling through equity lending

We now turn to decoupling via the equity lending market which is equivalent to the outright trade of voting rights. Formally, suppose that before the record date, the activist can make a public tioli-offer $p_L \in \mathbb{R}_+$ per share borrowed. As before, the activist can restrict her offer to β shares, but this does not affect the results. Shareholders observe the offer and lend their shares with probability $q_L : \mathbb{R}_+ \to [0, 1]$.

Lemma 1. *In any equilibrium, the activist offers $p_L^* = 0$, shareholders lend their share with probability $q_L^*(0) \geq \beta$, and the activist blocks the reform.*

Proof. See Appendix. □

When the activist uses the equity lending market, the economic exposure never leaves the shareholders. By lending their share, shareholders effectively only sell their voting right, so that their reservation value is determined by the impact of their lending decision on the aggregate vote outcome. Since there are many shareholders, they correctly anticipate that their individual decision is not going to change the outcome of the vote, so that shareholders do not value their voting rights. Shareholders are, therefore, willing to sell their voting rights at any positive price. On the other hand, the activist never assumes economic exposure herself, making it optimal for her to block the reform, for any private benefit $b > 0$. As a result, the activist can always acquire voting power for free and prevent the reform.

3.3 Votes are cheaper in the equity lending market

In comparison, we find that vote prices in the equity lending market are almost always lower than the implied vote prices from the option market, consistent with empirical evidence (Christoffersen et al. 2007, Kalay et al. 2014).

Corollary 1.

1. *In the option market, the implied price of a voting right is $p_S^V = p_S^* - v \in [0, \frac{b}{\beta}]$.*

2. *In the equity lending market, the implied price of a voting right is $p_L^* = 0$.*

Proof. Follows immediately from Proposition 1 and Lemma 1. □

When decoupling through the option market, the activist buys the bundle of cash flow and voting right of the shareholders. As a result, a shareholder demands at least the expected ex-post value of his share. In particular, conditional on not being rationed, a small shareholder will accept if the share price exceeds his expectation for the firm value conditional on not selling himself,

$$\underbrace{p_S}_{\text{sell}} \geq \underbrace{\mathbb{E}[\text{firm value}|\text{no sale}]}_{\text{don't sell}} \tag{4}$$
$$\iff p_S - \mathbb{E}[\text{firm value}] \geq 0.$$

Note that because there is a continuum of shareholders, an individual shareholder does not affect the probability that the reform passes, so that $\mathbb{E}[\text{firm value}|\text{no sale}] = \mathbb{E}[\text{firm value}]$. Nevertheless, the implied vote price may be strictly positive if A blocks the reform but $p_S > \mathbb{E}[\text{firm value}] = v$. If shareholders expect the reform to succeed at any lower share price, it is a best response not to sell so that the activist is forced to pay a premium of $p_S - v > 0$.

By contrast, voting and cash flow rights are traded separately in the equity lending market. Shareholders always retain their exposure so that their lending decision is determined by how they affect the collective vote outcome. Conditional on not being rationed, a shareholder will sell if his payoff from the cash flow right and the lending fee exceeds his payoff from the cash flow right alone, that is

$$\overbrace{\underbrace{\mathbb{E}[\text{firm value}|\text{sale}]}_{\text{cash flow right}} + \underbrace{p_L}_{\text{voting right}}}^{\text{sell}} \geq \overbrace{\mathbb{E}[\text{firm value}|\text{no sale}]}^{\text{don't sell}} \tag{5}$$
$$\iff p_L \geq 0.$$

Because there is a continuum of shareholders, no shareholder influences the outcome of the vote. As a result, $\mathbb{E}[\text{firm value}|\text{sale}] = \mathbb{E}[\text{firm value}|\text{no sale}]$ so that each shareholder accepts an arbitrarily small lending fee, which results in $p_L = 0$. Note that by the left side of Equation (5), shareholders always retain the cash-flow rights (in contrast to Equation (4)). Due to this separability of voting and cash flow rights, cash flow rights in (5) cancel out whereas (4) still depends on the expected value of the cash flow rights.

4 General model - finite number of shareholders

As most companies have multiple sizable blockholders (e.g., Edmans & Holderness 2017), it is essential to understand how large, and strategic blockholders affect vote prices for the different decoupling techniques. We now change the model to include any finite number of shareholders who consider the effect of their individual trading decision on the vote outcome.

Instead of a continuum of shares and shareholders, suppose that the company has $n \in \mathbb{N}$ shares outstanding with $n \geq 3$ and that each shareholder holds one share. The activist owns no shares. Now, the reform is blocked if at least $\beta n \in \mathbb{N}$ votes are cast against it. We assume that $\min\{1-\beta, \beta\}n > 1$, so that no shareholder can swing the outcome of the vote unilaterally.[13]

We first solve the model in which the activist uses the equity lending market and then the model in which she employs option contracts. In Section 5, we present a model in which she can use both techniques simultaneously.

4.1 Decoupling through equity lending

Suppose the activist can make a public tioli-offer $p_L \in \mathbb{R}_+$ per share borrowed. The offer is restricted to $m \in \{1, \ldots, n\}$ shares. Without loss of generality, we set $m = \beta n$, that is, the minimal amount of voting rights required to block the reform. Having observed the offer p_L, shareholders symmetrically decide with which probability $q_L(p_L)$ to lend, i.e., $q_L : \mathbb{R}_+ \to [0,1]$. As a result, the total number of shareholders who accept is a binomial random variable $M(n, q_L(p_L)) \sim Bin(n, q_L(p_L))$. Since shareholders are rationed when $M(n, q_L(p_L)) > m$, the activist borrows $\bar{M}(n, q_L(p_L)) = \min\{M(n, q_L(p_L)), m\}$ shares. We consider subgame perfect equilibria.

If the activist borrows fewer than m shares, the reform passes, the firm value rises to $v+\Delta$, and the activist does not obtain her private benefit b. By contrast, if $M(n, q_L(p_L)) \geq m$, the activist blocks the reform, receives the private benefit b, and the firm value remains at v. In either case, A pays p_L for the $\bar{M}(n, q_L(p_L))$ shares borrowed. Consequently, her payoff becomes

$$\Pi_A^L(p_L; q_L) = \Big[\sum_{i=m}^{n} \binom{n}{i} q_L^i \; (1-q_L)^{n-i}\Big] b - \mathbb{E}[\bar{M}(n, q_L)] p_L. \tag{6}$$

Any shareholder's payoff depends on her lending decision and the behavior of the other $n-1$ shareholders. Fix one shareholder, suppose that A offers price p_L, and that the other shareholders respond by mixing with probability $q_L(p_L)$. If the shareholder decides to lend his share, but fewer than $m-1$ other shareholders lend, the reform passes, but the shareholder still obtains p_L in addition to the security benefits of $(v+\Delta)/n$. Conversely, if at least $m-1$ of the other shareholders lend their voting rights, the reform is blocked so that the shareholder obtains security benefits of v/n. Further, if more than $m-1$ other shareholders also lend, i.e., $M(n-1, q_L(p_L)) > m-1$, the shareholder may be rationed so that he only obtains p_L with probability $\frac{m}{M(n-1, q_L(p_L))+1}$. His expected payoff from

[13] For very small and large values of β, this can imply that n will have to be strictly greater than 3.

lending, thus, is

$$\Pi_S^L(\text{lend}; p_L, q_L) = \frac{v+\Delta}{n} - \Big[\sum_{i=m-1}^{n-1} \binom{n-1}{i} q_L^i \, (1-q_L)^{n-1-i}\Big]\frac{\Delta}{n}$$
$$+ \Big[\sum_{i=0}^{m-2} \binom{n-1}{i} q_L^i \, (1-q_L)^{n-1-i} + \sum_{i=m-1}^{n-1} \binom{n-1}{i} q_L^i \, (1-q_L)^{n-1-i} \frac{m}{i+1}\Big] p_L. \tag{7}$$

If the shareholder does not lend his share, but at least m other shareholders do, the reform is blocked, and his payoff is v/n. Otherwise, it rises to $(v+\Delta)/n$. In expectation, the shareholder's payoff from keeping his share is

$$\Pi_S^L(\text{keep}; p_L, q_L) = \frac{v+\Delta}{n} - \Big[\sum_{i=m}^{n-1} \binom{n-1}{i} q_L^i \, (1-q_L)^{n-1-i}\Big]\frac{\Delta}{n}. \tag{8}$$

Since large shareholders may impact the aggregate outcome of the vote, retaining their shares can increase the likelihood that the reform passes and the firm value rises. Hence, large shareholders may demand a positive lending fee, which is determined by the likelihood that a shareholder's voting right will be pivotal in influencing the outcome of the vote.

Proposition 2.

- *The set of equilibrium prices is bounded above by*

$$\bar{p}_L^* = \min\{\max_{q_L} \frac{nq_L}{\mathbb{E}[\bar{M}(n, q_L)]} \binom{n-1}{m-1} q_L^{m-1}(1-q_L)^{n-m}\frac{\Delta}{n}, \frac{b}{m}\}. \tag{9}$$

- *Every price* $p_L^* \in [0, \bar{p}_L^*]$ *can be supported in equilibrium.*

Proof. See Appendix. □

When the price in the equity lending market is positive and b is very small relative to Δ, it may be too costly for the activist to make an offer. This is expressed in the upper bound $\frac{b}{m}$. However, if b (or n) is sufficiently large, the indifference condition based on Equation (7) and (8) determines the equilibrium vote price. After some re-arrangement, we can state shareholders' indifference condition as

$$p_L = \underbrace{\frac{nq_L}{\mathbb{E}[\bar{M}(n, q_L)]}}_{1/\mathbb{P}[\text{not rationed}]} \underbrace{\binom{n-1}{m-1} q_L^{m-1}(1-q_L)^{n-m}}_{\mathbb{P}[\text{pivotal}]} \frac{\Delta}{n}. \tag{10}$$

The implied vote price p_L^* in the equity lending market is equal to the probability that a shareholder's lending decision is pivotal for the aggregate vote outcome, weighted by his potential loss in share value and the probability that he is not rationed.

In equilibrium, the activist will only offer a positive $p_L^* > 0$ if a lower offer $p_L < p_L^*$ reduces her chances of obtaining the voting rights, $q_L^*(p_L) < q_L^*(p_L^*)$. Since shareholders only reject the offer with positive probability if the price is smaller or equal to the one derived in the indifference condition (10), the maximum value this condition can attain puts an upper bound on the vote prices attainable in equilibrium. For higher prices, all shareholders sell ($q_L = 1$) which is also a trivial best response for any positive offer. Based on the observation that (10) is continuous and equal to zero at $q_L = 0$, we show in the appendix that any price in $p_L^* \in [0, \bar{p}_L^*]$ can be supported in equilibrium.

While vote prices can be positive in the lending market if large, strategic shareholders exist, they are still low in the sense that they never compensate shareholders for their expected loss. In particular, the expected payment received by a shareholder selling her voting right is always smaller than the expected loss in cash flows she incurs.

Corollary 2. *In any equilibrium in which the activist acquires voting rights via the equity lending market, the shareholders are never fully compensated for their expected loss in security benefits:*

$$\underbrace{p_L^* \mathbb{E}[\bar{M}(n, q_L)]}_{=\mathbb{E}[\textit{total transfer to shareholders}]} < \underbrace{m \cdot \frac{\Delta}{n} \cdot \left[\sum_{i=m}^{n} \binom{n}{i} q_L^i \; (1 - q_L)^{n-i} \right]}_{\mathbb{E}[\textit{loss to } m \textit{ shareholders}]} . \tag{11}$$

Proof. Follows directly from (9) and $\binom{n-1}{m-1} q_L^{m-1} (1 - q_L)^{n-m} = \frac{m}{nq} \binom{n}{m} q_L^m (1 - q_L)^{n-m}$. □

The reason why shareholders are never fully compensated is twofold: First, the $n - m$ shareholders who's voting rights are not bought receive no compensation. Second, as Corollary 2 shows, even the m shareholders who receive a transfer are not fully compensated for their loss since no shareholder perceives himself as being *always* decisive for the vote; the individual pivotality probability is strictly below 1. As a result, any shareholder is willing to sell his voting right for a price below the expected loss in security benefits.

We now turn to the analysis of the option market and then compare the equilibrium vote prices of the two markets.

4.2 Decoupling through option trading

As in the contiuum model, A can buy a hedge that guarantees her a share value of $v + \Delta$ on m shares for the fair price. Afterward, the activist can make a public tioli-offer $p_S \in \mathbb{R}_+$ to buy shares. Without loss of generality, she restricts her offer to $m = \beta n$ shares. The activist conditions her offer on whether or not she acquired a hedge for m shares so that her strategy becomes $p_S : \{0, m\} \to \mathbb{R}_+$. Shareholders observe whether the activist hedged her position as well as the offer p_S and decide whether they want to sell their share. We again denote shareholders' symmetric strategy by $q_S : \{0, m\} \times \mathbb{R}_+ \to [0,1]$ and consider subgame perfect equilibria.

If the other shareholders sell with probability q_S, the expected profit of a shareholder who sells his share is

$$\Pi_S^O(\text{sell}; p_S, q_S) = \underbrace{\frac{nq_S}{\mathbb{E}[\bar{M}(n,q_S)]}}_{\mathbb{P}[\text{being rationed}]} \frac{v}{n} + \Big[\underbrace{1 - \frac{nq_S}{\mathbb{E}[\bar{M}(n,q_S)]}}_{\mathbb{P}[\text{not being rationed}]}\Big] p_S. \tag{12}$$

When selling, the shareholder only receives p_S if she is not rationed. If she is rationed, at least m other shareholders must have sold so that the reform is blocked and the shareholder receives security benefits of v/n.

If the shareholder does not sell, his expected payoff is

$$\Pi_S^O(\text{keep}; p_S, q_S) = \frac{v+\Delta}{n} - \left[\sum_{i=m}^{n-1} \binom{n-1}{i} q_S^i \, (1-q_S)^{n-1-i}\right] \frac{\Delta}{n}. \tag{13}$$

If the shareholder keeps his share, m other shareholders need to sell for A to be able to block the reform. In this case, the security benefits fall from $v+\Delta/n$ to v/n.

4.3 Votes are cheaper in the equity lending market

Since there exist multiple equilibria for both decoupling techniques when the number of shareholders is finite, we compare the sets of equilibrium prices. To this end, recall that p_S^V denotes the (implied) vote price as derived from the option market. That is, p_S^V is the share price minus the expected value of the cash flow right.

Proposition 3.

- *The set of implied equilibrium prices that can be supported with option trading are the same as in the continuous model,* $p_S^V \in [0, \bar{p}_S^V]$ *with* $\bar{p}_S^V = \frac{b}{m}$*;*
- *When b is sufficiently large, implied vote prices tend to be higher in the option market:* $[0, \bar{p}_L] \subset [0, \bar{p}_S^V]$*;*
- *When the ownership is more dispersed, the normalized implied vote price in the equity lending market converges to zero, whereas it is unaffected for option trading:*

$$\lim_{n\to\infty} n \, \bar{p}_L = 0 \qquad \lim_{n\to\infty} n \, \bar{p}_S^V = \frac{b}{\beta}.$$

Proof. See Appendix. $\square$

In the proof, we first note that the equilibrium construction for option trading directly translates from the continuum model. As in the continuum model, any equilibrium price that induces a lower total cost than A's willingness to pay to block the reform is attainable in a pure strategy equilibrium in which no shareholder is pivotal. Hence, the set of implied vote prices that can be supported in equilibrium is unaffected by the finite model.

Unless constrained by a low b, Proposition 3 shows that the set of equilibrium vote prices in the equity lending market is a strict subset of the set of equilibrium vote prices derived from the option market: the highest vote prices can only occur with option trading. This confirms our result that decoupling in the option market will tend to lead to higher vote prices. In the equity lending market, the probability with which shareholders perceive themselves as pivotal for the outcome of the vote constrains the maximal equilibrium vote price (Proposition 2). As a result, any shareholder is willing to sell his voting right for a price below the potential loss in security benefits of Δ/n so that $\bar{p}_L < min\{\Delta/n, b/m\}$. By contrast, when decoupling in the option market, positive vote prices need not depend on shareholders' pivotality for the vote outcome. As shown in the continuum model in Section 1, any $p_S^V \in [0, b/m]$ can be supported in equilibrium by shareholders' self-fulfilling beliefs about firm value. Analogously, and because no single shareholder individually holds a majority stake, any $p_S^{V*} \in [0, b/m]$ can be supported by $q_S = 0$ for all $p_S < p_S^* = p_S^{V*} + v/n$ and $q_S = 1$ for larger share prices.

The condition of a sufficiently high b that guarantees the strict ordering of price sets becomes innocuous when n is sufficiently large, i.e., when ownership in the firm is sufficiently dispersed. Since in a more dispersedly held firm each share has a claim to a smaller fraction of the firm's cash flows ($(v+\Delta)/n$ decreases), the value of a single voting or cash flow right clearly decreases. For a meaningful comparison across different ownership structures, we, therefore, consider the *normalized* vote prices $n\bar{p}_L$ and $n\bar{p}_S$, respectively. As n increases, any shareholder's individual probability to influence the vote outcome also converges to zero. In the equity lending market, this means that even if multiplied with n, the right-hand side of (10) converges to zero for all q_L, so that $n\bar{p}_L \to 0$. By contrast, the implied vote price from the option market is determined by shareholders' expectations of the outcome of the vote, which is unaffected by n. Thus, $\bar{p}_S^V = \frac{b}{\beta}$ for all n.

Our results provide an explanation for the empirical finding that higher ownership concentration yields higher equity lending fees around the record date (Porras Prado et al. 2016). Moreover, according to our model, the price difference between markets tends to be maximized for dispersedly held firms.

While generally not the case in the corporate control literature (Bagnoli & Lipman 1988), in fact, the continuum game is the limit of the finite shareholder game with respect to all of our results.

Corollary 3. *If $n \to \infty$, the set of equilibria of the finite model converges to the set of equilibria of the continuum model for both the lending and the option market.*

Proof. Follows from Proposition 2 and 3 . □

5 Simultaneous trading in the equity lending and option markets

Given the lower vote price in the equity lending market, one might ask why an activist would ever utilize option trading since these markets typically coexist. In this section, we establish that even when the activist has access to both forms of decoupling and voting rights are cheaper in the equity lending market, she may still choose to also decouple with options. Thus, despite the price differential, supply in the lending market may be slack in equilibrium, consistent with the empirical evidence (Aggarwal et al. 2015, Porras Prado et al. 2016).

Consider a setting in which the activist can acquire voting rights by equity lending or option trading. Again, the firm has n voting shares outstanding. A can acquire up to $n_L \geq 3$ and $n_S \geq 3$ of the votes in the equity lending and option markets, respectively. She needs $\beta n \leq n_S + n_L$ votes to block the reform. For simplicity, we further assume that $\frac{n_L}{2} < \beta n$, so that the equity lending market is not overwhelmingly large. Note, however, that we do *not* exclude the possibility that $n_L > \beta n$ so that A may exclusively use equity lending to block the reform.

For simplicity, we consider a timing where the activist simultaneously decides whether to hedge her portfolio and what offers to make to borrow and buy shares, respectively. In particular, the activist hedges her portfolio of shares at the fair price, or not. If she does, the hedge guarantees the activist a value of $\frac{v+\Delta}{n}$ for any share she may end up buying. Thus, with the hedge, A does not assume any economic exposure and is committed to block the reform. Simultaneously, the activist makes an offer to the shareholders for m_L, m_S shares borrowed or bought at prices $p_L \in \mathbb{R}_+$ and $p_S \in \mathbb{R}_+$, respectively. Finally, shareholders offering their shares in the equity lending market and the stock market respond by symmetric selling probabilities $q_L \in [0, 1]$ and $q_H \in [0, 1]$, respectively. We consider subgame perfect equilibria.

Proposition 4. *There exist equilibria in which A acquires βn voting rights, blocks the reform and trades in both markets, $m_L^*, m_S^* > 0$, although*

- *implied prices are lower in the lending market $p_L^* < p_S^* - {}^{v}/_{n}$,*
- *and supply in the equity lending market is slack, i.e., $m_L^* < n_L$.*

Proof. See Appendix. □

The main idea of the result is that by acquiring voting rights via option trading, the activist reduces the price for voting rights in the equity lending market. By splitting her order across both markets, the activist reduces the number of voting rights she needs to acquire in the equity lending market. This reduces the likelihood that an individual shareholder in the equity lending market is pivotal, causing a decrease in the price within the lending market. Consequently, acquiring an additional voting right through option trading, instead of the equity lending market, though potentially more expensive for the marginal vote, can reduce the overall cost by reducing the price for all inframarginal votes purchased in the lending market.

To provide an intuition, consider an extreme scenario where the activist acquires all voting rights available in the equity lending market, $m_L = n_L = \beta n - m_S$. In this case, similar to a unanimity vote, all shareholders in the equity lending market are pivotal and demand full compensation, resulting in a price of $p_L = \frac{\Delta}{n}$. However, by acquiring some voting rights through option trading at $p_S^V = \frac{\Delta}{n}$[14], the activist reduces the probability that shareholders in the equity lending market are pivotal. As a result, the equilibrium price in the equity lending market decreases, allowing her to purchase the voting rights in the lending market at a lower price of $p_L < p_S = \frac{\Delta}{n}$ – a strict improvement. We show in the appendix that this result extends even when $n_L > \beta n$.

In general, whenever the price in the option market is marginally fixed, the activist will never buy enough voting rights in the equity lending market to equalize prices across markets. To see this, note that at $p_L = p_S^V$, the overall cost for blocking the reform incurred by A is $(m_L^* + m_S^*)p_L$. Then, buying one fewer vote in the lending market strictly decreases the price for all inframarginal units in the equity lending market, say to $p'_L < p_L$. Note that the implied vote price in the option market relies on cash flow consideration and, therefore, does not change if the expected firm value is unaffected by this deviation. Hence, costs decrease to $(m_S^* + 1)p_L + (m_L^* - 1)p'_L < (m_L^* + m_S^*)p_L$: The activist splits her order across the markets precisely to create a price wedge between the equity lending and the option market to reduce her overall costs.

6 Extensions

In Appendix B, we consider three extensions. First, we show that the timing of hedging before acquiring the shares is optimal for the activist because it solves a commitment problem. If the shares were bought first, the long economic exposure would commit the activist not to buy a hedge or block the reform. As a result, shareholders do not sell at prices below $v + \Delta$ so the activist cannot extract rents from them.

Second, we consider how an initial share endowment $\alpha > 0$ of the activist affects implied vote prices. When $\alpha\Delta > b$, the stake commits the activist to support the reform. By contrast, if $\alpha\Delta < b$, our qualitative results are unchanged, and vote prices in the

[14] As shareholders always sell at any $p_S \geq \frac{v+\Delta}{n}$, this is always feasible.

equity lending market are lower than the ones implied in the option market.

Third, we consider dual-class shares. While the implied vote prices in the equity lending market remain zero, vote prices derived from the option market increase when the fraction of voting class shares decreases. This is consistent with the evidence of high vote prices derived in dual-class share settings as summarized in Adams & Ferreira (2008).

7 Implications for the empirical literature

Christoffersen et al. (2007) estimate the value of a voting right using U.S. and U.K. data from the equity lending market. They define the price of a voting right as the difference between the lending fee on the record date and the lending fee the days before and after the record date. They find that the average lending fee around the record date does not change significantly, concluding that the typical vote sells for a price of zero. Also using U.S. data, Aggarwal et al. (2015) find similar results: The lending fee increases by only 2 basis points on the record date, expressed as an annual fee.

In the option market, Kalay et al. (2014) develop a different methodology. They calculate the value of a synthetic stock, composed of a long call, a short put, and a long cash position. They subtract the value of the synthetic stock from the actual stock price — the difference is their measure of the vote price. The authors report average vote price of 16 bps, expressed as a fraction of the stock price, for a maturity of 38 days.[15] Based on the Internet Appendix to Kalay et al. (2014), one can calculate the annualized vote price required to compare to the annualized lending fee: $1 - (1 - 0.0016)^{\frac{365}{38}} \approx 0.0153 = 153$ bps. Also using option data, Kind & Poltera (2013) find an average implied vote price of 37 bps across all months in the sample period and 182 bps in months with shareholder meetings. Both of these studies imply vote prices that are substantially larger than the 0–2 bps findings in the equity lending literature.[16]

Our model explains the well-documented vote price difference between the lending and the option market. Moreover, consistent with the evidence in Porras Prado et al. (2016), vote prices in the lending market increase for more closely held firms according to our model.

Our results suggest that the implied prices for voting rights not only differ substantially between equity lending and option trading, but can also vary substantially within

[15] Although the vote price in the equity lending market is expressed as an annual borrowing fee and the vote price in the option market is expressed relative to the stock price, the two are comparable because the loan size in the equity lending market is approximately equal to the stock price.

[16] Kind & Poltera (2017) use U.S. option data to study vote prices around shareholder proposals. They find an average vote price of 28 bps just before the record date and around 2 bps (but statistically insignificant) briefly after the record date. Gurun & Karakaş (2022) show that vote prices (measured with options) increase after low earnings announcements. Jang et al. (2023) show that firms in control contests experience lower returns when vote prices are higher.

either technique – consistent with the empirical evidence. Which (equilibrium) price is selected depends on shareholders' beliefs about other shareholders' selling decisions and does not require any change in fundamentals such as firm size, age, ownership structure, and board composition.

The price difference may suggest a pecking order according to which activists would first utilize the lending market before resorting to the option market. We show that this is not necessarily the case. In particular, we establish that the different vote prices can emerge endogenously as a way for activists to split their orders and reduce the overall costs of vote buying. The optimal split of the order can lead to slack supply in the lending market - despite the price difference. This is also consistent with the slack supply documented for the lending market (Aggarwal et al. 2015, Porras Prado et al. 2016).

Last, when it comes to dual-class structures, we find that they will tend to systematically increase vote prices, consistent with the relatively high vote price estimates from studies based on firms with dual-class structures; for an overview of the evidence, see Adams & Ferreira (2008) and Levit et al. (2021).

8 Concluding remarks

Over the past two decades, decoupling of voting and cash flow rights has become a common practice among activist investors. We provide a theory of decoupling that helps to understand the diverging empirical findings on the implied vote price in these markets and offers guidance for policy design.

Lower vote prices in the equity lending market than the ones implied from the option market arise due to the different incentives these markets impose on shareholders. For equity lending, positive prices only arise to the extent that shareholders deem themselves pivotal for the collective vote outcome. By contrast, when activists buy shares and negate their economic exposure with options, positive vote prices arise because shareholders must be compensated for the expected value of their cash flow rights. While we focus on the two most prominent forms of decoupling, our theory can be applied to any decoupling technique in equity and debt markets.

In our model, the activist can use decoupling to reduce firm value at the expense of shareholders. This is a source of concern and may call for regulatory action. While the effects on overall welfare are not clear cut (e.g., think of an ESG investor reducing firm value but increasing social welfare), our model can help to understand which policy measures are effective in prohibiting decoupling if this was a policy goal.

First, decoupling through the equity lending or option market does not depend on hidden motives of the activist. We show that even under symmetric information, the activist can push her private agenda to the detriment of shareholder value. Therefore, policies aimed at increasing transparency—such as extended disclosure requirements[17] or rules of informed consent—do not suffice to prevent firm value-reducing decoupling.

Second, one way to substantially reduce the scope for decoupling would be to suspend the voting rights of borrowed or hedged shares. However, this exclusion would not be a comprehensive solution since an activist with a positive share endowment could still obtain control by eliminating the borrowed/hedged shares. If the activist already owns $\alpha > 0$ shares, she could borrow an additional fraction $\sigma > \frac{1-\alpha-\beta}{1-\beta}$ of shares, implicitly voting these σ shares as abstentions. Of the remaining $1 - \sigma$ shares with valid voting rights, the activist holds the β majority, allowing her to block the reform.

Third, a more reliable solution than excluding borrowed and hedged shares from voting would be to exclude the borrower/hedger from voting any of her shares. This solution not only has the same upsides as excluding borrowed or hedged votes but also prevents the acquisition of voting rights to void them.

A Appendix - Anatomy and classification

A.1 Anatomy

In the realm of modern financial markets, the shareholder voting process presents numerous opportunities for investors to acquire disproportionate voting rights compared to their actual economic exposure.

Before any shareholder meeting, the allocation of voting rights takes place at a predetermined record date, based on the shareholdings at that specific time. This allocation disregards any associated assets in an activist investor's portfolio. For instance, it does not consider hedges, thereby enabling activist investors to divest themselves of their economic exposure while retaining the right to vote. Additionally, the possession of shares at the record date determines the number of votes an investor is entitled to. Consequently, investors can exercise their voting rights using borrowed shares or shares that have already been sold with a future delivery date after the record date.

A.2 Classification

While there are multiple methods through which activists can gain disproportionate voting rights relative to their economic exposure, they can ultimately be distilled into two crucial steps:

The first step involves acquiring possession of shares for the record date, which can be achieved by either purchasing or borrowing them. Following this, if the shares were purchased, the investor must then divest themselves of the associated economic exposure.

[17]Compare Hu & Black (2006) for a discussion of disclosure requirements with the SEC.

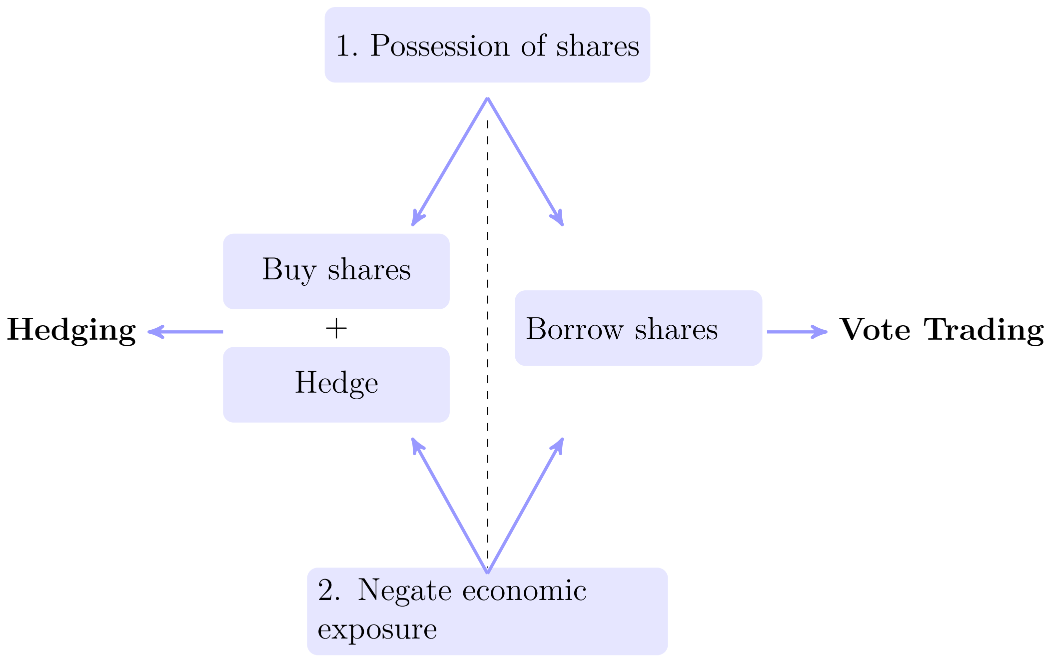


Figure 2: Classification

This can be accomplished by hedging through various derivative contracts and either before or after the acquisition of the shares. This gives rise to two salient categories.

Hedging In the first category of decoupling techniques, the activist buys the shares she wants to vote (before the record date) and hedges them to shed her economic exposure. As we show in Sections 3.1 and B.1, there is a crucial role of the timing of the activist's transactions: Hedging can only exacerbate the collective-decision problem if the activist hedges her economic exposure before acquiring the shares; otherwise, she suffers from a commitment problem.

Vote Trading The second category of decoupling techniques is composed of those techniques that are equivalent to the outright trading of voting rights. Essentially, in these techniques, the shares and hedge are both provided by the same party: the shareholder. Thereby, the economic exposure remains with the shareholder at all times; only the voting rights change hand. Most importantly, Vote Trading techniques include the common practice of borrowing shares over the record date, but also the usage of repos or synthetic assets.[18]

[18] For instance, in a repo contract, the shares posted as collateral are already set to be repurchased, such that only the voting rights are reallocated.

B Appendix - Extensions

B.1 Irrelevance of hedging an existing portfolio

Consider the model with a continuum of shareholders and decoupling through the option market. We flip the timing in order to show that the timing in Section 3.1 is indeed optimal from the activist's perspective.

Suppose that the activist can first make a public take-it-or-leave-it (tioli) offer $p_S \in \mathbb{R}_+$ for β shares. Shareholders observe the offer p_S and decide whether they want to sell. We again consider symmetric strategies, denoted by shareholders' mixing probability $q_S(p_S) : \mathbb{R}_+ \to [0, 1]$. Having acquired $\min\{q(p_S), \beta\}$ shares for p_S per share, the activist can hedge her position, guaranteeing her the "successful reform"-value $v + \Delta$. We analyze subgame perfect equilibria.

Solving the model from the back, consider the subgame in which the activist decides whether or not to acquire the hedge after having acquired shares at some price p_S.

If $q_S(p_S) < \beta$, the activist cannot block the reform such that the share value remains at $v + \Delta$. Since the hedge never pays out, its fair value is 0 and it is irrelevant.

By contrast, if $q_S(p_S) \geq \beta$, the activist can unilaterally determine the outcome of the vote. If she hedges her position, her share position commits the activist to not block the reform because $b < \beta\Delta$. Conversely, if A bought the hedge, the hedge negates her loss in share value, so that blocking the reform is strictly optimal. Anticipating this, the hedging market charges her $\beta\Delta$ so that the activists total payoff from buying the hedge and blocking the reform is $b - \beta\Delta < 0$. Hence, the activist will not buy a hedge.

Share buying Since shareholders know that the activist will not block the reform in any subgame, they know that the firm value will appreciate to $v + \Delta$. Hence, they will not sell at any lower price, so that the activist does not stand to gain from acquiring shares does not block the reform in any equilibrium.

Lemma 2. *Suppose that the activist first acquires shares and hedges afterwards. Then, the reform passes and the firm value increases to $v + \Delta$ in any equilibrium.*

Proof. Follows from text. □

B.2 Initial stakes

At first glance, it is not clear that the stake of an activist blockholder purchased at some prior date matters in a framework where investors can simply negate their economic exposure by acquiring a hedge. We show that may prevent her from engaging in vote trading for firm-value reducing purposes and that it lowers the attainable implied vote prices in the option market.

Consider the continuum shareholder model. Suppose that the activist has an initial stake of $\alpha \in (0, \beta)$ shares such that she only needs to buy $\beta - \alpha$ shares to block the reform. Without loss, we assume that if the activist acquires the hedge, she acquires a hedge for β shares such that she hedges her entire cash flow exposure conditional on obtaining control.

Corollary 4. *Suppose the activist blockholder with stake α first acquires hedges and shares afterwards, then*

- *If $b < \alpha\Delta$, the reform passes in any equilibrium. The equilibrium share price and firm value are at $v + \Delta$.*
- *If $b \in (\alpha\Delta, \beta\Delta)$, there are two types of equilibria:*
    1. *A buys the hedge for a price $\beta\Delta$, acquires $(\beta-\alpha)$ shares, and blocks the reform. In this case, the firm value remains at v. Shares trade at $p_S^* \in [v, v + \frac{b-\alpha\Delta}{\beta-\alpha}$ so that the implied vote price is in $[0, \frac{b-\alpha\Delta}{\beta-\alpha}]$*
    2. *A does not buy a hedge, the reform passes and the firm value increases to $v + \Delta$. The total shareholders' and the activist's payoffs are $(1-\alpha)(v+\Delta)$ and $\alpha(v+\Delta)$, respectively.*

Proof. Follows directly from the proof of Proposition 1 when changing the payoffs of the activist. □

Corollary 4 shows that if the activist is a blockholder, her stake provides an initial commitment to implement the reform, which cannot be overcome by hedging her portfolio. Thereby, if $b < \alpha\Delta$, the activist blockholder will not prevent the reform. If $b > \alpha\Delta$, the outcomes remain similar to our prior results. However, due to the reduced number of shares the activist needs to acquire and the skewed willingness to pay, the highest attainable vote price decreases.

Turning to decoupling through the equity lending market, the results are unchanged when $b > \alpha\Delta$—voting rights trade at zero prices and the activist always prevents the reform. If $b < \alpha\Delta$, the activist is better off when the reform passes, such that she does not engage in decoupling.

B.3 Dual-class structures

So far, we have assumed that all shares are identical voting shares. To incorporate dual-class structures, we consider the continuum model and introduce a proportion $\phi \in (0, 1]$ of voting shares and $1 - \phi$ of non-voting shares. Each shareholder owns either one type or the other.

Corollary 5. *All previous results from the continuum model remain valid for dual-class structures when replacing β by $\beta\phi$.*

Proof. The proofs hold verbatim, replacing β by $\beta\phi$. $\square$

Due to the dual-class structure, the (implied) vote price from the options market, measured as the difference between voting and non-voting shares, ranges from 0 to $\frac{b}{\beta\phi}$. This finding is interesting because it suggests that the introduction of non-voting shares alone can enhance the empirically measured vote premium—consistent with high but diverging empirical estimates of the vote premium derived from companies with dual-class structures (Adams & Ferreira 2008).

The price prediction for the equity lending market, on the other hand, is unaffected by a dual-class structures.[19]

C Appendix - Technical results

Lemma 3. $\mathbb{P}[M(n-1,q)=m-1]<1$ *and* $\lim_{n\to\infty}\mathbb{P}[M(n-1,q)=m-1]=0$.

Proof. The first assertion follows because $0<m<n-1$ such that $1=\sum_{i=0}^{n-1}\mathbb{P}[M(n-1,q)=i]>\mathbb{P}[M(n-1,q)=m-1]$. For the second assertion, note that $\mathbb{P}[M(n-1,q)=m-1]=\binom{n-1}{m-1}q^{m-1}(1-q)^{n-m}$ is maximized if

$$\begin{aligned} 0 &= \binom{n-1}{m-1}q^{m-2}(m-nq+q-1)(1-q)^{-m+n-1} \\ \iff q &= \frac{m-1}{n-1}. \end{aligned}$$

Thus,

$$\mathbb{P}[M(n-1,q)=m-1]\leq\binom{n-1}{m-1}\left(\frac{m-1}{n-1}\right)^{m-1}\left(\frac{n-m}{n-1}\right)^{n-m}. \tag{14}$$

Using Stirling's formula, $\binom{a}{b}=(1+o(1))\sqrt{\frac{a}{2\pi(a-b)b}}\frac{a^a}{(a-b)^{a-b}b^b}$, the right side of (14) becomes

$$=(1+o(1))\sqrt{\frac{n-1}{2\pi(n-m)(m-1)}}=(1+o(1))\sqrt{\frac{1}{2\pi(1-\eta)(n-1)\eta}}, \tag{15}$$

with $\eta=\frac{m-1}{n-1}$ (implying that $\eta\approx\beta$). When $n,m\to\infty$, the second result follows. $\square$

Lemma 4.

$$\sum_{i=m-1}^{n-1}\mathbb{P}[M(n-1,q)=i]\frac{m}{i+1}=\mathbb{P}[M(n,q)\geq m]\frac{m}{nq}. \tag{16}$$

[19] In the context of corporate takeovers, Hart (1995) points out that dual-class structures are irrelevant if voting rights and cash flow claims can be unbundled.

$$\sum_{i=0}^{m-2} \mathbb{P}[M(n-1,q)=i] + \sum_{i=m-1}^{n-1} \mathbb{P}[M(n-1,q)=i]\frac{m}{i+1} = \frac{\mathbb{E}[\bar{M}(n,q)]}{nq}. \tag{17}$$

$$\mathbb{P}[M(n-1,q)=m-1] = \frac{m}{nq}\mathbb{P}[M(n,q)=m]. \tag{18}$$

Proof.

$$\begin{aligned}
\sum_{i=m-1}^{n-1} \mathbb{P}[M(n-1,q)=i]\frac{m}{i+1} &= \sum_{i=m-1}^{n-1} \binom{n-1}{i} q^i (1-q)^{n-1-i}\frac{m}{i+1} \\
= \sum_{i=m-1}^{n-1} \frac{1}{nq}\binom{n}{i+1} q^{i+1}(1-q)^{n-(i+1)} m &= \sum_{k=m}^{n} \frac{1}{nq}\binom{n}{k} q^k (1-q)^{n-k} m \\
&= \frac{m}{nq} \cdot \mathbb{P}[M(n,q) \geq m].
\end{aligned}$$

$$\begin{aligned}
\mathbb{E}[\bar{M}(n,q)] &= \mathbb{P}[M(n,q)\geq m]m + \sum_{i=1}^{m-1} \binom{n}{i} q^i (1-q)^{n-i} i \\
&= \mathbb{P}[M(n,q)\geq m]m + \sum_{i=1}^{m-1} \binom{n-1}{i-1} n \cdot q \cdot q^{i-1}(1-q)^{n-i} \\
&= \mathbb{P}[M(n,q)\geq m]m + \sum_{k=0}^{m-2} \binom{n-1}{k} n \cdot q \cdot q^{k}(1-q)^{n-1-k} \\
&= nq\Big(\mathbb{P}[M(n,q)\geq m]\frac{m}{nq} - \mathbb{P}[M(n-1,q)\leq m-2]\Big),
\end{aligned}$$

and plugging (16) into the equation, (17) follows.

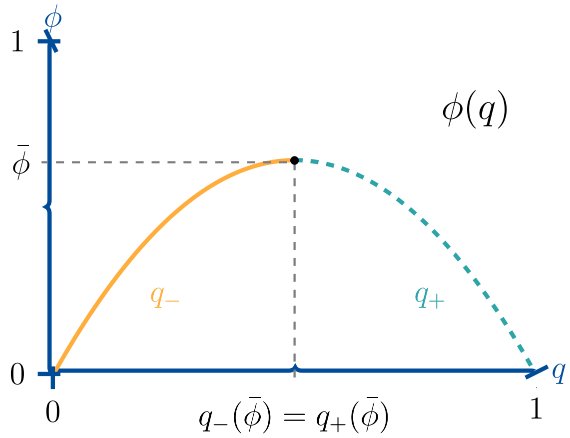
$$\begin{aligned}
\mathbb{P}[M(n-1,q)=m-1] &= \binom{n-1}{m-1} q^{m-1}(1-q)^{n-m} \\
= \frac{(n-1)!}{(n-m)!(m-1)!} q^{m-1}(1-q)^{n-m} &= \frac{(n)!}{(n-m)!(m)!}\frac{m}{nq} q^m (1-q)^{n-m} \\
&= \frac{m}{nq}\mathbb{P}[M(n,q)=m].
\end{aligned}$$

□

Lemma 5.

$$\phi(q) = \frac{\mathbb{P}[M(n-1,q)=m-1]nq}{\mathbb{E}[\bar{M}(n,q)]}$$

is continuous, strictly concave, with a unique maximum $\bar{\phi} < 1$ and $\phi(0) = \phi(1) = 0$. Also, $\lim_{n\to\infty} \phi(q) = 0$ for all q. Further, there are two continuous functions $q_-(\phi), q_+(\phi)$ on $[0, \bar{\phi}]$, of which q_- is strictly increasing and q_+ is strictly decreasing. For all $\phi \in [0, \bar{\phi})$ it holds that $q_-(\phi) < q_+(\phi)$ but $\bar{\phi} = \phi(q_-) = \phi(q_+)$. In particular $q_-(0) = 0$ and $q_+(0) = 1$.



Proof.

Figure 3: Form of $\phi(q)$ and definition of q_- and q_+.

Using (18), $\frac{1}{\phi(q)}$ can be rewritten as

$$\iff \frac{1}{\phi(q)} = \frac{\sum_{i=0}^{m} \mathbb{P}[M(n,q) = i]i + \mathbb{P}[M(n,q) > m]m}{m\mathbb{P}[M(n,q) = m]}$$
$$\iff \frac{1}{\phi(q)} = \frac{\sum_{i=1}^{m} \binom{n}{i} q^i (1-q)^{n-i} i + \sum_{i=m+1}^{n} \binom{n}{i} q^i (1-q)^{n-i} m}{m\binom{n}{m} q^m (1-q)^{n-m}}$$
$$\iff \frac{1}{\phi(q)} = \frac{1}{m\binom{n}{m}} [\sum_{i=1}^{m} \binom{n}{i} (\frac{q}{1-q})^{i-m} i + \sum_{i=m+1}^{n} \binom{n}{i} (\frac{q}{1-q})^{i-m} m].$$

Both summands are strictly convex in q such that $\frac{1}{\phi(q)}$ is strictly convex in q. Further, $\lim_{q\to 0} \frac{1}{\phi(q)} = \lim_{q\to 1} \frac{1}{\phi(q)} = \infty$, such that $\frac{1}{\phi(q)}$ is U-shaped. Since $\frac{1}{\phi(q)} \geq 0$, it follows that ϕ is hump-shaped, with $\phi(0) = \phi(1) = 0$ and a unique maximum $\bar{\phi}$. Further, because

$$\phi(q) = \frac{\mathbb{P}[M(n-1,q) = m-1]nq}{\mathbb{E}[\min\{m, M(n,q)\}]} < \frac{\mathbb{P}[M(n-1,q) = m-1]nq}{nq}, \tag{19}$$

Lemma 3 implies that $\bar{\phi} < 1$ and $\lim_{n\to\infty} \phi(q) = 0$.

Last, since ϕ is hump-shaped, with $\phi(0) = \phi(1) = 0$ and a unique maximum $\bar{\phi}$, for all $p < \bar{\phi}$ there are exactly two functions $q_-(p) < q_+(p)$ such that $p = \phi(q_-(p)) = \phi(q_+(p))$. Since ϕ is continuous, so are q_- and q_+. $\square$

Lemma 6. *$\frac{\mathbb{P}[M(n,q)=m]}{\mathbb{P}[M(n,q)\geq m]}$ is decreasing in q.*

Proof. Expanding gives

$$\frac{\mathbb{P}[M(n,q)=m]}{\mathbb{P}[M(n,q)\geq m]} = \frac{\binom{n}{m}q^m(1-q)^{n-m}}{\sum_{i=m}^{n}\binom{n}{i}q^i(1-q)^{n-i}} = \frac{\frac{n}{m}}{\sum_{i=m}^{n}\binom{n}{i}\frac{q}{1-q}^{i-m}}.$$

Since $i \geq m$, the denominator is increasing in q so that the expression is decreasing. □

Lemma 7. *$\mathbb{P}[M(n,\frac{m}{n})=m]$ is U-shaped in m with minium hat $\frac{n}{2}$.*

Proof.

$$\binom{n}{m+1}\Big(\frac{m+1}{n}\Big)^{m+1}\Big(1-\frac{m+1}{n}\Big)^{n-m+1} > \binom{n}{m}\Big(\frac{m}{n}\Big)^{m}\Big(1-\frac{m}{n}\Big)^{n-m}$$

$$\iff \frac{\left(\frac{m+1}{n}\right)^m\left(\frac{n-m-1}{n}\right)^{n-m-1}}{\left(\frac{m}{n}\right)^m\left(\frac{n-m}{n}\right)^{n-m-1}} >1$$

$$\iff \Big(\frac{m+1}{m}\Big)^m > \Big(\frac{n-m}{n-m-1}\Big)^{n-m-1}$$

The LHS and RHS are equal at $m=n-m-1$ and, thus, the inequality holds for all $m > {}^{(n-1)}/_2$. For all, $m > {}^{(n-1)}/_2$ the reverse holds true such that the aggregate transfer decreases.

Note that since $q=\frac{m}{n}$ maximizes $\mathbb{P}[M(n,q)=m]$ (Inequality (14)), the maximal pivotality probability is U-shaped in m. □

D Appendix - Proofs omitted in the main text

D.1 Proof of Lemma 1

Since no shareholder is pivotal with positive probability, the vote's outcome is independent of any individual shareholder's sale. As a result, no shareholder values his voting right, such that $q_L^*(p_L)=1$ for any $p_L>0$. It follows that $p_L^*=0$. Otherwise, $p_L'=\frac{p_L^*}{2}>0$ would be a profitable deviation for the activist because p' would also guarantee her the voting right, $q^*(p_L')=1$, but at a lower cost. Further, $q_L^*(0)\geq\beta$. If it was the case that $q_L^*(0)<\beta$, the activist would make zero profits. Hence, she could profitably deviate to a price p marginally above 0 at which $q_L^*(p_L)=1$, securing her all the voting rights at essentially zero cost, thereby guaranteeing her a profit.

D.2 Proof of Proposition 2

Proof. Shareholders are indifferent over lending if $\Pi_S(\text{lend};p_L,q_L)=\Pi_S(\text{keep};p_L,q_L)$. Using (7), (8), and (17), this rearranges to (10). Thus, in a symmetric equilibrium,

$q_L^* \in (0,1)$ requires that (q_L^*, p_L) satisfy (10). Further, $q_L^*(p_L) = 1$ is an equilibrium given any $p_L \geq 0$ and $q_L^*(p_L) = 0$ is the only equilibrium when $p_L = 0$.

Turning to the activist and using (6),

$$\Pi_A(p_L; q_L) \leq \Pi_A(p_L; 1) = b - p_L m$$

provides one first upper bound on any equilibrium offer p_L^*.

Now suppose that $p_L^* < \frac{b}{m}$, but

$$p_L^* > \max_{q_L} \frac{nq_L}{\mathbb{E}[\bar{M}(n, q_L)]} \binom{n-1}{m-1} q_L^{m-1}(1-q_L)^{n-m} \frac{\Delta}{n} = \max_{q_L} \phi(q_L) \frac{\Delta}{n},$$

where, by Lemma 5, $\phi(q_L)$ has a unique maximum. Consider a deviation to some $p_L' \in (\max_q \phi(q_L)\frac{\Delta}{n}, p_L^*)$. Since indifference condition (10) cannot be satisfied, the shareholders all sell, $q_L^*(p_L') = 1$, but at a lower price, so that the deviation is strictly profitable for the activist.

We note that when $b \approx \beta\Delta$, by (19),

$$\frac{nq_L}{\mathbb{E}[\bar{M}(n, q_L)]} \binom{n-1}{m-1} q_L^{m-1}(1-q_L)^{n-m} \frac{\Delta}{n} < \mathbb{P}[M(n-1, q_L) = m-1] \frac{\Delta}{n} < \frac{\Delta}{n} \approx \frac{b}{m} \quad (20)$$

so that the condition is always satisfied. The same is true for any b when n becomes large and the pivotality probability vanishes.

To construct an equilibrium with price $p_L^* \leq \bar{p}_L$, define the equilibrium strategy as

$$q_L^*(p_L) = \begin{cases} 1 & \text{if } \; p_L \geq p_L^*, \\ q_L(p_L) & \text{if } \; p_L < p_L^*, \end{cases}$$

where $q_L(p_L)$ is the increasing solution to the indifference condition, as described by Lemma 5. Thus, $q_L^*(p_L)$ is nondecreasing in p_L and 0 at $p_L = 0$.

Note that when (p_L, q_L) are such that (10) is satisfied, the activist's payoff is

$$\begin{aligned} \Pi_A(p_L; q_L) = & \mathbb{P}[M(n, q_L) \geq m]b - \mathbb{E}[\bar{M}(n, q_L)]p_L \\ & \mathbb{P}[M(n, q_L) \geq m]b - m\mathbb{P}[M(n, q_L) = m] \\ & \mathbb{P}[M(n, q_L) \geq m]\Big(b - m\frac{\mathbb{P}[M(n, q_L) = m]}{\mathbb{P}[M(n, q_L) \geq m]}\Big), \end{aligned}$$

which is increasing in q_L by Lemma 6. Since higher offers for $p_L \in [0, p_L^*)$ strictly increase q_L, A's profits increase and a downward deviation is not profitable. Clearly, any upward deviation only raises the cost and cannot be profitable either.

□

D.3 Proof of Proposition 3

Proof. As in the continuum model, any price $p_S^* \in [\frac{v}{n}, \frac{v+\Delta}{n}]$ can be supported in equilibrium after the activist acquired a hedge if $q_S^*(p_S) = 0$ if $p_S < p_S^*$ and $q^*(p_S) = 1$ if $p_S \geq p_S^*$. The implied payoff from owning the hedge is between 0 and b for A. The corresponding implied prices for voting rights are the continuum between 0 and $\frac{b}{m}$. Note that the equilibria can be supported for any admissible n.

The second statement follows immediately from the upper bound on the implied vote prices in the options market, $\frac{b}{m}$, the definition of $\bar{p}_L$ and (20).

For the third result, note that the construction for the options market is independent of n, so that $n\bar{p}_S^V = \frac{b}{\beta}$ for all n. By definition of $\bar{p}_L$, and (19), on the other hand,

$$n\bar{p}_L \leq n \max_{q_L} \frac{nq_L}{\mathbb{E}[\bar{M}(n, q_L)]} \binom{n-1}{m-1} q_L^{m-1}(1-q_L)^{n-m} \frac{\Delta}{n} \leq \max_{q_L} \mathbb{P}[M(n-1, q_L) = m-1]\Delta.$$

This, by Lemma 3, converges to zero.

□

D.4 Proof of Proposition 4

D.4.1 Notation

We denote the number of shares the activist wants to borrower as m_L and the ones she seeks to buy as m_S. Further, let $\bar{M}_L(n_L, q_L) = \min\{m_L, M(n_L, q_L)\}$ be the random variable of the shares she ends up borrowing and $\bar{M}_S(n_S, q_S)$ the number of shares she ends up buying.

D.4.2 Payoffs

Activist: Suppose the activist bought a hedge and blocks the reform whenever possible, then her payoff is (having deducted the cost and payout of the hedge)

$$\mathbb{P}[\bar{M}_S(n_S, q_S) + \bar{M}_L(n_L, q_L) \geq \beta n]b + \mathbb{E}[\bar{M}_S(n_S, q_S)](\frac{v}{n} - p_S) - \mathbb{E}[\bar{M}_L(n_L, q_L)]p_L, \quad (21)$$

where q_S and q_L are functions of the offer p_S, p_L, m_S, m_L and the decision to buy the hedge.

Shareholders in the equity lending market: Shareholders in the equity lending market are indifferent about lending their share if

$$\begin{aligned} p_L &= \sum_{i=\beta n - m_L}^{m_S} \mathbb{P}[\bar{M}_S(n_S, q_S) = i]\mathbb{P}[M(n_L - 1, q_L) = \beta n - i - 1]\frac{n_L q_L}{\mathbb{E}[\bar{M}_L(n_L, q_L)]} \\ &= \sum_{i=\beta n - m_L}^{m_S} \mathbb{P}[\bar{M}_S(n_S, q_S) = i]\mathbb{P}[M(n_L, q_L) = \beta n - i]\frac{\beta n - i}{\mathbb{E}[\bar{M}_L(n_L, q_L)]}. \end{aligned} \tag{22}$$

Shareholders in the share market: Shareholders are indifferent about selling their share if

$$\begin{aligned} &\mathbb{P}[M(n_S - 1, q_S) \le m_S - 1]p_S + \sum_{i=m_S}^{n_S - 1} \mathbb{P}[M(n_S - 1, q_S) = i]\frac{m_S}{i+1}p_S \\ &+ \sum_{i=m_S}^{n_S-1} \mathbb{P}[M(n_S - 1, q_S) = i](1 - \frac{m_S}{i+1})(\frac{v}{n} + \mathbb{P}[\bar{M}_L(n_L, q_L) + m_S \le \beta n - 1]\frac{\Delta}{n})) \\ &= \frac{v}{n} + \sum_{i=0}^{m_S - 1} \mathbb{P}[M(n_S - 1, q_S) = i]\mathbb{P}[\bar{M}_L(n_L, q_L) + i \le \beta n - 1]\frac{\Delta}{n} \\ &+ \sum_{i=m_S}^{n_S - 1} \mathbb{P}[M(n_S - 1, q_S) = i]\mathbb{P}[\bar{M}_L(n_L, q_L) + m_S \le \beta n - 1]\frac{\Delta}{n} \\ \iff & \frac{\mathbb{E}[\bar{M}_S(n_S, q_S)]}{n_S q_S}p_S - \frac{\mathbb{E}[\bar{M}_S(n_S, q_S)]}{n_S q_S}\frac{v}{n} \\ &= \sum_{i=m_S+1}^{n_S} \mathbb{P}[M(n_S, q_S) = i]\frac{m_S}{n_S q_S}\mathbb{P}[\bar{M}_L(n_L, q_L) + m_S \le \beta n - 1]\frac{\Delta}{n} \\ &+ \sum_{i=0}^{m_S} \mathbb{P}[M(n_S, q_S) = i]\frac{i}{n_S q_S}\mathbb{P}[\bar{M}_L(n_L, q_L) + i \le \beta n - 1]\frac{\Delta}{n} \\ \iff & p_S - \frac{v}{n} = \frac{\Delta}{n}\frac{\mathbb{E}[\bar{M}_S(n_S, q_S) | \bar{M}_S(n_S, q_S) + \bar{M}_L(n_L, q_L) \le \beta n - 1]}{\mathbb{E}[\bar{M}_S(n_S, q_S)]} \\ &* \mathbb{P}[\bar{M}_S(n_S, q_S) + \bar{M}_L(n_L, q_L) \le \beta n - 1]. \end{aligned} \tag{23}$$

We note that for $q_S = 0$ and $m_S > 0$ this is just

$$p_S - \frac{v}{n} = \mathbb{P}[\bar{M}_L(n_L, q_L) \le \beta n - 1]\frac{\Delta}{n},$$

and if $q_S = 1$ and $m_S > 0$ this is

$$p_S - \frac{v}{n} = \mathbb{P}[\bar{M}_L(n_L, q_L) + m_S \le \beta n - 1]\frac{\Delta}{n}.$$

D.4.3 Constructing the market equilibria

For every $\hat{p}$ and every offer by the hedged activist, (m_S, m_L, p_S, p_L), define the best response of the shareholders (q_L^*, q_S^*) as functions of this offer. We often omit the arguments to save on notation

$$q_S^*(p_S, p_L, m_S, m_L)$$
$$= \begin{cases} 1 \text{ if } p_S \geq \hat{p} \text{ and } p_S \geq \frac{v}{n} + \frac{\Delta}{n}\mathbb{P}[\bar{M}_L(n_L, q_L^*) + \bar{M}_S(n_S - 1, q_S^*) \leq \beta n - 1] \\ 0 \text{ if } p_S < \hat{p} \text{ or } p_S < \frac{v}{n} + \frac{\Delta}{n}\mathbb{P}[\bar{M}_L(n_L, q_L^*) + \bar{M}_S(n_S - 1, q_S^*) \leq \beta n - 1] \end{cases} \quad (24)$$

and

$$q_L^*(p_S, p_L, m_S, m_L) = \begin{cases} 1 \text{ if } p_L \geq p(p_S, p_L, m_S, m_L) \\ q_-^*(p_S, p_L, m_S, m_L) \text{ if } p_L < p(p_S, p_L, m_S, m_L) \end{cases} \quad (25)$$

with (note the $\frac{\beta n - m_S}{n_L}$)

$$p(p_S, p_L, m_S, m_L) = \sum_{i=\beta n - m_L}^{m_S} \mathbb{P}[M(n_L, \frac{\beta n - m_S}{n_L}) = \beta n - i]\mathbb{P}[\bar{M}_S(n_S, q_S^*) = i]\frac{\Delta}{n} \quad (26)$$

and

$$q_-^*(p_S, p_L, m_S, m_L) =$$
$$\min\{q_L : \sum_{i=\beta n - m_L}^{m_S} \frac{\mathbb{P}[M(n_L, q_L) = \beta n - i](\beta n - i)}{\mathbb{E}[\bar{M}_L(q_L, n_L)]}\mathbb{P}[\bar{M}_S(n_S, q_S^*) = i]\frac{\Delta}{n} = p_L\}, \quad (27)$$

unless the indifference condition has no solution, in which case we set $q_-^* = 0$.

Note that all functions are well defined and by (22) and (23) best responses. Further, q^* is nondecreasing in p_L and $q^* < 1$ if $p_L < p(p_S, p_L, m_S, m_L)$.[20] Further, if $p_S \geq \frac{v+\Delta}{n}$ then $q_S^*(p_S, p_L, m_S, m_L) = 1$ and $q_L^*(p_S, p_L, m_S, m_L)$ as defined by (25) form an equilibrium. If $p_S < \frac{v+\Delta}{n}$ and $\beta n - m_L \geq 0$, then $q_S^*(p_S, p_L, m_S, m_L) = 0$ and $q_L^*(p_S, p_L, m_S, m_L) = 1$ are an equilibrium. Last, if $\beta n - m_L < 0$, then $q_L^*(p_S, p_L, m_S, m_L) = 1$ and $q_S^*(p_S, p_L, m_S, m_L)$ are an equilibrium. Thus, the subgame always has an equilibrium.

D.4.4 Constructing the activist's equilibrium offer

Set $m_S^* = \{\beta n - n_L; 0\}$, $m_L^* = n_L$ and $p_S^* = p_L^* + \frac{v}{n} = \hat{p} = p(p_S^*, p_L^*, m_S^*, m_L^*) + \frac{v}{n}$. If $\hat{p} > \frac{b}{\beta n} + \frac{v}{n}$, which may happen if b and / or n are small, we proportionally decrease $p(p_S, p_L, m_S, m_L)$ at every point so that $\hat{p} = \frac{b}{\beta n} + \frac{\Delta}{n}$.

We now let the activist optimize over m_S^* and m_L^* subject to $\beta n = m_S^* + m_L^*$ and choosing a price $p_L^* = p(p_L^*, p_S^*, m_S^*, m_L^*)$. Note that along this frontier, $q_L^* = q_S^* = 1$, so

[20] The left side of the bracket of (27) is, by Lemma 5, a weighted sum of concave function and, hence, concave. By taking the minimum we choose the lower of two possible solutions for p_L which is bounded away from 1. Further, whenever $p_L < p(p_S, p_L, m_S, m_L)$ either q_-^* has a solution or 0 is a best response.

that the payoff (21) of the activist is given by

$$
\begin{aligned}
& b - m_L^* p_L^* - m_H^*(p_S^* - \frac{v}{n}) \\
& = b - (\beta n - m_H^*)\mathbb{P}[M(n_L, \frac{\beta n - m_S^*}{n_L}) = \beta n - m_S^*]\frac{\Delta}{n} - m_H^*(p_S^* - \frac{v}{n}).
\end{aligned} \tag{28}
$$

We observe that $\mathbb{P}[M(n_L, \frac{\beta n - m_S^*}{n_L}) = \beta n - m_S^*]$ is U-shaped in $\beta n - m_S^*$ with minimum at $\frac{n_L}{2}$, cf. Lemma 7. Hence, the optimal m_S^* has to be between 0 and $\beta n - \frac{n_L}{2}$. If the optimal $m_S^* = 0$, we decrease $\hat{p}$ and p_S^* until it is no longer is. Note that this is the case at some $p_S^* > p_L^*$.

Wrapping up, we have constructed a candidate equilibrium in which the activist makes an offers $p_S^* - \frac{v}{n} > p_L^*$ for $m_S^* > \max\{0, \beta n - n_L\}$ and $m_L^* = \beta n - m_S^*$. On path, shareholders react by selling: $q_L^* = 1$ and $q_S^* = 1$. We note that by construction, $p_S^* \leq \frac{b}{\beta n}$, so that the activist turns a profit.

D.4.5 Checking deviations by the activist

Since the activist turns a profit, any profitable deviation either needs to increase the rate of success or lower the cost. In the equilibrium constructed, the activist is always successful, so that we only need to consider deviations which reduce the cost, either by deviating downward from p_L^*, p_S^*, m_L^* and / or by lowering m_H^*.

Reducing p_S^*: By construction, a lower p_S^* is never accepted, independent of the actions in the equity lending market. Hence, such a deviation is equivalent to lowering m_H^* to zero. Lowering m_H^* without raising m_L^* will not allow the activist to prevent the reform and cannot be optimal. Thus, a potentially profitable downward deviation of p_S^* needs to be accompanied with setting $m_L^* \geq \beta n$ (if possible). Since we chose m_H^* and $m_L^* = \beta n - m_S$ optimally subject to staying on price path $p_L^* = p(p_S^*, p_L^*, 0, m_L^*)$, and any higher price is dominated, such a deviation can only be profitable when $p_L^* < p(p_S^*, p_L^*, 0, m_L^*)$. This is ruled in the next case. Last, any $m_L > \beta n$ gives a weakly lower payoff than $m_L = \beta n$ because the activist might only end up acquiring voting rights she does not need.

Reducing p_L^*: Any $p_L < p(p_S, p_L, m_S, m_L)$ implies that $q_L^* < 1$ is determined by equation (27) or (as its limit) zero. Plugging in (21), we can write the activist's utility as

$$
\begin{aligned}
& \sum_{i=\beta n - m_L}^{m_S} \mathbb{P}[\bar{M}(n_S, q_S^*) = i]\mathbb{P}[M(n_L, q_L^*) \geq \beta n - i](b - \frac{\Delta}{n}\frac{\mathbb{P}[M(n_L, q_L^*) = \beta n - i](\beta n - i)}{\mathbb{P}[M(n_L, q_L^*) \geq \beta n - i]}) \\
& - \mathbb{E}[\bar{M}_S(n_S, q_S^*)](p_S - \frac{v}{n}).
\end{aligned}
$$

In Lemma 6 we show that

$$\frac{\mathbb{P}[M(n_L, q_L) = \beta n - i]}{\mathbb{P}[M(n_L, q_L) \geq \beta n - i]}$$

is decreasing in q_L for any i. Further, because $\mathbb{P}[M(n_L, q_L) \geq \beta n - i]$ increases in q_L, decreasing p_L and thereby decreasing q_L is not profitable. Simultaneously changing m_L, m_S and / or p_S so that $p_L = p(p_S, p_L, m_S)$ is not optimal by construction.

Reducing m_S or m_L: Quantities m_S^* and m_L^* are chosen optimally given p_S^* and $p_L^*(p_S^*, p_L^*, m_S^*, m_L^*)$ and $\beta n = m_S^* + m_L^*$. Conditional on staying on $\beta n = m_S + m_L$, changing m_S or m_L can only be profitable if either p_S and / or p_L is reduced. This is ruled out above. Any deviation so that $\beta n > m_S + m_L$ is not profitable because the activist benefits from blocking the reform. We hence focus on $\beta n < m_S + m_L$. Consider reducing m_L, to m_L' so that $\beta n = m_S + m_L'$. Since $q_S^* \in \{0, 1\}$, changing m_L does not affect (26). Hence, if $p_L \geq p(p_S, p_L, m_S, m_L)$, the same is true for m_L' - the activist just saves money not buying unnecessary voting rights. Any $p_L < p(p_S, p_L, m_S, m_L)$ is not optimal by the argument above.

References

Adams, R. & Ferreira, D. (2008), 'One Share-One Vote: The Empirical Evidence*', *Review of Finance* **12**(1), 51–91.

Aggarwal, R., Saffi, P. A. & Sturgess, J. (2015), 'The role of institutional investors in voting: Evidence from the securities lending market', *The Journal of Finance* **70**(5), 2309–2346.

At, C., Burkart, M. & Lee, S. (2011), 'Security-voting structure and bidder screening', *Journal of Financial Intermediation* **20**(3), 458 – 476.

Bagnoli, M. & Lipman, B. L. (1988), 'Successful takeovers without exclusion', *The Review of Financial Studies* **1**(1), 89–110.

Bar-Isaac, H. & Shapiro, J. (2020), 'Blockholder voting', *Journal of Financial Economics* **136**(3), 695–717.

Blair, D. H., Golbe, D. L. & Gerard, J. M. (1989), 'Unbundling the voting rights and profit claims of common shares', *Journal of Political economy* **97**(2), 420–443.

Bolton, P. & Oehmke, M. (2011), 'Credit Default Swaps and the Empty Creditor Problem', *The Review of Financial Studies* **24**(8), 2617–2655.

Brav, A. & Mathews, R. D. (2011), 'Empty voting and the efficiency of corporate governance', *Journal of Financial Economics* **99**(2), 289 – 307.

Burkart, M., Gromb, D. & Panunzi, F. (1998), 'Why higher takeover premia protect minority shareholders', *Journal of Political Economy* **106**(1), 172–204.

Burkart, M. & Lee, S. (2008), 'One Share - One Vote: the Theory', *Review of Finance* **12**(1), 1–49.

Burkart, M. & Lee, S. (2015), 'Signalling to dispersed shareholders and corporate control', *The Review of Economic Studies* **82**(3), 922–962.

Bó, E. D. (2007), 'Bribing voters', *American Journal of Political Science* **51**(4), 789–803.

Casella, A., Llorente-Saguer, A. & Palfrey, T. R. (2012), 'Competitive equilibrium in markets for votes', *Journal of Political Economy* **120**(4), 593–658.

Christoffersen, S. E., Geczy, C. C., Musto, D. K. & Reed, A. V. (2007), 'Vote trading and information aggregation', *The Journal of Finance* **62**(6), 2897–2929.

Colonnello, S., Efing, M. & Zucchi, F. (2019), 'Shareholder bargaining power and the emergence of empty creditors', *Journal of Financial Economics* **134**(2), 297–317.

Danis, A. (2017), 'Do empty creditors matter? evidence from distressed exchange offers', *Management Science* **63**(5), 1285–1301.

Danis, A. & Gamba, A. (2018), 'The real effects of credit default swaps', *Journal of Financial Economics* **127**(1), 51–76.

Danis, A. & Gamba, A. (2023), 'Dark knights: The rise in firm intervention by cds investors', *Management Science* (Forthcoming).

Dekel, E. & Wolinsky, A. (2012), 'Buying shares and/or votes for corporate control', *The Review of Economic Studies* **79**(1), 196–226.

Edmans, A. & Holderness, C. G. (2017), 'Blockholders: A survey of theory and evidence', *The handbook of the economics of corporate governance* **1**, 541–636.

Eso, P., Hansen, S. & White, L. (2015), 'A theory of vote trading and information aggregation', Working Paper.

Feldhütter, P., Hotchkiss, E. & Karakaş, O. (2016), 'The value of creditor control in corporate bonds', *Journal of Financial Economics* **121**(1), 1–27.

Ferreira, D., Ornelas, E. & Turner, J. L. (2015), 'Unbundling ownership and control', *Journal of Economics & Management Strategy* **24**(1), 1–21.

Grossman, S. J. & Hart, O. D. (1988), 'One share-one vote and the market for corporate control', *Journal of Financial Economics* **20**, 175 – 202.

Gurun, U. & Karakaş, O. (2022), 'Earnings and the value of voting rights', *Available at SSRN 2793545* .

Harris, M. & Raviv, A. (1988), 'Corporate governance: Voting rights and majority rules', *Journal of Financial Economics* **20**, 203 – 235.

Hart, O. (1995), *Firms, contracts, and financial structure*, Clarendon Press; Oxford University Press.

Hu, H. & Black, B. (2006), 'The new vote buying: Empty voting and hidden (morphable) ownership', *Southern California Law Review* **79**(4), 811–908.

Hu, H. & Black, B. (2008*a*), 'Debt, equity and hybrid decoupling: Governance and systemic risk implications', *European Financial Management* **14**(4), 663–709.

Hu, H. & Black, B. (2008*b*), 'Equity and debt decoupling and empty voting ii: Importance and extensions', *University of Pennsylvania Law Review* **156**(3), 625–739.

Hu, H. & Black, B. S. (2007), 'Hedge funds, insiders, and the decoupling of economic and voting ownership: Empty voting and hidden (morphable) ownership', *Journal of Corporate Finance* **13**(2), 343–367.

Jang, I. J., Kim, H. & Mohseni, M. (2023), 'Alphas in trading with value of votes', *Available at SSRN: https://ssrn.com/abstract=3258211* .

Kalay, A., Karakas, O. & Pant, S. (2014), 'The market value of corporate votes: Theory and evidence from option prices', *The Journal of Finance* **69**(3), 1235–1271.

Kalay, A. & Pant, S. (2009), 'One share-one vote is unenforceable and sub-optimal', Working Paper.

Kind, A. & Poltera, M. (2013), 'The value of corporate voting rights embedded in option prices', *Journal of Corporate Finance* **22**, 16–34.

Kind, A. & Poltera, M. (2017), Shareholder proposals as governance mechanism: Insights from the market value of corporate voting rights, Technical report, Working Paper, University of Konstanz.

Levit, D., Malenko, N. & Maug, E. G. (2021), 'The voting premium', *European Corporate Governance Institute–Finance Working Paper* (720).

Levit, D., Malenko, N. & Maug, E. G. (2022), 'Trading and shareholder democracy', *Journal of Finance, Forthcoming* (631).

Lubben, S. J. (2007), 'Credit derivatives and the future of chapter 11', *Am. Bankr. LJ* **81**, 405.

Matvos, G. & Ostrovsky, M. (2008), 'Cross-ownership, returns, and voting in mergers', *Journal of Financial Economics* **89**(3), 391–403.

Neeman, Z. (1999), 'The freedom to contract and the free-rider problem', *Journal of Law, Economics, & Organization* **15**(3), 685–703.

Porras Prado, M., Saffi, P. A. & Sturgess, J. (2016), 'Ownership structure, limits to arbitrage, and stock returns: Evidence from equity lending markets', *The Review of Financial Studies* **29**(12), 3211–3244.

Voss, P. & Kulms, M. (2022), 'Separating ownership and information', *American Economic Review* **112**(9), 3039–62.