Delegated Blocks*

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Abstract

Will asset managers with large amounts of capital and high risk bearing capacity hold large blocks and monitor aggressively? Both block size and monitoring intensity are governed by the contractual incentives of institutional investors, which themselves are endogenous. We show that when high risk bearing capacity arises via optimal delegation, funds hold smaller blocks and monitor significantly less than proprietary investors with identical risk bearing capacity. This is because the optimal contract enables the separation of risk sharing and monitoring incentives. Our findings rationalize characteristics of real world asset managers and imply that block sizes will be a poor predictor of monitoring intensity.

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1 Introduction

The rise of asset managers has led to the concentration of vast amounts of capital in the hands of institutional investors.[1] How is this likely to affect corporate governance? These investors have large risk bearing capacity and thus are able—in principle—to hold large blocks and monitor portfolio firms aggressively. However, both block size and the extent of monitoring are endogenous to the contractual incentives of institutional investors. Such contractual incentives—in turn—are endogenously determined, and will anticipate institutional ownership and monitoring decisions. Will institutional investors hold large blocks commensurate to their risk bearing capacity in equilibrium? Conditional on holding such blocks, will they monitor firms aggressively? To help answer these questions, we study the economics of delegated blockholding. In particular, we characterize corporate governance and risk sharing in markets where equity ownership is optimally delegated and both equity block sizes and the level of monitoring are determined by endogenous contracts established between asset managers and their investors.

We benchmark our analysis against the influential characterization of risk sharing and monitoring in a market with *proprietary* ownership found in Admati, Pfleiderer, and Zechner (1994)—APZ henceforth. Taking as given the existence of a proprietary trader with high risk bearing capacity, APZ consider whether anticipated monitoring costs will limit the trader's willingness to hold large blocks. Under broad and plausible conditions, they find the answer is "no"—as long as traders with high risk bearing capacity cannot commit to limit their trading, they will trade to the competitive risk sharing allocation and monitor at a level consistent with that allocation. This is because the ability to trade repeatedly erodes the large trader's strategic advantage. APZ's striking finding is confirmed in the fully dynamic analysis of DeMarzo and Urosevic (2006). Overall, therefore, the existing

[1] See, e.g., Dasgupta, Fos, and Sautner (2021) for relevant stylized facts.

literature provides a reassuring view: risk sharing and monitoring can coexist happily in financial markets as long as ownership is proprietary.

We show that when high risk bearing capacity is instead attained endogenously via delegation, outcomes are dramatically different. First, the optimal fund holds less of the risky asset, i.e., a smaller block, than an investor with the same risk bearing capability would under the competitive risk sharing allocation. In other words, delegation hurts risk sharing. Second, delegation separates block sizes and monitoring incentives, because monitoring is undertaken by professional asset managers on behalf of the fund. It is *their* effective stake, not the fund's overall stake, that determines the fund's level of monitoring. The optimal delegation contract allocates an effective stake to these professional asset managers that results in a level of monitoring that would be privately optimal for fund investors at their *initial* endowment. These two effects combined imply that the optimal fund undertakes significantly less monitoring than a proprietary blockholder of identical risk bearing capacity. While delegation thus has negative implications for risk sharing and monitoring relative to the case with proprietary large traders, it does provide valuable risk sharing opportunities to agents who do not have full access to financial markets.

Model summary. We start with a minor variation of the APZ benchmark. Our version of their classical "CARA-Normal" model features a firm whose final-date equity cash flows are distributed Normally and a group of traders with CARA utility. There are two types of traders: a single large entity, L, with risk tolerance, i.e., risk bearing capacity, of λ, and a continuum of small traders with aggregate risk bearing capacity of $1 - \lambda$.[2] In addition to trading (potentially many times) in a Walrasian market at the initial date, L can also monitor at an intermediate date: such monitoring is costly for L but increases average

[2]For expositional ease, in the introduction we describe our model "as if" the economy has unit aggregate risk bearing capacity. However, our formal analysis is valid for any arbitrary aggregate risk bearing capacity.

final-date cash flows to all equity holders. The competitive equilibrium allocation in such an economy involves L holding λ fraction of the firm's equity.

Imagine that $L's$ initial endowment of the risky asset is $\omega < \lambda$. Will L trade from ω all the way to λ? There are several impediments. First, L knows that if she trades to λ she will then monitor at a commensurately higher intensity and all $1-\lambda$ other shareholders will benefit from such monitoring. Second, L knows that along the way to λ she must pay the full value of future monitoring when acquiring shares, i.e., she moves prices against herself as she trades. However, in a key result, APZ show that as long as L can't commit to limit her trading, she will nevertheless trade to λ and monitor at the high intensity corresponding to such large holdings. This arises because of an endowment effect. Counterfactually, if any sequence of trades led to a proposed final holding level for L that is strictly below λ, she would be tempted to buy a bit more because the current holding is now part of her endowment. Starting from this endowment, there will always be at least some incremental risk sharing gains by buying a bit more, despite having to pay the full value of future monitoring in making such purchases. This result implies that the anticipation of future monitoring costs does not act as an impediment to holding large positions in financial markets.

In our main analysis, we enrich the APZ framework to model delegated blockholding. We replace L with a measure of small, "unskilled," investors with an aggregate risk bearing capacity of λ and an aggregate endowment of $\omega < \lambda$ (the same as L above). These investors cannot trade or monitor. The remaining "skilled" investors, with an aggregate risk bearing capacity of $1 - \lambda$, can trade and monitor. They can also offer trading and monitoring services to the unskilled investors in return for fees. A fund is formed if unskilled investors (who we then call limited partners, or LPs) team up with an endogenously chosen subset of skilled investors with an aggregate risk bearing capacity of $\tau \in (0, 1 - \lambda]$ (who we then call general partners, or GPs) to pool their endowments. LPs are passive once the fund is formed, and GPs determine holding and monitoring levels subject to their contractual incentives.

We derive an optimal contract from the point of view of the LPs in two parts. First, we find the optimal allocation from the LPs' point of view *as if* they could act as a group and commit to a monitoring level and a trading strategy. Next, we show that under intuitive and plausible conditions a simple linear delegated fund contract exists that fully achieves this optimal outcome for the LPs, subject to the GPs' actual trading and monitoring decisions under the contract. The optimal linear contract specifies a fee f paid by LPs to join the fund and a skin in the game parameter $\phi \in [0, 1]$ representing the GPs' share of the fund's assets. Since GPs can choose to unilaterally deviate from the fund and benefit from the monitoring undertaken by the fund, the contractual payments must compensate GPs for their monitoring costs. Subject to compensating the GPs for their costs, the contract aims to induce them to trade and monitor so as to achieve the outcome desired by the LPs as a group.

We show that—despite the fact that the GPs cannot commit to limit the fund's trade (exactly as in APZ)—the optimal contract induces radically different trading and monitoring choices relative to the APZ benchmark. A key insight is that delegation separates monitoring incentives from overall holdings. This is because delegated monitoring is undertaken by professional asset managers on behalf of the fund: It is *their* effective stake, not the fund's overall stake, that determines the fund's level of monitoring. The optimal contract allocates a share of the fund's assets to GPs that induces monitoring at a level consistent with only the LPs' *initial* endowment; in other words, LPs do not have to compensate GPs for any monitoring that is excessive from the LP's private perspective. However, since LPs' initial endowment is $\omega < \lambda$, whereas the aggregate risk bearing capacity of the LP's is λ, the optimal fund monitors less than a proprietary trader with identical risk bearing capacity. Further, we show that the fund also holds too small an overall position in the asset: in particular, under the optimal contract, the LPs hold a position within the fund that fully reflects their market power as a strategic trader with aggregate risk bearing capacity λ. Overall, therefore, by separating monitoring incentives from risk sharing, the optimal contract enables LPs to attain their *privately* optimal, full-commitment, levels of both monitoring and risk sharing. But this is attained at the expense of lower overall levels of monitoring and risk sharing in the market. That said, the ability to access financial markets via delegation clearly enhances risk sharing relative to the case where unskilled agents are simply excluded from financial markets.

Applied implications. Our main results characterize the economics of monitoring and risk sharing in financial markets with delegated blockholding. Given the preponderance of delegated asset managers in modern financial markets, these results are relevant to interpreting key features of blockholding and monitoring that are prevalent today. Specifically, our model has three main applied implications for corporate governance and the role of the asset management industry.

Which asset managers will monitor. Our analysis of optimal delegation arrangements has implications for the degree to which different types of asset managers should be expected to engage in the monitoring of portfolio firms. In particular, we show that asset managers' (i.e., GPs') skin in the game, which determines their level of monitoring, is increasing in the endowment of each underlying investor (LP) in the fund. Thus, if fund investors have relatively high endowments, they will invest in funds in which managers take larger personal stakes and monitor aggressively. If, on the other hand, fund investors have relatively low endowments, they will invest in funds in which managers will take small personal stakes and monitor very little.

This depiction resonates with key characteristics of asset management firms observed in reality. Relatively poor real-world investors tend to invest in mutual funds. It is well documented that mutual fund managers have very little self-investment in their funds (Khorana, Servaes, and Wedge 2007), and mutual funds are notorious for being muted in their engagement efforts (e.g., Bebchuck et al 2017). In contrast, wealthy individuals tend to invest in hedge funds. Managers of these funds are well known to self-invest significantly and play an active role in the monitoring of their portfolio firms (Agarwal, Daniel, and Naik, 2009, Brav, Jiang, and Kim 2010).

Larger blocks may monitor less than small blocks. Our results imply that block size may not be a good predictor of monitoring intensity. With proprietary blocks as in APZ, larger stakes imply more monitoring because stake size directly determines monitoring intensity. However, with delegated blocks the fund's internal incentive structure separates monitoring incentives from stake size. In particular, in our model, the endogenous block size is increasing in both the number of fund investors and their initial endowments, whereas monitoring intensity is determined only by their initial endowment. As a result, blocks held by funds with many investors with low initial endowment may be larger but feature significantly less monitoring than those held by funds with a smaller number of investors with higher initial endowment. In this regard, our results are consistent with Nockher (2022), who shows that smaller blockholders tend to be more intensive monitors than larger blockholders.

The role of index funds in governance. At the broadest level, our analysis indirectly highlights a role for index funds in corporate governance. Our analysis speaks to the concentrated holding choices of active funds, who make deliberate portfolio decisions (as our GPs do). A key implication of our model is that such active asset managers do not utilize their full risk bearing capacity to hold concentrated positions, and expend suboptimally low levels of resources into monitoring. This finding must be viewed in the context of the evolution of the asset management industry and the emergence of passive, i.e., index, funds which—purely by virtue of their size—mechanically end up holding concentrated positions in firms. If active funds do not hold sufficiently concentrated stakes and thus limit their monitoring, as our results suggest, it becomes all the more important to understand the role of index funds in governance (Brav, Malenko, and Malenko 2022).

1.1 Related literature

Our paper relates most directly to APZ and papers that generalize and extend their findings. DeMarzo and Urosevic (2006) extend APZ to a fully dynamic setting, while Marinovic and Varas (2021) also incorporate private information. In contrast to these papers, which retain APZ's focus on proprietary blocks in their exploration of dynamic implications, we focus on the economics of delegated block ownership rather than on dynamics. More broadly, our work relates to a number of different literatures.

At the most basic level, our paper is connected to the significant theoretical literature that studies blockholder monitoring. This literature is surveyed by Edmans and Holderness (2017). Many papers within this literature (e.g., Shleifer and Vishny 1986, Faure-Grimaud and Gromb 2004) take block size as being exogenous. Others (e.g., Kyle and Vila 1991, Maug 1998, Kahn and Winton 1998, Back, Collin-Dufresne, Fos, Li, and Ljunqvist 2018) consider how proprietary blocks can emerge endogenously by focusing on the ability to generate short-term trading profits. Our analysis differs from all these prior papers by explicitly modeling the emergence of *delegated* equity blocks. Further, in contrast to the second strand discussed above, we assume fully transparent financial markets, so there are no trading profits; in this respect, our analysis has similarities to Bolton and von Thadden (1998), though they also focus purely on proprietary blocks.

More recently, a growing theoretical literature takes the delegated nature of equity ownership seriously, and considers the role of the incentives of asset managers in corporate governance. This literature is surveyed by Dasgupta, Fos, and Sautner (2021) (see, in particular, section 4 of that paper). While several papers within that literature (e.g., Dasgupta and Piacentino 2015) have highlighted the negative implications of agency frictions arising from the delegation of portfolio management on the level of monitoring at portfolio firms, none of those papers endogenize the presence of delegated blockholders.

Finally, our paper is related in spirit to the literature on the endogenous emergence of financial intermediaries, starting with the work of Diamond and Dybvig (1983), as well as the literature on optimal contracting in delegated portfolio management, starting with the work of Bhattacharya and Pfleiderer (1985). Relative to the former, which has focused on banking, we consider the emergence of asset managers. Relative to the latter, which considers optimal contracting with respect to trading by asset managers, we incorporate monitoring considerations as well.

2 A benchmark model

We start with a simplified, benchmark, version of the APZ model. Consider a financial market with a single firm with 1 infinitely divisible equity share outstanding, and a risk-free asset in perfectly elastic supply whose gross return is normalized to unity. There is a unit continuum of traders who have CARA utility, each with risk tolerance of ρ. To mirror the assumption of an exogenously specified large trader in APZ, we assume that a measure λ of such traders are aggregated into a single trading entity, L, who trades strategically taking her price impact into account, and can monitor the firm to improve its cash flows. The remaining $1-\lambda$ of atomistic traders act perfectly competitively. We assume that L has an endowment of $\omega \in (0, \lambda]$ shares while the remaining $1-\lambda$ traders have an aggregate endowment of $1-\omega$ shares, shared equally among them.

There are three dates. Potentially numerous rounds of trading opportunities are available at date 1 in a Walrasian market: in any given round of trade, traders submit demand functions and a market-clearing price is determined. At date 2, L can choose to monitor the firm as follows: at a cost of $c(m)$, where $c'(\cdot) > 0$ and $c''(\cdot) > 0$, she can exert monitoring effort $m \geq 0$ to generate a final equity payoff that is distributed according to $N\left(\mu(m), \sigma^2\right)$, where $\mu'(\cdot) > 0$ and $\mu''(\cdot) \leq 0$. At date 3, all payoffs are publicly realized. As in the bulk of the APZ analysis, L cannot commit to a final round of trade at date 1 or to a particular level of monitoring at date 2.[3]

Aggregate risk tolerance. In a Walrasian CARA-Normal market with symmetric information like ours, each competitive agent will have a demand function of $\rho\frac{\mu(m)-P}{\sigma^2}$, and thus the total demand of a measure x of atomistic competitive agents is given by $\rho x\frac{\mu(m)-P}{\sigma^2}$, which is equivalent to the demand of a *single* competitive agent with risk tolerance of ρx. In other words, the *aggregate risk tolerance* of a given measure of atomistic competitive agents is proportional to the measure of those agents. Accordingly, throughout the paper, we shall treat the $1-\lambda$ of atomistic traders as being represented by a single competitive trader with risk tolerance of $\rho(1-\lambda)$. For benchmarking purposes, we assume that L has the same risk tolerance as the aggregate risk tolerance of the measure of competitive agents he replaces, i.e., $\rho\lambda$. This assumption will be convenient when we generalize the model to explicitly model the large trader as an endogenous delegated trading vehicle, i.e., a fund, formed of a measure of investors and fund managers.

[3] APZ also consider the case of multiple assets as well as more general monitoring technologies; we use this baseline version of their model, as it is under these specific assumptions that APZ provide the most complete characterizations.

Competitive allocations with perfect risk sharing. Before analyzing the full equilibrium involving both strategic and competitive trading as well as monitoring, it is helpful to establish a benchmark in which all traders are competitive and monitoring cannot arise. In such a benchmark, risk sharing considerations are the sole determinants of equilibrium allocations. Denoting $L's$ equilibrium holdings by α, it is easy to see that the *competitive equilibrium allocation* is $\alpha = \lambda$. This is because the competitive equilibrium involves perfect risk sharing, under which L would hold $\frac{\rho\lambda}{\rho\lambda+\rho(1-\lambda)} = \lambda$ fraction of the risky asset while the atomistic traders would hold $\frac{\rho(1-\lambda)}{\rho\lambda+\rho(1-\lambda)} = 1 - \lambda$ of the risky asset, in accordance with their relative levels of risk tolerance.

Equilibrium trading and monitoring. In order to analyze $L's$ trading, taking into account both strategic and monitoring incentives, we follow APZ to outline a few baseline steps. First, given that L is unable to commit to a particular level of monitoring, the equilibrium monitoring level is determined by L's final holdings on date 2. If α is L's total ownership of the risky asset upon entering date 2, then m is given by:

$$m(\alpha) = argmax_m \alpha\mu(m) - c(m), \tag{1}$$

which is given implicitly by the solution to

$$\alpha = \frac{c'(m)}{\mu'(m)}. \tag{2}$$

Note that m does not affect the risk of L's portfolio, so risk adjustment does not affect this choice. Clearly, $m(\alpha)$ is increasing in α.

If $L's$ final ownership of the risky asset is expected to be α, the $1-\lambda$ atomistic investors have an aggregate demand of

$$\rho\,(1-\lambda)\,\frac{\mu(m(\alpha))-P}{\sigma^2},$$

giving rise to a market clearing price of

$$P\,(\alpha)=\mu(m(\alpha))-\frac{1-\alpha}{\rho(1-\lambda)}\sigma^2. \tag{3}$$

Finally, given that L is unable to commit to a final round of trade within date 1, we follow APZ in focusing on *globally stable allocations.* In the absence of the ability to commit to a given number of trades, APZ show that following any sequence of trades, L will wish to trade again ahead of her monitoring choice unless she has traded to an allocation which is globally stable. Such an allocation is defined as follows:

Definition 1. An allocation α_G is globally stable iff (i)

$$\alpha_G \in argmax_{\alpha}\Psi(\alpha)-\Psi(\alpha_G)-(\alpha-\alpha_G)P(\alpha_G),$$

and (ii) for every $\omega\in[0,1]$, such that $\omega\neq\alpha_G$,

$$\Psi(\alpha_G)-\Psi(\omega)-(\alpha_G-\omega)P(\alpha_G)>0,$$

where

$$\Psi(\alpha)=\alpha\mu(m(\alpha))-c(m(\alpha))-\frac{1}{2\rho\lambda}\alpha^2\sigma^2 \tag{4}$$

is the certainty equivalent for L of holding α units of the risky asset and monitoring accordingly.

In words, this means that: (i) once a globally stable allocation is reached, L will not wish to trade away from it at current prices; and (ii) L is willing to trade to the globally stable allocation from any other position at prices consistent with the globally stable allocation. In their central result, APZ show that:

Proposition 1. (Admati, Pfleiderer, and Zechner 1994) *As long as $\Psi(\alpha)$ is strictly concave, there exists a unique globally stable allocation, $\alpha_G = \lambda$, which coincides with the competitive equilibrium allocation.*

All proofs are in the Appendix. The restriction on the concavity of $\Psi(\alpha)$ is the same as in APZ (see APZ's Proposition 3 and 4).

This key result implies that the possibility of monitoring does not affect the degree of risk sharing in equilibrium. The reason, as APZ discuss, is that the lack of the ability to commit to a final round of trade erodes the strategic advantage of the large trader, who subsequently trades all the way to the competitive equilibrium allocation. Put another way, there is no trade-off between diversification and monitoring because an endowment effect induces L to trade all the way to the risk sharing optimum. Counterfactually, if any sequence of trades led to a proposed final holding level for L that is strictly below her risk sharing optimum, she would be tempted to buy a bit more because the current holding is now part of her endowment. Starting from this endowment, there will always be at least some incremental risk sharing gains by buying a bit more, despite having to pay the full value of future monitoring in making such purchases.

While the concept of global stability is essentially static, its relevance has been confirmed by DeMarzo and Urosevic (2006) in a fully dynamic version of the APZ model with continuous trading and monitoring opportunities. Indeed, their main result is that the large trader will ultimately trade to the competitive price-taking allocation, which generalizes and provides a dynamic micro-foundation for APZ's concept of global stability.

In reality, the concentration of ownership into the hands of a large trader is typically achieved by delegating portfolio management to professional asset managers who trade and monitor on behalf of their clients. Thus, in the remainder of the paper, we examine how incentives to monitor are determined when risk averse investors can optimally delegate to a professional fund manager, who can then trade freely in financial markets but cannot make prior commitments to monitor firms at any particular level of intensity.

3 Delegated blocks

We now introduce the possibility of delegated blockholding. Instead of assuming that a measure λ of investors exogenously acts as a single trading entity as above, we now assume that a measure λ of investors lacks the ability to either trade directly in the market or monitor, and are thus "unskilled." The remaining $1-\lambda$ fraction of investors are "skilled" and can trade freely in markets and monitor, as in the benchmark model. Such agents also have the ability to group themselves together to offer investment services to unskilled investors. In turn, if unskilled investors wish to trade the risky asset, they must pool their resources and employ a group of skilled investors, thus endogenously generating the possibility of delegated blockholding. To be consistent with the benchmark model, the λ measure of unskilled investors has aggregate endowment $\omega \leq \lambda$ of the risky asset (shared equally), while the $1-\lambda$ measure of skilled investors have the remaining $1-\omega$ endowment (also shared equally, as in the benchmark).[4]

A "fund" is formed when the λ measure of unskilled investors, who we also refer to as Limited Partners, or "LPs," decide to employ a chosen positive measure of skilled investors, who we also refer to as General Partners, or "GPs," to trade and monitor on their behalf.[5]

[4] The assumed equality between the measure of investors represented by L in the baseline model and the measure of unskilled investors in the main model is purely for expositional convenience. All our qualitative results hold for any $\lambda \in (0,1)$.

[5] Note that no funds would form without the participation of unskilled investors. In any fund with only skilled investors, some subset of those investors must monitor and thus pay costs. However, any investor

The GPs in a fund act collectively to make trading and monitoring decisions based on their joint incentives, while the LPs are passive once they have joined the fund. As in the benchmark model, the GPs cannot commit to a given trading strategy or monitoring level up front—they always behave opportunistically once the fund has been established. When a fund is formed, all GPs and LPs joining the fund contribute their endowments to the fund and agree to a contract.

Optimal delegation Since our interest is in optimal delegation, we aim to maximize the payoff of LPs. In other words, the contracting terms are chosen to optimize the payoff of the LPs while ensuring the participation of the requisite mass of GPs. We do this in two parts. First, we find the optimal allocation from the point of view of the LPs *as if* they could act as a group (as in our benchmark analysis), but *in addition* have the ability to commit ex ante to both a given monitoring level and a single round of trade. This corresponds to their optimal payoff with full commitment ability vis a vis both monitoring and trading. We then show that under intuitive and plausible conditions a simple linear delegated fund contract exists that fully achieves this optimal outcome for the LPs, subject to the GPs' actual trading and monitoring decisions under the contract. In other words, while no agents in the model actually have the ability to commit to a monitoring level or a trading strategy, we show that optimal delegation can achieve the full commitment optimum for the LPs.

If the LPs could act as a group and publicly commit to a monitoring level of m and a single round of trade, the price they would face if they traded to a final stake of α is given

that is supposed to be in the subset that monitors can choose not to join the fund, trade on their own, and enjoy exactly the same cash flow payoffs without paying the monitoring costs. So, to persuade them to join the fund, the subset that are in the fund but do not monitor must pay those that are expected to monitor. But this effectively means that monitoring costs are shared among all agents in the fund, and so the previous argument applies and individual investors who are not expected to monitor would prefer not to join the fund.

by $\mu(m) - \frac{1-\alpha}{\rho(1-\lambda)}\sigma^2$. Thus, they would have a joint optimization problem given by

$$\max_{m,\alpha} \alpha\mu(m) - c(m) - \frac{1}{2\rho\lambda}\alpha^2\sigma^2 - (\alpha - \omega)\left(\mu(m) - \frac{1-\alpha}{\rho(1-\lambda)}\sigma^2\right).$$

Solving this problem yields the following result.

Proposition 2. *The LPs' full commitment optimum has an optimal monitoring level, m^C, implicitly defined by $\omega = \frac{c'(m^C)}{\mu'(m^C)}$, and an optimal final stake of $\alpha^C \equiv \frac{\lambda(1+\omega)}{(1+\lambda)}$.*

Since $\omega < \lambda$, it is easy to see that the optimal final stake of the LPs lies between their initial endowment (ω) and their competitive allocation with perfect risk sharing (λ):

$$\omega < \alpha^C < \lambda.$$

The LPs want to increase their stake in the risky asset above their endowment to attain diversification benefits (so $\alpha^C > \omega$ is optimal). However, they avoid trading all the way to their competitive allocation so that they can fully exploit their strategic trading advantage, i.e., accounting for the fact that they move prices. Further, the LPs' full commitment optimal monitoring level does not depend on their final stake, α. Instead, by analogy to equation (2), it is clear that the LPs desire monitoring to occur "as if" their ownership was equal to their initial endowment ω. Thus, LPs want to hold more than their initial endowment for risk sharing purposes but wish to monitor only at their original endowment level. Intuitively, this is because they are aware that any increase in monitoring over and above the level implied by their endowment induces a higher price that offsets the benefits of the additional monitoring from the perspective of the LPs.

Let

$$\Pi_{LP}^C \equiv \alpha^C\mu(m^C) - c(m^C) - \frac{1}{2\rho\lambda}(\alpha^C)^2\sigma^2 - (\alpha^C - \omega)\left(\mu(m^C) - \frac{1-\alpha^C}{\rho(1-\lambda)}\sigma^2\right)$$

denote the LPs' aggregate equilibrium payoff at their full commitment optimum. Next we consider whether a delegated fund contract exists that can achieve a payoff of Π_{LP}^{C} for the LPs.

Fund formation At the outset, it is important to note that even if a fund contract can be designed that delivers exactly this optimal payoff to the LPs, it may not be feasible to form such a fund because individual LPs (who cannot actually coordinate their actions) will not be willing to join it. Indeed, we have the following result.

Lemma 1. *There exists a $\hat{\omega} \in (0, \lambda)$ such that for $\omega \leq \hat{\omega}$, LPs will join a fund that delivers an aggregate LP payoff of Π_{LP}^{C}, while for $\omega > \hat{\omega}$ they will not—i.e., delegated blockholding can arise only when unskilled agents have relatively low endowments of the risky asset.*

Intuitively, if an individual LP defects from the fund, they enjoy the full benefits of the fund's monitoring for free and lose only the diversification benefits of participation in the fund. Thus, if the endowment ω was sufficiently close to the competitive risk sharing level, and diversification benefits were therefore small, individual LPs would prefer to defect.

Now consider a proposed fund contract specified as follows: a chosen mass of GPs, $\tau \in (0, 1 - \lambda)$, invited into the fund, a skin in the game parameter, $\phi \in [0, 1]$, specifying the GPs' share of the fund's assets, and an up-front fee, f, which each participating LP must pay to join the fund. Overall, a fund formed under this contract can be represented as a linear contracting triple, (τ, ϕ, f), representing the measure of GPs, their skin in the game, and the per-LP fee, respectively.

We solve for the optimal fund by backward induction. We first assume that a fund with λ LPs and (an arbitrary positive measure of) τ GPs is formed, and proceed to compute the monitoring and trading decisions of the GPs for a given (τ, ϕ, f). We then solve for the optimal contracting terms that achieve a payoff of Π_{LP}^{C} for the LPs. While doing so, we ensure that all τ GPs are willing to join the fund. We denote the optimal set of linear contracting terms by the triple (τ^*, ϕ^*, f^*).

We reuse α to denote the final stake in the risky asset held by the fund. The GPs have an *effective stake* in the final payoff of the risky asset equal to their proportional share of the fund's stake, or $\phi\alpha$. Given that monitoring does not affect the risk of their payoff, they will optimally choose m as follows (all D-superscripts refer to functions defined for the delegated fund model):

$$m^D(\alpha) = argmax_m \phi\alpha\mu(m) - c(m), \tag{5}$$

which is given implicitly by the solution to $\phi\alpha = \frac{c'(m)}{\mu'(m)}$.

Since the GPs cannot commit to a given trading strategy, we again focus on globally stable trading allocations. We note first that the pricing function must be adjusted for the fact that the mass of competitive price-taking investors has been reduced from $1-\lambda$ to $1-\lambda-\tau$ given the formation of the fund. If the competitive investors expect the fund to end up with a stake of α, their aggregate demand will be

$$\rho\,(1-\lambda-\tau)\,\frac{\mu(m^D(\alpha)) - P}{\sigma^2},$$

giving rise to a market clearing price of

$$P^D\,(\alpha) = \mu(m^D(\alpha)) - \frac{1-\alpha}{\rho(1-\lambda-\tau)}\sigma^2. \tag{6}$$

The definition of a globally stable allocation must also be adjusted for our delegated fund model as follows, since GPs make decisions on behalf of the entire fund but enjoy only a ϕ proportion of its payoff.

Definition 2. An allocation α_G^D is globally stable iff (i)

$$\alpha_G^D \in argmax_\alpha \Psi^D(\alpha) - \Psi^D(\alpha_G^D) - \phi(\alpha - \alpha_G^D)P^D(\alpha_G^D),$$

and (ii) for every $\omega \in [0,1]$, such that $\omega \neq \alpha_G^D$,

$$\Psi^D(\alpha_G^D) - \Psi^D(\omega) - \phi(\alpha_G^D - \omega)P^D(\alpha_G^D) > 0,$$

where

$$\Psi^D(\alpha) = \phi\alpha\mu(m^D(\alpha)) - c(m^D(\alpha)) - \frac{1}{2\rho\tau}\phi^2\alpha^2\sigma^2 \tag{7}$$

is the certainty equivalent for the GPs if the fund holds a stake of α units of the risky asset and they monitor accordingly. We have the following result.

Lemma 2. *As long as $\Psi^D(\alpha)$ is strictly concave, there exists a unique globally stable allocation*

$$\alpha_G^D = \frac{\tau/\phi}{\tau/\phi + 1 - \lambda - \tau}. \tag{8}$$

The equilibrium stake of the fund α_G^D depends only on the sizes of the LP and GP populations, λ and τ, and the skin in the game parameter, ϕ. The expression is analogous to the globally stable allocation in the APZ benchmark derived in Proposition 1: it is part of the competitive equilibrium allocation in a market with two competitive traders, one of whom has risk tolerance of $\rho(1-\lambda-\tau)$ while the other has risk tolerance of $\rho\tau/\phi$. The former is simply the aggregation of the skilled investors who did not join the fund (i.e., did not become GPs). The latter investor aggregates the skilled investors inside the fund, i.e., the GPs. Recall that trading decisions in the fund are taken by a measure τ of GPs who have an aggregate risk tolerance of $\rho\tau$. However, these GPs are only exposed to a fraction ϕ of the holdings of the fund, giving them an *effective risk tolerance* of $\rho\tau/\phi$.

Notably, however, this allocation does not necessarily correspond to perfect risk sharing among all investors—which arose in the globally stable allocation of APZ (see Proposition 1)—as this would require an allocation of $\alpha_G^D = \tau + \lambda$ (since the measure of investors in the fund is the sum of λ LPs and τ GPs). The deviation from perfect risk sharing arises due a combination of two factors: First, only a measure of $\tau < \tau + \lambda$ agents make decisions on behalf of the whole fund; and second, those agents are exposed to only a fraction ϕ of the fund's holdings. Indeed, it is apparent that if $\tau/\phi = \tau + \lambda$ in the expression for α_G^D above, we obtain $\alpha_G^D = \tau + \lambda$. Given this deviation from perfect risk sharing in equilibrium, the determination of the optimal linear contracting parameters (τ^*, ϕ^*, f^*) is critical for determining both the level of monitoring and the degree of diversification in the model.

We assume for the remainder of the analysis that $\Psi^D(\alpha)$ is strictly concave, so that a globally stable allocation exists. Comparing the effective stakes of skilled investors inside and outside the fund yields the following result.

Lemma 3. *The GPs in the fund and the outside skilled investors end up with identical effective per-investor holdings of the risky asset.*

This result implies that there is perfect risk sharing over the part of the risky asset that is not (effectively) held by the LPs among the total $1-\lambda$ measure of skilled investors, whether inside or outside the fund. This is because the existence of multiple trading opportunities combined with the inability to commit to a particular trading strategy erodes the strategic advantage of the GPs, who subsequently trade to arrive at the point of perfect risk sharing between themselves (with effective risk tolerance τ/ϕ) and skilled investors outside the fund (with aggregated risk tolerance $1 - \lambda - \tau$), as discussed above.

Given the trading and implied monitoring choices of the GPs for a given (τ, ϕ, f), we now proceed to solve for the optimal linear contracting terms to determine (τ^*, ϕ^*, f^*). In our central result, we show that:

Proposition 3. *For $\omega \leq \hat{\omega}$, an optimal fund that achieves an aggregate payoff of Π^C_{LP} for the LPs exists and is characterized by:*

1. *a mass of GPs $\tau^* = \frac{(1-\lambda^2)\omega}{1-\lambda\omega}$,*

2. *a skin in the game parameter $\phi^* = \frac{(1+\lambda)\omega}{2\lambda\omega+\lambda+\omega}$, and*

3. *a fee*

$$f^* = \frac{1}{\lambda}\left[c(m^C) + P^{D*}(\alpha_G^{D*})\left((1-\phi^*)(\omega + \tau^*\frac{(1-\omega)}{1-\lambda}) - \omega\right)\right] \tag{9}$$

where the superscript $D*$ indicates that the associated function or variable is evaluated at ϕ^* and τ^*.

The proof proceeds in several steps. First, we solve for τ^* and ϕ^* such that (1) the GPs' equilibrium monitoring effort equals the LPs' optimal level, m^C, which requires that the GPs' effective stake in the risky asset, $\phi^*\alpha_G^{D*}$, equal ω, and (2) that the LPs' effective stake, $(1-\phi^*)\alpha_G^{D*}$, equals their optimal stake, α^C. Next, we set the fee f^* to just satisfy the participation constraint of individual GPs conditional on the existence of a fund involving τ^* GPs with a skin in the game parameter ϕ^*. Finally, we show that this fee level makes the LPs' aggregate payoff coincide exactly with Π^C_{LP}, which also ensures that they will participate given that $\omega \leq \hat{\omega}$.

We examine the implications of this result and the intuition behind it through a series of remarks and corollaries. First, we discuss the levels of risk sharing and monitoring implied by the equilibrium. The optimal delegation contract selects the skin in the game parameter (ϕ^*) and measure of GPs (τ^*) so as to render the effective holdings of the GPs to be equal to the endowment of the LPs (ω). LPs, however, do *not* hold their original endowment, but rather end up with a larger holding of the risky asset: $\lambda\frac{1+\omega}{1+\lambda} > \omega$. As explained previously, these outcomes correspond to what the LPs would like to do if they could act in concert and exert full commitment power over their monitoring and trading strategies.

With respect to monitoring, LPs prefer that monitoring occur at a level commensurate with their initial endowment due to the well-known free-riding phenomenon (first formalized by Grossman and Hart 1980). Effectively, the LPs cannot "enjoy" the monitoring benefits on anything more than their initial endowment, and thus would like to limit it. However, because of risk sharing they would like to hold a larger stake than their initial endowment. In the absence of commitment power, doing these things simultaneously is not possible in a proprietary blockholder model like APZ, where the final stake of the blockholder drives both monitoring incentives and risk sharing benefits. Thus, the key question is: how is this accomplished in our model, which still does not give investors any commitment power?

The answer is that, in our delegated case, LPs do not directly monitor—GPs monitor on their behalf. That is, delegation by definition splits roles: LPs own a fraction of the fund, $(1-\phi^*)\,\alpha_G^{D*}$, but monitoring is undertaken by the GPs who own the rest: $\phi^*\alpha_G^{D*}$. Thus, delegation enables the separation of (LP-) ownership and (GP-) monitoring. The contract that maximizes LP welfare ensures that monitoring occurs only at the level that is privately optimal for LPs absent risk sharing considerations, but gives them an effective stake in the fund that also optimizes their collective risk sharing and trading incentives without regard to any effect on monitoring. To summarize:

Remark 1. The delegation of blockholding breaks the link between diversification by LPs and the monitoring that occurs on their behalf. The optimal linear contract enables monitoring at a level privately optimal for LPs absent risk sharing motives, while also enabling LPs to trade to their privately optimal level of risk sharing absent monitoring motives, taking into account their collective market power.

Second, we consider the role of the fee, f^*. The fee is relevant to both GPs who receive it and LPs who pay it. We consider GPs first. Any individual skilled agent can choose between joining the measure τ^* of GPs inside the fund or remaining part of the measure

$1-\lambda-\tau^*$ of skilled agents who do not join the fund. As noted above, the fee f^* is set to make the payoffs of these two choices equal.

Remark 2. The fee f^* compensates GPs for their expected equilibrium monitoring costs as well as for the value of any endowment that they sacrifice when they join the fund.

Since, as shown in Lemma 3, GPs in the fund and skilled investors outside the fund end up holding the same effective stake per investor, their degree of diversification is not affected by joining the fund and the LPs do not need to compensate them for taking more or less risk. However, there are two ways in which GPs' payoffs differ from those of skilled agents who choose not to become GPs. First, skilled agents choosing to become GPs inside the fund share the cost of monitoring, while those remaining outside the fund enjoy the benefits of such monitoring for free. Thus, the fee f^* must fully compensate GPs for their monitoring costs at the fund's ultimate stake. Second, the fee must compensate GPs for giving up some of their endowment: When they join the fund, GPs are allocated an initial pre-trade endowment of $\frac{\phi^*}{\tau^*}\left(\omega+\tau^*\frac{1-\omega}{1-\lambda}\right)$, which is smaller than $\frac{1-\omega}{1-\lambda}$, the initial endowment of each skilled agent.

Next, we turn to the role of the fee in the LPs' payoff. Clearly, if GPs require full compensation for monitoring costs incurred inside the fund then LPs must pay for these. This is consistent with the LPs' full commitment optimum, in which they also pay the full monitoring cost. Further, in contrast to GPs, LPs start with an endowment in the fund that is *higher* than their initial endowment of $\frac{\omega}{\lambda}$. In line with the discussion above, this is a result of the reallocation of some of the GPs' initial endowment to the LPs. LPs must pay for this added initial endowment. Put another way, in order to achieve their full commitment optimum level of ownership, α^C, LPs effectively have to buy less as a result of joining the optimal fund than they would if, counterfactually, they traded independently to this allocation. The optimal fee charges them for this benefit, thus bringing their total payoff to Π^C_{LP}.

Remark 3. The fee f^*charges the LPs for the full expected monitoring costs expended by GPs as well as as for the value of any additional endowment that they are allocated when they join the fund.

Finally, in order to complete our analysis of delegated blockholding and compare it to the benchmark case of APZ, we compute the overall stake held by the fund under optimal delegated ownership. The optimal fund holds a stake in the risky asset equal to $\omega + \lambda\frac{1+\omega}{1+\lambda}$, i.e., the effective stake of the GPs plus the effective stake of the LPs. We show that this is less than its competitive risk sharing optimal allocation, $\lambda + \tau^*$, which is also what a proprietary blockholder representing the same measure of traders would hold under a globally stable allocation.

Corollary 1. *The optimal fund holds less of the risky asset than the corresponding competitive equilibrium allocation for a trader with the same overall risk tolerance.*

3.1 Risk sharing and monitoring: Delegated vs Proprietary Ownership

We are now in a position to compare our results on delegated blockholding to those of APZ's benchmark (presented in Section 2). Taking as given the existence of a proprietary trader with large risk bearing capacity, APZ ask whether the anticipation of monitoring costs affects the degree of risk diversification in the economy. Under broad and plausible conditions, they find the answer is "no"—concentrated blockholders still trade to the competitive risk sharing allocation and monitoring occurs at that allocation. However, we show that when blockholding is achieved by optimal delegation, the picture changes dramatically, in at least two ways.

First, the delegated vehicle that is formed holds *less* of the risky asset, i.e., a smaller block, than what is implied by perfect risk sharing, whereas in the proprietary APZ case perfect risk sharing is achieved. This is because, when delegating to form a fund, the optimal contractual terms account for the fact that the fund will affect prices when trading and thus ensures that the LPs ultimately hold an amount of the risky asset that reflects their market power (and thus shades its trades downwards), as shown in Proposition 2 and Corollary 1. Thus, in contrast to to the APZ proprietary benchmark, delegated blockholding results in *underdiversification* relative to the unconstrained optimum.

Second, delegation separates ownership and monitoring by allocating monitoring tasks only to a subset of participants in the fund, i.e., the GPs. The optimal delegation contract allocates an effective stake for GPs of ω, which results in a level of monitoring that would be privately optimal for LPs absent risk sharing considerations, i.e., corresponding purely to their initial endowments (see Remark 1). As a result, the optimal delegation contract achieves strictly less monitoring than a proprietary blockholding of comparable size.

3.2 Recontracting

In APZ, there is no trade-off between diversification and monitoring, because an endowment effect induces the large trader, L, to trade all the way to the risk sharing optimum. Counterfactually, if any sequence of trades led to a proposed final holding level for L that is strictly below her risk sharing optimum, she would be tempted to buy a bit more because the current holding is now part of her endowment. Starting from this endowment, there will always be at least some incremental risk sharing gains by buying a bit more, despite having to pay the full value of future monitoring in making such purchases. In the optimal contract solved above, LPs end up with a stake of $\frac{\lambda}{1+\lambda}(1+\omega)$, which is greater than their initial endowment of ω, but monitoring occurs at a level corresponding to an ownership of ω. Hence, a discerning reader may wonder whether a variant of the APZ endowment effect may come into play wherein the LPs now wish to recontract with GPs to reflect their new endowment. Could the possibility of such recontracting revive the APZ result, wherein the LPs achieve the risk sharing optimum holding of λ, and monitoring occurs at a commensurate level?

In principle, the LPs as a group would indeed like to recontract with GPs to form a fund that monitors more intensely. To see this, consider a situation where the equilibrium contract from above is signed and the fund trades to the stable allocation, but then an unexpected opportunity arises to dissolve the existing fund and start a new one prior to any monitoring taking place. We then have a repeat of the model above starting from an aggregate LP endowment of $\frac{\lambda}{1+\lambda}(1+\omega)$ instead of ω, which may lead to a new fund that will monitor at a level corresponding to ownership of $\frac{\lambda}{1+\lambda}(1+\omega)$.[6]

However, unlike repeated trading, which is always feasible, repeated contracting may be impossible because it is sensitive to free riding: as shown above in Lemma 1, as soon as LPs have an endowment higher than $\hat{\omega} < \lambda$, it is not possible to form a fund. For such high endowments, the risk sharing benefits to individual LPs is too small, and thus each individual LP would benefit by deviating and staying out of the fund (if it is formed), thus saving themselves the fees that must be paid to the GPs. As a result, any endowment level $\omega < \hat{\omega}$ for which a fund can be formed but $\frac{\lambda}{1+\lambda}(1+\omega) > \hat{\omega}$ holds would not be subject to recontracting. This clearly holds for some positive measure set of endowments $\Omega_S = \{\omega' > 0 : \omega' \leq \hat{\omega} < \frac{\lambda}{1+\lambda}(1+\omega') < \lambda\}$. Thus, the possibility of recontracting does not revive the APZ result.

[6] As noted in Lemma 3, GPs in the fund and skilled investors outside the fund hold the same effective stakes after trading, so the new fund formation problem is isomorphic to the original problem with different endowments.

4 Mutual Funds and Hedge Funds: Clientele, Fees, and Engagement

Our analysis of optimal delegation arrangements has implications for the asset management industry and the degree to which different types of asset managers engage in the monitoring of portfolio firms. Proposition 3 implies that the skin in the game of the asset managers (GPs), $\phi^* = \frac{(1+\lambda)\omega}{2\lambda\omega+\lambda+\omega}$, and their total effective investment in the risky asset, ω, are both increasing in the endowment of each underlying investor (LP) in the fund. In turn, the level of equilibrium monitoring undertaken by the fund, $\frac{\omega}{\gamma}$, increases in asset managers' effective stake. Thus, within the constraint under which delegated blockholding arises in equilibrium ($\omega < \hat{\omega} \in (0, \lambda)$), if fund investors have a relatively high endowment of the risky asset, they will invest in funds where managers take a larger personal stake in the fund; these funds monitor more aggressively. In contrast, if fund investors have a relatively small endowment of the risky asset, they will invest in funds where managers take a small personal stake in the fund; these funds monitor their portfolio firms very little.

The above depiction of asset management resonates with key characteristics of different types of asset management firms observed in reality. Relatively poor real world investors tend to invest in mutual funds. It is well documented that mutual fund managers invest very little in their funds: according to Khorana, Servaes, and Wedge (2007) some 57% of mutual fund managers do not invest at all in their constituent funds, and the average self-investment among the rest is 0.04%. Finally, mutual funds are notorious for being relatively muted in their engagement of portfolio firms (e.g., Bebchuck, Cohen, and Hirst 2017). In contrast, relatively wealthy individuals tend to invest in hedge funds, which typically have minimal net worth requirements. Hedge funds managers are well known to self-invest significantly in the fund (estimates in the literature range from 7% of fund assets under management in Agarwal, Daniel, and Naik 2009 to 20% in He and Krishnamurthy 2013). It is also well documented that hedge funds play a far more active role in the monitoring of their portfolio firms (Brav, Jiang, and Kim 2010).

Our results also imply that stake size may not be a good predictor of monitoring intensity. With proprietary blocks, larger stakes imply more monitoring because stake size directly determines monitoring intensity. However, with delegated blocks the fund's internal incentive structure separates monitoring incentives from stake size. As a result, funds with smaller stakes might actually monitor more intensively than those with larger stakes depending on their investor clientele. Specifically, the total delegated block size in our model is $\omega + \frac{\lambda}{1+\lambda}(1+\omega)$ which is increasing both in the number of LPs and in the aggregate endowment of the LPs. Potentially, therefore, funds with many investors holding limited endowments (large λ, small ω) can have large blocks with very little monitoring. In contrast, funds with fewer investors who hold high endowments (small λ, relatively large ω) can have relatively small blocks with significantly more monitoring.

As an illustrative example, compare a fund with a relatively large number ($\lambda = 15\%$) of investors with very limited endowments of the risky asset ($\omega = 0.1\%$) versus a fund with a relatively small number ($\lambda = 5\%$) of investors with relatively high endowments of the risky asset ($\omega = 0.5\%$). The former fund would hold approximately 13% of the firm, GPs would own a very small fraction—0.8%—of the fund's assets under management, and for $\gamma = 0.1$, monitoring would occur at a low intensity of $\frac{\omega}{\gamma} = 0.01$. The latter fund would hold approximately 5% of the firm, i.e., a much smaller stake; GPs would own a much larger fraction—9.5%—of the fund's assets under management, and for $\gamma = 0.1$, monitoring would occur at five-times the intensity of the other fund, $\frac{\omega}{\gamma} = 0.05$.

While our model is not ideally suited for calibration, it is noteworthy that these block sizes, GP-ownership stakes, and monitoring intensity are broadly in line with observations about mutual fund families and hedge funds. Large families like Blackrock or Fidelity often own well over 10% of US corporations but arguably do not monitor much, while their managers typically have very small stakes in the fund (as discussed above). In contrast, activist hedge funds hold a median stake of around 5-6% in target firms, monitor intensively, and—as discussed above—GPs often hold a personal stake of around 10% of the fund's assets under management. Our results are also consistent with Nockher (2022), who finds that smaller blockholders, and particularly those with a larger percentage of their fund invested in a given firm, tend to be more engaged monitors than larger blockholders.

5 Conclusion

Blockholder monitoring is important, but the determinants of long-term block sizes and the resulting implications for the degree of monitoring are not fully understood. The existing theoretical literature devoted to this question focuses only on proprietary blockholding, whereas modern markets are dominated by delegated asset managers. We present a simple model of delegated trading and monitoring to examine the economics of concentrated ownership and blockholder monitoring in financial markets dominated by institutional investors.

Our analysis shows that delegation has important consequences for both block sizes and monitoring. In particular, optimal delegation contracts allow for the separation of diversification and monitoring motives. This can lead to less monitoring and inferior risk sharing relative to proprietary blocks, but gives rise to monitoring and risk sharing benefits where proprietary blocks would not exist.

At an applied level, our model illustrates how some commonly observed characteristics of asset management firms—the clientele they serve, the extent of managerial self-investment, and the degree to which they monitor portfolio firms—can arise as a result of optimal contracting with fund investors. Further, our results imply that block size may not be a good predictor of monitoring intensity because the fund's internal incentive structure separates monitoring incentives from stake size. Finally, given that we conclude that active asset managers may endogenously avoid utilizing their full risk bearing capacity to hold concentrated positions, our analysis indirectly highlights the importance of the governance role of index asset managers—who mechanically hold concentrated stakes—in corporate governance.

Appendix

Proof of Proposition 1: We begin with condition (i) of the globally stable allocation. Combining definition (4) with the selected monitoring level (1) and the market clearing price (3), the optimization problem can be written as:

$$\max_{\alpha} \alpha\mu(m(\alpha)) - c(m(\alpha)) - \frac{1}{2\lambda}\alpha^2\sigma^2 - \Psi(\alpha_G) - (\alpha - \alpha_G)\left(\mu(m(\alpha_G)) - \frac{1-\alpha_G}{\rho(1-\lambda)}\sigma^2\right),$$

giving rise to the following first order condition:

$$\mu(m(\alpha)) + \alpha\mu'(m(\alpha))m'(\alpha) - c'(m(\alpha))m'(\alpha) - \frac{1}{\rho\lambda}\alpha\sigma^2 - \left(\mu(m(\alpha_G)) - \frac{1-\alpha_G}{\rho(1-\lambda)}\sigma^2\right) = 0.$$

Since $m(\alpha)$ satisfies $\alpha m'(\alpha) - c'(\alpha) = 0$, this simplifies to

$$\mu(m(\alpha)) - \frac{1}{\rho\lambda}\alpha\sigma^2 - \left(\mu(m(\alpha_G)) - \frac{1-\alpha_G}{\rho(1-\lambda)}\sigma^2\right) = 0$$

Now, setting $\alpha = \alpha_G$ above and solving gives:

$$\frac{1}{\rho\lambda}\alpha_G\sigma^2 = \frac{1-\alpha_G}{\rho(1-\lambda)}\sigma^2, \text{i.e., } \alpha_G = \lambda.$$

Now, we turn to condition (ii) of the globally stable allocation to verify that $\Psi(\lambda) - \Psi(\omega) - (\lambda - \omega)P(\lambda) > 0$ for all $\omega \neq \lambda$. This is equivalent to showing that $\omega = \lambda$ is a global maximum of the function

$$\Psi(\omega) - \omega P(\lambda) = \omega\mu(m(\omega)) - c(m(\omega)) - \frac{1}{2\rho\lambda}\omega^2\sigma^2 - \omega\left(\mu(m(\lambda)) - \frac{\sigma^2}{\rho}\right).$$

To verify this we first note that the simplified first order condition

$$\mu(m(\omega)) - \frac{1}{\rho\lambda}\omega\sigma^2 - \left(\mu(m(\lambda)) - \frac{\sigma^2}{\rho}\right) = 0$$

is satisfied at $\omega = \lambda$. We then evaluate the second order condition at $\omega = \lambda$: $\mu'(m(\lambda))m'(\lambda) - \frac{\sigma^2}{\rho\lambda}$. This is strictly negative as long as $\Psi(\alpha)$ is strictly concave as required.■

Proof of Proposition 2: In analyzing the full-commitment case, we assume that LPs collectively commit to a level of monitoring m which is publicly observed. Further, they also commit publicly to a single round of trade. Now, if they trade to a holding of α, then they will face a price of $\mu(m) - \frac{1-\alpha}{\rho(1-\lambda)}\sigma^2$, generating a payoff of

$$\alpha\mu(m) - c(m) - \frac{1}{2\rho\lambda}\alpha^2\sigma^2 - (\alpha - \omega)\left(\mu(m) - \frac{1-\alpha}{\rho(1-\lambda)}\sigma^2\right).$$

Taking the partial derivative of the objective function with respect to m yields a FOC of

$$\omega\mu'(m) - c'(m) = 0,$$

which does not depend on α. The SOC is clearly satisfied given our assumptions, so the solution is given implicitly by $\omega = \frac{c'(m^C)}{\mu'(m^C)}$.

Taking the partial derivative of the objective function with respect to α for a given m and simplifying yields the FOC

$$\frac{(1+\omega)\sigma^2}{\rho(1-\lambda)} - \alpha\left(\frac{\sigma^2}{\rho\lambda} + \frac{2\sigma^2}{\rho(1-\lambda)}\right) = 0,$$

and again the SOC is clearly satisfied. Solving for α yields an optimal stake of

$$\alpha^C = \frac{1+\omega}{1+\lambda}\lambda. \qquad ■$$

Proof of Lemma 1: In the full commitment optimum, each individual unskilled agent (LP) has a payoff of

$$\frac{1}{\lambda}\left(\alpha^C\mu(m^C) - c(m^C) - \frac{1}{2\rho\lambda}(\alpha^C)^2\sigma^2 - (\alpha^C-\omega)\left(\mu(m^C) - \frac{1-\alpha^C}{\rho(1-\lambda)}\sigma^2\right)\right).$$

If a single unskilled agent were to stay out of the fund and instead consume their endowment, they would receive a payoff of

$$\frac{1}{\lambda}\left(\omega\mu(m^C) - \frac{1}{2\rho\lambda}\omega^2\sigma^2\right).$$

Subtracting the latter from the former yields a difference of

$$\frac{1}{\lambda}\left((\alpha^C-\omega)\mu(m^C) - ((\alpha^C)^2-\omega^2)\frac{\sigma^2}{2\rho\lambda} - c(m^C) - (\alpha^C-\omega)\left(\mu(m^C) - \frac{1-\alpha^C}{\rho(1-\lambda)}\sigma^2\right)\right),$$

which is clearly negative when $\omega = \lambda$ (in which case $\alpha^C = \lambda$), and clearly positive when $\omega = 0$ (by virtue of the definition of α^C and m^C). Thus, if the difference decreases monotonically in ω, then by continuity there will be exactly one value of $\omega \in (0,\lambda)$ for which the difference is exactly zero. We take the $\omega-$derivative of the difference while accounting for the dependence of α^C and m^C on ω. This yields $\frac{1}{\lambda}$ times

$$\left(\frac{\lambda}{1+\lambda}-1\right)\mu(m^C)+\left(\frac{1+\omega}{1+\lambda}\lambda-\omega\right)\mu'(m^C)\frac{\partial m^C}{\partial\omega}-\frac{\sigma^2}{2\rho\lambda}\left(2\left(\frac{1+\omega}{1+\lambda}\lambda\right)\left(\frac{\lambda}{1+\lambda}\right)-2\omega\right)-c'(m^C)\frac{\partial m^C}{\partial\omega}$$

$$-\left(\frac{\lambda}{1+\lambda}-1\right)\left(\mu(m^C)-\frac{1-\frac{1+\omega}{1+\lambda}\lambda}{\rho(1-\lambda)}\sigma^2\right)-\left(\frac{1+\omega}{1+\lambda}\lambda-\omega\right)\left(\mu'(m^C)\frac{\partial m^C}{\partial\omega}+\frac{\lambda}{1+\lambda}\frac{\sigma^2}{\rho(1-\lambda)}\right)$$

or

$$-\frac{\sigma^2}{2\rho\lambda}\left(2\left(\frac{1+\omega}{1+\lambda}\lambda\right)\left(\frac{\lambda}{1+\lambda}\right)-2\omega\right)-c'(m^C)\frac{\partial m^C}{\partial\omega}$$

$$-\left(\frac{\lambda}{1+\lambda}-1\right)\left(-\frac{1-\frac{1+\omega}{1+\lambda}\lambda}{\rho(1-\lambda)}\sigma^2\right)-\left(\frac{1+\omega}{1+\lambda}\lambda-\omega\right)\left(\frac{\lambda}{1+\lambda}\frac{\sigma^2}{\rho(1-\lambda)}\right)$$

or

$$-\frac{\sigma^2(\lambda-\omega)}{\lambda(1-\lambda^2)\rho}-c'(m^C)\frac{\partial m^C}{\partial \omega}<0,\textbf{since}\frac{\partial m^C}{\partial \omega}>0. \qquad \blacksquare$$

.

Proof of Lemma 2: We begin with condition (i) of the globally stable allocation. Combining definition (7) with the selected monitoring level (5) and the market clearing price (6), the optimization problem can be written as:

$$\max_{\alpha} \phi\alpha\mu(m(\phi\alpha))-c(m(\phi\alpha))-\frac{\phi^2\alpha^2\sigma^2}{2\rho\tau}-\Psi(\alpha_G^D)-\phi(\alpha-\alpha_G^D)\left(\mu(m(\phi\alpha_G^D))-\frac{1-\alpha_G^D}{\rho(1-\lambda-\tau)}\sigma^2\right),$$

giving rise to the following first order condition:

$$\phi\mu(m(\phi\alpha))-\frac{1}{\rho\tau}\phi^2\alpha\sigma^2-\phi\left(\mu(m(\phi\alpha_G^D))-\frac{1-\alpha_G^D}{\rho(1-\lambda-\tau)}\sigma^2\right)=0.$$

Now, setting $\alpha=\alpha_G^D$ above and solving gives

$$\frac{1}{\rho\tau}\phi^2\alpha_G^D\sigma^2=\phi\frac{1-\alpha_G^D}{\rho(1-\lambda-\tau)}\sigma^2, \text{i.e., } \alpha_G^D=\frac{\tau/\phi}{\tau/\phi+1-\lambda-\tau}.$$

Now, we turn to condition (ii) of the globally stable allocation to verify that $\Psi^D(\frac{\tau/\phi}{\tau/\phi+1-\lambda-\tau})-\Psi^D(\omega)-\phi(\frac{\tau/\phi}{\tau/\phi+1-\lambda-\tau}-\omega)P^D(\frac{\tau/\phi}{\tau/\phi+1-\lambda-\tau})>0$ for all $\omega\neq\frac{\tau/\phi}{\tau/\phi+1-\lambda-\tau}$. This is equivalent to showing that $\omega=\frac{\tau/\phi}{\tau/\phi+1-\lambda-\tau}$ is a global maximum of the function $\Psi^D(\omega)-\phi\omega P^D(\frac{\tau/\phi}{\tau/\phi+1-\lambda-\tau})$, i.e.,

$$\phi\omega\mu(m(\phi\omega))-c(m(\phi\omega))-\frac{1}{2\rho\tau}\omega^2\phi^2\sigma^2-\phi\omega\left(\mu\left(m(\phi\frac{\tau/\phi}{\tau/\phi+1-\lambda-\tau})\right)-\frac{1}{\rho(\tau/\phi+1-\lambda-\tau)}\sigma^2\right).$$

To verify this we first note that the first order condition

$$\phi\mu(m(\phi\omega)) - \frac{1}{\rho\tau}\omega\phi^2\sigma^2 - \phi\left(\mu\left(m(\phi\frac{\tau/\phi}{\tau/\phi+1-\lambda-\tau})\right) - \frac{1}{\rho(\tau/\phi+1-\lambda-\tau)}\sigma^2\right) = 0$$

is satisfied at $\omega = \frac{\tau/\phi}{\tau/\phi+1-\lambda-\tau}$. We then evaluate the second order condition at $\omega = \frac{\tau/\phi}{\tau/\phi+1-\lambda-\tau}$: $\phi\mu'(m(\phi\frac{\tau/\phi}{\tau/\phi+1-\lambda-\tau}))m'(\phi\frac{\tau/\phi}{\tau/\phi+1-\lambda-\tau}) - \frac{\phi^2\sigma^2}{\rho\tau}$. This is strictly negative as long as $\Psi^D(\alpha)$ is strictly concave as required.■

Proof of Lemma 3: The per-GP effective allocation is $\frac{\phi\alpha_G^D}{\tau} = \frac{\phi}{\tau+\phi-\phi(\lambda+\tau)}$. The skilled investors outside the fund hold an aggregate stake of $1-\alpha_G^D = \frac{\phi-\phi(\lambda+\tau)}{\tau+\phi-\phi(\lambda+\tau)}$, leading to a per-investor allocation of $\frac{1}{1-\lambda-\tau}\frac{\phi-\phi(\lambda+\tau)}{\tau+\phi-\phi(\lambda+\tau)} = \frac{\phi}{\tau+\phi-\phi(\lambda+\tau)}$. ■

Proof of Proposition 3: To replicate a payoff of Π_{LP}^C for the LPs, (1) the fund, i.e., the GPs, must choose to monitor at level m^C, which they will only do if their own stake inside the fund is equal to ω units of the risky asset; and (2) the LPs must hold a final stake inside the fund of $\alpha^C = \frac{\lambda(1+\omega)}{(1+\lambda)}$ units of the risky asset. We choose ϕ and τ to achieve (1) and (2). For (1), we require that $\phi\alpha_G^D = \omega$. For (2), we require that $(1-\phi)\alpha_G^D = \frac{\lambda(1+\omega)}{(1+\lambda)}$. Plugging in the definition of α_G^D and solving these two equations for the two unknowns ϕ and τ yields ϕ^* and τ^* as given in the text of Proposition 3. From here forward, let the superscript $D*$ indicate that the associate function or variable is evaluated at τ^* and ϕ^*.

Next we determine the fee level f^* that just meets the participation constraint of individual GPs to ensure that the optimal mass τ^* will join the fund given the optimal skin in the game parameter ϕ^*. The fund's total endowment is given by $\omega + \tau^*\frac{1-\omega}{1-\lambda}$ (the LPs' endowment plus the GPs' share of the skilled investors' aggregate endowment of $1-\omega$).

The per-GP payoff for those who join the fund is given by

$$\frac{1}{\tau^*}\left[\Psi^{D*}(\alpha_G^{D*}) - \phi^*\left(\alpha_G^{D*} - \omega - \tau^*\frac{(1-\omega)}{1-\lambda}\right)P^{D*}(\alpha_G^{D*}) + \lambda f\right]. \tag{10}$$

The per-investor payoff for skilled investors who do not join the fund is

$$\frac{1}{1-\lambda-\tau^*}\left[\Psi_U^{D*}(\alpha_G^{D*}) - \left(1-\alpha_G^{D*} - \left(1-\omega-\tau^*\frac{(1-\omega)}{1-\lambda}\right)\right)P^{D*}(\alpha_G^{D*})\right],$$

where $\Psi_U^D(\alpha) = (1-\alpha)\mu(m^D(\alpha)) - \frac{(1-\alpha)^2\sigma^2}{2\rho(1-\lambda-\tau)}$ is the aggregate certainty equivalent payoff of the mass of $1-\lambda-\tau$ skilled investors outside of the fund who hold an aggregate stake of $1-\alpha$ given that the fund holds a stake of α. By defecting from the fund unilaterally, any given GP who is supposed to join the fund can realize the latter payoff. Thus, their participation constraint will be met as long as f is set to make these two payoffs equivalent. Defining

$$f^* = \frac{1}{\lambda}\left(\begin{array}{c} \frac{\tau^*}{1-\lambda-\tau^*}\left[\Psi_U^{D*}(\alpha_G^{D*}) - \left(1-\alpha_G^{D*} - \left(1-\omega-\tau^*\frac{(1-\omega)}{1-\lambda}\right)\right)P^{D*}(\alpha_G^{D*})\right] \\ -\left[\Psi^{D*}(\alpha_G^{D*}) - \phi^*\left(\alpha_G^{D*} - \omega - \tau^*\frac{(1-\omega)}{1-\lambda}\right)P^{D*}(\alpha_G^{D*})\right] \end{array}\right) \tag{11}$$

ensures the participation of the requisite mass of GPs. Later in this proof, we show that the above expression for $f^*$is equivalent to the expression shown in the statement of Proposition 3.

To complete the proof, we now show that the above contracting terms lead to an aggregate payoff for the LPs of Π_{LP}^C. First, we show that the price of the risky asset in the delegated fund equilibrium is equivalent to the price in the LPs' full commitment optimum. In the full commitment optimum, the price is given by $\mu(m^C) - \frac{1-\alpha^C}{\rho(1-\lambda)}\sigma^2$. Replacing α^C with $\frac{\lambda(1+\omega)}{(1+\lambda)}$ yields a price of $\mu(m^C) - \frac{1-\lambda\omega}{\rho(1-\lambda^2)}\sigma^2$. In the delegated fund equilibrium, the price, evaluated at the optimal fund parameters, is given by $P^{D*}(\alpha_G^{D*}) =$

$\mu(m^C) - \frac{1 - \frac{\tau^*/\phi^*}{\tau^*/\phi^* + 1 - \lambda - \tau^*}}{\rho(1-\lambda-\tau^*)}\sigma^2$. It is straightforward to show the algebraic equivalence of these two prices using the definitions of τ^* and ϕ^*.

Since the level of monitoring in the delegated fund equilibrium is identical to that in the LPs' full commitment optimum and the LPs' final holdings are identical, to complete our argument it suffices to show that LPs pay identical effective monitoring costs and trading costs across the two cases. With respect to the monitoring costs, note that the GPs directly pay the entirety of the actual costs in the delegated fund equilibrium while the LPs pay these costs in the full commitment optimum. Thus, the aggregate fee paid by the LPs must compensate GPs for their monitoring costs. With respect to trading costs, the costs incurred by the LPs in their full commitment optimum equal the equilibrium price times their aggregate trading quantity, or $P^{D*}(\alpha_G^{D*})(\alpha^C - \omega)$ (using the result above that the equilibrium price is equivalent to the full commitment price). In the delegated fund equilibrium they directly pay trading costs equal to the price times their proportional stake in the fund times its overall trading quantity, or $P^{D*}(\alpha_G^{D*})(1-\phi^*)(\alpha_G^{D*} - \omega - \tau^* \frac{(1-\omega)}{1-\lambda})$. Since, as shown previously, $(1-\phi^*)\alpha_G^{D*} = \alpha^C$, the *savings* in the LPs' direct trading costs for the delegated fund equilibrium relative to their full commitment equilibrium equal

$$P^{D*}(\alpha_G^{D*}) \left[(\alpha^C - \omega) - (1-\phi^*)(\alpha_G^{D*} - \omega - \tau^* \frac{(1-\omega)}{1-\lambda}) \right] = P^{D*}(\alpha_G^{D*}) \left[(1-\phi^*)(\omega + \tau^* \frac{(1-\omega)}{1-\lambda}) - \omega \right].$$

Thus, the aggregate fee must also transfer this amount from the LPs to the GPs.

To show that the equilibrium fee, f^* as defined in (11) accomplishes these requirements, first note that it is straightforward to show that $\frac{\tau^*}{1-\lambda-\tau^*}(1-\alpha_G^{D*}) = \omega$, and since we know that $\phi^*\alpha_G^{D*} = \omega$ also holds, we have $\frac{\tau^*}{1-\lambda-\tau^*}\Psi_U^{D*}(\alpha_G^{D*}) - \Psi^{D*}(\alpha_G^{D*}) = c(m^C)$. We can therefore rewrite the fee f^* as

$$\frac{1}{\lambda}\left[c(m^C)+P^{D*}(\alpha_G^{D*})\left(\phi^*(\alpha_G^{D*}-\omega-\tau^*\frac{(1-\omega)}{1-\lambda})-\frac{\tau^*}{1-\lambda-\tau^*}\left(1-\alpha_G^{D*}-\left(1-\omega-\tau^*\frac{(1-\omega)}{1-\lambda}\right)\right)\right)\right]$$

or

$$\frac{1}{\lambda}\left[c(m^C)+P^{D*}(\alpha_G^{D*})\left(\phi^*(-\omega-\tau^*\frac{(1-\omega)}{1-\lambda})-\frac{\tau^*}{1-\lambda-\tau^*}\left(-\left(1-\omega-\tau^*\frac{(1-\omega)}{1-\lambda}\right)\right)\right)\right]$$

or

$$\frac{1}{\lambda}\left[c(m^C)+P^{D*}(\alpha_G^{D*})\left(\phi^*(-\omega-\tau^*\frac{(1-\omega)}{1-\lambda})-\frac{\phi^*\alpha_G^{D*}}{(1-\alpha_G^{D*})}\left(-\left(1-\omega-\tau^*\frac{(1-\omega)}{1-\lambda}\right)\right)\right)\right]$$

or

$$\frac{1}{\lambda}\left[c(m^C)+P^{D*}(\alpha_G^{D*})\left((1-\phi^*)(\omega+\tau^*\frac{(1-\omega)}{1-\lambda})-\omega\right)\right],$$

as in Proposition 3. Thus, since the aggregate fee is λf^*, the LPs' obtain payoff Π_{LP}^C. Finally, note that Lemma 1 implies that all λ LPs find participation in the fund optimal as long as $\omega \leq \hat{\omega}$.■

Proof of Corollary 1: The fund holds $\alpha_G^{D*} = \frac{\tau^*/\phi^*}{\tau^*/\phi^*+1-\lambda-\tau^*}$ of the risky asset in equilibrium. The fund is made up of agents of measure $\lambda+\tau^*$ and thus the collective risk tolerance of this group of agents is $\lambda+\tau^*$. In a competitive equilibrium, such a collective of agents will hold $\lambda+\tau^*$ of the risky asset. Using the expressions in Proposition 3 we have:

$$\frac{\tau^*}{\phi^*}=\frac{\left(1-\lambda^2\right)\omega}{1-\lambda\omega}\frac{2\lambda\omega+\lambda+\omega}{(1+\lambda)\omega}=\frac{(1-\lambda)\left(2\lambda\omega+\lambda+\omega\right)}{1-\lambda\omega}.$$

We first show that $\tau^*/\phi^* < \lambda + \tau^*$. Assume the contrary. This implies that:

$$\frac{(1-\lambda)(2\lambda\omega+\lambda+\omega)}{1-\lambda\omega} \geq \lambda + \frac{(1-\lambda^2)\,\omega}{1-\lambda\omega},$$

which simplifies to $\lambda(\omega - \lambda) \geq 0$, which is a contradiction because $\lambda > 0$ and $\omega \leq \lambda$. Having shown that $\frac{\tau^*}{\phi^*} < \lambda + \tau^*$, we now observe that $\alpha_G^{D*} = \frac{\tau^*/\phi^*}{\tau^*/\phi^*+1-\lambda-\tau^*} < \frac{\lambda+\tau^*}{\lambda+\tau^*+1-\lambda-\tau^*} = \lambda + \tau^*$.■

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