Externalities of Responsible Investments*

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Abstract

When political institutions fail to control firm externalities, responsible investors can act as substitutes for government intervention. Individual investors, however, are unlikely to consider the aggregate effects of their choices, which raises the question of whether responsible capital is efficiently allocated across firms in the economy. In a general equilibrium model with heterogeneous social attitudes, we show that responsible investors tend to concentrate on a subset of firms while excluding others. This concentration of green capital can create product market power and crowd out the green investments of excluded firms. If this crowding-out dominates, aggregate CSR investments and welfare are higher without SRI.

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1 Introduction

Companies face increasing pressure to include environmental and social factors in their policies. One example of such pressure is the rising popularity of socially responsible investing (SRI). By the end of 2021, 4,375 investors managing $121 trillion have signed the United Nations Principles for Responsible Investment (UN PRI), pledging to incorporate corporate social responsibility (CSR) issues into their investment analysis and ownership policies.[1] The primary rationale is to change or divest from companies that exert negative externalities, to reduce their harm to society.

In a world where political institutions fail to control firm externalities (Hart and Zingales 2017; Bénabou and Tirole 2010), responsible investors can have a positive impact on society by acting as surrogates for government intervention. Compared to political institutions, however, individual investors are also less likely to internalize the aggregate effects of their choices. This failure to internalize aggregate effects may lead to inefficiencies in how responsible capital is allocated across firms in the economy, and generate negative spillovers to other stakeholders.

In this paper, we develop a framework to explore the inefficiencies of socially responsible investments (SRI). We argue that the practice of SRI itself can create *negative* externalities to (a) firms outside of responsible investors' portfolios and (b) consumers that incorporate CSR in their consumption decisions. Our model highlights a form of strategic complementarity in the portfolio choices of responsible investors, which leads to a concentration of green capital in only a few firms in an industry. Such concentration boosts the CSR investments of the targeted firms, but it also *crowds out* the CSR investments of those excluded, generating differentiation across firms, market power, and sometimes higher markups for green products. If these adverse effects are large, SRI reduces the overall greenness and the level of socially-motivated consumption in the economy.

We consider a market with N competing firms selling their products to consumers. Firms can produce with either a *brown* or a *green* technology. The brown technology generates negative externalities but is cheaper to implement. Thus, firms need incentives to invest in green technology. We model two sources of such incentives: SRI and socially responsible consumption (SRC).

[1] In 2006, only 63 investors managing a total of $6.5 trillion had signed the UN PRI.

Responsible and non-responsible investors trade the firms' shares in a secondary financial market. Non-responsible investors are purely profit-motivated, while responsible investors suffer disutility from investing in a firm that generates externalities. Firms internalize shareholders' social preferences in proportion to their ownership stakes when choosing CSR policies. This feature captures different channels through which shareholders influence firm behavior.[2] It follows that a firm invests more in CSR when responsible investors hold a larger fraction of its shares.

Like the financial market, the demand side of the product market is populated by responsible and non-responsible consumers. Responsible consumers suffer a disutility when buying from brown firms, so they are willing to pay a higher price for *green products*. The prospect of charging higher prices to these consumers represents the second source of incentives for CSR investments.

Our main results are driven by the interplay of two distinct strategic interactions: complementarity in responsible investors' portfolio choices and substitutability in firms' CSR investments. Responsible investors have a greater impact on CSR policies as a group, so their portfolio choices are strategic complements: They prefer to invest in firms with a large fraction of like-minded investors. The substitutability arises through a product market channel: Greater aggregate CSR leads to a more crowded market for green products, which reduces the return on CSR investments for each firm. The interaction between these two forces generates several interesting insights.

First, SRI generates differentiation in CSR investments across firms. To have more impact on firms' CSR policies, responsible investors concentrate on a small subset of firms in the industry. The presence of many responsible investors boosts the CSR investments of the targeted firms but reduces those of the excluded firms. The concentration of responsible capital thus generates dispersion in CSR policies. The dispersion makes firms with more green capital even more attractive for responsible investors so that they attract more SRI and induce more differentiation, in a self-reinforcing mechanism. Consistent with these predictions, we provide evidence suggesting that green capital is highly concentrated (Figure 1) and that there is a positive correlation between the concentrations of green capital and firms' CSR scores (Figure 2) in the data.

Second, the concentration of SRI creates market power for green products. Due to the pressure

[2]The most important channels in the context of CSR policies are voting and voice (see, e.g., Broccardo, Hart, and Zingales, 2022).

from responsible investors, targeted firms *overinvest* in CSR from the perspective of profit maximization. The overinvestment acts as a barrier to entry for the excluded firms, which may lead to an inefficiently low number of green firms and inflated prices for green products. Higher prices cause some responsible consumers to steer away from green products, reducing the level of SRC in the economy. Put differently, responsible investors may end up harming responsible consumers.

Third, SRI has an ambiguous effect on welfare. SRI primarily affects welfare through its impact on aggregate CSR. Perhaps surprisingly, we show that SRI may *decrease* aggregate CSR in equilibrium through the crowding out effect described above. Even when SRI increases aggregate CSR, it may still decrease welfare: the concentration in SRI may make green products more expensive, reducing the surplus of responsible consumers. For a region of the parameter space where SRI reduces welfare, there is another equilibrium with higher welfare (and higher aggregate CSR) where the responsible investors are crowded out. Notably, responsible investors are worse off in the equilibrium with higher welfare, as they do not hold any firm in this case. These results highlight the tension between social welfare and the private incentives of responsible investors.

Our results prove robust to various settings. In the main model, firms choose their CSR policies after investors trade their shares. We first examine the robustness of our results in a setting where this timing is reversed: firms choose their CSR policies to attract investors and maximize their stock prices. Even though the firms are ex-ante identical, we show that they may choose different CSR policies in equilibrium: some firms invest more in CSR, catering to responsible investors, while others invest less and focus on non-responsible investors. This separation helps firms to tailor their CSR policies to the investors' preferences so that neither group of firms wants to deviate from this strategy. Our main insights continue to hold since the presence of SRI causes firms to differentiate their CSR policies. We then show that our main results are also robust to different ways of modeling competition among firms, CSR investments, and firms' externalities.

It is worth emphasizing that, although we focus on the negative externalities of SRI, responsible investments can also increase aggregate CSR and welfare in our model. In that case, SRI generates a net positive externality to the economy. While the positive effects of SRI as a substitute for political institutions in controlling externalities have received much attention in the literature, its negative effects are less understood. Our analysis highlights a potential downside of SRI stemming from the investors' failure to internalize the consequences of their actions for the broader economy.

Next, we summarize the related theoretical literature. We discuss the novel empirical implications and existing evidence surrounding our results in Section 5.3.

Related literature. Our paper contributes to the growing literature on responsible investing. One strand of this literature focuses on the conditions under which SRI impacts firms' decisions. In Heinkel, Kraus, and Zechner (2001), socially responsible investors encourage firms to invest in CSR by excluding "brown" firms and increasing their cost of capital. Oehmke and Opp (2020), Landier and Lovo (2020), and Gupta, Kopytov, and Starmans (2022) study the implications of SRI on the production choices of financially constrained firms. Edmans, Levit, and Schneemeier (2022) show that, under certain conditions, holding a brown company that has taken a corrective action dominates blanket divestment.[3] A second strand of the literature focuses on the implications of SRI for financial market outcomes, like expected returns (Pedersen, Fitzgibbons, and Pomorski 2021, Pástor et al. 2021), price informativeness (Goldstein, Kopytov, Shen, and Xiang 2022), and the interplay between SRI and government regulation (Piatti, Shapiro, and Wang 2022).

The above papers consider socially responsible investors but abstract from socially responsible consumers. Two recent papers study the interactions between these two types of agents. Broccardo, Hart, and Zingales (2022) argue that voice (i.e., shareholder engagement) is more effective than exit (i.e., divestment and consumers' boycotts) in incentivizing firms' CSR investments. Hakenes and Schliephake (2021) analyze the interaction between socially responsible investments and consumption in a setting where firms' competition is exogenous.

Our contribution to this literature is to explore how green capital distributes across competing firms and analyze the implications for aggregate CSR and welfare. First, we explain the stylized fact that green capital is unevenly distributed across firms in an industry. Second, we explore how this concentration affects the CSR investments of firms with less green capital and, as a result, the

[3] Davies and Van Wesep (2018) show that divestment may also have the unintended consequence of inducing managerial short-termism.

competitive landscape and the level of socially-motivated consumption in the economy.

We also contribute to the vast literature on the objectives of the firm. The traditional view that firms should primarily maximize profits (Friedman, 1970) has been challenged by several recent papers (Elhauge, 2005; Bénabou and Tirole, 2010; Hart and Zingales, 2017). They argue that, when political institutions fail to control externalities, firms should internalize shareholders' social preferences, potentially pursuing social goals at the expense of profits. We highlight a novel downside of such internalization: in a world with heterogeneous social attitudes, internalizing shareholders' social preferences can generate differentiation and market power.

The substitutability in firms' CSR investments plays a key role in the negative externalities of SRI in our model, and is common to other papers on this topic. Aghion, Bénabou, Martin, and Roulet (2020) model CSR as investments in clean innovation that help firms escape price competition. In Albuquerque, Koskinen, and Zhang (2019), a firm's CSR increases the loyalty of its customers and allows the firm to charge higher prices. In both papers, an increase in the CSR investments of its rivals reduces a firm's incentives to invest in CSR. Of course, firms' CSR investments can also be complements when there are positive network effects in adopting green technologies, like imitative innovation and peer-pressure effects. Albuquerque and Cabral (2021) consider a model with this feature in which firms compete with each other. They show that firms' CSR commitments help solve coordination problems and generate positive externalities.

2 The Model

The model consists of two dates, $t \in \{1, 2\}$, and a discrete number of publicly traded firms, $j \in \mathcal{J} \equiv \{1, \ldots, N\}$. At time $t = 1$, investors trade claims to the firms' terminal values in a secondary market, which shapes their ownership structures. At time $t = 2$, firms first choose their CSR policies and then compete in selling their products to consumers. Finally, firms' terminal values realize and are distributed to shareholders. All agents in the model are rational and risk-neutral, and, for simplicity, we assume that there is no discounting.

Next, we describe our baseline model, which will be the focus of our analysis. Section 6 discusses the robustness of our main results to alternative modeling assumptions.

2.1 Product market

The product market consists of N firms, with $N \geq 3$, and a unit mass of consumers, indexed by $h \in [0, 1]$.[4] All firms offer identical products to consumers, but they may differ in their production technology. There are two possible production technologies: a *brown* technology, which generates a negative externality λ; a *green* technology, which generates no externality. One can interpret λ as the pollution generated by the production process or the societal cost of a negative corporate culture. To simplify the analysis in the baseline model, we assume that the externality depends on the production technology adopted by each firm but not on its production volume.

All firms have access to the brown technology for free. Firm j gains access to the green technology with probability $\sigma_j \in [0, 1]$ at a cost $C(\sigma_j) \equiv \frac{c}{2}\sigma_j^2$. The choice variable σ_j captures the intensity with which the firm invests in the green technology, and $C(\sigma_j)$ captures the investment cost. We let σ_j denote the CSR policy of firm j and $\vec{\sigma} \equiv (\sigma_1, \ldots, \sigma_N)$ the collection of CSR policies.

Investments in CSR policies may lead to different production technologies across firms. We use the notation $a_j = 1$ ($a_j = 0$) to signify that firm j uses the green (brown) technology and, therefore, offers a green (brown) product to consumers. The random vector $\vec{a} \equiv (a_1, \ldots, a_N)$ describes the type of product offered by each firm, where $\Pr(a_j = 1) = \sigma_j$ is independent across firms. For a given realization of $\vec{a}$, firms compete à la Bertrand in selling their products to consumers: First, the firms simultaneously set their prices, with ρ_j denoting firm j's price. Then, having observed the price vector $\vec{\rho} \equiv \{\rho_1, \ldots, \rho_N\}$ and product types $\vec{a}$, consumers choose which products to buy.

Consumers have heterogeneous social attitudes. A fraction $\chi_{co} \in (0, 1)$ of consumers are *socially responsible*: They internalize the externality generated by the brown technology, as they incur a disutility λ from consuming such products.[5] The disutility may capture consumers' concern with their social image or, more broadly, a distaste for brown products (Bénabou and Tirole 2010;

[4]The sub-game where firms compete in selling products to consumers does not admit pure-strategy equilibria when $N = 2$. Therefore, we consider $N \geq 3$ to simplify the analysis. In Online Appendix C.1, we consider a model with horizontally differentiated products that admits pure-strategy equilibria with $N = 2$; we show that our qualitative results carry through.

[5]The quadratic specification of the cost function and the assumption that the value of the disutility is equal to that of the externality simplify the exposition but do not affect our results. Any cost function $C(\sigma_j)$ satisfying $C(0) = C'(0) = 0$ and $C', C'' > 0$ and any non-increasing function of λ for the disutility would lead to the same qualitative results.

Albuquerque, Koskinen, and Zhang 2019).

For a given $\vec{\rho}$ and $\vec{a}$, consumer h's demand for firm j's product is denoted by x_{hj} and solves:

$$\max_{(x_{hj})_{j=1,\dots,N}\geq 0} u\left(\sum_{j=1}^{N} x_{hj}\right) - \sum_{j=1}^{N}[\rho_j + \mathbb{1}_{h,\mathcal{R}}\lambda(1-a_j)]x_{hj}, \tag{1}$$

where $\mathbb{1}_{h,\mathcal{R}} = 1$ for responsible consumers ($h \in \mathcal{R}$) and 0 otherwise ($h \in \mathcal{N}$). We denote the utility of consumption by $u(\cdot)$, with $u' > 0$ and $u'' < 0$.

The specification for consumers' utility of consumption in Program (1) embodies two main assumptions. First, aside from the disutility of consuming brown products for $\mathcal{R}$ types, the products are perfect substitutes in consumers' preferences. Second, consumers' budget constraints are not binding at their equilibrium consumption, so their choice is equivalent to an unconstrained problem. These assumptions simplify the exposition but do not affect any of our results.

The marginal cost of production is $\gamma \geq 0$ for all firms, regardless of whether they produce brown or green products. For a given $\vec{a}$, fixing the prices charged by its competitors, firm j chooses the product price ρ_j to maximize profits net of the CSR-related investment cost:

$$\max_{\rho_j \geq 0} \Pi_j \equiv (\rho_j - \gamma)\int_0^1 x_{hj}(\vec{a},\vec{\rho})\,dh - C(\sigma_j). \tag{2}$$

2.2 Ownership market

At time $t = 1$, the firms' ownership structure is determined. Each firm has a fixed supply of shares, normalized to one, traded in a secondary financial market.[6] A unit mass of atomistic investors, indexed by $i \in [0,1]$, simultaneously submit their demand schedules for the shares of each firm. The market-clearing price equates demand and supply.

Like consumers, investors have heterogeneous social attitudes. A fraction $\chi_{in} \in [0,1)$ of investors are socially responsible ($i \in \mathcal{R}$), as they internalize the externality λ when holding shares of firms that offer brown products. By contrast, non-responsible investors ($i \in \mathcal{N}$) only maximize the expected monetary return from their holdings. That is, the expected value of the claim to the firm's profits, net of its share price p_j, and trading cost K.

[6] We thus abstract from a potential impact of SRI on firm decisions through the cost of capital (Heinkel et al., 2001; Edmans et al., 2022).

Formally, for a given vector of CSR policies $\vec{\sigma}$, investor i solves:

$$\max_{s_{ij}\geq 0} \sum_j s_{ij}\mathbb{E}\left[\Pi_j - p_j - \mathbb{1}_{i,\mathcal{R}}\lambda(1-a_j)\right] - K(\iota'\vec{s}_i), \tag{3}$$

where s_{ij} is the number of shares investor i holds in firm j, $\mathbb{1}_{i,\mathcal{R}} = 1$ if i is responsible ($i \in \mathcal{R}$) and 0 otherwise ($i \in \mathcal{N}$), and $K(\iota'\vec{s}_i)$ is the trading cost of purchasing $\iota'\vec{s}_i = \sum_j s_{ij}$ shares, where $\vec{s}_i \equiv (s_{i1}, \ldots, s_{iN})$ and $\iota \equiv (1, \ldots, 1)$.

The expectation in Equation (3) is taken with respect to the vector of firm types $\vec{a}$, which determines both firms' profits and $\mathcal{R}$ investors' disutility. The trading cost $K(\cdot)$ reflects direct transaction costs as well as indirect costs, such as the borrowing or opportunity costs of raising funds for the investor. Consistent with this interpretation, $K(\cdot)$ depends on the total size of the investor's portfolio, not on the allocation of shares across firms. Finally, to obtain closed-form solutions for the investors' trading strategies, we posit $K(x) \equiv \frac{\kappa}{2}x^2$ as, e.g., in Banerjee et al. (2018).

At the beginning of $t = 2$, firms simultaneously choose their CSR policies given their ownership structure. They incorporate shareholders' social preferences into their objective function so that ownership influences the choice of CSR policies. Formally, given its ownership $\{s_{ij}\}$ for $i \in [0, 1]$, firm j chooses σ_j to solve:

$$\max_{\sigma_j\in[0,1]} \mathbb{E}\left[\Pi_j\right] - \lambda(1-\sigma_j)\eta s_j^{\mathcal{R}}, \tag{4}$$

where $s_j^{\mathcal{R}} \equiv \int_0^1 s_{ij}\mathbb{1}_{i,\mathcal{R}}di$ denotes the shares of firm j held by responsible investors and $\vec{s}^{\mathcal{R}} \equiv \left(s_1^{\mathcal{R}}, \ldots, s_N^{\mathcal{R}}\right)$ describes the distribution of SRI in the industry.

The objective in Program (4) is a weighted average of the expected payoff per share to investors, where the weights are the shares held by each shareholder.[7] This specification is commonly referred to as *proportional control* assumption (see, e.g., O'Brien and Salop, 1999; López and Vives, 2019),[8] and captures different channels through which shareholders can influence managerial decisions in proportion to their stake in the firm.[9] The parameter $\eta \in (0, 1]$ captures the extent to which firms

[7]At the beginning of time $t = 2$, investors have already paid the share price and transaction cost, but the firms' types are not yet realized. Therefore, an $\mathcal{N}$-type investor receives an expected payoff $\mathbb{E}[\Pi_j]$ from holding a share of firm j. An $\mathcal{R}$-type receives an expected payoff $\mathbb{E}[\Pi_j] - \lambda(1-\sigma_j)$, since the investor incurs the disutility λ when $a_j = 0$, which occurs with probability $1 - \sigma_j$.

[8]In these papers, firms maximize the weighted average of shareholders' portfolio payoffs, which may include the returns shareholders receive from other firms in their portfolios, and has thus implications for firms' competition. We abstract from anti-competitive effects and focus on the internalization of shareholders' social preferences.

[9]Examples of these channels include voting (see, e.g., Levit and Malenko, 2011; Levit, Malenko, and Maug, 2020), exit, and voice (see,

internalize shareholders' social preferences in their choice of CSR policies.

It is worth noticing that at $t = 1$, investors choose their portfolios based on their conjectures about firms' CSR policies, which are set at $t = 2$. Moreover, investors' portfolios shape firms' CSR policies, as they determine the firms' ownership and objective functions in Program (4). This two-way interaction between investors' portfolios and firms' CSR policies is the focus of the equilibrium analysis. In Online Appendix E, we show that our main insights continue to hold in a setting where firms choose their CSR policies first, and investors trade afterward.

2.3 Sequence of events

The timing of the model is summarized in what follows.

Time $t = 1$:

(i) Investors trade and form their portfolios $\{s_{ij}\}$ for $i \in [0, 1]$ and $j \in \mathcal{J}$.

Time $t = 2$:

(ii) Having observed the distribution of SRI in the industry ($\vec{s}^{\mathcal{R}}$), firms choose CSR policies $\vec{\sigma}$.

(iii) Firms' product types $\{a_j\}$ realize and are publicly observed. Given $\{a_j\}$, firms set their product prices $\vec{\rho}$, and consumers choose their demands $x_{hj}(\vec{a}, \vec{\rho})$ for $h \in [0, 1]$ and $j \in \mathcal{J}$.

(iv) Firms' terminal values realize and are distributed to shareholders.

We use *subgame perfect equilibrium* as the solution concept and restrict our attention to pure-strategy equilibria. An equilibrium of the game is a collection $\{\{s_{ij}\}, \vec{\sigma}, \vec{\rho}(\vec{a}), \{x_{hj}(\vec{a}, \vec{\rho})\}\}$, where $i, h \in [0, 1]$, and $j \in \mathcal{J}$. The equilibrium collection jointly solves Programs (1) and (2) for any realization of $\vec{a}$, Programs (3) and (4), and satisfies sequential rationality.

As we will see, depending on the parameters of the model, two types of equilibria may arise: *Symmetric equilibria*, in which firms have identical ownership structures and CSR policies; *Asymmetric equilibria*, in which firms have different ownership structures and CSR policies.

e.g., Edmans and Manso, 2011; Brav, Dasgupta, and Mathews, 2022).The aggregate shares held by $\mathcal{R}$ investors in any firm enter linearly under this assumption. However, our central results continue to hold under more general assumptions. For instance, we could have the more generic form $\mathbb{E}[\Pi_j] - \lambda(1 - \sigma_j) f(s_j^{\mathcal{R}})$, where $f : [0, 1] \to [0, 1]$ is an increasing function.

3 Product prices and CSR policies

We work our way backward by characterizing the product market equilibrium, taking as given firms' ownership structures. We first solve for the equilibrium pricing strategies and then, given these strategies, characterize firms' CSR policies.

3.1 Product market equilibrium

The equilibrium outcome at the final market competition stage depends on the number of firms that have successfully introduced the green technology. More specifically, there are three distinct cases. First, if no firm has implemented the green technology, all firms' products are homogeneous for all consumers. Perfect Bertrand competition implies that all firms set the price equal to marginal cost, i.e., $\rho_j^* = \gamma$ for all j. Hence, firms make zero profits in this case. Second, whenever at least two firms produce green goods, they again engage in perfect Bertrand competition and make zero profits. Third, suppose only one firm j introduces the green product. In that case, it sells a good that is considered *vertically differentiated* from those sold by its rivals with a brown technology by $\mathcal{R}$ consumers. All other firms ($-j \neq j$) still provide homogeneous goods and set $\rho_{-j}^* = \gamma$, leading to zero profits. Let $x^*(\rho_j)$ denote the demand of each $\mathcal{R}$ consumer obtained from Problem (1) given ρ_j and $\rho_{-j}^* = \gamma$. Then, firm j's equilibrium profit is given by:

$$\pi^m \equiv \max_{\rho_j \geq 0} \chi_{co}(\rho_j - \gamma)x^*(\rho_j). \tag{5}$$

To rule out trivial corner solutions, we assume that the investment cost is sufficiently high relative to the monopoly profit: $c > \eta\lambda + \pi^m$. Lemma 1 formalizes these insights; all proofs are given in Appendix B.

Lemma 1 (Product Market Equilibrium) *Firm j prices above marginal cost and makes positive profits π^m if and only if it is the only green firm in the market. Hence, each firm j's expected terminal value is*

$$\mathbb{E}[\Pi_j] = \sigma_j \prod_{-j \neq j}(1 - \sigma_{-j})\pi^m - C(\sigma_j). \tag{6}$$

3.2 CSR policies

Next, we characterize the firms' optimal choice of CSR policies. We take the first-order condition of Problem (4) and plug in firm j's expected profits based on Equation (6). The optimal CSR policy for firm j is then given by:

$$\sigma_j = \frac{1}{c}\left[\eta\lambda s_j^{\mathcal{R}} + \pi^m \prod_{-j\neq j}(1-\sigma_{-j})\right]. \tag{7}$$

Equation (7) shows that firm j's willingness to implement a greener CSR policy depends on two forces, the internalization of shareholder preferences and product market pressure. A greater share of responsible investors reduces the effective cost of greener CSR policies and, thus, encourages the firm to invest in them. This channel's strength scales with the externality's severity, λ. Moreover, firm j is *more* willing to implement CSR policies if other firms are *less* likely to implement them. In this case, firm j is more likely to capture the monopoly profits π^m from being the sole green firm. This result emphasizes that CSR policies are substitutes across firms in our settings.

Lemma 2 (Equilibrium CSR Policies) *For a given distribution of SRI ($\vec{s}^{\mathcal{R}}$), an equilibrium $\sigma_j^*(\vec{s}^{\mathcal{R}}) \in (0,1)$ of the subgame in which firms choose their CSR policies always exists. Moreover:*

1. *Suppose $s_j^{\mathcal{R}} \equiv s^{\mathcal{R}} \in [0,1]$ for all $j \in \mathcal{J}$. Then $\sigma_j^*(\vec{s}^{\mathcal{R}})$ is unique, and all firms choose the same CSR policy, that is, $\sigma_j \equiv \sigma^*$, which is increasing in $\eta s^{\mathcal{R}}$.*

2. *Suppose $s_{j'}^{\mathcal{R}} \geq s_j^{\mathcal{R}}$ for two firms $j, j' \in \mathcal{J}$. Then $\sigma_{j'}^* \geq \sigma_j^*$, where the inequality is strict if and only if $s_{j'}^{\mathcal{R}} > s_j^{\mathcal{R}}$.*

3. *Suppose $s_j^{\mathcal{R}} > 0$ only for firm $j \in \mathcal{J}$. Then $\sigma_j^*(\vec{s}^{\mathcal{R}})$ is unique and such that $\sigma_j^* > \sigma_0 > \sigma_{-j}^*$ for all $-j \neq j$, where σ_0 denotes the CSR policy when $s_j^{\mathcal{R}} = 0$ (or $\eta = 0$) for all $j \in \mathcal{J}$.*

Lemma 2 characterizes firms' optimal CSR policies for a given distribution of SRI and presents three main results. First, there is a unique equilibrium, which is symmetric, if responsible investors hold the same positions in all firms. If responsible ownership is symmetric, then an increase in green capital or a stronger internalization of shareholder preferences (i.e., a higher η) unambiguously translates into greener CSR policies by all firms.

Second, this subgame only admits asymmetric equilibria if the ownership structure is differentiated across firms. If $\mathcal{R}$ investors hold different shares in different firms, then firms implement different CSR policies in equilibrium, with firms with higher $s_j^{\mathcal{R}}$ choosing higher σ_j^*. Hence, the internalization of shareholder preferences may generate differentiation in firms' CSR policies. To better understand this result, suppose that initially, no responsible investors exist in any firm. Then, an increase in $s_j^{\mathcal{R}}$ for one firm j has the following two effects. First, there is a direct effect because management starts internalizing the negative externality caused by the brown production technology. As the perceived cost of being brown increases, this firm has stronger incentives to invest in green technologies. Second, there is an indirect effect because j's rivals optimally reduce their CSR investment levels in anticipation of j's investments. As a result, CSR investments are strategic substitutes.

Third, in the most extreme asymmetric equilibrium, only one firm has a positive fraction of responsible investors. This firm increases its CSR investment beyond σ_0 (i.e., beyond the level in the symmetric equilibrium without SRI). Firm j's rivals instead invest less than σ_0, increasing j's incentive to invest more. Hence, the direct and indirect effects reinforce each other so that asymmetries in green funds may generate substantial heterogeneity in CSR investments.

4 Equilibrium ownership

Having described how firms' equilibrium strategies depend on their ownership, we can now close the model by solving for the investors' portfolio choices and the distribution of SRI across firms.

4.1 Preliminaries

Since investors are atomistic, they take the vector of stock prices $\vec{p} \equiv (p_1, \ldots, p_N)$ as given when deciding on their asset holdings. Moreover, they rationally anticipate firms' CSR choices and expected profits and how these depend on the distribution of SRI in the economy.

Taking the first-order condition of Program (3) for investor i with respect to s_j yields:

$$\mathbb{E}[\Pi_j] - p_j - \mathbb{1}_{i,\mathcal{R}}\lambda(1-\sigma_j) \leq \kappa\iota'\vec{s}_i \qquad (8)$$

where the inequality is strict if and only if the investor does not invest in firm j, i.e., if $s_{ij} = 0$.

Note that the marginal cost, i.e., the right-hand side of Equation (8), is constant across firms. To break ties, we assume that investors incur a small cost from acquiring shares in multiple firms, so each individual investor prefers to hold shares of one firm only.[10]

We can then write investor i's demand of firm j's shares as:

$$s_{ij} = \begin{cases} \max\{\frac{1}{\kappa}\left[\mathbb{E}[\Pi_{j^*}] - p_{j^*} - \mathbb{1}_{i,\mathcal{R}}\lambda(1-\sigma_{j^*})\right], 0\} & \text{for } j^* = \text{argmax}_j\{\mathbb{E}[\Pi_j] - p_j - \mathbb{1}_{i,\mathcal{R}}\lambda(1-\sigma_j)\} \\ 0 & \text{for } j \neq j^*. \end{cases} \tag{9}$$

Hence, each investor i is willing to invest if the preference-adjusted return, $\mathbb{E}[\Pi_j] - p_j - \mathbb{1}_{i,\mathcal{R}}\lambda(1-\sigma_j)$, is positive for at least one firm. Otherwise, she does not invest in any firm.

The market clearing conditions determine the share prices in equilibrium. Let α_j^θ denote the fraction of $\theta \in \{\mathcal{R}, \mathcal{N}\}$ investors buying positive shares of firm j, where the purchased quantity is given in Equation (9) above. $\mathcal{R}$ investors suffer disutility from holding brown firms. Since each firm is brown with a positive probability in equilibrium (Lemma 2), these investors have lower valuations than $\mathcal{N}$ investors. Thus, all else equal, responsible investors have lower demand than non-responsible ones. This implies that, while $\mathcal{N}$ investors must always buy some shares (as otherwise, markets would not clear), $\mathcal{R}$ investors may choose not to invest in any firm. That is, in equilibrium we may have $\alpha_j^\mathcal{R} = 0 \forall j \in \mathcal{J}$, but we always have $\alpha_j^\mathcal{N} > 0$ for at least some $j \in \mathcal{J}$.

The market clearing condition for firm j is given by:

$$\chi_{in}\alpha_j^\mathcal{R}\frac{1}{\kappa}\left[\mathbb{E}[\Pi_j] - p_j - \lambda(1-\sigma_j)\right] + (1-\chi_{in})\alpha_j^\mathcal{N}\frac{1}{\kappa}\left[\mathbb{E}[\Pi_j] - p_j\right] = 1. \tag{10}$$

Solving this expression for j's equilibrium share price leads to:

$$p_j = \mathbb{E}[\Pi_j] - \underbrace{\frac{\kappa}{\chi_{in}\alpha_j^\mathcal{R} + (1-\chi_{in})\alpha_j^\mathcal{N}}}_{\text{liquidity discount}} - \underbrace{\frac{\lambda(1-\sigma_j)\chi_{in}\alpha_j^\mathcal{R}}{\chi_{in}\alpha_j^\mathcal{R} + (1-\chi_{in})\alpha_j^\mathcal{N}}}_{\text{brownness discount}}. \tag{11}$$

[10]Without this tie-breaking assumption, each of the equilibria we characterize coexists with observationally equivalent ones (i.e., equilibria with the same distribution of SRI) where investors hold diversified portfolios, that is, divide their optimal demands s_{ij} across multiple firms.

Equation (11) shows that p_j equals j's expected profits net of two distinct discounts. The first discount captures a standard liquidity discount, which is necessary to incentivize investors to trade in the asset. This term vanishes as the trading cost κ goes to zero. The second discount arises due to the presence of responsible investors and is thus increasing in their mass $\chi_{in}\alpha_j^{\mathcal{R}}$ and the externality λ. In the limit $\sigma_j \to 1$, firm j is always green, and this term disappears.

As we will show, depending on the parameters of the model, three types of equilibrium outcomes can arise:

(i) ***No SRI*** ($s_j^{\mathcal{R}} = 0\ \forall j \in \mathcal{J}$): no firm is held by $\mathcal{R}$ investors.

(ii) ***SRI without tilting*** ($s_j^{\mathcal{R}} = s^{\mathcal{R}} \in (0,1)\ \forall j \in \mathcal{J}$): each firm has the same positive share of $\mathcal{R}$ investors.

(iii) ***SRI with tilting*** ($\exists j, j' \in \mathcal{J}$ such that $s_j^{\mathcal{R}} \neq s_{j'}^{\mathcal{R}}$): firms differ in the share of $\mathcal{R}$ investors.

Before we formally characterize the different equilibrium outcomes, it is helpful to introduce the function $\widehat{\kappa}$, which will be used as a threshold for the transaction cost κ:

$$\widehat{\kappa}(y,z) \equiv \frac{\chi_{in}(1-\chi_{in})\lambda(1-y)}{N\chi_{in}-z}, \tag{12}$$

for $y \in [0,1]$ and $z \in \{0,\ldots,N\}$.

4.2 Symmetric equilibria (no SRI and SRI without tilting)

As a first step, we consider equilibria in which all firms choose the same CSR policy $\sigma^* \in (0,1)$. In this scenario, all firms generate identical expected profits ($\mathbb{E}[\Pi^*] = \sigma^*(1-\sigma^*)^{N-1}\pi^m - C(\sigma^*)$), so they must also have the same share prices p^* in equilibrium. Indeed, if two firms had different prices, all investors (responsible and non-responsible) would be strictly better off buying shares from the one with the lower price. Lemma 2 shows that equilibria, where firms have identical CSR policies, can exist only when all firms have the same fraction of $\mathcal{R}$ investors, that is, when $s_j^{\mathcal{R}}$ is constant for all j. Equation (9) implies that this is the case if and only if a fraction $\alpha_j^{\mathcal{R}} = 1/N$ of $\mathcal{R}$ investors buys an amount $\max\{\frac{1}{\kappa}\left[\mathbb{E}[\Pi^*] - p^* - \lambda(1-\sigma^*)\right], 0\}$ of shares in each firm $j \in \mathcal{J}$.

It follows that there can be only two types of equilibria where all firms have identical ownership structures and CSR policies (*symmetric equilibria*): equilibria in which all firms are solely held by $\mathcal{N}$ investors, and equilibria in which each firm has the same positive share of $\mathcal{R}$ investors.[11]

No SRI. We first consider equilibria where responsible investors do not invest so that we have $\alpha_j^{\mathcal{R}} = 0$ and $s_j^{\mathcal{R}} = 0$ for all j. In such equilibria, the CSR policy for each firm is given by $\sigma^* = \sigma_0$, which coincides with the equilibrium in which firms do not internalize shareholders' social preferences ($\eta = 0$). Let $\mathbb{E}[\Pi_0]$ denote the firms' expected profits and p_0 the equilibrium stock price. The symmetric equilibrium with no SRI exists if and only if $\mathcal{N}$ investors are willing to buy shares in any firm, whereas $\mathcal{R}$ investors are not. Therefore, the equilibrium share price must lie between the valuation of $\mathcal{R}$ and $\mathcal{N}$ investors:

$$p_0 \in [\mathbb{E}[\Pi_0] - \lambda(1 - \sigma_0), \mathbb{E}[\Pi_0]). \tag{13}$$

For the market clearing conditions to hold for all firms, it must be that $\alpha_j^{\mathcal{N}} = 1/N$. It then follows that $p_0 = \mathbb{E}[\Pi_0] - \frac{N\kappa}{1-\chi_{in}}$, so that the existence condition above boils down to $\kappa \leq \widehat{\kappa}(\sigma_0, 0)$. Intuitively, investors are willing to take large positions when the trading cost κ is sufficiently low. Since $\mathcal{N}$ investors have relatively higher valuations, their increased demand for shares completely crowds out $\mathcal{R}$ investors. In equilibrium, all firms are entirely held by $\mathcal{N}$ investors.

SRI without tilting. Next, we analyze the symmetric equilibria in which $\mathcal{R}$ investors participate in the financial market. This means that, for all firms, the equilibrium stock price must be lower than their valuation:

$$p^* < \mathbb{E}[\Pi^*] - \lambda(1 - \sigma^*). \tag{14}$$

As we discussed above, the share of $\mathcal{R}$ investors is identical across firms in these equilibria. Therefore, the market clearing condition for each firm j is given by Equation (10) for $\alpha_j^R = 1/N$ and imposing symmetry, which holds for all j if and only if $\alpha_j^{\mathcal{N}} = 1/N$ for all $j \in \mathcal{J}$. The equilibrium share price is thus $p^* = \mathbb{E}[\Pi^*] - \chi_{in}\lambda(1 - \sigma^*) - N\kappa$, where σ^* is the unique solution to the following

[11] Note that equilibria in which all firms are solely held by $\mathcal{R}$ investors cannot exist, since $\mathcal{N}$ investors have larger demand than $\mathcal{R}$ investors, and thus cannot be crowded out from the market.

system of equations:

$$\sigma^* = \frac{1}{c}\left[\eta\lambda s^{\mathcal{R}*} + \pi^m(1-\sigma^*)^{N-1}\right]; \tag{15}$$

$$s^{\mathcal{R}*} = \frac{\chi_{in}}{N}\frac{1}{\kappa}\left[N\kappa - \lambda(1-\chi_{in})(1-\sigma^*)\right]. \tag{16}$$

This system of equations is obtained by imposing symmetry in Equations (7) and (9), which describe firms' CSR policies and $\mathcal{R}$ investors' holdings, respectively. Note that firms' ownership and CSR policies are characterized by a fixed-point problem in equilibrium. The amount of shares each individual $\mathcal{R}$ investor demands depends on her expectation of the firm's CSR policy (σ^*), which in turn depends on the overall fraction of shares held by responsible investors ($s^{\mathcal{R}*}$).

Two opposing forces characterize the strategic interaction among $\mathcal{R}$ investors. On the one hand, the two-way feedback between σ^* and $s^{\mathcal{R}*}$ generates a source of strategic complementarity: when $\mathcal{R}$ investors demand more shares, $s^{\mathcal{R}*}$ and σ^* increase. The increase in σ^* reduces the expected disutility, and so, holding the stock price p^* fixed incentivizes each individual $\mathcal{R}$ investor to hold more shares. On the other hand, higher demand also gives rise to the traditional strategic substitutability across investors: when $s^{\mathcal{R}*}$ increases, the price increases, and trading profits decrease. Holding σ^* fixed, the increase in p^* reduces the incentives of each individual investor to invest.

We show that the second effect (strategic substitutability) always dominates in equilibrium so that the system of Equations (15) and (16) admits a unique solution.

Given the market clearing price and the firms' equilibrium investments, we show that the symmetric equilibrium with SRI exists if and only if $\kappa > \widehat{\kappa}(\sigma_0, 0)$.

Characterization. Collecting the results above, we have the following characterization of symmetric equilibria in Proposition 1.

Proposition 1 (Symmetric Equilibrium) *A symmetric equilibrium always exists and is unique:*

1. *If $\kappa \leq \widehat{\kappa}(\sigma_0, 0)$, the symmetric equilibrium features* ***No SRI****: $s_j^{\mathcal{R}} = 0$ and $\sigma_j = \sigma_0$ $\forall j \in \mathcal{J}$;*

2. *Otherwise, it features* ***SRI without tilting****: $s_j^{\mathcal{R}} = s^{\mathcal{R}} > 0$ and $\sigma_j = \sigma^* > \sigma_0$ $\forall j \in \mathcal{J}$.*

The threshold $\widehat{\kappa}(\sigma_0, 0)$ for the trading cost is defined in Equation (12).

Proposition 1 shows that the game always admits a symmetric equilibrium and that this equilibrium is unique.[12] If the trading cost is relatively low and financial markets are efficient, responsible investors are crowded out. In this case, firms are still willing to implement CSR policies but are only disciplined through responsible consumers. If the trading cost is high, responsible and non-responsible investors acquire positions in all firms. These firms are incentivized to implement greener CSR policies because managers internalize shareholders' social preferences.

Next, we describe how firms' CSR policies and SRI change with the parameters of the model.

Lemma 3 *In the symmetric equilibrium of the game:*

1. *Firms' CSR policy (σ^*) and the fraction of shares held by $\mathcal{R}$ investors ($s^{\mathcal{R}*}$) are both continuous and increasing in the trading cost κ and the fraction χ_{in} of $\mathcal{R}$ investors in the economy;*
2. *The threshold $\widehat{\kappa}(\sigma_0, 0)$ is increasing in λ, and decreasing in χ_{in} and χ_{co}.*

In the symmetric equilibrium, CSR policies and SRI are increasing and continuous in κ and χ_{in}. As argued above, an increase in the trading cost κ leads to more SRI in equilibrium and thus provides greater incentives for firms to implement CSR policies. Similarly, a larger fraction χ_{in} of responsible investors increases the willingness of an individual responsible investor to trade (through strategic complementarity) and hence encourages firms to increase their CSR investments. Finally, when the negative externality λ becomes smaller, the wedge between $\mathcal{R}$ and $\mathcal{N}$ investors' valuations falls and the equilibrium with SRI exists for a larger set of parameters. A similar effect arises when the fraction of $\mathcal{R}$ agents in either the product or the ownership markets increases.

4.3 Asymmetric equilibria (SRI with tilting)

In addition to the symmetric equilibria characterized above, the game may also admit asymmetric equilibria. These equilibria feature tilting so that firms differ in their share of responsible investors and, hence, their CSR policies.

To characterize all asymmetric equilibria of the game, consider any subset of firms in which $\mathcal{R}$ investors purchase a positive amount of shares. As firms are ex-ante identical, it is without loss of

[12] Since we have a continuum of agents, the symmetric equilibrium is unique up to permutations of investors' portfolios and consumers' demands. Such permutations do not affect the aggregate equilibrium outcomes.

generality to denote these firms by $j = 1, \ldots, \overline{n}$, with $\overline{n} \leq N$. To ensure market clearing, $\mathcal{N}$ investors must hold the shares of the other firms instead. That is, for all $\overline{n} < N$, firms $j = \overline{n}+1, \ldots N$ are held exclusively by $\mathcal{N}$ investors. Finally, there may be firms where both $\mathcal{R}$ and $\mathcal{N}$ investors hold shares. Without loss of generality, let these firms be $j = \underline{n}+1, \ldots, \overline{n}$, with $\underline{n} \leq \overline{n}$, so that the particular cases $\underline{n} = 0$ and $\underline{n} = \overline{n}$ correspond to equilibria where $\mathcal{N}$ investors hold shares in all firms, and only in firms without $\mathcal{R}$ investors, respectively.

We can then characterize all equilibria of the game by a pair $(\underline{n}, \overline{n})$, with $0 \leq \underline{n} \leq \overline{n} \leq N$,[13] such that (a) only $\mathcal{R}$ investors hold shares in firms $j \leq \underline{n}$; (b) both $\mathcal{R}$ and $\mathcal{N}$ investors hold shares in firms $j \in (\underline{n}, \overline{n}]$; (c) only $\mathcal{N}$ investors hold shares in firms $j \in (\overline{n}, N]$. The following Proposition characterizes existence conditions for asymmetric equilibria for any given pair $(\underline{n}, \overline{n})$.

Proposition 2 (Asymmetric Equilibria) *All asymmetric equilibria feature* ***SRI with tilting****. In these equilibria, $\mathcal{R}$ investors hold shares in firms $j \leq \overline{n}$ and $\mathcal{N}$ investors hold shares in firms $j > \underline{n}$, where $\overline{n} \geq \underline{n}$, and firm j's CSR policy is:*

$$\sigma_j = \begin{cases} \overline{\sigma} & \text{for } j \leq \underline{n} \\ \widehat{\sigma} & \text{for } \underline{n} < j \leq \overline{n} \\ \underline{\sigma} & \text{for } j > \overline{n}, \end{cases} \tag{17}$$

where $\overline{\sigma} > \widehat{\sigma} > \underline{\sigma}$. For any pair $(\underline{n}, \overline{n})$, these equilibria exist if and only if:

(i) For $\overline{n} = \underline{n} = n$, $\chi_{in} > \frac{n}{N}$ and $\widehat{\kappa}(\overline{\sigma}, n) < \kappa < \widehat{\kappa}(\underline{\sigma}, n)$.

(ii) For $\overline{n} > \underline{n}$, $\frac{\underline{n}}{N} < \chi_{in} \leq \frac{\overline{n}}{N}$ and $\kappa > \widehat{\kappa}(\widehat{\sigma}, \underline{n})$, or $\chi_{in} > \frac{\overline{n}}{N}$ and $\widehat{\kappa}(\widehat{\sigma}, \underline{n}) < \kappa < \widehat{\kappa}(\widehat{\sigma}, \overline{n})$.

The threshold values for the trading cost are defined in Equation (12), and the expressions for $\overline{\sigma}$, $\underline{\sigma}$, and $\widehat{\sigma}$ are described in Appendix B.5.

The equilibria described in Proposition 2 highlight a second source of strategic complementarity among $\mathcal{R}$ investors: complementarity in the choice of firms that are part of their portfolios.

[13] We rule out $(\underline{n}, \overline{n}) = \{(0,0), (0,N)\}$ as these correspond to the symmetric equilibria described in Proposition 1. Note also that there cannot be equilibria with $\underline{n} = \overline{n} = N$ as $\mathcal{N}$ investors are never crowded out from the market. All other inequalities can hold with equality.

To build intuition, consider two firms, j and $-j$, and start from an equilibrium in which $\mathcal{R}$ investors hold α% of the shares in both firms. Now consider switching some of the $\mathcal{R}$ investors' shares in firm $-j$ with those of $\mathcal{N}$ investors in firm j, so that now $\mathcal{R}$ investors hold more than α% of j's shares and less than α% in $-j$. Due to the complementarity in their effects on firms' CSR policies, the swap makes $\mathcal{R}$ investors better off: the increase in $s_j^{\mathcal{R}}$ spurs more green investments by firm j, reducing the investors' expected disutility from holding its shares. By contrast, due to the substitutability in CSR investments, firm $-j$ now has lower incentives to invest in CSR: its green investments decrease after the swap so that $-j$ becomes less attractive for $\mathcal{R}$ investors. The interaction of these two effects generates a self-reinforcing mechanism, such that more and more $\mathcal{R}$ investors want to move from firm $-j$ to j if they expect others to do so.

It is worth noticing that the argument described above does not involve any increase in the total demand for shares for either firm. It solely involves a reallocation of shares, i.e., which type of investor trades which firm. So the strategic substitutability in investors' demands that we discussed in the context of symmetric equilibria does not play a role here. Put differently, while the complementarity in the number of shares $\mathcal{R}$ investors hold in a given firm is offset by the strategic substitutability in their demands, the complementarity in the choice of which firms to hold is not. Instead, this complementarity is *reinforced* by the substitutability in firms' CSR policies, which further strengthens the incentives of $\mathcal{R}$ to concentrate in a subset of firms and leads to dispersion in CSR policies across firms.

The feedback loop between investors' portfolio choices and firms' CSR policies introduces two sources of non-fundamental variation in equilibrium outcomes. First, it leads to an abundance of potential asymmetric equilibria, which differ in their degree of concentration of responsible capital and dispersion of CSR policies. Such equilibria may coexist for a given set of parameters so that industries with similar fundamentals may have significant differences in equilibrium ownership structures and CSR investments. Second, the feedback loop may lead to jumps in equilibrium outcomes: small parameter changes can trigger a leap from the symmetric equilibrium without SRI to an asymmetric equilibrium, causing a sharp change in ownership and CSR policies.[14]

[14]This may occur, for example, when $\chi > n/N$ and κ crosses the threshold $\widehat{\kappa}(\overline{\sigma}, n)$ from the left so that an asymmetric equilibrium with

5 Implications

In this section, we derive the central model implications. First, we describe how green capital tends to concentrate in a subset of firms in equilibrium. We then derive the consequences of this concentration for firms' CSR policies and welfare. Finally, we describe existing empirical evidence and novel testable implications surrounding our results.

5.1 Comparative statics

Section 4 characterizes two types of equilibria: symmetric and asymmetric. The following proposition compares the existence conditions for these two types of equilibria.

Proposition 3 *Suppose $\frac{N-1}{N} > \chi > \frac{c}{N\eta\lambda}$. The following results hold:*

1. *If $\widehat{\kappa}(\sigma_0, 0) > \kappa > \widehat{\kappa}(\overline{\sigma}, 1)$, the only equilibria with SRI are asymmetric.*

2. *If $\kappa > \widehat{\kappa}(\sigma_0, 0)$, the symmetric equilibrium with SRI coexists with asymmetric equilibria.*

The thresholds $\widehat{\kappa}(\sigma_0, 0)$ and $\widehat{\kappa}(\overline{\sigma}, 1)$ are described in Proposition 1 and 2, respectively.

When the number of firms N is sufficiently large, the condition $\frac{N-1}{N} > \chi > \frac{c}{N\eta\lambda}$ becomes less stringent. As a result, the existence conditions for each equilibrium depend solely on the value of the trading cost κ if N is sufficiently large. In this case, $\mathcal{R}$ investors are more *likely* to participate in the financial market when the equilibrium is asymmetric. By concentrating on a subset of firms, these investors manage to have a larger impact on their CSR policies. The larger impact reduces the valuation gap between the two types of investors relative to the symmetric equilibrium, making it harder for $\mathcal{N}$ investors to crowd out $\mathcal{R}$ investors. Therefore, $\mathcal{R}$ investors hold shares of firms for a larger set of parameters in asymmetric equilibria.

Notice also that investors participate in the financial market only when their expected payoff is positive. Therefore, each $\mathcal{R}$ investor prefers any asymmetric equilibrium over the symmetric equilibrium without SRI. Moreover, while the symmetric equilibrium is always unique, the number of possible asymmetric equilibria steeply increases with N. In this sense, the characterization of

$\underline{n} = \overline{n} = n$ arises.

the equilibria suggests that $\mathcal{R}$ investors may tend to concentrate on a subset of firms, especially in relatively large or more competitive industries.

The following proposition explores the implications of increased concentration on two key equilibrium outcomes: the dispersion in CSR policies and firms' market power.

Proposition 4 *Consider the asymmetric equilibria in which $\mathcal{R}$ investors hold firms $j \leq n$ and $\mathcal{N}$ investors hold firms $j > n$, with $n \in \mathcal{J}$, and suppose $c > \underline{c}$. The following statements hold:*

1. *These equilibria admit a unique collection of CSR policies, $\sigma_j = \overline{\sigma}(n)$ for all $j \leq n$ and $\sigma_j = \underline{\sigma}(n)$ for all $j > n$, where $\overline{\sigma}(n) > \underline{\sigma}(n)$.*
2. *Suppose that the equilibrium changes from $n = n''$ to $n = n'$, with $n', n'' \in \mathcal{J}$ and $n' < n''$ (e.g., κ decreases). In the equilibrium where green capital is more concentrated, that is, where n is smaller:*
   (a) The dispersion in CSR policies is larger, i.e., $\overline{\sigma}(n') - \underline{\sigma}(n') > \overline{\sigma}(n'') - \underline{\sigma}(n'')$.
   (b) Firms held by $\mathcal{R}$ investors charge higher prices in expectation.
   (c) Conditional on at least one firm offering a green product (i.e., $a_j = 1$ for some $j \in \mathcal{J}$), the expected price of green products may be higher or lower when n is larger.

The threshold value $\underline{c}$ for the green investment cost is described in Appendix B.7.

To simplify the analysis, Proposition 4 focuses on the most extreme type of asymmetric equilibria, in which responsible and non-responsible investors concentrate in disjoint subsets of firms. In particular, $\mathcal{R}$ investors concentrate in a group of $n \leq N$ firms and exclude all others. We can therefore interpret n as an inverse measure of the concentration of responsible capital in equilibrium.[15] The trading cost κ determines the existence of these types of equilibria (equilibria with higher n exist only for higher values of κ), but has no direct effect on the equilibrium CSR policies. Therefore, we can think of a change in κ as only changing the equilibrium through its effect on n.

A higher concentration of green capital generates both dispersion in CSR policies and concentration in the market for green products. Suppose green capital becomes more concentrated

[15]In general, we may have multiple equilibrium CSR policies for a given value of n. Proposition 4 focuses on values of c such that the equilibrium CSR policies are unique for all n, so we do not need to worry about equilibrium selection when we compare the equilibrium outcomes for different values of n.

because the economy moves from an equilibrium with $n = n' + 1$ to one with $n = n'$. Since firm $j = n' + 1$ loses its $\mathcal{R}$ investors, this firm invests less in CSR. The reduction in $\sigma_{n'+1}$ potentially motivates other firms to invest more in CSR since each of them faces a higher chance of being the only green firm in the market (in which case, it can price above the marginal cost and make positive profits). Importantly, firms still targeted by $\mathcal{R}$ investors (i.e., $j \leq n'$) react more to this change. They have higher CSR investments and, thus, a higher chance of having monopoly power for the green product. Hence, they benefit more from the reduction in $\sigma_{n'+1}$. While $\underline{\sigma}$ may increase or decrease, the difference $\overline{\sigma} - \underline{\sigma}$, i.e., the dispersion in CSR policies always increases when green capital is more concentrated (i.e., n is lower).

The expected price of green products may increase or decrease with the concentration of green capital. This price depends on the aggregate probability that only one firm offers this product, which is a non-monotonic function of the differences in CSR policies. So, even though the increase in concentration leads to a reallocation of market power in favor of the firms held by $\mathcal{R}$ investors, who charge higher expected prices, it has an ambiguous effect on the overall competitiveness in the economy.

5.2 Welfare analysis

This section explores the welfare implications of our analysis. We begin by describing the economy's total surplus when investors' portfolio choices, firms' production decisions, and consumers' consumption levels are at their equilibrium values.

We can write the expected total surplus S as follows:

$$S = \int_0^1 \mathbb{E}\left[u(\iota'\vec{x}_h) - \iota'\vec{x}_h\gamma\right] dh - \frac{\kappa}{2}\int_0^1 \left(\iota'\vec{s}_i\right)^2 di - \sum_{j=1}^{N}\left[\lambda(1-\sigma_j) + \frac{c}{2}\sigma_j^2\right], \tag{18}$$

where $\vec{x}_h \equiv \{x_{h1}(\vec{a}, \vec{\rho}), \ldots, x_{hN}(\vec{a}, \vec{\rho})\}$ represents the vector of consumer h's consumption for a given realization of product types $\vec{a}$ and prices $\vec{\rho}$.

Since prices (for products and shares) in the model are transfers, they do not affect the total surplus. Our welfare measure includes the negative externality generated by firms but, to avoid double counting, excludes the expected disutilities incurred by the responsible agents (investors and consumers). It follows that S comprises three main components. First, the expected surplus in the product market, which is the difference between the sum of consumers' utilities and the firms' production costs. Second, the investors' total trading cost. Third, the sum of the expected negative externality and the green investment cost.

In equilibrium, firms' CSR investments may exceed or fall short of the socially optimal level. On the one hand, firms do not fully internalize the negative externality. This feature of the model pushes towards underinvestment. On the other hand, their private incentives are partly driven by the prospect of charging high markups to responsible consumers. This second feature might lead to overinvestment since these rents are not part of the social planner's objective.

SRI has a direct and an indirect effect on total surplus S. The direct effect represents the contribution of $\mathcal{R}$ investors to the total trading costs. The indirect effect operates through the impact of SRI on firms' CSR policies $\vec{\sigma}$, which in turn affect the other two components of S. When firms underinvest in CSR, then $\mathcal{R}$ investors can potentially improve welfare by moving firms' CSR policies closer to the social optimum. To highlight the most interesting results of our model, we focus on this case in the following proposition.

Proposition 5 *Suppose $\lambda > c$ so that firms' CSR investments in equilibrium fall short of the socially optimal level. Suppose the equilibrium without SRI coexists with an asymmetric equilibrium:*

1. *Welfare and aggregate greenness may be higher in the equilibrium without SRI.*

2. *$\mathcal{R}$ investors may be better off in the equilibrium where welfare and aggregate greenness are lower.*

Responsible investors boost the greenness of the firms in their portfolios but also crowd out the CSR investments of excluded firms. We show that the overall effect on aggregate CSR investments may be negative in equilibria where green capital is particularly concentrated, and the crowding-out effect is strong. The economy's aggregate greenness is then larger in the equilibrium in which $\mathcal{R}$ investors do not hold any firm. The concentration of $\mathcal{R}$ investors creates dispersion in CSR policies (Proposition 4), even when it does not reduce aggregate greenness. The dispersion generates market power and may lead to higher prices for green products, reducing the expected surplus in this market.

The two negative externalities described above hamper the positive effect of SRI and may lead to higher welfare in settings where $\mathcal{R}$ investors do not hold any firm. $\mathcal{R}$ investors do not consider the negative externalities of their investments to the broader economy but solely the expected externality of the firms in their portfolio. They are better off when participating in the financial market, even though welfare is lower in this equilibrium.

5.3 Empirical implications

The main premise of our model is that firms' CSR policies reflect pressure from investors and consumers. Since these two sources of incentives interact, frameworks that consider them in isolation may fail to capture the full economic implications of SRI. A growing empirical literature finds that shareholders' social preferences shape their governance and firms' ESG policies (Flammer 2015; Kim, Wan, Wang, and Yang 2019; Bolton, Li, Ravina, and Rosenthal 2020; Naaraayanan, Sachdeva, and Sharma 2021). Moreover, there is evidence that boycotts from both consumers and prospective employees affect firms' profits and stock prices (Davidson, Worrell, and El-Jelly 1995; Ashenfelter, Ciccarella, and Shatz 2007; Hacamo 2022), and, more broadly, that consumers incorporate firms' CSR into their shopping habits (Mazar and Zhong 2010; Albuquerque et al. 2020).

The complementarity in responsible investors' portfolio choices is the central economic mechanism in our framework. Figure 1 provides suggestive empirical evidence for this mechanism, showing that green funds are more concentrated than non-green funds in 20 out of 21 industry groups. Starks, Venkat, and Zhu (2017) find evidence that investors with longer horizons have preferences for, and tend to group in, firms with high ESG scores. Similarly, Dimson, Karakaş, and Li (2021) document the prevalence of *coordinated* engagements: groups of institutional investors who cooperate in promoting CSR issues.[16] These findings suggest that complementarity among responsible investors indeed plays a role in the data.

A specific test of the strategic complementarity would require an exogenous shock to the proportion of green investors at the firm level: The complementarity works as a multiplier of

[16] Examples of such networks of investors include the Climate Action 100+ campaign, which is backed by 518 global investors with 47 trillion in assets, and the Coalition for Inclusive Capitalism, with 31 organizations representing over 30 trillion in assets

exogenous shocks to green capital so that a $y\%$ exogenous increase in green capital leads to an $m * y\%$ increase in equilibrium, where $m > 1$ represents the multiplier. We are not aware of any such shock in empirical work on ESG. An increasing number of papers use shocks to institutional investors' holdings — e.g., index inclusions and reconstitutions (Antón, Ederer, Giné, and Schmalz, 2022) and consolidations in the asset management industry (Azar, Schmalz, and Tecu, 2018; He, Huang, and Zhao, 2019) — to investigate other outcomes. It would be interesting to use a similar type of shock to investigate the complementarity among responsible investors.

The second main driver of our results is the strategic substitutability in firms' CSR policies. The underlying rationale is that increased competition for socially motivated consumers reduces an individual firm's return from CSR investments. If we interpret CSR as investments in "clean" innovation, the mechanism described above is consistent with the findings in Aghion, Bloom, Blundell, Griffith, and Howitt (2005), who show that competition reduces innovation in industries where it affects mostly post-innovation rents. Starks, Venkat, and Zhu (2017) use shocks to firms' reputation to investigate investors' preferences for CSR. The same shocks may be used to test the strategic substitutability in the broader context of firms' ESG scores. We expect that a negative shock to firm j's reputation/ESG scores will lead to higher ESG scores for its direct competitors. It is worth mentioning that Cao, Liang, and Zhan (2019) find evidence of peer-pressure effects in CSR proposals, which is consistent instead with strategic complementarity.

Finally, our model links the concentration of green capital to dispersion in firms' CSR policies and product market position. We predict that increased concentration leads to a larger dispersion of CSR policies and a reallocation of market shares in favor of the firms targeted by green investors. Figure 2 and Table 1 suggest a positive correlation between the concentration of green capital and firms' ESG scores. Importantly, the causality runs in both directions in our model because increased dispersion in CSR policies also incentivizes $\mathcal{R}$ investors to tilt their portfolio towards firms with greener policies. Therefore, similar to before, a test of the causal effect of concentration on the equilibrium outcomes would require an exogenous shock to green capital.

6 Extensions and robustness

This section briefly discusses the robustness of our results to alternative assumptions. A complete analysis of each variation of the baseline model is in the Online Appendix.

6.1 Product market

Horizontal differentiation. In our baseline model, we assume that, aside from the disutility of consuming brown products for $\mathcal{R}$ consumers, the products offered by the firms are perfect substitutes. We relax this assumption in Online Appendix C.1, where we consider a model with two firms and horizontally differentiated products à la Hotelling.[17] First, we prove that our main results carry through in this extension. Second, we show that this additional (exogenous) dimension of differentiation across firms amplifies SRI's negative externality on consumer surplus.

The intuition for this second result is as follows. With horizontally differentiated products, a brown firm also prices above marginal costs and charges a higher price if its rival is green. In this case, the green firm is a monopolist in the segment of $\mathcal{R}$ consumers and charges a higher price to profit from them. Competition for brown consumers is then less fierce, and the brown firm can also charge a higher price. The concentration of SRI increases the dispersion in CSR policies, making it more likely that firms offer different products. As a result, SRI raises the expected price of both types of products, generating a larger negative externality than in the baseline model.

Heterogeneous marginal costs. In the baseline model, we assume that firms' marginal cost (γ) is invariant with respect to the production technology, so the only extra cost of being green is the investment cost incurred to develop the green technology. In Online Appendix C.2, we consider a variation of the model where green products are more expensive to produce. The main difference with the baseline model is that now a firm also makes profits when it is the only one to offer a brown product (as it can price slightly below the marginal cost of the green firms and serve $\mathcal{N}$ consumers at a profit). This feature of the model strengthens the strategic substitutability: when

[17] Unlike in our base model, with horizontally differentiated products, the market competition game admits a pure-strategy equilibrium with $N = 2$ firms.

σ_{-j} increases, the probability that firm j ends up being a brown monopolist goes up, further reducing the incentives to invest in CSR. As a result, the crowding-out effect of the concentration of green capital on the CSR policies of excluded firms is stronger.

Robustness of strategic substitutability in CSR investments. A driving force for our main results is that firms' CSR policies are strategic substitutes. The baseline model captures this channel in a simple framework featuring perfect Bertrand competition. In Online Appendix C.3, we show that the strategic substitutability generalizes to a broad range of settings. A sufficient condition for CSR policies to be strategic substitutes is that the (average) extra profit that any firm obtains from being green decreases in the number of green rivals. This condition is satisfied in many models of competition where the products are vertically differentiated along the CSR dimension, including the Hotelling model, which we analyze in Online Appendix C.1.

6.2 Modeling of externalities

Proportional externality. Our main model considers the case where the negative externality generated by brown firms does not depend on how much these firms produce. In Online Appendix D.1, we explore an extension of the model where this externality is proportional to output (e.g., producing each unit generates a fixed amount of pollution). In equilibrium, each brown firm's output depends on how many other firms are brown and sell to nonresponsible consumers. Since the probability that each firm is brown depends on its CSR investment, the expected externality generated by each firm thus also depends on the other firms' CSR policies. Given that $\mathcal{R}$ investors internalize such externalities, the equilibrium analysis is more cumbersome than in the main model. However, our main qualitative results carry through in this extension.

Curtailment of public bads vs. provision of public goods. In our main model, the social role of CSR investments is the curtailment of public bads (e.g., pollution or a negative corporate culture): firms have access to a brown technology that generates a negative externality, but they can eliminate such externality by developing a green technology. Since each firm is brown with a positive probability in equilibrium, $\mathcal{R}$ investors have lower valuations than $\mathcal{N}$ investors, so they are relatively less likely to participate in the financial market (they are sometimes crowded out by $\mathcal{N}$ investors).

In Online Appendix D.2, we consider a variation of the model where CSR investments represent public goods (e.g., donations to charity or employees' satisfaction): the status-quo technology generates no externality, but implementing the new technology brings about a positive externality, which is internalized by responsible agents. Our main insights (that is, the concentration of green capital and the negative externalities of SRI) continue to hold in such a setting. However, now $\mathcal{R}$ investors have higher valuations than $\mathcal{N}$ investors, so $\mathcal{R}$ investors are more likely to trade than $\mathcal{N}$ investors. Hence, whether CSR is modeled as curtailment of public bads or provision of public goods determines the relative propensity of $\mathcal{R}$ investors to participate in the financial market.

6.3 Financial market

Reverse timing. In the main model, firms choose their CSR policies after investors trade their shares. In Online Appendix E, we examine the robustness of our results in a setting where this timing is reversed: firms choose their CSR policies to attract investors and maximize their stock prices. This setting represents contexts where firms have a credible way to commit to certain CSR policies.[18] Before trading takes place, each firm j commits to a CSR investment σ_j. Investors observe firms' CSR policies and submit their demands for shares. The game is akin to competitive screening games (e.g., Rothschild and Stiglitz, 1976) since investors have heterogeneous values for CSR policies, and firms can use their CSR policies to target a specific group of investors.

We show that, although the firms are ex-ante identical, they may select different CSR policies in equilibrium: some firms invest more in CSR to attract responsible investors, while others invest less and focus on non-responsible investors. This differentiation makes it harder for each firm to deviate and attract both types of investors since each equilibrium policy is tailored to the preferences of a specific type of investor. Our main insights then carry through since the presence of SRI causes firms to differentiate their CSR policies.

[18]See Albuquerque et al. (2019) for a discussion of the evidence surrounding firms' CSR commitments.

Discussion of socially responsible preferences. In our model, responsible investors (consumers) internalize the pollution externalities produced by the firms they hold (buy from). This way of modeling socially responsible agents is consistent with traditional explanations of individuals' demand for CSR (e.g., warm-glow giving (Andreoni 1990); and image concerns (Bénabou and Tirole 2010)). The specification of $\mathcal{R}$ investors' preferences embodies two main assumptions.

First, $\mathcal{R}$ investors do not internalize consumers' utility, so they do not advocate for lower product prices. This assumption is consistent with the observation that, in practice, firms' pricing strategies do not enter their ESG scores. Relaxing this assumption (similar to Magill, Quinzii, and Rochet (2015)) is unlikely to change our main results. Selling green products would still be less profitable in a more crowded market, so the strategic substitutability in CSR investments carries through. Moreover, the objective of lowering product prices is likely to strengthen the strategic complementarity in $\mathcal{R}$ investors' portfolio choices. Second, $\mathcal{R}$ investors do not internalize the aggregate effects of their choices (consistent with each investor being relatively small compared to the economy), which leads to the concentration of green capital and the crowding out of the CSR investments of the excluded firms. This assumption differs from models featuring large investors (or coalitions of small investors) that internalize firms' pollution independently of their ownership (Oehmke and Opp (2020); Gupta, Kopytov, and Starmans (2022)).

7 Conclusion

This paper has analyzed the equilibrium consequences of socially responsible investments (SRI). Responsible investors consider the externalities firms create in their portfolios and, thus, tend to invest in firms with greener CSR policies. A greater fraction of responsible investors, in turn, incentivizes firm management to implement greener policies, generating a feedback loop between SRI and CSR policies. We have shown that, in equilibrium, SRI tends to be concentrated in a small subset of firms, leading to unintended spillovers to firms that are primarily held by profit-motivated investors. This concentration generates differentiation and market power in the market for green products. Furthermore, it crowds out CSR investments of excluded firms. If this effect is particularly strong, then the presence of SRI *decreases* aggregate CSR investments and social welfare.

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A Data Appendix

A.1 Data Construction and Definition

We obtain mutual funds' stock holding information from Thomson/Refinitiv S12 (S12 hereafter). Since S12 does not have an indicator for ESG funds, following Dikolli et al. (2022), we rely on Morningstar's classification to identify ESG funds in S12 data (https://www.morningstar.com/esg-screener). We classify a fund as an ESG fund if Morningstar states that the fund's management identifies the fund as sustainability-focused in public filings ("Sustainable Investment by Prospectus"). Otherwise, we classify the fund as a non-ESG fund.

We match S12 data and Morningstar data using fund tickers. Since tickers can be reused, we manually compare fund names in S12 and Morningstar and verify the match if it can be inferred from the fund names that the same sponsor manages two funds. Throughout this process, we identify 82 ESG funds in S12 data. We define $GreenCapital_{it}$ as the aggregated value of stock holdings in firm i at year t by ESG funds. Similarly, $Non-GreenCapital_{it}$ is defined as the aggregated stock holdings in firm i at year t by non-ESG funds.

We obtain Refinitiv's ESG Combined Scores for US firms listed in NYES and NASDAQ during 2002-2020. The list of NYSE/NASDAQ-listed stocks and stock prices is obtained from CRSP.

A.2 Figures and Tables

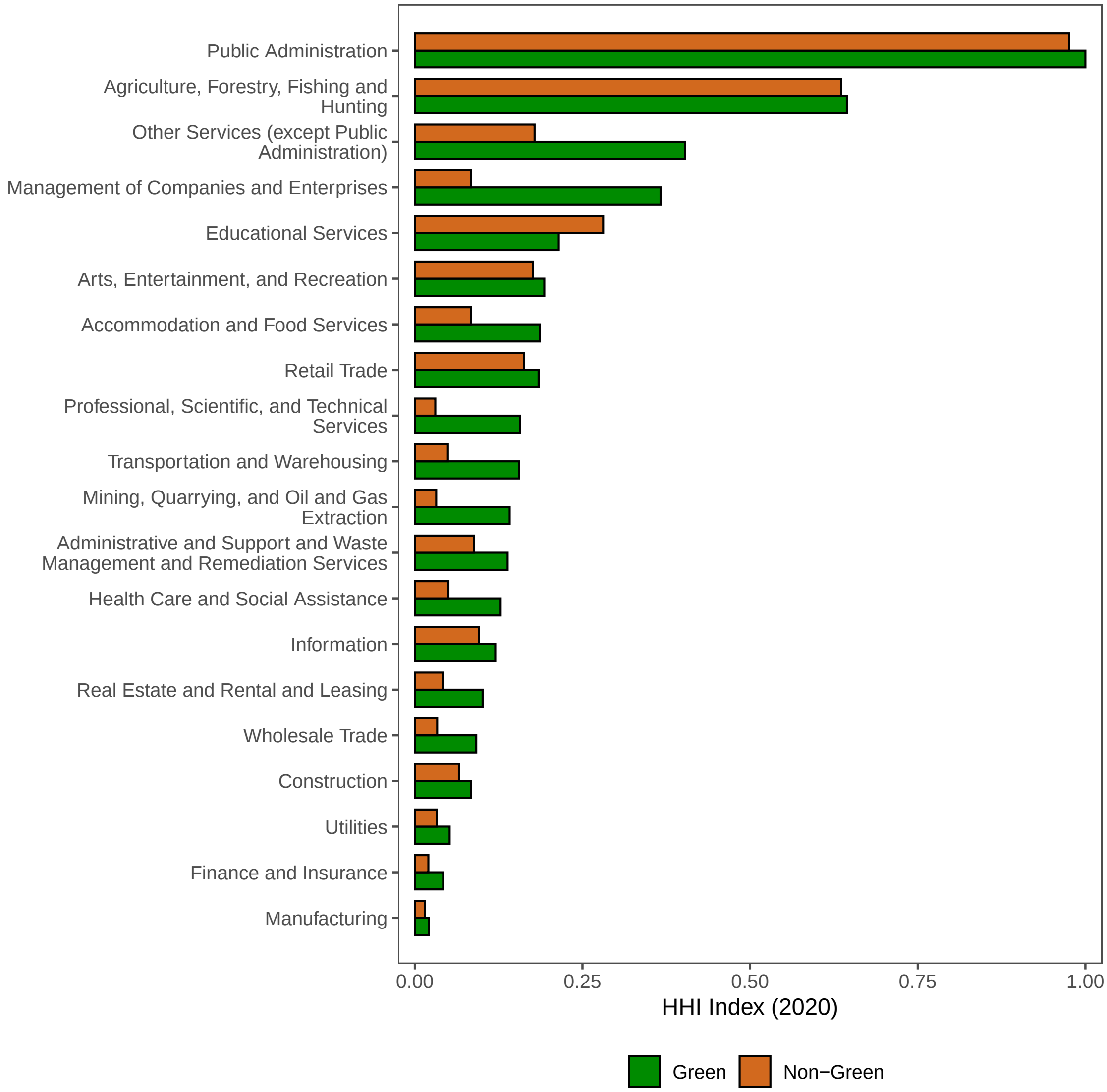
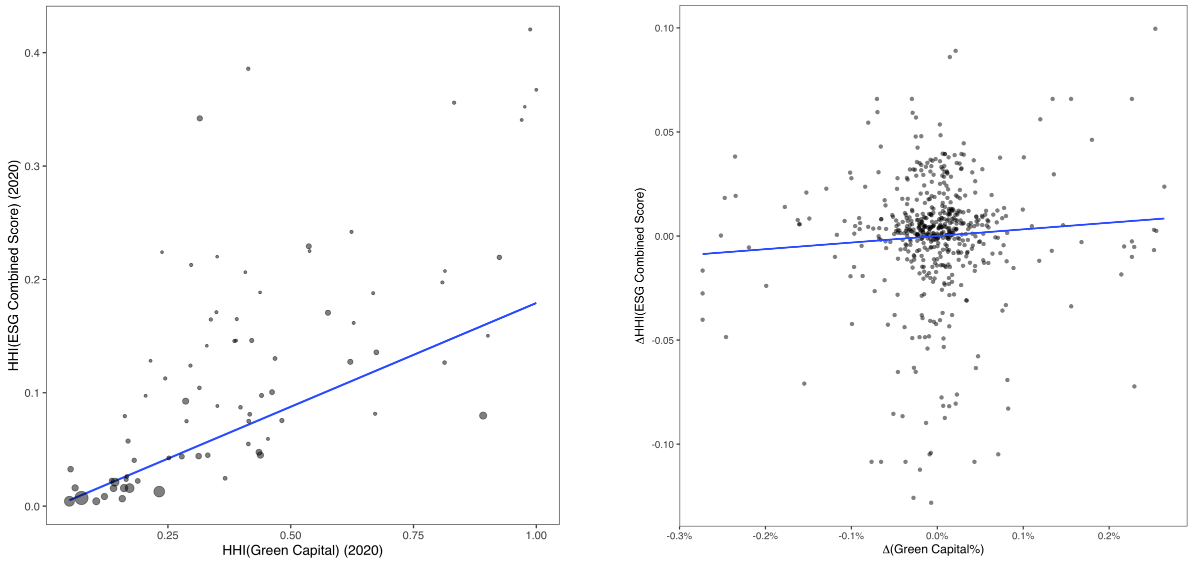


Figure 1: This figure plots HHI indexes of Green and Non-Green Capital at the industry level for NYSE/NASDAQ-listed stocks as of 2020. Industries are defined by NAICS 2-digit codes. NYSE/NASDAQ-listed stocks are from CRSP.



(a) Cross-sectional.

(b) Time series.

Figure 2: This figure summarizes the relationship between the concentration of Green Capital and ESG Combined Scores at the industry level. Panel (a) shows HHI indexes of Green Capital and ESG Combined Score at the industry level for NYSE/NASDAQ-listed stocks as of 2020. The size of the circles denotes the amount of green capital in the industry. Panel (b) shows changes in Green Capital (scaled by industry market capitalization) and changes in HHI indexes of ESG Combined Score at the industry level for NYSE/NASDAQ-listed stocks during the period 2012-2020. Both changes are demeaned by year. Industries are defined by NAICS 3-digit codes. All variables are winsorized at 1 percent. The blue lines denote a fitted line from a WLS regression of the y-axis on the x-axis.

	$\Delta HHI(ESG\ Combined\ Score)\ (t)$				$\Delta Green\ Capital\%\ (t)$	
	(1)	(2)	(3)	(4)	(5)	(6)
$\Delta Green\ Capital\%\ (t)$	6.495***	3.181**				
	(2.284)	(1.439)				
$\Delta Green\ Capital\%\ (t-1)$			1.950	2.938		
			(2.746)	(2.697)		
$\Delta HHI(ESG\ Combined\ Score)\ (t-1)$					0.001	0.000
					(0.001)	(0.001)
Constant	−0.012***		−0.013***		0.000	
	(0.001)		(0.001)		(0.000)	
Year Fixed Effects		Y		Y		Y
Observations	545	545	488	488	468	468
Adjusted R^2	0.014	0.282	−0.001	0.277	−0.001	0.027

Table 1: This table examines changes in green capital (scaled by industry market capitalization) and changes in HHI indexes of ESG Combined Score. Columns (1)-(4) use changes in HHI indexes of ESG Combined Score as the dependent variable. In columns (5)-(6), the dependent variable is a change in green capital. (t) denotes a variable is concurrent, while (t-1) denotes a variable is one year lagged. All variables are winsorized at 1 percent. Standard errors are clustered by industry and reported in parentheses. *, **, and *** indicate statistical significance at the 10%, 5%, and 1%, respectively.

B Proofs

B.1 Proof of Lemma 1.

This proof is given in the main text.

B.2 Proof of Lemma 2.

As a first step, we establish equilibrium existence. For all $j \in \mathcal{J}$, define

$$f_j(\vec{\sigma}) \equiv \frac{1}{c}\left[\eta\lambda s_j^{\mathcal{R}} + \pi^m \prod_{-j\neq j}(1-\sigma_{-j})\right], \tag{B.1}$$

with $s_j^{\mathcal{R}} \in [0,1]$ for all j. Given the parametric assumption given in the main text, $f_j : [0,1]^N \to [0,1]$ is a continuous function. Hence, the vector-valued function

$$F(\vec{\sigma}) \equiv (f_j(\vec{\sigma}))_{j=1,\dots,N} \tag{B.2}$$

is a continuous function mapping $[0,1]^N$, a non-empty, compact and convex subset of $\mathbb{R}^N$, into itself. By Brouwer's fixed point theorem, $F(\cdot)$ admits at least one fixed point — i.e., vectors $\vec{\sigma}^*$ such that $\vec{\sigma}^* = F(\vec{\sigma}^*)$. The definition of $F(\cdot)$ implies that fixed points correspond to the Nash equilibria of the investment game.

We next prove results (i)-(iii):

(i) Suppose $s_j^{\mathcal{R}} \equiv s^{\mathcal{R}} \in [0,1]$ for all $j \in \mathcal{J}$. Then, suppose by contradiction that the game admits an asymmetric equilibrium: without loss of generality, say that $\sigma_1 \neq \sigma_2$. Then, from the FOC (7) for firm 1 we obtain

$$\eta s^{\mathcal{R}} = c\sigma_1 - \pi^m(1-\sigma_2)\prod_{k\neq 1,2}(1-\sigma_k). \tag{B.3}$$

Substituting this into the FOC of firm 2 and rearranging gives

$$1 = \underbrace{\frac{\pi^m}{c}}_{<1}\underbrace{\prod_{k\neq 1,2}(1-\sigma_k)}_{\leq 1}, \tag{B.4}$$

which can never hold given the assumption that $c > \eta\lambda + \pi^m$. This establishes that any equilibrium must be symmetric. Imposing symmetry (i.e., $\sigma_j = \sigma^*$ for all $j \in \mathcal{J}$) and rearranging, the system of FOCs (7) gives

$$c\sigma^* - \pi^m(1-\sigma^*)^{N-1} = \eta s^{\mathcal{R}}. \tag{B.5}$$

The LHS of this equation is increasing in σ^*, whereas the RHS is a constant (w.r.t. σ^*) in $[0,\eta]$. For $\sigma^* = 0$, the LHS equals $-\pi < \eta s^{\mathcal{R}}$; for $\sigma^* = 1$, the LHS equals $c > \eta s^{\mathcal{R}}$. Taken together, these results imply the existence of a unique symmetric equilibrium $\sigma^* \in (0,1)$, which is increasing in $\eta s^{\mathcal{R}}$.

(ii) For any vector $\vec{s}^{\mathcal{R}}$, the FOCs (7) of any two firms j and j' can be written as

$$\sigma_j = \frac{1}{c}\left[\eta\lambda s_j^{\mathcal{R}} + \pi^m(1-\sigma_{j'})\mathcal{Y}\right] \tag{B.6}$$

$$\sigma_{j'} = \frac{1}{c}\left[\eta\lambda s_{j'}^{\mathcal{R}} + \pi^m(1-\sigma_j)\mathcal{Y}\right] \tag{B.7}$$

with

$$\mathcal{Y} \equiv \prod_{y\neq j,j'}(1-\sigma_y). \tag{B.8}$$

For any given $\mathcal{Y}$ — i.e., holding fixed all other firms' strategies — solving this system gives

$$\sigma_j = \frac{c(\pi^m\mathcal{Y} + \eta\lambda s_j^{\mathcal{R}}) - \pi^m\mathcal{Y}(\pi^m\mathcal{Y} + \eta\lambda s_{j'}^{\mathcal{R}})}{c^2 - (\pi^m\mathcal{Y})^2} \tag{B.9}$$

$$\sigma_{j'} = \frac{c(\pi^m\mathcal{Y} + \eta\lambda s_{j'}^{\mathcal{R}}) - \pi^m\mathcal{Y}(\pi^m\mathcal{Y} + \eta\lambda s_j^{\mathcal{R}})}{c^2 - (\pi^m\mathcal{Y})^2} \tag{B.10}$$

from which it follows that

$$\sigma_j - \sigma_{j'} = \frac{\eta\lambda(s_j^{\mathcal{R}} - s_{j'}^{\mathcal{R}})}{c - \pi^m\mathcal{Y}} \geq 0 \Leftrightarrow s_j^{\mathcal{R}} \geq s_{j'}^{\mathcal{R}}, \tag{B.11}$$

where the inequality is strict if and only if $s_j^{\mathcal{R}} > s_{j'}^{\mathcal{R}}$.

(iii) Suppose $s_j^{\mathcal{R}} > 0$ only for firm $j \in \mathcal{J}$. From (ii), we know that any equilibrium is such that $\sigma_j^* \equiv \overline{\sigma}^* > \sigma_{-j}^* \equiv \underline{\sigma}^*$ for all $-j \neq j$. Hence, $(\overline{\sigma}^*, \underline{\sigma}^*)$ solve the system

$$\overline{\sigma}^* = \frac{1}{c}\left[\eta\lambda + \pi^m(1-\underline{\sigma}^*)^{N-1}\right] \tag{B.12}$$

$$\underline{\sigma}^* = \frac{\pi^m}{c}(1-\overline{\sigma}^*)(1-\underline{\sigma}^*)^{N-2} \tag{B.13}$$

Given that $\overline{\sigma}^* > \underline{\sigma}^*$, we have

$$\underline{\sigma}^* < \frac{\pi^m}{c}(1-\underline{\sigma}^*)^{N-1}, \tag{B.14}$$

Let σ_0 denote the CSR policy when $s_j^{\mathcal{R}} = 0$ for all $j \in \mathcal{J}$, which is obtained from

$$\sigma_0 = \frac{\pi^m}{c}(1-\sigma_0)^{N-1}. \tag{B.15}$$

Then, Equation (B.13) implies $\underline{\sigma}^* < \sigma_0$, which, in turn, implies $\overline{\sigma}^* > \sigma_0$.

Finally, we can show that the equilibrium is unique. To this end, note that Equation (B.13) can be written as

$$g(\underline{\sigma}^*) \equiv \frac{\underline{\sigma}^*}{(1-\underline{\sigma}^*)^{N-2}} = \frac{\pi^m}{c}(1-\overline{\sigma}^*) \Longrightarrow \underline{\sigma}^*(\underline{\sigma}) = g^{-1}\left(\frac{\pi^m}{c}(1-\overline{\sigma}^*)\right), \tag{B.16}$$

where the inverse $g^{-1}(\cdot)$ is well defined because $g(\cdot)$ is strictly increasing. Moreover, as $g(\cdot)$ is strictly convex,[19] in the $(\underline{\sigma}, \overline{\sigma})$-plane the best response function $\underline{\sigma}^*(\overline{\sigma})$ is decreasing and concave. From the Equation (B.12), the best response $\overline{\sigma}^*(\underline{\sigma})$ is a decreasing and convex function in the $(\underline{\sigma}, \overline{\sigma})$-plane.[20] Hence, as $\underline{\sigma}^*(1) = 0$, $\overline{\sigma}^*(0) < 1$, and $\underline{\sigma}^*(0) < 1$, $\overline{\sigma}^*(1) > 0$, these two best response functions intersect exactly once, with $\underline{\sigma}^*(\overline{\sigma})$ intersecting $\overline{\sigma}^*(\underline{\sigma})$ from above.

B.3 Proof of Proposition 1.

The proof that the symmetric equilibrium with no SRI exists for all $\kappa \leq \widehat{\kappa}(\sigma_0, 0)$ is given in the text.

In what follows, we derive the existence condition for the symmetric equilibrium with SRI. Substituting the market clearing price from Equation (11) for $\alpha_j^{\mathcal{R}} = \alpha_j^{\mathcal{N}} = 1/N$ into the individual demand of each $\mathcal{R}$ investor given in Equation (9), hereafter denoted by $s_{ij}^{\mathcal{R}}$, and rearranging, we have that any such symmetric equilibrium must satisfy

$$\Gamma(s_{ij}^{\mathcal{R}}) \equiv \kappa(N - s_{ij}^{\mathcal{R}}) - (1-\chi_{in})\lambda(1-\sigma^*(s_{ij}^{\mathcal{R}})) = 0, \tag{B.17}$$

[19]Indeed

$$\frac{\mathrm{d}^2 g(\cdot)}{\mathrm{d}\underline{\sigma}^2} = (N-2)(1-\underline{\sigma}^*)^{-N}(2+(N-3)\underline{\sigma}^*) > 0,$$

which, $g(\cdot)$ being strictly increasing, implies that its inverse is strictly increasing and concave.

[20]Indeed,

$$\frac{\mathrm{d}^2}{\mathrm{d}\underline{\sigma}^2}\left[\frac{1}{c}\left[\eta\lambda + \pi^m(1-\underline{\sigma}^*)^{N-1}\right]\right] = \frac{\pi^m}{c}(N-1)(N-2)(1-\underline{\sigma}^*)^{N-3} > 0.$$

where, imposing symmetry in Equation (7), $\sigma^*(s_{ij}^{\mathcal{R}})$ is implicitly defined from[21]

$$\eta\lambda\frac{\chi_{in}}{N}s_{ij}^{\mathcal{R}} + \pi^m(1-\sigma^*)^{N-1} - c\sigma^* = 0. \tag{B.18}$$

For the market clearing conditions $\frac{\chi_{in}}{N}s_{ij}^{\mathcal{R}} + \frac{1-\chi_{in}}{N}s_{ij}^{\mathcal{N}} = 1$ (with $s_{ij}^{\mathcal{N}}$ denoting individual shares demand of each $\mathcal{N}$ investor) to hold, it must be $s_{ij}^{\mathcal{R}} < \frac{N}{\chi_{in}}$, and $s_{ij}^{\mathcal{R}} \geq 0$.

We have

$$\Gamma\left(\frac{N}{\chi_{in}}\right) < 0, \tag{B.19}$$

and

$$\Gamma(0) < 0 \Leftrightarrow \kappa < \widehat{\kappa}(\sigma_0, 0), \tag{B.20}$$

given that $\sigma^*(0) = \sigma_0$ — i.e., $\Gamma(0) < 0$ if and only if there exists the symmetric equilibrium with no SRI.

Differentiating $\Gamma(s_{ij}^{\mathcal{R}})$ twice gives

$$\frac{d^2\Gamma(s_{ij}^{\mathcal{R}})}{d(s_{ij}^{\mathcal{R}})^2} = (1-\chi_{in})\lambda\frac{d^2\sigma^*(s_{ij}^{\mathcal{R}})}{d(s_{ij}^{\mathcal{R}})^2}, \tag{B.21}$$

where, from Equation (B.18), applying the implicit function theorem twice, we find

$$\frac{d^2\sigma^*(s_{ij}^{\mathcal{R}})}{d(s_{ij}^{\mathcal{R}})^2} = \frac{\eta^2\lambda^2\pi^m(N-2)(N-1)\chi_{in}^2(1-\sigma^*)^{N+1}}{N^2\left(c+\pi^m(N-1)(1-\sigma^*)^{N-2}\right)^3} > 0. \tag{B.22}$$

Hence, $\Gamma(s_{ij}^{\mathcal{R}})$ is strictly convex. Therefore, we have:

1. If $\Gamma(0) < 0$, given that $\Gamma(\frac{N}{\chi_{in}}) < 0$, $\Gamma(s_{ij}^{\mathcal{R}})$ being strictly convex implies $\Gamma(s_{ij}^{\mathcal{R}}) < 0$ for all $s_{ij}^{\mathcal{R}} \in [0, \frac{N}{\chi_{in}}]$: hence, for all $\kappa < \widehat{\kappa}(\sigma_0, 0)$, there is no symmetric equilibrium with SRI;
2. If $\Gamma(0) > 0$, given that $\Gamma(\frac{N}{\chi_{in}}) < 0$ and $\Gamma(s_{ij}^{\mathcal{R}})$ is strictly convex, $\Gamma(s_{ij}^{\mathcal{R}}) = 0$ admits a unique solution $s_{ij}^{\mathcal{R}*}$, which, from Lemma 2 (i), implies that there is a unique σ^*: hence, for all $\kappa > \widehat{\kappa}(\sigma_0, 0)$ there exists a unique symmetric equilibrium with SRI.

We can thus conclude that a symmetric equilibrium featuring SRI without tilting exists if and only if $\kappa > \widehat{\kappa}(\sigma_0, 0)$, and that this equilibrium is unique.

[21] Note that, by symmetry across $\mathcal{R}$ investors and given that $\alpha_j^{\mathcal{R}} = 1/N$ for all j: $s_j^{\mathcal{R}} = \frac{\chi_{in}}{N}s_{ij}^{\mathcal{R}}$.

B.4 Proof of Lemma 3.

We have shown in Proposition 1 that for $\kappa \leq \widehat{\kappa}(\sigma_0, 0)$, $s_j^{\mathcal{R}} = 0$ and $\sigma_j = \sigma_0$. For $\kappa > \widehat{\kappa}(\sigma_0, 0)$, we obtain $s_j^{\mathcal{R}} = s^{\mathcal{R}}$ and $\sigma_j = \sigma^*$. To show continuity, we first consider the limit $\kappa \searrow \widehat{\kappa}(\sigma_0, 0)$. In this case, we find from the system of equations (Equations (15) and (16)) stated in the main text that $\lim_{\kappa \searrow \widehat{\kappa}(\sigma_0,0)} s_j^{\mathcal{R}} = 0$, which implies that $\sigma^* = \sigma_0$.

Next, we show that σ^* and $s^{\mathcal{R}}$ are increasing in the trading cost κ. Equation (15) implies that σ^* depends on κ only indirectly through $s^{\mathcal{R}}$. Moreover, σ^* is increasing in $s^{\mathcal{R}}$. As a result, we know that the sign of $\frac{d\sigma^*}{d\kappa}$ is the same as the sign of $\frac{ds^{\mathcal{R}}}{d\kappa}$. Hence,

$$\frac{ds^{\mathcal{R}}}{d\kappa} = \frac{\frac{\partial s^{\mathcal{R}}}{\partial \kappa}}{1 - \frac{\partial \sigma^*}{\partial s^{\mathcal{R}}} \frac{\partial s^{\mathcal{R}}}{\partial \sigma^*}} \tag{B.23}$$

$$\frac{d\sigma^*}{d\kappa} = \frac{\partial \sigma^*}{\partial s^{\mathcal{R}}} \frac{ds^{\mathcal{R}}}{d\kappa} \tag{B.24}$$

It then follows that $\frac{ds^{\mathcal{R}}}{d\kappa} > 0$ is equivalent to:

$$\frac{\eta\lambda^2 (1 - \sigma^*)(1 - \chi_{in})\chi_{in}}{c\kappa N((N-2)\sigma^* + 1) - \eta\lambda(N-1)\chi_{in}(\kappa N - \lambda(1 - \chi_{in})(1 - \sigma^*))} < 1 \tag{B.25}$$

which is true, given the equilibrium condition for σ^* and the parametric assumptions stated in the main text.

Following the same steps, we obtain a similar condition such that $\frac{ds^{\mathcal{R}}}{d\chi_{in}} > 0$ and $\frac{d\sigma^*}{d\chi_{in}} > 0$:

$$\frac{\eta\lambda^2 (1 - \sigma^*)^2 (1 - \chi_{in})\chi_{in}}{\kappa N \left(c(1 - \sigma^*)^2 + \pi^m (N-1)(1 - \sigma^*)^N\right)} < 1. \tag{B.26}$$

We can again confirm that this inequality holds. As a result, we conclude that σ^* and $s^{\mathcal{R}}$ are strictly increasing in κ and χ_{in} for $\kappa > \widehat{\kappa}(\sigma_0, 0)$. For lower values of κ, $\sigma^* = \sigma_0$ and $s^{\mathcal{R}} = 0$ are instead constant w.r.t. κ and χ_{in}.

Finally, the comparative statics of $\widehat{\kappa}(\sigma_0, 0)$ is as follows:

- Since σ_0 does not depend neither on λ nor on χ_{in}, it follows that $\widehat{\kappa}(\sigma_0, 0)$ is increasing in λ, and

$$\frac{\partial \widehat{\kappa}(\sigma_0, 0)}{\partial \chi_{in}} = -\frac{\lambda(1 - \sigma_0)}{N} < 0.$$

- Since σ_0 is increasing in π^m, which, in turn, is proportional to χ_{co}, it follows that $\widehat{\kappa}(\sigma_0, 0)$, being decreasing in σ_0, is decreasing in χ_{co}.

B.5 Proof of Proposition 2.

Let $\overline{\alpha}_j^{\mathcal{R}} > 0$ and $\underline{\alpha}_j^{\mathcal{R}} > 0$ be the fraction of $\mathcal{R}$ investors who buy shares in any firm $j \leq \underline{n}$ and $j \in (\underline{n}, \overline{n}]$, respectively, with $\sum_{j \leq \underline{n}} \overline{\alpha}_j^{\mathcal{R}} + \sum_{j \in (\underline{n}, \overline{n}]} \underline{\alpha}_j^{\mathcal{R}} = 1$. Similarly, let $\overline{\alpha}_j^{\mathcal{N}} > 0$ and $\underline{\alpha}_j^{\mathcal{N}} > 0$ be the fraction of $\mathcal{N}$ investors who buy shares in any firm $j > \overline{n}$ and $j \in (\underline{n}, \overline{n}]$, respectively, with $\sum_{j > \overline{n}} \overline{\alpha}_j^{\mathcal{N}} + \sum_{j \in (\underline{n}, \overline{n}]} \underline{\alpha}_j^{\mathcal{N}} = 1$.

Then, for any firm $j \leq \underline{n}$, the market clearing condition is

$$\chi_{in} \overline{\alpha}_j^{\mathcal{R}} \frac{1}{\kappa} \left[\mathbb{E}[\Pi_j] - p_j - \lambda(1 - \sigma_j) \right] = 1. \tag{B.27}$$

As $\mathcal{R}$ investors are indifferent between holding shares in any of these firms, $\mathbb{E}[\Pi_j] - p_j - \lambda(1 - \sigma_j)$ must be constant across these firms. This implies that the considered market clearing condition can hold for all $j \leq \underline{n}$ if and only if $\overline{\alpha}_j^{\mathcal{R}} \equiv \overline{\alpha}^{\mathcal{R}}$ for all such j.

Similarly, for any firm $j > \overline{n}$, the market clearing condition is

$$(1 - \chi_{in}) \overline{\alpha}_j^{\mathcal{N}} \frac{1}{\kappa} \left[\mathbb{E}[\Pi_j] - p_j \right] = 1. \tag{B.28}$$

As $\mathcal{N}$ investors are indifferent between holding shares in any of these firms, $\mathbb{E}[\Pi_j] - p_j$ must be constant across these firms. This implies that the considered market clearing condition can hold for all $j > \overline{n}$ if and only if $\overline{\alpha}_j^{\mathcal{N}} \equiv \overline{\alpha}^{\mathcal{N}}$ for all such j.

Finally, for any firm $j \in (\underline{n}, \overline{n}]$, the market clearing condition is

$$\chi_{in} \underline{\alpha}_j^{\mathcal{R}} \frac{1}{\kappa} \left[\mathbb{E}[\Pi_j] - p_j - \lambda(1 - \sigma_j) \right] + (1 - \chi_{in}) \underline{\alpha}_j^{\mathcal{N}} \frac{1}{\kappa} \left[\mathbb{E}[\Pi_j] - p_j \right] = 1. \tag{B.29}$$

In what follows, we first consider the case with $\underline{n} = \overline{n}$, and then move to $\underline{n} < \overline{n}$.

Proof of (i). For $\underline{n} = \overline{n} \equiv n$, from the above results it follows $\overline{\alpha}^{\mathcal{R}} = \frac{1}{n}$ and $\overline{\alpha}^{\mathcal{N}} = \frac{1}{N-n}$. For this equilibrium to exist for any given n, $\mathcal{R}$ investors should be indifferent between buying shares in any of firms $j = 1, \ldots, n$, and strictly prefer doing so to buying shares in other firms or not buying any shares:

$$\mathbb{E}[\Pi_1] - p_1 - \lambda(1-\sigma_1) = \ldots = \mathbb{E}[\Pi_n] - p_n - \lambda(1-\sigma_n) > \max\{0, \max_{j'>n} \mathbb{E}[\Pi_{j'}] - p_{j'} - \lambda(1-\sigma_{j'})\}. \quad \text{(B.30)}$$

By contrast, $\mathcal{N}$ investors should be indifferent between buying shares in any of firms $j = n+1, \ldots, N$, and strictly prefer doing so to buying shares in other firms or not buying any shares:

$$\mathbb{E}[\Pi_{n+1}] - p_{n+1} = \ldots = \mathbb{E}[\Pi_N] - p_N > \max\{0, \max_{j'\leq n} \mathbb{E}[\Pi_{j'}] - p_{j'}\}. \quad \text{(B.31)}$$

In this equilibrium, as $s_j^{\mathcal{R}} = 1$ for $j \leq n$ and $s_j^{\mathcal{R}} = 0$ for $j > n$, by the results in Lemma 2 (ii) it follows that

$$\sigma_1^* = \ldots = \sigma_n^* \equiv \overline{\sigma} > \sigma_{n+1}^* = \ldots = \sigma_N^* \equiv \underline{\sigma},$$

where $(\overline{\sigma}, \underline{\sigma})$ solve the system obtained from the FOCs (7) for $s_j^{\mathcal{R}} = 1$ for $j \leq n$ and $s_j^{\mathcal{R}} = 0$ for $j > n$:

$$\overline{\sigma} = \frac{1}{c}\left[\eta\lambda + \pi^m (1-\overline{\sigma})^{n-1}(1-\underline{\sigma})^{N-n}\right] \quad \text{(B.32)}$$

$$\underline{\sigma} = \frac{\pi^m}{c}(1-\underline{\sigma})^{N-n-1}(1-\overline{\sigma})^n \quad \text{(B.33)}$$

Then, using the market clearing conditions (B.27)-(B.28), the existence conditions (B.30)-(B.31) yield

$$\frac{n\kappa}{\chi_{in}} > \frac{\kappa(N-n)}{1-\chi_{in}} - \lambda(1-\underline{\sigma}), \quad \text{(B.34)}$$

and

$$\frac{\kappa(N-n)}{1-\chi_{in}} > \frac{n\kappa}{\chi_{in}} + \lambda(1-\overline{\sigma}). \quad \text{(B.35)}$$

These conditions are simultaneously satisfied if and only if $\chi_{in} > \frac{n}{N}$ and $\widehat{\kappa}(\overline{\sigma}, n) < \kappa < \widehat{\kappa}(\underline{\sigma}, n)$.[22]

Proof of (ii). For any $\underline{n} < \overline{n}$, from Equation (B.27) and Equation (B.28) we find

$$\underline{n}\overline{\alpha}^{\mathcal{R}} = 1 - \sum_{j\in(\underline{n},\overline{n}]} \underline{\alpha}^{\mathcal{R}} = \frac{\underline{n}\kappa}{\chi_{in}\left[\mathbb{E}[\Pi_j] - p_j - \lambda(1-\sigma_j)\right]} \quad \forall j \leq \overline{n}, \quad \text{(B.36)}$$

and

$$(N-\overline{n})\overline{\alpha}^{\mathcal{N}} = 1 - \sum_{j\in(\underline{n},\overline{n}]} \underline{\alpha}^{\mathcal{N}} = \frac{(N-\overline{n})\kappa}{(1-\chi_{in})\left[\mathbb{E}[\Pi_j] - p_j\right]} \quad \forall j > \underline{n}, \quad \text{(B.37)}$$

[22]Note that, since $\underline{\sigma}$ and $\overline{\sigma}$ do not depend on κ, the latter condition properly defines an interval for κ.

respectively. Summing the market clearing conditions (B.29) for $j \in (\underline{n}, \overline{n}]$ and using these results, after simple manipulations we get

$$\chi_{in}\left[\mathbb{E}[\Pi_j] - p_j - \lambda(1-\sigma_j)\right] + (1-\chi_{in})\left[\mathbb{E}[\Pi_j] - p_j\right] = N\kappa, \quad \forall j \in (\underline{n}, \overline{n}]. \tag{B.38}$$

Since $s_j^{\mathcal{R}} = 1$ for all $j \leq \underline{n}$, and $s_j^{\mathcal{R}} = 0$ for all $j > \overline{n}$, from Lemma 2 (ii) it follows that in equilibrium $\sigma_1^* = \ldots = \sigma_{\underline{n}}^* \equiv \overline{\sigma}$, and $\sigma_{\overline{n}+1}^* = \ldots = \sigma_N^* \equiv \underline{\sigma} < \overline{\sigma}$. Moreover, $\mathbb{E}[\Pi_j] - p_j - \lambda(1-\sigma_j)$ must be constant for all $j \in (\underline{n}, \overline{n}]$ (by $\mathcal{R}$ investors' indifference conditions), and $\mathbb{E}[\Pi_j] - p_j$ must also be constant across these j (by $\mathcal{N}$ investors' indifference conditions). Taken together, these indifference conditions imply that also σ_j must be constant for $j \in (\underline{n}, \overline{n}]$: $\sigma_{\underline{n}+1}^* = \ldots = \sigma_{\overline{n}}^* \equiv \widehat{\sigma}$. From Lemma 2 (ii), this is the case if and only if, for all $j \in (\underline{n}, \overline{n}]$: $s_j^{\mathcal{R}} \equiv \widehat{s}^{\mathcal{R}}$, which in turn (by symmetry across $\mathcal{R}$ investors) holds if and only if $\underline{\alpha}_j^{\mathcal{R}} \equiv \underline{\alpha}^{\mathcal{R}}$ for all $j \in (\underline{n}, \overline{n}]$. Specifically, we have

$$\underline{\alpha}^{\mathcal{R}} = \frac{1 - \underline{n}\overline{\alpha}^{\mathcal{R}}}{\overline{n} - \underline{n}} = \frac{1}{\overline{n} - \underline{n}}\left[1 - \frac{\underline{n}\kappa}{\chi_{in}\left[N\kappa - \lambda(1-\chi_{in})(1-\widehat{\sigma})\right]}\right], \tag{B.39}$$

where the second equality uses the market clearing condition (B.27) and $\mathcal{R}$ investors indifference condition. Then, for the market clearing condition (B.29) to hold for all firms $j \in (\underline{n}, \overline{n}]$, it must also be $\underline{\alpha}_j^{\mathcal{N}} \equiv \underline{\alpha}^{\mathcal{N}}$ for all $j \in (\underline{n}, \overline{n}]$.

Next, the investment levels $(\overline{\sigma}, \widehat{\sigma}, \underline{\sigma})$ in equilibrium are obtained from

$$\begin{aligned}
\overline{\sigma} &= \frac{1}{c}\left[\eta\lambda + \pi^m(1-\overline{\sigma})^{\underline{n}-1}(1-\widehat{\sigma})^{\overline{n}-\underline{n}}(1-\underline{\sigma})^{N-\overline{n}}\right] \\
\widehat{\sigma} &= \frac{1}{c}\left[\eta\lambda\widehat{s}^{\mathcal{R}}(\widehat{\sigma}) + \pi^m(1-\overline{\sigma})^{\underline{n}}(1-\widehat{\sigma})^{\overline{n}-\underline{n}-1}(1-\underline{\sigma})^{N-\overline{n}}\right] \\
\underline{\sigma} &= \frac{\pi^m}{c}(1-\overline{\sigma})^{\underline{n}}(1-\widehat{\sigma})^{\overline{n}-\underline{n}}(1-\underline{\sigma})^{N-\overline{n}-1}
\end{aligned} \tag{B.40}$$

where $\widehat{s}^{\mathcal{R}}(\widehat{\sigma}) \equiv \chi_{in}\underline{\alpha}^{\mathcal{R}}\frac{1}{\kappa}\left[N\kappa - (1-\chi_{in})\lambda(1-\widehat{\sigma})\right] \in (0,1)$; hence, from Lemma 2 (ii), $\overline{\sigma} > \widehat{\sigma} > \underline{\sigma}$. It is immediate to check that, together with the indifference conditions of $\mathcal{R}$ and $\mathcal{N}$ investors, these inequalities imply that $\mathcal{R}$ investors strictly prefer firms $j \leq \overline{n}$ to firms $j > \overline{n}$, and $\mathcal{N}$ investors strictly prefer firms $j > \underline{n}$ to firms $j \leq \underline{n}$, as required for the existence of such equilibria.

Moreover, using (B.38) we find that $\mathcal{R}$ investors prefer buying shares in any firm $j \leq \overline{n}$ to not buying at all if and only if

$$\mathbb{E}[\Pi_j] - p_j - \lambda(1-\sigma_j) > 0 \Leftrightarrow \kappa > \widehat{\kappa}(\widehat{\sigma}, 0). \tag{B.41}$$

Under this condition, it follows that $\mathcal{N}$ investors participate in the financial market.

Hence, to characterize existence conditions for these equilibria, we only need to impose conditions under which all shares $\underline{\alpha}^\theta$ and $\overline{\alpha}^\theta$, for $\theta \in \{\mathcal{R}, \mathcal{N}\}$, are positive. The market clearing conditions (B.27)-(B.28) immediately imply $\overline{\alpha}^\theta > 0$ whenever (B.41) holds. The condition $\underline{\alpha}^{\mathcal{R}} > 0$ is then equivalent to $\underline{n}\overline{\alpha}^{\mathcal{R}} < 1$. Using (B.36) and substituting the market clearing prices, after simple manipulations, we obtain that this condition holds if and only if

$$\chi_{in} > \frac{\underline{n}}{N} \quad \text{and} \quad \kappa > \widehat{\kappa}(\widehat{\sigma}, \underline{n}) \geq \widehat{\kappa}(\widehat{\sigma}, 0), \tag{B.42}$$

with equality at $\underline{n} = 0$ only.

Similarly, $\underline{\alpha}^{\mathcal{N}} > 0$ is equivalent to $(N - \overline{n})\overline{\alpha}^{\mathcal{N}} < 1$. Using (B.37) and substituting the market clearing prices, after simple manipulations, we obtain that this condition holds if and only if

$$\chi_{in} \leq \frac{\overline{n}}{N} \quad \text{or} \quad \left\{\chi_{in} \leq \frac{\overline{n}}{N} \text{ and } \kappa < \widehat{\kappa}(\widehat{\sigma}, \overline{n})\right\}. \tag{B.43}$$

Putting these conditions together, for any given $(\underline{n}, \overline{n})$, with $\underline{n} < \overline{n}$, we finally have:[23]

1. For $\chi_{in} \leq \frac{\underline{n}}{N}$, the considered equilibria do not exist;
2. For $\frac{\underline{n}}{N} < \chi_{in} \leq \frac{\overline{n}}{N}$, the considered equilibria exist if and only if $\kappa > \widehat{\kappa}(\widehat{\sigma}, \underline{n})$;
3. For $\chi_{in} > \frac{\overline{n}}{N}$, the considered equilibria exist if and only if $\widehat{\kappa}(\widehat{\sigma}, \underline{n}) < \kappa < \widehat{\kappa}(\widehat{\sigma}, \overline{n})$.

B.6 Proof of Proposition 3.

Proof of 1. Consider asymmetric equilibria with $\underline{n} = \overline{n} = 1$, which, from the results in Proposition 2, exist if $\chi_{in} > 1/N$ and $\widehat{\kappa}(\overline{\sigma}, 1) < \kappa < \widehat{\kappa}(\underline{\sigma}, 1)$. From Lemma 2 (iii) we know that $\underline{\sigma} < \sigma_0$, which implies $\widehat{\kappa}(\underline{\sigma}, 1) > \widehat{\kappa}(\sigma_0, 0)$. Next, a sufficient condition for $\widehat{\kappa}(\overline{\sigma}, 1) < \widehat{\kappa}(\sigma_0, 0)$ is

$$\chi_{in} > \frac{1}{N(\overline{\sigma} - \sigma_0)}.$$

[23]Note that, since $\widehat{\sigma}$ is itself a function of κ, the following inequalities do not properly define upper/lower bounds on the value of κ.

In turn, to obtain a sufficient condition for this inequality to hold, note that (since $\underline{\sigma} < \sigma_0$)

$$\overline{\sigma} = \frac{1}{c}\left[\eta\lambda + \pi^m(1-\underline{\sigma})^{N-1}\right] > \frac{1}{c}\left[\eta\lambda + \pi^m \underbrace{(1-\sigma_0)^{N-1}}_{=\frac{\sigma_0 c}{\pi^m}}\right] \Longrightarrow \overline{\sigma} - \sigma_0 > \frac{\eta\lambda}{c}. \tag{B.44}$$

Hence, we find that

$$\chi_{in} > \frac{c}{N\eta\lambda} \tag{B.45}$$

is a sufficient condition for $\widehat{\kappa}(\overline{\sigma}, 1) < \widehat{\kappa}(\sigma_0, 0)$.[24] Under this condition, for $\widehat{\kappa}(\sigma_0, 0) > \kappa > \widehat{\kappa}(\overline{\sigma}, 1)$, the only equilibria with SRI are asymmetric.

Proof of 2. Consider asymmetric equilibria with $\underline{n} = 0 < \overline{n} = N-1$. Suppose $\chi_{in} \leq \frac{N-1}{N}$. Then these equilibria exist for

$$\kappa > \widehat{\kappa}(\widehat{\sigma}, 0). \tag{B.46}$$

In what follows we prove that the last condition is satisfied for all $\kappa > \widehat{\kappa}(\sigma_0, 0)$.

Substituting the market clearing price from Equation (B.38) into the individual demands $s_{ij}^{\mathcal{R}}$ of each $\mathcal{R}$ investor given in Equation (9) and rearranging, we have that any such equilibrium must satisfy

$$\Gamma(s_{ij}^{\mathcal{R}}) \equiv \kappa(N - s_{ij}^{\mathcal{R}}) - (1-\chi_{in})\lambda(1 - \widehat{\sigma}(s_{ij}^{\mathcal{R}})) = 0, \tag{B.47}$$

where $\widehat{\sigma}(\cdot)$ solves

$$\frac{\eta\lambda\chi_{in}}{N-1}s_{ij}^{\mathcal{R}} + \pi^m(1-\widehat{\sigma})^{N-1}\left(1 - \frac{\pi^m}{c}(1-\widehat{\sigma})^{N-2}\right) - c\widehat{\sigma} = 0. \tag{B.48}$$

This equation is obtained from the system (B.40), for $\underline{n} = 0$ and $\overline{n} = N-1$ and, accordingly, $\underline{\alpha}^{\mathcal{R}} = 1/(N-1)$, substituting the third equation into the second one and rearranging.

For the market clearing conditions $\frac{\chi_{in}}{N-1}s_{ij}^{\mathcal{R}} + (1-\chi_{in})\underline{\alpha}^{N}s_{ij}^{N} = 1$ to hold, it must be $s_{ij}^{\mathcal{R}} < \frac{N-1}{\chi_{in}}$, and $s_{ij}^{\mathcal{R}} \geq 0$.

We have

$$\Gamma\left(\frac{N-1}{\chi_{in}}\right) < 0 \quad \forall \chi_{in} \leq \frac{N-1}{N}, \tag{B.49}$$

[24]Note that this sufficient condition imposes an upper bound on c, which is however in general compatible with our parametric restrictions.

and

$$\Gamma(0) > 0 \Leftrightarrow \kappa > \widehat{\kappa}(\sigma_0, 0), \tag{B.50}$$

given that $\widehat{\sigma}(0) = \sigma_0$. This is because, as $s_{iN}^{\mathcal{R}} = 0$ in this equilibrium, if $s_{ij}^{\mathcal{R}} = 0$ for all $j = 1, \ldots, N-1$ as well, then $\mathcal{R}$ investors do not hold shares in any of the firms, so we obtain the equilibrium with no SRI.

Hence, for all $\kappa > \widehat{\kappa}(\sigma_0, 0)$ (i.e., in the region of parameters where there exists the symmetric equilibrium with SRI): $\Gamma(0) > 0 > \Gamma\left(\frac{N-1}{\chi_{in}}\right)$, which implies that asymmetric equilibria with $\underline{n} = 0 < \overline{n} = N-1$ exist for all $\chi_{in} \leq \frac{N-1}{N}$.

B.7 Proof of Proposition 4.

The proof proceeds in two steps. First, we prove that the equilibria mentioned in Proposition 4 admit a unique collection of CSR policies. Second, we prove comparative statics for dispersion in CSR policies and the probability that a green firm prices above marginal cost.

Part 1. We know from Lemma 2 (ii) that an equilibrium exists. Consider the equilibrium CSR policies $\overline{\sigma}^*$ and $\underline{\sigma}^*$. We have shown in the proof of Proposition 2 that this equilibrium is characterized by:

$$\Gamma_1(\overline{\sigma}^*, \underline{\sigma}^*) = \eta\lambda + \pi^m(1-\overline{\sigma}^*)^{n-1}(1-\underline{\sigma}^*)^{N-n} - c\overline{\sigma}^* = 0, \tag{B.51}$$

$$\Gamma_2(\overline{\sigma}^*, \underline{\sigma}^*) = \pi^m(1-\underline{\sigma}^*)^{N-n-1}(1-\overline{\sigma}^*)^{n} - c\underline{\sigma}^* = 0. \tag{B.52}$$

Combining Equations (B.51) and (B.52) yields:

$$\eta\lambda(1-\overline{\sigma}^*) - c(\overline{\sigma}^* - \underline{\sigma}^*)(1-\overline{\sigma}^* - \underline{\sigma}^*) = 0. \tag{B.53}$$

Note that Γ_1 and Γ_2 are decreasing in both $\overline{\sigma}$ and $\underline{\sigma}$. Then, any additional equilibrium $(\overline{\sigma}^{**}, \underline{\sigma}^{**})$ must satisfy either (i) $\overline{\sigma}^{**} > \overline{\sigma}^*$ and $\underline{\sigma}^{**} < \underline{\sigma}^*$ or (ii) $\overline{\sigma}^{**} < \overline{\sigma}^*$ and $\underline{\sigma}^{**} > \underline{\sigma}^*$.

However, Equation (B.53) increases in $\underline{\sigma}^*$ and decreases in $\overline{\sigma}^*$, if $\overline{\sigma}^* < \frac{1}{2}$. A sufficient condition to get $\overline{\sigma}^* < 1/2$ is that:

$$\Gamma_1(1/2, 0) < 0 \Leftrightarrow \eta\lambda + \pi^m \left(\frac{1}{2}\right)^{n-1} - \frac{1}{2}c < 0 \Leftrightarrow c > 2\eta\lambda + 2\pi^m \left(\frac{1}{2}\right)^{n-1} \equiv \underline{c}. \tag{B.54}$$

Hence condition (B.53), which holds at $\overline{\sigma}^*$ and $\underline{\sigma}^*$, can also hold at $\overline{\sigma}^{**}$ and $\underline{\sigma}^{**}$ if either (i) $\overline{\sigma}^{**} > \overline{\sigma}^*$ and $\underline{\sigma}^{**} > \underline{\sigma}^*$ or (ii) $\overline{\sigma}^{**} < \overline{\sigma}^*$ and $\underline{\sigma}^{**} < \underline{\sigma}^*$, which is not consistent with conditions (i) and (ii) mentioned above. Hence, we conclude that asymmetric equilibria in which responsible investors hold firms $j \leq n$ and non-responsible investors hold firms $j > n$ admit unique CSR policies $\sigma_j = \overline{\sigma}(n)$ for $j \leq n$ and $\sigma_j = \underline{\sigma}(n)$ for $j > n$.

Part 2. We first define the dispersion in CSR policies for a given n as $\mathcal{D}(n) \equiv \overline{\sigma}(n) - \underline{\sigma}(n)$. We can use Equation (B.53) to write:

$$\eta\lambda(1 - \overline{\sigma}(n)) - c\mathcal{D}(n)(1 - 2\overline{\sigma}(n) + \mathcal{D}(n)) = 0, \tag{B.55}$$

which leads to

$$\mathcal{D}(n)' = \frac{2c\mathcal{D}(n) - \eta\lambda}{c(1 - 2\underline{\sigma}(n))}\overline{\sigma}(n)'. \tag{B.56}$$

It follows from $\underline{\sigma} < (\overline{\sigma} <)\frac{1}{2}$ that $c(1 - 2\underline{\sigma}(n)) > 0$ for all $c > \underline{c}$. Similarly, using Equation (B.53) we have

$$2c\mathcal{D}(n) - \eta\lambda = \frac{2\eta\lambda(1 - \overline{\sigma}(n)}{1 - \overline{\sigma}(n) - \underline{\sigma}(n)} - \eta\lambda = \eta\lambda\frac{1 - \overline{\sigma}(n) + \underline{\sigma}(n)}{1 - \overline{\sigma}(n) - \underline{\sigma}(n)} > 0.$$

Hence, the sign of $\mathcal{D}(n)'$ is equal to the sign of $\overline{\sigma}(n)'$. It follows from the implicit function theorem that $\overline{\sigma}(n)' < 0$. To see this, consider Equation (B.51), plug in $\underline{\sigma}(\overline{\sigma})$ (which does not depend on n as shown in Equation (B.53)), and differentiate this expression with respect to $\overline{\sigma}$ and n:

$$\overline{\sigma}' = \frac{\frac{d\Gamma_1}{dn}}{-\frac{d\Gamma_1}{d\overline{\sigma}}} \tag{B.57}$$

with

$$\frac{d\Gamma_1}{dn} = \pi^m (1-\bar{\sigma}^*)^{n-1} \left(1-\underline{\sigma}^*\right)^{N-n} \left(\log(1-\bar{\sigma}^*) - \log\left(1-\underline{\sigma}^*\right)\right) < 0 \tag{B.58}$$

$$\frac{d\Gamma_1}{d\overline{\sigma}} = -c - \pi^m (N-n)(1-\bar{\sigma}^*)^{n-1} \left(1-\underline{\sigma}^*\right)^{N-n-1} \frac{d\underline{\sigma}}{d\overline{\sigma}} - \pi^m (n-1)(1-\bar{\sigma}^*)^{n-2} \left(1-\underline{\sigma}^*\right)^{N-n}. \tag{B.59}$$

Using Equation (B.53), we have that

$$\frac{d\underline{\sigma}}{d\overline{\sigma}} = \frac{\pi^m (1-2\bar{\sigma}^*) + \eta\lambda}{\pi^m \left(1-2\underline{\sigma}^*\right)} > 0 \tag{B.60}$$

which implies that $\frac{d\Gamma_1}{d\overline{\sigma}} < 0$ and, hence, $\overline{\sigma}' < 0$. Thus we conclude that dispersion decreases in n, which implies that it increases with the concentration of SRI.

Next, we prove that firms held by responsible investors charge higher expected prices if the concentration is higher (i.e., n is smaller). As $\rho_j = \gamma$ unless $a_j = 1$ and $a_{-j} = 0 \; \forall -j \neq j$, we need to show that the probability that firm $j \leq n$ is decreasing in n. This probability is given by:

$$\mathcal{P}(n) \equiv \overline{\sigma}(n)\left(1-\overline{\sigma}(n)\right)^{n-1} \left(1-\underline{\sigma}(n)\right)^{N-n}.$$

Using Equation (B.51), we can re-write this probability as:

$$\mathcal{P}(n) = \frac{\overline{\sigma}(n)}{\pi^m}\left(c\overline{\sigma}(n) - \eta\lambda\right). \tag{B.61}$$

It follows that

$$\mathcal{P}'(n) = \frac{2c\overline{\sigma}(n) - \eta\lambda}{\pi^m}\overline{\sigma}'(n), \tag{B.62}$$

which is negative because $\overline{\sigma}'(n) < 0$ (as shown above) and $2c\overline{\sigma}(n) > c\overline{\sigma}(n) > \eta\lambda$, where the latter inequality immediately follows from Equation (B.51).

Finally, we show that the expected price of green products may be higher or lower when n is larger, conditional on $a_j = 1$ for at least one $j \in \mathcal{J}$. We let n_g denote the number of green firms and derive the following expression for this conditional expectation:

$$\mathbb{E}[\rho_j | n_g \geq 1] = \Pr(n_g = 1 | n_g \geq 1)\rho^m + \Pr(n_g \geq 2 | n_g \geq 1)\gamma = \gamma + \Pr(n_g = 1 | n_g \geq 1)\left(\rho^m - \gamma\right), \tag{B.63}$$

where $\rho^m > \gamma$ denotes the monopoly price. The probability for $n_g = 1$ given $n_g \geq 1$ is given by:

$$\Pr(n_g = 1 | n_g \geq 1) = \frac{\Pr(n_g = 1)}{\Pr(n_g \geq 1)} = \frac{\Pr(n_g = 1)}{1 - \Pr(n_g = 0)} = \frac{\sum_{j=1}^{N} \sigma_j \cdot \prod_{y \neq j}(1 - \sigma_y)}{1 - \prod_{j=1}^{N}(1 - \sigma_j)}. \quad \text{(B.64)}$$

Figure 3 plots this conditional probability for a set of parameters. The figure confirms the non-

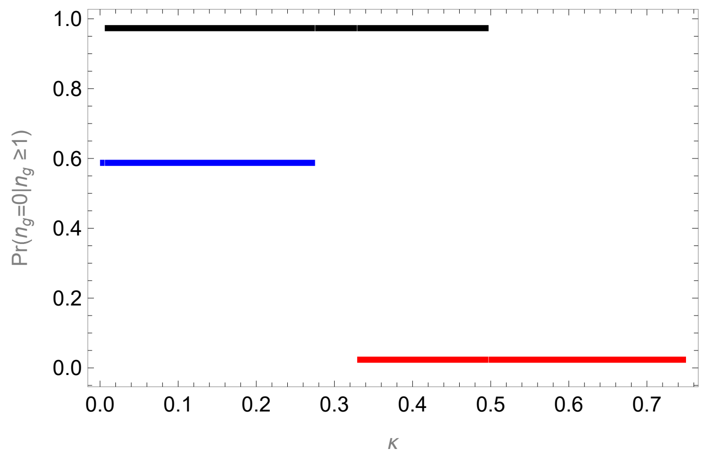


Figure 3: This figure shows the conditional probability for $n_g = 1$ given $n_g \geq 1$ as a function of the trading cost κ. Model parameters: $N = 6$, $c = 8/15$, $\eta = 1/10$, $\gamma = 0$, $\lambda = 4$, $\pi^m = 1/3$ and $\chi_{in} = 1/2$. The blue line represents the symmetric equilibrium without SRI ($n = 0$); the black line represents the asymmetric equilibrium with $n = 1$, and the red line represents the asymmetric equilibrium with $n = 2$.

monotone relationship between the expected price and n: the expected price for $n = 0$ is lower than that for $n = 1$ but higher than that for $n = 2$.

B.8 Proof of Proposition 5.

Suppose a welfare-maximizing planner can choose shareholdings, CSR policies, and product prices. Then, from the definition of social welfare in Equation (18), it follows that the maximization of expected surplus in the product market surplus requires having at least one green firm and that all firms price at marginal cost. The minimization of trading costs requires that all individual investors (both $\mathcal{R}$ and $\mathcal{N}$ types) purchase the same total amount of shares, which is independent of $\vec{\sigma}$. Finally, the third term is minimized when $\sigma_j = \min\{\lambda/c, 1\} \forall j \in \mathcal{J}$. From the remarks above, it follows that the social optimum prescribes $\sigma_j = 1 \forall j \in \mathcal{J}$ for all $c < \lambda$.[25] Whenever this holds, under-investment unambiguously emerges in all equilibria of the game.

We prove the remaining results in the lemma by examples. We consider the following specifi-

[25]Note that this is compatible with $c > \eta\lambda + \pi^m$ as long as $c > \pi^m$ and η is small enough.

cation of consumers' (gross) utility function:

$$u(x) \equiv -\frac{\alpha}{2}x^2 + \beta x, \tag{B.65}$$

with $\alpha > 0$, and $\beta \in (\gamma, \gamma + \lambda)$. The restriction $\beta > \gamma$ is needed to ensure positive demand, whereas $\beta < \lambda + \gamma$ implies that responsible consumers *boycott* brown products — i.e., they are unwilling to purchase them even if priced at marginal cost.

Letting ρ denote the product price, this utility yields a linear demand function $x^*(\rho) \equiv \frac{\beta-\rho}{\alpha}$, whereby consumer surplus is

$$CS(\rho) \equiv u(x^*(\rho)) - \rho x^*(\rho) = \frac{(\beta - 3\rho)(\beta - \rho)}{2\alpha}. \tag{B.66}$$

In our setting, ρ is the minimum price at which the relevant goods are sold, and x is the total demanded quantity of these goods, the relevant goods being only those sold by green firms for $\mathcal{R}$ consumers.

In equilibrium, $\rho = \gamma$, hence $\pi_j = 0$ for all $j \in \mathcal{J}$, under competition; and $\rho = \rho^m \equiv \frac{\beta+\gamma}{2}$ under monopoly: a monopolist green firm thus obtains a profit $\pi^m = \chi_{co}\frac{(\beta-\gamma)^2}{4\alpha}$.

The expected surplus in the product market can thus be written as:

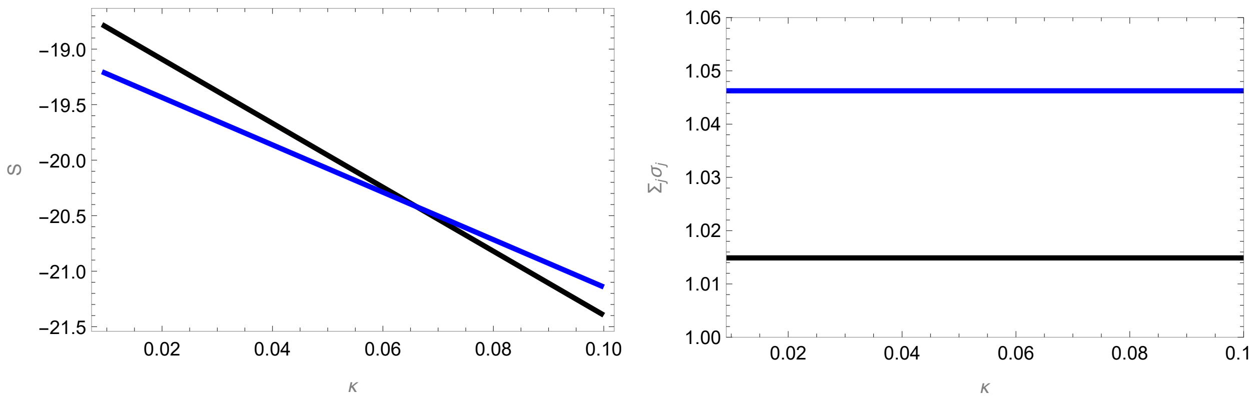
$$\underbrace{\prod_{j=1}^{N}(1-\sigma_j)}_{\Pr(n_g=0)}(1-\chi_{co})CS(\gamma) + \underbrace{\sum_{j=1}^{N}\sigma_j \cdot \prod_{y\neq j}(1-\sigma_y)}_{\Pr(n_g=1)}\left[(1-\chi_{co})CS(\gamma) + \chi_{co}CS(\rho^m) + \pi^m\right] +$$

$$+ \underbrace{\left[1 - \prod_{j=1}^{N}(1-\sigma_j) - \sum_{j=1}^{N}\sigma_j \cdot \prod_{y\neq j}(1-\sigma_y)\right]}_{\Pr(n_g\geq 2)} CS(\gamma).$$

We can rearrange the equation above as:

$$(1-\chi_{co})CS(\gamma) + \sum_{j=1}^{N}\sigma_j \cdot \prod_{y\neq j}(1-\sigma_y)(\chi_{co}CS(\rho^m)+\pi^m) + \left(1 - \prod_{j=1}^{N}(1-\sigma_j) - \sum_{j=1}^{N}\sigma_j \cdot \prod_{y\neq j}(1-\sigma_y)\right)\chi_{co}CS(\gamma). \tag{B.67}$$

The two panels in Figure 4 display a numerical example, based on the outlined model specification (and satisfying all the mentioned parametric conditions), in which the symmetric equilibrium without SRI coexists with an asymmetric equilibrium (with tilting). Total surplus and aggregate greenness are lower in the latter equilibrium, as shown in Panel (a) and Panel (b), respectively. In the equilibrium without SRI, $\mathcal{R}$ investors obtain zero utility because $s_{ij} = 0$ for $i \in \mathcal{R}$. It follows that $\mathcal{R}$ investors must obtain higher utility if they are willing to trade.



(a) Total Surplus

(b) Aggregate Greenness

Figure 4: This figure shows total surplus S (Panel (a)) and aggregate greenness (Panel (b)) as a function of the trading cost κ. Model parameters: $N = 6$, $c = 8/15$, $\eta = 1/10$, $\alpha = 9/32$, $\beta = 1$, $\gamma = 0$, $\lambda = 4$, and $\chi_{co} = \chi_{in} = 3/8$. The blue line represents the symmetric equilibrium without SRI; the black line represents the asymmetric equilibrium with $n = 1$.

Online Appendix

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C Product market extensions

C.1 Horizontal differentiation

In this section, we consider a duopoly model with horizontal differentiation *à la* Hotelling. We show that as in the main model: (i) asymmetric equilibria in which $\mathcal{R}$ investors participate in the financial market are likely to exist for a larger set of parameter values than symmetric equilibria in which $\mathcal{R}$ participate in the financial market; (ii) asymmetric equilibria may be *less green* — i.e., entail a lower level of aggregate investments — than symmetric equilibria.

Model Setup

Suppose consumers, both responsible and non-responsible ones, are uniformly located on a unit Hotelling line $h \in [0, 1]$. Two firms are located at the ends of the line: firm $j = 1$ is located at $h_1 = 0$; firm $j = 2$ is located at $h_2 = 1$. Each consumer located at h has unit demand and derives utility

$$u_{hj} \equiv v - t|h - h_j| - \mathbb{1}_{h,\mathcal{R}}\lambda(1 - a_j) - \rho_j,$$

from buying firm j's product, with $v > 0$ and $t > 0$ being the gross utility from consuming any product and the unit transportation cost, respectively. In what follows, we normalize production costs to zero and assume

$$\lambda > v > \frac{2t}{3}\frac{3-\chi}{1-\chi}.$$

The first inequality implies that responsible consumers *boycott* brown firms' products — i.e., they are unwilling to purchase brown products, regardless of their price. The second inequality ensures full market coverage whenever there is at least one green firm in the market. Moreover, similar to the base model, to rule out corner solutions at the investment stage, we assume

$$c > \frac{(27-7\chi)(9\lambda(1-\chi)+4t(3-\chi)\chi)}{72(3-\chi)(1-\chi)} > 0.$$

Finally, we posit $\chi_{in} = \chi_{co} \equiv \chi$ and normalize $\eta \equiv 1$.

Equilibrium Analysis

Product market equilibrium. We first characterize equilibria in the product market for any $(a_1, a_2) \in \{0,1\}^2$:

- If both products are green ($a_1 = a_2 = 1$), the two firms engage in standard Hotelling competition for all consumers. Hence, $\rho^*(1,1) = t$, and each firm makes a profit (gross of investment costs) $\pi^*(1,1) = t/2$.
- If both products are brown ($a_1 = a_2 = 0$), the two firms engage in standard Hotelling competition for non-responsible consumers, whereas responsible consumers boycott both firms. Hence, $\rho^*(0,0) = t$, and each firm makes profit $\pi^*(0,0) = (1-\chi)t/2$.
- If firm j sells a green product and the other firm $-j$ a brown product ($a_j = 1, a_{-j} = 0$), then, assuming full market coverage in the segment of non-responsible consumers, firm $-j$ solves

$$\max_{\rho_{-j}} (1-\chi)\left[\frac{1}{2} + \frac{\rho_j - \rho_{-j}}{2t}\right]\rho_{-j},$$

yielding the standard best response function

$$\rho_{-j} = \frac{t + \rho_j}{2}.$$

Assuming that it serves all responsible consumers, firm j solves

$$\max_{\rho_j} \left[(1-\chi)\left[\frac{1}{2} - \frac{\rho_j - \rho_{-j}}{2t}\right] + \chi\right]\rho_j,$$

which yields

$$\rho_j = \frac{\rho_{-j}}{2} + t\frac{1+\chi}{2(1-\chi)}.$$

From these best responses, we find the equilibrium prices $\rho_j^*(a_j, a_{-j})$:

$$\rho_j^*(1,0) = \frac{t}{3}\frac{3+\chi}{1-\chi} > \rho_{-j}^*(1,0) = \frac{t}{3}\frac{3-\chi}{1-\chi},$$

which, under the above parametric restrictions, satisfy the mentioned full market coverage conditions. The corresponding profits $\pi_j^*(a_j, a_{-j})$ are

$$\pi_j^*(1,0) = \frac{t}{18}\frac{(3+\chi)^2}{1-\chi} > \pi_{-j}^*(1,0) = \frac{t}{18}\frac{(3-\chi)^2}{1-\chi}.$$

Note that, unlike under perfect substitutability, here the presence of product differentiation, entailed by the different production technology employed by the two firms, reduces consumer surplus in the segment of non-responsible consumers, since $\rho_{-j}^*(1,0) > t$. Moreover, it also induces a misallocation of non-responsible consumers across firms, since $\rho_j^*(1,0) > \rho_{-j}^*(1,0)$ (thereby some non-responsible consumers located closer to firm j will patronize the cheaper rival), which reduces total welfare.

Given firms' CSR policies (σ_j, σ_{-j}), firm j's expected profit (gross of investment costs) is thus

$$\Pi_j = \sigma_j(\sigma_{-j}\pi^*(1,1) + (1-\sigma_{-j})\pi_j^*(1,0)) + (1-\sigma_j)(\sigma_{-j}\pi_j^*(0,1) + (1-\sigma_{-j})\pi^*(0,0)),$$

with

$$\pi_j^*(1,0) - \pi^*(0,0) > \pi^*(1,1) - \pi^*(0,1) > 0.$$

In words, when products are horizontally differentiated, being a green firm always increases market profits compared to producing brown products, but still does so to a larger extent when facing a brown rival than when also the rival produces green products.

CSR policies. Given the shares $s_j^{\mathcal{R}}$ of responsible investors in each firm $j = 1, 2$, maximizing firm j's expected profit w.r.t. σ_j gives

$$\sigma_j = \frac{\lambda}{c} s_j^{\mathcal{R}} + \frac{t\chi(8(3-\chi) - \sigma_{-j}(27-7\chi))}{18c(1-\chi)},$$

from which it is easy to find the unique pair of equilibrium CSR policies for all pairs $(s_j^{\mathcal{R}}, s_{-j}^{\mathcal{R}})$:

$$\sigma_j^*(s_j^{\mathcal{R}}, s_{-j}^{\mathcal{R}}) = \frac{36c(1-\chi)(9\lambda(1-\chi)s_j^{\mathcal{R}} + 4t(3-\chi)\chi) - 2t\chi(27-7\chi)(9\lambda(1-\chi)s_{-j}^{\mathcal{R}} + 4t(3-\chi)\chi)}{324c^2(1-\chi)^2 - t^2(27-7\chi)^2\chi^2},$$

with

$$\frac{\partial \sigma_j^*}{\partial s_j^{\mathcal{R}}} > 0 > \frac{\partial \sigma_j^*}{\partial s_{-j}^{\mathcal{R}}} \quad \text{and} \quad \frac{\partial \sigma_j^*}{\partial s_j^{\mathcal{R}}} > \left| \frac{\partial \sigma_j^*}{\partial s_{-j}^{\mathcal{R}}} \right|.$$

An increase in $s_j^{\mathcal{R}}$ makes firm j's management more willing to invest, so to reduce (in expectation) the negative externality caused by the brown technology. As CSR investments are, for the same reasons explained in the main model, strategic substitutes, a larger $s_j^{\mathcal{R}}$ induces a drop in investment levels by the rival firm $-j$. Yet, in the duopoly model, the *direct effect* of an increase in $s_j^{\mathcal{R}}$ on σ_j^* always outweighs in magnitude its *strategic effect* on σ_{-j}^*: as a result, aggregate CSR investments are always increasing in $\sum_{j=1,2} s_j^{\mathcal{R}}$.

Equilibrium ownership. By the same steps of the main analysis, it follows that the equilibria of the game are as follows:

- *Symmetric equilibrium with no SRI.* An equilibrium where $\mathcal{R}$ investors do not participate in the financial market, and $\mathcal{N}$ investors buy shares in both firms, exist if and only if $\kappa \leq \widehat{\kappa}(\sigma_0, 0)$, where $\sigma_0 \equiv \sigma_j^*(0,0))$ and $\widehat{\kappa}(\cdot)$ is the function defined in (12), for $N = 2$.

- *Symmetric equilibrium with SRI.* In the equilibrium with SRI without tilting, both firms invest

$$\sigma^* = \chi \frac{8\kappa t(3-\chi) + 18\kappa\lambda(1-\chi) - 9\lambda^2(1-\chi)^2}{18c\kappa(1-\chi) - 9\lambda^2(1-\chi)^2\chi + \kappa t\chi(27-7\chi)}.$$

After some algebra, we find that this equilibrium exists if and only if $\kappa > \widehat{\kappa}(\sigma_0, 0)$.

- *Asymmetric equilibria with $\underline{n} = \overline{n} = 1$.* Asymmetric equilibria in which all $\mathcal{R}$ investors buy shares in the same firm j, and all $\mathcal{N}$ investors buy shares in the other firm $-j$, exist if and only if

$$\chi > 1/2 \quad \text{and} \quad \widehat{\kappa}(\sigma_j^*(1,0),1) < \kappa < \widehat{\kappa}(\sigma_{-j}^*(1,0),1)$$

- *Asymmetric equilibria with $\underline{n} = 0$ and $\overline{n} = 1$.* In the asymmetric equilibria where all $\mathcal{R}$ investors buy shares in the same firm j and $\mathcal{N}$ investors buy shares in both firms, firms' CSR policies are

$$\sigma_j^* = \frac{4\chi(-2\kappa t^2(3-\chi)\chi(27-7\chi) + 9c(1-\chi)(4\kappa t(3-\chi) + 18\kappa(1-\chi)\lambda - 9(1-\chi)^2\lambda^2))}{324c^2\kappa(1-\chi)^2 - \kappa t^2(27-7\chi)^2\chi^2 - 324c(1-\chi)^3\chi\lambda^2},$$

and

$$\sigma_{-j}^* = \frac{2t\chi(72c\kappa(3-\chi)(1-\chi) + 9(1-\chi)^2\chi(3+\chi)\lambda^2 - 2\kappa\chi(27-7\chi)(2t(3-\chi) + 9(1-\chi)\lambda))}{324c^2\kappa(1-\chi)^2 - \kappa t^2(27-7\chi)^2\chi^2 - 324c(1-\chi)^3\chi\lambda^2}.$$

After some algebra, we find that these equilibria exist if either

$$\chi < 1/2 \quad \text{and} \quad \kappa > \widehat{\kappa}(\sigma_0, 0),$$

or

$$\chi > 1/2 \quad \text{and} \quad \widehat{\kappa}(\sigma_0, 0) < \kappa < \widehat{\kappa}(\sigma_j^*, 1).$$

However, as σ_j^* depends on κ nonlinearly, it is impossible to solve the latter inequality for κ analytically.

- *Asymmetric equilibria with $\underline{n} = 1$ and $\overline{n} = 2$.* In the asymmetric equilibria where $\mathcal{R}$ investors buy shares in both firms, and all $\mathcal{N}$ investors buy shares in one firm j only, firms' CSR investments are

$$\sigma_j^* = \frac{-2\kappa t\chi(27-7\chi)(4t(3-\chi)\chi + 9(1-\chi)\lambda) + 36c(1-\chi)(4\kappa t(3-\chi)\chi + 9\kappa(1-\chi)(2\chi-1)\lambda + 9(1-\chi)^2\chi\lambda^2)}{324c^2\kappa(1-\chi)^2 - \kappa t^2(27-7\chi)^2\chi^2 - 324c(1-\chi)^3\chi\lambda^2},$$

and

$$\sigma^*_{-j} = \frac{36c\kappa(1-\chi)(4t(3-\chi)\chi + 9(1-\chi)\lambda) + 2\chi(9(1-\chi)^2\lambda^2(t\chi(3+\chi) - 18(1-\chi)\lambda) - \kappa t(27-7\chi)(4t(3-\chi)\chi + 9(1-\chi)(2\chi-1)\lambda))}{324c^2\kappa(1-\chi)^2 - \kappa t^2(27-7\chi)^2\chi^2 - 324c(1-\chi)^3\chi\lambda^2}.$$

After some algebra, we find that these equilibria exist if and only if

$$\chi > 1/2 \quad \text{and} \quad \kappa > \widehat{\kappa}(\sigma^*_{-j}(1,0),1).$$

It is straightforward to see that $\widehat{\kappa}(\sigma^*_{-j}(1,0),1) > \widehat{\kappa}(\sigma_0,0)$, and

$$\widehat{\kappa}(\sigma^*_j(1,0),1) < \widehat{\kappa} \Leftrightarrow \lambda > \widehat{\lambda}, \tag{C.1}$$

where[1]

$$\widehat{\lambda} \equiv \frac{c}{2\chi} + t\frac{14\chi + t^2(27-7\chi)^2\chi^2(3+\chi)(2\chi-1) + 324c^2(1-\chi)^2(3-53\chi+14\chi^2) - 18ct(1-\chi)\chi(27-7\chi)(27-55\chi+16\chi^2) - 54}{11664c^2\lambda(1-\chi)^3}.$$

Hence, we have to distinguish two cases:

Case (i): $\chi < 1/2$. In this case, we have:

- For $\kappa \leq \widehat{\kappa}(\sigma_0,0)$, the unique equilibrium of the game is the symmetric equilibrium with no SRI.
- For $\kappa > \widehat{\kappa}(\sigma_0,0)$, the game exhibits a symmetric equilibrium with SRI, and the asymmetric equilibria with $\underline{n} = 0$ and $\overline{n} = 1$.

When multiple equilibria coexist, we have:

- Aggregate investments $\sum_{j=1,2}\sigma^*_j$ are higher, and the probability with which both firms are brown $(1-\sigma^*_1)(1-\sigma^*_2)$ is lower, in the asymmetric equilibria than in the symmetric equilibrium.
- $\mathcal{R}$ investors are better off in the asymmetric equilibria than in the symmetric equilibrium, whereas the opposite holds for $\mathcal{N}$ investors.[2] Total investor welfare, defined as $\chi\sum_{j=1,2}\alpha^{\mathcal{R}}_j u^{\mathcal{R}}_j + (1-\chi)\sum_{j=1,2}\alpha^{\mathcal{N}}_j u^{\mathcal{N}}_j$ is higher in the symmetric equilibrium.

[1] This restriction is in general compatible with the parametric assumptions stated above.
[2] The payoff of $\mathcal{R}$ and $\mathcal{N}$ investors in firm j in (any) equilibrium write as

$$u^{\mathcal{R}}_j = \frac{1}{2\kappa}\left(\frac{\kappa - (1-\chi)\alpha^{\mathcal{N}}_j\lambda(1-\sigma^*_j)}{\chi\alpha^{\mathcal{R}}_j + (1-\chi)\alpha^{\mathcal{N}}_j}\right)^2, \quad \text{and} \quad u^{\mathcal{N}}_j = \frac{1}{2\kappa}\left(\frac{\kappa + \chi\alpha^{\mathcal{R}}_j\lambda(1-\sigma^*_j)}{\chi\alpha^{\mathcal{R}}_j + (1-\chi)\alpha^{\mathcal{N}}_j}\right)^2,$$

respectively. The average (or expected) payoffs are then $\sum_{j=1,2}\alpha^{\mathcal{R}}_j u^{\mathcal{R}}_j$ and $\sum_{j=1,2}\alpha^{\mathcal{N}}_j u^{\mathcal{N}}_j$, respectively.

Case (ii): $\chi > 1/2$. Omitting the asymmetric equilibria with $\underline{n} = 0$ and $\overline{n} = 1$ considered in the previous case (whose existence conditions cannot be characterized in this region of parameters), and assuming (C.1), we have:

- For $\kappa < \widehat{\kappa}(\sigma_j^*(1,0),1)$, the unique equilibrium of the game features no SRI.
- For $\kappa \in (\widehat{\kappa}(\sigma_j^*(1,0),1), \widehat{\kappa}(\sigma_0,0))$, the symmetric equilibrium with no SRI coexists with asymmetric equilibria with $\underline{n} = \overline{n} = 1$. In these asymmetric equilibria: aggregate investments $\sum_{j=1,2}\sigma_j^*$ are higher, the probability with which exactly one firm is green $\sum_{j=1,2}\sigma_j^*(1-\sigma_{-j}^*)$ is higher, and the probability with which both firms are brown $(1-\sigma_1^*)(1-\sigma_2^*)$ is lower, compared to the symmetric equilibrium with no SRI.
- For $\kappa \in (\widehat{\kappa}(\sigma_0,0), \widehat{\kappa}(\sigma_{-j}^*(1,0),1)$, the asymmetric equilibria with $\underline{n} = \overline{n} = 1$ coexist with the symmetric equilibrium with SRI. In the asymmetric equilibria, aggregate investments $\sum_{j=1,2}\sigma_j^*$ are lower compared to the symmetric equilibrium if and only if

$$\kappa > \widetilde{\kappa} \equiv \frac{(1-\chi)\chi\lambda(18c(1-\chi)+t\chi(3+\chi)-9(1-\chi)\lambda)}{(2\chi-1)(18c(1-\chi)+t\chi(27-7\chi))} \in (\widehat{\kappa}(\sigma_0,0), \widehat{\kappa}(\sigma_{-j}^*(1,0),1)).$$

- For $\kappa > \widehat{\kappa}(\sigma_{-j}^*(1,0),1)$, the symmetric equilibrium with SRI coexist with asymmetric equilibria with $\underline{n} = 1$ and $\overline{n} = 2$. In the asymmetric equilibria, aggregate investments $\sum_{j=1,2}\sigma_j^*$ are lower than in the symmetric equilibrium.

Hence, as in the main model, we find that, under (C.1), asymmetric equilibria in which $\mathcal{R}$ investors participate in the financial market exist for a larger set of parameter values than symmetric equilibria in which $\mathcal{R}$ participate in the financial market. Moreover, asymmetric equilibria entail a lower level of aggregate investments than symmetric equilibria when (i) $\chi > 1/2$, and (ii) $\kappa > \widetilde{\kappa}$.

C.2 Heterogeneous marginal costs

In the baseline model, we assume that firms' marginal cost is invariant with respect to the production technology. This assumption simplifies the analysis because the only extra cost of being green is the investment cost incurred to develop the green technology. In a more general model, the green and the brown product could have different marginal costs. Suppose, for example, that firm j's unit cost is γ_{a_j}, with $\gamma_1 > \gamma_0$ so that the green product is more expensive to produce. Whenever all firms use the same technology, or both technologies are employed by at least two firms, all firms keep making zero profits. If there is one green firm (and multiple brown firms), then the green monopolist is the only firm to make positive profits provided it has a comparative advantage over its brown rivals for $\mathcal{R}$ consumers, i.e., $\gamma_1 - \gamma_0 < \lambda$. If, on the contrary, there is a monopolist brown firm (and multiple green firms), then this brown firm would be the only one making positive profits for all $\gamma_1 > \gamma_0$ (as it can price slightly below γ_1 and serve $\mathcal{N}$ consumers). Hence, for $\gamma_1 - \gamma_0 \in (0, \lambda)$, each firm's expected profit equals

$$\mathbb{E}[\Pi_j] = \pi_1^m \sigma_j \prod_{-j \neq j} (1 - \sigma_{-j}) + \pi_0^m (1 - \sigma_j) \prod_{-j \neq j} \sigma_{-j} - C(\sigma_j), \tag{C.2}$$

with π_a^m denoting the monopoly profit when the monopolist has technology $a \in \{0, 1\}$. The optimal CSR policy is then given as

$$\sigma_j = \frac{1}{c} \left[\eta \lambda s_j^{\mathcal{R}} + \pi_1^m \prod_{-j \neq j} (1 - \sigma_{-j}) - \pi_0^m \prod_{-j \neq j} \sigma_{-j} \right]. \tag{C.3}$$

The base model corresponds to the case with $\pi_0^m = 0$. As π_0^m grows larger, an increase in any σ_{-j} has a stronger effect in reducing the optimal σ_j, and so the strategic substitutability strengthens. As a result, the crowding-out effect of the concentration of SRI in a subset of firms on the CSR policies of excluded firms is even stronger.

C.3 Robustness of strategic substitutability in CSR investments

A driving force for our main results is that firms' CSR policies are strategic substitutes. In the baseline model, we capture this channel in a stark framework featuring perfect Bertrand competition. However, we can easily relax this assumption and consider more general settings. A sufficient condition for CSR policies to be strategic substitutes is that the (average) extra profit that any firm obtains from being green decreases in the number of green rivals. This condition is satisfied in many settings, including the Hotelling model analyzed in the Online Appendix.

To see this formally, we can express the expected profit for firm j as

$$\mathbb{E}[\Pi_j] = \sigma_j \mathbb{E}[\tilde{\Pi}_j | a_j = 1] + (1 - \sigma_j)\mathbb{E}[\tilde{\Pi}_j | a_j = 0] - C(\sigma_j), \tag{C.4}$$

where $n_{-j} \in \{0, \ldots, N-1\}$ denotes the number of green firm j's rivals. We have that

$$\mathbb{E}[\tilde{\Pi}_j | a_j = a] = \sum_{z=0}^{N-1} \Pr[n_{-j} = z]\mathbb{E}[\tilde{\Pi}_j | a_j = a, n_{-j} = z], \tag{C.5}$$

for $a \in \{0, 1\}$.[3] Maximizing $\mathbb{E}[\Pi_j]$ with respect to σ_j then gives

$$\sigma_j = \frac{1}{c} \sum_{z=0}^{N-1} \Pr[n_{-j} = z]\Delta\pi(z), \tag{C.6}$$

where

$$\Delta\pi(z) \equiv \mathbb{E}[\tilde{\Pi}_j | a_j = 1, n_{-j} = z] - \mathbb{E}[\tilde{\Pi}_j | a_j = 0, n_{-j} = z] \tag{C.7}$$

is the (average) extra profit from being green when z rivals are green.

We obtain strategic substitutability whenever the expression in the RHS of Equation (C.6) is decreasing in any $\sigma_{j'}$. Denoting by $n_{-j,j'}$ the number of green firms excluding j and j', whose probability distribution does not depend on $(\sigma_j, \sigma_{j'})$, we have

$$\Pr[n_{-j} = z] = \sigma_{j'} \Pr[n_{-j,j'} = z - 1] + (1 - \sigma_{j'}) \Pr[n_{-j,j'} = z] \quad \forall z \geq 1, \tag{C.8}$$

$$\Pr[n_{-j} = 0] = (1 - \sigma_{j'}) \Pr[n_{-j,j'} = 0] \quad \text{for } z = 0. \tag{C.9}$$

Differentiating the RHS of Equation (C.6) with respect to $\sigma_{j'}$ and rearranging gives

$$\frac{\partial \sigma_j}{\partial \sigma_{j'}} = -\frac{1}{c} \sum_{z=0}^{N-2} \Pr[n_{-j,j'} = z] \left[\Delta\pi(z) - \Delta\pi(z+1)\right]. \tag{C.10}$$

A sufficient condition for this derivative to be negative is thus $\Delta\pi(z) \geq \Delta\pi(z+1)$, with strict inequality for at least one z.

[3] We still take expectations in this expression because the product market competition model may also include additional sources of uncertainty, such as uncertain demand or production costs.

D Alternative modeling of externalities

D.1 Proportional externality

In the base analysis, we supposed for simplicity that the negative externality generated by brown firms does not depend on how much they produce. Here we show that considering an output-related externality instead does not alter the qualitative properties of the equilibrium configuration of the game: in particular, under some parametric restrictions, there is a region of parameters where the only equilibria with SRI exhibit tilting (i.e., are asymmetric equilibria).

Model Setup

Suppose that the negative externality generated by brown firms is proportional to their output — i.e., of magnitude $\lambda(1-a_j)x_j^*$, with x_j^* denoting the equilibrium demand for firm j's product. As each brown firm's output in equilibrium depends on how many of its rivals are green or brown, under this assumption, the expected negative externality (internalized by responsible consumers and investors) generated by each firm also depends on its rivals' CSR policies, which considerably complicates the analysis.

Hence, in what follows, we consider for simplicity a model with $N = 2$ strategic firms and a fringe of brown firms. In this setting, it is easy to see that, as in the base model, brown goods are always priced at cost in equilibrium. To make things interesting, we consider the following tie-breaking condition: (non-responsible) consumers prefer the brown goods sold by strategic firms to those sold by fringe firms (otherwise, each strategic brown firm sells a negligible output and generates negligible externality). Finally, we assume responsible consumers *boycott* brown products to simplify the analysis.

Equilibrium Analysis

Product market equilibrium and CSR policies. Let $x^*(\rho_j)$ denote each non-responsible consumer's demand for brown products. In equilibrium, those products are always priced at cost: $\rho_j^* = \gamma$. Hence, we have:

- If both firms are brown, assuming that non-responsible consumers are equally split across brown firms, each of them sells

$$x_j^* = \frac{1-\chi_{co}}{2} x^*(\gamma)$$

- If one firm, say firm $-j$, is green, it optimally charges $\rho_{-j}^* > \gamma$ and attracts only responsible consumers. Hence, the brown firm j serves all non-responsible consumers, i.e. sells

$$x_j^* = (1-\chi_{co}) x^*(\gamma).$$

The CSR investment stage is, of course, as in the base model. For $N = 2$, we obtain a unique equilibrium for all pairs $(s_1^{\mathcal{R}}, s_2^{\mathcal{R}}) \in [0,1]^2$, given by

$$\sigma_j^* = \frac{c(\pi^m + \eta\lambda s_j^{\mathcal{R}}) - \pi^m(\pi^m + \eta\lambda s_{-j}^{\mathcal{R}})}{c^2 - (\pi^m)^2}, \quad \forall j = 1,2,$$

with σ_j^* being increasing in $s_j^{\mathcal{R}}$ and decreasing in $s_{-j}^{\mathcal{R}}$. As seen in the Hotelling model, in a duopoly, aggregate greenness is always increasing in $\sum_{j=1,2} s_j^{\mathcal{R}}$.

Equilibrium ownership. Given the product market equilibrium outcome, the expected penalty per share suffered by $\mathcal{R}$ investors in firm j equals

$$\lambda(1-\sigma_j)\left[\sigma_{-j}(1-\chi_{co})x^*(\gamma) + (1-\sigma_{-j})\frac{1-\chi_{co}}{2}x^*(\gamma)\right] = \lambda\frac{x^*(\gamma)}{2}(1-\sigma_j)(1+\sigma_{-j}).$$

Hence, each $\mathcal{R}$ investor demand shares

$$s_{ij}^{\mathcal{R}} = \max\left\{\frac{1}{\kappa}\left[\mathbb{E}[\Pi_j] - p_j - \lambda\frac{1-\chi_{co}}{2}x^*(\gamma)(1-\sigma_j)(1+\sigma_{-j})\right], 0\right\},$$

in the firm j that maximizes the expression in square brackets. The shares demand by $\mathcal{N}$ investors is, of course, as in the base model.

Note that if other $\mathcal{R}$ investors are expected to target a firm j, under this model specification, there is a further reason why any $\mathcal{R}$ investor has stronger incentives to buy shares in j as well: if this firm is expected to be green with high probability, then, contingent on being brown, its rival will sell more in expectation (as it will serve all non-responsible consumers with higher probability) and thereby will generate more externality. By exacerbating strategic complementarities in $\mathcal{R}$ investors' choices, this alternative model specification may widen the scope for the existence of asymmetric equilibria.

To simplify notation, in what follows, we denote

$$z \equiv \frac{1-\chi_{co}}{2} x^*(\gamma).$$

Given the fractions $\alpha_j^{\mathcal{R}}$ and $\alpha_j^{\mathcal{N}}$ of $\mathcal{R}$ and $\mathcal{N}$ investors that buy shares in each firm j, the market clearing condition of firm $j \in \{1,2\}$ is

$$\chi_{in}\alpha_j^{\mathcal{R}}\frac{1}{\kappa}\left[\mathbb{E}[\Pi_j] - p_j - \lambda z(1-\sigma_j)(1+\sigma_{-j})\right] + (1-\chi_{in})\alpha_j^{\mathcal{N}}\frac{1}{\kappa}\left[E[Pi_j] - p_j\right] = 1.$$

The equilibrium characterization then mirrors that of the base model:

- *Symmetric equilibrium with no SRI.* A symmetric equilibrium with $\alpha_j^{\mathcal{R}} = s_j^{\mathcal{R}} = 0$ for $j = 1,2$, and

  $$\sigma_j = \sigma_0 \equiv \frac{\pi^m}{c+\pi^m} \quad \forall j = 1,2,$$

  exists if and only if

  $$\kappa \leq \kappa_0 \equiv \frac{\lambda z(1-\chi)(1-\sigma_0^2)}{2}$$

- *Symmetric equilibrium with SRI.* A symmetric equilibrium in which $\mathcal{R}$ investors participate in the stock market, where $\alpha_j^{\mathcal{R}} = \alpha_j^{\mathcal{N}} = 1/2$ for $j = 1,2$, and firms choose in equilibrium

  $$\sigma_j^* = \sigma^* \equiv \frac{(c+\pi^m)\kappa - \frac{1}{2}\sqrt{4(c+\pi^m)^2\kappa^2 - 4\eta\lambda^2(1-\chi)\chi z\left(2\kappa(\eta\lambda\chi + \pi^m) - \eta\lambda^2(1-\chi)\chi z\right)}}{\eta\lambda^2(1-\chi)\chi z} \quad \forall j = 1,2,$$

  exists if and only if $\kappa > \kappa_0$.

- *Asymmetric equilibria with $\underline{n} = \overline{n} = 1$.* Asymmetric equilibria with $\alpha_j^{\mathcal{R}} = s_j^{\mathcal{R}} = 1$ and $\alpha_{-j}^{\mathcal{R}} = s_{-j}^{\mathcal{R}} = 0$ (i.e., $\alpha_j^{\mathcal{N}} = 0$ and $\alpha_{-j}^{\mathcal{N}} = 1$), where

  $$\sigma_j^* = \overline{\sigma} \equiv \frac{c(\pi^m + \eta\lambda) - (\pi^m)^2}{c^2 - (\pi^m)^2}, \quad \sigma_{-j}^* = \underline{\sigma} \equiv \frac{\pi^m(c - (\pi^m + \eta\lambda))}{c^2 - (\pi^m)^2},$$

exist if and only if

$$\chi_{in} > \frac{1}{2} \quad \text{and} \quad \frac{\chi_{in}(1-\chi_{in})\lambda z(1-\overline{\sigma})(1+\underline{\sigma})}{2\chi_{in}-1} \equiv \underline{\kappa} < \kappa < \overline{\kappa} \equiv \frac{\chi_{in}(1-\chi_{in})\lambda z(1-\underline{\sigma})(1+\overline{\sigma})}{2\chi_{in}-1},$$

with $\overline{\kappa} > \widehat{\kappa}$, and $\underline{\kappa} < \widehat{\kappa}$ if and only if

$$\chi > \frac{(c-\pi^m)^2(c+2\pi^m)}{2\eta\lambda(\pi^m(3\pi^m+\eta\lambda)+c(2\pi^m-c)}, \quad \text{and} \quad \eta > \frac{(c-\pi^m)(c+3\pi^m-\sqrt{c^2+4c\pi^m+5(\pi^m)^2})}{2\pi^m\lambda} > 0 \tag{D.1}$$

- *Asymmetric equilibria with $\underline{n}=0$ and $\overline{n}=1$.* Asymmetric equilibria where $\mathcal{N}$ investors are in both firms, and all $\mathcal{R}$ investors buy shares in the same firm j, exist if and only if: either

$$\chi_{in} < \frac{1}{2} \quad \text{and} \quad \kappa > \frac{(1-\chi_{in})\lambda z(1-\sigma_j^*)(1+\sigma_{-j}^*)}{2}$$

or

$$\chi_{in} > \frac{1}{2} \quad \text{and} \quad \frac{(1-\chi_{in})\lambda z(1-\sigma_j^*)(1+\sigma_{-j}^*)}{2} < \kappa < \frac{\chi_{in}(1-\chi_{in})\lambda z(1-\sigma_j^*)(1+\sigma_{-j}^*)}{2\chi_{in}-1}$$

with $(\sigma_j^*, \sigma_{-j}^*)$ being the CSR policies chosen in equilibrium by the two firms, which depend themselves on κ through $\mathcal{R}$ investors' shares demand.

- *Asymmetric equilibria with $\underline{n}=1$ and $\overline{n}=2$.* Asymmetric equilibria where $\mathcal{R}$ investors are in both firms, and all $\mathcal{N}$ investors buy shares in the same firm j, exist if and only if: either

$$\chi_{in} < \frac{1}{2} \quad \text{and} \quad \frac{(1-\chi_{in})\lambda z(1-\sigma_j^*)(1+\sigma_{-j}^*)}{2} < \kappa < \frac{\chi_{in}(1-\chi_{in})\lambda z(1-\sigma_j^*)(1+\sigma_{-j}^*)}{1-2\chi_{in}},$$

or

$$\chi_{in} > \frac{1}{2} \quad \text{and} \quad \kappa > \frac{(1-\chi_{in})\lambda z(1-\sigma_j^*)(1+\sigma_{-j}^*)}{2},$$

where the equilibrium CSR policies $(\sigma_j^*, \sigma_{-j}^*)$ again depend themselves on κ.

Thus, under the conditions given in (D.1), there is a region of parameters where the only equilibria with SRI are asymmetric equilibria.

D.2 Provision of public goods

In the main model, $a_j = 0$ corresponds to a brown production process that generates a negative externality of absolute value λ, which is eliminated by developing a green production technology ($a_j = 1$). In such a setting, responsible agents have lower valuations than non-responsible ones for brown firms' products and shares. This implies that in equilibrium $\mathcal{N}$ investors can crowd out $\mathcal{R}$ investors, but not the other way around.

Here we show that our results are robust under the opposite assumption. Namely, suppose the status-quo technology ($a_j = 0$) entails no externality, but developing a novel technology ($a_j = 1$) brings up a positive externality λ, which is internalized by responsible agents. As in this setting, they accordingly have higher valuations for firms' shares, compared to $\mathcal{N}$ investors, in this model, $\mathcal{R}$ investors can crowd out $\mathcal{N}$ investors from the market.

Model Setup

For given prices $\vec{\rho}$ and types $\vec{a}$, consumer h's demand for firm j's product, denoted by x_{hj}, now solves:

$$\max_{(x_{hj})_{j=1,\dots,N} \geq 0} u\left(\sum_{j=1}^{N} x_{hj}\right) + \sum_{j=1}^{N} [\mathbb{1}_{h,\mathcal{R}} \lambda a_j - \rho_j] x_{hj}.$$

Similarly, for a given vector of CSR policies $\vec{\sigma}$, investor i now solves:

$$\max_{s_{ij} \geq 0} \sum_j s_{ij} \mathbb{E}\left[\Pi_j - p_j + \mathbb{1}_{i,\mathcal{R}} \lambda a_j\right] - K(\iota' \vec{s}_i).$$

Hence, the proportional control assumption implies that firm j chooses CSR policies solving:

$$\max_{\sigma_j \in [0,1]} \mathbb{E}\left[\Pi_j\right] + \lambda \sigma_j \eta s_j^{\mathcal{R}},$$

where expected profits are as in the main model.

Equilibrium Analysis

In the product market, developing the new technology still allows a firm to sell a product perceived as vertically differentiated by a fraction of consumers. Hence, a firm makes positive profits, denoted by π^m, if and only if it is the unique firm producing with the new technology.

Moving backward to the investment stage, it is straightforward to see that firm j's profit is still maximized for σ_j solving Equation (7).

Finally, in the investment stage, we obtain the following shares' demand:

$$s_{ij} = \begin{cases} \max\{\frac{1}{\kappa}\left[\mathbb{E}[\Pi_{j^*}] - p_{j^*} + \mathbb{1}_{i,\mathcal{R}}\lambda\sigma_{j^*}\right], 0\} & \text{for } j^* = \text{argmax}_j\{\mathbb{E}[\Pi_j] - p_j + \mathbb{1}_{i,\mathcal{R}}\lambda\sigma_j\} \\ 0 & \text{for } j \neq j^*. \end{cases}$$

The market clearing conditions determine the share prices in equilibrium:

$$p_j = \mathbb{E}[\Pi_j] - \underbrace{\frac{\kappa}{\chi_{in}\alpha_j^{\mathcal{R}} + (1-\chi_{in})\alpha_j^{\mathcal{N}}}}_{\text{liquidity discount}} + \underbrace{\frac{\lambda\sigma_j\chi_{in}\alpha_j^{\mathcal{R}}}{\chi_{in}\alpha_j^{\mathcal{R}} + (1-\chi_{in})\alpha_j^{\mathcal{N}}}}_{\text{greenium}},$$

where $\sum_j \alpha_j^{\mathcal{R}} = 1$ in all equilibria, whereas $\sum_j \alpha_j^{\mathcal{N}} \in \{0,1\}$, as $\alpha_j^{\mathcal{N}} = 0$ for all j in an equilibrium where $\mathcal{N}$ investors, having lower valuations than $\mathcal{R}$ investors, are crowded out from the market.

The characterization of equilibria then mirrors the one in the base model. Define

$$\kappa_1 \equiv \frac{\lambda\chi_{in}\sigma_1}{N},$$

where $\sigma_1 \in (0,1)$ is the unique solution of

$$\sigma_1 = \frac{1}{c}\left[\pi^m(1-\sigma_1)^{N-1} + \eta\lambda\right],$$

and corresponds to firms' symmetric CSR investment if $\mathcal{N}$ investors are crowded out (hence, $s_j^{\mathcal{R}} = 1$ for all j).

Then, by the very same steps of the main analysis, it follows that a symmetric equilibrium always exists and is unique:

- For $\kappa \leq \kappa_1$, $\mathcal{N}$ investors are crowded out ($\alpha_j^{\mathcal{N}} = 0$ for all j), $\mathcal{R}$ investors are equally split across firms ($\alpha_j^{\mathcal{R}} = 1/N$ for all j), and all firms choose $\sigma_j = \sigma_1$.

- For $\kappa > \kappa_1$, both $\mathcal{N}$ and $\mathcal{R}$ investors are equally split across firms ($\alpha_j^{\mathcal{R}} = \alpha_j^{\mathcal{N}} = 1/N$ for all j), $\mathcal{R}$ investors overall hold shares $s_j^{\mathcal{R}} = s^{\mathcal{R}*}$ in all firms, and all firms choose $\sigma_j = \sigma^*$, where $(\sigma^*, s^{\mathcal{R}*})$

solve

$$\begin{cases} \sigma^* = \frac{1}{c}\left[\eta\lambda s^{\mathcal{R}*} + \pi^m(1-\sigma^*)^{N-1}\right]; \\ s^{\mathcal{R}*} = \frac{\chi_{in}}{N}\frac{1}{\kappa}\left[N\kappa + \lambda(1-\chi_{in})\sigma^*\right]. \end{cases}$$

Also, asymmetric equilibria are characterized as in the main model. Consider the most extreme asymmetric equilibria, namely those in which $\mathcal{R}$ investors entirely own firms $j \le n$, whereas $\mathcal{N}$ investors entirely own the other firms $j > n$, for $n \in \{1, \dots, N-1\}$. These equilibria feature the same CSR policies of the main model: namely, $\sigma_j = \overline{\sigma}$ for $j \le n$ and $\sigma_j = \underline{\sigma}$ for $j > n$, obtained from System (B.33), which implies that the results of Proposition 4 hold, and now exist for

$$\chi_{in} < \frac{n}{N} \quad \text{and} \quad \frac{\chi_{in}(1-\chi_{in})\lambda\underline{\sigma}}{n - N\chi_{in}} \equiv \underline{\kappa} < \kappa < \overline{\kappa} \equiv \frac{\chi_{in}(1-\chi_{in})\lambda\overline{\sigma}}{n - N\chi_{in}}.$$

Thus, the concentration of $\mathcal{R}$ investors in a subset of firms may still arise in equilibrium even when these investors have higher valuations compared to $\mathcal{N}$ investors.

Complete characterization for N=2. Consider the game with $N = 2$ firms (and a fringe of non-strategic rivals who always employ the status-quo technology) and assume $\chi_{in} < 1/2$ and $c < \sqrt{2}\pi^m$.[4]

For $\kappa < \min(\underline{\kappa}, \kappa_1)$,[5] the unique equilibrium features exclusion of $\mathcal{N}$ investors and, accordingly, $\sigma_j = \sigma_1$ for all j.

For $\kappa \in (\min(\underline{\kappa}, \kappa_1), \max(\underline{\kappa}, \kappa_1))$:

- If $\underline{\kappa} < \kappa_1$,[6] the symmetric equilibrium with exclusion coexists with the extreme asymmetric equilibria characterized above — i.e., $\alpha_j^{\mathcal{R}} = \alpha_{-j}^{\mathcal{N}} = 1$, and $(\sigma_j, \sigma_{-j}) = (\overline{\sigma}, \underline{\sigma})$;
- If instead $\underline{\kappa} > \kappa_1$, the symmetric equilibrium without exclusion of $\mathcal{N}$ investors[7] coexists with

[4]The restriction $c < \sqrt{2}\pi^m$ is just a sufficient condition for $\widetilde{\kappa} \in (\kappa_1, \overline{\kappa})$, with $\widetilde{\kappa}$ defined below.

[5]With $N = 2$,

$$\kappa_1 \equiv \frac{\lambda\chi_{in}(\pi^m + \eta\lambda)}{2(c + \pi^m)}.$$

[6]That is, for

$$\lambda > \frac{\pi^m(c - \pi^m)}{c(1 - 2\chi_{in}) + \pi^m} \quad \text{and} \quad \eta > \frac{\pi^m(c - \pi^m)}{\lambda(c(1 - 2\chi_{in}) + \pi^m)}.$$

[7]For $N = 2$, we have

$$\sigma^* = \frac{2\kappa(\eta\lambda\chi_{in} + \pi^m)}{2(c + \pi^m)\kappa - \eta\lambda^2\chi_{in}(1 - \chi_{in})},$$

asymmetric equilibria where $\mathcal{R}$ investors buy shares in both firms, and all $\mathcal{N}$ investors buy shares of the same firm.[8]

For $\kappa > \max(\underline{\kappa}, \kappa_1)$, defining

$$\widetilde{\kappa} \equiv \frac{\pi^m \lambda(1-\chi_{in})\chi_{in}(c-\eta\lambda)}{c(c+\pi^m)(1-2\chi_{in})},$$

the symmetric equilibrium without exclusion of $\mathcal{N}$ investors coexists with the following equilibria:

- for $\kappa < \widetilde{\kappa}$, the extreme asymmetric equilibria, and the equilibria where $\mathcal{R}$ investors buy shares in both firms and $\mathcal{N}$ investors only buy shares in one firm;
- for $\kappa \in (\widetilde{\kappa}, \overline{\kappa})$, the extreme asymmetric equilibria only;
- for $\kappa > \overline{\kappa}$, asymmetric equilibria where $\mathcal{N}$ investors buy shares in both firms, and all $\mathcal{R}$ investors buy shares of the same firm.[9]

Hence, provided the fraction of $\mathcal{R}$ investors is not too large,[10] and trading costs are non-negligible, the game is likely to admit equilibria where $\mathcal{R}$ investors concentrate in a subset of firms.

E Reverse timing

In the baseline model, the firms' objective is determined by their ownership so that CSR policies are chosen after the trading stage. In this Appendix, we consider an alternative timing of the game and suppose that firms choose CSR policies to attract investors.

in this symmetric equilibrium without exclusion.

[8]In this equilibrium, $\alpha^{\mathcal{R}}_{-j} \equiv \alpha^{\mathcal{R}}$ and $\alpha^{\mathcal{N}}_{-j} = 0$, $\alpha^{\mathcal{R}}_{j} \equiv 1-\alpha^{\mathcal{R}}$ and $\alpha^{\mathcal{N}}_{j} = 1$, with

$$\alpha^{\mathcal{R}} = \frac{(c+\pi^m)\left(c\kappa - \eta\lambda^2\chi_{in}(1-\chi_{in})\right)}{\chi_{in}\left(2c^2\kappa + c\lambda(1-\chi_{in})(\pi^m-\eta\lambda) + 2\pi^m c\kappa - 2\pi^m\eta\lambda^2(1-\chi_{in})\right)},$$

and CSR policies

$$\sigma_j = \frac{\kappa(c(\eta\lambda(2\chi_{in}-1)+\pi^m) - 2\pi^m\eta\lambda(1-\chi_{in}))}{(c+\pi^m)\left(c\kappa - \eta\lambda^2\chi_{in}(1-\chi_{in})\right)}, \quad \sigma_{-j} = \frac{\eta\lambda+\pi^m}{c+\pi^m}.$$

[9]In this equilibrium, $\alpha^{\mathcal{R}}_{-j} = 0$ and $\alpha^{\mathcal{N}}_{-j} \equiv \alpha^{N}$, $\alpha^{\mathcal{R}}_{j} = 1$ and $\alpha^{\mathcal{N}}_{j} = 1-\alpha^{N}$, with

$$\alpha^{\mathcal{N}} = \frac{c^2\kappa - c\eta\lambda^2(1-\chi_{in})\chi_{in} - (\pi^m)^2\kappa}{(1-\chi_{in})\left(2c^2\kappa - c\lambda\chi_{in}(2\eta\lambda+\pi^m) - (\pi^m)^2(2\kappa-\lambda\chi_{in})\right)},$$

and CSR policies

$$\sigma_j = \frac{\kappa\left(c(2\eta\lambda\chi_{in}+\pi^m) - (\pi^m)^2\right)}{c^2\kappa - c\eta\lambda^2(1-\chi_{in})\chi_{in} - (\pi^m)^2\kappa}, \quad \sigma_{-j} = \frac{\pi^m\left(c\kappa - \eta\lambda^2(1-\chi_{in})\chi_{in} - \kappa(2\eta\lambda\chi_{in}+\pi^m)\right)}{c^2\kappa - c\eta\lambda^2(1-\chi_{in})\chi_{in} - (\pi^m)^2\kappa}.$$

[10]For $\chi_{in} > 1/2$ (no matter the value of c), the only asymmetric equilibria are those in which $\mathcal{R}$ investors buy shares in both firms and all $\mathcal{N}$ investors buy shares of the same firm, which exist for all $\kappa > \kappa_1$. Hence, there is never concentration of $\mathcal{R}$ investors in equilibrium.

E.1 Model Setup

Each firm j's owner has one share to sell and commits to σ_j to maximize the share price. After each owner commits to σ_j, shares are traded. Finally, CSR policies are implemented, product types $a_j \in \{0, 1\}$ realize, and firms set prices.

As we shall see, the equilibrium analysis becomes much more cumbersome under this alternative timing. Hence, in what follows, we again consider $N = 2$ strategic firms and a non-strategic fringe of brown firms, and we content ourselves to show that an equilibrium where all $\mathcal{R}$ investors buy shares in firm j and all $\mathcal{N}$ investors buy shares in the other firm $-j$ can also exist under this alternative timing of the game. Owners of ex-ante identical firms may optimally commit to different CSR policies to target different investors.

E.2 Equilibrium Analysis

The product market equilibrium is unchanged, as it only depends on the realized product types. Moving backward, we shall derive investors' demand for any given vector $\vec{\sigma}$ of CSR policies to which firms' owners committed in the first stage. It turns out that investors' optimal choices are also as in the main model. This is because, in both models, they take $\vec{\sigma}$ as given:[11] the fact that here $\vec{\sigma}$ is observed, whereas in the main model it is correctly conjectured in equilibrium, plays no role on the equilibrium path (and off-path events are immaterial to the analysis, given the continuum of investors).

Given the two firms' CSR policies (σ_j, σ_{-j}), in an asymmetric equilibrium where all $\mathcal{R}$ investors buy shares in firm j, and all $\mathcal{N}$ investors buy shares in firm $-j$ (i.e., $\alpha_j^{\mathcal{R}} = \alpha_{-j}^{\mathcal{N}} = 1$ and $\alpha_{-j}^{\mathcal{R}} = \alpha_j^{\mathcal{N}} = 0$), market clearing prices are thus

$$p_j^*(\sigma_j, \sigma_{-j}) = \mathbb{E}[\Pi_j(\sigma_j, \sigma_{-j})] - \frac{\kappa}{\chi_{in}} - \lambda(1 - \sigma_j),$$

and

$$p_{-j}^*(\sigma_j, \sigma_{-j}) = \mathbb{E}[\Pi_{-j}(\sigma_j, \sigma_{-j})] - \frac{\kappa}{1 - \chi_{in}}.$$

[11] Moreover, in both models, consumers take as given the shares' market clearing price and correctly anticipate firms' expected profits given $\vec{\sigma}$.

In such a candidate equilibrium, firm j's owner solves

$$\max_{\sigma_j} p_j^*(\sigma_j, \sigma_{-j}),$$

and similarly firm $-j$'s owner. As $\mathbb{E}[\Pi_j(\sigma_j, \sigma_{-j})] = \sigma_j(1-\sigma_{-j})\pi^m - \frac{c}{2}\sigma_j^2$, we obtain the FOCs

$$\sigma_j = \frac{1}{c}\left[(1-\sigma_{-j})\pi^m + \lambda\right],$$

and

$$\sigma_{-j} = \frac{1}{c}(1-\sigma_j)\pi^m,$$

from which we derive the candidate equilibrium CSR policies

$$\sigma_j^* = \frac{c(\pi^m+\lambda) - (\pi^m)^2}{c^2 - (\pi^m)^2}, \quad \sigma_{-j}^* = \frac{\pi^m(c - \pi^m - \lambda)}{c^2 - (\pi^m)^2},$$

with $1 > \sigma_j^* > \sigma_{-j}^* > 0$. Note that these correspond to the equilibrium CSR policies of the base model for $\eta = 1$. Hence, provided this equilibrium exists, it has the same implications emphasized in the main model.

Accordingly, firms' share prices in this candidate equilibrium are

$$p_j^* = \frac{-2c^4\lambda + c^3(\lambda+\pi^m)^2 + 2(\pi^m)^2c^2(\lambda - \pi^m) + (\pi^m)^4 c - 2(\pi^m)^4\lambda}{2\left(c^2 - (\pi^m)^2\right)^2} - \frac{\kappa}{\chi_{in}},$$

and

$$p_{-j}^* = \frac{(\pi^m)^2 c(c - \lambda - (\pi^m))^2}{2\left(c^2 - (\pi^m)^2\right)^2} - \frac{\kappa}{1-\chi_{in}},$$

with $p_j^* - p_{-j}^*$ being increasing in χ_{in}.

In the remainder of the analysis, we derive the existence conditions for this equilibrium. First, as seen in the paper, for $\mathcal{R}$ investors to optimally buy shares in firm j, and $\mathcal{N}$ investors to optimally buy shares in firm $-j$, it must be

$$\chi_{in} > \frac{1}{2} \quad \text{and} \quad \widehat{\kappa}(\sigma_j^*, 1) < \kappa < \widehat{\kappa}(\sigma_{-j}^*, 1). \tag{E.1}$$

Unlike in the base model, however, here, this is just a necessary existence condition for the equilibrium. The reason is that while choosing σ_j^* is the best response to σ_{-j}^* *locally*, i.e., when taking as granted that all $\mathcal{R}$ (resp. $\mathcal{N}$) investors will buy shares in firm j (resp. $-j$), each firm's owner may have profitable *global deviations*, consisting in triggering different purchasing behaviors by investors. All possible global deviations are listed below:

1. Firm j's global deviations consist in attracting $\mathcal{N}$ investors, and
   (a) either keep attracting also $\mathcal{R}$ investors, or
   (b) attract only $\mathcal{N}$ investors: in this case, $\mathcal{R}$ investors may either buy shares in the other firm $-j$, or be crowded out.
2. Firm $-j$'s global deviations consist in attracting $\mathcal{R}$ investors, and
   (a) either keep attracting also $\mathcal{N}$ investors, or
   (b) attract only $\mathcal{R}$ investors: in this case, $\mathcal{N}$ investors buy shares in the other firm j.

In what follows, we assume that whenever (following a global deviation) there is no demand for a firm j, its shares are sold at zero ($p_j = 0$) — this firm is still present in the product market, though. Note that deviations 1(a) and 2(a), which, as we shall see, are the more problematic for equilibrium existence, could be ignored by imposing market clearing conditions for both firms, which rule out the possibility for one firm to attract all investors (this entails removing the zero lower bound constraint on share prices).

Moreover, similar deviations should also be considered starting from a candidate symmetric equilibrium. We can thus argue that, rather than undermining the prevalence of asymmetric equilibria in the game, this alternative timing reduces the scope for equilibrium multiplicity.

We shall now consider each possible global deviation separately and provide conditions that rule out each. When all these conditions are jointly satisfied, the considered asymmetric equilibrium exists.

Deviation 1(a). If firm j can attract all investors, its market clearing price is[12]

$$p_j^d(\sigma_j, \sigma_{-j}^*) = \mathbb{E}[\Pi_j(\sigma_j, \sigma_{-j}^*)] - \kappa - \lambda(1 - \sigma_j)\chi_{in}.$$

[12]The superscript d, standing for *deviation*, is consistently used throughout.

Given that $p^d_{-j} = 0$, all investors optimally buy shares in firm j if both

$$\mathbb{E}[\Pi_j(\sigma_j, \sigma^*_{-j})] - p^d_j(\sigma_j, \sigma^*_{-j}) > \mathbb{E}[\Pi_{-j}(\sigma_j, \sigma^*_{-j})],$$

and

$$\mathbb{E}[\Pi_j(\sigma_j, \sigma^*_{-j})] - p^d_j(\sigma_j, \sigma^*_{-j}) - \lambda(1-\sigma_j) > \mathbb{E}[\Pi_{-j}(\sigma_j, \sigma^*_{-j})] - \lambda(1-\sigma^*_{-j}),$$

are satisfied. These two conditions boil down to

$$\kappa > \kappa_{j,1}(\sigma_j) \equiv \sigma^*_{-j}(1-\sigma_j)\pi^m - \frac{c}{2}(\sigma^*_{-j})^2 - \lambda\chi_{in}(1-\sigma_j),$$

and

$$\kappa > \kappa_{j,2}(\sigma_j) \equiv \sigma^*_{-j}(1-\sigma_j)\pi^m - \frac{c}{2}(\sigma^*_{-j})^2 - \lambda[1-\sigma^*_{-j} - (1-\chi_{in})(1-\sigma_j)].$$

The condition for the deviation to be profitable, $p^d_j(\sigma^*_j, \sigma^*_{-j}) > p^d_j(\sigma_j, \sigma^*_{-j})$, boils down to

$$\kappa > \kappa_{j,3}(\sigma_j) \equiv \frac{c\chi_{in}\left(c^2\sigma_j - c(\lambda+\pi^m) + (\pi^m)^2(1-\sigma_j)\right)^2}{2\left(c^2-(\pi^m)^2\right)^2}.$$

Hence, for a given vector of parameters, this deviation destroys the considered equilibrium if there exists a value σ^d_j such that $\kappa > \max\{\kappa_{j,1}(\sigma^d_j), \kappa_{j,2}(\sigma^d_j), \kappa_{j,3}(\sigma^d_j)\}$.

Deviation 1(b). If firm j attracts $\mathcal{N}$ investors only, its market clearing price is

$$p^d_j(\sigma_j, \sigma^*_{-j}) = \mathbb{E}[\Pi_j(\sigma_j, \sigma^*_{-j})] - \frac{\kappa}{1-\chi_{in}}.$$

Supposing such deviation is implementable, it would be profitable if σ_j satisfies $p^*_j(\sigma^*_j, \sigma^*_{-j}) < p^d_j(\sigma_j, \sigma^*_{-j})$. This admits no solution for all

$$\kappa > \underline{\kappa} \equiv \frac{\lambda\chi_{in}(1-\chi_{in})\left(2c^3 - c^2(\lambda+2\pi^m) - \lambda(\pi^m)^2\right)}{2c(2\chi_{in}-1)\left(c^2-(\pi^m)^2\right)} \in (\widehat{\kappa}(\sigma^*_j, 1), \widehat{\kappa}(\sigma^*_{-j}, 1)).$$

It follows that $\kappa > \underline{\kappa}$ is a sufficient condition to rule out the considered deviation.

Deviation 2(a). If firm $-j$ can attract all investors, its market clearing price is

$$p^d_{-j}(\sigma^*_j, \sigma_{-j}) = \mathbb{E}[\Pi_{-j}(\sigma^*_j, \sigma_{-j})] - \kappa - \lambda(1-\sigma_{-j})\chi_{in}.$$

Given that $p_j^d = 0$, all investors optimally buy shares in firm $-j$ if both

$$\mathbb{E}[\Pi_j(\sigma_j^*, \sigma_{-j})] < \mathbb{E}[\Pi_{-j}(\sigma_j^*, \sigma_{-j})] - p_{-j}^d(\sigma_j^*, \sigma_{-j}),$$

and

$$\mathbb{E}[\Pi_j(\sigma_j^*, \sigma_{-j})] - \lambda(1 - \sigma_j^*) < \mathbb{E}[\Pi_{-j}(\sigma_j^*, \sigma_{-j})] - p_{-j}^d(\sigma_j^*, \sigma_{-j}) - \lambda(1 - \sigma_{-j}),$$

are satisfied. These two conditions boil down to

$$\kappa > \kappa_{-j,1}(\sigma_{-j}) \equiv \sigma_j^*(1 - \sigma_{-j})\pi^m - \frac{c}{2}(\sigma_j^*)^2 - \lambda\chi_{in}(1 - \sigma_{-j}),$$

and

$$\kappa > \kappa_{-j,2}(\sigma_{-j}) \equiv \sigma_j^*(1 - \sigma_{-j})\pi^m - \frac{c}{2}(\sigma_j^*)^2 - \lambda[1 - \sigma_j^* - (1 - \chi_{in})(1 - \sigma_{-j})].$$

Moreover, the deviation is profitable if and only if σ_{-j} is such that $p_{-j}^d(\sigma_j^*, \sigma_{-j}) > p_{-j}^d(\sigma_j^*, \sigma_{-j}^*)$, which gives

$$\kappa > \kappa_{-j,3}(\sigma_{-j}) \equiv \frac{(1 - \chi_{in})\left(c\left(c^2\sigma_{-j} - c\pi^m + \pi^m(\lambda - \pi^m\sigma_{-j} + \pi^m)\right)^2 + 2\lambda(1 - \sigma_{-j})\chi_{in}\left(c^2 - (\pi^m)^2\right)^2\right)}{2\chi_{in}\left(c^2 - (\pi^m)^2\right)^2}.$$

Hence, for a given vector of parameters, this deviation destroys the considered equilibrium if there exists a value σ_{-j}^d such that $\kappa > \max\{\kappa_{-j,1}(\sigma_{-j}^d), \kappa_{-j,2}(\sigma_{-j}^d), \kappa_{-j,3}(\sigma_{-j}^d)\}$.

Deviation 2(b). If firm $-j$ attracts $\mathcal{R}$ investors only, its market clearing price is

$$p_{-j}^d(\sigma_j^*, \sigma_{-j}) = \mathbb{E}[\Pi_{-j}(\sigma_j^*, \sigma_{-j})] - \frac{\kappa}{\chi_{in}} - \lambda(1 - \sigma_{-j}),$$

whereas the market clearing price of firm j, who ends up attracting $\mathcal{N}$ investors, is

$$p_j^d(\sigma_j^*, \sigma_{-j}) = \mathbb{E}[\Pi_j(\sigma_j^*, \sigma_{-j})] - \frac{\kappa}{1 - \chi_{in}}.$$

In order for firm $-j$ to attract $\mathcal{R}$ investors only, it must be

$$\mathbb{E}[\Pi_j(\sigma_j^*, \sigma_{-j})] - p_j^d(\sigma_j^*, \sigma_{-j}) > \mathbb{E}[\Pi_{-j}(\sigma_j^*, \sigma_{-j})] - p_{-j}^d(\sigma_j^*, \sigma_{-j}),$$

and

$$\mathbb{E}[\Pi_j(\sigma_j^*, \sigma_{-j})] - p_j^d(\sigma_j^*, \sigma_{-j}) - \lambda(1 - \sigma_j^*) < \mathbb{E}[\Pi_{-j}(\sigma_j^*, \sigma_{-j})] - p_{-j}^d(\sigma_j^*, \sigma_{-j}) - \lambda(1 - \sigma_{-j}).$$

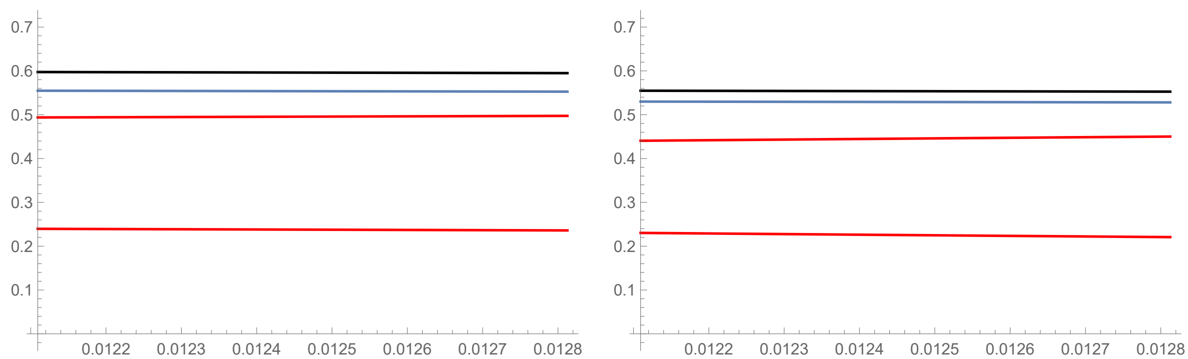
These are simultaneously satisfied if and only if

$$\widehat{\kappa}(\sigma_{-j}^d, 1) < \kappa < \widehat{\kappa}(\sigma_j^*, 1),$$

which is not the case in the region of parameters defined in (E.1), where this equilibrium can exist.

Existence. The above results imply that deviation 2(b) can never be implemented in the relevant region of parameters defined in (E.1), and deviation 1(b) is ruled out for $\kappa > \underline{\kappa}$.

The following figures show that also deviations 1(a) and 2(a) cannot be profitable and implementable at the same time for all $\kappa \in [\underline{\kappa}, \widehat{\kappa}(\sigma_{-j}^*, 1)$, considering a numerical example with parameters' values $c = 2, \pi^m = 1, \lambda = 1/20, \chi_{in} = 3/4$. This establishes a possibility result concerning the existence of asymmetric equilibria also under this alternative timing of the game.



(a) Firm j. The condition for the deviation to be profitable ($\kappa > \kappa_{j,3}$) requires σ_j to be within the two red lines. This is not compatible with $\kappa > \kappa_{j,1}$, which requires σ_j to be above the black line, nor with $\kappa > \kappa_{j,2}$, which requires σ_j to be above the blue line.

(b) Firm $-j$. The condition for the deviation to be profitable ($\kappa > \kappa_{-j,3}$) requires σ_{-j} to be within the two red lines. This is not compatible with $\kappa > \kappa_{-j,1}$, which requires σ_{-j} to be above the black line, nor with $\kappa > \kappa_{-j,2}$, which requires σ_{-j} to be above the blue line.

Figure OA1: X-axis: $\kappa \in [\underline{\kappa}, \widehat{\kappa}(\sigma_{-j}^*, 1)]$, Y-axis: σ_j (panel a), σ_{-j} (panel b).