Reducing Carbon using Regulatory and Financial Market Tools

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Abstract

We study the conditions under which debt securities that make the cost of debt contingent on the issuer's carbon emissions, similar to sustainability-linked loans and bonds, can be equivalent to a carbon tax. We propose a model in which standard and environmentally-oriented agents can adopt polluting and non-polluting technologies, with the latter being less profitable than the former. A carbon tax can correct the laissez-faire economy in which the polluting technology is adopted by standard agents, but requires sufficient political support. Carbon-contingent securities provide an alternative price incentive for standard agents to adopt the non-polluting technology, but require sufficient funds to fully substitute the regulatory tool. Absent political support for the tax, carbon-contingent securities can only improve welfare, but the same is not true when some support for a carbon tax exists. Understanding the conditions under which the regulatory and capital market tool are substitutes or complements within one economy is an important stepping stone in thinking about carbon pricing globally. It sheds light, for instance, on how developed economies can deploy finance to curb carbon emissions in developing economies where support for a carbon tax does not exist.

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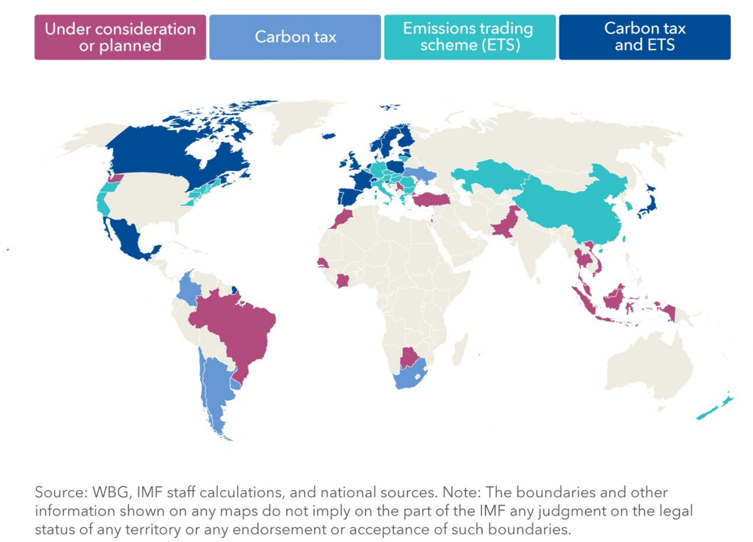
1 Introduction

There is widespread scientific consensus that Earth's climate has warmed significantly since the late 1800s and human activities, primarily greenhouse gas emissions, are the primary cause. Consequently, the issue of reducing and pricing emission has risen on the agenda of policymakers and has been the subject of numerous debates. As illustrated in Figure 1, there is considerable heterogeneity across countries with respect to whether or not carbon pricing regulation is implemented and the form that it takes, with some countries adopting a carbon tax, others a cap-and-trade system, and a few others having adopted both.[1] There are many reasons behind this fragmented regulation. At the international level, there are complex considerations around what would constitute an equitable climate transition that takes into account the fact that the countries most exposed to climate damages are the ones that have contributed the least to global emissions, and are also the ones least equipped with the resources to finance the climate transition.[2] At the domestic level, the policy design and implementation are critically affected by a series of political constraints which depend on electoral preferences and concern for the environment, expectations of energy costs, and policymakers' incentives.[3]

Figure 1. Carbon Pricing Regulation

The figure captures the current state of carbon pricing regulation worldwide as downloaded from the up-to-date carbon pricing dashboard developed by the World Bank Group. Source *https://carbonpricingdashboard.worldbank.org*, accessed November 2022.



Even when regulation has been implemented, the carbon prices implied by the adopted regulatory tools are largely below the consensus level needed to incentivize the achievement of the Paris Agreement goal to remain below the 1.5°C degree rise in global temperature. Furthermore, the investment estimates needed

[1]A carbon tax involves charging a tax on each unit of pollution. A cap-and-trade system involves capping the total quantity of emissions allowed, distributing rights to emitters within this total, and allowing them to trade the permits among themselves.

[2]A comprehensive discussion around these issues can be found in Nordhaus [2020].

[3]Prominent examples are the Washington State's two failed carbon tax referendums from 2016 and 2018, which are studied in detail in a recent work by Anderson, Marinescu, and Shor [2019].

to achieve this goal are significant and range from \$5 trillion per year by 2030 (World Resource Institute, 2021) to \$6.9 trillion per year (OECD, 2018). Many developing countries such as India, argue that developed countries that have been responsible for large emissions during their industrialisation over many years should be responsible for bearing most of the costs of the transition. Indeed, in 2009 developed countries committed to jointly mobilize \$100 billion a year by 2020 to help developing countries adapt to climate change, but these funds have been slow to come by and as of 2020 were still about \$17 billion short.[4]

The amount of financial resources that needs to be mobilized in order to support the climate transition is significant, and well beyond the scope of what governments can provide. Financial markets are now playing an increasingly important role by providing a platform through which investors can channel funds towards projects with environmental, social and sustainability-related outcomes. A prominent example is the market for sustainable debt securities, which has grown exponentially in recent years from a total issuance volume of \$109 billion pre-2012, to \$5,910 billion as of 2022 (see Figures 2 and 3 below).[5] Of these, \$1,611 billion consist of sustainability-linked debt, a new class of instruments introduced only in 2018 which have an interest rate that is contingent on the issuer's performance against a sustainability-related target, which in most cases is represented by greenhouse gas emissions.[6]

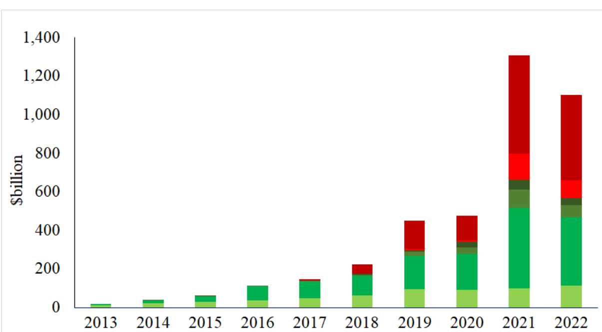


Figure 2. Sustainable Debt Issuance per Year

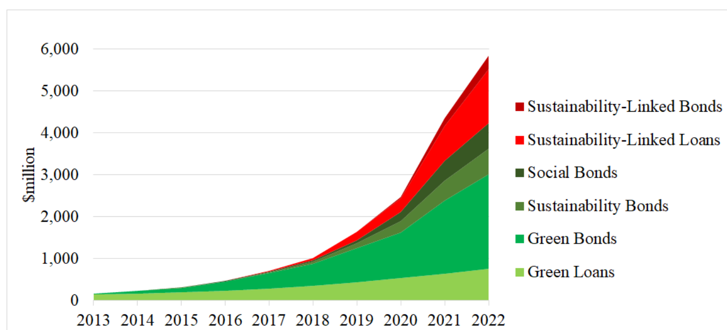


Figure 3. Cumulative Sustainable Debt Issuance

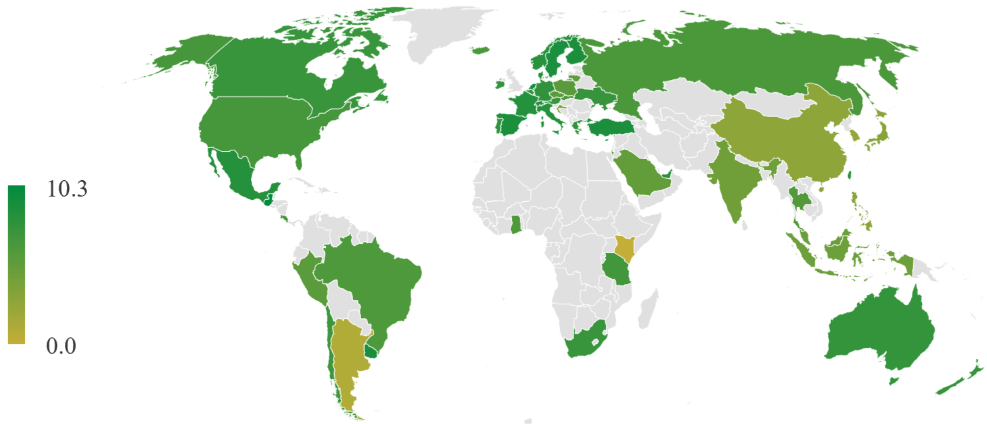
Importantly, the capital mobilized through sustainability-linked debt is orders of magnitude larger than the \$100 billion pledge to developing countries, and this form of carbon-contingent financing has a wider reach, being implemented in countries where support for regulation has been insufficient (see Figure 4 below). By combining the global nature of capital markets with the carbon-price incentives of regulation, these securities have the potential to be an important tool for reducing carbon.

[4]Details can be found in the OECD report `https://www.oecd.org/climate-change/finance-usd-100-billion-goal/`

[5]This market comprises project-based securities such as green, social and sustainable bonds and loans, as well as outcome-based securities such as sustainability-linked loans and bonds which make the cost of debt contingent on outcomes such as the issuers' reduction in carbon emissions. Barbalau and Zeni [2022] provide a detailed analysis of the market and analyse the incentives for issuing these security classes.

[6]Details regarding the targets underlying sustainability-linked debt instruments can be found in, for instance, Barbalau and Zeni [2022] and Kölbel and Lambillon [2022].

Figure 4. Percentage of Sustainability-Linked Debt Issuance
This figure shows the geographical distribution of sustainability-linked debt (which includes corporate and government issued sustainability-linked loans and bonds) relative to all debt (corporate and government issued loans and bonds) issued since 2013. Data are collected from Bloomberg. A more intense shade of green indicates a higher proportion of sustainability-linked debt relative to total debt.



Motivated by this stylized evidence, in this paper we study the interaction between regulatory and financial market tools for pricing carbon within one economy, focusing on the role of its population's wealth and environmental concerns. The regulatory tool we focus on is a carbon tax that can be implemented by the domestic regulator subject to a median voter political constraint that at least half of the voters are better off with the tax. The financial market tool is represented by carbon-contingent securities which have a payoff that increases (decreases) if the issuer's carbon emissions are in excess (deficit) of a predetermined target, in a manner that resembles the one observed in sustainability-linked debt instruments. The focus on a single economy is a necessary first step to study how regulation and financial markets jointly shape incentives to reduce emissions while abstracting from cross-country considerations such as international agreements and carbon leakage effects.

We start by proposing a baseline model which features standard and environmentally-oriented agents that are risk-neutral and behave atomistically. Both standard and environmental agents are exposed to climate shocks caused by global carbon emissions, but environmental agents also internalize the negative impact of emissions associated with their actions.[7] Each agent has endowments which she can either invest in polluting and non-polluting production technologies, with the latter being less profitable than the former, or lend to other agents through carbon-contingent debt securities. There is a regulator that sets a carbon tax to maximize social welfare and who is subject to a median voter political constraint which means that

[7]These are similar to the warm-glow non-consequentialist investors in, for instance, Inderst and Opp [2022], who have a preference for investing in sustainable firms rather than a concern about the investment's ultimate impact, or the deontological agents documented in Hart, Thesmar, and Zingales [2022], who care about doing the right thing irrespective of consequences. Empirical evidence on such preferences for sustainable investing is provided by Riedl and Smeets [2017], Bonnefon, Landier, Sastry, and Thesmar [2022], Humphrey, Kogan, Sagi, and Starks [2021].

it can only implement a tax if it is admissible for at least half of the population. To focus on real effects in our welfare analysis, we treat the agents' preference for taking the right action as a purely decisional utility, and do not include in our notion of welfare.

The model predicts that in a laissez-faire economy without financial markets, standard agents will invest in the polluting technology and environmental ones in the green, non-polluting technology. If exposure to climate shocks is higher than the profitability loss brought about by investing in the less profitable green technology, the regulator will find it optimal to implement a carbon tax and by doing so correct the laissez-faire economy, improve welfare and reduce emissions. However, the extent to which the tax can be enforced is subject to a political constraint.

If the carbon tax is not implemented, carbon-contingent financing from environmental to standard agents arises. Carbon-contingent securities offer an alternative price incentive for standard agents to switch to the green technology, but the extent to which these securities can fully substitute regulation depends on the funds of environmental agents, who are effectively financing the transition. When the funds deployed are sufficiently large, the financial market solution can fully substitute regulation and achieve the same level of welfare and emissions reduction as the carbon tax. As a result, in an economy where there is no support for a carbon tax, the introduction of financial markets circumvents the political constraint and creates welfare gains. However, financial markets for pricing carbon have the effect of decreasing support for regulation in equilibrium, and can thus shift the economy from one that supports a carbon tax to one that does not. When this happens, and the capital deployed through carbon-contingent financing is not enough to finance the transition of all standard agents, financial markets generate welfare losses and achieve lower emissions reduction than the carbon tax.

The baseline model delivers useful insights, but cannot rationalize the empirical evidence that carbon-contingent financing co-exists with carbon pricing regulation (as suggested from Figures 1 and 4). Formally modelling the intensive-margin interaction between market-based and regulatory tools is necessary if one wants to derive an optimal carbon tax policy which accounts for the role of green finance in a realistic way. Therefore, we extend the model to allow for a continuum of agents with heterogeneous environmental preferences and heterogeneous beliefs about their exposure to climate shocks. These agents can invest in a continuum of production technologies with a carbon intensity that can be reduced by incurring a convex carbon abatement cost.

In the continuous model, the regulator can implement a revenue-neutral tax which involves redistributing the revenues from the tax equally across all agents. We first show that the issuance of carbon-contingent securities, the market-implied price of carbon, and the resulting emission abatement generated by financial markets are a decreasing function of the tax, suggesting again that the two tools can be used as substitutes. We then show that, in line with the linear model, the presence of financial markets makes the regulation less appealing for the median voter, thereby reducing the probability of implementation of a given tax. Subject to the new median voter constraint, we thus solve for the optimal revenue-neutral carbon tax which takes into account the presence of financial markets for pricing carbon.

The model shows that the optimal carbon tax in the presence of financial markets is strictly lower than the optimal tax if financial markets did not exist. If the political constraint is binding and there is no support for the optimal carbon tax, then introducing financial markets can improve welfare and reduces emissions. However, the presence of financial markets makes the political constraint more binding, and can potentially switch the economy from one that supports an optimally higher carbon tax to one that does not support an optimally lower carbon tax. When this happens, financial markets reduce real welfare and achieve a lower level of emissions reduction than the carbon tax alone. Notably, the ex-post redistribution rule of the tax revenues plays an important role in determining the sensitivity of the median voter's preference for a given tax, suggesting that in this scenario there is scope for a welfare-improving design of the ex-post compensations such as tax rebates. When political support for the optimal tax is strong, the introduction of financial markets does not alter the emissions reduction achieved in equilibrium but may generate welfare losses in the presence of convex abatement costs. Under the financial market solution abatement is not distributed equally across agents, as would be the case with a uniform carbon tax, but only a fraction of agents reduce emissions while the others are financing this. Thus, while markets are equivalent to an optimal tax in terms of the emissions reduction they can achieve, they do so inefficiently.

The extended model is not only able to generate the observed co-existence of a carbon tax policy and carbon contingent finance, but it also rationalizes why countries with environmentally-oriented voters have both high carbon taxes and active sustainable finance markets. Importantly, our model predicts that in those highly regulated countries the share of emissions reduction achieved by financial markets is low relative to that achieved by the regulation, suggesting that carbon-contingent funds are best directed to markets without carbon taxes, where there is more abatement potential.

The rest of the paper is organized as follows: in Section 2 we provide a brief review of the related literature, underlying the original contribution of the work; in Section 3 we present and solve the baseline model; in Section 4 we present and solve the extended model; in Section 5 we conclude and discuss future directions of research in our framework.

2 Literature

This paper contributes to understanding how security design can enable financial markets to complement government regulation in addressing the sustainability issues faced by the world. Our paper can be broadly framed at the intersection between finance and environmental/climate economics.

The literature studying the interaction between financial markets and corporate behaviour has largely focused on understanding the conditions under which and channels through which investments by agents with pro-social and/or pro-environmental preferences can have an impact by reforming the firms. The channel most studied is the cost of capital channel. Notable papers in this literature stream include Heinkel, Kraus, and Zechner [2001] who study how exclusionary ethical investing impacts corporate behavior, Pastor, Stambaugh, and Taylor [2020] who study how shifts in customers' tastes for green products and investors' tastes for green holdings produce positive social impact, Oehmke and Opp [2022a] who study the conditions for impact in a context in which investors can relax firms' financial constraints for responsible production, and Landier and Lovo [2020] who study how ESG funds should invest to maximize social welfare in a setup in which financing markets are subject to a search friction. Chowdhry, Davies, and Waters [2019] also study the conditions under which impact investments improve social outcomes, but they focus on the role of contracting and security design when firms that cannot commit to social goals are jointly financed by profit and socially-motivated investors. Broccardo, Hart, and Zingales [2022] emphasize a governance rather than a cost of capital channel, in a setup in which investors' preferences are alike those of a social planner internalizing global externalities.[8] In most of these papers investors are big or they act as if they are big, whereas we examine atomistic investors that take global emissions as given. We are also effectively studying a cost of capital channel, looking at debt rather than equity financing, and abstract from corporate governance and a firm's decision to reform by taking the technologies as given and only looking at which will be financed in equilibrium. Our focus is instead on the interaction between security design and regulation, which is absent in all the works cited above.

The agents in our model are effectively implementing a Coasian solution by subsiding the technology shift of agents that would otherwise pollute. Adrian, Bolton, and Kleinnijenhuis [2022] estimate the gains that could be realized from phasing out coal, and make a case for a Cosean bargain whereby we would be better off by paying the polluter to stop polluting. They discuss the role of international agreements that feature compensation conditional on phasing out coal, as well as blended finance which would leverage public funds to de-risk investments in renewable energy and catalyze investments from capital markets. In contrast to this paper, we propose a model that features a purely decentralized market solution that does not rely on international agreements which are subject to political frictions.

[8] Dyck, Lins, Roth, and Wagner [2019] provide empirical evidence on the role of institutional investors in driving corporate environmental and social performance.

The literature stream that our paper is most related to is the one at the intersection of finance and corporate behavior, but which also brings regulation into the picture. Heider and Inderst [2021] examine optimal environmental policy when firms need costly external financing, and derive implications for the leniency of emissions caps or emissions pricing as a function of firms' financial constraints and their potential to become green. Döttling and Rola-Janicka [2022] study environmental and financial regulation in a setup with financially constrained firms and endogenous climate-related transition and physical risks, which captures the fact that climate risks affect asset and collateral values, and the way in which taxes interact with financial constraints. Oehmke and Opp [2022b] study the role of green capital requirements as a tool to incentivize bank lending to green firms when emissions taxes are not available. Hong, Wang, and Yang [2021] study the welfare implications of investment mandates which involve restricting a fixed fraction of the representative investor's portfolio to hold firms that meet sustainability guidelines. Inderst and Opp [2022] study the interaction between financial regulation, taking the form of a taxonomy for sustainable investment products, and traditional tools for environmental regulation such as taxes on externalities or production standards. Biais and Landier [2022] study the complementarity between firms, which can invest in green technologies, and government, which can impose emission caps but has limited commitment power. They find a role for a large fund that can tilt the equilibrium towards caps by engaging with firms to foster investment in green technologies. Ramadorai and Zeni [2021] find that firms' abatement actions depend greatly on their beliefs about climate regulation, and that both informational frictions and reputational concerns can amplify responses to climate regulation, increasing its effectiveness. Huang and Kopytov [2022] show that in the presence of socially responsible investors, pollution can increase with regulation stringency because regulation reshapes firms' shareholders composition and makes polluting firms' shareholders less averse to holding polluting shares.

While previous literature has focused on market responses to (the anticipation) of regulation, our focus is on how markets affect the implementation of regulation, placing emphasis on political constraints and voting. We are, to the best of out knowledge, the first ones to note that a specific security design can substitute regulation. In doing so we build on the work of Barbalau and Zeni [2022] who study the trade-offs related to designing green debt securities as project-based contracts that specify ex-ante the projects that the proceeds will be allocated to, and outcome-based contracts that do not impose constraints on the use of proceeds but embed contingencies that ensure commitment to outcomes. We show that a carbon-contingent security design can fully substitute a carbon tax for which there is insufficient political support if the capital deployed through such instruments is sufficiently high.

While our paper delivers the insight that financial markets for pricing carbon can substitute regulation when political support is missing, we also find that it weakens support for regulation and under certain conditions lead to less efficient outcomes. Thus, our paper also relates to the literature showing that socially responsible investments can have counterproductive implications. Green and Roth [2021] show that ESG investing strategies that focus on the social value of the companies included in their portfolio, with no regard for the implications of these investments on total welfare, allocate their capital inefficiently from the perspective of generating impact and financial returns. Gupta, Kopytov, and Starmans [2022] highlight that socially responsible investors who value acquiring firms with high negative production externalities that they can reform, create trading gains that can actually cause a potential delay in reform. Bisceglia, Piccolo, and Schneemeier [2022] point to the failure of socially responsible investors to internalize the impact of their investment on product market competition, resulting concentration of green capital to few firms and increased market power.

More broadly, our paper relates to the large literature in climate economics that tackles the issue of pricing carbon, by emphasizing the value of using prices to reduce carbon emissions.[9] Stavins [2020] provides a very good overview of price (tax) and quantity (cap-and-trade) instruments for pricing carbon, discussing the dimensions along which these instruments differ and the features that make them equivalent. Goulder and Schein [2013] make a distinction between endogenous carbon pricing tools such as "pure" cap-and-trade systems that imply a market-based volatile carbon price, and exogenous pricing tools such as a carbon tax and a "hybrid" option (a cap-and-trade system with a price ceiling and/or price floor). They discuss the relationship between these tools, exploring the dimensions along which they are equivalent as well as when they have different impacts. Our contribution is to bring the financial sector into the conventional carbon picing analysis.

3 Simple Model

We start with a simple linear model featuring two technologies, two time periods $t = 0, 1$, two types of agents (standard and environmentally-oriented), and a regulator which sets a carbon tax to maximize social welfare subject to a median voter constraint.

There are two technologies, which take as input capital I to produce output y and carbon emissions e. They differ as follows:

(i) the polluting technology, indexed by π, yields output y_π and emissions e_π given by

$$y_\pi = \pi I \qquad \text{and} \qquad e_\pi = I,$$

where $\pi > 1$ is a production parameter;

[9] This is based on integrated assessment models which describe the global interplay between the economy and the climate, and are aimed at calculating the social cost of carbon, as well as quantifying mitigation scenarios for policy-making.

(ii) the non-polluting or green technology, indexed by g, yields output y_g and zero emissions

$$y_g = gI \qquad \text{and} \qquad e_g = 0,$$

with g a green production parameter which satisfies $1 < g < \pi$.

There are two types of risk-neutral entrepreneurs indexed by $i = 1, 2$, namely:

(i) standard entrepreneurs, indexed by $i = 1$, who form a proportion θ of the population, are each endowed with capital h_1, and have utility

$$U_1 = C_1 - \lambda E,$$

where λ is a climate exposure parameter which captures the impact of the total emissions in the economy, $E = \theta e_1 + (1-\theta)e_2$, which are a weighted sum of the emission produced by standard and environmental entrepreneurs, e_1 and e_2 respectively;

(ii) environmentally-oriented or green entrepreneurs, indexed by $i = 2$, who form a proportion $1-\theta$ of the population, have capital h_2 and utility

$$U_2 = C_2 - \lambda E - \eta e_2,$$

where e_2 are emissions associated with their actions, η is a green preference parameter which is assumed to satisfy $\eta > \pi - g$. Note that whereas the environmental entrepreneurs dislike the emissions associated with their actions and which they feel responsible for[10], both types of entrepreneurs are affected by total carbon emissions. The latter can be conceptualized as capturing a global climate shock that affects them irrespective of their preferences and over which they have no control.[11] Thus, entrepreneurs are atomistic with respect to the global climate shock, which can be thought as a natural disaster or the negative effects of pollution on health which affect the entire population.

There is a regulator which maximizes utilitarian social welfare given by

$$W = \theta C_1 + (1-\theta)C_2 - \lambda E. \tag{1}$$

Note that the individual green preference parameter η does not enter the regulator's welfare function, which allows us to capture only real effects while excluding obvious, mechanical effects resulting from

[10] This assumption is in line with work by Hart and Zingales [2017] that assumes that individuals put some weight on doing the right or socially efficient thing if they feel responsible for the action in question.

[11] In the extended version of this model, we introduce a more general case where the climate exposure parameter λ_i varies with the agent type i, reflecting its belief about the economic damage caused by carbon emissions. The more general setting allows us to study scenarios in which the economic damage of climate change is not recognized by a share of the population, i.e. the economy features a share of "climate deniers". Importantly, while the equilibrium solution changes depending on the input parameters, the qualitative predictions of the paper are not altered in this more general framework.

agents' perception of warm glow. This modelling choice follows Broccardo et al. [2022] and Inderst and Opp [2022] and involves regarding η as merely decisional utility, since it remains relatively unaffected by the actually achieved aggregate outcomes.[12]

3.1 Laissez-Faire Benchmark

In the decentralized economy, agents choose to produce output using the polluting or non-polluting technologies. Denote capital investment in the polluting and non-polluting technology by I_π and I_g, respectively, and denote agent i's green preference using η_i, which for the standard agent $i = 1$ takes the value $\eta_1 = 0$ and for the environmental agent $i = 2$ takes the value $\eta_2 = \eta$. Recall that emissions are only produced by the investment in the polluting technology, that is $e_\pi = I_\pi$ and $e_g = 0$. Thus, agent i's problem of allocating its endowment to the polluting and the green technology, is

$$U_i^* = \max_{I_\pi, I_g} \ \pi I_\pi + g I_g - \eta_i I_\pi - \lambda E \ \text{ such that } I_\pi + I_g \leq h_i. \tag{2}$$

Given we assumed $\pi > g > 1$, and $\eta > \pi - g$, it is immediate to see that the standard agent $i = 1$ will invest all available capital in the polluting technology, $I_\pi^* = h_1$, whereas the environmental agent will invest all capital in the non-polluting technology, $I_g^* = h_2$.

Taking account of such choices, aggregate emissions are $E^* = \theta e_1^* + (1-\theta) e_2^* = \theta h_1$, the utility of the standard agent is

$$U_1^* = \pi h_1 - \lambda \theta h_1, \tag{3}$$

the utility of the green agent is

$$U_2^* = g h_2 - \lambda \theta h_1 \tag{4}$$

and the utilitarian social welfare is

$$W^* = \theta \pi h_1 + (1-\theta) g h_2 - \lambda \theta h_1. \tag{5}$$

3.2 Carbon Tax

Suppose that the regulator can alter the laissez-faire economy by imposing a tax τ on the emissions produced by the polluting technology π, and by doing so alter the investment decisions of the agents. Denoting $R^\tau = \tau E^\tau$ the total revenues collected from the tax, with $E^\tau = \theta e_1^\tau + (1-\theta) e_2^\tau$ the sum of the

[12] Including the preference parameter η in the regulator's welfare function has the main implications that financial markets alone can achieve a higher welfare than the carbon tax even when the latter is not subject to political constraints.

optimal emissions of the standard and environmental agents given the tax, the utilitarian social welfare is

$$W^{\tau} = \theta C_1^{\tau} + (1-\theta)C_2^{\tau} - \lambda E^{\tau} + R^{\tau} \tag{6}$$

with C_1^{τ} and C_2^{τ} the consumption of the standard and environmental agents, respectively, evaluated at their optimal investment choices given the tax τ.

It is straightforward to show that any tax $\tau \geq 0$ will not change the actions of the environmental agent relative to the benchmark laissez-faire economy in which the green technology is adopted. It is therefore sufficient to focus on the standard agent's problem, which in the presence of the tax becomes

$$U_1^{\tau} = \max_{I_\pi, I_g} \; gI_g + (\pi - \tau)I_\pi - \lambda E^{\tau} \;\; \text{such that} \;\; I_\pi + I_g = h_1. \tag{7}$$

Optimal investment choices given the tax τ are

$$\begin{aligned} I_g^{\tau} &= h_1 \qquad \text{and} \qquad I_\pi^{\tau} = 0 \qquad \text{if} \quad \tau \geq \pi - g \\ I_g^{\tau} &= 0 \qquad \text{and} \qquad I_\pi^{\tau} = h_1 \qquad \text{otherwise,} \end{aligned} \tag{8}$$

and the emissions associated with the standard agent's choices are $e_1^{\tau} = 0$ if $\tau \geq \pi - g$, and $e_1^{\tau} = h_1$ otherwise. Substituting the utilities U_1^{τ} and U_2^{τ} into the utilitarian social welfare in (6) and re-arranging, we have

$$W = \begin{cases} W^{\tau} = \theta g h_1 + (1-\theta) g h_2 & \text{if} \quad \tau \geq \pi - g \\ W^{*} = \theta \pi h_1 + (1-\theta) g h_2 - \lambda \theta h_1 & \text{otherwise.} \end{cases} \tag{9}$$

Thus, implementing a carbon tax that is sufficiently high to incentivize the transition to the green technology, i.e. $\tau \geq \pi - g$, yields a higher welfare if $\lambda > \pi - g$. Therefore, the optimal tax is $\tau = \pi - g$ if $\lambda > \pi - g$, and $\tau = 0$ otherwise.

We focus henceforth on the case in which $\lambda > \pi - g$, such that the tax should be optimally implemented. In this case, aggregate emissions are zero, $E^{\tau} = 0 < E^{*}$, and utilitarian social welfare is higher relative to the laissez-faire economy

$$W^{\tau} = \theta g h_1 + (1-\theta) g h_2 > W^{*}. \tag{10}$$

Political Constraint. As discussed in the introduction, an important issue is the requirement that the regulation has political support. The regulator is subject to a political constraint in the sense that it can only implement a tax τ that makes at least half of the population better off.

Formally, the regulator must solve a constrained maximization problem of the type

$$\max_{\tau} \; W^{\tau} \;\; \text{such that } \tau \leq \bar{\tau}_{0.5} \tag{11}$$

which states that the optimal tax should be at most equal to that supported by the median voter, denoted as $\bar{\tau}_{0.5}$. We now outline the voting problem and derive an explicit expression for $\bar{\tau}_{0.5}$.

The voting problem. The agents' utilities if they were to vote in favour of the carbon tax is

$$U_i^{\tau} = gh_i - \lambda E^{\tau}. \tag{12}$$

Recalling that in the laissez-faire economy the utility of the standard agent is $U_1^* = \pi h_1 - \lambda E^*$, we can re-write its utility as

$$U_1^{\tau} = U_1^* - (\pi - g)h_1 + \lambda(E^* - E^{\tau}), \tag{13}$$

which can be lower than the laissez-faire if $\lambda\theta < \pi - g$. On the other hand, following the same arguments, the utility of the environmental agent can be re-written as

$$U_2^{\tau} = U_2^* + \lambda(E^* - E^{\tau}), \tag{14}$$

which is strictly higher than the laissez-faire utility $U_2^* = gh_2 + \lambda E^*$ since $E^* - E^{\tau} = \theta h_1$.

Taking account of these voting choices, the threshold $\bar{\tau}_{0.5}$ defining the maximum tax that makes the median voter indifferent between supporting or not the regulation can be expressed as

$$\bar{\tau}_{0.5} = \begin{cases} \pi - g \;\; \text{if} \;\; \theta < 0.5 \\ \pi - g \;\; \text{if} \;\; \theta > 0.5 \quad \text{and} \quad \pi - g < \lambda\theta \\ 0 \;\; \text{otherwise.} \end{cases} \tag{15}$$

We can therefore introduce the following

PROPOSITION 1. *Suppose that $\lambda > \pi - g$ such that the implementation of the carbon tax is desirable. Then if the median voter is an environmentally-oriented type $\theta < 0.5$, then the tax $\tau^o = \pi - g$ achieves the unconstrained optimum in* (11) *and social welfare is higher relative to the laissez-faire*

$$W^{\tau^o} = \theta g h_1 + (1 - \theta) g h_2 > W^*.$$

If the median voter is a standard type $\theta > 0.5$, then either $\lambda\theta > \pi - g$, in which case $\tau^o = \pi - g$, or $\tau^o = 0$, *in which case*

$$W^{\tau^o} = \theta\pi h_1 + (1-\theta)gh_2 - \lambda\theta h_1 = W^*.$$

We have thus derived the optimal carbon tax policy in the absence of financial markets. In the next subsection we will consider the regulator's problem when financial markets for pricing carbon exists, and agents can lend or borrow through carbon-contingent debt securities.

3.3 Carbon-Contingent Financing

So far, we have studied each agent's decisions assuming access to own capital only, represented by their endowments $h_i, i = 1, 2$. In what follows, we allow for external financing. Specifically, we introduce carbon-contingent debt securities similar to those observed in the market for sustainable finance and allow agents to borrow and lend by issuing and purchasing these securities, respectively. In this setup, we assume that agent i can issue, at time $t = 0$, a debt security with principal value d_i and payoff at time $t = 1$ given by

$$\bar{r}d_i - \rho(\bar{e}_i - e_i), \tag{16}$$

where $\bar{r}$ is a fixed interest rate, e_i denotes agent i's emissions at time $t = 1$, and $\bar{e}_i$ the benchmark emissions set at time $t = 0$. These benchmark emissions $\bar{e}_i$ are essentially the counterfactual of what emissions would be in the absence of external financing. This return specification is analogous to that underlying sustainability-linked loans and bonds, which feature a fixed interest rate component and a variable component that is contingent on the deviation of realized emissions from a benchmark that is agreed at contract issuance. If realized emissions are higher than the benchmark, i.e. $e_i > \bar{e}_i$, then the interest rate in (16) increases and viceversa.

We first outline the issuer's problem and the lender's problem. Then, we derive the equilibrium fixed rate $\bar{r}$ and the contingent rate ρ as a function of agents' preferences and endowments.

The Issuer's Problem. Consider first the case of the environmentally-oriented agent $i = 2$, whose benchmark emissions are $\bar{e}_2 = 0$. Upon issuing the debt security, she faces the following investment problem

$$U_2 = \max_{I_\pi, I_g} \; \pi I_\pi + gI_g - (\eta+\tau)I_\pi - \bar{r}d_2 - \rho I_\pi - \lambda E \;\text{ such that } I_g + I_\pi \leq h_2 + d_2. \tag{17}$$

Since $\eta > \pi - g$, the environmental agent will continue to invest only in the green technology (i.e. $I_g = h_2 + d_2$) for any tax $\tau \geq 0$ or contingent rate $\rho \geq 0$, and so there will be no contingent component associated with the payoff in (16), which will simply degenerate into a fixed payoff $\bar{r}d_2$. In this economy, the supply of capital is provided by the standard agents, the interest rate $\bar{r}$ is set such that these standard agents are just indifferent between lending to the environmental agents or investing in their preferred technology. Therefore, we have that $\bar{r} = \pi$ if $\tau = 0$, and $\bar{r} = g$ if $\tau = \pi - g$. Hence, for the environmental agent $i = 2$, it is never strictly optimal to borrow external funds from the standard agent $i = 1$ because the interest rate to be repaid is at least as much as the return on their preferred investment, i.e. $\bar{r} \geq g$. We assume henceforth that when indifferent on the extensive margin, that is, when indifferent about raising or not external finance, the environmental agent always prefers to use internal finance only.

Consider now the case of the standard agent $i = 1$. If there is a carbon tax $\tau = \pi - g$, then the agent's benchmark emissions are $\bar{e}_1 = 0$, and the problem is similar to that of the environmental agent $i = 2$, implying that it is never strictly optimal for the standard agent $i = 1$ to raise external financing. On the other hand, if there is no carbon tax $\tau = 0$, then benchmark emissions are $\bar{e}_1 = h_1$ and the standard agent can profit if she produces less emissions relative to this benchmark $e_1 < \bar{e}_1$. The problem solved by the standard agent if the latter were to borrow d_1 through the issuance of carbon-contingent debt is

$$U_1 = \max_{I_\pi, I_g} \pi I_\pi + g I_g - \bar{r} d_1 + \rho(h_1 - I_\pi) - \lambda E \text{ such that } I_g + I_\pi \leq h_1 + d_1, \tag{18}$$

which yields solution $I_g = h_1 + d_1$ if $\rho \geq \pi - g$, and $I_g = 0$ otherwise.[13] When determining whether or not to issue a carbon-contingent security, each agent acts as atomistic anticipating that its own investment choices have a negligible impact on cumulative emissions E. If the price of carbon implied by the carbon-contingent debt contract is sufficiently high to incentivize the transition to the green technology, i.e. $\rho \geq \pi - g$, the standard agent's utility is

$$U_1 = g(h_1 + d_1) - \bar{r} d_1 + \rho h_1 - \lambda E \geq \pi h_1 + (g - \bar{r}) d_1 - \lambda E, \tag{19}$$

which is higher than the utility from using internal finance only, $\pi h_1 - \lambda \theta h_1$, provided $\bar{r} \leq g$.
In the case where the contingent rate is not sufficiently high to incentivize switching to the green technology, i.e. $\rho < \pi - g$, the standard agents' utility is

$$U_1 = \pi(h_1 + d_1) - \bar{r} d_1 - \rho d_1 - \lambda E = \pi h_1 - (\bar{r} - \pi + \rho) d_1 - \lambda E \tag{20}$$

which is lower than the utility from using internal finance only if $\bar{r} > \pi - \rho > g$.

We now determine the equilibrium market price of carbon implied by the lending rate ρ, the baseline return $\bar{r}$, and the supply of credit to the standard agent $i = 1$ by solving the lender's problem.

[13] Here we implicitly assume that when indifferent on the intensive margin, the agent always prefers to implement the green technology. Relaxing the assumption does not change the equilibrium outcome.

The Lender's Problem. Environmental agents $i = 2$ decide the optimal amount of lending d_2, and invest the remainder $h_2 - d_2$ in the green technology. From the point of view of the lender, the carbon-contingent security entails providing capital d_2 at time zero, and receiving at time $t = 1$ a fixed return $\bar{r}d_2$ net of a variable return which depends on the carbon emissions reduction or "carbon credits" generated by the security, denoted as q_2, so that the total payoff associated with the security reads

$$\bar{r}d_2 - \rho q_2. \tag{21}$$

Recalling that the emissions generated by investing $h_2 - d_2$ in the green technology are $e_2 = 0$, the environmental agent who is acting as a lender will only internalize the emission reduction generated by the security, q_2, so the agent's problem is

$$U_2 = \max_{d_2 \leq h_2} g(h_2 - d_2) + \bar{r}d_2 - \rho q_2 + \eta q_2 - \lambda E, \tag{22}$$

where the first term is the return from investing in the green technology, the next two terms are the cash flows associated with the contingent security, and the subsequent term captures green preferences reflecting the emissions associated with the agent's actions, subject to the financing constraint

$$g(h_2 - d_2) + \bar{r}d_2 - \rho q_2 \geq 0, \tag{23}$$

so while this class of investors may be willing to reward emission reductions they will only do so up to the point that they deplete their wealth.

From (22), it follows that the fixed indifference rate at which the environmental agent is willing to lend any amount $d_2 \in [0, h_2]$ is $\bar{r} = g$. On the other hand, note that if the lending choice generates emissions reduction (increase) $q_2 > 0$ ($q_2 < 0$), the utility of the agent decreases (increases) via the financial channel i.e. the variable part of the contingent-security payoff, but it increases (decreases) via the green preference channel. Therefore, if lending through the contingent security increases emissions (i.e. $q_2 < 0$), then environmental agents would require compensation at a minimum rate $\rho \geq \eta$. Since $\eta > \pi - g$, recalling agent $i = 1$'s investment problem in (18), this implies that an equilibrium in which the standard agent is willing to borrow through the contingent security and implement the polluting technology does not exists. If, on the other hand, lending generates emissions reduction (i.e. $q_2 > 0$), then environmental agents will be willing to forgo $\rho \leq \eta$ for each unit of emissions reduction achieved, provided the financing constraint (23) is verified. Since $\eta > \pi - g$, then an equilibrium in which the standard agent $i = 1$ is willing to borrow through the contingent security and implement the green technology can arise for any contingent rate $\rho \in [\pi - g, \eta]$.

In equilibrium, the total carbon credits delivered to environmental agents must meet the physical emissions reduction supplied by standard agents, that is

$$(1-\theta)q_2 = \theta(\bar{e}_1 - e_1), \tag{24}$$

This implies that each environmental agent's lending activity is responsible for an equilibrium emissions reduction $q_2 = \frac{\theta}{1-\theta}(\bar{e}_1 - e_1) = \frac{\theta}{1-\theta}h_1$.[14]

In such an equilibrium, financial returns in (23) are thus $gh_2 - \rho\frac{\theta}{1-\theta}h_1$ and they are non-negative provided

$$\rho \leq \bar{\rho} = g\frac{h_2}{h_1}\frac{1-\theta}{\theta}. \tag{25}$$

It follows that if the endowments of environmental agents satisfy

$$h_2 \geq \frac{\pi - g}{g}\frac{\theta}{1-\theta}h_1, \tag{26}$$

then $\bar{\rho} \geq \pi - g$ and an equilibrium with a constraint-admissible rate $\rho \in [\pi - g, min(\bar{\rho}, \eta)]$ always exists. However, if lenders' endowments are such that the budget constraint in (26) is violated, then the carbon-contingent financing solution is not enough to incentivize the technology switch of the entire population of standard agents. In such a case, a smaller share $\theta_d \in [0, \theta)$ of standard agents, given by $\theta_d = \frac{g(1-\theta)h_2}{(\pi-g)h_1}$,[15] could still borrow at the limit rate $\rho = \pi - g$, and switch to the green technology g, whereas the remainder of standard agents would continue to invest in the polluting technology π using internal finance only.

For completeness, note that if the standard agent were to act as lenders, then their problem would be

$$U_1 = \max_{d_1 \leq h_1} (\pi - \tau)(h_1 - d_1) + \bar{r}d_1 - \rho q_1 - \lambda E, \tag{27}$$

with $q_1 = 0$ since emissions produced by environmental agents are zero in the counterfactual scenario without carbon-contingent financing. Hence, only non-contingent lending could occur at the fixed interest rate $\bar{r} = g$ in the presence of the tax $\tau = \pi - g$, or at the rate $\bar{r} = \pi$ if there is no tax $\tau = 0$. Since the standard agents can do at least as well by investing in their preferred technology, it is optimal for them not to lend independently of the carbon tax policy.

We formalize these results in the following proposition:

[14]This comes from the fact that $\bar{e}_1 = h_1$ since the counterfactual economy is a laissez-faire whereas $e_1 = 0$ with contingent securities since it is never optimal to borrow and not switch to the non-polluting technology.

[15]This is determined such that the constraint binds at $\rho = \bar{\rho} = \pi - g$.

PROPOSITION 2. *If there is no carbon tax, then a market for carbon-contingent financing arises in which environmental agents act as lenders and standard agents as borrowers. In such case*

- *if environmental agents' endowments h_2 are sufficiently large to satisfy the inequality in* (26)*, emissions are priced at least $\rho = \pi - g$ and carbon-contingent debt financing enables all standard agents in the economy to adopt the green technology;*
- *otherwise, emissions are priced at exactly $\rho = \pi - g$ and only a share $\theta_d = \frac{g(1-\theta)h_2}{(\pi-g)h_1} < \theta$ of standard agents can access carbon-contingent debt financing and switch to the green technology, whereas the remainder $\theta - \theta_d$ continue to adopt the polluting technology.*

The existence of a market for carbon-contingent securities depends on whether the tax is implemented. If the carbon tax is implemented, then all emissions are priced at the tax rate $\tau = \pi - g$ and all agents adopt the green technology, so there is no scope for pricing carbon via the financial market solution. On the other hand, if there is no tax, then carbon contingent finance arises and the extend to which it enables a complete technology switch depends on the share of environmental agents' endowments.

3.4 Carbon-Contingent Financing and Political Constraints

The previous section has shown that carbon-contingent financing emerges only in the absence of the carbon tax. Borrowing through the issuance of carbon contingent securities is optimal for standard agents, whereas lending via these securities is optimal for environmental agents. We now take a step back and show how the possibility of being a lender (borrower) of carbon-contingent debt affects agents' willingness to vote in favour of a carbon tax $\tau = \pi - g$, derive the constrained optimal tax and welfare in presence of financial markets, and compare it with the benchmark results outlined in Proposition 1.

If there are sufficient funds to finance the technology switch of all standard agents, the utility of environmental agents given the rate ρ is

$$U_2^\rho = gh_2 + (\eta - \rho)q_2 - \lambda E^\rho = gh_2 + (\eta - \rho)\frac{\theta}{1-\theta}h_1 \tag{28}$$

since $q_2 = \frac{\theta}{1-\theta}h_1$ and $E^\rho = 0$. Recalling that $\rho \leq \eta$, we have that $U_2^\rho \geq U_2^\tau = gh_2$, so these agents are better off with securities rather than the tax, since their preference for contributing to reducing emissions is stronger than the price paid to incentivize standard investors to reform.[16] On the other hand, the utility of standard borrowers that switch to the green technology

$$U_1^\rho = gh_1 + \rho h_1, \tag{29}$$

[16]Without loss of generality, here we make the implicit assumption that when indifferent between the tax and carbon contingent finance, i.e. when $\rho = \eta$, they choose the finance option.

is higher than their utility with the tax $U_1^\rho \geq U_1^\tau = gh_1$ since $\rho \geq \pi - g > 0$. So these agents are also better off with securities rather than the tax because they are financially rewarded for reducing their emissions. In this case, recalling that aggregate emissions are zero, i.e. $E^\rho = 0$, welfare is

$$W^\rho = \theta(gh_1 + \rho h_1) + (1-\theta)(gh_2 - \rho\frac{\theta}{1-\theta}h_1) = \theta gh_1 + (1-\theta)gh_2$$

which is equal to the welfare achieved by the optimal tax $W^\tau = \theta gh_1 + (1-\theta)gh_2$.

Consider now the case in which the funds deployed through carbon-contingent finance are insufficient to fund the transition of all standard agents but can only fund a fraction θ_d. In such a case, aggregate emissions are $(\theta - \theta_d)h_1$ and the environmental lender's utility becomes

$$U_2^\rho = gh_2 + (\eta - \rho)\frac{\theta_d}{1-\theta}h_1 - \lambda(\theta - \theta_d)h_1 \tag{30}$$

which recalling that $\rho = \pi - g$, can be higher than the utility with the tax, i.e. $U_2^\rho > U_2^\tau = gh_2$, if $(\eta - \rho)\frac{\theta_d}{1-\theta} > \lambda(\theta - \theta_d)$. If this inequality is verified, the environmental lender's private benefits from reforming a fraction of the population through the provision of carbon-contingent finance are higher than the benefit that would be provided by the tax in terms of reducing the carbon emissions of those agents that cannot be reformed using finance. Consequently, they are willing to tolerate some aggregate emissions, as long as the reduction achieved is mediated through them. On the other hand, the standard borrower's utility becomes

$$U_1^\rho = gh_1 + \rho h_1 - \lambda(\theta - \theta_d)h_1, \tag{31}$$

which recalling that $\rho = \pi - g$, can be higher than the utility $U_1^\tau = gh_1$ if the net financial profits from issuing the security are higher than the environmental loss $\pi - g > \lambda(\theta - \theta_d)$. In sum, financial markets decrease both standard and environmental agents' support for a carbon tax $\tau = \pi - g$ by creating a more appealing counterfactual than the benchmark in which investments are funded using personal endowments only. When standard or environmental agents vote against the tax and carbon-contingent markets can only financing the transition of a share θ_d of standard entrepreneurs, total emissions are $E^\rho = (\theta - \theta_d)h_1$, welfare becomes

$$W^\rho = \theta_d(gh_1 + \rho h_1) + (1-\theta)(gh_2 - \rho\frac{\theta_d}{1-\theta}h_1) + (\theta - \theta_d)(\pi - \lambda)h_1 \tag{32}$$

so the difference relative to the welfare with the optimal tax in (10) is $W^\rho - W^\tau = -(\theta - \theta_d)(\lambda - (\pi - g))h_1$, which is strictly negative since $\lambda > \pi - g$.

The equilibrium implications of the introduction of financial markets in a politically constrained regulatory framework depend on the model parameters, as we formalize in the following

PROPOSITION 3. *Suppose that $\lambda > \pi - g$ such that the implementation of the carbon tax is desirable. If environmental endowments satisfy the inequality in* (26)*, then there is never voting in favour of a carbon tax and all standard agents issue carbon-contingent securities at the market rate $\rho \in [\pi - g, \eta]$. Otherwise, if environmental endowments do not satisfy the inequality in* (26) *the following scenarios are possible:*

- *if the median voter is an environmental type $\theta < 0.5$, then either there is voting in support of the carbon tax $\tau^o = \pi - g$ if $(\eta - \rho)\frac{\theta_d}{1-\theta} < \lambda(\theta - \theta_d)$, or there is no carbon tax $\tau^o = 0$ and a fraction $\theta_d < \theta$ of standard agents issue carbon-contingent securities at the market rate $\rho = \pi - g$.*
- *if the median voter is a standard type $\theta < 0.5$, then either there is voting in support of the carbon tax $\tau^o = \pi - g$ if $\pi - g < \lambda(\theta - \theta_d)$, or there is no carbon tax $\tau^o = 0$ and only a share $\theta_d < \theta$ of standard agents issue carbon-contingent securities at the market rate $\rho = \pi - g$.*

Proposition 3 is stating that when endowments of environmental agents are sufficiently large, financial markets fully substitute the carbon tax. Effectively, the presence of financial markets allows to circumvent any political constraint associated with the regulatory tool and redistribute resources efficiently across agents. On the other hand, when environmental agents' endowments are not very large, financial markets can only partially substitute the regulatory tool, and thus achieve only partial emissions reduction. This is because when environmental agents' endowments do not satisfy the inequality in (26), carbon-contingent securities provide a financially-constrained alternative to a politically-constrained carbon tax. Importantly, while doing so, they also tilt the median voter's preferences away from the carbon tax, potentially generating welfare losses as we outline in the following

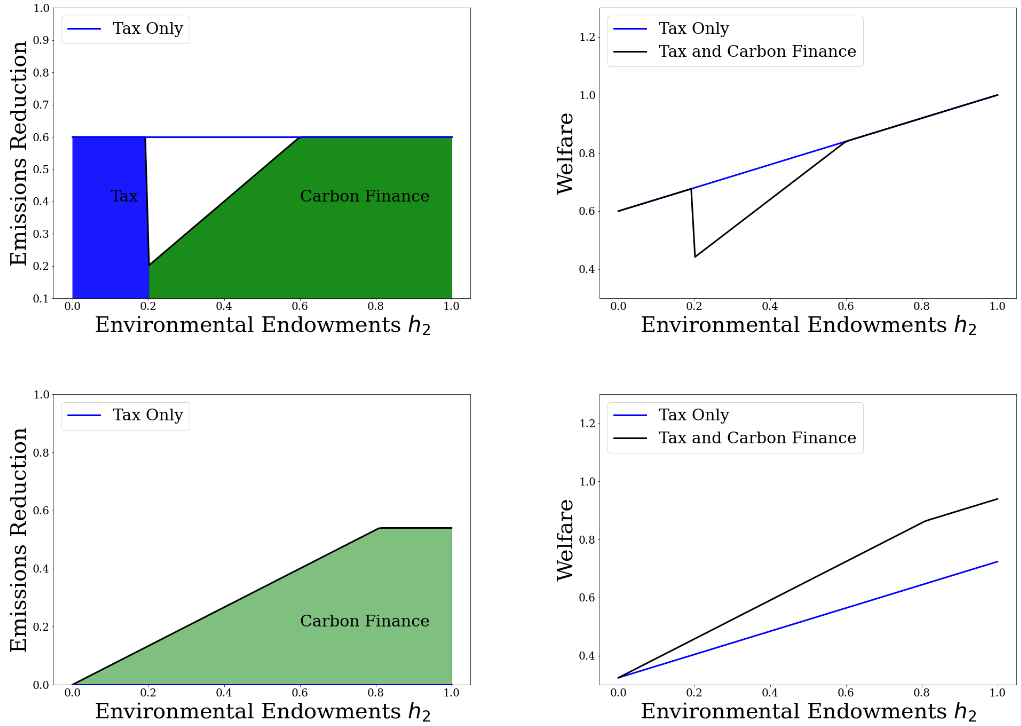
COROLLARY 1. *Suppose that $\lambda > \pi - g$ such that the implementation of the carbon tax is desirable. Then introducing carbon-contingent securities has the following equilibrium implications on welfare and carbon emissions:*

- *If h_2 satisfies* (26) *and $\pi - g > \lambda\theta$ with $\theta > 0.5$, then introducing carbon-contingent securities improves welfare and achieve a higher level of emissions reduction than the carbon tax.*
- *If h_2 violates* (26) *and either $\lambda(\theta - \theta_d) < \pi - g < \lambda\theta$ with $\theta > 0.5$, or $\eta > \pi - g + \lambda(1 - \theta)\frac{\theta - \theta_d}{\theta_d}$ with $\theta < 0.5$, then carbon contingent securities reduce welfare and achieve a lower level of emissions reduction than the carbon tax.*
- *In any other case, carbon contingent securities achieves the same level of emissions reduction and the same welfare than the carbon tax.*

The corollary outlines the regions of the model parameters where introducing financial markets improves welfare, deteriorates welfare, and achieves the same welfare as the carbon tax respectively. The first case is the one in which political constraints for the tax are binding even in absence of financial markets. Such a case is depicted in the bottom plots in Figure 5, showing that financial markets improve welfare and

Figure 5. Reducing Carbon: Carbon Tax vs Carbon-Contingent Financing

The plots show welfare and emissions reduction achieved by the tax alone and by the combined presence of the tax and financial markets. Top (bottom) plots refer to the case in which the political constraint for the tax is not binding (binding respectively). The top and bottom left plots show the emissions reduction achieved by the constrained carbon tax only (area under the blue line), and by the combined presence of the tax and financial markets (area under the black line). The green/blue regions refer to the case when in presence of financial markets, emissions reduction is achieved through carbon-contingent securities/the carbon tax respectively. The top and bottom right plots refer to the welfare in the presence of the tax only (blue line) and in the combined presence of the tax and financial markets (black line). The x-axis represents environmental agents endowments. Other model parameters are $\eta = 0.4$, $\lambda = 1$, $\theta = 0.6$, $h_1 = 1$, $\pi - g = 0.4$ (top plots) and $\eta = 0.5$, $\lambda = 1$, $\theta = 0.6$, $h_1 = 1$, $\pi - g = 0.6$ (bottom plots) respectively.



achieve a higher level of emissions reduction than the tax. The second case is the detrimental one in which financial markets switch the economy from one that supports the tax from one that does not, but cannot fully substitute the emissions reduction achieved by the tax. Such a case is depicted in the top plots in Figure 5, when endowments of environmental entrepreneurs h_2 are high enough to eliminate support for the tax, but not high enough to reform the investments of all standard agents. Finally, the third case is the case in which financial markets and the carbon tax are perfect substitutes, which is the case depicted in the top plots in Figure 5 for high environmental endowments h_2.

It is worth noting that, although we have framed the security payoff in (16) as the sum of a fixed term (interest on the principal d_i), and a carbon-contingent term (difference between actual and counterfactual emissions, $e_i - \bar{e}_i$), in this stylized risk-neutral model without frictions, the role played by the former is marginal. Specifically, in equilibrium, lending any positive amount $d_2 \in (0, h_2]$ from environmental to standard entrepreneurs can occur only if the latter invest the borrowed capital in the green technology, and at an interest rate $\bar{r} = g$ which is the rate of return on the green technology. Therefore, none of the equilibrium results would change if the debt notional was normalized to $d_2 = 0$, and the environmental entrepreneurs would enter the contract at time $t = 0$ without providing the capital upfront but only exchanging the contingent term in (16) at time $t = 1$, while continuing to invest in their preferred green technology at time $t = 0$. In the extended model with a continuum of entrepreneurs and non-linear technologies, we will make use of this property and study a simpler version of the security design where the notional d_i at time $t = 0$ is normalized to zero.

4 Extended Model

The simple model, in light of being linear delivers either-or type of predictions and cannot rationalize the empirical evidence showing that contingent finance co-exists with carbon pricing regulation. To understand the interaction between market-based and regulatory tools on the intensive margin, we extend the model to allow for a continuum of agents with heterogeneous environmental preferences, as well as a continuum of production technologies with a convex carbon abatement cost. Specifically, instead of assuming either a polluting or a non-polluting production technology, we allow for the possibility of reducing the emission intensity of the production technology at a cost.

There is a continuum of carbon abatement technologies, parameterized by $\delta \in [0, 1]$, which for an investment input I deliver output and emissions

$$y(I, \delta) = (\pi - \phi(\delta))I \quad \text{and} \quad e(I, \delta) = I(1 - \delta),$$

where $\phi(\delta)$ denotes the cost of abatement, assumed to be convex $\phi(\delta) = \frac{1}{2}\phi\delta^2$.

There is a mass one of agents $i \in [0, 1]$ with endowments h_i, environmental preferences η_i increasing monotonically in i, and utility

$$U_i = C_i - \eta_i e_i - \lambda_i E, \tag{33}$$

where e_i denotes emissions associated with the actions of agent i, $E = \int_0^1 e_i di$ the total emissions in the economy, and λ_i an agent-specific "climate exposure" parameter representing agent i's beliefs about the damage associated with the carbon emissions in the economy, which can differ from the actual exposure, λ, known by the regulator. While we do not specify a functional form, we assume that the parameter λ_i is positively correlated with preferences η_i and is therefore an increasing function of the type $i \in [0, 1]$.[17] This specification allows for the presence of "climate deniers" in the economy, namely agents which do not

[17] It is plausible to assume that an agent that suffers a higher disutility from the emissions associated with its actions, perceives a higher exposure to climate shocks.

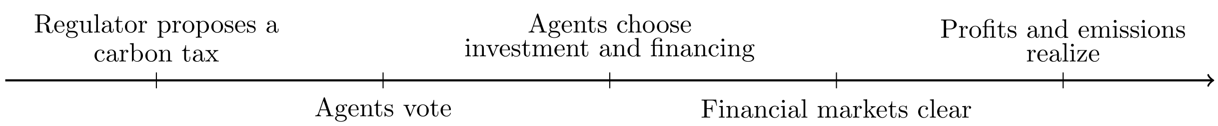
recognize the economic damage associated with carbon emissions. According to our modelling assumption, these agents also value less any environmental benefit that derives from their own actions.

The regulator maximizes utilitarian social welfare, which is given by

$$W = \int_0^1 C_i di - \lambda E. \tag{34}$$

with λ the actual climate exposure parameter which is known by the regulator.

The timeline below summarizes the sequence of actions in the model:



As in the case of the simpler model, our aim is to determine the conditions under which financial markets as a tool for pricing carbon can substitute the regulatory tool and improve welfare. To do so, we follow a backward induction approach and first determine the agents' optimal investment and financing choices in the joint presence of a given carbon tax and a market for carbon-contingent securities. We then input these choices into the agents' utilities at the timing of voting, and derive the maximum admissible tax that each agent can support assuming each fully internalizes the behaviour of others and the adjustment of financial markets. Solving for the maximum tax as a function of the agent's type will allow us to determine the median-voter constraint, which we then input into the regulator problem of finding the constrained-optimal tax which maximizes the utilitarian social welfare in the presence of financial markets.

As a useful benchmark, we outline the investment choices, utilitarian social welfare, and cumulative emissions in a laissez-faire economy with no financial markets and no carbon taxes.

4.1 Laissez-Faire Benchmark

In a decentralized economy without financial markets nor taxes, each agent chooses investment I_i and abatement δ_i to maximize the utility in (33), with consumption given by investment output $C_i = y(I_i, \delta_i)$ and emissions $e_i = e(I_i, \delta_i)$. The investment problem is

$$U_i^* = \max_{I_i, \delta_i} y(I_i, \delta_i) - \eta_i e(I_i, \delta_i) - \lambda_i E \;\text{ such that } I_i \leq h_i. \tag{35}$$

The optimal abatement choice is given by the individual environmental preference scaled by the cost of abatement

$$\delta_i^* = \frac{\eta_i}{\phi},$$

while the optimal investment given optimal abatement is

$$I_i^* = h_i \text{ if } \pi - \eta_i(1 - \frac{1}{2}\frac{\eta_i}{\phi}) > 0 \tag{36}$$

$$I_i^* = 0 \text{ otherwise.} \tag{37}$$

Assuming that the profitability of the most polluting technology π is large enough, we focus on the case in which condition (36) is always satisfied and it is optimal for each agent i to invest and by doing so to produce some emissions (i.e. we focus on internal solutions only). Furthermore, unless otherwise stated, we assume that all agents have equal endowments $h_i = \$1$, so that the utility of each agent i is

$$U_i^* = (\pi - \frac{1}{2}\frac{\eta_i^2}{\phi}) - \eta_i e_i^* - \lambda_i E^*, \tag{38}$$

with $e_i^* = (1 - \frac{\eta_i}{\phi})$ and $E^* = \int_0^1 e_i^* di$. The utilitarian social welfare in this economy is given by

$$W^* = \int_0^1 C_i^* di - \lambda E^* = \int_0^1 (\pi - \frac{1}{2}\frac{\eta_i^2}{\phi} - \lambda(1 - \frac{\eta_i}{\phi}))di. \tag{39}$$

4.2 Carbon Tax

The regulator considers to impose a tax τ on the emissions e_i produced by each agent i to maximize the utilitarian social welfare. To preserve consistency with the previous framework where tax revenues are never effectively collected, and also motivated by extensive empirical evidence on implemented carbon tax policies, we assume that the carbon tax is *revenue-neutral*.[18] Under a revenue-neutral carbon tax, the government taxes every ton of carbon emitted and redistributes the collected tax revenues to taxpayers in the form of lump-sum payments. We assume that the redistribution rule is of a fixed type, that is, the regulator redistributes revenues as a fixed proportion α of the tax and thus makes a payment $\$\alpha\tau$ to each agent i after the revenues are collected. The design of ex-post compensations (i.e., tax rebates) is extensively studied in the context of incomplete environmental regulation and carbon leakage risk (see, for example, Martin, Muûls, De Preux, and Wagner [2014], Fowlie and Reguant [2022]). We show below that, even when considering a single economy, tax rebates have important implications on the equilibrium level of carbon-contingent financing and the voting decisions of agents.

[18]Examples of revenue-neutral carbon taxes include both those applied to firms and those applied to consumers. As far as the former group is concerned, a popular one is the carbon tax implemented since 2001 in the United Kingdom, the Climate Change Levy. For the case of carbon taxes applied to consumers, a popular example is the tax implemented by the Canadian province of British Columbia in 2001, the first North American revenue-neutral carbon tax applied to the purchase or use of fuel in British Columbia.

Consider a situation in which the agents can finance investments with own funds only and there are no financial markets. Maintaining the assumption that the productivity of the most polluting technology π is sufficiently large so that investment is non-zero, i.e. $I_i = h_i = \$1$ for each i, we have that the agent i's problem in the presence of the tax becomes

$$U_i^\tau = \max_{\delta_i} \pi - \phi(\delta_i) - \eta_i e_i(\delta_i) - \tau e_i(\delta_i) + \alpha\tau - \lambda_i E^\tau \tag{40}$$

with $e_i(\delta_i) = 1 - \delta_i$ and $E^\tau = \int_0^1 e_i(\delta_i^\tau)di$. The optimal abatement choice for agent i given the tax is

$$\delta_i^\tau = \frac{\eta_i + \tau}{\phi} = \delta_i^* + \frac{\tau}{\phi}, \tag{41}$$

where we assume that $\eta_i < \phi - \tau$ for each $i \in [0,1]$,[19] which substituting into the utility (40) yields

$$U_i^\tau = \pi - \frac{1}{2}\frac{(\eta_i + \tau)^2}{\phi} - (\eta_i + \tau)(1 - \frac{\eta_i + \tau}{\phi}) + \alpha\tau - \lambda_i E^\tau. \tag{42}$$

Recalling that the regulator redistributes the tax revenues equally, namely

$$R^\tau = \tau E^\tau = \tau \int_0^1 e_i(\delta_i^\tau)di = \tau\alpha \tag{43}$$

one obtains that the tax redistribution rule

$$\alpha = 1 - \frac{\bar{\eta} + \tau}{\phi}, \tag{44}$$

with $\bar{\eta} = \int_0^1 \eta_i di$ the average green preference. Substituting α into (20) yields after some re-arrangement

$$U_i^\tau = U_i^* - \frac{1}{2}\frac{\tau^2}{\phi} + \tau\frac{\eta_i - \bar{\eta}}{\phi} + \lambda_i(E^* - E^\tau). \tag{45}$$

with U_i^* as in (38) and $E^* - E^\tau = \int_0^1 (e_i^* - e_i^\tau)di = \frac{\tau}{\phi}$. The utility of agent i in (45) is equal to the laissez-faire benchmark U_i^* when $\tau = 0$, and otherwise is concave in the tax τ with a maximum at $\tau_i = \lambda_i + \eta_i - \bar{\eta}$. Importantly, net gains from the introduction of the tax, $U_i^\tau - U_i^*$, are monotonically increasing in the type $i \in [0,1]$. Therefore, in order to find the tax that receives support from at least half of the population, it is sufficient to look at the maximum admissible tax that the median voter $i = 0.5$ is prepared to support. From (45), we can derive that such maximum acceptable tax is

$$\tau \leq \bar{\tau}_{0.5} = max(0, 2\tau_{0.5}). \tag{46}$$

[19] This ensures that the abatement technology lies in the admissible region $\delta_i^\tau \in [0,1]$ for any tax chosen by the regulator. We discuss the case in which the technology constraint is violated in the Appendix, when introducing financial markets and carbon contingent securities.

with $\tau_{0.5} = \lambda_{0.5} + \eta_{0.5} - \bar{\eta}$. Any tax above the threshold $\bar{\tau}_{0.5}$ in (46) makes the median voter strictly worse off than the laissez-faire benchmark, i.e. $U^{\tau}_{0.5} < U^{*}_{0.5}$. In this economy where financial markets are not taken into account, we show in the Appendix that utilitarian social welfare is

$$W^{\tau} = W^{*} - \frac{1}{2}\frac{\tau^2}{\phi} - \frac{\tau\bar{\eta}}{\phi} + \lambda(E^{*} - E^{\tau}), \tag{47}$$

The regulator chooses the optimal tax by maximizing the social welfare in (47) subject to the constraint that the chosen tax is below the threshold of the median voter $\bar{\tau}_{0.5}$. Thus, we prove the following:

PROPOSITION 4. *For a given median-voter constraint $\bar{\tau}_{0.5}$ as in* (46)*, the tax τ^{o} that maximizes the regulator problem*

$$\max_{\tau} W^{\tau} \quad \textit{such that} \quad \tau \leq \bar{\tau}_{0.5} \tag{48}$$

with utilitarian social welfare as in (47) *is given by $\tau^{o} = \min(\lambda - \bar{\eta}, \bar{\tau}_{0.5})$.*

The proposition shows that the unconstrained optimal tax equates the Pigouvian benchmark net of the average green preference, i.e. $\tau^{o} = \lambda - \bar{\eta}$, where the latter term appears as the regulator accounts for the average effect of environmental preferences η_i on agent i's investment choices. However, when the political constraint is binding, the optimal tax is the one that makes the median voter indifferent between supporting or not the regulation, namely $\tau^{o} = \bar{\tau}_{0.5} = 2(\lambda_{0.5} + \eta_{0.5} - \bar{\eta})$. This implies that if the median voter's belief about the climate exposure parameter is well below the actual one, i.e. $\lambda_{0.5} << \lambda$, then the constrained-optimal tax can be strictly below the optimal one. On the other hand, in the case considered in the simple model where each agent i's belief about the climate exposure parameter $\lambda_i = \lambda$, the optimal *revenue-neutral* tax will always receive support from at least half of the population. Finally, it is important to stress that, in addition to the belief about the climate parameter $\lambda_{0.5}$, the median voter threshold is a function of the difference between the median voter and the average green preference, i.e. $\eta_{0.5} - \bar{\eta}$. This term follows from the choice of the tax redistribution rule, which in general depends on the distribution of preferences η_i and endowments h_i across types. Allowing for heterogenous endowments $h_i \neq \$1$, the median-voter threshold would become $\bar{\tau}_{0.5} = 2(\lambda_{0.5} + \eta_{0.5} - \int_0^1 w_i \eta_i di)$, with $w_i = \frac{h_1}{\int_0^1 h_i di}$ the share of agent i's endowments in the economy.[20] In practice, this implies that the political constraint becomes more (less) binding if the majority of the endowments are concentrated among agents with green preference above (below) average. The importance of tax redistribution rules has received particular attention in relation to the issue of carbon leakage, as a way to incentivize firms not to relocate production elsewhere. We highlight the importance of the tax redistribution rule within one economy, through its effect on median voter preferences and support for a carbon tax.

[20] Specifically, with heterogeneous endowments, equation (43) becomes $\tau(\int_0^1 h_i(1 - \frac{\eta_i + \tau}{\phi})di = \tau\alpha \int_0^1 h_i di$, which re-arranging implies $\alpha = 1 - \frac{\int_0^1 w_i \eta_i}{\phi} - \frac{\tau}{\phi}$ with $w_i = \frac{h_i}{\int_0^1 h_i di}$. Substituting thus into we can solve for the medan voter threshold as before.

4.3 Carbon-Contingent Financing

Given a certain carbon tax τ, we derive the conditions under which a market for carbon-contingent financing exists and the equilibrium price of carbon implied by this market. We introduce carbon-contingent securities along the lines of those studied in the previous baseline model. Specifically, we assume that each agent i can issue a carbon-contingent security which effectively rewards the issuer for reducing emissions and imposes a monetary penalty for increasing emissions relative to a benchmark agreed at security issuance. Without loss of generality, we assume a zero principal notional at time $t = 0$ and focus on the carbon-contingent part of the security payoff.[21] Under this simplified security structure, the payoff to the security issuer at time $t = 1$ is given by

$$\rho(e_i^\tau - e_i), \tag{49}$$

where e_i the issuer i's actual emissions at time $t = 1$ and e_i^τ the benchmark emissions given by the counterfactual scenario where the security is not issued and there is a carbon tax τ, determined at time $t = 0$. Thus, the security issuer is rewarded with a positive payoff if it reduces emissions below the benchmark, and vice-versa.

We first derive the issuer and lender problem and then outline the conditions under which the net gains from issuing the security are a monotonically decreasing function of the type. We then solve for a cutoff type which is indifferent between selling or buying a carbon-contingent security, and derive the equilibrium price of carbon emissions implied by the contract, ρ, as a function of this type. The equilibrium will allow us to determine the financial market response to the tax τ, which we will then input into the regulator's problem.

The Issuer's Problem. Denote $\mathcal{I} \subset [0, 1]$ the set of agents that issue the carbon-contingent contract and thus act as sellers in this market. Denote $\mathcal{I}_i^\tau(\rho)$ issuer i's utility for a given tax τ and security-implied carbon price ρ, which is given by

$$\mathcal{I}_i^\tau(\rho) = \max_{\delta_i} U_i^\tau(\delta_i) + \rho(e_i^\tau - e_i(\delta_i)) \quad \text{such that} \quad \delta_i \leq 1 \tag{50}$$

with utility under the tax $U_i^\tau(\delta_i) = \pi - \phi(\delta_i) - \eta_i e_i(\delta_i) - \tau e_i(\delta_i) + \tau\alpha - \lambda_i E^\tau$, emissions $e_i(\delta_i) = 1 - \delta_i$, and subject to the technology constraint that the maximum abatement is one which reduces emissions to

[21] Note that for simplicity of the analysis, and following the discussion in the previous section, we have normalized the notional d_i in (16) to zero and decided to only focus on the equilibrium pricing of the contingent term of the security. The security could also be interpreted as a carbon swap which has zero price at issuance and an exchange of a variable component ρe_i for a fixed component $\rho\bar{e}_i$ at time $t = 1$.

zero. The optimal carbon abatement choice for a given tax τ and security-implied carbon price ρ is

$$\delta_i^\tau(\rho) = \delta_i^\tau + \frac{\rho}{\phi} \tag{51}$$

where $\delta_i^\tau = \frac{\eta_i+\tau}{\phi}$ is the optimal abatement technology choice in the counterfactual scenario where a tax exists and the security is not issued, derived in (41).[22] Substituting the optimal technology back into the utility in (50), we show in the Appendix that

$$\mathcal{I}_i^\tau(\rho) = U_i^\tau(\delta_i^\tau) + \frac{1}{2}\frac{\rho^2}{\phi}, \tag{52}$$

so issuing a carbon-contingent security yields strictly positive profits with respect to a benchmark utility with the carbon tax only. Furthermore, the profits are only a function of the market price of carbon ρ and independent of agent i's preferences.

The Lender's Problem. Denote now the set of lenders, which act as buyers of carbon-contingent contracts, as $\mathcal{L} \subset [0,1] - \mathcal{I}$. Denote the emissions reduction financed through the carbon-contingent contract, which can also be thought of as the quantity of carbon credits purchased by agent $i \in \mathcal{L}$ as

$$q_i = \int_{j\in\mathcal{I}_i} (e_j^\tau - e_j)dj, \tag{53}$$

with $\mathcal{I}_i$ the set of issuers whose contingent securities are purchased by agent i, with $\int_{i\in\mathcal{L}} \mathcal{I}_i = \mathcal{I}$. Agent i continues to invest in the abatement technology δ_i^τ, and only decides the optimal quantity q_i to purchase, by solving the problem

$$\mathcal{L}_i^\tau(\rho) = \max_{q_i} U_i^\tau(\delta_i^\tau) + \eta_i q_i - \rho q_i \text{ such that } C_i^\tau(\rho) - \rho q_i \geq 0, \tag{54}$$

where $U_i^\tau(\delta_i^\tau)$ is the benchmark utility in the presence of the carbon tax, and the consumption $C_i^\tau(\rho) = \pi - \frac{1}{2}\frac{\eta_i^2}{\phi} - \frac{1}{2}\frac{\tau^2}{\phi} - \frac{\tau(\bar{\eta}+\int_{i\in\mathcal{I}}\rho di)}{\phi}$ is the amount of output that agent i can consume after the tax, whose derivation is outlined in the Appendix. The constraint is the equivalent of the budget constraint introduced in the previous section. From the linearity of the problem, it follows that

$$\begin{aligned} q_i^\tau(\rho) &= \frac{C_i^\tau(\rho)}{\rho} \quad \text{if} \quad \rho \leq \eta_i \\ q_i^\tau(\rho) &= 0 \quad \text{otherwise.} \end{aligned} \tag{55}$$

Substituting the optimal quantity back into the utility in (54), the utility of lender i given the tax τ and

[22] Here we continue to abstract from corner solutions in which optimal abatement would be above the technology constraint $\delta \in [0,1]$. The condition for an interior solution is $\eta_i < \phi - \tau - \rho$, and we already assumed that ϕ is large enough so that each agent i's preferences verify $\eta_i < \phi - \tau$. In the Appendix, we outline the equilibrium in which the technology constraint is violated.

the security-implied carbon price ρ is given by

$$\mathcal{L}_i^\tau(\rho) = U_i^\tau(\delta_i^\tau) + (\eta_i - \rho)\frac{C_i^\tau(\rho)}{\rho}1\{\eta_i \geq \rho\}, \tag{56}$$

meaning that agent i is strictly better off purchasing the security if its green preference is stronger than the market-implied price of carbon $\eta_i > \rho$, realizing profits that depend on this difference and the net return on the technology $C_i^\tau(\rho)$. Otherwise, it has the same utility as in the benchmark scenario where the security is not issued.

Define the net gains from issuing the security as the difference between the agent i's utility associated with issuing a carbon-contingent security, given in (52), and the utility associated with acting as a lender in carbon-contingent security markets, given in (56) as

$$\Pi_i^\tau(\rho) = \mathcal{I}_i^\tau(\rho) - \mathcal{L}_i^\tau(\rho). \tag{57}$$

For the net gains in (57) to be monotonically decreasing in the agent's type i, it is sufficient to show that

$$\frac{\partial}{\partial \eta_i}(\eta_i - \rho)\frac{\pi_i^\tau(\rho)}{\rho} > 0. \tag{58}$$

Below we outline a sufficient condition for the inequality in (58) to hold for each $i \in [0,1]$ given the set of issuers $\mathcal{I} = [0, i)$ and the set of lenders $\mathcal{L} = [i, 1]$.

Single-crossing condition. *For a given abatement cost ϕ, profitability π, carbon tax τ and green preferences $\eta_i \in \mathcal{C}^1([0,1])$ with $\eta_i^{'} > 0$, a sufficient condition for the net gains in* (57) *to decrease monotonically with the type i is that*

$$\pi > \frac{1}{2}\frac{\tau^2}{\phi} + \frac{1}{2}\frac{\eta_i^2}{\phi} + \tau\frac{\bar{\eta}}{\phi} + \tau i\frac{\rho}{\phi} + (\frac{\eta_i}{\rho} - 1)(\eta_i + \frac{\tau\rho}{\eta_i^{'}}). \tag{59}$$

with $\bar{\eta} = \int_0^1 \eta_i di$ the average green preference.

The proof is provided in the Appendix. When the single-crossing property is verified, we can solve for an internal cutoff type $x \in (0,1)$ verifying $\Pi_x^\tau(\rho) = 0$ such that the set of issuers is $\mathcal{I} = [0, x)$ and the set of lenders is $\mathcal{L} = [x, 1]$. The higher the cutoff type x the higher the fraction of agents that are borrowing through carbon-contingent securities and the higher the level of carbon-contingent financing in the economy. Formally, we introduce the following:

Definition. *For a given carbon tax τ, the pair (ρ, x) constitutes an equilibrium if* *a) the market clearing condition*

$$\int_0^x (e_i^\tau - e_i(\delta_i^\tau(\rho)))di = \int_x^1 q_i^\tau(\rho)di \tag{60}$$

is satisfied, with $\delta_i^\tau(\rho)$ as in (51) and $q_i^\tau(\rho)$ the optimal quantity purchased as in (55), and

b) the net gains in (57) with $i = x$ satisfies the indifference condition

$$\Pi_x^\tau(\rho) = 0. \tag{61}$$

We prove in the Appendix the following equilibrium for the market-implied carbon price ρ and the level of carbon-contingent financing in the economy, pinned down by the cutoff type $x \in (0, 1)$.

PROPOSITION 5. *For a given abatement cost ϕ, profitability π, green preferences $\eta_i \in \mathcal{C}^1([0, 1])$ with $\eta_i^{'} > 0$ which satisfy condition (59), and carbon tax τ, the pair (ρ, x) which solves*

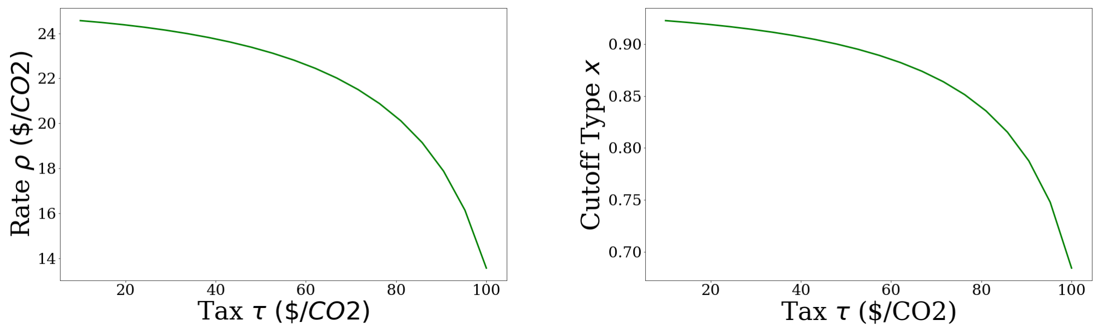
$$\rho = \frac{-\tau(1-x) + \sqrt{(\tau(1-x))^2 + 4\phi k(x)}}{2} \quad \textit{and} \quad \frac{1}{2}\frac{\rho^2}{\phi} = (\eta_x - \rho)\frac{C_x^\tau(\rho)}{\rho} \tag{62}$$

with $k(x) = \frac{1}{x}(\pi - \frac{1}{2}\frac{\tau^2}{\phi} - \frac{\tau\bar{\eta}}{\phi})(1-x) - \frac{1}{x}\int_x^1 \frac{1}{2}\frac{\eta_i^2}{\phi}di$, constitutes an equilibrium.

Figure 6 shows the equilibrium carbon rate ρ^τ (left plot) and the cutoff type x^τ (right plot) against the tax τ in \$/CO_2t. Preferences are assumed to be convex $\eta_i = \eta i^2$ in the agent type $i \in [0, 1]$, with $\eta_i \leq$\$40/CO_2t for each $i \in [0, 1]$. As observed, the equilibrium rate ρ^τ, which represents the market-implied price of carbon expressed in \$/CO_2t, as well as the cutoff type x^τ, are a decreasing function of the tax τ. This implies that the equilibrium carbon abatement financed through carbon-contingent securities is also a decreasing function of the tax τ.[23] In such a setting, the negative correlation is due to the fact that a higher tax increases the cost of delivering further emissions reduction, making contingent securities less appealing in equilibrium. Importantly, the effect is a function, among other things, of the ex-post tax redistribution rule chosen by the regulator, suggesting that different designs of tax rebates alter the utilitarian social welfare of the regulated economy. Worth noting is that the equilibrium cutoff type x is informative about whether the median voter is typically an issuer, i.e. $x^\tau > 0.5$, or a lender, i.e. $x^\tau < 0.5$, of carbon-contingent securities. For what follows, it is useful to stress that when the tax has moderate values (for example, has a similar magnitude as the green preference of the highest type $\tau \approx$\$40/CO_2t), Figure 6 suggests that the equilibrium cutoff type is well above the median voter, with the implication that the median voter is typically a borrower of carbon contingent securities.

[23] That is because $\int_0^x (e_i^\tau - e_i(\delta_i^\tau(\rho^\tau)))di = \int_0^x (\frac{\rho}{\phi})di = \frac{\rho x}{\phi}$.

Figure 6. Equilibrium carbon-contingent financing as a function of the tax
The plots show the equilibrium rate ρ (left plot) and the cutoff type x (right plot) in (95) as a function of the tax τ when preferences are convex $\eta_i = \eta i^2$ in the type $i \in [0, 1]$. Endowments are $h_i = \$1$ for each i, and the other model parameters are $\eta = 40$, $\phi = 150$, and $\pi = 50$.



4.4 The Voting Problem

We now solve for the agents' voting problem and determine the maximum admissible tax that a regulator can enforce without loosing political support from the majority. We do so by taking into account that the existence of financial markets for pricing carbon affects political support for regulation. Since preferences for the tax τ increase monotonically with the type $i \in [0, 1]$, as we show below, the maximum admissible tax is the one that makes the median type $i = 0.5$ indifferent between voting or not for the carbon tax. In what follows, we focus on the case in which the magnitude of the tax is comparable to the green preference of the highest type, which allows us to abstract from corner solutions in which the equilibrium relationship between the tax and the price of carbon-contingent securities is determined by the technology constraint. Under such assumption, the median voter is typically an issuer of carbon-contingent securities (i.e. the case in which the equilibrium cutoff type $x^\tau > 0.5$ as in Figure 6). We leave in the Appendix a discussion of the case in which the median voter is a lender (i.e. $x^\tau < 0.5$), which becomes more likely when the technology constraint is violated.

To assess support for regulation, we contrast the median voter's utility in an economy where she issues carbon-contingent securities in absence of a carbon tax, against one in which she issues carbon-contingent securities while subject to a carbon tax τ.

Recall $\mathcal{I}_i^\tau(\rho)$ as the utility of issuer i in (52) in an economy with contingent securities priced at ρ and a carbon tax equal to τ, and define the total emissions in such economy as $E^\tau(\rho) = E^\tau - \frac{\rho}{\phi}$. Now for a given tax τ, denote as (ρ^τ, x^τ) the equilibrium price and level of contingent financing as in (95), and as (ρ^0, x^0) the equilibrium price and level of contingent financing when the tax $\tau = 0$. We show in the

Appendix that the issuer i's utility as a function of the tax τ is

$$\mathcal{I}_i^\tau(\rho^\tau) = \mathcal{I}_i^0(\rho^0) - \frac{1}{2}\frac{\tau^2}{\phi} + \tau\frac{\eta_i - \bar{\eta} - \rho^\tau x^\tau}{\phi} - \frac{1}{2}\frac{(\rho^0)^2 - (\rho^\tau)^2}{\phi} + \lambda_i(E^0(\rho^0) - E^\tau(\rho^\tau)) \tag{63}$$

meaning that the utility is equal to the one achieved in a laissez-faire economy with financial markets only, denoted as $\mathcal{I}_i^0(\rho^0)$, net of the financial cost introduced by the tax adjusted for the lump-sum redistribution, captured by the terms $-\frac{1}{2}\frac{\tau^2}{\phi} + \tau\frac{\eta_i - \bar{\eta} - \rho^\tau x^\tau}{\phi}$, net of the decrease in profits from issuance of the carbon contingent security (the security price decreases as seen in Figure 6), captured by the term $-\frac{1}{2}\frac{(\rho^0)^2 - (\rho^\tau)^2}{\phi}$, and accounting for the difference in climate exposure introduced by the tax, captured by the last term $\lambda_i(E^0(\rho^0) - E^\tau(\rho^\tau))$.

Similarly to the simple model case in (45), the utility gain from the tax $\mathcal{I}_i^\tau(\rho^\tau) - \mathcal{I}_i^0(\rho^0)$ are monotonically increasing as a function of the agent's type i, allowing us to express the political constraint as a function of the maximum admissible tax $\bar{\tau}_{0.5}$ which can be supported by the median voter. Rearranging (63) with $i = 0.5$, the maximum admissible tax is either the solution to

$$\bar{\tau}_{0.5} + 2\rho^{\bar{\tau}_{0.5}} x^{\bar{\tau}_{0.5}} + \frac{(\rho^0)^2 - (\rho^{\bar{\tau}_{0.5}})^2}{\bar{\tau}_{0.5}} + 2\frac{\lambda_{0.5}}{\bar{\tau}_{0.5}}(\rho^0 x^0 - \rho^\tau x^\tau) = 2(\lambda_{0.5} + \eta_{0.5} - \bar{\eta}) \tag{64}$$

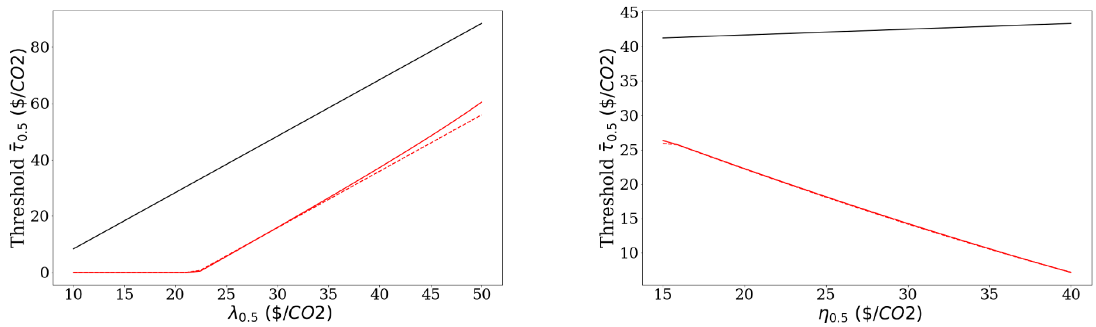
if $\bar{\tau}_{0.5} > 0$, otherwise is $\bar{\tau}_{0.5} = 0$.

Note that the median voter threshold in (64) differs from the case without financial markets, $\bar{\tau}_{0.5} = 2(\lambda_{0.5} + \eta_{0.5} - \bar{\eta})$, for three additional terms. The first term accounts for the fact that the presence of financial markets decreases the carbon tax revenues, thereby reducing the ex-post lump-sum transfer to agent i by an amount $\rho^\tau x^\tau$. The second term accounts for the fact that the tax reduces the price of carbon implied by the contingent security (i.e. $\rho^\tau < \rho^0$), thereby reducing the median voter (borrower)'s profits from the issuance of the carbon contingent security. Finally, the third term accounts for the fact that the equilibrium contribution of financial markets to cumulative emissions reduction is reduced by the tax. The second and third effects are only tangible when the equilibrium adjustment of financial markets to the tax is very large, i.e., $\frac{d\rho^\tau}{d\tau} << 0$. When those are negligible (for example, when the tax has not large values as reported in Figure 6), the threshold simplifies to an explicit solution

$$\bar{\tau}_{0.5} \approx 2(\lambda_{0.5} + \eta_{0.5} - \bar{\eta} - \rho^0 x^0). \tag{65}$$

Figure 7 shows, in red, the threshold $\bar{\tau}_{0.5}$ in (64) as a function of the climate exposure parameter $\lambda_{0.5}$ (left plot) and the median-voter green preference $\eta_{0.5}$ (right plot). The black line is the reference threshold in absence of financial markets, i.e., $\bar{\tau}_{0.5} = 2(\lambda_{0.5} + \eta_{0.5} - \bar{\eta})$, which indeed is a specific case of (64) when $\rho^\tau = 0$ for each $\tau \geq 0$. Finally, the red dotted line is the approximate threshold in (65) assuming

Figure 7. Voting Problem The plot shows the indifference threshold $\bar{\tau}_{0.5}$ which solves (64) (red line) along with the baseline solution in absence of financial markets (black line). The red dotted line is the simplified threshold in (65) which does not take into account of equilibrium adjustments in the financial markets as a function of the tax. The x-axis refers to the median-voter belief of the climate exposure parameter $\lambda_{0.5}$(right plot) and the green preference parameter η (right plot) respectively. The distribution of preferences is convex as $\eta_i = \eta i^2$ and endowments are equal $h_i = 1$. Other model parameter values are: $\phi = 100$, $\pi = 50$, and $\eta = 20$ (left plot); $\phi = 120$, $\pi = 50$, and $\lambda_{0.5} = 20$ (right plot).



that the marginal effect of the tax τ on the equilibrium price and level of contingent financing is negligible.

The first thing to note when comparing the red lines against the black line is that the threshold is much lower when the presence of financial markets is taken into account in the voting problem. This implies that more often than not, the presence of financial markets shifts the economy from one that supports a tax (black line) to one that does not (red lines). Second, we note that the thick and dashed red lines in both plots, which outline the median voter threshold in (64) and the approximate solution in (65) respectively, are virtually equivalent, meaning that in the voting problem equilibrium price adjustments in the carbon finance market (i.e., intensive margin considerations) are not relevant (see also Figure 6 for smaller values of the tax τ).

Perhaps most importantly, we note while the relationship between the median-voter threshold and the climate belief parameter $\lambda_{0.5}$ remains unaltered in presence of financial markets, the relationship between the median-voter threshold and the median-voter preference $\eta_{0.5}$ changes drastically when financial markets are taken into account. Specifically, absent financial markets, a higher median-voter preference $\eta_{0.5}$ implies a higher support for the carbon tax (as reflected in an increasing threshold, Figure 7, right plot, black line). In contrast, in presence of financial markets, the median-voter thresold decreases as a function of the preference $\eta_{0.5}$. This because the financial markets response to higher green preferences, which enters the median voter threshold negatively via the term ρx is stronger than the direct positive effect of green preferences, captured by η, and it effectively generates the inverted relationship outlined by the red line in Figure 7, right plot.

4.5 The Regulator Problem

As in the baseline model, the regulator is subject to a political constraint in that it must propose a tax which is supported by at least half of the population. The regulator utilitarian welfare in the presence of financial markets is

$$W^{\tau}(\rho) = \int_0^x C^{\tau}_{\mathcal{I}_i}(\rho)di + \int_x^1 C^{\tau}_{\mathcal{L}_i}(\rho)di - \lambda E^{\tau}(\rho). \tag{66}$$

where $C^{\tau}_{i,\mathcal{I}}(\rho^{\tau})$ and $C^{\tau}_{i,\mathcal{L}}(\rho^{\tau})$ is the consumption of the agent i in the group of issuers and lenders of carbon contingent securities, outlined in the appendix, and the equilibrium carbon price ρ and the indifference type x which pins down the level of contingent financing. As discussed, we limit the analysis to the case in which the equilibrium pair (ρ, x) satisfies the interior condition in (95), which ultimately amounts to assuming that the externality λ is not too large so that a certain amount of pollution is optimal for each agent i. We show in the Appendix that the welfare in (67) can be expressed as

$$W^{\tau}(\rho) = W^* - \frac{1}{2}\frac{\tau^2}{\phi} - \frac{1}{2}\frac{\rho^2 x}{\phi} - \frac{\tau(\bar{\eta} + \rho x)}{\phi} - \frac{\bar{\eta}\rho x}{\phi} + \lambda\frac{\tau + \rho x}{\phi} \tag{67}$$

from which it derives the following

PROPOSITION 6. *For a given threshold $\bar{\tau}_{0.5}$ as in* (64)*, the optimal tax τ^o which maximizes the constrained regulator problem*

$$\max_{\tau} W^{\tau} \quad \textit{such that} \;\; \tau \leq \bar{\tau}_{0.5}$$

with utilitarian social welfare as in (67)*, is given by*

$$\tau^o = \min\Big(\lambda - \bar{\eta} - \frac{\rho^o x^o + \rho^o \rho^o_{\tau} x + (0.5\rho^o)^2 x^o_{\tau}}{1 + \rho^o_{\tau} x^o_{\tau}}), \bar{\tau}_{0.5}\Big) \tag{68}$$

with (ρ^o, x^o) the equilibrium pair in (95) *evaluated at the tax $\tau = \tau^o$, and $(\rho^o_{\tau}, x^o_{\tau})$ the derivative of (ρ^o, x^o) in* (95) *with respect to τ evaluated at $\tau = \tau^o$.*

Since in our range of model parameters the marginal effect of the tax on the security-implied carbon price and the cutoff type is small, i.e. the derivative of ρ and x with respect to τ are small, the unconstrained optimal tax can be conveniently approximated by

$$\tau^o \approx \lambda - \bar{\eta} - \rho^o x^o, \tag{69}$$

meaning that the optimal tax τ^o differs from the one in absence of financial markets by an amount that reflects the abatement enabled by financial markets in equilibrium. Under such approximation, we derive the following corollary, which summarizes the effect of introducing financial markets on utilitarian welfare and carbon emissions reduction. As for the case of the simple model, we focus on the plausible scenario in which the average green preferences are low enough so that $\lambda - \bar{\eta} > 0$, meaning that the implementation of a carbon tax is always desirable in absence of financial markets.

COROLLARY 2. *Suppose that $\lambda - \bar{\eta} > 0$ so that the implementation of a carbon tax in absence of financial markets is always desirable. Then introducing carbon-contingent securities has the following equilibrium implications on welfare and carbon emissions:*

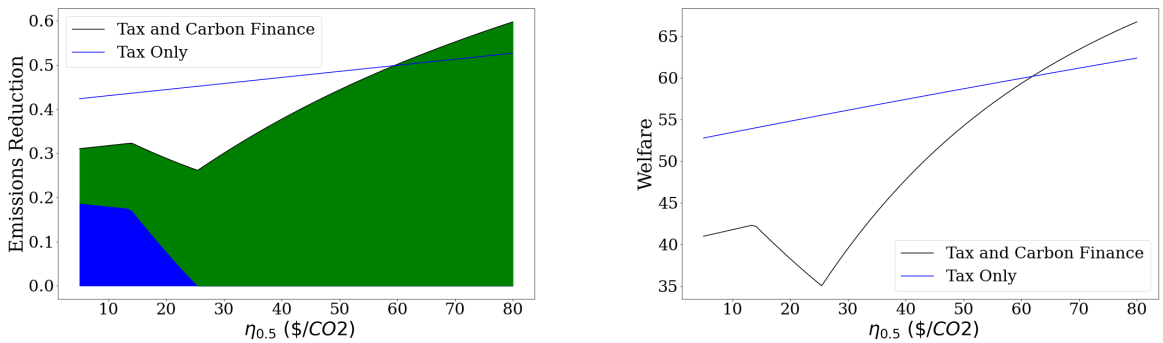
- *If the median voter threshold in the absence of financial markets satisfies $\bar{\tau}_{0.5} > \lambda - \bar{\eta}$, then introducing financial markets will either provide the same amount of emissions reduction or a lower amount of emissions reduction than with the optimal tax only. Meanwhile, welfare will only be lower.*
- *If the median voter threshold in the absence of financial markets $\bar{\tau}_{0.5} < \lambda - \bar{\eta}$, then*
  - *either $\rho^0 x^0 < \lambda - \bar{\eta}$, in which case introducing financial markets will always provide a lower amount of emissions reduction and a lower welfare than with the constrained tax only;*
  - *or $\rho^0 x^0 > \lambda - \bar{\eta}$, in which case introducing financial markets will always provide in a higher amount of emissions reduction than with the constrained tax only. Meanwhile, welfare could be higher than with the tax if λ is large enough.*

The proof of the corollary is outlined in the Appendix. If the carbon tax in absence of financial markets receives enough political support (i.e., the median-voter constraint does not bind $\bar{\tau}_{0.5} > \lambda - \bar{\eta}$), then financial markets can at best provide the same amount of emissions reduction but a lower welfare than with the tax only. This because if financial markets do not reduce support for the optimal tax in (69), then there is perfect substitution in terms of emissions abatement achieved, but lower welfare brought by the fact that there are convex costs in emissions abatement.[24] On the other hand if financial markets reduce support for the tax, then not only they reduce welfare but also decrease the emissions reduction achieved by the former. On the other hand, if the tax alone lacks enough political support, then there is a scope for financial markets to increase welfare and emissions reduction. This only occurs in those cases where abatement generated by financial markets alone is very high so that the tax is *optimally* equal to zero (that is, if $\rho^0 x^0 > \lambda - \bar{\eta}$). In such a case, financial markets circumvent the political constraint and achieve (alone) higher emissions reduction than the regulatory tool. While welfare is still penalized by the fact that the abatement is distributed unequally across agents, it can be higher than the one with the regulatory tool provided the climate parameter λ is high enough.

Figure 8 summarizes this discussion by showing the equilibrium emissions reduction (left plot) and welfare (right plot) as a function of the median voter preference $\eta_{0.5}$, which moves both the optimal tax as well as the equilibrium level of carbon contingent financing. As for the simple model, the blue line represents the emissions reduction (left plot) and welfare (right plot) achieved by the tax only, whereas the black line is the emissions reduction (left plor) and welfare (right plot) achieved with the equilibrium combination of

[24]With convex abatement costs, it is optimal to achieve a certain level of abatement by implementing each the same amount rather than to split abatement heterogeneously across groups of agents.

Figure 8. Reducing Carbon: Carbon Tax vs Carbon-Contingent Financing
The y-axis shows the equilibrium emissions reduction (left plot) and welfare (right plot) achieved by an economy with the carbon tax only (blue line) and by an economy with the combined presence of financial markets and the tax (black line). The x-axis shows the median voter preference $\eta_{0.5}$. Preferences are assumed to be convex in types $\eta_i = \eta i^2$, and endowments are homogeneous $h_i = 1$ for each $i \in [0, 1]$. The green region of the left plot is the amount of emission abatement achieved through issuance of carbon-contingent securities. The blue region is the residual abatement achieved through the tax. The area below the blue line represents the baseline emissions abatement achieved by the tax only, i.e. when financial markets are not present. Other model parameters are $\phi = 150$, $\lambda = 150$, $\pi = 50$, $\lambda_{0.5} = 25$.



carbon-contingent securities and the tax. When plotting emissions reduction, the relative contribution of the two tools is represented by the blue region and the green region, respectively. According to the model predictions, financial markets are beneficial only in those economies where climate regulation receive low support, suggesting that carbon-contingent funds are best directed to markets without carbon taxes, where there is more abatement potential.

5 Concluding Remarks

We start by proposing a baseline model in which financially- and environmentally- motivated agents can invest their endowments in polluting or non-polluting technologies, with the latter being less profitable than the former. We show that a carbon tax corrects the laissez-faire allocation in which the polluting technology is adopted by standard agents, and has the effect of increasing welfare and decreasing emissions. If there is no political support for a carbon tax, carbon-contingent financing provided by environmentally-motivated agents can effectively substitute the carbon tax. Whether the financial market solution partially or fully substitutes regulation depends importantly on the endowments of environmental agents who, by lending to financially-motivated agents via carbon-contingent contracts, are essentially subsidizing their investment in the non-polluting technology. We show that when environmental agents are endowed with sufficiently large funds, financial markets are a superior alternative to the regulatory tool in that they achieve higher welfare than the carbon tax alone independently of the stringency of the political constraint. Pricing emissions through financial markets creates welfare gains also when environmental funds are small, provided that the political constraint is also binding. However, when financial markets shift an economy from one that supports a carbon tax to one that does not, and the capital deployed through carbon-contingent financing is small, there can be welfare losses.

We then extend the model to a continuum of agents with heterogeneous environmental preferences and production technologies the emission intensity of which can be reduced at a convex abatement cost. We derive the optimal tax when the regulator is politically constrained in implementing a revenue-neutral carbon tax which involves redistributing the revenues from the tax across voters. Solving for the agents' financing and investment decisions while taking account of the financial market's response to the tax, we show that taxation and carbon-contingent financing can co-exist and derive the conditions under which together they achieve higher welfare than the carbon tax alone. They are still characterized by a substitution relationship, and the share of emission reduction enabled through carbon-contingent financing is smaller the higher the tax, suggesting that such capital flows are best directed to unregulated markets where they can have more impact.

References

Mr Tobias Adrian, Mr Patrick Bolton, and Alissa M Kleinnijenhuis. The great carbon arbitrage. Technical report, International Monetary Fund, 2022.

Soren T Anderson, Ioana Marinescu, and Boris Shor. Can Pigou at the polls stop us melting the poles? Technical report, National Bureau of Economic Research, 2019.

Adelina Barbalau and Federica Zeni. The optimal design of green securities. *Available at SSRN 4155721*, 2022.

Bruno Biais and Augustin Landier. Emission caps and investment in green technologies. *Available at SSRN 4100087*, 2022.

Michele Bisceglia, Alessio Piccolo, and Jan Schneemeier. Externalities of responsible investments. *Available at SSRN 4183855*, 2022.

Jean-François Bonnefon, Augustin Landier, Parinitha R Sastry, and David Thesmar. The moral preferences of investors: Experimental evidence. Technical report, National Bureau of Economic Research, 2022.

Eleonora Broccardo, Oliver Hart, and Luigi Zingales. Exit versus voice. *Journal of Political Economy*, 130(12):3101–3145, 2022.

Bhagwan Chowdhry, Shaun William Davies, and Brian Waters. Investing for impact. *The Review of Financial Studies*, 32(3):864–904, 2019.

Robin Döttling and Magdalena Rola-Janicka. Too levered for pigou? a model of environmental and financial regulation. *Available at SSRN 4024366*, 2022.

Alexander Dyck, Karl V Lins, Lukas Roth, and Hannes F Wagner. Do institutional investors drive corporate social responsibility? International evidence. *Journal of Financial Economics*, 131(3):693–714, 2019.

Meredith L Fowlie and Mar Reguant. Mitigating emissions leakage in incomplete carbon markets. *Journal of the Association of Environmental and Resource Economists*, 9(2):307–343, 2022.

Lawrence H Goulder and Andrew R Schein. Carbon taxes versus cap and trade: a critical review. *Climate Change Economics*, 4(03):1350010, 2013.

Daniel Green and Benjamin Roth. The allocation of socially responsible capital. *Available at SSRN 3737772*, 2021.

Deeksha Gupta, Alexandr Kopytov, and Jan Starmans. The pace of change: Socially responsible investing in private markets. *Available at SSRN 3896511*, 2022.

Oliver Hart and Luigi Zingales. Companies should maximize shareholder welfare not market value. *Journal of Law*, 2:247–274, 2017.

Oliver D Hart, David Thesmar, and Luigi Zingales. Private sanctions. Technical report, National Bureau of Economic Research, 2022.

Florian Heider and Roman Inderst. A corporate finance perspective on environmental policy. *SAFE Working Paper No. 345*, 2021.

Robert Heinkel, Alan Kraus, and Josef Zechner. The effect of green investment on corporate behavior. *Journal of Financial and Quantitative Analysis*, pages 431–449, 2001.

Harrison Hong, Neng Wang, and Jinqiang Yang. Welfare consequences of sustainable finance. *National Bureau of Economic Research No. w28595*, 2021.

Shiyang Huang and Alexandr Kopytov. Sustainable finance under regulation. *Available at SSRN 4231723*, 2022.

Jacquelyn Humphrey, Shimon Kogan, Jacob Sagi, and Laura Starks. The asymmetry in responsible investing preferences. Technical report, National Bureau of Economic Research, 2021.

Roman Inderst and Marcus M Opp. Socially optimal sustainability standards with non-consequentialist. 2022.

Julian F Kölbel and Adrien-Paul Lambillon. Who pays for sustainability? an analysis of sustainability-linked bonds. *Swiss Finance Institute Research Paper No. 23-07*, 2022.

Augustin Landier and Stefano Lovo. Esg investing: How to optimize impact? *HEC Paris Research Paper No. FIN-2020-1363*, 2020.

Ralf Martin, Mirabelle Muûls, Laure B De Preux, and Ulrich J Wagner. Industry compensation under relocation risk: A firm-level analysis of the eu emissions trading scheme. *American Economic Review*, 104(8):2482–2508, 2014.

William Nordhaus. The climate club: how to fix a failing global effort. *Foreign Affairs*, 99:10, 2020.

Martin Oehmke and Marcus M Opp. A theory of socially responsible investment. *Swedish House of Finance Research Paper No. 20-2*, 2022a.

Martin Oehmke and Marcus M Opp. Green capital requirements. *Swedish House of Finance Research Paper No. 22-16*, 2022b.

Lubos Pastor, Robert F Stambaugh, and Lucian A Taylor. Sustainable investing in equilibrium. 2020.

Tarun Ramadorai and Federica Zeni. Climate regulation and emissions abatement: Theory and evidence from firms' disclosures. *European Corporate Governance Institute–Finance Working Paper*, (730), 2021.

Arno Riedl and Paul Smeets. Why do investors hold socially responsible mutual funds? *The Journal of Finance*, 72(6):2505–2550, 2017.

Robert N Stavins. The future of us carbon-pricing policy. *Environmental and energy policy and the economy*, 1(1):8–64, 2020.

Appendix

Proof. [Corollary 1]. The proof of the corollary follows immediately from the discussion in the paper. Financial markets improve welfare and achieve strictly higher emissions reduction than the tax only when the tax cannot pass in absence of financial markets. This can only happen when the median voter is a standard type, i.e. when $\theta > 0.5$, and when the profitability loss is high enough to make this voter willing to forego economic benefits deriving from the reduction in total emissions, i.e. $\pi - g > \lambda\theta$. Conversely, financial markets deteriorate welfare and achieve a strictly lower amount of emissions reduction than the tax only when i) they switch the economy from one that supports the tax to one that does not, and ii) they do not have enough funds to reform investments of all standard agents. A necessary condition for the latter scenario to occur is that environmental endowments h_2 do not satisfy the inequality in (26). When this happens, financial markets are detrimental provided that in the baseline economy there is enough support for the tax, i.e., either when $\theta < 0.5$ or when $\theta > 0.5$ but $\lambda\theta > \pi - g$, but in the economy with financial markets support for the tax disappears, i.e. either $\eta > \pi - g + \lambda(1-\theta)\frac{\theta - \theta_d}{\theta_d}$ when $\theta < 0.5$ or $\theta(\theta - \theta_d) < \pi - g$ when $\theta > 0.5$.

Proof. [Proposition 4] The utilitarian social welfare in the laissez-faire reads

$$W^* = \int_0^1 C_i^* di - \lambda E^* \tag{70}$$

which substituting each agent i's investment and abatement choices (I_i^*, δ_i^*) and recalling that $h_i = 1$ for each i gives

$$\begin{aligned} W^* &= \int_0^1 (\pi - \phi(\delta_i^*))I_i^* di - \lambda \int_0^1 e_i(\delta_i^*)di \\ &= \int_0^1 (\pi - \frac{1}{2}\frac{\eta_i^2}{\phi})di - \lambda(1 - \int_0^1 \frac{\eta_i}{\phi}di) \\ &= \int_0^1 (\pi - \frac{1}{2}\frac{\eta_i^2}{\phi} - \lambda(1 - \frac{\eta_i}{\phi})di. \end{aligned} \tag{71}$$

In presence of the revenue-neutral carbon tax, the social welfare becomes

$$W^\tau = \int_0^1 C_i^\tau di - \lambda \int_0^1 e_i^\tau di \tag{72}$$

where each agent i's consumption in presence of the tax reads $C_i^\tau = (\pi - \phi(\delta_i^\tau) - \tau e_i^\tau + \alpha\tau)I_i^\tau$, with $I_i^\tau = 1$, $\delta_i^\tau = \delta_i^* + \frac{\tau}{\phi}$, and $e_i^\tau = e_i(\delta_i^*) - \frac{\tau}{\phi}$. Substituting we get

$$W^\tau = \int_0^1 (\pi - \phi(\delta_i^* + \frac{\tau}{\phi}) - \tau e_i^\tau + \alpha\tau)di - \lambda \int_0^1 (e_i(\delta_i^*) - \frac{\tau}{\phi})di \tag{73}$$

which recalling the definition of α becomes

$$
\begin{aligned}
W^\tau &= \int_0^1 (\pi - \phi(\delta_i^* + \frac{\tau}{\phi}))di - \lambda \int_0^1 (e_i(\delta_i^*) - \frac{\tau}{\phi})di \\
&= \int_0^1 (\pi - \phi(\delta_i^*) - \frac{1}{2}\frac{\tau^2}{\phi} - \frac{\tau\eta_i}{\phi})di - \lambda \int_0^1 (e_i(\delta_i^*) - \frac{\tau}{\phi})di \\
&= \int_0^1 (\pi - \phi(\delta_i^*))di - \lambda \int_0^1 e_i(\delta_i^*)di - \frac{1}{2}\frac{\tau^2}{\phi} - \frac{\tau\bar{\eta}}{\phi} + \lambda\frac{\tau}{\phi} \\
&= W^* - \frac{1}{2}\frac{\tau^2}{\phi} - \frac{\tau\bar{\eta}}{\phi} + \lambda\frac{\tau}{\phi}
\end{aligned}
\tag{74}
$$

with $\lambda\frac{\tau}{\phi} = \lambda(E^* - E^\tau)$. It is therefore immediate to note that

$$
\frac{d^2W^\tau}{d\tau^2} < 0 \quad \text{and} \quad \frac{dW^\tau}{d\tau}\Big|_{\tau=\lambda-\bar{\eta}} = 0 \tag{75}
$$

from which follows that there is a unique unconstrained optimum in $\tau^o = \lambda - \bar{\eta}$ and $W^\tau > W^*$ for $\tau \in (0, \lambda - \bar{\eta}]$.

The Issuer and the Lender problem. From the point of view of an issuer of the carbon-contingent security, for a given carbon tax τ, the abatement problem reads

$$
\mathcal{I}^\tau(\rho) = \max_{\delta} \pi - \phi(\delta_i) - \eta_i e_i(\delta_i) - \tau e_i(\delta_i) + \rho(e_i^\tau - e_i(\delta)) + \alpha\tau - \lambda E^\tau \quad \text{such that } \delta_i \leq 1. \tag{76}
$$

with $e_i^\tau = e_i(\delta_i^*) - \frac{\tau}{\phi}$ the amount of emissions reduction delivered if the security was not issued, E^τ the total emissions produced in the economy, and $\alpha\tau$ the lump-sum payment that derives from the collection of tax revenues. As for the simple model, the implicit assumption is that when determining whether or not to issue the security, the agent i acts as atomistic and takes global emissions and the lump-sum payment as given. This is because a change in agent i's financing strategy will have negligible impact on total emissions and therefore tax revenues. Solving for the optimal abatement we get

$$
\delta_i^\tau(\rho) = \delta_i^\tau + \frac{\rho}{\phi} \text{ if } \frac{\rho}{\phi} + \delta_i^\tau < 1 \tag{77}
$$

and $\delta_i^\tau(\rho) = 1$ otherwise.

We first consider the case in which the technology constraint is not violated (i.e. the problem admits an internal solution), which is the case we focus on in the paper. Substituting the optimal abatement choice in the utility problem, and noting that $e_i(\delta_i^\tau + \frac{\rho}{\phi}) = e_i^\tau - \frac{\rho}{\phi}$, we get

$$
\begin{aligned}
\mathcal{I}^\tau(\rho) &= \pi - \phi(\delta_i^\tau + \frac{\rho}{\phi}) - (\eta_i + \tau)(e_i^\tau - \frac{\rho}{\phi}) + \rho\frac{\rho}{\phi} + \alpha\tau - \lambda E^\tau \\
&= \pi - \phi(\delta_i^\tau) - \frac{1}{2}\frac{\rho^2}{\phi} - \frac{\rho(\eta_i + \tau)}{\phi} - (\eta_i + \tau)(e_i^\tau - \frac{\rho}{\phi}) + \rho\frac{\rho}{\phi} + \alpha\tau - \lambda E^\tau \\
&= \pi - \phi(\delta_i^\tau) - (\eta_i + \tau)e_i^\tau + \frac{1}{2}\frac{\rho^2}{\phi} + \alpha\tau - \lambda E^\tau \\
&= U_i^\tau + \frac{1}{2}\frac{\rho^2}{\phi}.
\end{aligned}
\tag{78}
$$

which is strictly positive provided $\rho > 0$ and constant as a function of the type i.

On the other hand, if the constraint is violated $\delta_i^\tau(\rho) = 1 = \delta_i^\tau + (1 - \delta_i^\tau)$, the utility reads

$$
\begin{aligned}
\mathcal{I}^\tau(\rho) &= \pi - \phi(\delta_i^\tau + (1 - \delta_i^\tau)) - (\eta_i + \tau)(e_i^\tau - (1 - \delta_i^\tau)) + \rho(1 - \delta_i^\tau) + \alpha\tau - \lambda E^\tau \\
&= \pi - \phi(\delta_i^\tau) - \frac{1}{2}\frac{(1 - \delta_i^\tau)^2}{\phi} - \frac{(1 - \delta_i^\tau)(\eta_i + \tau)}{\phi} - (\eta_i + \tau)(e_i^\tau - (1 - \delta_i^\tau)) + \rho(1 - \delta_i^\tau) + \alpha\tau - \lambda E^\tau \\
&= \pi - \phi(\delta_i^\tau) - (\eta_i + \tau)e_i^\tau + (\eta_i + \tau)(1 - \delta_i^\tau)(1 - \frac{1}{\phi}) + (1 - \delta_i^\tau)(\rho - \frac{1}{2}\frac{(1 - \delta_i^\tau)}{\phi}) + \alpha\tau - \lambda E^\tau \\
&= U_i^\tau + (1 - \delta_i^\tau)((\eta_i + \tau)(1 - \frac{1}{\phi}) + \rho - \frac{1}{2}\frac{(1 - \delta_i^\tau)}{\phi}) \\
&= U_i^\tau + (1 - \delta_i^\tau)(\phi\delta_i^\tau - \delta_i^\tau + \rho - \frac{1}{2\phi} + \frac{1}{2\phi}\delta_i^\tau)
\end{aligned}
\tag{79}
$$

which is strictly positive and monotonically decreasing in the type i provided $\rho > \frac{1}{\phi} + 1 - 2\delta_i^\tau(\phi - 1 + \frac{1}{\phi})$.

From the point of view of the lender, abatement strategies are kept fixed and the investment problem simply reads

$$
\mathcal{L}^\tau(\rho) = \max_{q_i} U_i^\tau + \eta_i q_i - \rho q_i \quad \text{such that } C_i^\tau(\rho) - \rho q_i \geq 0 \tag{80}
$$

The term $C_i^\tau(\rho)$ is the maximum amount that agent i can consume if she does buy carbon-contingent securities. The latter is given by the output delivered by the technology in which she invests net of the carbon tax and lump-sum payment received by the regulator. This reads

$$
C_i^\tau(\rho) = \pi - \phi(\delta_i^\tau) - \tau e_i^\tau + \tau\alpha(\rho) \tag{81}
$$

where $\alpha(\rho)$ is the lump-sum payment received after financial markets are cleared

$$
\alpha(\rho) = \int_0^1 e_i^\tau di - \int_{\mathcal{I}} (e_i^\tau - e_i(\delta_i^\tau(\rho)))di = 1 - \frac{\bar{\eta} + \tau}{\phi} - \int_{\mathcal{I}} (e_i^\tau - e_i(\delta_i^\tau(\rho)) \tag{82}
$$

which agent i takes as given. Expanding the abatement costs $\phi(\delta_i^\tau) = \frac{\phi}{2}(\delta_i^\tau)^2$, the emissions $e_i^\tau = 1 - \delta_i^\tau$, with $\delta_i^\tau = \frac{\eta_i+\tau}{\phi}$, one gets

$$\begin{aligned} C_i^\tau(\rho) &= \pi - \frac{1}{2}\frac{(\eta_i+\tau)^2}{\phi} - \tau(1 - \frac{\eta_i+\tau}{\phi}) + \tau(1 - \frac{\bar{\eta}+\tau}{\phi} - \int_{\mathcal{I}}(e_i^\tau - e_i(\delta_i^\tau(\rho)))di) \\ &= \pi - \frac{1}{2}\frac{(\eta_i+\tau)^2}{\phi} + \tau(\frac{\eta_i - \bar{\eta}}{\phi}) - \tau\int_{\mathcal{I}}(e_i^\tau - e_i(\delta_i^\tau(\rho)))di \\ &= \pi - \frac{1}{2}\frac{\tau^2}{\phi} - \frac{1}{2}\frac{\eta_i^2}{\phi} - \tau\frac{\bar{\eta}}{\phi} - \tau\int_{\mathcal{I}}(e_i^\tau - e_i(\delta_i^\tau(\rho)))di. \end{aligned} \tag{83}$$

From the linearity of the problem, it follows that the optimal utility of the lender is

$$\mathcal{L}_i^\tau(\rho) = U_i^\tau + (\eta_i - \rho)\frac{C_i^\tau(\rho)}{\rho}1\{\eta_i \geq \rho\}. \tag{84}$$

Proof. [Single-Crossing Condition] Let us focus on the case in which the abatement problem of the issuer admits an internal solution (i.e., the technology constraint is not violated). The net gains from issuance of the carbon-contingent security are

$$\Pi_i^\tau(\rho) = \mathcal{I}_i^\tau(\rho) - \mathcal{L}_i^\tau(\rho) = \frac{\rho^2}{2\phi} - (\eta_i - \rho)\frac{C_i^\tau(\rho)}{\rho}1\{\eta_i \geq \rho\} \tag{85}$$

with $\mathcal{I}_i^\tau(\rho)$ as in (78) and $\mathcal{L}_i^\tau(\rho)$ as in (84). We want to prove that

$$\frac{\partial}{\partial i}\Pi_i^\tau(\rho) < 0 \tag{86}$$

for each $i \in [0,1]$. The first term in (85) is a constant function of the type i. On the other hand, the second term in (85) is equal to zero if $\eta_i < \rho$ and equal to $-(\eta_i - \rho)\frac{C_i^\tau(\rho)}{\rho}$ for $\eta_i > \rho$. To prove (86), it is therefore sufficient to prove that

$$\frac{d}{di}\big((\frac{\eta_i}{\rho} - 1)C_i^\tau(\rho)\big) > 0. \tag{87}$$

From (83), recalling that in the interior solution case $\delta_i^\tau(\rho) = \delta_i^\tau + \frac{\rho}{\phi}$, we get

$$C_i^\tau(\rho) = \pi - \frac{1}{2}\frac{\tau^2}{\phi} - \frac{1}{2}\frac{\eta_i^2}{\phi} - \tau\frac{\bar{\eta}}{\phi} - \tau(\int_{\mathcal{I}}\frac{\rho}{\phi}di)) \tag{88}$$

so the condition becomes

$$\frac{d}{di}\big((\frac{\eta_i}{\rho} - 1)\big(\pi - \frac{1}{2}\frac{\tau^2}{\phi} - \frac{1}{2}\frac{\eta_i^2}{\phi} - \tau\frac{\bar{\eta}}{\phi} - \tau(\int_{\mathcal{I}}\frac{\rho}{\phi}di)\big)\big) > 0 \tag{89}$$

Now because we are looking for an equilibrium where i is the cutoff type $\mathcal{I} = [0, i)$, then the expression further simplifies to

$$\frac{d}{di}\big((\frac{\eta_i}{\rho} - 1)\big(\pi - \frac{1}{2}\frac{\tau^2}{\phi} - \frac{1}{2}\frac{\eta_i^2}{\phi} - \tau\frac{\bar{\eta}}{\phi} - \tau i\frac{\rho}{\phi}\big)\big) > 0. \tag{90}$$

Expanding the derivative, we get

$$\frac{\eta_i^{'}}{\phi}(\pi - \frac{1}{2}\frac{\tau^2}{\phi} - \frac{1}{2}\frac{\eta_i^2}{\phi} - \tau\frac{\bar{\eta}}{\phi} - \tau i\frac{\rho}{\phi}) - (\frac{\eta_i}{\rho} - 1)(\frac{\eta_i}{\phi}\eta_i^{'} + \frac{\tau\rho}{\phi}) > 0 \tag{91}$$

dividing everything by $\eta_i^{'}/\phi$, with $\eta_i^{'} > 0$, we have

$$(\pi - \frac{1}{2}\frac{\tau^2}{\phi} - \frac{1}{2}\frac{\eta_i^2}{\phi} - \tau\frac{\bar{\eta}}{\phi} - \tau i\frac{\rho}{\phi}) > (\frac{\eta_i}{\rho} - 1)(\eta_i + \frac{\tau\rho}{\eta_i^{'}}) \tag{92}$$

that is profits of the most polluting technology verify

$$\pi > \frac{1}{2}\frac{\tau^2}{\phi} + \frac{1}{2}\frac{\eta_i^2}{\phi} + \tau\frac{\bar{\eta}}{\phi} + \tau i\frac{\rho}{\phi} + (\frac{\eta_i}{\rho} - 1)(\eta_i + \frac{\tau\rho}{\eta_i^{'}}). \tag{93}$$

It is worth noting that this condition is in line with the preliminary assumption of large π so that it is always optimal to produce some level of emissions for each type i.

A similar argument applies to the case in which the technology constraint is violated, resulting in a threshold for the profit of the most polluting technology equal to

$$\pi > \frac{1}{2}\frac{\tau^2}{\phi} + \frac{1}{2}\frac{\eta_i^2}{\phi} + \tau\frac{\bar{\eta}}{\phi} + \tau(i - \frac{\int_0^i(\tau + \eta_j)dj}{\phi}) + (\frac{\eta_i}{\rho} - 1)(\eta_i + \frac{\tau\rho}{\eta_i^{'}}). \tag{94}$$

Proof. [Proposition 5] From the definition of equilibrium, we look for a cutoff type $x \in (0,1)$ and a rate ρ such that the following conditions are jointly verified

$$\int_0^x \frac{\rho}{\phi}h_i di = \int_x^1 \frac{C_i^\tau(\rho)}{\rho}h_i di \quad \text{and} \quad \Pi_i^\tau(\rho) = 0 \quad \text{for} \quad i = x. \tag{95}$$

Recalling $h_i = \$1$ for each i and that $C_i^\tau(\rho)$ is as in (88), the market clearing condition rearranged becomes

$$\frac{\rho^2 x}{\phi} + \rho\frac{\tau x(1-x)}{\phi} = (\pi - \frac{1}{2}\frac{\tau^2}{\phi} - \frac{\tau\bar{\eta}}{\phi})(1-x) - \int_x^1 \frac{1}{2}\frac{\eta_i^2}{\phi}di, \tag{96}$$

which solving for ρ gives

$$\rho = \frac{-\tau(1-x) + \sqrt{(\tau(1-x))^2 + 4\phi k_x}}{2} \tag{97}$$

with $k_x = \frac{1}{x}(\pi - \frac{1}{2}\frac{\tau^2}{\phi} - \frac{\tau\bar{\eta}}{\phi})(1-x) - \frac{1}{x}\int_x^1 \frac{1}{2}\frac{\eta_i^2}{\phi}di$. The indifference condition implies that

$$\frac{1}{2}\frac{\rho^2}{\phi} = (\eta_x - \rho)\frac{C_x^\tau(\rho)}{\rho}, \tag{98}$$

which rearranging gives the result.

Now if $\rho + \eta_x > \phi - \tau$, then the equilibrium of this type cannot exist in that we have reached a corner solution in which the rate ρ is so high that the optimal abatement technologies of some issuers $0 < i < x$ go beyond the available cleanest technology $\delta = 1$. In such a corner solution, the utility of the issuer of carbon-contingent securities is defined as in (79). An equilibrium that admits an interior solution (i.e. that verifies the single-crossing property) must verify $\rho > \frac{1}{\phi} + 1 - 2\delta_x^\tau(\phi - 1 + \frac{1}{\phi})$ with x the equilibrium cutoff type. Under such condition, profits from issuing the security are strictly positive[25] and the indifference condition becomes

$$(1 - \delta_x^\tau)(\phi\delta_x^\tau - \delta_x^\tau + \rho - \frac{1}{\phi} + \frac{1}{\phi}\delta_x^\tau) = (\eta_x - \rho)\frac{C_x^\tau(\rho)}{\rho}. \tag{99}$$

On the other hand, market clearing condition becomes

$$\int_0^j \frac{\rho}{\phi} di + \int_j^x (1 - \delta_i^\tau) di = \int_x^1 \frac{C_i^\tau(\rho)}{\rho} di \tag{100}$$

with $j < x$ the type whose optimal abatement $\delta_j^\tau(\rho) = \delta_j^\tau + \frac{\rho}{\phi} = 1$.

Provided that the pair (ρ, x) solution to (99) and (100) verify $\rho > \frac{1}{\phi} - \delta_x^\tau(\phi - 1 + \frac{1}{\phi})$, then an equilibrium still exists.

Voting problem when median voter is an issuer. Let the median voter be a seller of carbon-contingent securities. From the previous discussion, the utility for a given tax τ is given by

$$\mathcal{I}_i^\tau(\rho) = U_i^\tau(\rho) + \frac{1}{2}\frac{\rho^2}{\phi} \tag{101}$$

where $U_i^\tau(\rho)$ is the utility of agent i in an economy with carbon contingent securities priced at ρ and the tax τ, given by

$$\begin{aligned} U_i^\tau(\rho) &= \pi - \phi(\delta_i^\tau) - \eta_i e_i^\tau - \tau e_i^\tau + \tau\alpha(\rho) - \lambda_i E^\tau(\rho) \\ &= \pi + \frac{1}{2}\frac{\eta_i^2}{\phi} - \frac{1}{2}\frac{\tau^2}{\phi} + \tau\frac{\eta_i - \bar{\eta} - \rho x}{\phi} - \lambda_i(E^\tau - \frac{\rho x}{\phi}). \end{aligned} \tag{102}$$

with $E^\tau(\rho) = (E^\tau - \frac{\rho x}{\phi})$ emissions in such economy and $\alpha(\rho) = 1 - \frac{\bar{\eta}+\tau}{\phi} - \frac{\rho x}{\phi}$ the lump-sum tax redistribution payment in such economy. Now when deciding whether or not to vote in favour of the tax, agent i compares her utility in presence of the tax with her utility in absence of the tax. In doing so, agent i takes into account the equilibrium adjustment to the price and level of carbon-contingent financing. That is, denoting (ρ^τ, x^τ) the equilibrium solution in (95) to a given tax τ, agent i is interested in the sign of the following

$$\mathcal{I}_i^\tau(\rho^\tau) - \mathcal{I}_i^0(\rho^0) = U_i^\tau(\rho^\tau) - U_i^0(\rho^0) - \frac{1}{2}\frac{((\rho^0)^2 - (\rho^\tau)^2)}{\phi}, \tag{103}$$

where (ρ^0, x^0) are the equilibrium price and level of contingent financing if the tax does not pass. Substi-

[25]For the utility in (79) to be strictly higher than U_i^τ it is sufficient that $\rho > \frac{1}{\phi} - \delta_x^\tau(\phi - 1 + \frac{1}{\phi})$.

tuting the utility in (102) evaluated in the equilibrium prices ρ^τ and ρ^0 into (103) we get

$$\begin{aligned}\mathcal{I}_i^\tau(\rho^\tau) - \mathcal{I}_i^0(\rho^0) &= U_i^\tau(\rho^\tau) - U_i^0(\rho^0) - \frac{1}{2}\frac{((\rho^0)^2 - (\rho^\tau)^2)}{\phi} \\ &= -\frac{1}{2}\frac{\tau^2}{\phi} + \tau\frac{\eta_i - \bar{\eta} - \rho^\tau x^\tau}{\phi} + \lambda_i(E^0(\rho^0) - E^\tau(\rho^\tau)) - \frac{1}{2}\frac{((\rho^0)^2 - (\rho^\tau)^2)}{\phi} \\ &= -\frac{1}{2}\frac{\tau^2}{\phi} + \tau\frac{\eta_i - \bar{\eta} - \rho^\tau x^\tau}{\phi} + \lambda_i(\frac{\rho^\tau x^\tau + \tau}{\phi} - \frac{\rho^0 x^0}{\phi}) - \frac{1}{2}\frac{((\rho^0)^2 - (\rho^\tau)^2)}{\phi}\end{aligned} \tag{104}$$

which is zero in $\tau = 0$, concave in τ with a maximum at $\tau_i \approx \lambda_i + (\eta_i - \bar{\eta}) - \rho^0 x^0$. Solving for the maximum τ such that $\mathcal{I}_i^\tau(\rho^\tau) - \mathcal{I}_i^0(\rho^0) = 0$, we get

$$\tau + 2\rho^\tau x^\tau + 2\frac{\lambda_i}{\tau}(\rho^0 x^0 - \rho^\tau x^\tau) + \frac{1}{\tau}((\rho^0)^2 - (\rho^\tau)^2) = 2(\lambda_i + \eta_i - \bar{\eta}) \tag{105}$$

which gives the result in (64) for $i = 0.5$.

Voting problem when median voter is a lender. Consider now the case in which the median voter is a buyer of carbon-contingent securities i.e. the equilibrium type $x < 0.5$. As discussed in the paper, this equilibrium is less likely when the technology constrain is not violated, but may arise for large values of the tax which triggers the technology constraint in presence of financial markets. The utility for a given tax τ is

$$\mathcal{L}_i^\tau(\rho) = U_i^\tau(\rho) + (\eta_i - \rho)\frac{C_i(\rho)}{\rho} \tag{106}$$

with $U_i^\tau(\rho)$ and $C_i^\tau(\rho)$ as outlined in (102) and (83), respectively. The net profits from voting in favour of the tax read

$$\begin{aligned}\mathcal{L}_i^\tau(\rho^\tau) - \mathcal{L}_i^0(\rho^0) &= U_i^\tau(\rho^\tau) - U_i^0(\rho^0) + (\frac{\eta_i}{\rho^\tau} - 1)C_i^\tau(\rho^\tau) - (\frac{\eta_i}{\rho^0} - 1)C_i^0(\rho^0) \\ &= U_i^\tau(\rho^\tau) - U_i^0(\rho^0) + (\frac{\eta_i}{\rho^\tau} - 1)(C_i^0(\rho^0) - \frac{1}{2}\frac{\tau^2}{\phi} - \tau\frac{\bar{\eta}}{\phi} - \tau\frac{\rho^\tau x^\tau}{\phi}) - (\frac{\eta_i}{\rho^0} - 1)C_i^0(\rho^0) \\ &= U_i^\tau(\rho^\tau) - U_i^0(\rho^0) - (\frac{\eta_i}{\rho^\tau} - 1)(\frac{1}{2}\frac{\tau^2}{\phi} + \tau\frac{\bar{\eta}}{\phi} + \tau\frac{\rho^\tau x^\tau}{\phi}) + (\frac{\eta_i}{\rho^\tau} - \frac{\eta_i}{\rho^0})C_i^0(\rho^0).\end{aligned} \tag{107}$$

Unlike the case of the issuer, the net profits from voting in favour of the tax can be decreasing in the type i. Intuitively, this happens because of the fact that by reducing the amount of resources that they can lend, the tax reduces agent i's ability to "do good" for the environment. Specifically, as in the simple model, because of warm glow preferences lenders are willing to forgo cumulative emissions reduction in exchange for a higher capacity to contribute to emissions reduction themselves.

If voting profits start to decrease with the type i, then we cannot identify the tax that receives support by at least half of the population by looking at the media-voter problem. However noting that $\frac{d}{di}C_i^0(\rho^0) =$

$-\frac{\eta_i \eta_i'}{\phi} < 0$, a sufficient condition for the net profits to continue to monotonically increase in the type i is

$$-(\frac{1}{\rho^\tau})(\frac{1}{2}\frac{\tau^2}{\phi} + \tau\frac{\bar{\eta}}{\phi} + \tau\frac{\rho^\tau x^\tau}{\phi}) + (\frac{1}{\rho^\tau} - \frac{1}{\rho^0})C_i^0(\rho^0) - \frac{\eta_i \eta_i'}{\rho}(\frac{\eta_i}{\rho^\tau} - \frac{\eta_i}{\rho^0}) > 0 \quad (108)$$

this is most likely verified under the assumption that $\pi >> 0$ is very large (which gives that $C_i^0(\rho^0)$ is large enough).

Proof. **[Proposition 6]** From the utility of the lender and issuer of carbon contingent securities, the utilitarian welfare reads

$$W^\tau(\rho) = \int_0^x (C^\tau_{\mathcal{I}_i}(\rho) + \rho(e_i^\tau - e_i(\delta_i^\tau(\rho))))di + \int_x^1 (C^\tau_{\mathcal{L}_i}(\rho) - \rho q_i(\rho)) - \lambda E^\tau(\rho) \quad (109)$$

with $C^\tau_{\mathcal{I}_i}(\rho)$ the issuer's consumption excluding the profits from the security payoff, defined as

$$C^\tau_{\mathcal{I}_i}(\rho) = \pi - \phi(\delta_i^\tau(\rho)) - \tau e_i(\delta_i^\tau(\rho)) + \tau\alpha(\rho) \quad (110)$$

and $C^\tau_{\mathcal{L}_i}(\rho)$ the lender's consumption before purchasing securities, defined as in (83), and $E^\tau(\rho)$ the equilibrium emissions in the economy for a given tax τ and carbon market price ρ. From the market clearing condition, we have that $\int_0^x (e_i^\tau - e_i(\delta_i^\tau(\rho))di = \int_x^1 q_i(\rho)di$ and thus welfare simplifies to

$$W^\tau(\rho) = \int_0^x C^\tau_{\mathcal{I}_i}(\rho)di + \int_x^1 C^\tau_{\mathcal{L}_i}(\rho)di - \lambda E^\tau(\rho). \quad (111)$$

Substituting the expression for consumption of the issuer and the lender, we get

$$W^\tau(\rho) = \int_0^x (\pi - \phi(\delta_i^\tau(\rho)) - \tau e_i(\delta_i^\tau(\rho)) + \tau\alpha(\rho))di + \int_x^1 (\pi - \phi(\delta_i^\tau) - \tau e_i^\tau + \tau\alpha(\rho))di - \lambda E^\tau(\rho) \quad (112)$$

now recalling that $\alpha(\rho)$ is defined such that the tax is revenue-neutral, the welfare becomes

$$W^\tau(\rho) = \int_0^x (\pi - \phi(\delta_i^\tau(\rho)))di + \int_x^1 (\pi - \phi(\delta_i^\tau))di - \lambda E^\tau(\rho). \quad (113)$$

Expanding the abatement costs of issuers and lenders, we get

$$W^\tau(\rho) = \int_0^x (\pi - \frac{1}{2}\frac{(\eta_i + \tau + \rho)^2}{\phi})di + \int_x^1 (\pi - \frac{1}{2}\frac{(\eta_i + \tau)^2}{\phi})di - \lambda E^\tau(\rho) \quad (114)$$

solving the square

$$W^\tau(\rho) = \int_0^1 (\pi - \frac{1}{2}\frac{(\eta_i + \tau)^2}{\phi})di - \frac{1}{2}\frac{\rho^2 x}{\phi} - \frac{\tau\rho x}{\phi} - \frac{\bar{\eta}\rho x}{\phi} - \lambda(1 - \frac{\eta_i + \tau + \rho}{\phi})di \quad (115)$$

Recalling welfare in the laissez-faire $W^* = \int_0^1 (\pi - \frac{1}{2}\frac{\eta_i^2}{\phi} - \lambda(1 - \frac{\eta_i}{\phi}))di$ we have

$$W^\tau(\rho) = W^* - \frac{1}{2}\frac{\tau^2}{\phi} - \frac{1}{2}\frac{\rho^2 x}{\phi} - \frac{\tau(\bar{\eta} + \rho x)}{\phi} - \frac{\bar{\eta}\rho x}{\phi} + \lambda\frac{\tau + \rho x}{\phi}. \tag{116}$$

Now the regulator problem is to maximize $W^\tau(\rho)$ by taking into account the equilibrium response of ρ and x to the tax τ. In the simplified case where equilibrium adjustments of the carbon market price are very small and can be ignored, the optimal tax becomes

$$\tau^o \approx \lambda - \bar{\eta} - \rho^o x^o \tag{117}$$

where ρ^o and x^o denote the equilibrium market-implied carbon price and level of financing evaluated at the optimal tax τ^o. Otherwise the tax solves

$$\tau^o = \lambda - \bar{\eta} - \frac{\rho^o x^o + \rho^o \rho^o_\tau x + (0.5\rho^o)^2 x^o_\tau}{1 + \rho^o_\tau x^o_\tau} \tag{118}$$

which proves the result.

Proof.[Corollary] We prove it in the case in which the tax in the economy with finnaical markets and the tax is well approximated by $\tau^o \approx \lambda - \bar{\eta} - \rho^o x^o$. The benchmark against which we assess welfare and emissions is an economy in which markets do not exist and the carbon tax is the only tool for reducing emission. In this economy without financial markets welfare is

$$W^\tau = W^* - \frac{1}{2}\frac{\tau^2}{\phi} - \frac{\tau\bar{\eta}}{\phi} + \lambda\frac{\tau}{\phi} \tag{119}$$

and the tax can be either unconstrained $\tau^o = \lambda - \bar{\eta}$ or constrained $\tau^o = \bar{\tau}_{0.5}$.

Consider first the unconstrained tax case case, which occurs when $\bar{\tau}_{0.5} > \lambda - \bar{\eta}$, such that the optimal tax is $\tau^o = \lambda - \bar{\eta}$ and welfare becomes

$$W^{\tau^o} = W^* + \frac{1}{2}\frac{(\lambda - \bar{\eta})^2}{\phi} \tag{120}$$

With financial markets, the optimal tax can be either the unconstrained one $\tau^o = \lambda - \bar{\eta} - \rho^o x^o$, or the constrained one $\tau^o = \bar{\tau}_{0.5}$. In the unconstrained case (which dominates the constrained one), welfare becomes

$$
\begin{aligned}
W^{\tau^o}(\rho^o) &= W^* + \frac{1}{2}\frac{(\lambda-\bar{\eta}-\rho^o x^o)^2}{\phi} + (\lambda-\bar{\eta}-\frac{\rho^o}{2})\frac{\rho^o x^o}{\phi} \\
&= W^* + \frac{1}{2}\frac{(\lambda-\bar{\eta})^2}{\phi} + \frac{1}{2}\frac{(\rho^o x^o)^2}{\phi} - \frac{(\lambda-\bar{\eta})\rho^o x^o}{\phi} + (\lambda-\bar{\eta}-\frac{\rho^o}{2})\frac{\rho^o x^o}{\phi} \\
&= W^* + \frac{1}{2}\frac{(\lambda-\bar{\eta})^2}{\phi} + \frac{1}{2}\frac{(\rho^o x^o)^2}{\phi} - \frac{1}{2}\frac{(\rho^o)^2 x^o}{\phi} \\
&= W^* + \frac{1}{2}\frac{(\lambda-\bar{\eta})^2}{\phi} + \frac{1}{2}\frac{(\rho^o)^2 x^o}{\phi}(x^o-1).
\end{aligned}
\tag{121}
$$

Since $x^o < 1$, welfare is strictly lower than with the optimal tax W^{τ^o}. As far as emissions are concerned, we have that in presence of financial markets

$$
E^{\tau^o}(\rho^o) = E^* - \frac{\tau^o}{\phi} - \frac{\rho^o x^o}{\phi} = E^* - \frac{\lambda-\bar{\eta}}{\phi}
\tag{122}
$$

which are equivalent to those achieved by the optimal tax $\tau^o = \lambda - \bar{\eta}$ in absence of financial markets. In the economy with financial markets, welfare and emissions obtained with the unconstrained tax dominate those obtained when the tax is constrained. So the constrained case clearly achieves lower welfare and lower emissions reduction relative to the tax only economy.

Consider now the case in which in absence of financial markets, the carbon tax is constrained $\tau^o = \bar{\tau}_{0.5} = 2(\lambda_{0.5} + \eta_{0.5} - \bar{\eta})$, which occurs when$^- \leq -\bar{\eta}$. Recall that this necessarily implies that $\lambda_{0.5} < \lambda$. In such a case, welfare with the tax only is

$$
W^{\bar{\tau}_{0.5}} = W^* + \frac{1}{\phi}(\lambda-\bar{\eta})\bar{\tau}_{0.5} - \frac{1}{2\phi}(\bar{\tau}_{0.5})^2 = W^* + \frac{1}{\phi}\bar{\tau}_{0.5}(\lambda-\bar{\eta}-\frac{1}{2}\bar{\tau}_{0.5}).
\tag{123}
$$

This has two be compared against two possible cases: i) the tax in presence of financial markets is also constrained, i.e. $\tau^o = 2(\lambda_{0.5} - \bar{\eta} + \eta_{0.5}) - 2\rho^o x^o$, and ii) the tax in presence of financial markets is *optimally* equal to zero, i.e. when $\rho^o x^o > \lambda - \bar{\eta}$. In the latter case, welfare with financial markets reads

$$
W^{\tau^o}(\rho^o) = W^* + \frac{1}{\phi}\rho^o x^o(\lambda-\bar{\eta}-\frac{\rho^o}{2}),
\tag{124}
$$

which is higher than the one in (123) if

$$
\bar{\tau}_{0.5}(\lambda-\bar{\eta}-\frac{1}{2}\bar{\tau}_{0.5}) < \rho^o x^o(\lambda-\bar{\eta}-\frac{\rho^o}{2}).
\tag{125}
$$

Recalling that $\rho^o x^o > \lambda - \bar{\eta}$, it is sufficient that

$$
\bar{\tau}_{0.5} < (\lambda-\bar{\eta})\frac{(\lambda-\bar{\eta}-\frac{\rho^o}{2})}{(\lambda-\bar{\eta}-\frac{1}{2}\bar{\tau}_{0.5})}
\tag{126}
$$

and since $\bar{\tau}_{0.5} < \lambda - \bar{\eta}$, it is sufficient that $\frac{(\lambda - \bar{\eta} - \frac{\rho^o}{2})}{(\lambda - \bar{\eta} - \frac{1}{2}\bar{\tau}_{0.5})} < 1$ implying that $\rho^o > \bar{\tau}_{0.5}$. This can be generally achieved for a range of model parameters.

As for the emissions reduction, we have

$$E^{\tau^o}(\rho^o) = E^* - \frac{\rho^o x^o}{\phi} < E^* - \frac{(\lambda - \bar{\eta})}{\phi} < E^* - \frac{\bar{\tau}_{0.5}}{\phi} \tag{127}$$

meaning that emissions in this scenario are always lower. When the carbon tax in presence of financial markets is $\tau^o = 2(\lambda_{0.5} - \bar{\eta} + \eta_{0.5}) - 2\rho^o x^o = \bar{\tau}_{0.5} - 2\rho^o x^o > 0$, welfare with financial markets becomes

$$\begin{aligned}
W^{\bar{\tau}_{0.5}}(\rho^o) &= W^* - \frac{1}{2}\frac{(\bar{\tau}_{0.5} - 2\rho^o x^o)^2}{\phi} - \frac{1}{2}\frac{(\rho^o)^2 x^o}{\phi} - \frac{(\bar{\tau}_{0.5} - 2\rho^o x^o)(\bar{\eta} + \rho^o x^o)}{\phi} - \frac{\bar{\eta}\rho^o x^o}{\phi} + \lambda\frac{\bar{\tau}_{0.5} - \rho^o x^o}{\phi} \\
&= W^* - \frac{1}{2}\frac{\bar{\tau}_{0.5}^2}{\phi} - 2\frac{(\rho^o x^o)^2}{\phi} + 2\frac{\bar{\tau}_{0.5}\rho^o x^o}{\phi} - \frac{1}{2}\frac{(\rho^o)^2 x^o}{\phi} - \frac{(\bar{\tau}_{0.5} - 2\rho^o x^o)(\bar{\eta} + \rho^o x^o)}{\phi} - \frac{\bar{\eta}\rho^o x^o}{\phi} + \lambda\frac{\bar{\tau}_{0.5} - \rho^o x^o}{\phi} \\
&= W^* - \frac{1}{2}\frac{\bar{\tau}_{0.5}^2}{\phi} + \frac{\bar{\tau}_{0.5}\rho^o x^o}{\phi} - \frac{1}{2}\frac{(\rho^o)^2 x^o}{\phi} - \frac{\tau_{0.5}\bar{\eta}}{\phi} + \frac{\bar{\eta}\rho^o x^o}{\phi} + \lambda\frac{\tau_{0.5} - \rho^o x^o}{\phi} \\
&= W^{\bar{\tau}_{0.5}} + \frac{\bar{\tau}_{0.5}\rho^o x^o}{\phi} - \frac{1}{2}\frac{(\rho^o)^2 x^o}{\phi} + \frac{\bar{\eta}\rho^o x^o}{\phi} - \lambda\frac{\rho^o x^o}{\phi} \\
&= W^{\bar{\tau}_{0.5}} - \frac{\rho^o x^o}{\phi}(\lambda - \bar{\eta} - \bar{\tau}_{0.5}) - \frac{1}{2}\frac{(\rho^o)^2 x^o}{\phi}
\end{aligned} \tag{128}$$

which is strictly below $W^{\bar{\tau}_{0.5}}$ since $\lambda - \bar{\eta} > \bar{\tau}_{0.5}$. Emissions in this economy are

$$E^{\bar{\tau}_{0.5}}(\rho^o) = E^* - \frac{\bar{\tau}_{0.5} - \rho^o x^o}{\phi} > E^* - \frac{\bar{\tau}_{0.5}}{\phi} \tag{129}$$

which are strictly higher than with the constrained tax only.