Dynamic Trading and Asset Pricing with Time-Inconsistent Agents

Stig R. H. Lundeby Zhaneta K. Tancheva *

BI Norwegian Business School

August 8, 2023

Abstract

We examine the implications of time inconsistency and time-varying partial naivete for dynamic trading, asset prices, and portfolio choices. We show that the time-varying degree of naivete causes time variation in discount rates, which creates uncertainty about the future consumption of the time-consistent agents and a drop in their consumption level when discount rates are high and market value is low. As a result, the presence of partially naive time-inconsistent investors generates extra risk on the market and increases the risk premium in the economy. In addition, we show that sophisticated agents who are aware of their bias and their higher future consumption when upcoming market value is low, tend to borrow more in order to invest in risky assets. In contrast, time-consistent and fully naive agents, who believe they will become time-consistent, tend to sell risky assets and save more. Finally, time-consistent and fully naive agents prefer to invest in value stocks that are less sensitive to discount rate shocks, while sophisticated agents tend to invest more in growth stocks.

JEL Codes: D53, E21, G10, G41
Keywords: Time inconsistency, naivete, asset pricing, portfolio choice

*Stig R. H. Lundeby: BI Norwegian Business School, stig.r.lundeby@bi.no; Zhaneta K. Tancheva: BI Norwegian Business School, zhaneta.k.tancheva@bi.no. We appreciate helpful comments from Paul Ehling, Christian Heyerdahl-Larsen, Lars Lochstoer, Bas Werker, Costas Xiouros, and participants at the Bulgarian Council for Economic Analysis Conference and BI Norwegian Business School Seminar. Any remaining errors are our own.

1. Introduction

A myriad of experimental and field studies in economics show evidence that people tend to behave in a dynamically inconsistent way due to a preference for receiving immediate rewards and avoiding immediate costs (Loewenstein and Thaler, 1989; Ainslie, 1992). Importantly, this present bias is also prevalent when making financial decisions. For instance, despite their plan to start saving aggressively from next period on, some people still overconsume and save too little when next period comes. Economic theory has proposed to model such time-inconsistent preferences using hyperbolic discounting, represented by higher levels of impatience in the short run compared to the long run, that give rise to a disagreement between temporal selves (Phelps and Pollak, 1968; Laibson, 1997; Harris and Laibson, 2001).

Even though empirical evidence shows that approximately 50% of people are time inconsistent (Halevy, 2015), theoretical asset pricing models usually feature agents with time-consistent preferences, using standard exponential discounting of the future streams of consumption. Prominent exceptions are the studies by Khapko (2015) and Andries et al. (2018) who investigate the effect of time-inconsistent discounting and risk aversion on asset prices. However, these models feature a single representative investor. Heterogeneous agents contract-theoretic models with time inconsistency, on the other hand, typically have a (monopolistic) principal-agent setting, which differs from the competitive financial markets where all agents are price takers. In addition, even though these models show interesting interactions between time-consistent and time-inconsistent agents, they do not investigate the implications for trading and asset prices. In this paper we fill in this gap and capture the effect of time inconsistency on the trading, asset prices, and portfolio choice in a general equilibrium model with competitive market economy and heterogeneous agents.

Another main point of differentiation from previous asset pricing models is that we assume the time-inconsistent agent to be partially aware of her bias as in O'Donoghue and Rabin (2001). In particular, following Eliaz and Spiegler (2006), the partially naive agent mispredicts her changing tastes and believes that with probability $1-\theta$ she will become time consistent from next period on and with probability θ she will remain time inconsistent and more impatient. This can be caused by not completely appreciating the set of circumstances in which her preferences and beliefs change.

However, in contrast to Eliaz and Spiegler (2006) we allow the agents to have time-varying naivete. Since the changes in degree of naivete cause variation in discount rates, this introduces uncertainty to her future consumption level of the time-consistent agents and a drop in their consumption when discount rates are high and market value is low. This generates a novel source of risk on the market, even if there is no aggregate uncertainty. We show that even in a simple setting with log utility, the time-consistent agent requires to be compensated for this risk with a substantial extra risk premium. In addition, the premium is time-varying and increases with the level of naivete of the time-inconsistent agents: from 0% with sophisticated agent to about 4% with fully naive agents when the time-inconsistent agent holds a 60% share of the market. In contrast, in a fully rational economy with time-consistent agents and log utility the risk premium is 0.09% per year and does not vary over time.

In addition to this, the interaction between time-consistent and time-inconsistent agents with time-varying naivete provides a novel perspective to understand their portfolio choices and risky asset positions. Previous contract-theoretic models provide evidence that on the one hand, in a monopolistic three-period setting partially naive agents can accept exploitative contracts due to their changing tastes, while sophisticated agents prefer non-exploitative contracts that can serve as a commitment device (Eliaz and Spiegler, 2006), implying that they may choose different portfolio positions. On the other hand, Gottlieb and Zhang (2022) show that welfare inefficiency due to time inconsistency naivete vanishes in the case of long-term contracting and is independent of the agents' naivete.

In this paper, however, we show that in the case of dynamic trading, even over long horizons, (partially) naive and sophisticated agents make different portfolio choices. First, since sophisticated agents realize that their future consumption level increases when discount rates are high and market value is low (opposite of the consumption level of time-consistent agents), they tend to borrow in order to finance risky asset purchases. In contrast, believing that they will become time consistent from next period on, fully naive agents sell down risky assets and begin saving in the risk-free asset. Second, we examine the positions of these agents in value and growth stocks that are exposed differently to discount rate shocks. We find that time-consistent and fully naive agents are averse to holding growth stocks that are very sensitive to discount rate shocks and prefer holding value stocks, whereas sophisticated agents prefer holding growth stocks.

An interesting follow up analysis can show whether the degree of naivete of agents can explain why investors tend to trade excessively, a puzzle first documented by Odean (1999) and Barber and Odean (2000). Since partially naive agents tend to accept exploitative contracts, as shown by Eliaz and Spiegler (2006), they may be inclined to also make portfolio decisions which decrease their utility and wealth. A particularly interesting aspect to consider is that partially naive agents hold perceived information advantage about the degree of their naivete and may be willing to take gambles that look attractive from today's point of view, but are exploitative ex post. These two trading motives have been documented as the leading trading motives by Liu et al. (2022), so in the next version of the paper we plan to investigate whether partial naivete about time inconsistency can serve as a microfoundation of the excessive trading of investors compared to their plan.

The rest of the paper is organized as follows. Section 2 outlines the model setup and preferences of the agents. Sections 3 shows the equilibrium conditions. Section 4 and 5 present the numerical solution and main results in a two-risky assets economy with log utility. Section 6 concludes the paper.

2. Model setup

2.1 Investor preferences

We consider a discrete time general equilibrium model with a continuum of two types of agents, each with unit mass: time-consistent (TC) and time-inconsistent (TI). Both types have CRRA preferences with identical risk aversion parameters, γ, and time discount factors, β. Let $W_{TI,t}$ and $W_{TC,t}$ denote the wealth of the time-inconsistent and time-consistent agent, respectively. Furthermore, the sum of the wealth of both agents represents aggregate wealth, $W_t = W_{TI,t} + W_{TC,t}$. We denote the wealth share of the time-inconsistent agent as $s_t = \frac{W_{TI,t}}{W_t}$, such that:

$$W_{TI,t} = s_t W_t \tag{1}$$

$$W_{TC,t} = (1 - s_t) W_t. \tag{2}$$

The time-consistent investor uses standard exponential discounting with a time discount factor β for each period both in the short and the long run. In comparison, the time-inconsistent agent is present-biased and has quasi-hyperbolic discounting: she is more impatient about the immediate future (discounting it at a rate $\beta\delta$) than about the distant future [1]. The additional discount factor in the short run $\delta \in (0, 1)$ denotes the degree of the time inconsistency bias. To capture the present bias, it is convenient to model the time-inconsistent investor as an agent with different intertemporal selves.

As a main point of differentiation from previous literature focusing on the effects of dynamic inconsistency on asset prices[2], we assume that the time-inconsistent agent is partially naive and underestimates the extent of her bias, as defined by O'Donoghue and Rabin (2001). In particular, following Eliaz and Spiegler (2006), the partially naive agent believes that with probability $1 - \theta$

[1]In other words, the short-run discount rate ($-\ln \delta\beta$) of the time-inconsistent agent is larger than her long-run discount rate ($-\ln \beta$). This means that self t gives larger relative weight to the immediate consumption $C_{N,t}$ at time t than she plans to give to immediate consumption $C_{N,t,t+1}$ at time $t + 1$.

[2]Khapko (2015) and Andries, et al. (2018) consider the cases when the time-inconsistent agent is either sophisticated (fully aware of her bias) or naive (completely unaware of her bias))

her future self $t+1$ will become time consistent with utility $V_{TC,t+1}$ and with probability θ she will remain time inconsistent and more impatient with utility $V_{TI,t+1}$. The reason is that she does not fully appreciate the set of circumstances or investment opportunities in which her preferences change. In contrast to Eliaz and Spiegler (2006), who assume that θ is constant, we allow θ_t to be time-varying. Thus, the utility of the time-inconsistent agent at time t is given by:

$$\begin{aligned} V_{TI}(W_{TI,t}, s_t, \theta_t, t) &= \max_{C_{TI,t}, \varphi_{TI,t}} \frac{C_{TI,t}^{1-\gamma}}{1-\gamma} + \beta\delta\mathbb{E}_t\Big[\theta_t V_{TI}(W_{TI}, s_{t+1}, \theta_{t+1}, t+1) \\ &+ (1-\theta_t)V_{TC}(W_{TI,t+1}, s_{t+1}, \theta_{t+1}, t+1)\Big], \end{aligned} \tag{3}$$

where V_{TC} represents the value function of the time-consistent agent:

$$V_{TC}(W_{TC,t}, s_t, \theta_t, t) = \max_{C_{TC,t}, \varphi_{TC,t}} \frac{C_{TC,t}^{1-\gamma}}{1-\gamma} + \beta\mathbb{E}_t\left[V_{TC}(W_{TC,t+1}, s_{t+1}, \theta_{t+1}, t+1)\right]. \tag{4}$$

Equation (3) shows that the perceived $t+1$ utility according to self t, $V_{TI,t,t+1}$, is a weighted average of the respective utility in case the agent becomes time consistent or remains time inconsistent. The first subindex of the planned value function denotes the time at which the agent makes the plan and the second subindex shows the time at which the value takes place.[3]:

$$\begin{aligned} V_{TI,t,t+1}(W_{TI,t+1}, s_{t+1}, \theta_{t+1}, t+1) &= \theta_t V_{TI,t+1}(W_{TI,t+1}, s_{t+1}, \theta_{t+1}, t+1) \\ &+ (1-\theta_t)V_{TC,t+1}(W_{TI,t+1}, s_{t+1}, \theta_{t+1}, t+1). \end{aligned} \tag{5}$$

However, the rational time-consistent agent holds the true beliefs in the model and knows that the partially naive agent will remain time inconsistent with probability 1. The time-consistent agent observes the time-inconsistency parameter δ, but only knows the distribution of the degree of naivete θ. Thus, they can infer θ from prices. We refer to agents with $\theta = 0$ as fully naive and agents with $\theta = 1$ as sophisticated.

Note that each (representative) naive agent believes that while she may become time consis-

[3] $V_{TI,t,t+1}$ denotes the value function at time $t+1$ according to the plan of self t at time t.

tent with probability θ_t at $t+1$, the rest of the agents of her type will remain time-inconsistent with probability 1. This assumption is consistent with recent experimental evidence showing that agents show little awareness of their own bias, but anticipate the bias of others (Fedyk, 2022). Since we consider a competitive market economy, where each individual agent is a price-taker, the time-inconsistent investor believes that the wealth of the entire continuum of investors of her type will not be affected in case she turns time consistent. This means that each time-inconsistent agent understands the actual wealth she will have, $W_{TI,t+1}$, even if her utility becomes $V_{TC,t+1}(W_{TI,t+1}, s_{t+1}, \theta_{t+1}, t+1)$. As a result, the planned value function $V_{TI,t,t+1}$ is a function of the actual wealth $W_{TI,t+1}$. This also ensures that the actual consumption shares of the two types of agents sum up to the aggregate consumption, even though each individual naive agent mispredicts her own planned consumption level at time $t+1$ according to the plan of her self t, $C_{TI,t,t+1}$.

2.2 The economy

Consider a simple model with $N+1$ assets: a 1-period risk free asset in zero net supply and N risky assets with one share each that pays dividends $D_{n,t}$ every period. We denote the ex-dividend price of asset n as $P_{n,t}$. The aggregate dividend and market capitalization in period t can then be written as:

$$\bar{D}_t \equiv \sum_{n=1}^{N} D_{n,t} \tag{6}$$

$$\bar{P}_t \equiv \sum_{n=1}^{N} P_{n,t}. \tag{7}$$

Aggregate financial wealth at the beginning of time t is therefore

$$W_t \equiv W_{TI,t} + W_{TC,t} = \bar{P}_t + \bar{D}_t \tag{8}$$

and aggregate consumption is equal to the sum of the consumption of the two agents and to the aggregate dividends as markets clear:

$$C_t \equiv C_{TI,t} + C_{TC,t} = \bar{D}_t. \tag{9}$$

Note that we can write the return on the market as

$$\bar{R}_{t+1} \equiv \frac{\bar{P}_{t+1} + \bar{D}_{t+1}}{\bar{P}_t} = \frac{W_{t+1}}{W_t - \bar{D}_t}. \tag{10}$$

2.3 Dividend Processes

2.3.1 Aggregate dividends

Suppose aggregate dividend growth is given by

$$\Delta d_{t+1} \equiv log(\bar{D}_{t+1}) - log(\bar{D}_t) = \mu + \varepsilon_{t+1} \tag{11}$$

where the shocks are distributed i.i.d. and can take values

$$\hat{\varepsilon} = \begin{pmatrix} \hat{\varepsilon}_1 \\ \vdots \\ \hat{\varepsilon}_L \end{pmatrix} \tag{12}$$

with probabilities $\Pi_{\varepsilon,l}$. The attractiveness of specification (11) is that it ensures aggregate dividend, and therefore aggregate consumption, is always positive. Furthermore, it makes aggregate expected growth rate and risk time-invariant. Thus, any time variation in risk premia and risk free rate must come from our heterogeneous agent set-up.

2.3.2 Individual dividends

Assume that individual assets' dividends are given by

$$D_{n,t} = \frac{\bar{D}_t}{N}(1 + a_n x_t) \tag{13}$$

s.t. $\sum_{n=1}^{N} a_n = 0$. The stochastic variable x_t represents time variation in expected dividend growth for individual assets. If the autocorrelation in x_t is high, it represents a form of "long-run" risk for individual assets. The idea is to capture that some assets are "growth" assets, whereas others are "cash cows" and they will be differently exposed to variation in discount rates. Note that this condition ensure that

$$\sum_{n=1}^{N} D_{n,t} = \bar{D}_t, \qquad \forall\ t \tag{14}$$

The multiplication of $\frac{\bar{D}_t}{N}$ in front of x_t ensures that x_t does not vanish as a source of uncertainty as aggregate dividends grow.

$$\frac{D_{n,t+1}}{D_{n,t}} = \frac{\bar{D}_{t+1}}{\bar{D}_t}\frac{1 + a_n x_{t+1}}{1 + a_n x_t} \tag{15}$$

We assume that x_t follows a Markow process that can take the values

$$\hat{x} = \begin{pmatrix} \hat{x}_1 \\ \vdots \\ \hat{x}_M \end{pmatrix} \tag{16}$$

with transition probabilities

$$\Pi_{x,m,j} \equiv \mathbb{P}(x_{t+1} = \hat{x}_j | x_t = \hat{x}_m) \tag{17}$$

Note that with this specification, assets with high current dividends will have low expected dividend growth and vice-versa.

2.4 Time-varying time inconsistency

In addition to the uncertainty in the aggregate dividend process, the time-varying subjective probability (belief) of the partially naive agent's changing tastes, θ, represents another source of randomness in the model. The parameter θ captures the level of naivete of the time-inconsistent agent with which she mispredicts her changing tastes.

We assume that θ_t follow independent discrete Markov processes, which can take values $\hat{\theta}$, where

$$\hat{\theta} \equiv \begin{pmatrix} \hat{\theta}_1 \\ \vdots \\ \hat{\theta}_K \end{pmatrix}. \tag{18}$$

Assume that the transition probabilities are time-invariant and given by

$$\Pi_{\theta,k,l} \equiv \mathbb{P}(\theta_{t+1} = \hat{\theta}_l | \theta_t = \hat{\theta}_k) \tag{19}$$

$$. \tag{20}$$

Note that $\sum_{l=1}^{K} \Pi_{\theta,k,l} = 1$ and the probability of moving from state $\hat{\theta}_k$ to state $\hat{\theta}_l$ is $\Pi_{\theta,k,l}$.

3. Equilibrium

3.1 Consumption-savings problem

The optimization problem that the time-consistent and time-inconsistent agents solve is to maximize their utility, subject to their budget constraints:

$$\begin{aligned} V_{TI}(W_{TI,t}, s_t, \theta_t, t) &= \max_{C_{TI,t},\varphi_{TI,t}} \frac{C_{TI,t}^{1-\gamma}}{1-\gamma} + \beta\delta\mathbb{E}_t\Big[\theta_t V_{TI}(W_{TI}, s_{t+1}, \theta_{t+1}, t+1) \\ &+ (1-\theta_t)V_{TC}(W_{TI,t+1}, s_{t+1}, \theta_{t+1}, t+1)\Big] \end{aligned} \tag{21}$$

$$V_{TC}(W_{TC,t}, s_t, \theta_t, t) = \max_{C_{TC,t},\varphi_{TC,t}} \frac{C_{TC,t}^{1-\gamma}}{1-\gamma} + \beta\mathbb{E}_t\left[V_{TC}(W_{TC,t+1}, s_{t+1}, \theta_{t+1}, t+1)\right] \tag{22}$$

$$\text{s.t.} \quad W_{TI,t+1} = (W_{TI,t} - C_{TI,t})(R_{f,t} + \varphi_{TI,t}^{\top} R_{t+1}^e) \tag{23}$$

$$W_{TC,t+1} = (W_{TC,t} - C_{TC,t})(R_{f,t} + \varphi_{TC,t}^{\top} R_{t+1}^e), \tag{24}$$

where $\varphi_{TI,t}$ and $\varphi_{TC,t}$ are the time-inconsistent and time-consistent agents' risky asset shares and the return on wealth of each type of agents is:

$$R_{TI,t+1} \equiv R_{f,t} + \varphi_{TI,t} R_{t+1}^e \tag{25}$$

$$R_{TC,t+1} \equiv R_{f,t} + \varphi_{TC,t} R_{t+1}^e. \tag{26}$$

In addition, in equilibtium markets must clear, such that aggregate consumption equals aggregate dividends and the two agents together hold the market. We consider perception-perfect equilibria as in O'Donoghue and Rabin (1999, 2001), based on which the naive agent chooses her optimal consumption plan according to the predictions of her current self t, while the behavior of her future selves must be consistent with the extent of their naivete.

The challenge in finding the equilibrium is that, even though the time-inconsistent agent realizes her actual future wealth $W_{TI,t+1}$, she mispredicts her planned value function $V_{TI,t,t+1}$ and therefore believes her planned consumption, $C_{TI,t,t+1}$, will be lower than her actual consumption $C_{TI,t+1}$.[4] Therefore, we solve the optimization problem using backwards recursion. It is convenient to denote the wealth-consumption ratios of the two agents by:

$$\phi_{TI,t} \equiv \frac{W_{TI,t}}{C_{TI,t}} \tag{27}$$

$$\phi_{TC,t} \equiv \frac{W_{TC,t}}{C_{TC,t}}. \tag{28}$$

The first-order conditions for consumption are therefore

$$\left(\frac{W_{TI,t}-C_{TI,t}}{C_{TI,t}}\right)^{\gamma} = \beta\delta\mathbb{E}_t\Big[\Big(\theta_t\phi^{\gamma}_{TI,t+1}+(1-\theta_t)\phi^{\gamma}_{TC,t+1}\Big)R^{1-\gamma}_{TI,t+1}\Big] \tag{29}$$

$$\left(\frac{W_{TC,t}-C_{TC,t}}{C_{TC,t}}\right)^{\gamma} = \beta\mathbb{E}_t\Big[\phi^{\gamma}_{TC,t+1}R^{1-\gamma}_{TC,t+1}\Big]. \tag{30}$$

The following proposition shows that we can express the value functions of both agents at any period t independent of the unknown planned value function of the time-inconsistent agent and we provide a general recursive form of the wealth-consumption ratios:

Proposition 1. *The maximized value functions of the time-consistent and time-inconsistent agents at any period t are given by:*

$$V_{TI}(W_{TI,t},s_t,\theta_t,t) = \phi^{\gamma}_{TI,t}\frac{W^{1-\gamma}_{TI,t}}{1-\gamma} \tag{31}$$

$$V_{TC}(W_{TC,t},s_t,\theta_t,t) = \phi^{\gamma}_{TC,t}\frac{W^{1-\gamma}_{TC,t}}{1-\gamma} \tag{32}$$

and the wealth-consumption rations take the following form:

$$\phi_{TI,t} \equiv 1+(\beta\delta)^{\frac{1}{\gamma}}\mathbb{E}_t\left[\left(\theta_k\phi^{\gamma}_{TI,t+1}+(1-\theta_k)\phi^{\gamma}_{TC,t+1}\right)\times R^{1-\gamma}_{TI,t+1}\right]^{\frac{1}{\gamma}} \tag{33}$$

$$\phi_{TC,t} \equiv 1+\beta^{\frac{1}{\gamma}}\mathbb{E}_t\left[\phi^{\gamma}_{TC,t+1}R^{1-\gamma}_{TC,t+1}\right]^{\frac{1}{\gamma}}. \tag{34}$$

In Appendix A, we prove the proposition by recursion.

[4] In other words, the time-inconsistent agent is not aware that she will overconsume in the next period.

The market clearing condition is that aggregate consumption equals aggregate dividends, which gives us a condition for aggregate wealth:

$$\bar{D}_t = C_{TI,t} + C_{TC,t} = \left(\frac{s_t}{\phi_{TI,t}} + \frac{1 - s_t}{\phi_{TC,t}} \right) W_t. \tag{35}$$

Another useful result, shown in Appendix A, follows from the backwards recursion:

Proposition 2. *In a competitive market with time-consistent and partially naive time-inconsistent agents, it takes at least four periods in order to capture the effect of time-varying naivete.*

Intuitively, at the final period T, both types will consume their entire wealth, which implies that the time T value functions are just the utility of wealth $V_{i,T} = \frac{W_{i,T}^{1-\gamma}}{1-\gamma}$. As a consequence, both agents face the exact same first-order conditions for optimal portfolios at time $T-1$, thus giving rise to both agents simply holding the market portfolio $\bar{R}$.

In every period t where $t \leq T-3$, the agent faces an additional uncertainty due to future θ being random. Therefore, the agents face a non-trivial portfolio decision about how much of their savings to allocate to the risky asset versus the risk-free asset. Thus, the next period wealth-consumption ratios are functions of s_t, θ_t, and x_t, and the portfolio returns are functions of s_t, θ_t, x_t, θ_{t+1}, x_{t+1}, η_{t+1}, and ε_{t+1}.

3.2 Portfolio choice

To find the optimal allocation between the risky and risk-free assets in the portfolio, we find the first order conditions w.r.t. $\varphi_{TI,t}$ and $\varphi_{TC,t}$:

$$0 = \mathbb{E}_t\left[\left(\theta_t \phi_{TI,t+1}^{\gamma} + (1-\theta_t)\phi_{TC,t+1}^{\gamma}\right) R_{TI,t+1}^{-\gamma} R_{t+1}^{e}\right] \tag{36}$$

$$0 = \mathbb{E}_t\left[\phi_{TC,t+1}^{\gamma} R_{TC,t+1}^{-\gamma} R_{t+1}^{e}\right]. \tag{37}$$

Combining (29), (30), (36), and (37) gives us conditions for the risk-free rate:

$$\left(\frac{W_{TI,t} - C_{TI,t}}{C_{TI,t}}\right)^{\gamma} = \beta\delta\mathbb{E}_t\Big[\Big(\theta_t\phi^{\gamma}_{TI,t+1} + (1-\theta_t)\phi^{\gamma}_{TC,t+1}\Big)R^{-\gamma}_{TI,t+1}\Big]R_{f,t} \tag{38}$$

$$\left(\frac{W_{TC,t} - C_{TC,t}}{C_{TC,t}}\right)^{\gamma} = \beta\mathbb{E}_t\Big[\phi^{\gamma}_{TC,t+1}R^{-\gamma}_{TC,t+1}\Big]R_{f,t}. \tag{39}$$

Substituting the wealth-consumption ratios and returns on wealth of each agent and considering that $R_{f,t} = \frac{1}{\mathbb{E}_t[M_{TI,t,t+1}]}$ we can find the pricing kernels (SDF's) of the time-consistent and the partially naive time-inconsistent agents.

Proposition 3. *The pricing kernels of the time-consistent and partially naive time-inconsistent agents are given by:*

$$M_{TI,t,t+1} = \theta_t\beta\delta\left(\frac{C_{TI,t+1}}{C_{TI,t}}\right)^{-\gamma} + (1-\theta_t)\beta\delta\left(\frac{W_{TC,t+1}}{W_{TI,t+1}}\right)^{\gamma}\left(\frac{C_{TC,t+1}}{C_{TI,t}}\right)^{-\gamma} \tag{40}$$

$$M_{TC,t,t+1} = \beta\left(\frac{C_{TC,t+1}}{C_{TC,t}}\right)^{-\gamma}. \tag{41}$$

Note that at time t the naive agent mispredicts her planned value function, $V_{TI,t,t+1}$ but not her actual future wealth, $W_{TI,t+1}$. Thus, even though it is different from the actual consumption $C_{TI,t+1}$, the planned consumption $C_{TI,t,t+1}$ is the one that maximizes $V_{TI,t,t+1}$. In that sense the envelope theorem w.r.t. wealth holds. However, since the partially naive agent is partially aware that she may remain time-inconsistent, her SDF captures an adjustment regarding the disagreement between selves t and $t+1$, similar to the case of a fully sophisticated agent, as in Khapko (2015) and Andries, et al. (2018).

The market-clearing condition for the financial assets is that the agents in aggregate hold the market portfolio, i.e.

$$\begin{aligned}\frac{P_t}{W_t - \bar{D}_t} &= \frac{W_{TI,t} - C_{TI,t}}{W_t - \bar{D}}\varphi_{TI,t} + \frac{W_{TC,t} - C_{TC,t}}{W_t - \bar{D}}\varphi_{TC,t} \\ &= \tilde{s}_t\varphi_{TI,t} + (1-\tilde{s}_t)\varphi_{TC,t},\end{aligned} \tag{42}$$

where $\tilde{s}_t \equiv \frac{W_{TI,t}-C_{TI,t}}{W_t-\bar{D}}$ is the fraction of total reinvested wealth that belongs to the time-inconsistent agent.

3.3 Special case: Log utility

It is useful to consider the special case of log-utility, i.e. $\gamma = 1$ that follow directly from Proposition 1. In this case, the dependence on portfolio returns drop out, and the wealth-consumption ratio of the time-inconsistent agent will be a function of θ and time only, whereas for the time-consistent agent the wealth-consumption ratios are constant. Due to the simplified functional forms, we explicitly highlight the dependence on θ below. Thus, if $\theta_t = \hat{\theta}_k$, we have:

Corollary 1. *In the case of a finite horizon model time-consistent and time-inconsistent agents with log utility the wealth-consumption ratio expressions are given as follows:*

$$\phi_{TI,t}(\hat{\theta}_k) \equiv 1 + (1-\hat{\theta}_k)\beta\delta\phi_{TC,t+1} + \hat{\theta}_k\beta\delta\sum_{l=1}^{K}\Pi_{\theta,k,l}\phi_{TI,t+1}(\hat{\theta}_l) \tag{43}$$

$$\phi_{TC,t} \equiv 1 + \beta + \beta^2 + \cdots + \beta^{T-t} = \frac{1-\beta^{T+1-t}}{1-\beta}. \tag{44}$$

Note that $\phi_{TI,t}$ is a function only of θ_t and t, whereas ϕ_{TC} is a function of t only. In particular, the wealth share s_t does not enter either function. This is unique to the log-utility case, as the return on the agents' optimal portfolios drop out of the expressions. However, the wealth share will still affect aggregate wealth and expected return through the market clearing condition (35) and individual asset prices through the market clearing condition for portfolio choice (42). In this special case, we see that the time-consistent agent will consume a deterministic fraction of her wealth every period, whereas the time-inconsistent agent will consume a stochastic fraction of her wealth. Thus, whenever the time-inconsistent agent wants to consume more, aggregate wealth must fall in order to induce the time-consistent agent to consume less. How much aggregate wealth has to drop, will depend on wealth share s through (35).

To get even neater expressions, consider the limit as $T \to \infty$, i.e. the infinite horizon problem.

In this case, the dependence on time drops out, and the expressions take the simplified form, which can be solved out completely.

Corollary 2. *In the case of an infinite horizon model with log utility the wealth-consumption ratio expressions are given as follows:*

$$\phi_{TI}(\hat{\theta}_k) = 1 + (1 - \hat{\theta}_k)\frac{\beta\delta}{1-\beta} + \hat{\theta}_k \beta\delta \sum_{l=1}^{K} \Pi_{k,l}\phi_{TI}(\hat{\theta}_l) \tag{45}$$

$$\phi_{TC} = \frac{1}{1-\beta}, \tag{46}$$

We can see that for a fully naive agent, i.e. $\hat{\theta}_k = 0$, the function takes the intuitive form

$$\phi_{TI}(0) = 1 + \frac{\beta\delta}{1-\beta} = \frac{1 - \beta + \beta\delta}{1-\beta} = \frac{1}{1-\beta} - \frac{(1-\delta)\beta}{1-\beta}. \tag{47}$$

In other words, the wealth-consumption ratio of the fully naive agent is lower than that of the rational agent. How much lower, depends on how large the δ-discounting is.

4. Numerical solution: Two risky assets

We solve the model recursively assuming a terminal date T for the economy. In order to solve the model, we define a grid with N points between 0 and 1 for the wealth-share of the time inconsistent agent.

4.1 Time $T - 1$

At time $T - 1$ we solve the following problem for the wealth-consumption ratios $\phi_{TI,T-1,n}$ and $\phi_{TC,T-1,n}$:

$$\phi_{T-1,n} = \Big(\frac{s_n}{\phi_{TI,T-1,n}} + \frac{1 - s_n}{\phi_{TC,T-1,n}}\Big)^{-1} \tag{48}$$

$$\phi_{TI,T-1,n} = 1 + (\beta\delta)^{\frac{1}{\gamma}} \Big[\sum_{l=1}^{L} \Pi_{\varepsilon,l} \Big(\frac{e^{\mu+\varepsilon_l}}{\phi_{T-1,n} - 1}\Big)^{1-\gamma}\Big]^{\frac{1}{\gamma}} \tag{49}$$

$$\phi_{TC,T-1,n} = 1 + \beta^{\frac{1}{\gamma}} \Big[\sum_{l=1}^{L} \Pi_{\varepsilon,l} \Big(\frac{e^{\mu+\varepsilon_l}}{\phi_{T-1,n} - 1}\Big)^{1-\gamma}\Big]^{\frac{1}{\gamma}} \tag{50}$$

$$\tag{51}$$

for each wealth-share s_n on the grid. Note that in the case of log-utility the expressions for the individual wealth-consumption ratios do not depend on the wealth-share s_n, but the aggregate wealth-consumption ratio still depends on s_n.

This gives us the following market share for asset 1 in state $x_{T-1} = x_m$ and risk-free rate

$$\omega_{T-1,1,m,n} = \sum_{m=1}^{M} \sum_{l=1}^{L} \Pi_{x,m,j} \Pi_{\varepsilon,l} Q_{T,l,n} \frac{e^{\mu+\varepsilon_l}(1 + x_j)}{\phi_{T-1,n} - 1} \tag{52}$$

$$R_{f,T-1} = \Big(\sum_{l=1}^{L} \Pi_{\varepsilon,l} Q_{T,l,n}\Big)^{-1}, \tag{53}$$

where

$$Q_{T,l,n} = \beta\delta(\phi_{TI,T-1,n} - 1)^{-\gamma} \Big(\frac{e^{\mu+\varepsilon_l}}{\phi_{T-1,n} - 1}\Big)^{-\gamma} = \beta(\phi_{TC,T-1,n} - 1)^{-\gamma} \Big(\frac{e^{\mu+\varepsilon_l}}{\phi_{T-1,n} - 1}\Big)^{-\gamma}. \tag{54}$$

4.2 Time $t < T - 1$

At a general time t, we solve the following problems for a given wealth-share s_n

5. Results: A two-asset economy with log utility

In this section we report numerical results for a $T = 100$ period model where the agents have log utility. We assume that $\beta = 0.99$, $\delta = 0.8$, the expected log-growth rate μ is 2% per year, and that the aggregate shock ε_t can take the values -3% or 3% with equal probabilities. Furthermore, θ_t can take the values 0, 0.5, or 1, with the the transition probabilities being symmetric with diagonal elements 0.95 and off-diagonal elements of 0.025. Finally, we assume that there are two risky assets in the economy - the first asset pays dividends $\bar{D}_t(1 + x_t)$ and the second pays dividends $\bar{D}_t(1 - x_t)$, where $x_t \in \{-0.3, 0.3\}$, with the transition probabilities being symmetric with diagonal elements 0.96 and off-diagonal elements 0.04. We refer to the asset with a high (low) current dividend as a "value" ("growth") asset respectively. With the given parameters, the expected dividend growth of the value and growth asset is 0.2% and 5.6% per year respectively.

5.1 Conditional market risk premium

From Table 1 Panel A (in Appendix B), we see that the conditional market risk premium in year 0 in percent per year varies quite a bit with the level of naivete of the agents. In the case where the wealth share of the time-inconsistent agent is 60%, the market premium is close to 4% per year when the time-inconsistent agents are fully naive, and close to 0% when they are fully sophisticated. These are large variations in conditional market risk premia considering that the agents have log utility. For comparison, in the case of a fully rational market, the risk premium is 0.09% per year and does not vary over time. This means that the uncertainty about the time-varying naivete of the time-inconsistent agents, introduces variation in discount rates and extra risk on the market, compensated by a higher risk premium.

To make sense of these results, we note that as the time-inconsistent agents become more sophisticated (higher θ), they become more aware that they will likely consume a larger fraction of their wealth next year. To equalize marginal utilities between today and next period, they therefore wish to increase their consumption already today. However, the time-consistent agents want to consume a fraction $\frac{1}{\phi_{TC}} = \frac{1-\beta}{1-\beta^{T+1}}$ of their wealth. Since the only way the time-inconsistent agents can consume more as a group is that time-consistent agents consume less as a group, wealth must fall today through higher discount rates. This makes the market particularly risky for a time-consistent agent as the discount rate will be high, and therefore the market value low, whenever the consumption share of the time-consistent agent is low. When the time-inconsistent agents are fully sophisticated, they realize that their consumption share is positively correlated with discount rates, i.e. their consumption share is high whenever the market value is low. As a consequence, they are very willing to buy risky assets thereby driving the risk-premium down. However, when the time-inconsistent agents are fully naive, they believe that they will consume as a rational agent in the future, and therefore they are unwilling to hold risky assets unless compensated for discount rate risk.

5.2 Portfolio choice

5.2.1 Risky vs. risk-free assets

This intuition also shows up in the portfolio decisions of the agents. Recall that agent $i \in \{TC, TI\}$, chooses their optimal portfolio $R_{i,t+1} = R_{f,t} + \varphi_{i,t}^{\top} R_{t+1}^{e}$, where R_{t+1}^{e} denotes the vector of excess returns. Define the leverage factor $\kappa_{i,t} = \sum_{n=1}^{N} \varphi_{i,n,t}$. If $\kappa_{i,t} > 1$ the agents of type i borrow to finance risky asset purchases, if $\kappa_{i,t} = 1$, they neither borrow nor save in the riskless asset, and if $\kappa_{i,t} < 1$, they save in the riskless asset. From Table 2 (Appendix B) we see that the leverage factor is 1 for all agents when time-inconsistent agents are fully naive. However, as the time-inconsistent agents become more sophisticated, they begin to borrow to finance risky asset purchases, whereas the time-consistent agents sell down in risky assets and begin saving in the risk-free asset.

5.2.2 Risky portfolio weights: Value vs. growth stocks

Given our discussion so far, it is clear that time-consistent agents view risky assets as particularly risky due to the negative co-movement between their consumption and discount rates. However, not all risky assets are equally exposed to discount rate shocks. In particular, assets with a high expected dividend growth, will be more exposed to discount rate news than assets with a low expected dividend growth. As a result, we would expect the time-consistent agent to be particularly averse to holding "growth" assets, and more willing to hold "value" assets, whereas the opposite would be the case for sophisticated time-inconsistent agents.

We define the risky portfolio of agent i as $\frac{\varphi_{i,t}}{|\kappa_{i,t}|}$. Tables 3 and 4 (Appendix B) report the risky portfolio weights in the growth and value assets, respectively. We see that when time-inconsistent agents are fully naive, everyone simply holds the market portfolio. The reason is that the fully naive agents incorrectly believe that they will become time consistent from next period on and therefore will make the same portfolio decisions as them. However, as time-inconsistent agents become more sophisticated (θ increases), they begin to slightly overweight growth assets and underweight value assets. The active risky portfolio choice of the time-consistent agent is even more striking - they heavily overweight value assets and heavily underweight (even short) growth assets, consistent with our prediction.

6. Conclusion

This paper examines the role of dynamic trading with time inconsistency and time-varying partial naivete for the asset prices, dynamic trading, and portfolio choices of investors. We show that the time-varying naivete causes time variation in discount rates, which creates uncertainty about the future consumption of the time-consistent agents. Since the time-inconsistent agents overconsume compared to their plan, the time-consistent agents realize that they will experience a drop in their future consumption level when discount rates are high and market value is low. This generates extra risk on the market and increases the risk premium in the economy. In addition, we show that since sophisticated agents who are aware of their bias and their higher future consumption when upcoming market value is low, they tend to increase their leverage ratios in order to invest in risky assets. Time-consistent and fully naive agents, who believe they will become time-consistent, on the other hand, tend to sell risky assets and save more. Finally, we show that the degree of naivete affects the risky asset allocation of investors. While time-consistent and fully naive agents are averse to investing in growth stocks that are more sensitive to discount rate shocks and prefer to buy value stocks, sophisticated agents tend to invest more in growth stocks.

References

[1] Abel, A. B. 1999. Risk Premia and Term Premia in General Equilibrium. *Journal of Monetary Economics* 43, 3-33.

[2] Ainslie, G. W. 1992. Picoeconomics (Cambridge, UK: Cambridge University Press).

[3] Andries, M., T. M. Eisenbach, and M. C. Schmalz. 2018. Horizon-Dependent Risk Aversion and the Timing and Pricing of Uncertainty. Working Paper.

[4] Backus, D., B. Routledge, and S. Zin. 2009. Who Holds Risky Assets? Working Paper Stern School of Business.

[5] Barber, B.M., and T. Odean. 2000. Trading is Hazardous to Your Wealth: The Common Stock Investment Performance of Individual Investors. *Journal of Finance* 55, 773-806.

[6] Barberis, N., R. Greenwood, L.J. Jin, and A. Schleifer. 2018. Extrapolation and Bubbles. *Journal of Financial Economics* 129, 203-227.

[7] Barberis, N., and W. Xiong. 2009. What Drives the Disposition Effect? An Analysis of a Long-Lasting Preference-Based Explanation. *Journal of Finance* 64, 751-784.

[8] Blanchard, O. J. 1985. Debt, Deficits, and Finite Horizons. *Journal of Political Economy* 93, 223-247.

[9] Borovička, J. 2019. Survival and Long-Run Dynamics with Heterogeneous Beliefs under Recursive Preferences. *Journal of Political Economy, Forthcoming.*

[10] Cochrane, J. 2007. Financial Markets and the Real Economy. Handbook of the Equity premium, pp. 237-325. Amsterdam, The Netherlands: Elsevier.

[11] Daniel, K., D. Hirshleifer, and A. Subrahmanyam. 1998. Investor psychology and security market under- and overreaction. *Journal of Finance* 53, 1839-1885.

[12] Eliaz, K., and R. Spiegler. 2006. Contracting with Diversely Naive Agents. *Review of Economic Studies* 73, 689-714.

[13] Epstein, L., E. Farhi, and T. Strzalecki 2014. How Much Would You Pay to Resolve Long-Run Risk? *American Economic Review* 104, 2680-2697.

[14] Epstein, L., and S. Zin. 1989. Substitution, Risk Aversion, and the Temporal Behavior of Consumption and Asset Returns: A Theoretical Framework. *Econometrica* 57, 937-969.

[15] Fedyk, A. 2022. Asymmetric Naivete: Beliefs about Self-Control. Working paper.

[16] Friedman, M. 1953. *Essays in Positive Economics*. University of Chicago Press, Chicago.

[17] Garleanu, N., and S. Panageas. 2015. Young, Old, Conservative, and Bold: The Implication of Finite Lives and Heterogeneity for Asset Pricing. *Journal of Political Economy* 123, 670-695.

[18] Gottlieb, D., and X. Zhang. 2021. Long-Term Contracting with Time-Inconsistent Agents. *Econometrica* 123, 793-824.

[19] Grinblatt, D., and M. Keloharju. 2009. Sensation Seeking, Overconfidence, and Trading Activity. *Journal of Finance* 64, 549-578.

[20] Grinblatt, D., M. Keloharju, and J. Linnainmaa. 2011. IQ and Stock Market Participation. *Journal of Finance* 66, 2121-2164.

[21] Halevy, Y. 2015. Time Inconsistency: Stationarity and Time Invariance. *Econometrica* 83, 335-352.

[22] Hansen, L. P., J. C. Heaton, and J. Lee. 2005. Consumption Strikes Back? Measuring Long Run Risk. *Journal of Political Economy* 116, 3967-4056.

[23] Heidehues, P., and B. Kőszegi. 2010. Exploiting Naivete About Self-Control in the Credit Market. *American Economic Review* 100, 2279-2303.

[24] Khapko, M. 2015. Asset Pricing with Dynamically Inconsistent Agents. Working paper.

[25] Koopmans, T. C. 1960. Stationary Ordinary Utility and Impatience. *Econometrica* 28, 287-309.

[26] Kreps, D. M., and E. Porteus. 1978. Temporal Resolution of Uncertainty and Dynamic Choice Theory. *Econometrica* 46, 185-200.

[27] Kyle, A. S., and F. A. Wang. 1997. Speculation Duopoly with Agreement to Disagree: Can Overconfidence Survive the Market Test? *Journal of Finance* 52, 2073-2090.

[28] Laibson, D. 1997. Golden Eggs and Hyperbolic Discounting. *Quarterly Journal of Economics* 112, 443-477.

[29] Liu, H., C. Peng, W.A. Xiong, and W. Xiong. 2022. Taming the Bias Zoo. *Journal of Financial Economics* 143, 716-741.

[30] Loewenstein G., and Thaler, R. H. 1989. Anomalies: Intertemporal Choice. *Journal of Economic Perspectives* 3, 181-193.

[31] Lucas, R., and N. Stokey. 1984. Optimal Growth with Many Consumers. *Journal of Economic Theory* 32, 139-171.

[32] Odean, T. 1999. Do Investors Trade too Much? *American Economic Review* 89, 1279-1298.

[33] O'Donoghue, T., and M. Rabin. 1999 Doing Now or Later. *American Economic Review* 89, 103-124.

[34] O'Donoghue, T., and M. Rabin. 2001 Choice and Procrastination. *Quarterly Journal of Economics* 116, 121-160.

[35] Phelps, E. S., and R. A. Pollak. 1968. On Second-best National Saving and Game-equilibrium Growth. *Review of Financial Studies* 35, 185-199.

[36] Pohl, W., K. Schmedders, and O. Wilms. 2018. Asset Pricing with Heterogeneous Agents and Long-Run Risk. Working Paper.

[37] Pollak, R. A. 1968. Consistent Planning. Review of Financial Studies 35, 201-208.

[38] Saez, E., and G. Zucman. 2016. Wealth Inequality in the United States Since 1913: Evidence from Capitalized Income Tax Data. *Quarterly Journal of Economics* 131, 519-578.

[39] Thaler, R. H., and S. Benartzi. 2004. Save More Tomorrow: Using Behavioral Economics to Increase Employee Saving. *Journal of Political Economy* 112, 164-187.

[40] Weil, P. 1989. The Equity Premium Puzzle and the Risk-Free Rate Puzzle. *Journal of Monetary Economics* 24, 401-421.

[41] Wilcox, D. 1992. The Construction of the U.S. Consumption Data: Some Facts and Their Implications for Empirical Work. *American Economic Review* 82, 922-941.

A Proofs of main results

1.1 Proofs of Propositions 1 and 2

1.2 Time T

At the last period T, both agents optimally consume their entire wealth as there is no tomorrow. Therefore, the maximized value functions become:

$$V_{TI}(W, s_T, \theta_T, T) = V_{TC}(W, s_T, \theta_T, T) = \frac{W_T^{1-\gamma}}{1-\gamma} \tag{55}$$

and aggregate wealth is $W_T = D$ which is deterministic.

1.3 Time $T - 1$

Substituting the return definition (eq. (10)) and the value function at time T (eq. (55)) into the value function at time $T - 1$, the agent solves:

$$\begin{aligned} V_{TI}(W_{TI,T-1}, s_{T-1}, \theta_{T-1}, T-1) \equiv & \max_{C_{TI,T-1}} \frac{C_{TI,T-1}^{1-\gamma}}{1-\gamma} + \beta\delta\Big[\theta_{T-1} V_{TI}(W_{TI,T}, s_T, \theta_T, T) \\ & + (1-\theta_{T-1}) V_{TC}(W_{TI,T}, s_T, \theta_T, T)\Big] \\ = & \max_{C_{TI,T-1}} \frac{C_{TI,T-1}^{1-\gamma}}{1-\gamma} + \beta\delta \frac{(W_{TI,T-1} - C_{TI,T-1})^{1-\gamma} R_T^{1-\gamma}}{1-\gamma} \end{aligned} \tag{56}$$

$$\begin{aligned} V_{TC}(W_{TC,T-1}, s_{T-1}, \theta_{T-1}, T-1) \equiv & \max_{C_{TC,T-1}} \frac{C_{TC,T-1}^{1-\gamma}}{1-\gamma} + \beta V_{TC}(W_{TC,T}, s_T, \theta_T, T) \\ = & \max_{C_{TC,T-1}} \frac{C_{TC,T-1}^{1-\gamma}}{1-\gamma} + \beta \frac{(W_{TC,T-1} - C_{TC,T-1})^{1-\gamma} R_T^{1-\gamma}}{1-\gamma}. \end{aligned} \tag{57}$$

Since the ex-dividend price of the market in period T is 0, the return on the market R_T is risk-free. Thus both agents must earn the same return (i.e. the riskless rate) on their portfolios. Therefore, the problem at time $T-1$ becomes one of how much to consume or save, whereas the portfolio choice is trivial.

Finding and rearranging the first order conditions w.r.t. $C_{TI,T-1}$ and $C_{TC,T-1}$, we can express the wealth-consumption ratio as follows:

$$\left(\frac{W_{TI,T-1} - C_{TI,T-1}}{C_{TI,T-1}}\right)^{\gamma} = \beta\delta R_T^{1-\gamma} \tag{58}$$

$$\left(\frac{W_{TC,T-1} - C_{TC,T-1}}{C_{TC,T-1}}\right)^{\gamma} = \beta R_T^{1-\gamma}. \tag{59}$$

Rearranging, we can express the wealth-consumption ratio as follows:

$$\phi_{TI,T-1} \equiv \frac{W_{TI,T-1}}{C_{TI,T-1}} = 1 + (\beta\delta)^{\frac{1}{\gamma}} R_T^{\frac{1-\gamma}{\gamma}} \tag{60}$$

$$\phi_{TC,T-1} \equiv \frac{W_{TC,T-1}}{C_{TC,T-1}} = 1 + (\beta\delta)^{\frac{1}{\gamma}} R_T^{\frac{1-\gamma}{\gamma}}. \tag{61}$$

Note that these functions can only be a function of s_{T-1} since the problems (56) and (57) are invariant to θ_{T-1}. Furthermore, ϕ_{TI} and ϕ_{TC} can only depend on s_{T-1} through R_T. But since next period wealth is known, R_T can only depend on s_{T-1} through current aggregate wealth $W_{T-1}(s_{T-1})$. We find aggregate wealth through the market clearing condition that aggregate consumption equals aggregate dividend:

$$D = \left(\frac{s_{T-1}}{\phi_{TI}} + \frac{1 - s_{T-1}}{\phi_{TC}}\right) W_{T-1}(s_{T-1}). \tag{62}$$

Observe that since variation in θ_t is the only source of uncertainty, the fact that wealth in $T-1$ does not depend on θ_{T-1} implies that it is known (deterministic) already at $T-2$.

Using (60) and (61) in (56) and (57) yields an expression for the maximized value functions:

$$\begin{aligned}
V_{TI}(W_{TI,T-1}, s_{T-1}, \theta_{T-1}, T-1) &= \left[\left(\frac{1}{\phi_{TI,T-1}}\right)^{1-\gamma} + \beta\delta\frac{(\phi_{TI,T-1}-1)^{1-\gamma}R_T^{1-\gamma}}{\phi_{TI,T-1}^{1-\gamma}}\right]\frac{W_{TI,T-1}^{1-\gamma}}{1-\gamma} \\
&= \left[1+\beta\delta(\phi_{TI,T-1}-1)^{1-\gamma}R_T^{1-\gamma}\right] \\
&\times \left(\frac{1}{\phi_{TI,T-1}}\right)^{1-\gamma}\frac{W_{TI,T-1}^{1-\gamma}}{1-\gamma} \\
&= \phi_{TI,T-1}^{\gamma}\frac{W_{TI,T-1}^{1-\gamma}}{1-\gamma} \qquad (63)
\end{aligned}$$

$$\begin{aligned}
V_{TC}(W_{TC,T-1}, s_{T-1}, \theta_{T-1}, T-1) = \Bigg[\left(\frac{1}{\phi_{TC,T-1}}\right)^{1-\gamma} \\
+ \beta\frac{(\phi_{TC,T-1}-1)^{1-\gamma}R_T^{1-\gamma}}{\phi_{TC,T-1}^{1-\gamma}}\Bigg]\frac{W_{TC,T-1}^{1-\gamma}}{1-\gamma} \\
= \phi_{TC,T-1}^{\gamma}\frac{W_{TC,T-1}^{1-\gamma}}{1-\gamma}, \qquad (64)
\end{aligned}$$

where the last equalities uses the definitions (60) and (60) which gives us

$$\begin{aligned}
1+\beta\delta(\phi_{TI,T-1}-1)^{1-\gamma}R_T^{1-\gamma} &= 1+(\beta\delta)(\beta\delta)^{\frac{1-\gamma}{\gamma}}R_T^{\frac{(1-\gamma)^2}{\gamma}}R_T^{1-\gamma} \\
&= 1+(\beta\delta)^{\frac{1}{\gamma}}R_T^{\frac{1-\gamma}{\gamma}} = \phi_{TI,T-1} \qquad (65) \\
1+\beta(\phi_{TC,T-1}-1)^{1-\gamma}R_T^{1-\gamma} &= 1+\beta\times\beta^{\frac{1-\gamma}{\gamma}}R_T^{\frac{(1-\gamma)^2}{\gamma}}R_T^{1-\gamma} \\
&= 1+\beta^{\frac{1}{\gamma}}R_T^{\frac{1-\gamma}{\gamma}} = \phi_{TC,T-1}. \qquad (66)
\end{aligned}$$

1.3.1 Time $T-2$

Note that since the market clearing condition at $T-1$ implies that aggregate wealth is known at $T-2$, the market return must be equal to the risk-free rate between $T-2$ and $T-1$ as well. Thus, the time $T-2$ problem again simplifies to a simple consumption/savings problem, with a trivial portfolio choice:

$$
\begin{aligned}
V_{TI}(W_{TI,T-2}, s_{T-2}, \theta_{T-2}, T-2) \equiv & \max_{C_{TI,T-2}} \frac{C_{TI,T-2}^{1-\gamma}}{1-\gamma} + \beta\delta\Big[\theta_{T-2} V_{TI}(W_{TI,T-1}, s_{T-1}, \theta_{T-1}, T-1) \\
& + (1-\theta_{T-1}) V_{TC}(W_{TI,T-1}, s_{T-1}, \theta_{T-1}, T-1)\Big] \\
& = \max_{C_{TI,T-2}} \frac{C_{TI,T-2}^{1-\gamma}}{1-\gamma} + \beta\delta\Big(\theta_{T-2}\phi_{TI,T-1}^{\gamma} + (1-\theta_{T-2})\phi_{TC,T-1}^{\gamma}\Big) \\
& \times \frac{(W_{TI,T-2} - C_{TI,T-2})^{1-\gamma} R_{T-1}^{1-\gamma}}{1-\gamma} \qquad (67) \\
V_{TC}(W_{TC,T-2}, s_{T-2}, \theta_{T-2}, T-2) \equiv & \max_{C_{TC,T-2}} \frac{C_{TC,T-2}^{1-\gamma}}{1-\gamma} + \beta V_{TC}(W_{TC,T-1}, s_{T-1}, \theta_{T-1}, T-1) \\
& = \max_{C_{TC,T-2}} \frac{C_{TC,T-2}^{1-\gamma}}{1-\gamma} + \beta\phi_{TC,T-1}^{\gamma} \frac{(W_{TC,T-2} - C_{TC,T-2})^{1-\gamma} R_{T-1}^{1-\gamma}}{1-\gamma} \qquad (68)
\end{aligned}
$$

The first order conditions are

$$
\Big(\frac{W_{TI,T-1} - C_{TI,T-1}}{C_{TI,T-1}}\Big)^{\gamma} = \beta\delta\Big(\theta_{T-2}\phi_{TI,T-1}^{\gamma} + (1-\theta_{T-2})\phi_{TC,T-1}^{\gamma}\Big) R_{T-1}^{1-\gamma} \qquad (69)
$$

$$
\Big(\frac{W_{TC,T-1} - C_{TC,T-1}}{C_{TC,T-1}}\Big)^{\gamma} = \beta\phi_{TC,T-1}^{\gamma} R_{T-1}^{1-\gamma} \qquad (70)
$$

The wealth-consumption ratios can be expressed as follows:

$$
\phi_{TI,T-2} \equiv 1 + (\beta\delta)^{\frac{1}{\gamma}} \times \Big(\theta_{T-2}\phi_{TI,T-1}^{\gamma} + (1-\theta_{T-2})\phi_{TC,T-1}^{\gamma}\Big)^{\frac{1}{\gamma}} R_{T-1}^{\frac{1-\gamma}{\gamma}} \qquad (71)
$$

$$
\phi_{TC,T-2} \equiv 1 + \beta^{\frac{1}{\gamma}} \phi_{TC,T-1} R_{T-1}^{\frac{1-\gamma}{\gamma}} \qquad (72)
$$

Since both $\phi_{TI,T-1}$ and $\phi_{TC,T-1}$ depend only on s_{T-1}, they are known at time $T-2$. However, note that $\phi_{TI,T-2}$ depends directly on θ_{T-2}, which will cause the aggregate wealth at time $T-2$ to depend on both s_{T-2} and θ_{T-2}. Thus, R_{T-1} will depend on both s_{T-2} and θ_{T-2} as well.

From the market clearing condition that aggregate consumption equals aggregate dividend gives us an implied function for aggregate wealth at time $T-2$:

$$0 = \left(\frac{s_{T-2}}{\phi_{TI,T-2}} + \frac{1 - s_{T-2}}{\phi_{TC,T-2}} \right) W_{T-2}(s_{T-2}, \theta_{T-2}) - D \tag{73}$$

Similar algebra as in the previous section gives us the following expressions for the maximized value functions at time $T-2$

$$V_{TI}(W_{TI,T-2}, s_{T-2}, \theta_{T-2}, T-2) = \phi_{TI,T-2}^{\gamma} \frac{W_{TI,T-2}^{1-\gamma}}{1-\gamma} \tag{74}$$

$$V_{TC}(W_{TC,T-2}, s_{T-2}, \theta_{T-2}, T-2) = \phi_{TC,T-2}^{\gamma} \frac{W_{TC,T-2}^{1-\gamma}}{1-\gamma} \tag{75}$$

1.4 Time $t \leq T-3$

In every period t where $t \leq T-3$, the agent faces uncertainty due to future θ being random. Thus, the return on the market portfolio will be random, and in particular not equal the risk-free rate. The agents therefore face a non-trivial portfolio decision about how much of their savings to allocate to the risky asset versus the riskless asset. We will begin by assuming that next period's value functions can be written on the form

$$V_{TI}(W_{TI,t+1}, s_{t+1}, \theta_{t+1}, t+1) = \phi_{TI,t+1}^{\gamma} \frac{W_{TI,t+1}^{1-\gamma}}{1-\gamma} \tag{76}$$

$$V_{TC}(W_{TC,t+1}, s_{t+1}, \theta_{t+1}, t+1) = \phi_{TC,t+1}^{\gamma} \frac{W_{TC,t+1}^{1-\gamma}}{1-\gamma}, \tag{77}$$

which we already know to be the case for $t = T-3$. Then, we will show that this implies that the value functions in period t can be written on the same form, thus proving by induction our assumption.

The agents' problems in period t are

$$V_{TI}(W_{TI,t}, s_t, \hat{\theta}_k, t) = \max_{C_{TI,t}, \varphi_{TI,t}} \frac{C_{TI,t}^{1-\gamma}}{1-\gamma} + \beta\delta \sum_{l=1}^{K} \Pi_{k,l} \Big[\hat{\theta}_k \phi_{TI,t+1}^{\gamma} + (1-\hat{\theta}_k)\phi_{TC,t+1}^{\gamma}\Big] R_{TI,t+1}^{1-\gamma} \frac{(W_{TI,t} - C_{TI,t})^{1-\gamma}}{1-\gamma} \tag{78}$$

$$V_{TC}(W_{TC,t}, s_t, \hat{\theta}_k, t) = \max_{C_{TC,t}, \varphi_{TC,t}} \frac{C_{TC,t}^{1-\gamma}}{1-\gamma} + \beta \sum_{l=1}^{K} \Pi_{k,l} \phi_{TC,t+1} R_{TC,t+1}^{1-\gamma} \frac{(W_{TI,t} - C_{TI,t})^{1-\gamma}}{1-\gamma}, \tag{79}$$

where

$$R_{TI,t+1} \equiv R_{f,t} + \varphi_{TI,t} R_{t+1}^{e} \tag{80}$$

$$R_{TC,t+1} \equiv R_{f,t} + \varphi_{TC,t} R_{t+1}^{e}. \tag{81}$$

1.4.1 Time $t \leq T-3$: Proof of initial assumption about the value functions

To prove the initial assumption about the form of the value functions (76) and (77), we use:

$$W_{TI,t} - C_{TI,t} = (\phi_{TI,t} - 1)\frac{W_{TI,t}}{\phi_{TI,t}} \tag{82}$$

$$W_{TC,t} - C_{TC,t} = (\phi_{TC,t} - 1)\frac{W_{TC,t}}{\phi_{TC,t}} \tag{83}$$

We can therefore write the maximized value functions as follows

$$\begin{aligned} V_{TI}(W_{TI,t}, s_t, \hat{\theta}_k, t) = & \Big[1 + \Big(\phi_{TI,t} - 1\Big)^{1-\gamma} \beta\delta \sum_{l=1}^{K} \Pi_{k,l} \Big[\hat{\theta}_k \phi^{\gamma}_{TI,t+1} \\ & + (1 - \hat{\theta}_k)\phi^{\gamma}_{TC,t+1}\Big]\Big] \frac{1}{1-\gamma}\Big(\frac{W_{TI,t}}{\phi_{TI,t}}\Big)^{1-\gamma} \\ & = \phi^{\gamma}_{TI,t} \frac{W^{1-\gamma}_{TI,t}}{1-\gamma} \end{aligned} \tag{84}$$

$$\begin{aligned} V_{TC}(W_{TC,t}, s_t, \hat{\theta}_k, t) = & \Big[1 + \Big(\phi_{TC,t} - 1\Big)^{1-\gamma} \beta \sum_{l=1}^{K} \Pi_{k,l} \phi^{\gamma}_{TC,t+1} R^{1-\gamma}_{TC,t+1}\Big] \\ & \times \frac{1}{1-\gamma}\Big(\frac{W_{TC,t}}{\phi_{TC,t}}\Big)^{1-\gamma} \\ & = \phi^{\gamma}_{TC,t} \frac{W^{1-\gamma}_{TC,t}}{1-\gamma} \end{aligned} \tag{85}$$

$$\tag{86}$$

where we used

$$\begin{aligned} & 1 + \Big(\phi_{TI,t} - 1\Big)^{1-\gamma} \beta\delta \sum_{l=1}^{K} \Pi_{k,l} \Big[\hat{\theta}_k \phi^{\gamma}_{TI,t+1} + (1-\hat{\theta}_k)\phi^{\gamma}_{TC,t+1}\Big] R^{1-\gamma}_{TI,t+1} \\ & = 1 + (\beta\delta)^{\frac{1-\gamma}{\gamma}} \Big\{ \sum_{l=1}^{K} \Pi_{k,l} \Big[\hat{\theta}_k \phi^{\gamma}_{TI,t+1} + (1-\hat{\theta}_k)\phi^{\gamma}_{TC,t+1}\Big] R^{1-\gamma}_{TI,t+1} \Big\}^{\frac{1-\gamma}{\gamma}} \\ & \times (\beta\delta) \Big\{ \sum_{l=1}^{K} \Pi_{k,l} \Big[\hat{\theta}_k \phi^{\gamma}_{TI,t+1} + (1-\hat{\theta}_k)\phi^{\gamma}_{TC,t+1}\Big] R^{1-\gamma}_{TI,t+1} \Big\} \\ & = 1 + (\beta\delta)^{\frac{1}{\gamma}} \Big\{ \sum_{l=1}^{K} \Pi_{k,l} \Big[\hat{\theta}_k \phi^{\gamma}_{TI,t+1} + (1-\hat{\theta}_k)\phi^{\gamma}_{TC,t+1}\Big] R^{1-\gamma}_{TI,t+1} \Big\}^{\frac{1}{\gamma}} \\ & = \phi_{TI,t} \end{aligned} \tag{87}$$

and

$$
\begin{aligned}
\phi_{TI} \frac{1}{1-\gamma} \Big(\frac{W_{TI,t}}{f_{TI}(s_t, \hat{\theta}_k, t)^{\frac{1}{\gamma}}} \Big)^{1-\gamma} &= \phi_{TI,t} \times \phi_{TI,t}^{\gamma-1} \times \frac{W_{TI,t}^{1-\gamma}}{1-\gamma} \\
&= \phi_{TI,t}^{\gamma} \frac{W_{TI,t}^{1-\gamma}}{1-\gamma} \qquad (88)
\end{aligned}
$$

The same is easy to show for the time-consistent case. We have therefore shown that the assumption about the form of the value function was correct.

B Tables and figures

Table 1: Conditional market risk premium and risk-free rate

This table reports conditional market risk premium (Panel A) and risk-free rate (Panel B) in percent per year. θ represents the different level of naivete of the time-inconsistent (TI) agent.

Panel A: Conditional Market Risk-Premium

	$\theta = 0$	$\theta = 0.5$	$\theta = 1$
$s = 0.0$	0.09	0.09	0.09
$s = 0.2$	1.01	0.45	0.03
$s = 0.4$	2.44	1.16	0.05
$s = 0.6$	3.85	1.84	0.07
$s = 0.8$	5.18	2.50	0.09

Panel B: Risk-free Rate

	$\theta = 0$	$\theta = 0.5$	$\theta = 1$
$s = 0.0$	3.00	3.00	3.00
$s = 0.2$	0.89	1.97	16.53
$s = 0.4$	-0.95	1.18	22.27
$s = 0.6$	-2.58	0.50	25.41
$s = 0.8$	-4.04	-0.08	27.39

Table 2: Portfolio choice: Leverage factors

This table reports the leverage factor for the time-consistent TC agent (Panel A) and time-inconsistent TI agent with different level of naivete θ (Panel B).

Panel A: Leverage factor TC agent

	$\theta = 0$	$\theta = 0.5$	$\theta = 1$
$s = 0.0$	1.00	1.00	1.00
$s = 0.2$	1.00	0.89	0.03
$s = 0.4$	1.00	0.87	0.01
$s = 0.6$	1.00	0.86	0.01
$s = 0.8$	1.00	0.85	0.00

Panel B: Leverage factor TI agent

	$\theta = 0$	$\theta = 0.5$	$\theta = 1$
$s = 0.0$	-	-	-
$s = 0.2$	1.00	1.43	5.21
$s = 0.4$	1.00	1.19	2.60
$s = 0.6$	1.00	1.09	1.72
$s = 0.8$	1.00	1.04	1.27

Table 3: Risky portfolio weights: Growth asset

This table reports risky portfolio weights for the growth asset for the time-consistent (TC) agent (Panel A), time-inconsistent (TI) agent with different level of naivete θ (Panel B), and the market weights (Panel C) respectively.

Panel A: TC agent risky portfolio weight in growth asset

	$\theta = 0$	$\theta = 0.5$	$\theta = 1$
$s = 0.0$	0.48	0.48	0.48
$s = 0.2$	0.47	0.47	0.02
$s = 0.4$	0.47	0.47	-0.70
$s = 0.6$	0.47	0.47	-1.47
$s = 0.8$	0.47	0.46	-2.02

Panel B: TI agent risky portfolio weight in growth asset

	$\theta = 0$	$\theta = 0.5$	$\theta = 1$
$s = 0.0$	-	-	-
$s = 0.2$	0.47	0.48	0.48
$s = 0.4$	0.47	0.47	0.47
$s = 0.6$	0.47	0.47	0.46
$s = 0.8$	0.47	0.47	0.46

Panel C: Market weight in growth asset

	$\theta = 0$	$\theta = 0.5$	$\theta = 1$
$s = 0.0$	0.48	0.48	0.48
$s = 0.2$	0.47	0.47	0.47
$s = 0.4$	0.47	0.47	0.46
$s = 0.6$	0.47	0.47	0.46
$s = 0.8$	0.47	0.47	0.45

Table 4: Risky portfolio weights: Value asset

This table reports risky portfolio weights for the value asset for the time-consistent (TC) agent (Panel A), time-inconsistent (TI) agent with different level of naivete θ (Panel B), and the market weights (Panel C) respectively.

Panel A: TC agent risky portfolio weight in value asset

	$\theta = 0$	$\theta = 0.5$	$\theta = 1$
$s = 0.0$	0.52	0.52	0.52
$s = 0.2$	0.53	0.53	0.98
$s = 0.4$	0.53	0.53	1.70
$s = 0.6$	0.53	0.53	2.47
$s = 0.8$	0.53	0.54	3.02

Panel B: TI agent risky portfolio weight in value asset

	$\theta = 0$	$\theta = 0.5$	$\theta = 1$
$s = 0.0$	-	-	-
$s = 0.2$	0.53	0.52	0.52
$s = 0.4$	0.53	0.53	0.53
$s = 0.6$	0.53	0.53	0.54
$s = 0.8$	0.53	0.53	0.54

Panel C: Market weight in value asset

	$\theta = 0$	$\theta = 0.5$	$\theta = 1$
$s = 0.0$	0.52	0.52	0.52
$s = 0.2$	0.53	0.53	0.53
$s = 0.4$	0.53	0.53	0.54
$s = 0.6$	0.53	0.53	0.54
$s = 0.8$	0.53	0.53	0.55