# Gambling for Redemption or Ripoff, and the Impact of Superpriority*

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**ABSTRACT**

Myers (1977) described how firms can gamble using asset substitution, which is switching to a less efficient and more volatile project. Gambling using derivatives is a sharper instrument, allowing the owners to gamble just to what is needed, and with negligible efficiency loss. In our model, "gambling for redemption" operates at small scale and is socially beneficial, while "gambling for ripoff" operates at large scale and is socially inefficient but benefits firm owners (at the expense of bondholders). Superpriority laws grant Qualified Financial Contracts (QFCs) bankruptcy law exemptions, which make more funds available for gambling. This reduces firm value due to difficulty borrowing in the face of more gambling for ripoff.

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# 1 Introduction

In the early days of Federal Express, the company's cash once dwindled to \$5,000, too little to cover the \$24,000 jet fuel bill due the following Monday. With the firm hanging on the edge, the founder Frederick Smith flew to Las Vegas over the weekend and played blackjack to convert the \$5,000 into \$32,000, enough to keep the company afloat for another week.[1] While this gamble was obviously beneficial to the firm's owners by providing a positive probability to avoid bankruptcy, it was probably also beneficial for other claimants, including the fuel company, who would have received little in bankruptcy. Gambling by a firm can also benefit owners at the expense of creditors, as in the asset substitution studied in Myers (1977), where the upside benefit is received by the owners, and the downside is borne by creditors. In this paper, we study pure gambling by a firm using derivatives, which allows more control over the payoff distribution and negligible inefficiency of investment compared to asset substitution. We can understand the impact of gambling through two polar cases in a single-period model. Gambling for redemption, as exemplified by the Federal Express scenario, involves gambling just enough to stay in business. Such gambling is good for the owners, the creditors, and for overall efficiency. Gambling for ripoff, which operates at a larger scale, benefits the owners at the expense of the creditors and overall economic efficiency.

Gambling for ripoff is of special current interest because of controversial legislation before the financial crisis that exempts qualified financial contracts (QFCs), such as repos and financial derivatives, from important provisions of bankruptcy law, including automatic stay and clawbacks. These claims, referred to as superpriority[2] claims by Roe (2010), are claimed to have accelerated the financial crisis.[3] Superpriority laws make it easier for firms to gamble with their assets, allowing for gambling at a larger scale and in the presence of accounting controls. In subsection 1.1 we will discuss superpriority laws in detail. For now, it suffices to note that superpriority laws increase

[1] Frock (2006)

[2] Priority is the promised order of satisfaction of claims in bankruptcy. The QFCs technically do not have a priority, since they are exempt from bankruptcy law, but in effect their exemption is like having higher priority than all other claims, hence the term "superpriority."

[3] Roe (2010) argues that these laws undermined creditors' incentives to monitor the firm and creating a too-big-to-fail problem.

the scale of available gambling, which makes gambling for ripoff more appealing to the owners. We start with a single-period model in which superprioprity laws seem good for firm owners, but perhaps only because the amount of debt and the continuation value are both exogenous. In the multi-period model, superpriority is reflected in bond pricing and bond investors understand that superpriority increases the likelihood of gambling for ripoff. As a result, bond investors demand higher returns to compensate for the increased risk, leading to a decrease in what can be borrowed and a decrease in the firm's overall value.

Our single-period model considers a firm that has debt coming due, and the required payment may or may not be covered by the incoming cash flow. In this model, the costs of bankruptcy include the loss of continuation value and an administrative cost paid out of the surviving assets. The loss of continuation value is borne primarily by the owners, while the administrative cost is borne primarily by the bondholders. To avoid bankruptcy, the owners can choose to undertake a fair gamble to handle the shortfall.[4] The single-period model shows that if the net gain from continuation is positive, the owners gamble for redemption, just to the level needed to repay the debt, and the gambling can benefit both the owners and the bondholders by minimizing both costs. This is because, in the absence of bankruptcy, the owners can keep the continuation value, and the bondholders receive the required payment without administrative costs. Notice that the presence of administrative costs is not crucial for this result, as shown in the following numerical example. Furthermore, if some of the debt has been rolled over, bondholders may benefit more from gambling for redemption. The reason is that the owners may gamble to cover a lower required due payment, increasing the probability that bondholders will be paid in full.

We now show a numerical example to demonstrate gambling for redemption. Let's consider firm A with continuation value of 120 (excluding cash flow), liquidation value of 60 (excluding cash flow), cash flow of 20, and total debt of 100. The firm can roll over debt of 60, so that the required payment is 40. The parameters and values are listed in Table 1. If the firm does

[4]"Owners" and "bondholders" are two players in the model. The management of the firm is assumed to be aligned with the owners, working towards maximizing equity value. This is to say, our firms are more like proprietorships than corporations, which is a common feature of models in corporate finance. This simplification allow us to focus on the role of gambling.

| | Firm A ($F < C$) | Firm B ($F > C$) |
|---|---|---|
| Continuation value ($C$) | 120 | 80 |
| Liquidation value ($L$) | 60 | 60 |
| Cash flow ($\pi$) | 20 | 20 |
| Total debt ($F$) | 100 | 100 |
| Debt rolled over ($B$) | 60 | 60 |
| Required payment ($F - B$) | 40 | 40 |

Table 1: Numerical examples: parameter values

not gamble, it will default since $20 < 40$, and the owners will receive nothing in bankruptcy, and the bondholders receive the cash plus the liquidation value, 80. Consider the following two fair gambles:

(a) gambling for redemption, winning 20 with probability 0.5 and losing cash flow 20 with probability 0.5, and

(b) gambling for ripoff, that is, winning 180 with probability 0.1 and losing 20 with probability 0.9

In this paper, we assume either risk neutrality or the expectations are interpreted as the risk-neutral (martingale) probabilities based on Cox and Ross (1976). In gamble (a), the owners win 20 half of the time to cover the required payment of 40, and the continuation value of 120 is maintained (and subtract the debt rolled over). The owners have an expected value of $0.5 \times (120 - 60) = 30$, which is strictly better than receiving zero in bankruptcy. Interestingly, this gamble does not hurt bondholders since receiving 100 with probability 0.5 and 60 with probability 0.5 (with gambling) has the same expected value of 80 compared to receiving 80 with probability 1 (without gambling). If we also consider bankruptcy costs paid by bondholders, gambling for redemption makes bondholders strictly better off. This is consistent with Federal Express's gambling. In this case, the owners are not interested in larger gambles that rip off bondholders. For instance, taking gamble (b) gives the owners an expected value of $0.1 \times (120 + 180 + 20 - 100) = 22$, which is less than the expected value of 30 if they gamble for redemption. Hence, the owners prefer gambling for redemption over gambling for ripoff.

However, whenever the net gain from continuation is negative, the owners prefer not to continue because the cost of lost continuation value is trumped by the obligation to repay the debt. In this case, the owners may simply "take the money and run," and gambling can provide a legitimate means of doing so. To illustrate, let's assume firm B's continuation value is 80 (instead of 120), which is lower than the total debt of 100. With other parameter values the same, a larger gamble (b) allows the owners to obtain an expected value of $0.1 \times (80 + 180 + 20 - 100) = 18$, which is higher than the value of $0.5 \times (80 - 60) = 10$ obtained from gamble (a), gambling for redemption. However, in gamble (b), bondholders only get fully repaid 10% of the time, with an expected value of $0.1 \times 100 + 0.9 \times 60 = 64 < 80$. This means that bondholders are being ripped off compared to the case of gambling for redemption or no gambling at all. It is important to note that in the previous example, continuation is good for the owners because the continuation value 120 exceeds the total debt 100; while in this example, continuation is bad for the owners because the total debt 100 exceeds the continuation value 80.

We now show that if superpriority allows the firm to gamble with the liquidation value, gambling for ripoff can be more attractive because it enables the transfer of that liquidation value to the owners as well, reducing net gain from continuation. Let's consider firm A again, with a continuation value of 120 (excluding cash), and the firm's liquidation value 60 can be pledged to gamble with. We consider two more fair gambles that are not available absent superpriority, but are available now:

(c) gambling for redemption with gambling away liquidation value, that is, winning 20 with probability 0.8 and losing 80 (cash flow plus liquidation value) with probability 0.2, and

(d) gambling for ripoff with gambling away liquidation value, that is, winning 720 with probability 0.1 and losing 80 (cash flow plus liquidation value) with probability 0.9.

Gambles (a) and (b) are still available, and the owners of firm A prefer gamble (a) to gamble (b) as we previously discussed. However, gamble (c) makes the owners better off, as they obtain an expected value of $0.8 \times (120 - 60) = 48$, which is higher than the owners' expected value in gamble (a), and bondholders receive an expected value of $0.8 \times 100 = 80$. Gamble (d) is even better for the owners, who would receive an expected value of $0.1 \times (120 + 720 + 20 - 100) = 76$. This value is higher than that of gamble (c), which is 48, and also higher than the expected value of gamble (a), which is 30. Bondholders, however, receive an expected value of only $0.1 \times 100 = 10$, which is lower than that of gambles (a) and (b). This example illustrates that being able to gamble away the liquidation value of assets can push the owners towards gambling for ripoff, even when gambling for redemption is a viable option. This is because without the option to gamble the liquidation value of the assets, the net gain from continuation for the owners is $120 - 100 = 20$, which is positive, and hence gambling for redemption prevails. However, if the liquidation value can be used, the net gain from continuation becomes $120 - 100 - 60 = -40$, which is negative, making gambling for ripoff appealing.

In the first example of gambling for redemption, gambling "to win 20 with a probability of 0.5 and lose the cash flow of 20 with a probability of 0.5" is indeed the optimal gamble for the owners. It may seem strange that bondholders also prefer this gambling, given that we have learned from Myers (1977) that in a single-period model, their payoff is concave in firm value, so Jensen's inequality implies that gambling makes them worse off. However, the bond payoff is linear below the face value of debt, and linear above, so the strict concavity is only at the point of meeting the required payment exactly. For gambles that stay (weakly) on one side or the other of the point of strict concavity, bondholders are risk neutral. When gambling for redemption, owners gamble precisely the amount needed to make the required payment, which results in bondholders feeling indifferent absent bankruptcy costs, and actually benefiting if costs are involved. By contrast, gambling for ripoff crosses the point of concavity and the bondholders are worse off if the scale of gambling is large enough.

In Myers (1977), asset substitution is deemed inefficient because firms undertake wasteful activities to add noise to the payoff distribution. Asset substitution is also imprecise because the added noise can move the cash across the threshold, making bondholders worse off. Previous literature has mostly added noise to the entire distribution, typically using normal or lognormal distributions as proxy for risk (Ericsson (1997), Ross et al. (1998), Leland (1998), Gong (2004) and Della Seta et al. (2020)).[5] Under the assumption of these normal distributions, taking on more risk increases the probability of payoffs on both tails. However, gambling with derivatives is a sharper tool to achieve the desired outcome. The cost can be very small, and it is also possible to be more precise and gamble only on the left tail. For example, a firm could buy a digital option paying off exactly the amount due on their debt, which makes the choice of gambles more flexible in our framework.

The single-period model has some implications. In normal times when it is beneficial for the firm owners to keep the firm running, gambling would not be a problem because the firm with sufficient cash to pay debt would not gamble. Even if the firm experiences temporary negative shocks on cash flow, the owners prefer gambling for redemption, the owners would still prefer gambling for redemption, minimizing the probability of bankruptcy. Gambling becomes a problem when the firm's continuation value is small compared to its debt that cannot be rolled over. In such cases, the owners would prefer gambling for ripoff, as it maximizes their benefits by looting the value that should have gone to the bondholders. Interestingly, the owners favor such extreme gambling regardless having enough cash to cover debt or not. When cash is insufficient, gambling for ripoff transfers value from the bondholders to the owners. But if there is enough cash to cover the debt, gambling for ripoff also dissipates the equity's continuation value. In this scenario, providing liquidity to save the firm may help keep it running temporarily, but it may not be sufficient to change the risk-taking behavior of the owners. Instead, policies that increase the firm's continuation value or prevent large-scale gambling may be more socially efficient.

In the single-period model, the face value of maturing debt is assumed to be exogenous. This might be a good assumption at the time of the superpriority legislation, if the legislation is a surprise to the bondholders with existing debt. However, to understand the impact of the law once it is understood by bondholders and priced into the debt, a multi-period model is more useful. In each

[5]Ericsson (1997) studies firm's one-time choice of risk which is either at a high level or at a low level. Gong (2004), Ross et al. (1998), Leland (1998) and Della Seta et al. (2020) extend the choice of variance to an interval.

period of the multi-period model, the owners choose gambling and new financing after a capital shock is realized. It's worth noting that the optimal gambling strategy may not always be the polar cases of gambling for redemption or ripoff, but instead, somewhere in between. When continuation is sufficiently large, gambling tilts towards gambling for redemption, and vice versa. Overall, the benefit of gambling depends on the frequency of gambling tilting towards gambling for redemption and ripoff. Our main result of the multi-period model shows that if there is significant liquidation value can be gambled with (for example due to superpriority), gambling reduces the maximum amount the owners can borrow, and also reduces the market value of equity. This suggests that superpriority can benefit firm owners if it is a surprise at the time of passage but may not be beneficial once lenders understand that the law can make large gambles more attractve to owners.

If the firm owners are potentially worse off due to superpriority laws, as suggested in our multi-period model, they may resort to more defensive measures (operating leverage, secured debt, short-term debt, and even repos) to protect against the laws. Furthermore, negative pledge covenants in bankruptcy may no longer offer sufficient protection for bondholders, and they may need to rely more on perfected security interests under UCC Article 9. This is supported by some empirical evidence. For example, (Benmelech et al., 2020, Figure 8a) documents an increase of secured debt over total debt since 1995 and an upward jump in 2005. (Baily et al., 2008, Figure 6) shows that the issuance of total value of short term (with 1-4 days maturity) asset-backed commercial paper has increased significantly from 2005 to mid 2007, whereas the commercial paper with longer terms (with 21-40 days and $>$ 40 days maturities) stayed steady during the period. There was also a surge in the growth in the market for repurchase agreements, a much higher growth rate compared to the total debt in the financial sector, particularly after 1999 (Roe (2010)). Ganduri (2016) finds a surge of the number of repurchase agreements after BAPCPA went into effect in 2005, whereas the number of loans plummeted during the same period; Lewis (2020) provides causal evidence of expansion of repo collateral rehypothecation as a result of the law and estimates a money multiplier of private-label mortgage collateral to be 4.5 times that of Treasuries. However, relying on perfected collateral to prevent large gambles carries its own costs, including the inflexibility of assets redeployment and constraints on future borrowing and investment. This has been discussed in Donaldson et al. (2019, 2020).[6]

The paper is structured as follows. The fowllowing subsection 1.1 provides more details on superpriority laws. Section 2 focuses on the two polar cases of "gambling for redemption" and "gambling for ripoff" by examining a stripped-down single-period model, and with discussions of other potential applications of our single-period model. Section 3 develops a multi-period model using the building block in Section 2 and incorporates endogenous decision making to study the ex-ante effects of gambling with and without superpriority. Section 4 characterizes equilibrium properties of the model and provides numerical examples to illustrate the results, and Section 5 concludes.

## 1.1 Superpriority of QFCs and gambling

Traditionally, redeploying assets to compensate claimants with amounts exceeding what they would have received through bankruptcy proceedings has been difficult. While common law permits asset seizure to satisfy debts, seizure or sales can be clawed back in bankruptcy. Specifically, if an asset transfer occurs within 90 days (or longer period in some instances)[7] of the filing of bankruptcy, it is considered a preferential treatment if the firm is unable to equally satisfy all the creditors, and the transfer are subject to reversal by the court. The purpose of these provisions is to prevent a frenzied competition among creditors to grab assets in the firm, and to provide a fair resolution for all parties involved. In addition, bond covenants can be used to trigger bankruptcy in response to asset seizure or sales. Such covenants often contain clauses that limit the firm's ability to sell its assets, typically placing the firm in default on its loans if the covenants are violated. Moreover, cross-default clauses in bond agreements which stipulate that a default on one bond triggers on all outstanding bonds, normally result in the firm entering bankruptcy. All these provisions tend to

[6] In turn, these costs can be mitigated somewhat by issuing collateralized debt with a call provision or a short maturity.

[7] The clawback extends back one year for a preferential transfer to an insider, or up to two years for a fraudulent conveyance.

make asset seizure or sale pointless.

Hence, in the absence of superpriority, any promise by the firm to transfer assets to cover losses from gambles would not be credible for the gambling counterparties unless they are sure that the firm will not be forced into bankruptcy. This lack of credibility constrains the scale of gambling, as counterparties are less likely to engage in high-stake gambling unless the firm is in good shape.

However, the exemption from bankruptcy law for "superpriority" claims bypasses these legal rules that protect assets, which in general prioritize the contractual rights of derivatives counterparties to "terminate, liquidate, or accelerate" derivatives contracts before the commencement of bankruptcy. While bankruptcy procedures for different firms are governed by different laws, these laws generally grant or expand superpriority rights to the derivatives contracts.[8] These rights provide a significant advantage to gambling counterparties, as these laws ensure that the assets can be collected without being stayed in the firm's estate during bankruptcy proceedings. Consequently, the firm and their gambling counterparties can make commitments to pledge the firm's assets in gambling, knowing that these commitments will not be hampered by bankruptcy law.[9]

The "superpriority" claims we are talking about obtained their exemption from bankruptcy in a series of laws passed between 1978 and 2006. See Schwarcz and Sharon (2014) for a detailed history of the law. The game changer appears to have been the 2005 amendment to the bankruptcy

[8]Several provisions govern the bankruptcy process in the United States, each with their own specific rules and exceptions, but these provisions basically have similar superpriority rules. Chapter 7 and Chapter 11 of the Bankruptcy Code apply to most individuals and entities, with some exceptions. See 11 U.S. Code §362(b)(6), §546(e). Chapter 15 specifically addresses bankruptcy of foreign debtors and includes provisions that offer safe harbor to QFCs. The Federal Deposit Insurance Act (FDIA), the Federal Credit Union Act (FCUA), and the Housing and Economic Recovery Act of 2008 (HERA) each provide superpriority rules for financial institutions defined under the FDICIA. Similarly, the Securities Investor Protection Corporation (SIPC) governs the liquidation of stockholders under SIPA, with similar rules to the Code. See 15 U.S. Code §78eee(b)(2)(C).

The Orderly Liquidation Authority (OLA) oversees bankruptcy procedures for systemically important financial companies. Unlike under the Bankruptcy Code, resolutions under OLA grant a 24-hour stay of assets, and superpriority rights are enforced unless the receiver transfers all QFCs to another financial institution and provides notice to counterparties within this 24-hour period. For more details, see *Qualified Financial Contracts and Netting under U.S. Insolvency Laws* by Cleary Gottlieb Steen & Hamilton LLP downloaded from `https://www.clearygottlieb.com/-/media/organize-archive/cgsh/files/2017/publications/qualified-financial-contracts-and-netting-under-us-insolvency-laws.pdf` on August 15, 2022. For details of OLA, see Treasury (2018). In 2018, the Treasury proposed a new Chapter of the Bankruptcy Code, Chapter 14, to replace the OLA in handling bankruptcy procedures for large, interconnected firms.

[9]Gambling with assets is also possible due to poor specification or enforcement of property rights and bankruptcy law in under-developed countries.

code, known as the Bankruptcy Abuse Prevention and Consumer Protection Act (BAPCPA), which broadened the scope of the exemption to include all derivative securities. which started with some commodity futures and previously extended to repos and swaps, to all derivative securities. Taken together, these laws exempt qualified financial contracts (QFCS)- which encompass a wide range of financial instruments such as securities contracts, commondity contracts, forward contracts, repos, and swaps - with immunity from the automatic stay and clawbacks that typically arise in bankruptcy proceedings.[10] In addition, BAPCA and the subsequent 2006 Act introduced further protections for "master netting agreements" relating to the QFCs mentioned above. These agreements allow counterparties to offset mutual obligations. For instance, if two firms owed each other one dollar, without netting, when one firm is in bankruptcy the counterparty has to repay the one dollar and may receive only 50 cents out of the dollar from the firm. With a netting agreement, the counterparty can set off the debt and be paid 100 cents out of a dollar. This treatment makes gambling even easier.

The superpriority treatment has drawn a lot of attention since the 2008 financial crisis. Roe (2010) observes a soaring volume of interest rate derivatives from \$13 trillion in 1994 to \$430 trillion in 2009, representing almost a forty-fold increase. During the same period, private business debt only tripled from \$11 trillion to \$34 trillion.[11] Baily et al. (2008) also shows an exponential growth in the value of outstanding CDS since 2001. Roe argues that this is because superpriority provides a cheaper way of financing, facilitating greater liquidity that would not have occurred otherwise. This shift away from traditional financing also reduces the incentives of derivatives counterparties to monitor the firm, exacerbating the "too big to fail" problem if systemically important firms rely heavily on the these derivatives. In addition to the costs, Duffie and Skeel (2012) highlights the benefits of the safe harbor exemption on QFCs, such as ensuring the redemption of critical hedges and reducing self-fulfilling security runs. Though it is also possible that superpri-

[10] Supepriority also favors derivatives by exempting clawbacks of constructive (but not actual) fraudulent transfers. See Vasser (2005). Nonetheless, it is worth noting that the exemption may not be applicable in cases of gambling, where the transfer is made in satisfaction of a pre-existing claim and represents a fair value exchange, which probably cannot be defined as fraudulent.

[11] Roe (2010) Figure 1.

ority causes runs to grab assets that were previously deterred by the automatic stay and clawbacks. Previous economic literature has focused on the fire sales in the repo market, which dilute the collateral value for the secured creditors.[12] Our paper excludes the above factors related to superpriority but instead focuses on gambling.

# 2 Optimal gambling: the single-period model

We start with a stripped-down model to identify two different types of gambling: gambling for redemption and gambling for ripoff. Gambling for redemption occurs when the firm cannot immediately pay off its debt, and bankruptcy would result in a net loss for the owners. In this case, the owners will only take necessary risks to meet their debt obligations and avoid bankruptcy, benefitting both the owners and the bondholders. In contrast, gambling for ripoff occurs when the owners would gain in net from bankruptcy. In this case, owners will gamble to a large payoff to maximize the probability of bankruptcy, allowing them to evade debt obligations while still collecting the firm's unprotected liquidation value, benefiting the owners at the expense of the bondholders. Superpriority claims can reduce the net loss to owners by enabling them to directly collect a portion of the firm's asset value without having to pay. This result in bigger gambles.

We are looking at a snapshot when a firm has debt of $F > 0$ maturing now. The crucial outcome is whether this firm, if socially valuable, continues or not. If the debt cannot be fully repaid, the firm undergoes a liquidation (as in Chapter 7 bankruptcy), and all assets are sold to repay the debt. In this section, we assume that $F$ is positive so that bankruptcy is possible. In the multi-period model, debt can be negative, representing a firm's funds in the bank, but that is not relevant for this section's purpose. Bankruptcy has costs, as owners receive nothing, and therefore they typically prefer to pay off the debt and continue. The owners can repay the debt using a combination of (1) cash flow $\pi \geq 0$ generated from operations, (2) proceeds from rolling over some of the debt, and (3) net gambling proceeds $G$. We assume that the maximum debt that can be rolled over, denoted

[12] See Infante (2013), Oehmke (2014), Antinolfi et al. (2015) and Auh et al. (2018).

by $B$, is smaller than the total debt amount $F$, or else gambling would not be necessary. The owners are only allowed to choose gambles with negative payoffs that can be credibly repaid. We implement this requirement as a constraint on the owners' optimization problem, which depends on whether gambling is available and on the fraction of liquidation value that can be used to settle superpriority claims.

We show the economic balance sheets of the firm in Table 2 given a gambling payoff $G$. Balance sheet (1) is a general sheet, with some values left blank and to be calculated based on whether the firm continues or not. If the sum of cash flow $\pi$ and net gambling outcome $G$ is greater than or equal to the debt amount $F$ minus the maximum debt that can be rolled over $B$, i.e., $\pi + G \geq F - B$, then the firm can continue. In this case, the entire value of the firm before raising any new funds includes the continuation value $C$, cash flow $\pi$ and net outcome from gambling $G$. The existing bond has a value of $F$, and the remaining value, $C + \pi + G - F$, goes to the owners. However, if the sum of cash $\pi + G$ cannot cover the required amount $F - B$, all bondholders and owners bear a fractional bankruptcy cost $c \in [0, 1]$ of their respective values. The owners always receive nothing, and bondholders receive the residual value, which is equal to the liquidation value plus any cash after paying the bankruptcy costs, i.e., $(1 - c)(L + \pi + G)$.

(1)
general

| | | | |
|---|---|---|---|
| Cash | $\pi+G$ | Old bonds | ? (face $F$)[13] |
| Continuation value | ? | Lawyers and accountants | ? |
| Liquidation value | ? | Equity | ? |

continues
$\pi+G \geq F-B$

fails
$\pi+G < F-B$

(2)
can pay off $F$

| | | | |
|---|---|---|---|
| Cash | $\pi+G$ | Old bonds | $F$ |
| Continuation value | C | Lawyers and accountants | 0 |
| Liquidation value | 0 | Equity | $C+\pi+G-F$ |

(3)
cannot pay off $F$

| | | | |
|---|---|---|---|
| Cash | $\pi+G$ | Old bonds | $(\pi+G+L)(1-c)$ |
| Continuation value | 0 | Lawyers and accountants | $(\pi+G+L)c$ |
| Liquidation value | L | Equity | 0 |

In balance sheet (3), we also make the following assumptions to focus on the interesting cases:

Table 2: ECONOMIC BALANCE SHEETS
The balance sheets are given in economic value, which is at a snapshot when a gambling payoff $G$ is realized and before raising funds $B$ from new bondholders or paying old bondholders the face value $F$. Balance sheet (1) is the general sheet, and the missing values filled with question marks are to be calculated based on whether the firm continues or not. For example, the existing bonds have a face value $F$, but the bondholders receive $F$ only if the firm continues.
Balance sheet (2) shows the corresponding values when the firm continues. That is, when $\pi + G \geq F - B$, the total cash can cover the required net payment.
Balance sheet (3) shows the corresponding values when the firm fails. That is, when $\pi + G < F - B$, the total cash cannot cover the required net payment. In this case, all bondholders and owners bear a fractional bankruptcy cost $c \in [0,1]$ of their values. The owners always receive zero, and bondholders receive the residual value, which is the liquidation value plus any cash, $(1-c)(L+\pi+G)$.

**Assumption 1** $C \geq B \geq L \geq 0$.

In this assumption, $C \geq B$ ensures that the maximum amount of debt can be rolled over (new liability) should not be greater than the continuation value of the firm, otherwise the owners would simply collect the cash, abandon the firm and walk away, making it pointless to discuss whether the firm will continue or not. The assumption of $L \leq B$ focuses our analysis on the interesting case because otherwise the owners could probably borrow for at least a little while to defer part of the current payment $F - B$ if $L > B$. $L \leq C$ indicates that it is always socially efficient to continue the firm. Under these assumptions, we show the balance sheets in economic value after the realization of gambling payoff $G$.

The only uncertainty in the model is the gambling, i.e., $F$, $\pi$, $B$, $C$, $c$, $L$, and $\gamma$ are all constants known to the agents. Taking all these variables to be constant is a better approximation than it may seem. At the time of a short gamble, the agents may not know what would come from a full liquidation of the assets, but they may know how much they would raise for an as-is sale (perhaps to the counterparty) at this point of time. The owners choose a gambling payoff $\mathbf{G}(\tilde{x})$ to maximize the expected equity value

$$\mathrm{E}\big[\big(\pi + B + \mathbf{G}(\tilde{x}) \geq F\big)\big(C + \pi + \mathbf{G}(\tilde{x}) - F\big)\big]. \tag{1}$$

All agents in our model are risk neutral, and $\mathrm{E}[\cdot]$ in (1) indicates either the common beliefs or risk-neutral expectations given agents' shared valuations. We assume that $\tilde{x} \sim_d U(0,1)$ is the underlying randomness for gambling. The gambling function $\mathbf{G}$ maps the support of randomness to a set of the feasible gambling outcomes $\mathscr{O}$. Formally, the feasible gambling set is

$$\mathscr{G} \equiv \left\{ \text{non-increasing } \mathbf{G} : [0,1] \to \mathscr{O} \,\middle|\, \mathrm{E}[\mathbf{G}(\tilde{x})] = 0 \right\} \tag{2}$$

given the feasible gambling outcomes

$$\mathscr{O} = \begin{cases} \{0\}, & \text{no gambling} \\ [-\gamma L - \pi, \bar{G}], & \text{otherwise} \end{cases} \tag{3}$$

To ensure a unique solution, the feasible gambling set in (2) requires $\mathbf{G}$ to be non-increasing. The fair gambling requirement, $\mathrm{E}[\mathbf{G}(\tilde{x})] = 0$, assumes negligible transaction costs, which is a reasonable assumption given the tiny costs associated with trading derivatives in liquid markets. The distribution of gambling outcomes can be any that satisfies these constraints, since derivatives are much more flexible instruments for gambling compared to asset substitution (as in Myers (1977)).

We also have in mind that the gambling is very short-term, which is essential if the proceeds are to be used to pay current liabilities. Very short-term gambles are definitely possible when gambling uses derivatives. Very short-term gambling means it is reasonable that the liquidation value after the gamble is assumed to be known before the gamble. As noted above, short-term gambling motivates making $L$ an exogenous constant which follows from the firm's gambling counterparties understanding the firm's marketability in the short run.

The constraint of gambling outcomes in (3) defines the maximum amount the firm can lose in the gamble. If gambling is not available, the outcome is always 0. If the gambling is possible, the gambling counterparties would only gamble with the firm if the firm can credibly repay, that is to say, there is a limit that the firm can promise to lose in a gamble. This limit, $\pi$ plus the fraction $\gamma$ of the liquidation value, is determined by the the priority of the gambling counterparties.

Absent superpriority, the firm can always promise the cash flow $\pi$ to the gambling counterparties by paying upfront, but the other assets go to the bondholders, so that $\gamma = 0$. With superpriority, gambling counterparties are paid before the existing debt $F$, and would receive a fraction $\gamma$ of assets not serving as perfected collateral. Hence, superpriority increases $\gamma$, allowing larger gambles. In our simple example, we assume there is no perfected collateral, so that superpriority increases the amount available for gambling from 0 to the entire liquidation value $L$:

$$\gamma \equiv \begin{cases} 1, \text{ with superpriority} \\ 0, \text{ absent superpriority.} \end{cases}$$

In (3), we assume an upper bound $\bar{G} >> 0$ to avoid a closure problem in some cases if there is no upper limit of gambling, but we can compute limits of expected payoffs as $\bar{G} \uparrow \infty$. We think of $\bar{G} >> 0$, but we require at a minimum that $\bar{G} \geq F - B - \pi$, or $\bar{G} + \pi \geq F - B$, meaning that gambling to cover the required payment is possible.

Then, given the owners' choice of gambling $\mathbf{G}(\tilde{x})$, the bond value is

$$\textit{bond value} = \mathrm{E}\Big[(\pi + B + \mathbf{G}(\tilde{x}) < F)(1-c)(\pi + \mathbf{G}(\tilde{x}) + L) + (\pi + B + \mathbf{G}(\tilde{x}) \geq F)F\Big].$$

In the above function, $\pi + \mathbf{G}(\tilde{x}) + L \geq \pi - \gamma L - \pi + L \geq (1-\gamma)L \geq 0$. Since $L \leq B$ by assumption 1, we can show that $\pi + \mathbf{G}(\tilde{x}) + L \leq F$ when $\pi + \mathbf{G}(\tilde{x}) < F - B$. This suggests that the bondholders are better off if the firm survives. This is crucial for the result that gambling for redemption makes bondholders at least indifferent. Below we show that the gambling behavior of the owners is quite different depending on whether $F$ is greater than $C - \gamma L$. In particular, if $F < C - \gamma L$, the owners will gamble at a small scale which benefits bondholders; whereas if $F > C - \gamma L$, the owners will gamble at a large scale which rips off the bondholders. With superpriority, $\gamma$ is larger, implying that the owners gambles for ripoff more often.

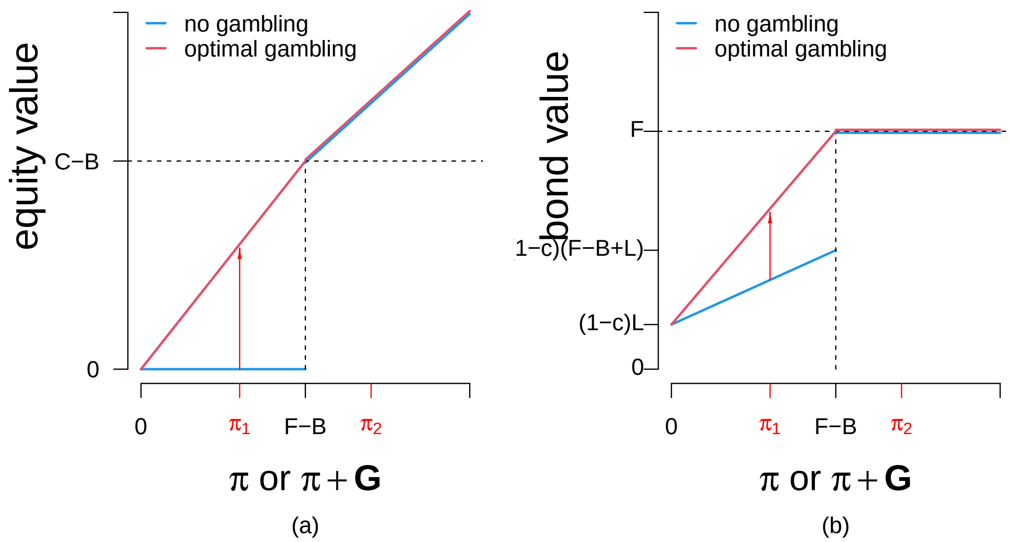



Figure 1: *Gambling for redemption, which is just enough to pay off debt, makes both equity and bonds more valuable. The owners prefer to gamble for redemption when the required debt payment is less than equity's share of the firm's continuation value. To stay alive, cash (= cash flow $\pi$+ any gambling proceeds* **G***) must cover the required debt payment (maturing debt $F$ less value of new debt $B$). If the firm stays alive, the equity value is the continuation value $C$ plus any leftover cash $\pi + G - F$, while the existing bondholders get $F$. If the firm fails, the bondholders receive the liquidation value of the assets $L$ plus any cash $\pi + g$, all subject to a proportional bankruptcy cost $c$.*
*Gambling using derivatives is precise. The optimal gambling demonstrated by the red lines "concavifies" the equity's value function. The red arrow shows that both equity and bond values increase given cash flow $\pi_1 < F - B$ (the cash flow cannot cover the required debt payment). For $\pi_2 \geq F - B$ (the firm has enough cash flow to cover required debt payment), no arrow is shown because optimal gambling does not change equity and bond values.*

## Example 1: gambling for redemption (absent superpriority, $\gamma = 0$)

Without superpriority, $\gamma = 0$. Figure 1 demonstrates the equity value and bond value as functions of cash flow when $F < C$, i.e., the face value of debt is less than the firm's continuation value. The blue lines represent the values without gambling: if cash flow $\pi$ is below $F - B$, the firm loses all the continuation value in bankruptcy and bondholders lose a fraction $1 - c$ of the remaining assets $\pi + L$; if cash flow $\pi$ is above $F - B$, the net continuation value $C - B$ is maintained and the total bond value is $F$.

To reach the maximal expected equity value, the value of an optimal gambling strategy, shown by the red lines, should "concavify" the blue curves. As shown in Figure 1, if the firm starts with cash flow $\pi_2 \geq F-B$, the firm is sound and the owners will only gamble along the 45 degree segment and will never gamble down below $F-B$, and none of those gambles change the expected payoff for the owners or bondholders. However, if the firm starts with cash flow $\pi_1 < F-B$, equity is worthless unless there is gambling. In this case, optimal gambling retains the continuation value as often as possible, and the after-gamble cash flow $\pi_1 + \mathbf{G}(\tilde{x})$ achieves $F-B$, with probability $\frac{\pi_1}{F-B}$, and 0, with probability $1-\frac{\pi_1}{F-B}$. The owners get $C-B$ with probability $\frac{\pi_1}{F-B}$, expected value $\frac{\pi_1}{F-B}(C-B) > 0$. The bondholders get $F$ with probability $\frac{\pi_1}{F-B}$ and $(1-c)L$ with probability $1-\frac{\pi_1}{F-B}$, expected value $\frac{\pi_1}{F-B}F + (1-\frac{\pi_1}{F-B})(1-c)L \geq (1-c)L + \frac{\pi_1}{F-B}(1-c)(F-B)$. The right-hand-side is the bond value without gambling. The owners and bondholders will achieve values along the "concavified" value functions and are better off than not gambling.

To conclude, gambling for redemption adds value to the owners and the bondholders due to less frequent value loss in bankruptcy. When there is no fractional bankruptcy cost ($c=0$) and the face value of ongoing debt is equal to the liquidation value ($B=L$), the bondholders are indifferent to whether the owners gamble or not.

## Example 2: gambling for ripoff (absent superpriority, $\gamma = 0$)

However, when the face value of debt $F$ is greater than the firm's continuation value $C$, *"gambling for redemption"* is no longer optimal. The dashed red lines in Figure 2 give the payoffs of fair Bernoulli gambles. An example is

$$\mathbf{G}(\tilde{x}) = \begin{cases} \xrightarrow{\pi/(\bar{G}+\pi)} \bar{G} \\ \xrightarrow[\bar{G}/(\bar{G}+\pi)]{} -\pi \end{cases}$$

As the payoff $\bar{G}$ increases, the probability of winning declines but the owners have a larger value because not paying $F-B$ is more important to them than not receiving $C-B$. The above Bernoulli gamble concavifies the owners' original value function and is the optimal gamble. In

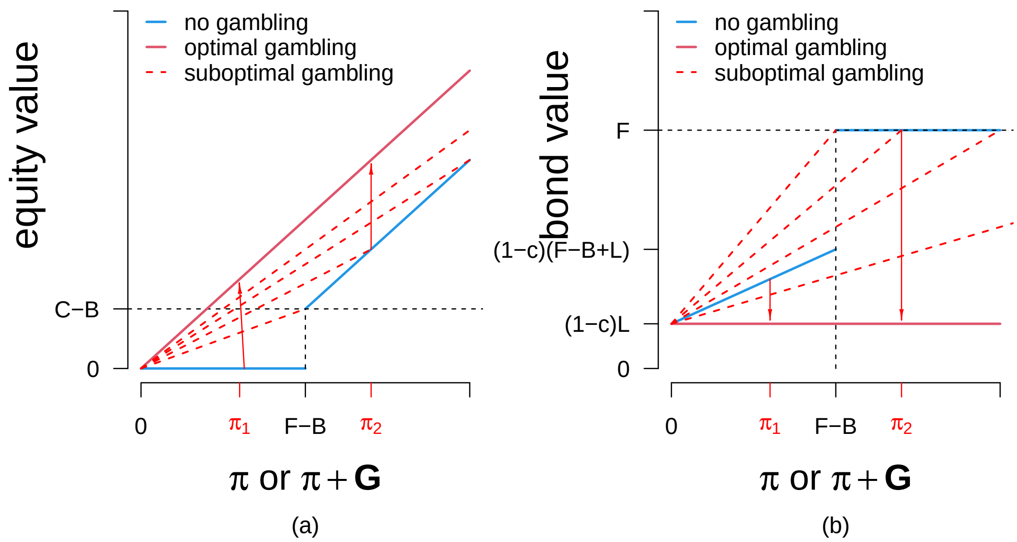



Figure 2: *Gambling for ripoff, which is gambling to the largest scale to fail as often as possible, benefits the owners at the expense of bondholders. The owners prefer to gamble for ripoff when the required debt payment is greater than the equity's share of the firm's continuation value. Similar to gambling for redemption, to stay alive, cash ($=$ cash flow $\pi+$ gambling proceeds $G$) must cover the required debt payment (maturing debt $F$ less value of new debt $B$). If the firm stays alive, the equity value is the continuation value $C$ plus any leftover cash $\pi+G-F$, while the bondholders get $F$. If the firm fails, the existing bondholders receive the liquidation value $L$ plus cash $\pi+g$, subject to a proportional bankruptcy cost $c$.*
*Equity value after optimal gambling (red lines) is a concavification of the original value function (blue lines). The bond value decreases to $(1-c)L$, the amount that cannot be gambled away by the owners. Interestingly, whether the firm is out of money ($\pi=\pi_1$) or in the money ($\pi=\pi_2$), the owners always choose to gamble for ripoff.*

this gamble, the owners obtain $\bar{G}$ with probability $\frac{\pi}{\bar{G}+\pi}$ and 0 with probability $1-\frac{\pi}{\bar{G}+\pi}$. Therefore, the maximum that the owners can achieve in gambling in the limit is $\lim_{\bar{G}\uparrow\infty}\frac{\pi}{\bar{G}+\pi}(\bar{G}-F+C)=\pi$ and the bond value is $\lim_{\bar{G}\uparrow\infty}\left(\frac{\pi}{\bar{G}+\pi}F+\frac{\bar{G}}{\bar{G}+\pi}(1-c)L\right)=(1-c)L$. The bondholders almost always only receive part of the liquidation value. In this case, *gambling for redemption* would increase the total value of bond and equity because the continuation value would be preserved as often as possible, but the owners would rather choose a larger gamble which gives them a higher value at the expense of bondholders. We have shown that $\pi=\pi_1$, if the firm has cash flow $< F-B$, gambling for ripoff transfers value from the bondholders to the owners. Interestingly, gambling for ripoff is also optimal for equity in this example even if $\pi=\pi_2>F-B$. so the firm has enough to payoff the debt $F-B$ without gambling.

## Example 3: with superpriority $C-L<F<C$

Positive available liquidation value to gamble will change the shape of gambling if $C-L<F<C$, as illustrated in Figure 3. Superpriority makes the liquidation value available for gambling, allowing firm owners to gamble down to $-L$ instead of 0. In Figure 3, the continuation value $C$ is greater than the face value of debt $F$, so that the owners will gamble its cash flow for redemption of the value of the payment due absent superpriority and obtain $\frac{\pi}{F-B}(C+\pi-F)$, as depicted by Figure 3(a). Figure 3(b) shows the relevant bond value and bondholders are also better off. However, when gambling further down to $-L$ is available, gambling for ripoff yields greater benefits as shown in Figure 3(c). However, this larger gambles make bondholders worse as in Figure 3(d). In the graph, whether $L+F-B$ is greater than $C-B$ (or $F$ is greater than $C-L$) determines the optimal gambling, where $C-L$ is the owners' bankruptcy cost, but they benefit from bankruptcy by not paying $F$.

These graphic observations are formally stated by the following propositions:

**PROPOSITION 2.1** *when $F<C-\gamma L$ (the payment due now is less than the value lost in bankruptcy), it is optimal to* ***gamble for redemption****. Under this parameter restriction, gambling increases the value of both bond and equity when $\pi<F-B$, and leaves both unchanged when $\pi\geq F-B$.*

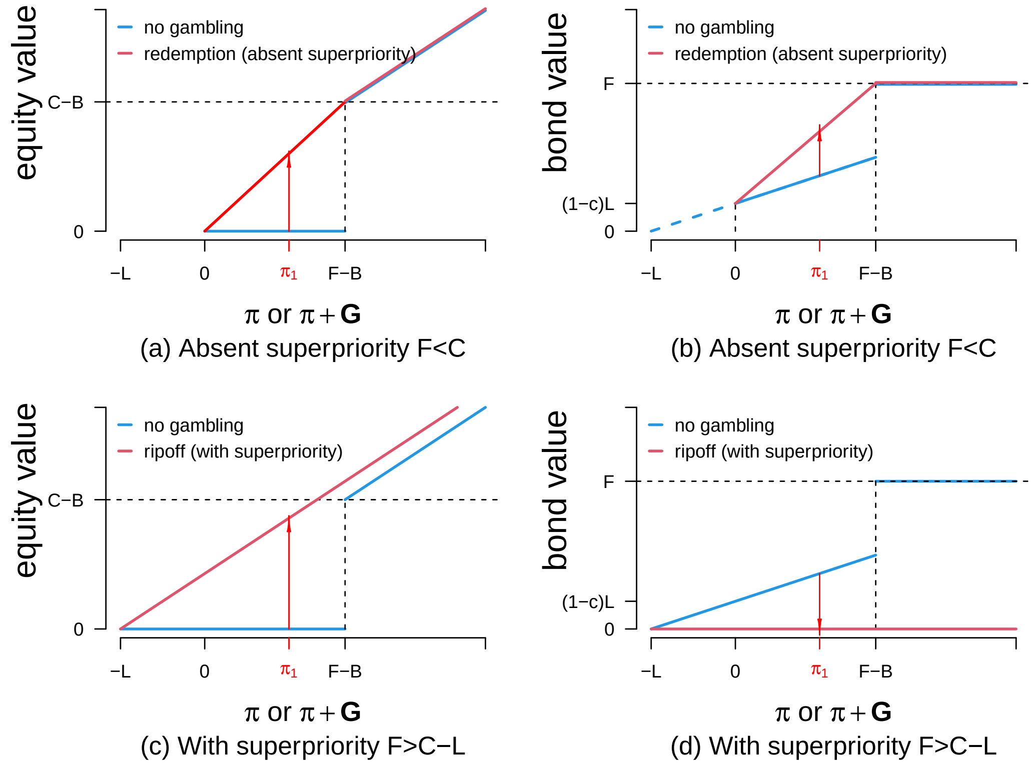



Figure 3: ***When $C-L<F<C$, superpriority makes gambling for ripoff more appealing to the owners.***
*Absent superpriority, the owners can only gamble down to 0 and will gamble for redemption since $F<C$, shown by (a)(b); with superpriority, the owners can gamble down to $-L$ and will choose to gamble for ripoff since $L+F>C$, shown by (c)(d).*

*Specifically,*

*(1) If the cash flow before gambling is insufficient to make the current debt payment ($\pi<F-B$), all optimal gambles have the same distribution. In particular, an optimal gamble is*

$$\mathbf{G}^*(\tilde{x}) = \begin{cases} F-B-\pi, & \text{for } 0<x\leq \frac{\pi+\gamma L}{F-B+\gamma L} \\ -\gamma L-\pi, & \text{for } \frac{\pi+\gamma L}{F-B+\gamma L}<x<1. \end{cases} \tag{4}$$

*(2) If the cash flow before gambling is sufficient to make the current debt payment ($\pi\geq F-B$), the optimal gambles are the feasible gambles that never reduce cash below the current debt*

*payment $F-B$. The set of solutions is*

$$\left\{ \mathbf{G} : [0,1] \to \mathscr{O} \,\middle|\, \pi + \mathbf{G}(\tilde{x}) \geq F - B \text{ and } \mathrm{E}[\mathbf{G}(\tilde{x})] = 0 \right\}.$$

*In particular, not gambling ($\mathbf{G}^*(\tilde{x}) \equiv 0$) is always optimal, and it is the only solution if $\pi = F - B$.*

*(3) The payoffs are*

$$\textit{equity value} = \begin{cases} \pi - F + C, & \textit{for } \pi \geq F - B \\ \frac{\pi+\gamma L}{F-B+\gamma L}(C-F), & \textit{for } \pi < F - B \end{cases}$$

$$\textit{bond value} = \begin{cases} F, & \textit{for } \pi \geq F - B \\ \frac{\pi+\gamma L}{F-B+\gamma L}F + (1 - \frac{\pi+\gamma L}{F-B+\gamma L})(1-c)(1-\gamma)L, & \textit{for } \pi < F - B \end{cases}$$

$$\textit{bond+equity} = \begin{cases} \pi + C, & \textit{for } \pi \geq F - B \\ \frac{\pi+\gamma L}{F-B+\gamma L}C + (1 - \frac{\pi+\gamma L}{F-B+\gamma L})(1-c)(1-\gamma)L, & \textit{for } \pi < F - B \end{cases}$$

**Proof.** (Sketch) Following Aumann and Perles (1965), we first concavify the objective function and use Kuhn-Tucker conditions to solve the concavified problem. Since the constructed solution for the concavified problem is also feasible for the original problem, and the concavified objective function is greater than the original function, we can conclude that the solution(s) for the concavified problem also solves the original problem. Details see Appendix A. ■

In Proposition 2.1(2), if we believe that gambling is costly, or if we are not using risk neutral probabilities (the owners are risk averse), then $\mathbf{G}^*(\tilde{x}) \equiv \pi$ (no gambling) should be the unique solution. However, in Proposition 2.1(1), gambling is still optimal in the face of a sufficiently small cost.

**PROPOSITION 2.2** *when $F > C - \gamma L$, it is optimal for the owners to* ***gamble for ripoff.*** *Gambling for ripoff transfers value from bondholders to the owners when $\pi < F - B$, and also destroys* *continuation value when $\pi \geq F - B$. Specifically,*

*(1) The optimal gambling is*

$$\mathbf{G}^*(\tilde{x}) = \begin{cases} \bar{G}, & \text{for } 0 < x \leq \frac{\pi+\gamma L}{\bar{G}+\pi+\gamma L} \\ -\gamma L - \pi, & \text{for } \frac{\pi+\gamma L}{\bar{G}+\pi+\gamma L} < x < 1 \end{cases} \tag{5}$$

*(2) Owners' payoff is $\frac{\pi+\gamma L}{\bar{G}+\pi+\gamma L}(C+\pi+\bar{G}-F)$, which increases to $\pi+\gamma L$ as $\bar{G}\to\infty$. The value of the bond is $\frac{\pi+\gamma L}{\bar{G}+\pi+\gamma L}F+\frac{\bar{G}}{\bar{G}+\pi+\gamma L}(1-c)(1-\gamma)L$, and declines to $(1-c)(1-\gamma)L$ as $\bar{G}\to\infty$. For any $\pi>0$, the total value of bond and equity is always $\pi+L-(1-\gamma)cL$ when $\bar{G}\to\infty$.*

**Proof.** See Appendix B. ∎

Since it is efficient to continue the firm, gambling for redemption maximizing the probability of continuation is socially beneficial while gambling for ripoff minimizes this probability is socially damaging. In the trade-offs between *gambling for redemption* and *gambling for ripoff*, superpriority plays an important role. It transfers owners the liquidation value which should go to bondholders, making *gambling for ripoff* more appealing to the owners. With more "ripoff" cases, continuation value is lost more often.

There is also a knife-edge case when $F = C - \gamma L$. In this case, any fair gamble with outcomes distributed long the 45-degree linear segment would yield the same expected value. That is to say, *gambling for redemption* and *gambling for ripoff* give the same outcome for the owners, and anything in between the two polar cases is also optimal. Though these optimal gambles generate different values for the bondholders (for example, we still have *gambling for redemption* makes the bondholders better-off and *gambling for redemption* worse-off), we don't want to go into the details of what the equilibrium(equilibria) is(are) because it is reasonable to believe that $F = C - \gamma L$ almost never happen.

In this paper we assume the absence of workouts before or during bankruptcy. We can think of workouts as infeasible if there is a large number of diverse claimants. Even if workouts are possible, gambling for ripoff tends to be robust because the owners would prefer gambling to workouts and bankruptcy. Under gambling for ripoff, the bondholders receive little assets especially when superpriority is available. In a hypothetical workout, this threat of gambling for ripoff allows the owners to appropriate most of the bondholders' value, making the payoffs similar to gambling for ripoff even if the gambling does not actually happen in equilibrium. Gambling for redemption is less robust to the availability of workouts, but the advantage of workouts can be undermined by the high costs.

## 2.1 Applications

The single-period model, despite its simplicity, can be a useful tool for understanding gambling behaviors under various economic and legislative conditions. We present four cases where this framework may be useful to elucidate the observed data.

Traczynski (2019) documents empirical evidence that in states where antidiscrimination laws permit married firm owners to select asset protection at times of failure, firms receive smaller loans without taking on additional risks. This finding aligns with our model. While asset protection increases the benefit of failure, it restricts the amount that the firm can risk in gambling, which is contrary to superpriority laws. As a result, the tradeoff between gambling for redemption or ripoff remains unchanged. But because owners accumulate more assets upon failure, bondholders would receive less in either case, resulting in reduced borrowing.

In another empirical work which concerns Silicon Valley Bank's failure to hedge interest rate risk, Jiang et al. (2023) discovered that despite having high exposure to interest rate risk, many banks (including SVB) significantly reduced their hedging during periods of monetary tightening, thereby increasing their likelihood of bankruptcy. As per our theory, when rising interest rates reduce a bank's continuation value, the banks is motivated to take large gambles. In this case, a massive regulatory failure allowed them to take enormous risks. Even though the subsequent bailout resolved the run, but if the banks can obtain full deposit insurance coverage for free or borrow at above the fair value (according to BTFP), it not only places an unjust burden on those who bear the losses, but it also increases the firm's value upon failure, further encouraging banks to engage in gambling for ripoff for their own benefit.

The introduction of superpriority for retiree medical benefits may also increase the incentives for gambling. After Congress enacted Chapter 11 section 1114 in 1988, granting special priority to retiree medical benefits, gambling for ripoff could become more attractive because an increase in debt obligations would reduce the net gain that the owners receive in continuation. Consequently, gambling for ripoff would wipe out most of the firm's assets, and the dilution effect of the assets could be more severe than merely taking on additional debt. Such high-risk behavior might make it more difficult for the firm to obtain funding, and bondholders may only be willing to lend if the firm promises to file for Chapter 7 liquidation to circumvent the legislation with underfunded retiree insurance benefits, exacerbating the problems highlighted by Keating (1990, 1991).

Dambra et al. (2023) provides empirical evidence that multiemployer pension plans in the United States have exhibited increased risk-taking behavior since the enactment of the American Rescue Plan Act in 2021. This legislation infused funds into specific underfunded multiemployer pension plans, and its impact aligns with our model prediction. Specifically, for underfunded pension plans that face difficulties in meeting their funding obligations, a bailout could amplify the perceived value of failure, incentivizing plan management to engage in riskier investments.

# 3 The Dynamic Model with Endogenous Debt and Continuation Value

Our analysis so far has been based on a single-period model, which is meant to capture the key conditions for gambling for redemption and ripoff. While the assumptions are intended to look like a snapshot of a dynamic model, the exogenous amount of borrowing in the model may not reflect optimal choice. But we can think of the single-period model as a representation of owners' risk-taking behavior when superpriority laws are a surprise. If the superpriority legislation induces gambling for ripoff in the single-period model, it is likely to do so in a multi-period model as well.

Given that, the continuation value is likely to be small, and this will affect how much the firm can borrow and the terms on which it can borrow.

We now shift our focus to a dynamic model in which continuation value and the amount of borrowing are endogenous. Both are affected by the availability of gambling in general and the presence or absence of superpriority law in particular. Lenders are able to anticipate the owners' behavior in response to superpriority, and we find that a significant availability of superpriority tends to reduce firm value. This is because the increased incentive to gamble is anticipated and ultimately reflected in the terms on which the owners can borrow.

In this section, the firm is liquidated and ceases to exist either due to bankruptcy resulting from failure to meet debt payments, or due to an exogenous disappearance of the firm's market. All debt has a one-period duration and is priced at a fair value considering any dilution. To avoid bankruptcy, owners are required to fully repay the face value of the debt. This approach finesses the leverage ratchet effect generalized by Admati et al. (2018), which suggests that existing bondholders are the only ones who benefit from a debt buyback. The exogenous disappearance of the firm's market occurs with a conditional probability $\rho$, after gambling and before borrowing, with randomness drawn independently of the other shocks in the model. We write the objective function as an expectation taken over gambling and the distribution of exogenous ending dates (due to disappearance of the industry) and endogenous ending dates (due to bankruptcy). We consider the value function at two types of choice nodes: choice of gambling (before gambling) and proposal of new debt (after gambling). The choice variables, state variables, and realization of shocks are conditional on the firm still existing.

Here is the timeline of the model at time $t$:

$t$ — I. capital $K_t$ and face value of debt $F_t$ from time $t-1$

II. exits with exogenous probability $\rho$

III. cash flow $vK_t$ arrives

$C^{b4}(K_t, F_t)$ $\leftarrow$ IV. owners choose gambling $\mathbf{G}(\tilde{x})$

V. gambling outcome $\mathbf{G}(x_t)$, "paper surplus" $S_t \equiv vK_t - F_t + \mathbf{G}(x_t)$

$C^{after}(K_t, S_t)$ $\leftarrow$ VI. owners choose borrowing $(B_t, F_{t+1})$ acceptable to investors with $K_t + S_t + B_t \leq (1+g)K_t$

VII. if $S_t + B_t < 0$, not enough cash to cover $F_t$, terminate

VIII. purchase of new capital and shock $\delta_t$, capital after the shock $K_{t+1} \equiv \delta_t(K_t + S_t + B_t)$

$t+1$ — IX. carry capital $K_{t+1}$ and debt $F_{t+1}$ into the next period

Figure 4: Timeline

In period $t$, the firm begins with capital $K_t$ and a maturing debt $F_t$ which represents cash if negative. Nature chooses whether to terminate the firm with exogenous probability $\rho$ and in this case, the owners receive a payout equal to the positive difference between the value of capital and debt, or $(K_t - F_t)^+$, while bondholders receive $\min K_t, F_t$. If the firm is not terminated (with probability $1-\rho$), capital pays a cash flow $vK_t > 0$ to the owners, where $v$ is a constant representing the return on capital.

After exogenous continuation, the owners have the option to participate in a frictionless competitive gambling market. The value function $C^{b4}(K_t, F_t)$ is the owners' continuation value "before" gambling, as indicated by the arrow before step *IV* in Figure 4. In a frictionless competitive gambling market, the gambling choice $\mathbf{G}(\tilde{x})$ has a mean of 0, representing a fair gamble as in the single-period model, and $\mathbf{G}(\tilde{x}) \equiv 0$ if there is no gambling.

Without superpriority, the owners can gamble with only the cash flow $vK_t + (F_t)^-$,[14] which includes savings (negative borrowings) from the previous period. With superpriority, the owners can gamble with $vK_t + (F_t)^- + \gamma\theta K_t$, where $\gamma$ is a constant parameter representing the proportion

[14] We follow the convention that for any $a \in \mathbb{R}$, $a^+ \equiv \max\{a, 0\}$ and $a^- \equiv (-a)^+$. Note that $a = a^+ - a^-$, $a^+, a^- \geq 0$, and $a^+a^- = 0$.

of assets not protected, such as those not pledged as perfected collateral.[15] The liquidation value per unit of capital is denoted by $\theta \in [0,1)$. So, $\gamma\theta K_t$ is the value of capital that the owners can gamble away.

Additionally, we assume that the proceeds from new borrowing cannot be used to pay off gambling debts. Similar to the single-period model, $\bar{G}K_t$ is the upper bound for gambling to prevent a closure problem. In the numerical solution, we take the upper bound to infinity. Note that $\gamma = 0$ if there is no superpriority. Formally, the set $\mathscr{G}_t$ of feasible gambles is given by

$$\mathscr{G}_t \equiv \left\{ \text{non-increasing } \mathbf{G} : [0,1] \to \mathscr{O}_t \,\middle|\, \mathrm{E}[\mathbf{G}(\tilde{x})] = 0 \right\}, \tag{6}$$

given the feasible gambling outcomes

$$\mathscr{O}_t = \begin{cases} \{0\}, & \text{no gambling} \\ [-vK_t - (F_t)^- - \gamma\theta K_t, \bar{G}K_t], & \text{otherwise} \end{cases} \tag{7}$$

It is worth emphasizing that gambling in this paper has a short duration. We think that it is optimal for the owners to use short-maturity derivatives, as they would otherwise need to manage the risk of the various positions over the course of the gamble's duration.

After the gambling outcome is realized, the owners have a net cash of $S_t \equiv vK_t - F_t + \mathbf{G}(x_t)$. The value function $C^{after}(K_t, S_t)$ represents the owners' continuation value "after" gambling, denoted by an arrow after step *IV* in Figure 4.

Next, the owners propose a bond offer with borrowing $B_t$ and face value $F_{t+1}$. If $B_t < 0$, it is interpreted as risk-free investment. The bond market is also frictionless and competitive, which means that the owners can always propose an offer that will be accepted by the investors. Any offer that is not accepted can be replaced by $B_t = 0$ and $F_{t+1} = 0$.

If there is not enough cash after borrowing to cover all the debt due, i.e. $S_t + B_t < 0$, bankruptcy

[15]For our analysis $\gamma$ is exogenous, but in a richer model superpriority laws could induce firms to undertake otherwise ineddicient actions to increase $\gamma$ since the owners and bondholders would have incentives to seek protection of the firm's assets.

occurs. In this case, the new debt issuance is cancelled, and the owners receive $(1-c)(\theta K_t + S_t)^+$ which is the value of assets after selling at a discounted price and a deduction of fractional bankruptcy cost. Existing bondholders obtain $(1-c)[F_t \wedge (\mathbf{G}(x_t) + \theta K_t + \nu K_t)]$ if borrowing is positive.[16]

If $S_t + B_t \geq 0$, there is enough cash to clear all debt obligations, the firm continues and may also increase the capital at a growth rate capped by $g$ per period. We impose the cap $g$ to rule out infinite borrowing and to reflect the fact that firms typically have limited capacity to expand within a period of time. We also require $(1+g)(1-\rho)\mathrm{E}[\tilde{\delta}_t] < 1$ to ensure that firm's value is finite. The new capital after augmentation but before the shock $\tilde{\delta}$ is $K_t + S_t + B_t$, which is the sum of remaining capital and new investment that comes from net cash and new borrowing. The firm must ensure that it can repay the face value of the new debt after borrowing, while also respecting the maximum growth rate $g$ of capital so that

$$K_t \leq K_t + S_t + B_t \leq (1+g)K_t \tag{8}$$

At the end of the period, capital is subject to an i.i.d. multiplicative shock $\tilde{\delta}_t > 0$, so capital after the shock is given by

$$K_{t+1} = \delta_t(K_t + S_t + B_t), \tag{9}$$

which is the capital the owners carry on into the next period. The shock $\tilde{\delta}_t$ captures any depreciation and other exogenous factors that affect the firm's asset value, such as changes in the economy, technology, or market conditions, and it is assumed to be independent and identically distributed over time. The owners then face the same decision problems as before, starting with the new level of capital and new debt obligations.

We look for an equilibrium that is Markov in a short list of state variables. Specifically, the owners' value function $C^{b4}$ before gambling depends on the firm's capital after a shock, $K_t$, and

[16] If $F_t$ is negative, the firm lends the money and will recover $F_t$ in full. If $F_t$ is positive, we assumes that bondholders are also subject to a fractional bankruptcy cost even after they received full repayment. Since $\mathbf{G}(\tilde{x}) \geq -\nu K_t - (F_t)^- - \gamma\theta K_t$, the bondholders at least receive $(1-c)[F_t \wedge (1-\gamma)\theta K_t]$ where $(1-\gamma)\theta K_t$ can be seen as the protected collateral not eligible for gambling.

its outstanding debt, $F_t$. The value function $C^{after}$ after gambling depends on the capital after the shock, $K_t$, and the net cash after gambling realization, $S_t$.

There are several points in a period where we could examine the equity's continuation values, but it is simplest to focus on those right before and after gambling. We will present the owners' problems sequentially in the form of a Bellman equation.

## 3.1 Bellman equations

We analyze a subgame perfect Nash equilibrium in which all the optimal debt offers are accepted. We state the owners' problems before gambling (gambling node) and after gambling but before borrowing (borrowing proposal node):

***(Gambling node)*** At time $t$ after surviving the exogenous termination shock, given capital $K_t$ and debt outstanding $F_t$, the owners choose adapted gambling $\mathbf{G}(\tilde{x}) \in \mathscr{G}$ to maximize expected value. The Bellman equation is

$$C^{b4}(K_t, F_t) = \max_{\mathbf{G} \in \mathscr{G}_t} \mathrm{E}\Big[C^{after}(K_t, vK_t - F_t + \mathbf{G}(\tilde{x}))\Big], \tag{10}$$

subject to the set of feasible gambles defined in (6) and (7).

***(Borrowing proposal node)*** At time $t$ after gambling is realized, given capital after gambling $K_t$ and net cash after gambling $S_t \equiv vK_t - F_t + \mathbf{G}(x_t)$, the owners choose adapted new borrowing and face value $(B_t, F_{t+1})$ to maximize expected value. The Bellman equation is

$$\begin{aligned} C^{after}(K_t, S_t) = \max_{(B_t, F_{t+1})} \mathrm{E}\Big[&\big(S_t + B_t < 0\big)(1-c)(\theta K_t + S_t)^+ + \big(S_t + B_t \geq 0\big)\Big\{\rho(K_{t+1} - F_{t+1})^+ \\ &+ (1-\rho)C^{b4}(K_{t+1}, F_{t+1})\Big\}\Big], \end{aligned} \tag{11}$$

subject to the borrowing constraint (8), capital augmentation (9), and the bondholders' willingness to lend

$$
\begin{cases}
F_{t+1} \geq B_t, \text{ if } B_t \leq 0 \\
\mathrm{E}\Big[\rho(F_{t+1} \wedge K_{t+1}) \quad +(1-\rho)\Big\{\big(\widetilde{S^*}_{t+1} + B^*_{t+1}(K_{t+1}, \widetilde{S}_{t+1}) < 0\big)(1-c)[F_{t+1} \wedge (\mathbf{G}^*(\tilde{x}, K_{t+1}, F_{t+1}) + \theta K_{t+1} + \nu K_{t+1})] \\
\qquad\qquad\qquad\qquad + \big(\widetilde{S^*}_{t+1} + B^*_{t+1}(K_{t+1}, \widetilde{S}_{t+1}) \geq 0\big)F_{t+1}\Big\}\Big] \geq B_t, \text{ if } B_t > 0
\end{cases}
\tag{12}
$$

where $B^*_{t+1}(K_{t+1}, \widetilde{S}_{t+1})$, $\mathbf{G}^*(\tilde{x}, K_{t+1}, F_{t+1})$, and $\widetilde{S^*}_{t+1}$ are the result of owners' optimal choice in the next period. Specifically,

$$\widetilde{S^*}_{t+1} \equiv \nu K_{t+1} - F_{t+1} + \mathbf{G}^*(\tilde{x}, K_{t+1}, F_{t+1}).$$

We assume that the bond market does not deal with a firm that is facing bankruptcy in the current period. That is, when the proposed new borrowing does not cover the shortfall, we let $\iota = 0$. We also assume that the firm does not have other sources of financing.

To simplify the analysis, we normalize the problems by dividing $K_t$ throughout, assuming homogeneity of the problems. This allows us to expressed everything as value per unit of capital.

## 3.2 Normalization

In this section, we look for a homogenous solution in our short list of state variables in each node. We define the following

$$
\begin{aligned}
&\delta \equiv \delta_t,\ \delta' \equiv \delta_{t+1};\ \ s \equiv \frac{S_t}{K_t},\ s' \equiv \frac{S_{t+1}}{K_{t+1}}; \\
&\beta \equiv \frac{B_t}{K_t},\ \phi \equiv \frac{F_t}{K_t},\ \phi' \equiv \frac{F_{t+1}}{K_{t+1}};\ \ b \equiv \frac{B_t}{K_{t+1}/\delta_t},\ f' \equiv \frac{F_{t+1}}{K_{t+1}/\delta_t} = \delta\phi',
\end{aligned}
$$

and the ratio of capital augmentation is

$$\frac{K_{t+1}/\delta_t}{K_t} = 1 + s + \beta \in [1, 1+g] \tag{13}$$

To clarify the notation, we use a prime symbol ($'$) to denote the variables in the next period, and the variables without a prime represents the current period. For example, $s'$ is net cash per capital after gambling in the next period and $s$ in this period. The choice variables are denoted as $(\beta, f')$, representing the borrowing per new capital and the face value per new capital, respectively.

By the assumption of homotheticity, we have

$$
\begin{aligned}
&C^{b4}(K_t, F_t) = K_t C^{b4}(1, \frac{F_t}{K_t}) = K_t C^{b4}(1, \phi) \equiv K_t C^{b4}(\phi),\\
&C^{after}(K_t, S_t) = K_t C^{after}(1, \frac{S_t}{K_t}) = K_t C^{after}(1, s) \equiv K_t C^{after}(s),\\
&\mathbf{G}(\tilde{x}, K_t, F_t) = K_t \mathbf{G}(\tilde{x}, 1, \frac{F_t}{K_t}) = K_t \mathbf{G}(\tilde{x}, 1, \phi) \equiv K_t \mathbf{g}(\tilde{x}, \phi)
\end{aligned}
$$

Now we restate the firm's problem:

***(Gambling node)*** Given debt per unit of capital $\phi$, the owners choose adapted gambling $\mathbf{g}(\tilde{x}) \in \mathscr{G}_\phi$ to maximize expected value. The Bellman equation is

$$
C^{b4}(\phi) = \max_{\mathbf{g} \in \mathscr{G}_\phi} \mathrm{E}\left[ C^{after}(v - \phi + \mathbf{g}(\tilde{x})) \right], \tag{14}
$$

the set $\mathscr{G}_\phi$ of feasible gambles is given by

$$
\mathscr{G}_\phi \equiv \left\{ \text{non-increasing } \mathbf{g} : [0,1] \to \mathscr{O}_\phi \,\middle|\, \mathrm{E}[\mathbf{g}(x)] = 0 \right\},
$$

given the feasible gambling outcomes

$$
\mathscr{O}_\phi = \begin{cases} \{0\}, & \text{no gambling} \\ [-v - \phi^- - \gamma\theta, \bar{G}], & \text{otherwise} \end{cases}
$$

Note that $\gamma = 0$ if there is no superpriority.

***(Borrowing proposal node)*** We conjecture that there exists a set $\mathscr{S}$ of values of $s$ for which the firm owners will choose to continue operating the firm. If we assume that both the owners and bondholders follow the equilibrium policy functions in each period and that the firm's value is increasing in $s$, then for a Markov equilibrium, $\mathscr{S}$ will take the form $\mathscr{S} = s|s \geq \underline{s}$, where $\underline{s}$ is a constant threshold representing the level of net cash flow below which the firm defaults.

Given net cash per capital after gambling $s$, the owners choose adapted new borrowing and face value $(\beta, f')$ to maximize expected value. The Bellman equation is

$$C^{after}(s) = \max_{(\beta,f')} \big(s < \underline{s}\big)(1-c)(\theta+s)^{+} + \big(s \geq \underline{s}\big)(1+s+\beta)\mathrm{E}\Big[\rho\tilde{\delta}(1-\tilde{\phi}')^{+} + (1-\rho)\tilde{\delta}C^{b4}(\tilde{\phi}')\Big], \tag{15}$$

subject to the borrowing constraint derived from (8):

$$-s \leq \beta \leq g - s \tag{16}$$

and the bondholders agreeing to lend

$$\begin{cases} f' \geq b, \text{ if } \beta \leq 0 \\ \mathrm{E}\Big[\rho\tilde{\delta}(\tilde{\phi}' \wedge 1) + (1-\rho)\tilde{\delta}\Big[\Big(\tilde{s}'^{*} < \underline{s}\Big)(1-c)\big[\tilde{\phi}' \wedge \big(\mathbf{g}^{*}(\tilde{x},\tilde{\phi}') + \theta + v\big)\big] + \Big(\tilde{s}'^{*} \geq \underline{s}\Big)\tilde{\phi}'\Big]\Big] \geq b, \text{ if } \beta > 0 \end{cases} \tag{17}$$

where

$$\tilde{\phi}' = \frac{f'}{\tilde{\delta}},\ b = \frac{\beta}{1+s+\beta}$$
$$\tilde{s}'^{*} = v - \tilde{\phi}' + \mathbf{g}^{*}(\tilde{x}, \phi')$$

and $\mathbf{g}^{*}(\tilde{x}, \phi')$ solves the firm's gambling problem as in (14) for each realization $\phi'$ in the next period.

# 4 Equilibrium and Graphic Illustration

The owners' problem does not have a closed-form solution because of the interdependence between gambling and the continuation value function, but the equilibrium properties and the numerical results provide useful insights into firms' gambling in a dynamic setting. We present a set of propositions that follow directly from the model assumptions.

**PROPOSITION 4.1** *Given the cap of growth rate $g$, cash flow $v$ per unit of capital, and ratio $\rho$ that the industry dies, the equity value per unit of capital $C^{after}(s)$ and $C^{b4}(\phi)$ are bounded by $1+s+\frac{(1-\rho)(1+g)}{\rho-g+\rho g}v$, and $\lim_{s\to\infty} C(s) = 1+s+\frac{(1-\rho)(1+g)}{\rho-g+\rho g}v$.*

**Proof.** Since firm's growth is bounded by $g$ in each period, the value is capped by growing at maximum in each period perpetually, which means that the firm is debt-free and obtains cash flow $(1+g)^t v$ in period $t$ with probability $(1-\rho)^t$, and the present value of the cash flow from time 1 is $\sum_{t=1}^{\infty}(1-\rho)^t(1+g)^t v$. Adding the intial value of capital and cash, $1+s$, the total equity value is equivalent to

$$1+s+\frac{(1-\rho)(1+g)}{\rho-g+\rho g}v.$$

As the net cash $s$ grows, equity value $C^{after}(s)$ converges to the cap, and an increment of $s$ raises firm's value at (almost) a one-for-one rate.

Since for any $\phi$, $C^{b4}(\phi) = \mathrm{E}\Big[C^{after}(v-\phi+\mathbf{g}^*(\tilde{x},\phi))$, and therefore $C^{b4}(\phi)$ is also capped by $1+s+\frac{(1-\rho)(1+g)}{\rho-g+\rho g}v$. ■

## 4.1 Bond pricing and policy functions

For further analysis we assume that $\tilde{\delta}'$ follows a uniform distribution in the interval $(\underline{\delta},\bar{\delta})$, without loss of generality. The range of the capital shock is set to be large because we are interested in analyzing the case with risky borrowing. Table 3 presents the parameter values used for the numerical exercise in this section. Note that the hazard ratio $\rho$ is set to be large to ensure that the curves converge more quickly. We can vary the parameters value as long as the condition

| | |
|---|---|
| $g$ | 0.03 |
| $\rho$ | 0.15 |
| $c$ | 0.05 |
| $\tilde{\delta}$ | $U(0.05, 1.95)$ |
| $v$ | 0.05 |
| $\theta$ | 0.8 |

Table 3: Parameter values for the numerical exercise.

$(1+g)(1-\rho)\mathrm{E}[\tilde{\delta}_t] < 1$ is satisfied to insure finite firm value.

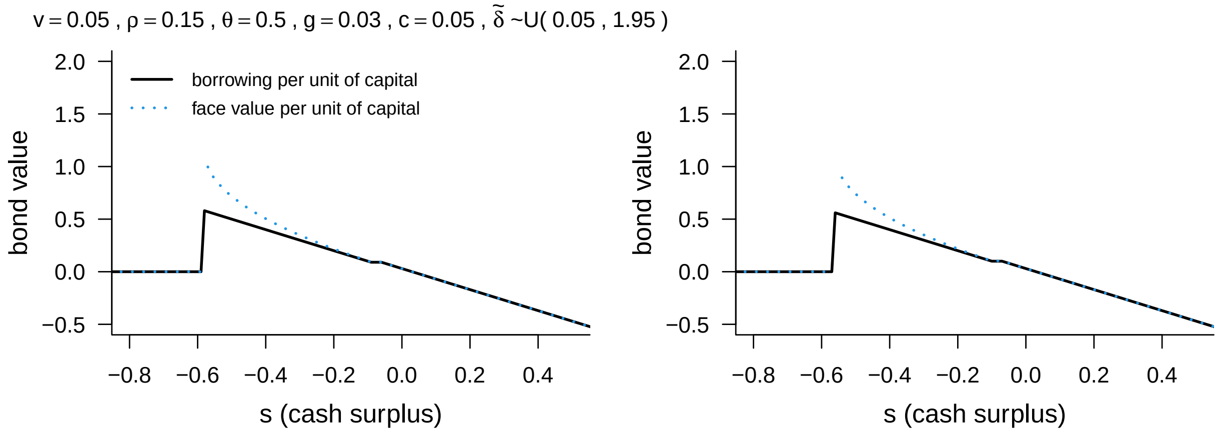
The maximization of the owners' wealth should always offer a fair bond price in equilibrium that equals to the face value subtracts the loss from bankruptcy. According to (17), when borrowing is positive, borrowing per unit of new capital $b$ is a function of face value $f'$:

$$\begin{aligned} b(f') =& f' - \rho \mathrm{E}\big[\big(\tilde{\delta} - f'\big)^{-}\big] \\ & - (1-\rho)\mathrm{E}\left[\left(\widetilde{s'^{*}} < \underline{s}\right)\big[(1-c)\big(\tilde{\delta}(g^{*}+v+\theta) - f'\big)^{-} + cf'\big]\right], \end{aligned}$$

where $\widetilde{s'^{*}} = v - \frac{f'}{\tilde{\delta}} + \mathbf{g}^{*}$.

In the bond pricing equation, $\mathrm{E}\big[\big(\tilde{\delta} - f'\big)^{-}\big]$ is the expected loss from bankruptcy when the industry dies with probability $\rho$, and $\mathrm{E}\left[\left(\widetilde{s'^{*}} < \underline{s}\right)\big[(1-c)\big(\tilde{\delta}(g^{*}+v+\theta) - f'\big)^{-} + cf'\big]\right]$ is the expected loss when the industry survives but the owners go into bankruptcy, with probability $1-\rho$. More precisely, $(1-c)\big(\tilde{\delta}(g^{*}+v+\theta) - f'\big)^{-}$ is the loss from not receiving the full face value, and $cf'$ is cost from lawyers and accountants.

Without gambling, the bond pricing equation shows a hump shaped pricing curve, which is consistent with traditional bond pricing theory that states that bankruptcy costs make it costly to borrow as face value of the debt increases. When borrowing is small enough, $f' \leq \underline{\delta}[(v+\theta) \wedge 1]$, the bondholders are always paid in full, but they have bankruptcy costs paid to lawyers and accountants so that the bond has an actual value of $b(f') = f' - (1-\rho)cf'\mathrm{E}\big[\big(\widetilde{s'^{*}} < \underline{s}\big)\big]$. As $f'$ increases, the probability of bankruptcy increases, and the amount received from bankruptcy is smaller, making bankruptcy cost disproportionately higher. When $f'$ is high enough, the bondholders are always the residual investors, and the borrowing can be close to zero. Therefore, there should be an endogenous borrowing limit.



(a) bond pricing without gambling

(b) bond pricing with gambling: gamma = 0

(c) bond pricing with gambling: gamma = 0.5

(d) bond pricing with gambling: gamma = 1

Figure 5: ***Borrowing and face value in equilibrium*** All variables are normalized by dividing by $K_t$. The solid black curves in the figures represent the optimal borrowing per unit of capital as a function of the net cash surplus per unit of capital, while the blue dashed curves show the face value per unit of capital. As the net cash surplus becomes more negative, borrowing increases to cover the shortfall and keep the firm afloat, with face value increasing disproportionately. However, if the shortfall is too large, the owners will not borrow and the firm will enter bankruptcy. Conversely, when there is a net cash surplus, borrowing can be negative since there is a growth cap.

Gambling can either increase or decrease bond value by changing the probability of survival. In Figure 5, we can see the policy functions for borrowing and face value as functions of cash surplus, per unit of capital. The borrowing function is represented by the black curve, which illustrates that borrowing increases as the shortfall becomes more negative, indicating that the firm needs more funds to cover its debt obligations. The dashed lines show that the face value of debt increases disproportionately due to its increased risk. When superpriority is introduced, debt becomes even riskier, particularly when the firm can gamble away more assets, as shown in panels (c) and (d) where $\gamma$ is larger. This makes it more difficult for the owners to borrow, which leads to a decrease in the maximum amount the firm can borrow and a higher likelihood of failure since $\underline{s}$ is larger.

## 4.2 Equity value

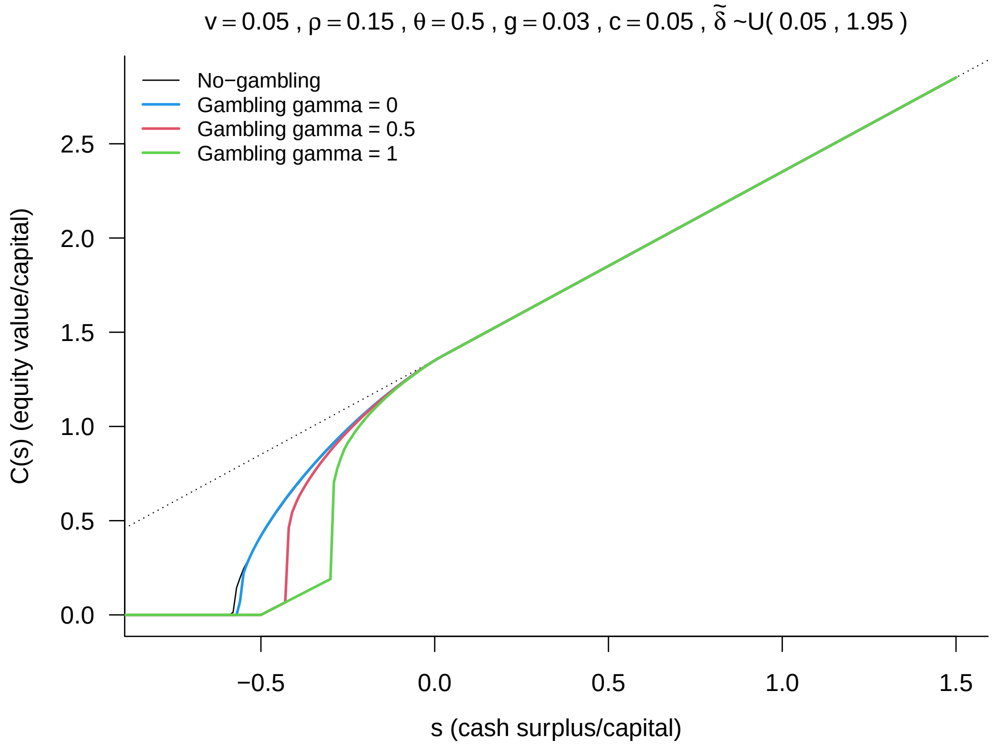



Figure 6: ***(Equity value per unit of capital as a function of net cash per unit of capital)*** Increasing $\gamma$ implies that more assets are available for gambling, which reduces equity value when the firm is in distress. For example, when $s = -0.3$, the owners can still borrow to cover the shortfall if they cannot gamble too much of the assets (at least when $\gamma \leq 0.5$). However, if the firm can gamble away all of its assets, even if the remaining assets still have some value after paying the debt, the owners have to liquidate the assets and the firm will fail.

Figure 6 displays the equity value per unit of capital as a function of cash surplus, and is representative of the overall effect of gambling on firm's value. Since the bond is fairly priced, all the value of gambling is reflected in the equity value. Changing parameter values would not affect our main conclusions. The comparison of the black curves and the blue curves allows us to disentangle the pure gambling effect from the effect of superpriority. Without superpriority (as shown by the blue curve), gambling does not significantly affect equity value and can even increase it when $v$ is higher. This is because higher cash flow increases the probability of winning a fair gamble, further reducing borrowing costs and bankruptcy risk.

However, with superpriority, increasing $\gamma$ significantly reduces equity value. This is due to two factors: first, assets are diluted because owners can redeploy them to gamble; second, gambling for ripoff becomes more likely to prevail.

With superpriority, when $\gamma$ increases, equity value is largely reduced. Two effects contribute to the dissipation: first, the assets are diluted because the ability of asset redeployment by the firm owners; second, "gambling for ripoff" would be more likely to prevail. These raise the cost of borrowing and may even make it impossible for the firm to borrow. The linear slope in the figure represents the forced liquidation region, where the firm is forced to sell off its assets to pay off the debt. If the firm cannot gamble too much of its assets, which is the case when $\gamma \leq 0.5$, it can still borrow to cover the shortfall and continue. However, if the firm can gamble away all its assets, even if the remaining assets still have value after paying the debt, the owners have no choice but to liquidate the assets and push the firm into bankruptcy because they are unable to borrow enough.

## 4.3 Optimal gambling

The optimal gambling can be found following the same procedure as in the single-period model, that is, by "concavifying" the value function $C(s)$ for each contingency. Figure 7 shows an example to illustrate gambling. In the following, we provide formal equations of optimal gambling.

Note that $s \equiv v - \phi + g$ is the post-gambling cash surplus in this period, and it can be used to target where can the owners gamble to. Assume that $s_p \equiv v - \phi$ is the pre-gambling cash surplus.

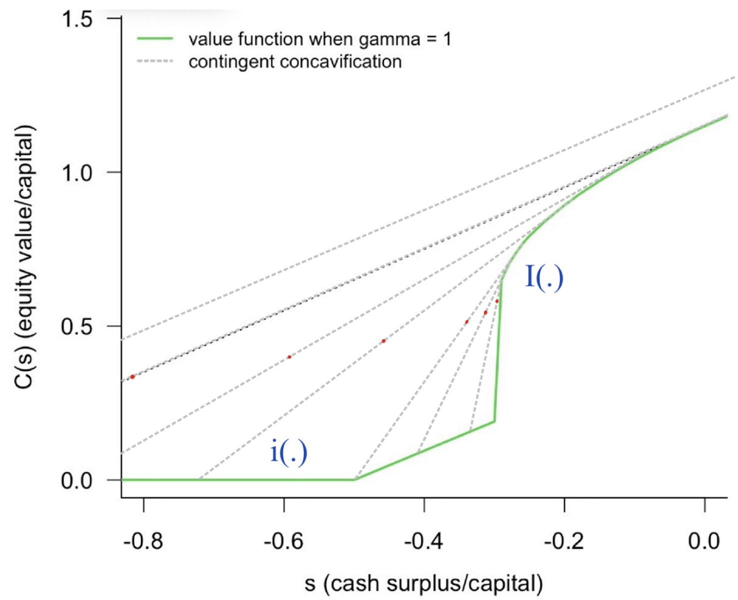



Figure 7: ***An example of gambling*** gambling has a mixed feature of "gambling for redemption" and "gambling for ripoff" and is continuous rather than jumping between extremes. $i(.)$ is the set where the owners gamble down to, and $I(.)$ is the set where the owners gamble up to.

Given any $s_p$ and face value $\phi$, the firm can gamble down to $-v-\gamma\theta$.[17] The optimal gambling always concavifies the value function $C(s)$, that is, to gamble down to the lowest point $-\phi-\gamma\theta$, and up to a tangent point of $C(s)$ and a linear line going through $-\phi-\gamma\theta$. By using the optimal gambling and the value function, we can compute the pre-gambling value functions of equity value and bond value, denoted as $\widehat{C}(s_p,\phi)$ and $\widehat{\beta}(s_p,\phi)$, respectively.

We first define

$$s_0 \equiv -1-\frac{(1-\rho)(1+g)}{\rho-g+\rho g}v, \quad s_1 \equiv \underline{s}-\frac{C(\underline{s})}{C'(\underline{s})},$$

where $s_0$ is the interception of the upper bound of value function (see Prop. 4.1) and the $x$ axis, and $s_1$ is the smallest $s$ on the $x$ axis through which the tangent point on the value function $C(s)$ is the "kink" (around $s=-0.3$ in Figure 7). Also, define

$$i(\phi) \equiv -\phi-\gamma\theta$$

[17]In this section we only consider positive borrowing, $\phi \geq 0$, for simplicity of notation. Negative borrowing can be similarly derived with slightly different notation.

and

$$I(\phi) \equiv \begin{cases} +\infty, & \text{if } i(\phi) \leq s_0 \\ \arg\max_s \frac{C(s)}{s-i(\phi)}, & \text{if } s_0 < i(\phi) \end{cases}$$

where $i(\phi)$ is the minimum value that the owners can gamble to. $I(\phi)$ is the tangent point of $C(s)$ and the straight line going through $i(\phi)$, if not infinite.

If $i(\phi)$ falls on the left of $s_0$, we cannot find a tangent point of $i(\phi)$ along the value function curve $C(s)$, and hence the owners will gamble for ripoff, the biggest gambling. If $i(\phi)$ falls between $s_1$ and $\underline{s}$, then the tangent point is exactly the "kink" and it is gamble for redemption. Any point in between $s_0$ and $s_1$ always has tangent point(s) on $C(s)$, and the tangent point(s) should be the point(s) that the owners gamble towards. Proposition 4.2 formally states the gambling feature:

**PROPOSITION 4.2** ***(Optimal gambling)*** *Given $s_p, \phi$,*

1. *the optimal gambling for the owners is*

$$\mathbf{g}^*(\tilde{x}, s_p, \phi) = \begin{cases} (I(\phi) - s_p)^+, & \text{if } 0 < x < w(s_p, \phi) \\ i(\phi) - s_p, & \text{if } w(s_p, \phi) \leq x < 1 \end{cases}$$

   *where $w(s_p, \phi) \equiv \frac{s_p - i(\phi)}{I(\phi) \vee s_p - i(\phi)}$ is the probability or weight that the equity value goes up. This result indicates that if $I(\phi) \leq s_p$, then $\mathbf{g}^*(\tilde{x}, s_p, \phi) \equiv 0$ (i.e., the owners do not choose to gamble); otherwise, if $I(\phi) > s_p$, the owners gambles up to $I(\phi) - s_p$, and net cash after gambling is $s = I(\phi)$; down to $i(\phi) - s_p$, and $s = i(\phi)$.*

2. *for $I(\phi) > s_p$, we can compute equity and bond value respectively. We know that when $s = I(\phi)$ the bondholders obtain full face value, and when $s = i(\phi)$, equity value is $C(i(\phi))$, and bond value is*

$$\Phi_L(i(\phi)) \equiv \begin{cases} \phi, & \text{if } \underline{s} \geq i(\phi) \\ (1-c)(1-\gamma)\theta - C(i(\phi)), & \text{if } w(s_p, \phi) \leq x < 1 \end{cases}$$

*Then,*

$$\textbf{\textit{equity value:}}\ \widehat{C}(s_p,\phi) = w(s_p,\phi)C\big(I(\phi)\big) + (1-w(s_p,\phi))C\big(i(\phi)\big), \tag{18}$$

$$\textbf{\textit{bond value:}}\ \widehat{\phi}(s_p,\phi) = w(s_p,\phi)\phi + (1-w(s_p,\phi))\Phi_L(i(\phi)). \tag{19}$$

One notable feature of our optimal gambling strategy is that the firm does not always choose extreme risks. Instead, it has a continuous feature in terms of where to gamble to, rather than jumping between extremes.

Our model of Gambling is highly adaptable and can easily adjust to different assumptions. For example, if the value function is more concave, gambling for extreme ripoff may not happen since we can probably always find a finite tangent point on the value function.

# 5 Conclusions

We provided a simple framework to analyze gambling by firms. "Gambling for redemption" is a Pareto improvement and occurs when the firm owners are eager to maintain the firm, whereas "gambling for ripoff" can be socially costly and occurs when continuing a firm is beneficial socially but not to the owners. By making gambling some of the assets possible, superpriority law lowers the value lost to owners in bankruptcy and increases the incentives for the firm owners to gamble for ripoff. In the more realistic intertemporal model with endogenous borrowing and endogenous continuation value, the owners choices of gambling are intermediate between gambling for redemption and ripoff. We find that superpriority increases the scale of gambling taken by the owners and makes funding more difficult. Our results suggest an interesting empirical question: how do we distinguish "gambling for redemption" and "gambling for ripoff" ex post since they both wipe out the firm's assets in the case of failure? To know the exact gambling, we can instead look at their bets in place, compare the risk of their assets and the amount of matured debt in place.

One possible implication of superpriority law will be the adoption of financing that reduces the scale of superpriority gambling. One possibility is the adoption by bond issuers of more defensive measures that protect against superpriority claims. For example, it may be more common to protect bonds to specific perfected collateral instead of passive covenants claiming the preclusion of asset sales and security transfers. It may also incentivize firms to issue short-term bonds which have less exposure to a stay in bankruptcy, or even use repos which are also protected against bankruptcy. The substitution away from traditional financing to repo financing can cause an asset grab race which undermines the purposes of bankruptcy law to facilitate an orderly liquidation (or reorganization) and to give breathing space for the firm owners to resolve financial difficulties.

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# A Proof of Optimal Gambling: redemption

Given constants $F, B, C, \pi \in \mathbb{R}_{++}$, $L \in \mathbb{R}_{+}$, $\bar{G} \geq F - B - \pi$ and $\gamma \equiv \begin{cases} 1, \text{ with superpriority} \\ 0, \text{ absent superpriority,} \end{cases}$ the question becomes

$$\max_{\mathbf{G}(x)} \mathrm{E}\Big[(\pi + \mathbf{G}(\tilde{x}) \geq F - B)(\pi + C + \mathbf{G}(\tilde{x}) - F)\Big]$$
$$\text{s.t. } \mathrm{E}[\mathbf{G}(\tilde{x})] = 0, \text{ and } -\gamma L - \pi \leq \mathbf{G}(\tilde{x}) \leq \bar{G}$$

Since $\tilde{x}$ is the underlying randomness: $\tilde{x} \sim_d U(0,1)$, then w.l.o.g. we assume that $\mathbf{G}(x)$ is non-increasing in $x$. To get the necessary conditions for the solution, we first concavify the function

$$(\pi + \mathbf{G}(\tilde{x}) \geq F - B)(\pi + C + \mathbf{G}(\tilde{x}) - F) \tag{*}$$

to make it continuous.

**Gambling for redemption:** When $F < C - \gamma L$, define $H(\mathbf{G}(x))$ as the concavified function of (*)

$$H(\mathbf{G}(x)) \equiv \begin{cases} \frac{C-B}{F-B+\gamma L}(\pi + \mathbf{G}(x) + \gamma L), & \text{for } \pi + \mathbf{G}(x) < F - B \\ C + \pi + \mathbf{G}(x) - F, & \text{for } \pi + \mathbf{G}(x) \geq F - B \end{cases}$$

The subgradient of $H(\mathbf{G})$ is

$$\nabla H(\mathbf{G}) = \begin{cases} (-\infty, 1], & \text{for } \mathbf{G} = \bar{G} \\ \{1\}, & \text{for } F-B-\pi < \mathbf{G} < \bar{G} \\ [1, \frac{C-B}{F-B+\gamma L}], & \text{for } \mathbf{G} = F-B-\pi \\ \{\frac{C-B}{F-B+\gamma L}\}, & \text{for } -\pi-\gamma L < \mathbf{G} < F-B-\pi \\ [\frac{C-B}{F-B+\gamma L}, +\infty], & \text{for } \mathbf{G} = -\pi-\gamma L \end{cases}$$

Assume $\lambda, w_1, w_2 \geq 0$, and the first order condition of the problem is

$$\lambda - w_1 + w_2 \in \nabla H(\mathbf{G})$$

with

$$w_1 \geq 0,\ (\mathbf{G}+\pi+\gamma L)w_1 = 0$$
$$w_2 \geq 0,\ (\mathbf{G}-\bar{G})w_2 = 0$$

We ignore the case when $\mathbf{G} \in (-\pi-\gamma L, F-B-\pi)$ since it has measure zero and is not on the original function. We then have

$$\mathbf{G} = \begin{cases} \bar{G}, & \text{for } -\infty < \lambda - w_1 + w_2 \leq 1 \\ [F-B-\pi, \bar{G}], & \text{for } \lambda - w_1 + w_2 = 1 \\ F-B-\pi, & \text{for } 1 \leq \lambda - w_1 + w_2 \leq \frac{C-B}{F-B+\gamma L} \\ -\pi-\gamma L, & \text{for } \frac{C-B}{F-B+\gamma L} \leq \lambda - w_1 + w_2 \leq +\infty \end{cases}$$

(1) If $\pi < F-B$, $\mathbf{G}(x) = -\pi-\gamma L$ is the only $\mathbf{G}(x)$ that is smaller than 0. For $\mathrm{E}[\mathbf{G}(x)] = 0$, there must be some $x$ such that $\mathbf{G}(x) = -\pi-\gamma$. Therefore, $\lambda - w_1 + w_2 \geq \frac{C}{F-B+\gamma L} > 1$ since

$C - \gamma L > F - B$. This implies that $\lambda - w_1 + w_2 = \frac{C-B}{F-B+\gamma L}$ and $\mathbf{G}(x) = F - B - \pi$ or $\mathbf{G}(x) = -\pi - \gamma L$. Thus, by solving

$$0 = \int_{x=0}^{t} (F - B - \pi) dx + \int_{x=t}^{1} (-\pi - \gamma L) dx,$$

we have $t = \frac{\pi + \gamma L}{F - B + \gamma L}$. The optimal gambling is

$$\mathbf{G}^*(x) = \begin{cases} F - B - \pi, & \text{for } 0 < x \leq \frac{\pi+\gamma L}{F-B+\gamma L} \\ -\gamma L - \pi, & \text{for } \frac{\pi+\gamma L}{F-B+\gamma L} < x < 1. \end{cases}$$

Since $\mathbf{G}^*(x)$ solves the concavified problem and is also feasible for the original problem, and the concavified objective function is greater than the original function, we can conclude that $\mathbf{G}^*(x)$ also solves the original problem. If we relax the condition that $\mathbf{G}$ is decreasing, then any gamble with the same distribution would also be optimal.

(2) If $\pi > F - B$, then we must have that for some $x$, $\mathbf{G}(x) \geq F - B$ and $\lambda - w_1 + w_2 \geq 1$. Then any $\mathbf{G}(x) \in [F - B - \pi, \bar{G}]$ that satisfies

$$\int_{x=0}^{1} \mathbf{G}(x) dx = 0$$

would be a possible solution.

Now we prove that these candidate solutions are the actual solutions. For any candidate solutions $\{\mathbf{G}^*(x) | \pi + \mathbf{G}^*(x) \geq F - B \text{ and } \mathrm{E}[\mathbf{G}^*(x)] = 0\}$,

$$\mathrm{E}[H(\mathbf{G}^*(x))] = C + \pi - F.$$

Since for any feasible solutions $\mathrm{E}[H(\mathbf{G}(x))] \leq C + \pi - F = \mathrm{E}[H(\mathbf{G}^*(x))]$, the candidate solutions are the actual solutions. For the same argument as above, the solutions for the concavified problem are also the solutions for the original problem.

# B Proof of Optimal Gambling: ripoff

**Gambling for ripoff:** When $F > C - \gamma L$, similarly define $H(\mathbf{G}(x))$ as the concavified function of (*)

$$H(\mathbf{G}(x)) \equiv \frac{\pi + \bar{G} - F + C}{\pi + \bar{G} + \gamma L}(\pi + \mathbf{G} + \gamma L)$$

The subgradient of $H(\mathbf{G})$ is

$$\nabla H(\mathbf{G}) = \begin{cases} (-\infty, \frac{\pi+\bar{G}-F+C}{\pi+\bar{G}+\gamma L}], & \text{for } \mathbf{G} = \bar{G} \\ \frac{\pi+\bar{G}-F+C}{\pi+\bar{G}+\gamma L}, & \text{for } -\pi - \gamma L < \mathbf{G} < \bar{G} \\ [\frac{\pi+\bar{G}-F+C}{\pi+\bar{G}+\gamma L}, +\infty), & \text{for } \mathbf{G} = -\pi - \gamma L \end{cases}$$

The first order condition is the same as before. Ignoring the case in which $-\pi - \gamma L < \mathbf{G} < \bar{G}$ since the measure is zero, we have

$$\mathbf{G} = \begin{cases} \bar{G}, & \text{for } \lambda - w_1 + w_2 \leq \frac{\pi+\bar{G}-F+C}{\pi+\bar{G}+\gamma L} \\ -\pi - \gamma L, & \text{for } \lambda - w_1 + w_2 \geq \frac{\pi+\bar{G}-F+C}{\pi+\bar{G}+\gamma L} \end{cases}$$

Therefore, the Lagrange multipliers satisfy $\lambda - w_1 + w_2 = \frac{\pi+\bar{G}-F+C}{\pi+\bar{G}+\gamma L}$. By solving

$$0 = \int_{x=0}^{t} \bar{G} dx + \int_{x=t}^{1} (-\pi - \gamma L) dx,$$

we have $t = \frac{\pi+\gamma L}{\bar{G}+\pi+\gamma L}$. The optimal gambling is

$$\mathbf{G}^*(x) = \begin{cases} \bar{G}, & \text{for } 0 < x \leq \frac{\pi+\gamma L}{\bar{G}+\pi+\gamma L} \\ -\gamma L - \pi, & \text{for } \frac{\pi+\gamma L}{\bar{G}+\pi+\gamma L} < x < 1 \end{cases}$$

For the same argument as above, the solutions for the concavified problem are also the solutions for the original problem.