How Does Benchmarking Affect Market Efficiency? — The Role of Learning Technology *

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ABSTRACT

We study the impact of investors' benchmarking concerns on market efficiency and asset pricing. Both separative and integrative learning technologies are examined as investors allocate limited attention across assets. We show that benchmarking can increase the price informativeness of benchmarked asset as investors optimally adopt integrative learning to observe a combined signal about asset payoffs. This is contrary to the result from the existing literature that assumes separative learning. Benchmarking can also increase the overall market efficiency with either type of learning. Yet, the implications for asset prices and comovements can be qualitatively different under different learning technology.

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1 Introduction

With the dramatic growth of asset management industry, asset managers become dominant market players (Appel, Gormley, and Keim (2016), Gerakos, Linnainmaa, and Morse (2021)). They receive implicit and explicit incentives (e.g., bonus) based on their performance relative to prespecified benchmark portfolios (Chevalier and Ellison (1997, 1999), Ma, Tang, and Gómez (2019)). As institutional investors, they can gather information in various ways and speculate on private signals to beat their performance benchmarks which motivate their hedging needs (Cremers and Petajisto (2009), Jiang, Verbeek, and Wang (2014)). Thus, an important question is how investors' benchmarking concerns affect their attention allocation, information acquisition, and portfolio choice across multiple assets. This is crucial for understanding how benchmarking affects market efficiency and asset pricing. In this paper, we show that answers to those questions strongly depend on the type of learning technology adopted by investors.

Breugem and Buss (2018) show that benchmarking harms price informativeness by reducing the information-sensitive asset supply and discouraging investors' information acquisition. In a multi-asset market, this argument relies on a restrictive learning technology implicitly assumed for all investors. With an unrestrictive learning technology, we find that benchmarking can improve both firm-specific price informativeness and market-wide informational efficiency when investors optimally allocate limited attention across risky assets. Also, the impacts of benchmarking on asset prices can be qualitatively different under different type of learning.

The two types of learning technologies arise in the economic theory of rational inattention. The first one imposes a structural restriction on the signal form so that investors can collect information only about an individual asset or asset class at a time; see Peng (2005), Peng and Xiong (2006), and Van Nieuwerburgh and Veldkamp (2010). We call it the *separative* learning technology to distinguish it from the other which we call the *integrative* learning technology. The latter removes the restriction on signal form and allows investors to observe a combined signal about multiple assets; see Mondria (2010) and Miao, Wu, and Young (2022). Both learning technologies seem theoretically legitimate and empirically relevant. For example, Hameed, Morck, Shen, and Yeung (2015) find that analysts tend to follow assets that contain more valuable market- and industry-wide information (integrative signals). To the best of our knowledge, the critical role of learning technology in our question has not been studied in the literature.

We study the competitive rational expectations equilibrium under either type of learning technology for a continuum of benchmarked investors. This allows us to conduct a detailed comparative analysis that help explain different equilibrium implications of separative versus integrative learning. We consider two risky assets with independent payoffs and noisy supplies. Each investor's performance is evaluated relative to a preassigned benchmark which may include a single asset or both assets. The weight of an asset in an investor's benchmark reflects how strongly he is concerned about his performance relative to that asset. The sum of such weights across all investors defines the aggregate benchmarking level of that asset. Each investor first makes his optimal information choice by determining the structure and precision of his private signal, subject to a capacity constraint on his information processing as in Sims (2003, 2006). Since asset prices are publicly observable, each investor can infer others' private information from prices before choosing his optimal portfolio.

With the separative learning technology, investors can only observe signals about individual asset payoffs separately. In equilibrium, investors choose to allocate all of their learning capacity to the asset that has the highest marginal value of private information; investors are indifferent about two assets that have equal marginal values of information. We show that the endogenous fraction of investors who specialize in learning one asset decreases in the aggregate benchmarking level of this asset and increases in the benchmarking level of the other asset. Consequently, the price informativeness of an asset always decreases in the benchmarking level of this asset and increases in that of the other asset. The mechanism is that investors' benchmarking concerns directly drive up their hedging demand and reduce the information-sensitive (effective) supply of the benchmarked asset. As a result, the marginal value of information decreases for that asset and investors acquire less information about it. Thus, benchmarking reduces the price informativeness of the benchmarked asset, similar to Breugem and Buss (2018) who consider investors' costly information acquisition about individual assets. In contrast, we study investors' attention allocation across two assets under alternative learning technology.

Both firm-specific price informativeness and market-wide informational efficiency can be quantified by the *mutual information* of asset price(s) and payoff(s). In statistics and information theory (Cover (1999)), mutual information is a standard measure of reduced uncertainty about one random variable or vector given the knowledge of another random variable or vector. Market efficiency measured in this way reflects the overall quality of information processing in terms of how much information about all asset payoffs can be extracted from all asset prices. In a single-asset economy, the mutual information measure of market efficiency is the same as that of price informativeness. In a two-asset economy with separative learning, this measure of market efficiency is equal to the sum of price informativeness of individual assets. This is because each investor gathers a separative private signal about one asset and thus the equilibrium prices are independent across assets. Benchmarking reduces the price informativeness of the benchmarked asset but increases that of the other asset. Consequently, the measure of market efficiency is a non-monotonic function of the benchmarking level of each asset.

With the integrative learning technology, investors are not restricted to collect information about each asset separately. Each investor's best response is not to specialize in learning about one asset but to observe a private signal about a linear combination of asset payoffs (Mondria (2010)). We provide new analytical results about the linear symmetric equilibrium where benchmarked investors make the same optimal information choice. We show that the impact of benchmarking on price informativeness can be quite different from the result in the separative learning case: if one asset is much more uncertain than the other in terms of payoff or supply, an increase in the benchmarking level of the more uncertain asset may improve its price informativeness. This is because the marginal value of private information now contains two terms, representing two channels through which the aggregate benchmarking concerns affect investors' attention choice. One is due to the reduction in the effective asset supply, as in the separative learning case. The other reflects the cross-learning effect since each asset price contains information about the other asset. When the supply effect dominates, benchmarking harms price informativeness, as in the separative learning case. When the cross-learning effect dominates, investors optimally put more attention to the riskier asset when its benchmarking level increases. This makes the price more informative about the payoff of the riskier asset.

With integrative learning, the overall market efficiency, as measured by the mutual information of prices and payoffs, is greater than the sum of price informativeness of individual assets. This measure of market efficiency also reflects the amount of information one can extract from prices about a hypothetical portfolio in which the weight of each asset exactly matches the weight of investors' attention allocated to that asset. In general, the attention-implied hypothetical portfolio differs from an average investor's speculative portfolio which is determined by the effective supplies of assets. The two portfolios coincide only when the two assets are equally uncertain like perfect substitutes. In this case, variations in benchmarking levels still affect investors' attention allocation through affecting the ratio of effective supplies. While this attention shift can affect the price informativeness of individual assets, it has no impact on the overall market informational efficiency. Investors' attention allocation in this special case can be treated as a nominal baseline, relative to which we can define a measure of investors' real attention. We find that investors' real attention always leans toward the more uncertain asset when the benchmarking level of either asset increases. This shift of real attention can explain the improvement of market informational efficiency. However, as the benchmarking level of the riskier asset increases further, the attention-implied portfolio can deviate far away from the average speculative portfolio. In other words, investors' attention is disproportionately allocated to the riskier asset even though its effective supply diminishes to zero. In this regime, market informational efficiency declines and will hit its lower bound when the much riskier asset occupies all investors' attention.

We also show that the impacts of benchmarking on expected asset prices, return volatility, and dispersion of portfolio returns can reverse directions under different learning technology.

As an interesting implication, the two assets in the original model can be interpreted as a common risk factor and an idiosyncratic risk factor. By the separative learning technology, investors can only observe signals about each factor separately. We find that a higher benchmarking level on the common risk factor always makes investors put less attention on this factor, leading to a higher volatility of the common risk factor and hence a stronger asset return comovement. In contrast, by integrative learning each investor optimally chooses to observe a signal about a linear combination of both risk factors. We find that when the common risk factor becomes much more volatile than the idiosyncratic one, a higher benchmarking level of the common risk factor can make investors allocate more attention to it and reduce its posterior variance. As a result, the asset return comovement can decrease in the benchmarking level of the common risk factor. In other words, a higher benchmarking level of the common risk factor may help dampen asset comovement, contrary to the implication of separative learning. This may happen, for example, during recessions as the common risk factor becomes highly volatile.

Our results highlight the critical role of learning technology in a multi-asset economy. These can enrich the ongoing debate on how benchmarking or passive investing affects the informational efficiency of financial markets. As a key takeaway, though benchmarking reduces price informativeness under separative learning, it may improve price informativeness under integrative learning, especially when assets have significantly different uncertainty levels. Moreover, benchmarking can improve the overall market efficiency under either type of learning.

There are a set of empirical implications. Take integrative learning for example, our model predicts: (1) An increase in the benchmarking level of an asset with high (resp. low) uncertainties can make investors put more (resp. less) attention to learn about this asset and increase (resp. decrease) its price informativeness. (2) An increase in the benchmarking level of the less uncertain asset may increase the *dispersion of portfolio excess returns* defined by Kacperczyk, Van Nieuwerburgh, and Veldkamp (2016). (3) A higher benchmarking level of the common risk factor may dampen asset comovements in recessions when this factor has heightened volatility.

Our results shed light on how to measure price efficiency. There are various measures of firm-specific price informativeness.[1] However, there is a caveat if one directly extends such firm-specific measures to proxy market-wide informational efficiency because the aggregate of firm-specific price informativeness may not adequately reflect the information content revealed by all asset prices, unless most investors are separative learners (a strong assumption).

[1]These include *price non-synchronicity* (Roll (1988); Morck, Yeung, and Yu (2000); Durnev, Morck, and Yeung (2004); Chen, Goldstein, and Jiang (2007)), *forecasting or revelatory price efficiency* (Bond, Edmans, and Goldstein (2012)), and *welfare-based price informativeness* (Bai, Philippon, and Savov (2016)). Also, Gârleanu and Pedersen (2018) define a measure of *price inefficiency* based on a ratio of conditional variances. Dávila and Parlatore (2018, 2021) use the precision of the signal about asset payoffs revealed by asset prices. Farboodi, Matray, Veldkamp, and Venkateswaran (2022) propose a measure from a structural model that links price informativeness to firms' data.

Any measures that treat assets separately may miss valuable information from learning across assets if integrative learning is dominantly adopted by investors. The mutual information measure used in this paper does not make implicit assumptions about investors' learning technology. In principle, this can be a general measure of market-wide informational efficiency. It could be an interesting research agenda to study how to implement this measure empirically.

This paper adds to the theoretical literature on the implications of benchmarked investors. Brennan (1993) develops a two-factor asset pricing model with a benchmarked fund manager. Cuoco and Kaniel (2011) show that managers' relative performance concerns increase the prices of the benchmarked assets. Basak and Pavlova (2013) show that as the institutional investor optimally holds more of assets included in his benchmark, these assets become more expensive, volatile, and correlated. Buffa, Vayanos, and Woolley (2022) show that benchmarking is part of an optimal contract in the presence of agency frictions. Buffa and Hodor (2022) show that heterogeneous benchmarking can result in negative spillovers across asset returns. Kacperczyk, Nosal, and Sundaresan (2022) study the impact of asset ownership on price informativeness when investors have market power. Different from the above papers, we focus on how benchmarking affects investors' attention allocation under two alternative learning technologies. Our work is most closely related to Breugem and Buss (2018). Different from their paper, we find that the price informativeness of an asset may increase in the benchmarking level of this asset.

There is a burgeoning empirical literature on benchmarks in asset management; see Ma et al. (2019), Gerakos et al. (2021), and Evans, Gómez, Ma, and Tang (2022) for example. Pavlova and Sikorskaya (2022) empirically measure the benchmarking intensity to capture investors' inelastic demand for a stock. They find that both active and passive funds buy additions to their benchmarks and sell deletions. Their novel measure of benchmarking intensity may help test various implications from our model.

The implications of investors' benchmarks are also relevant to passive investing and indexing. Among others, Bond and Garcia (2022) and Baruch and Zhang (2022) show that an increase in index investors reduces the price informativeness of the index. Liu and Wang (2019) show that the effect critically depends on the causes of the rise of indexing and an increase in indexing may increase the price informativeness of the index. Lee (2020) extends the model of Gârleanu and Pedersen (2018) by considering asset managers' strategic trading. Coles, Heath, and Ringgenberg (2022) report evidence that indexing does not change price informativeness.

The rest of this paper is structured as follows. Section 2 presents the model. Section 3 considers the separative learning case that investors can only learn about each asset separately. Section 4 studies the integrative learning case that each investor optimally chooses to observe a combined signal about both assets. Section 5 concludes. Proofs are presented in the Appendix.

2 Model

Consider a three-period economy with dates $t = 1, 2$, and 3. At $t = 1$, investors choose the precisions of signals about asset payoffs, subject to an information processing capacity. They decide how to allocate their limited learning capacity across assets. At $t = 2$, investors observe the private signals about asset payoffs and then choose their optimal stock holdings. At $t = 3$, investors receive compensations based on their performances relative to preassigned benchmarks.

Asset Market. There are two independent risky assets and one risk-free asset which has zero net supply and exogenous gross return $R_f = 1$. The risky asset $j \in \{1, 2\}$ has a final payoff of $\tilde{V}_j$ at $t = 3$ and a random supply $\tilde{Z}_j$, where $\tilde{V}_1$, $\tilde{V}_2$, $\tilde{Z}_1$, and $\tilde{Z}_2$ are independent normal random variables with distributions denoted $\tilde{V}_j \sim \mathcal{N}(\overline{v}_j, \tau_{v,j}^{-1})$ and $\tilde{Z}_j \sim \mathcal{N}(\overline{z}_j, \tau_{z,j}^{-1})$. We can write them as vectors, $\tilde{V} := (\tilde{V}_1, \tilde{V}_2)'$ and $\tilde{Z} := (\tilde{Z}_1, \tilde{Z}_2)'$ with $\tilde{V} \sim \mathcal{N}(\overline{v}, \Sigma_v)$ and $\tilde{Z} \sim \mathcal{N}(\overline{z}, \Sigma_z)$, where

$$\overline{v} := \begin{pmatrix} \overline{v}_1 \\ \overline{v}_2 \end{pmatrix}, \qquad \Sigma_v := \begin{pmatrix} \tau_{v,1}^{-1} & 0 \\ 0 & \tau_{v,2}^{-1} \end{pmatrix}, \qquad \overline{z} := \begin{pmatrix} \overline{z}_1 \\ \overline{z}_2 \end{pmatrix}, \qquad \Sigma_z := \begin{pmatrix} \tau_{z,1}^{-1} & 0 \\ 0 & \tau_{z,2}^{-1} \end{pmatrix}.$$

Benchmarked Investors. The economy is populated with one unit mass of investors (i.e., asset managers) whose individual performance is evaluated against an individually designated benchmark. It is a salient feature of the asset management industry that asset managers care about their performance relative to certain preassigned benchmarks (e.g., index portfolios). Their different investment styles can be reflected by the composition of their designated benchmarks which are exogenously given in this paper.

For each $i \in [0, 1]$, we define a vector $\gamma^i := (\gamma_1^i, \gamma_2^i)'$ to represent investor-i's benchmarking level with respect to the two assets, where $\gamma_j^i \geq 0$ captures the strength of investor i's benchmarking concerns regarding asset j. In other words, the benchmark portfolio for investor i consists of γ_1^i shares of asset 1 and γ_2^i shares of asset 2. $\gamma_j^i > 0$ means that investor i is concerned about his performance relative to a benchmark portfolio involving asset j. For example, if $\gamma_1^i > 0$ and $\gamma_2^i = 0$, then investor-i only has asset 1 in his benchmark; if $\gamma_2^i > 0$ and $\gamma_1^i > 0$, then he has both assets in his benchmark; and if $\gamma_1^i = \gamma_2^i = 0$, then investor-$i$ is not benchmarked.

Portfolio Choice. Each investor is endowed with the same initial wealth ($W_0^i = W_0$) and has the same risk-aversion coefficient $\lambda > 0$. Let $\mathrm{E}^i[\cdot]$ and $\mathrm{Var}^i(\cdot)$ denote investor i's posterior expectations and variances conditional on his information set at $t = 2$, which includes public prices $\tilde{P} := (\tilde{P}_1, \tilde{P}_2)'$ and private signals $\tilde{Y}^i$ (to be defined shortly).

Following Van Nieuwerburgh and Veldkamp (2009, 2010), Mondria (2010), and Breugem and Buss (2018), we assume that investors have a preference for early resolution of uncertainty.[2]

[2] See footnote 10 of Van Nieuwerburgh and Veldkamp (2010) for more details. The utility function in equation

Specifically, their expected utility at $t = 1$ is $\mathrm{E}\big[-\ln \mathrm{E}^i\big[\exp(-\lambda\tilde{C}^i)\big]\big]$. Therefore, at $t = 2$, each investor i chooses the optimal asset holdings, $\theta^i := (\theta_1^i, \theta_2^i)'$ to maximize an objective function equivalent to the expected mean-variance utility:

$$U_2^i(\gamma^i, \tilde{P}, \tilde{Y}^i) := \max_{\theta^i} \lambda \mathrm{E}^i[\tilde{C}^i] - \tfrac{1}{2}\lambda^2 \mathrm{Var}^i(\tilde{C}^i), \tag{1}$$

where investor i's compensation $\tilde{C}^i$ depends on the terminal value $\tilde{W}^i$ of his managed portfolio relative to his benchmark portfolio,[3] $\tilde{C}^i = \tilde{W}^i - (\gamma^i)'(\tilde{V} - \tilde{P})$. Since the terminal value of his portfolio is $\tilde{W}^i = W_0 + \sum_j \theta_j^i(\tilde{V}_j - \tilde{P}_j) = W_0 + (\theta^i)'(\tilde{V} - \tilde{P})$, we can write his compensation as

$$\tilde{C}^i = W_0 + \sum_{j=1,2} (\theta_j^i - \gamma_j^i)(\tilde{V}_j - \tilde{P}_j) = W_0 + (\theta^i - \gamma^i)'(\tilde{V} - \tilde{P}). \tag{2}$$

The equilibrium prices $\tilde{P}_1$ and $\tilde{P}_2$ are determined by the market-clearing conditions,

$$\int \theta_j^i(\gamma_j^i, \tilde{Y}^i, \tilde{P}_j)\,di = \tilde{Z}_j, \quad \text{for } j = 1, 2. \tag{3}$$

Information Choice. Suppose investors are able to observe private information in the form:

$$\tilde{Y}^i = \Lambda^i \tilde{V} + \tilde{\varepsilon}^i, \quad \text{with } \tilde{\varepsilon}^i \sim \mathcal{N}(0, \Sigma^i) \quad \text{and} \quad \Sigma^i := \begin{pmatrix} (\tau_1^i)^{-1} & 0 \\ 0 & (\tau_2^i)^{-1} \end{pmatrix}. \tag{4}$$

Here, Λ^i is a 2×2 matrix that reflects how investor i chooses to learn about the two risky assets. The private information contains the noise $\tilde{\varepsilon}^i := (\tilde{\varepsilon}_1^i, \tilde{\varepsilon}_2^i)'$ which is orthogonal to $\tilde{V}$ and independent across different investors. A signal is more precise when more attention is paid to it. Without loss of generality, we assume that the matrix Σ^i is diagonal and investor i can choose the entries of Σ^i at no cost but subject to some constraint on his learning capacity. Thus, Λ^i and Σ^i represent an investor's information choice made at $t = 1$ to maximize his expected utility:

$$U_1^i = \max_{\Lambda^i, \Sigma^i} \mathrm{E}[U_2^i(\gamma^i, \tilde{P}, \tilde{Y}^i)]. \tag{5}$$

Information Constraint. Following Sims (2003, 2006), we use a measure from information theory to quantify the amount of information that a signal contains about the asset payoffs. The entropy $H(\tilde{X})$ of a random variable $\tilde{X}$ is a measure of its uncertainty. With a continuous prob-

(5) is equivalent to $U_1 = \mathrm{E}_1[u_1(\mathrm{E}_2[u_2(.)])]$ where $u_1(x) = -\ln(-x)$ and $u_2(x) = -\exp(-\lambda x)$. The inner utility u_1 is convex, corresponding to a preference for early resolution of uncertainty. This specification is equivalent to that investors maximize a mean-variance objective function.

[3] This benchmark concerns capture a performance-based fee that adjusts up or down based on outperforming or underperforming a benchmark, as in Breugem and Buss (2018).

ability density $p(x)$, the entropy is defined as $\mathrm{H}(\tilde{X}) = -\int p(x)\ln p(x)dx$. We can also define the conditional entropy $\mathrm{H}(\tilde{X}|\tilde{Y}) = -\iint p(x,y)\ln p(x|y)dxdy$, where $p(x,y)$ and $p(x|y)$ are joint and conditional density functions, respectively. The standard information-theoretic measure for uncertainty reduction is the *mutual information* defined as

$$\mathrm{I}(\tilde{X};\tilde{Y}) = \mathrm{H}(\tilde{X}) - \mathrm{H}(\tilde{X}|\tilde{Y}) = \iint p(x,y)\ln\frac{p(x,y)}{p(x)p(y)}dxdy. \tag{6}$$

This measure is nonnegative $\mathrm{I}(\tilde{X};\tilde{Y}) \geq 0$, symmetric $\mathrm{I}(\tilde{X};\tilde{Y}) = \mathrm{I}(\tilde{Y};\tilde{X})$, and invariant under any linear transformations of random variables, i.e., $\mathrm{I}(\tilde{X};\tilde{Y}) = \mathrm{I}(a\tilde{X}+b; c\tilde{Y}+d)$ with $ac \neq 0$.

By acquiring private information, investors can reduce their uncertainty about asset payoffs. We assume investors face the following information processing constraint

$$\mathrm{I}(\tilde{V};\tilde{Y}^i) = \mathrm{H}(\tilde{V}) - \mathrm{H}(\tilde{V}|\tilde{Y}^i) \leq \tfrac{1}{2}\ln(K). \tag{7}$$

where the parameter $K > 1$ sets the upper limit on how much information each trader can learn. Given the normal distributions of $\tilde{V}$ and $\tilde{Y}^i$, the above constraint is equivalent to

$$\left|\mathrm{Var}(\tilde{V} \mid \tilde{Y}^i)\right| \geq K^{-1}\left|\mathrm{Var}(\tilde{V})\right|. \tag{8}$$

Without loss of generality, we will take K as given and focus on how each investor chooses to allocate his learning capacity between asset 1 and asset 2.[4]

Learning Technology. Given the general form of the private signal $\tilde{Y}^i := (\tilde{Y}_1^i, \tilde{Y}_2^i)'$ in equation (4), there are different equilibrium implications, depending on whether Λ^i can be non-diagonal. If $\mathit{\Lambda}^i$ is restricted to be a diagonal matrix, investor i will collect information about each asset separately because $\tilde{Y}_1^i$ is orthogonal to $\tilde{Y}_2^i$. This implies that investor i's belief about asset 1 will be independent of his belief about asset 2. When every trader has this constrained learning technology, the prices of the two assets (with independent fundamentals) are independent. In contrast, if the matrix $\mathit{\Lambda}^i$ is non-diagonal, then $\tilde{Y}_1^i$ can be correlated with $\tilde{Y}_2^i$. In this case, each investor i will find it optimal to observe a signal on a linear combination of the two assets. The prices of the two assets are correlated. This induces a cross-learning effect. Hereafter, we distinguish the following two learning technologies:

(1) *separative 1earning*, where Λ^i is *constrained* to be diagonal for all i.

(2) *integrative learning*, where Λ^i is *unconstrained* as it can be non-diagonal.

[4]For a choice problem with an information cost function $c(\cdot)$ such that $U = U_1 - c(K)$, there is an equivalent endowment of capacity K_c that delivers the same portfolio predictions. We can assume that $c(\cdot)$ is increasing and sufficiently convex to deliver an interior optimal level of K_c. We can then take K_c as given for the rest of our analysis.

For either learning technology, the equilibrium is defined as follows:

Definition of Equilibrium. A rational-expectations equilibrium is defined by investors' portfolio choices $\{\theta^i\}$ and information choices $\{\Lambda^i, \Sigma^i\}$ as well as market-clearing prices $\tilde{P}$ such that

1. taking Λ^i, Σ^i, and $\tilde{P}$ as given, θ^i solves investor i's optimal portfolio choice problem (1);
2. given $\{\theta^i\}$, the prices $\tilde{P}$ satisfy the market-clearing condition (3);
3. taking the aggregate benchmarking level $\overline{\gamma}_j := \int_0^1 \gamma_j^i di$ for $j = \{1,2\}$ as given, Λ^i and Σ^i solve investor i's optimal information choice problem (5) subject to the constraint (8).

3 Separative Learning Equilibrium

We first study the equilibrium under separative learning, where Λ^i is restricted to be diagonal and traders can only observe signals about individual assets. As Σ^i is a diagonal matrix chosen by trader i, the information choice problem is equivalent to the one with $\Lambda^i = I_2$, a 2×2 identity matrix. Therefore, each trader's private information takes the simple form below:

$$\tilde{Y}^i := (\tilde{Y}_1^i, \tilde{Y}_2^i)' \qquad \text{where} \qquad \tilde{Y}_j^i = \tilde{V}_j + \tilde{\varepsilon}_j^i, \quad \tilde{\varepsilon}_j^i \sim \mathcal{N}\left(0, (\tau_j^i)^{-1}\right) \tag{9}$$

Proposition 1. *Given traders' information choices $\{\tau_1^i, \tau_2^i\}$, the market-clearing price for asset j is*

$$\tilde{P}_j = \left(\tau_{v,j} + \tau_{p,j} + \overline{\tau}_j\right)^{-1} \left[\tau_{v,j}\overline{v}_j + (\tau_{p,j} + \overline{\tau}_j)\tilde{s}_{p,j} - \lambda(\overline{z}_j - \overline{\gamma}_j)\right], \tag{10}$$

where $\overline{\tau}_j := \int_0^1 \tau_j^i di$ measures the total signal precision about asset j and $\overline{\gamma}_j := \int_0^1 \gamma_j^i di$ measures the aggregate benchmarking level of asset j. This price reveals a signal $\tilde{s}_{p,j}$ about the payoff $\tilde{V}_j$:

$$\tilde{s}_{p,j} := \tilde{V}_j - \lambda(\tilde{Z}_j - \overline{z}_j)/\overline{\tau}_j, \tag{11}$$

and the precision of this signal is given by

$$\tau_{p,j} := \left(\mathrm{Var}(\tilde{s}_{p,j} - \tilde{V}_j)\right)^{-1} = \overline{\tau}_j^2 \tau_{z,j}/\lambda^2. \tag{12}$$

Given his private information and benchmarking needs, trader i's optimal holding on asset j is

$$\theta_j^i = \gamma_j^i + \frac{\mathrm{E}^i[\tilde{V}_j] - \tilde{P}_j}{\lambda \mathrm{Var}^i(\tilde{V}_j)}, \tag{13}$$

which is based on his posterior belief about asset j:

$$\mathrm{E}^i[\tilde{V}_j] = \left(\tau_{v,j} + \tau_j^i + \tau_{p,j}\right)^{-1}\left(\tau_{v,j}\overline{v}_j + \tau_j^i \tilde{Y}_j^i + \tau_{p,j}\tilde{s}_{p,j}\right), \qquad \mathrm{Var}^i(\tilde{V}_j) = \left(\tau_{v,j} + \tau_j^i + \tau_{p,j}\right)^{-1}. \tag{14}$$

Proof. See Appendix A.1. □

Equation (10) shows that the price $\tilde{P}_j$ increases in the composite signal $\tilde{s}_{p,j}$ and decreases in the effective supply $(\overline{z}_j - \overline{\gamma}_j)$ that remains available for information-sensitive speculation. The equilibrium prices only depend on these aggregate variables because the economy has a continuum of traders whose individual signals or benchmarking concerns have negligible price impacts. Equation (10) also shows the separation of asset prices: $\tilde{P}_j$ only depends on variables related to asset j. This is also true for traders' optimal holdings θ_j^i in equation (13) which only depends on variables related to asset j. In other words, the two risky assets are fully separated in terms of their pricing and informational content. With the results in Proposition 1, each trader i chooses the optimal precision levels of his signals, τ_1^i and τ_2^i, to maximize his expected utility at $t = 1$. The information choice problem (5) can be simplified as

$$\max_{\tau_1^i,\tau_2^i}\left[\sum_{j=1,2}\frac{\left(\mathrm{E}^i[\tilde{V}_j]-\tilde{P}_j\right)^2}{\mathrm{Var}^i(\tilde{V}_j)}\right] \qquad \text{s.t.} \quad \prod_{j=1,2}(\tau_{v,j}+\tau_j^i)\le K\prod_{j=1,2}\tau_{v,j} \quad \text{and} \quad \tau_j^i\ge 0.$$

For each asset, we define the following quantity as its *Adjusted Squared Sharpe Ratio* (ASSR):

$$\mathrm{ASSR}_j := \frac{\mathrm{E}[(\tilde{V}_j-\tilde{P}_j)^2]}{\mathrm{Var}(\tilde{V}_j)} = \frac{(\mathrm{E}[\tilde{V}_j-\tilde{P}_j])^2}{\mathrm{Var}(\tilde{V}_j)} + \frac{\mathrm{Var}(\tilde{V}_j-\tilde{P}_j)}{\mathrm{Var}(\tilde{V}_j)}, \tag{15}$$

which is a type of reward-to-risk ratio.

Proposition 2. *With separative learning, each investor optimally allocates all of his learning capacity to the asset that has a higher* ASSR. *The optimal precision of his signal on asset j is*

$$\tau_j^i = \begin{cases} (K-1)\tau_{v,j} & \text{if} \quad \mathrm{ASSR}_j = \max\{\mathrm{ASSR}_1,\mathrm{ASSR}_2\}, \\ 0 & \text{if} \quad \mathrm{ASSR}_j \neq \max\{\mathrm{ASSR}_1,\mathrm{ASSR}_2\}. \end{cases} \tag{16}$$

Proof. See Appendix A.2. □

When $\mathrm{ASSR}_1 = \mathrm{ASSR}_2$, each investor is indifferent to the learning choice between asset 1 and asset 2. ASSR_j depends on the total precision of signals about asset j and this precision depends on the fraction of traders who choose to learn about asset j. Let $\Gamma_1 \in [0,1]$ denote the fraction of investors who learn about asset 1 only. Then $\Gamma_2 = 1-\Gamma_1$ is the fraction of investors who learn about asset 2. Thus, investors' optimal choice of signal precision can be written as

$$\begin{aligned} &\tau_1^i = (K-1)\tau_{v,1}, \quad \tau_2^i = 0, \quad \text{for} \quad i\in[0,\Gamma_1], \\ &\tau_1^i = 0, \quad \tau_2^i = (K-1)\tau_{v,2}, \quad \text{for} \quad i\in(\Gamma_1,1]. \end{aligned} \tag{17}$$

Theorem 1. *With separative learning, there exists a unique linear equilibrium. It has three possible cases for investors' overall attention allocation, denoted by* (Γ_1, Γ_2)*:*

(1) If $\mathrm{ASSR}_1 > \mathrm{ASSR}_2$ *holds for any* $\Gamma_1 \in [0,1]$*, then* $\Gamma_1 = 1$ *and* $\Gamma_2 = 0$*;*

(2) If $\mathrm{ASSR}_1 = \mathrm{ASSR}_2$ *holds at some* $\Gamma_1^* \in (0,1)$*, then* $\Gamma_1 = \Gamma_1^*$ *and* $\Gamma_2 = 1 - \Gamma_1^*$*;*

(3) If $\mathrm{ASSR}_1 < \mathrm{ASSR}_2$ *holds for any* $\Gamma_1 \in [0,1]$*, then* $\Gamma_1 = 0$ *and* $\Gamma_2 = 1$*.*

The equilibrium prices are governed by (10). *Each investor's optimal portfolio follows* (13). *The optimal information choice is given by equation* (16) *for either case (1) or case (3), and by equation* (17) *for case (2). The total precision of investors' signals on asset* j *is* $\overline{\tau}_j = \Gamma_j (K-1)\tau_{v,j}$*.*

In this equilibrium, each investor's attention is allocated exclusively to one asset. Depending on their aggregate benchmarking concerns about either asset, a fraction of them focus on learning about asset 1 while the rest focus on learning about asset 2. In Theorem 1, Case (1) is a corner equilibrium which occurs if the inequality, $\mathrm{ASSR}_1 > \mathrm{ASSR}_2$, holds no matter how many traders choose to learn about asset 1. In this case, all traders choose to learn about asset 1 and ignore asset 2. Case (2) is an interior equilibrium where a fraction (Γ_1) of investors learn about asset 1 only, and the rest mass of investors learn about asset 2 only. This equilibrium features $\mathrm{ASSR}_1 = \mathrm{ASSR}_2$ so that a marginal investor is indifferent to the learning choice between asset 1 and asset 2. Case (3) is another corner equilibrium, similar to Case (1) but the other way around.

There are different ways to measure price informativeness. We propose to use the measure of mutual information whose general definition is given by equation (6). Specifically, we will use the measure $\mathrm{I}(\tilde{V}_j; \tilde{P}_j)$ since it quantifies the amount of information about the payoff $\tilde{V}_j$ revealed by the asset price $\tilde{P}_j$. In a linear Gaussian model, this measure is informationally equivalent to the correlation measure, $\mathrm{Corr}(\tilde{V}_j, \tilde{P}_j)$ or the R-squared measure, $R^2 = 1 - \frac{\mathrm{Var}(\tilde{V}_j|\tilde{P}_j)}{\mathrm{Var}(\tilde{V}_j)}$. In general, mutual information takes into account all linear and nonlinear dependence between two random variables (or vectors of random variables). This is different from the correlation measure which only captures linear dependence and the R-squared measure which requires a specific regression model. As a standard information-theoretic measure, mutual information is model-free and thus can quantify the efficiency of information transmission in arbitrary models.

In this separative learning equilibrium, the price informativeness for asset j can be quantified by the mutual information of asset j's payoff and price:

$$\mathrm{I}(\tilde{V}_j; \tilde{P}_j)_{\mathrm{sep}} = \frac{1}{2}\ln\left(\frac{\mathrm{Var}(\tilde{V}_j)}{\mathrm{Var}(\tilde{V}_j \mid \tilde{P}_j)}\right) = \frac{1}{2}\ln\left(\frac{\tau_{v,j} + \tau_{p,j}}{\tau_{v,j}}\right) = \frac{1}{2}\ln\left(1 + \Gamma_j^2\left(\frac{K-1}{\lambda}\right)^2 \tau_{v,j}\tau_{z,j}\right), \tag{18}$$

where $\tau_{p,j} = \left(\Gamma_j \tau_{v,j}\frac{K-1}{\lambda}\right)^2 \tau_{z,j}$. We have the following proposition.

Proposition 3. *Suppose the effective asset supply is positive ($\overline{z}-\overline{\gamma}>0$). Then*

$$\frac{\mathrm{d}\Gamma_j}{\mathrm{d}\overline{\gamma}_j}\le 0, \qquad \frac{\mathrm{d}\Gamma_j}{\mathrm{d}\overline{\gamma}_{-j}}\ge 0, \qquad \frac{\mathrm{dI}(\tilde{V}_j;\tilde{P}_j)_{\mathrm{sep}}}{\mathrm{d}\overline{\gamma}_j}\le 0, \qquad \frac{\mathrm{dI}(\tilde{V}_j;\tilde{P}_j)_{\mathrm{sep}}}{\mathrm{d}\overline{\gamma}_{-j}}\ge 0. \tag{19}$$

Proof. See Appendix A.3. The subscript "$-j$" denotes the other asset. For example, if $j=1$, then "$-j$" refers to 2. □

An increase in the benchmarking level of asset 1 can reduce investors' attention to asset 1 and increase their attention to asset 2, resulting in lower price informativeness of asset 1 and higher price informativeness of asset 2.

By Proposition 2, each investor optimally allocates all his attention to the asset with the highest ASSR. Given Proposition 1 and that $\overline{\tau}_j=\Gamma_j(K-1)\tau_{v,j}$, it is straightforward to calculate

$$\mathrm{ASSR}_j(\Gamma_j,\overline{\gamma}_j):=\frac{\mathrm{E}[(\tilde{V}_j-\tilde{P}_j)^2]}{\mathrm{Var}(\tilde{V}_j)}=\frac{\tau_{v,j}+\left(\overline{\tau}_j+\lambda^2/\tau_{z,j}\right)^2\tau_{z,j}/\lambda^2+\lambda^2(\overline{z}_j-\overline{\gamma}_j)^2}{(\overline{\tau}_j+\tau_{v,j}+\overline{\tau}_j^2\tau_{z,j}/\lambda^2)^2}\tau_{v,j}, \tag{20}$$

which can be interpreted as the marginal value of private information about asset j. It can be shown that ASSR_j decreases in Γ_j (the fraction of investors who learn about asset j only) and $\overline{\tau}_j$ (the total precision of investors' signals on asset j):

$$\frac{\partial\,\mathrm{ASSR}_j}{\partial\,\Gamma_j}<0, \qquad \frac{\partial\,\mathrm{ASSR}_j}{\partial\,\Gamma_{-j}}>0, \qquad \frac{\partial\,\mathrm{ASSR}_j}{\partial\,\overline{\tau}_j}<0, \qquad \frac{\partial\,\mathrm{ASSR}_j}{\partial\,\overline{\tau}_{-j}}>0. \tag{21}$$

This is due to the *strategic substitutability effect* as noted in Grossman and Stiglitz (1980). More and more information acquisition makes the asset price more and more informative such that it is less valuable for a marginal investor to acquire new information. Consider the interior solution in Theorem 1 where the marginal value of private information is equalized across assets:

$$\mathrm{ASSR}_1(\Gamma_1,\overline{\gamma}_1)=\mathrm{ASSR}_2(\Gamma_1,\overline{\gamma}_2). \tag{22}$$

We can take the total derivative with respect to $\overline{\gamma}_1$ on both sides and obtain

$$\frac{\mathrm{d}\Gamma_1}{\mathrm{d}\overline{\gamma}_1}=-\Big(\underbrace{\frac{\partial\mathrm{ASSR}_1}{\partial\Gamma_1}}_{-}-\underbrace{\frac{\partial\mathrm{ASSR}_2}{\partial\Gamma_1}}_{+}\Big)^{-1}\underbrace{\frac{\partial\mathrm{ASSR}_1}{\partial\overline{\gamma}_1}}_{-}<0. \tag{23}$$

Here, $\frac{\partial\,\mathrm{ASSR}_1}{\partial\Gamma_1}<0$ and $\frac{\partial\,\mathrm{ASSR}_2}{\partial\Gamma_1}>0$ are from the *strategic substitutability effect* in equation (21). The partial derivative $\partial\,\mathrm{ASSR}_1/\partial\bar{\gamma}_1<0$ can be seen from equation (20). This is because a higher benchmarking level $\bar{\gamma}_1$ reduces the *effective supply* $\bar{z}_1-\bar{\gamma}_1$ of asset 1 and thus reduces its expected squared return, $\mathrm{E}[(V_1 - P_1)^2]$. Therefore, as $\bar{\gamma}_1$ increases, the marginal benefit of learning about asset 1 becomes smaller than that of asset 2. As a result, some investors switch from learning about asset 1 to learning about asset 2, which makes asset 2's aggregate signal precision higher and thus reduces its marginal benefit of learning. This continues until the marginal benefits of private information about two assets become equal again and investors are in a new equilibrium, which occurs at a lower value of endogenous parameter Γ_1.

Since the fraction of investors who choose to learn about asset j always (weakly) decreases in the benchmarking level of asset j, the price informativeness of asset j also decreases in $\overline{\gamma}_j$. Given investors' limited attention, an increase in the other asset $-j$'s benchmarking level can drive more investors to learn about asset j and hence improve its price informativeness.

Corollary 1. *Suppose the effective supply is positive for each asset ($\overline{z} - \overline{\gamma} > 0$). The more volatile an asset is (in terms of its payoff or noisy supply), the more investors choose to learn about it:*

$$\frac{\mathrm{d}\Gamma_j}{\mathrm{d}\tau_{v,j}} \leq 0, \qquad \frac{\mathrm{d}\Gamma_j}{\mathrm{d}\tau_{z,j}} \leq 0, \qquad \frac{\mathrm{d}\Gamma_j}{\mathrm{d}\tau_{v,-j}} \geq 0, \qquad \frac{\mathrm{d}\Gamma_j}{\mathrm{d}\tau_{z,-j}} \geq 0. \tag{24}$$

Proof. See Appendix A.4. □

The above results are intuitive because the private information about more uncertain outcomes should be more valuable and attractive, consistent with Kacperczyk et al. (2016).

Based on the price informativeness measure (18) for individual assets, we can extend the Shannon entropy measure to evaluate the overall informational efficiency in this market.

Corollary 2. *With separative learning, the overall market efficiency can be measured by the mutual information of asset prices and payoffs, which is also equal to the sum of price informativeness of individual assets:*

$$\begin{aligned} \mathrm{I}(\tilde{V};\tilde{P})_{\mathrm{sep}} &= \mathrm{I}(\tilde{V}_1;\tilde{P}_1)_{\mathrm{sep}} + \mathrm{I}(\tilde{V}_2;\tilde{P}_2)_{\mathrm{sep}} \\ &= \frac{1}{2}\ln\left(1 + \Gamma_1^2\left(\frac{K-1}{\lambda}\right)^2 \tau_{v,1}\tau_{z,1}\right) + \frac{1}{2}\ln\left(1 + (1-\Gamma_1)^2\left(\frac{K-1}{\lambda}\right)^2 \tau_{v,2}\tau_{z,2}\right), \end{aligned} \tag{25}$$

Proof. This follows from Proposition 1, Theorem 1, and Proposition 3. Equation (25) holds because the two assets have independent payoffs and their prices are also independent. □

The measure $\mathrm{I}(\tilde{V};\tilde{P})_{\mathrm{sep}}$ is a simple but non-monotonic function of the endogenous variable Γ_1, which decreases in $\overline{\gamma}_1$ and increases in $\overline{\gamma}_2$ (Proposition 3). In Section 4.2, we will discuss more about how benchmarking affects the market informational efficiency under different learning technology.

We can compute the impacts of benchmarking on the (unconditional) expected asset prices:

$$\frac{\mathrm{dE}[\tilde{P}_j]}{\mathrm{d}\overline{\gamma}_j} = \frac{\lambda}{\tau_{v,j}+\tau_{p,j}+\overline{\tau}_j}\left(1+\frac{(1+2\overline{\tau}_j\tau_{z,j}/\lambda^2)(\overline{z}_j-\overline{\gamma}_j)(K-1)\tau_{v,j}}{\tau_{v,j}+\tau_{p,j}+\overline{\tau}_j}\frac{\mathrm{d}\Gamma_j}{\mathrm{d}\overline{\gamma}_j}\right), \tag{26}$$

$$\frac{\mathrm{dE}[\tilde{P}_j]}{\mathrm{d}\overline{\gamma}_{-j}} = \frac{\lambda\left(1+2\overline{\tau}_j\tau_{z,j}/\lambda^2\right)(\overline{z}_j-\overline{\gamma}_j)(K-1)\tau_{v,j}}{\left(\tau_{v,j}+\tau_{p,j}+\overline{\tau}_j\right)^2}\frac{\mathrm{d}\Gamma_j}{\mathrm{d}\overline{\gamma}_{-j}} > 0. \tag{27}$$

On average, benchmarking can affect asset prices both directly (through the increased demand) and indirectly (through decreased price informativeness). While higher demands drive prices higher, lower price informativeness can cause higher risk premium and lower prices. Numerically, we find that the direct demand effect typically dominates in equation (26) and thus the expected price of an asset tends to increase with its own benchmarking level.

Only the indirect information effect matters in equation (27) . An increase of benchmarking on asset "$-j$" increases the fraction of traders who choose to learn about the other asset j. As the total precision of signals about asset j increases, the risk premium of asset j decreases and thus the expected price of asset j increases. Therefore, an increase in the benchmarking level of an asset tends to increase the expected prices of the other asset.

We conclude this section by presenting the following observation:

Corollary 3. *An increase in the benchmarking level of an asset always increases the return volatility of this asset and decreases the return volatility of the other asset:*

$$\frac{\mathrm{dVar}(\tilde{V}_j-\tilde{P}_j)}{\mathrm{d}\overline{\gamma}_j} \geq 0, \qquad \frac{\mathrm{dVar}(\tilde{V}_j-\tilde{P}_j)}{\mathrm{d}\overline{\gamma}_{-j}} \leq 0. \tag{28}$$

Regardless of how these assets are benchmarked, their prices or returns are always uncorrelated:

$$\mathrm{Cov}(\tilde{P}_1,\tilde{P}_2)=0, \qquad \mathrm{Cov}(\tilde{V}_1-\tilde{P}_1,\tilde{V}_2-\tilde{P}_2)=0. \tag{29}$$

Proof. See Appendix A.5. □

Corollary 3 states that, in the separative learning equilibrium, an increase in the benchmarking level of asset j increases asset j's return volatility and decreases asset $-j$'s return volatility. This is consistent with the impact of benchmarking on investors' attention or the price informativeness of an asset (Proposition 3). In addition, the asset prices are uncorrelated because investors collect information about each asset separately. Independent sources of ex-ante uncertainty remain independent ex-post.

4 Integrative Learning Equilibrium

Now we turn to the case that investors are not restricted to observe two separate signals. In other words, we consider a general specification that the matrix Λ^i in equation (4) is allowed to be non-diagonal and also possibly degenerate (with rank one) for all investors $i \in [0,1]$.

If Λ^i is non-diagonal and full rank, we can use eigen-decomposition to diagonalize this matrix and transform the two correlated signals into two independent ones. This transformation reduces the problem to a separative learning equilibrium which has been solved in Section 2.

If Λ^i is non-diagonal but degenerate, it means that investor i chooses to observe a signal which is a linear combination of the two payoffs plus a noise term. In this case, it is equivalent to set $\Lambda^i = (1, \omega^i)$ such that the private information can be expressed as a scalar:

$$\tilde{Y}^i = \tilde{V}_1 + \omega^i \tilde{V}_2 + \tilde{\varepsilon}^i, \qquad \tilde{\varepsilon}^i \sim \mathcal{N}(0, 1/\tau^i), \tag{30}$$

where both ω^i and τ^i are chosen by investor i. As shown by Mondria (2010), it is indeed optimal for each agent to choose a degenerate Λ^i and thus observe a combined signal in the form of (30). We refer it as *integrative learning*, in contrast to the *separative learning* discussed before.

Following Admati (1985) and Mondria (2010), we can solve for the equilibrium in three steps. First, conjecture the existence of a linear equilibrium where each investor i optimally chooses the learning technology with a degenerate $\Lambda^i = (1, \omega^i)$ and thus observes a combined signal as in (30). Second, given investors' information choices, derive their optimal asset holdings and the market-clearing prices. Third, given these results, find the optimal information choice for each investor, including the optimal attention weight ω^i and the optimal signal precision τ^i.

Proposition 4. *Given traders' information choices $\{\omega^i, \tau^i\}$, the market-clearing price (vector) is*

$$\tilde{P} = C + B\left(\Omega\tilde{V} - \lambda\left(\tilde{Z} - \overline{z}\right)\right), \qquad \Omega := \int \tau^i \begin{pmatrix} 1 & w^i \\ w^i & (w^i)^2 \end{pmatrix} di, \tag{31}$$

where C is a constant vector given by equation (A-27) and B is a full-rank matrix given by:

$$B := \left(\Sigma_v^{-1} + \lambda^{-2}\Omega\Sigma_z^{-1}\Omega + \Omega\right)^{-1}\left(I_2 + \lambda^{-2}\Omega\Sigma_z^{-1}\right). \tag{32}$$

Given his private information and benchmarking needs, trader i's optimal portfolio choice is

$$\theta^i = \gamma^i + (\lambda\hat{\Sigma}_v^i)^{-1}(\hat{V}^i - \tilde{P}). \tag{33}$$

The expressions of $\hat{\Sigma}_v^i$ and $\hat{V}^i$ are given by (A-30) and (A-31). See Appendix A.6 for the proof.

Similar to the case of separative learning, the equilibrium prices under integrative learning do not depend on individual investors' information precision or benchmarking concerns. Instead, they depend on the total signal precision and the aggregate benchmarking levels. Therefore, each individual investor solves the same information allocation problem.

Using the above results, we can evaluate the matrix $\mathrm{E}[(\tilde{V}-\tilde{P})^2]$. Define $A_{jj} := \mathrm{E}[(\tilde{V}_j-\tilde{P}_j)^2]$ and $A_{12} := \mathrm{E}[(\tilde{V}_1-\tilde{P}_1)(\tilde{V}_2-\tilde{P}_2)]$. This allows us to simplify each investor's information problem:

Proposition 5. *The information choice problem for each investor is equivalent to*

$$\max_{\omega^i,\tau^i} \frac{(\omega^i)^2 A_{22} + 2\omega^i A_{12} + A_{11}}{\tau_{v,1}^{-1} + (\omega^i)^2 \tau_{v,2}^{-1}} \qquad s.t. \qquad 0 \le \tau^i \le \frac{K-1}{\tau_{v,1}^{-1} + (\omega^i)^2 \tau_{v,2}^{-1}}. \tag{34}$$

The first order condition for the choice variable ω^i is

$$A_{12}(\omega^i)^2 + \left(A_{11} - A_{22}\tau_{v,1}^{-1}\tau_{v,2}\right)\omega^i - A_{12}\tau_{v,1}^{-1}\tau_{v,2} = 0. \tag{35}$$

As long as $A_{12} \neq 0$, the best response of investor i is to choose the attention weight

$$\omega^i = \frac{\mathrm{ASSR}_2 - \mathrm{ASSR}_1}{2A_{12}\tau_{v,1}} + \sqrt{\left(\frac{\mathrm{ASSR}_2 - \mathrm{ASSR}_1}{2A_{12}\tau_{v,1}}\right)^2 + \frac{\tau_{v,2}}{\tau_{v,1}}}, \tag{36}$$

where $\mathrm{ASSR}_j = A_{jj}\tau_{v,j}$ is the adjusted squared Sharpe ratio for asset j as defined in equation (15).

Proof. See Appendix A.7. □

Equation (36) shows that investors' attention allocation depends on the difference of ASSR values between these two assets as well as the expected product of their returns A_{12}. In fact, when investors have access to the integrative learning technology, there does not exist an equilibrium where a fraction of traders choose to specialize in learning only about one asset.

Corollary 4. *If Λ_i is not restricted to be diagonal, then there does not exist an equilibrium where a finite fraction of investors specialize in learning only about one asset. Instead, the best response for each investor is to choose integrative learning by observing a signal about the linear combination of asset payoffs in the form of equation* (30).

Proof. See Appendix A.8. □

Corollary 4 implies that the restriction on Λ_i to be a diagonal matrix is critical for the separative learning equilibrium. Without this restriction on Λ_i, we always obtain an integrative learning equilibrium where investors optimally learn about a linear combination of assets.

Theorem 2. *If Λ^i is not restricted to be diagonal, then there exists a linear symmetric equilibrium where each investor optimally chooses the integrative learning technology to observe a combined private signal, $\tilde{Y}^i = \tilde{V}_1 + \omega \tilde{V}_2 + \tilde{\varepsilon}^i$, where $\tilde{\varepsilon}^i \sim \mathcal{N}(0, 1/\tau)$ is independent noise across investors. In this equilibrium, investors choose the same attention weight and thus the same signal precision:*

$$\omega^i = \omega = \alpha + \sqrt{\alpha^2 + \frac{\tau_{v,2}}{\tau_{v,1}}}, \qquad \tau^i = \tau = \frac{K-1}{\tau_{v,1}^{-1} + \omega^2 \tau_{v,2}^{-1}}. \tag{37}$$

The optimal attention weight ω increases in the parameter α which is given by

$$\alpha := \frac{\mathrm{ASSR}_2 - \mathrm{ASSR}_1}{2A_{12}\tau_{v,1}} = \frac{\tau_{v,2}^{-1}\tau_{z,2}^{-1} + \tau_{v,2}^{-1}(\bar{z}_2 - \bar{\gamma}_2)^2 - \tau_{v,1}^{-1}\tau_{z,1}^{-1} - \tau_{v,1}^{-1}(\bar{z}_1 - \bar{\gamma}_1)^2}{2(\bar{z}_1 - \bar{\gamma}_1)(\bar{z}_2 - \bar{\gamma}_2)\tau_{v,2}^{-1}}. \tag{38}$$

Given this optimal information choice (ω, τ), the equilibrium prices are determined by equation (31) and the optimal portfolio choice for each investor is given by equation (33). This equilibrium exists if and only if the expected value of the product of two asset returns is positive: $A_{12}(\omega) > 0$.

Proof. See Appendix A.9. □

The above symmetric equilibrium exists as long as $A_{12}(\omega) := \mathrm{E}[(\tilde{V}_1 - \tilde{P}_1)(\tilde{V}_2 - \tilde{P}_2)] > 0$. This condition can be verified by first computing the value ω by equation (37). We show that this condition is satisfied as long as investors' risk aversion λ is not too small or too large; see Appendix A.9 for proofs.[5] In this paper, we focus on analyzing the symmetric equilibrium in Theorem 2, considering its uniqueness and tractability.

The aggregate benchmarking level of each asset affects both the numerator and denominator of α in equation (38). Note that $\tau_{v,j}^{-1}\tau_{z,j}^{-1} + \tau_{v,j}^{-1}(\bar{z}_j - \bar{\gamma}_j)^2 = \mathrm{Var}\big((\tilde{V}_j - \bar{v}_j)\cdot(\tilde{Z}_j - \bar{\gamma}_j)\big)$ is the total payoff uncertainty of asset j's supply effectively available for investors' speculation. Thus, the parameter α captures the relative attractiveness of asset 2 against asset 1. When $\alpha \to +\infty$, we have $\omega \to \infty$ such that all investors only learn about asset 2. When $\alpha \to -\infty$, we have $\omega \to 0$ such that all investors only learn about asset 1. As before, we consider positive effective supplies for both assets, $\bar{z}_j - \bar{\gamma}_j > 0$. Equation (38) suggests that $\alpha \to \pm\infty$ whenever $\bar{\gamma}_j \to \bar{z}_j$ for each asset j, and the sign of α is determined by the sign of $\mathrm{ASSR}_2 - \mathrm{ASSR}_1$ in such limits. For example, as the average effective supply of asset 1 vanishes (i.e., $\bar{\gamma}_1 \to \bar{z}_1$), all investors will choose to learn about asset 1 (i.e., $\alpha \to -\infty$ and $\omega \to 0$) if its noisy supply is so volatile that $\tau_{v,1}^{-1}\tau_{z,1}^{-1} > \tau_{v,2}^{-1}\tau_{z,2}^{-1} + \tau_{v,2}^{-1}(\bar{z}_2 - \bar{\gamma}_2)^2$ holds; otherwise, all investors choose to learn about asset 2.

[5] When the result of ω in (37) cannot support the condition $A_{12}(\omega) > 0$, we may have an equilibrium asymmetric across investors in terms of their choices of ω^i. The equilibrium solution is less tractable and not unique.

4.1 Asset-specific Price Informativeness

We still use the mutual information, denoted $\mathrm{I}(\tilde{V}_j;\tilde{P}_j)_{\text{int}}$, to measure price informativeness. Given the equilibrium attention weight ω, we can directly calculate $\mathrm{I}(\tilde{V}_j;\tilde{P}_j)_{\text{int}}$ for asset $j=1,2$. Their expressions are given by (A-72) and (A-73) in Appendix A.13. When ω goes to either 0 or ∞, the price informativeness in the integrative learning equilibrium converges to the separative learning case presented in equation (18) with either $\Gamma_1=1$ or $\Gamma_1=0$. It is easy to verify that

$$\lim_{\omega\to 0}\mathrm{I}(\tilde{V}_1;\tilde{P}_1)_{\text{int}}=\frac{1}{2}\ln\left(1+\left(\frac{K-1}{\lambda}\right)^2\tau_{v,1}\tau_{z,1}\right),\quad \lim_{\omega\to\infty}\mathrm{I}(\tilde{V}_2;\tilde{P}_2)_{\text{int}}=\frac{1}{2}\ln\left(1+\left(\frac{K-1}{\lambda}\right)^2\tau_{v,2}\tau_{z,2}\right). \tag{39}$$

The impact of benchmarking on price informativeness mostly depends on how investors' aggregate benchmarking concerns affect their attention allocation ω. To understand the direct effect on ω, it is convenient to define

$$\Delta:=\mathrm{Var}(\tilde{V}_2)\mathrm{Var}(\tilde{Z}_2)-\mathrm{Var}(\tilde{V}_1)\mathrm{Var}(\tilde{Z}_1)=\tau_{v,2}^{-1}\tau_{z,2}^{-1}-\tau_{v,1}^{-1}\tau_{z,1}^{-1}, \tag{40}$$

which measures the difference of uncertainty between asset 2 and asset 1. We also define

$$\xi:=\mathrm{Var}\left(\tilde{V}_1\mathrm{E}[\tilde{Z}_1-\overline{\gamma}_1]\right)+\mathrm{Var}\left(\tilde{V}_2\mathrm{E}[\tilde{Z}_2-\overline{\gamma}_2]\right)=\tau_{v,1}^{-1}(\overline{z}_1-\overline{\gamma}_1)^2+\tau_{v,2}^{-1}(\overline{z}_2-\overline{\gamma}_2)^2, \tag{41}$$

which measures the total payoff uncertainty of the average speculative portfolio with $(\overline{z}_1-\overline{\gamma}_1)$ shares of asset 1 and $(\overline{z}_2-\overline{\gamma}_2)$ shares of asset 2. We have the following results.

Proposition 6. *Depending on the sign of Δ, investors' optimal attention allocation satisfies*

$$\omega\big|_{\Delta=0}=\frac{\overline{z}_2-\overline{\gamma}_2}{\overline{z}_1-\overline{\gamma}_1},\qquad \omega\big|_{\Delta>0}>\frac{\overline{z}_2-\overline{\gamma}_2}{\overline{z}_1-\overline{\gamma}_1},\qquad \omega\big|_{\Delta<0}<\frac{\overline{z}_2-\overline{\gamma}_2}{\overline{z}_1-\overline{\gamma}_1}. \tag{42}$$

The impact of aggregate benchmarking on investors' attention is characterized by

$$\frac{d\omega}{d\overline{\gamma}_1}=\frac{(\Delta+\xi)\omega}{2(\omega-\alpha)(\overline{z}_1-\overline{\gamma}_1)^2(\overline{z}_2-\overline{\gamma}_2)\tau_{v,2}^{-1}},\qquad \frac{d\omega}{d\overline{\gamma}_2}=\frac{(\Delta-\xi)\omega}{2(\omega-\alpha)(\overline{z}_1-\overline{\gamma}_1)(\overline{z}_2-\overline{\gamma}_2)^2\tau_{v,2}^{-1}}. \tag{43}$$

Depending on the magnitude of Δ, there are three possible cases for the impact of $\overline{\gamma}_j$ on ω:

Case (1): If $\Delta\ge\xi$, then $\frac{d\omega}{d\overline{\gamma}_1}>0$, $\frac{d\omega}{d\overline{\gamma}_2}\ge 0$;

Case (2): If $\Delta\le-\xi$, then $\frac{d\omega}{d\overline{\gamma}_1}\le 0$, $\frac{d\omega}{d\overline{\gamma}_2}<0$;

Case (3): If $|\Delta|<\xi$, then $\frac{d\omega}{d\overline{\gamma}_1}>0$, $\frac{d\omega}{d\overline{\gamma}_2}<0$.

Proof. See Appendix A.10. □

Equation (42) shows that if two assets are equal in their total prior uncertainty ($\Delta = 0$), then investors' optimal attention weight is exactly equal to the ratio of effective supplies, $\omega = \frac{\overline{z}_2 - \overline{\gamma}_2}{\overline{z}_1 - \overline{\gamma}_1}$. If asset 2 becomes more uncertain than asset 1 (i.e., $\Delta > 0$), then more attention will be allocated to asset 2 such that $\omega > \frac{\overline{z}_2 - \overline{\gamma}_2}{\overline{z}_1 - \overline{\gamma}_1}$. Similarly, asset 1 can attract more attention if it is more uncertain.

By Proposition 6, when the magnitude of Δ is larger than ξ, the impact of benchmarking on investors' attention is different from the result in the separative learning case. Take Case (1) for example, if asset 2 is significantly more uncertain than asset 1, investors will allocate more attention to learn about asset 2 even when its benchmarking level increases. This cannot happen in the separative learning equilibrium according to (19).

Why is the impact of benchmarking different here? Proposition 5 shows that the marginal value of information has a cross-learning term and the optimal attention allocation is obtained when the marginal value of information is equalized across assets,

$$\text{ASSR}_1 + A_{12}\tau_{v,1}\omega = \text{ASSR}_2 + A_{12}\tau_{v,2}/\omega, \tag{44}$$

where $\text{ASSR}_j = \text{E}[(\tilde{V}_j - \tilde{P}_j)^2]\tau_{v,j}$ and $A_{12} = \text{E}\left[(\tilde{V}_1 - \tilde{P}_1)(\tilde{V}_2 - \tilde{P}_2)\right]$. Equation (44) can be written as

$$\tau_{v,1}\omega - \frac{\tau_{v,2}}{\omega} = \frac{\text{ASSR}_2 - \text{ASSR}_1}{A_{12}}. \tag{45}$$

Benchmarking can affect the two assets' ASSRs and the expected product of their returns (A_{12}). We examine the first derivative of equation (45) with respect to $\overline{\gamma}_2$ and obtain

$$\left(\tau_{v,1} + \frac{\tau_{v,2}}{\omega^2}\right)\frac{\text{d}\omega}{\text{d}\overline{\gamma}_2} = \frac{1}{A_{12}}\Bigg(\underbrace{\frac{\text{d}(\text{ASSR}_2 - \text{ASSR}_1)}{\text{d}\overline{\gamma}_2}}_{\text{effect of reduced supply}} + \underbrace{\frac{\text{ASSR}_1 - \text{ASSR}_2}{A_{12}}\frac{\text{d}A_{12}}{\text{d}\overline{\gamma}_2}}_{\text{effect of cross learning}}\Bigg). \tag{46}$$

Similar to the result for separative learning, an asset's ASSR tends to decrease in its benchmarking level and increase in the other asset's benchmarking level: $\frac{\text{dASSR}_2}{\text{d}\overline{\gamma}_2} < 0$ and $\frac{\text{dASSR}_1}{\text{d}\overline{\gamma}_2} > 0$. This implies the negative sign of the first term in (46), given the equilibrium condition $A_{12} > 0$. Benchmarking reduces the effective supply of the benchmarked asset, decreases the marginal value of information about this asset, and thus shifts investors' attention to the other asset. The second term relates to the cross-learning effect and its sign is set by the sign of ($\text{ASSR}_2 - \text{ASSR}_1$). We can prove $\frac{\text{d}A_{12}}{\text{d}\overline{\gamma}_j} < 0$ in multiple special cases; see Appendix A.11. This is further verified by extensive numerical tests. Thus, if $\text{ASSR}_2 < \text{ASSR}_1$, the second term in (46) is negative and thus reduces investors' attention to asset 2. If $\text{ASSR}_2 > \text{ASSR}_1$, we have the opposite effect.

When $\text{ASSR}_2 \gg \text{ASSR}_1$ (i.e., asset 2 is much more uncertain than asset 1), the cross-learning effect dominates the negative impact of reduced supply. In this case, a higher benchmarking level of asset 2 can make investors shift more attention to asset 2, because it is more valuable for investors to learn about asset 2 than asset 1. As investors increase their attention to asset 2, the marginal value of private information decreases due to the *strategic substitutability* effect. Meanwhile, the marginal value of information about asset 1 increases. This process continues until the marginal benefits of private information become equalized across assets again. Investors are then in a new equilibrium with a higher value of ω (i.e., more attention to asset 2).

Corollary 5. *When* $\frac{\mathrm{ASSR}_2-\mathrm{ASSR}_1}{2A_{12}\tau_{v,1}} > \frac{\overline{z}_2-\overline{\gamma}_2}{\overline{z}_1-\overline{\gamma}_1}$*, the cross-learning effect dominates such that investors allocate more attention to asset 2 while its benchmarking level increases,* $\frac{d\omega}{d\overline{\gamma}_2} > 0$.

Similarly, when $\frac{\mathrm{ASSR}_1-\mathrm{ASSR}_2}{2A_{12}\tau_{v,2}} > \frac{\overline{z}_1-\overline{\gamma}_1}{\overline{z}_2-\overline{\gamma}_2}$*, the cross-learning effect dominates such that investors allocate more attention to asset 1 while its benchmarking level increases,* $\frac{d\omega}{d\overline{\gamma}_1} < 0$.

Proof. See Appendix A.12. □

When the cross-learning effect dominates, investors increase their attention to the asset with the higher ASSR if the benchmarking level of this asset increases. In contrast, benchmarking always reduces investors' attention to the asset in the separative learning equilibrium.

The exact dependence of $\mathrm{I}(\tilde{V}_j;\tilde{P}_j)$ on ω involves tedious algebra. Numerically, we find that when ω increases, $\mathrm{I}(\tilde{V}_1;\tilde{P}_1)$ decreases and $\mathrm{I}(\tilde{V}_2;\tilde{P}_2)$ increases. This monotonic dependence is intuitive. Since each investor i chooses to observe a signal $\tilde{Y}^i = \tilde{V}_1 + \omega\tilde{V}_2 + \tilde{\varepsilon}^i$, we can derive that

$$\mathrm{Var}(\tilde{V}_1 \mid \tilde{Y}^i) = \tau_{v,1}^{-1} - \frac{K-1}{K}\left(\tau_{v,1}^{-1} + \omega^2\tau_{v,2}^{-1}\right)^{-1}, \qquad \mathrm{Var}(\tilde{V}_2 \mid \tilde{Y}^i) = \tau_{v,2}^{-1} - \frac{K-1}{K}\left(\omega^{-2}\tau_{v,1}^{-1} + \tau_{v,2}^{-1}\right)^{-1}.$$

When more attention is allocated to asset j, the signal $\tilde{Y}^i$ is more precise about $\tilde{V}_j$ and investors can learn more about $\tilde{V}_j$ from the equilibrium price $\tilde{P}_j$ that aggregates all private information.

Proposition 6 then predicts that $\mathrm{I}(\tilde{V}_j;\tilde{P}_j)_{\mathrm{int}}$ can increase with the same asset's benchmarking level $\overline{\gamma}_j$, either when $\Delta > \xi$ if $j=2$ or when $\Delta < -\xi$ if $j=1$. An example is shown in Figure 1 (left) where the solid line represents $\mathrm{I}(\tilde{V}_1;\tilde{P}_1)_{\mathrm{int}}$ which increases in $\overline{\gamma}_1$. As asset 1 has a much higher ASSR than asset 2 (Corollary 5), investors keep putting more attention to asset 1 even when its benchmarking level increases. While the impact is positive on $\mathrm{I}(\tilde{V}_1;\tilde{P}_1)$, it is negative on the other asset's price informativeness. Figure 1 (right) shows that $\mathrm{I}(\tilde{V}_2;\tilde{P}_2)_{\mathrm{int}}$ decreases in $\overline{\gamma}_1$. Obviously, the results are opposite to those under separative learning (dotted lines). Thus, the conjecture in Breugem and Buss (2018) that benchmarking would still harm price informativeness in a multi-asset market crucially depends on their implicit assumption of separative learning. We provide clear economic conditions under which this conjecture holds or fails.[6]

[6]Price informativeness in Breugem and Buss (2018) is measured by the variance reduction or R-squared, $\mathrm{R}_j^2 := \frac{\mathrm{Var}[\tilde{V}_j]-\mathrm{Var}[\tilde{V}_j|\tilde{P}_j]}{\mathrm{Var}[\tilde{V}_j]}$. This is just a monotonic function of our mutual information measure: $\mathrm{R}_j^2 = 1-\exp(-2\mathrm{I}(\tilde{V}_j;\tilde{P}_j))$.

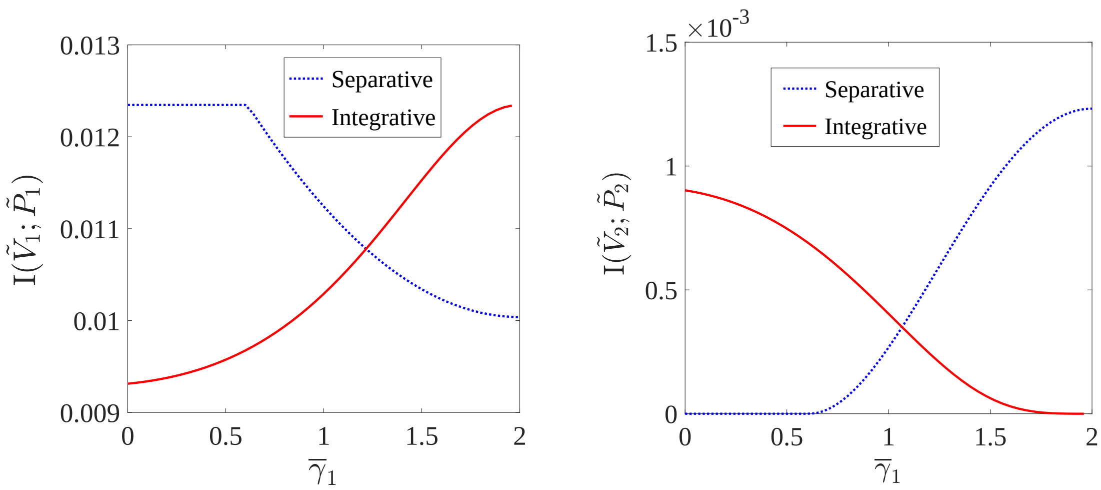

Figure 1. Price Informativeness $\mathrm{I}(\tilde{V}_1;\tilde{P}_1)$ and $\mathrm{I}(\tilde{V}_2;\tilde{P}_2)$ versus asset 1's benchmarking level, $\overline{\gamma}_1$. Parameters: $\lambda = 1$, $K = 1.5$, $\tau_{v,1} = \tau_{v,2} = \tau_{z,2} = 1$, $\tau_{z,1} = 0.1$, $\overline{z}_1 = \overline{z}_2 = 2$, $\overline{\gamma}_2 = 0$.

4.2 Market Informational Efficiency

The measure $\mathrm{I}(\tilde{V}_j;\tilde{P}_j)_{\text{int}}$ excludes the other asset price which contains private information about asset j. We can use a more inclusive measure, $\mathrm{I}(\tilde{V}_j;\tilde{P})_{\text{int}}$, where $\tilde{P} = (\tilde{P}_1, \tilde{P}_2)'$ is the price vector. The formula of $\mathrm{I}(\tilde{V}_j;\tilde{P})_{\text{int}}$ is given by (A-77) in Appendix A.13. In general, both $\mathrm{I}(\tilde{V}_j;\tilde{P}_j)_{\text{int}}$ and $\mathrm{I}(\tilde{V}_j;\tilde{P})_{\text{int}}$ are non-monotonic functions of $\overline{\gamma}_j$. There is an obvious information inequality, $\mathrm{I}(\tilde{V}_j;\tilde{P})_{\text{int}} \geq \mathrm{I}(\tilde{V}_j;\tilde{P}_j)_{\text{int}}$. This can be seen from Figure 2 where the gap, $\mathrm{I}(\tilde{V}_j;\tilde{P})_{\text{int}} - \mathrm{I}(\tilde{V}_j;\tilde{P}_j)_{\text{int}}$, reflects the information gain by including the other asset price.

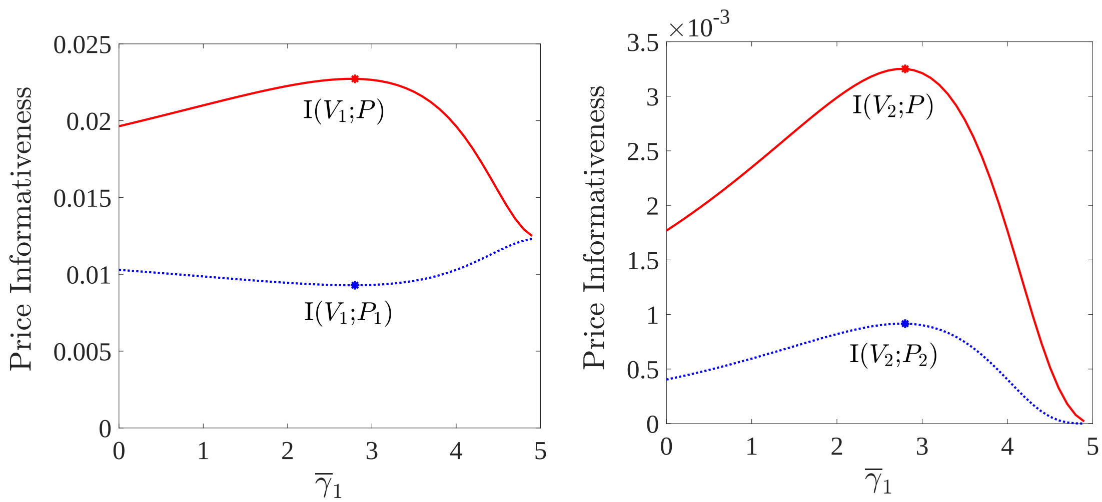

Figure 2. Price informativeness $\mathrm{I}(\tilde{V}_j;\tilde{P}_j)_{\text{int}}$ and $\mathrm{I}(\tilde{V}_j;\tilde{P})_{\text{int}}$ versus asset 1's benchmarking level $\overline{\gamma}_1$. Parameters: $\lambda = 1$, $K = 1.5$, $\tau_{v,1} = \tau_{v,2} = \tau_{z,2} = 1$, $\tau_{z,1} = 0.1$, $\overline{z}_1 = \overline{z}_2 = 5$, and $\overline{\gamma}_2 = 3$.

While both $\mathrm{I}(\tilde{V}_j;\tilde{P}_j)_{\text{int}}$ and $\mathrm{I}(\tilde{V}_j;\tilde{P})_{\text{int}}$ can measure the asset-specific price efficiency, they may not reflect the overall information quality in multi-asset economy. The informational measure of market efficiency should reflect how much uncertainty one can reduce about the random payoffs of all assets after observing the market prices of all assets. The dependence between two random vectors can be evaluated by their mutual information. We define the *market informational efficiency* by the mutual information of the payoff vector and the price vector:

$$\mathrm{I}(\tilde{V};\tilde{P}) := \mathrm{H}(\tilde{V}) - \mathrm{H}(\tilde{V}|\tilde{P}) = \frac{1}{2}\ln\left(\frac{|\mathrm{Var}(\tilde{V})|}{|\mathrm{Var}(\tilde{V}|\tilde{P})|}\right) = \frac{1}{2}\ln\left(\frac{|\Sigma_v|}{|\Sigma_v - \Theta|}\right), \tag{47}$$

where $\Theta := \mathrm{Var}(\tilde{V}) - \mathrm{Var}(\tilde{V}|\tilde{P})$ is the reduction in variance of $\tilde{V}$ conditional on $\tilde{P}$. This can be computed using the standard Kalman filtering result; see equation (A-75). The determinant of the (conditional) covariance matrix is often called the *generalized variance* of multivariate random variables. Regardless of the learning technology, the value of $\mathrm{I}(\tilde{V};\tilde{P})$ measures the amount of information that we can obtain about one random vector by observing the other random vector. Recall that the information processing constraint (8) for each investor is $\mathrm{I}(\tilde{V};\tilde{Y}^i) \le \frac{1}{2}\ln K$.

Theorem 3. *In the integrative learning equilibrium of Theorem 2, the informational efficiency is*

$$\mathrm{I}(\tilde{V};\tilde{P})_{\text{int}} = \frac{1}{2}\ln\left(1 + \left(\frac{K-1}{\lambda}\right)^2 \frac{\tau_{z,1} + \omega^2\tau_{z,2}}{\tau_{v,1}^{-1} + \omega^2\tau_{v,2}^{-1}}\right) = \mathrm{I}(\tilde{V}_1 + \omega\tilde{V}_2;\tilde{P})_{\text{int}}. \tag{48}$$

This is strictly greater than the sum of price informativeness of individual assets:

$$\mathrm{I}(\tilde{V};\tilde{P})_{\text{int}} > \mathrm{I}(\tilde{V}_1;\tilde{P})_{\text{int}} + \mathrm{I}(\tilde{V}_2;\tilde{P})_{\text{int}} > \mathrm{I}(\tilde{V}_1;\tilde{P}_1)_{\text{int}} + \mathrm{I}(\tilde{V}_2;\tilde{P}_2)_{\text{int}}. \tag{49}$$

In the separative learning equilibrium of Theorem 1, the market informational efficiency is

$$\mathrm{I}(\tilde{V};\tilde{P})_{\text{sep}} = \frac{1}{2}\ln\left(1 + \Gamma_1^2\left(\frac{K-1}{\lambda}\right)^2 \tau_{v,1}\tau_{z,1}\right) + \frac{1}{2}\ln\left(1 + \Gamma_2^2\left(\frac{K-1}{\lambda}\right)^2 \tau_{v,2}\tau_{z,2}\right). \tag{50}$$

This is exactly equal to the sum of price informativeness of individual assets:

$$\mathrm{I}(\tilde{V};\tilde{P})_{\text{sep}} = \mathrm{I}(\tilde{V}_1;\tilde{P})_{\text{sep}} + \mathrm{I}(\tilde{V}_2;\tilde{P})_{\text{sep}} = \mathrm{I}(\tilde{V}_1;\tilde{P}_1)_{\text{sep}} + \mathrm{I}(\tilde{V}_2;\tilde{P}_2)_{\text{sep}}. \tag{51}$$

Proof. See Appendix A.14. □

Investors' choice of integrative learning is aligned with their objectives of portfolio management. Their primary concern is about the performance of their managed portfolios rather than a specific asset. Equation (48) shows that the market informational efficiency takes a simple form under integrative learning. If we define $\tau_{v,\omega}^{-1} := \tau_{v,1}^{-1} + \omega^2 \tau_{v,2}^{-1}$ and $\tau_{z,\omega} := \tau_{z,1} + \omega^2 \tau_{z,2}$, then (48) can be rewritten as $\mathrm{I}(\tilde{V};\tilde{P})_{\text{int}} = \frac{1}{2}\ln\left(1+\left(\frac{K-1}{\lambda}\right)^2 \tau_{v,\omega}\tau_{z,\omega}\right)$. In a market with only one risky asset, the price efficiency is $\mathrm{I}(\tilde{V}_0;\tilde{P}_0) = \frac{1}{2}\ln\left(1+\left(\frac{K-1}{\lambda}\right)^2 \tau_v \tau_z\right)$. Thus, for any value of ω, the two-asset economy looks similar to a one-asset economy with the payoff and supply uncertainties characterized by $\tau_{v,\omega}^{-1}$ and $\tau_{z,\omega}^{-1}$, respectively. The key difference is that the market efficiency, $\mathrm{I}(\tilde{V};\tilde{P})_{\text{int}}$, in a two-asset economy depends on investors' attention allocation, ω, which is affected by their aggregate benchmarking concerns. This is not the case in the one-asset economy where there is no attention allocation and price efficiency is unaffected by the asset's benchmarking level.

The second equality in (48), $\mathrm{I}(\tilde{V};\tilde{P})_{\text{int}} = \mathrm{I}(\tilde{V}_1+\omega\tilde{V}_2;\tilde{P})_{\text{int}}$, shows that the vector of asset payoffs is informationally equivalent to a hypothetical portfolio that pays $\tilde{V}_1+\omega\tilde{V}_2$. The weight $(1,\omega)$ exactly matches investors' equilibrium attention allocation. Our measure of market efficiency thus reflects the overall quality of information processing in this economy, from the information choice by individual investors to the price formation of all assets. Though the hypothetical portfolio is consistent with investors' attention allocation ω, it may not match the average speculative portfolio which holds $(\overline{z}_j - \overline{\gamma}_j)$ shares of each asset $j = 1,2$. These two portfolios coincide only when $\Delta = 0$ with $\omega = \frac{\overline{z}_2-\overline{\gamma}_2}{\overline{z}_1-\overline{\gamma}_1}$. In this special case, the two assets are equally uncertain and the market efficiency becomes invariant with investors' benchmarking concerns. Only in this case the two-asset economy (under integrative learning) is informationally equivalent to the one-asset economy and investors' benchmarking concerns have no impact on market efficiency.

With integrative learning, *the whole is greater than the sum of the parts*, according to (49). The first inequality, $\mathrm{I}(\tilde{V};\tilde{P})_{\text{int}} > \mathrm{I}(\tilde{V}_1;\tilde{P})_{\text{int}} + \mathrm{I}(\tilde{V}_2;\tilde{P})_{\text{int}}$, is due to the fact that the sum has missed the information in off-diagonal terms of the variance-reduction matrix, $\Theta := \mathrm{Var}(\tilde{V}) - \mathrm{Var}(\tilde{V}|\tilde{P})$. The aggregate of price informativeness of two assets is insufficient to reflect the market informational efficiency. The magnitude of this information loss scales with the effective asset supplies and hence declines in the benchmarking level of each asset.

With separative learning, *the whole is equal to the sum of the parts*. The equality (51) holds because prices and payoffs are independent across assets and there is no information loss when one aggregates the price informativeness across assets to measure the market informational efficiency. Unless it is in a corner equilibrium, the two-asset economy under separative learning is not informationally equivalent to the one-asset economy even when $\Delta = 0$.

As separative learning is not investors' best response, one may believe that integrative learning would dominate separative learning in terms of improving market informational efficiency. This thinking of totality and optimality seems plausible but remains unverified. Based on the two equilibrium solutions, we are able to compare their overall informational efficiency for a full range of benchmarking levels and economic regimes. Our analysis in Appendix A.15 shows that *integrative learning does not always dominate separative learning*, as we explain below.

Corollary 6. *Under integrative learning, the market information efficiency is bounded as follows*

$$\frac{1}{2}\ln\left(1+\left(\frac{K-1}{\lambda}\right)^2\min\left\{\tau_{v,1}\tau_{z,1},\tau_{v,2}\tau_{z,2}\right\}\right)\leq \mathrm{I}(\tilde{V};\tilde{P})_{\mathrm{int}}\leq\frac{1}{2}\ln\left(1+\left(\frac{K-1}{\lambda}\right)^2\max\left\{\tau_{v,1}\tau_{z,1},\tau_{v,2}\tau_{z,2}\right\}\right), \tag{52}$$

where the equality holds only when $\overline{\gamma}_j=\overline{z}_j$ so that $\omega\to\infty$ or $\omega\to 0$. In such limits, the two-asset economy becomes informationally equivalent to the one-asset economy. The lower (resp. upper) bound in (52) is determined by the riskiness of the more (resp. less) uncertain asset.

Proof. This follows from (38) in Theorem 2 and (48) in Theorem 3. □

Under separative learning, $\mathrm{I}(\tilde{V};\tilde{P})_{\mathrm{sep}}$ does not obey the inequality (52) since the lower limit of $\mathrm{I}(\tilde{V};\tilde{P})_{\mathrm{sep}}$ can be lower than that in (52). $\mathrm{I}(\tilde{V};\tilde{P})_{\mathrm{sep}}=\frac{1}{2}\ln\left(1+\left(\frac{K-1}{\lambda}\right)^2\max\left\{\tau_{v,1}\tau_{z,1},\tau_{v,2}\tau_{z,2}\right\}\right)$ when the effective supply $(\overline{z}_j-\overline{\gamma}_j)$ of the more uncertain asset j is small enough to support the corner solution (Proposition 2). Thus, the separative learning equilibrium may produce a higher level of market informational efficiency than the integrative learning equilibrium.

Next, we discuss the impact of benchmarking on market efficiency:

Proposition 7. *Under integrative learning, $\mathrm{I}(\tilde{V};\tilde{P})_{\mathrm{int}}$ is an increasing function of ω when $\Delta<0$, and a decreasing function of ω when $\Delta>0$. Moreover, $\mathrm{I}(\tilde{V};\tilde{P})_{\mathrm{int}}$ is invariant with ω when $\Delta=0$.*

The impact of benchmarking on market informational efficiency is characterized by

$$\frac{d\mathrm{I}(\tilde{V};\tilde{P})_{\mathrm{int}}}{d\overline{\gamma}_j}=\frac{d\mathrm{I}(\tilde{V};\tilde{P})_{\mathrm{int}}}{d\omega}\frac{d\omega}{d\overline{\gamma}_j}=\frac{-\omega\tau_{z,1}\tau_{z,2}\Delta}{\left(\tau_{v,1}^{-1}+\omega^2\tau_{v,2}^{-1}\right)\left(\tau_{z,1}+\omega^2\tau_{z,2}\right)+\left(\frac{\lambda}{K-1}\right)^2\left(\tau_{v,1}^{-1}+\omega^2\tau_{v,2}^{-1}\right)^2}\cdot\frac{d\omega}{d\overline{\gamma}_j}, \tag{53}$$

where the analytical expression of $\frac{d\omega}{d\overline{\gamma}_j}$ is given by equation (43) in Proposition 6.

Depending on the sign and magnitude of $\Delta:=\tau_{v,2}^{-1}\tau_{z,2}^{-1}-\tau_{v,1}^{-1}\tau_{z,1}^{-1}$, there are three scenarios:

(1) If asset 1 is moderately more volatile such that $-\xi\leq\Delta<0$, then $\frac{d\mathrm{I}(\tilde{V};\tilde{P})_{\mathrm{int}}}{d\overline{\gamma}_1}\geq 0$, $\frac{d\mathrm{I}(\tilde{V};\tilde{P})_{\mathrm{int}}}{d\overline{\gamma}_2}<0$;

(2) If asset 2 is moderately more volatile such that $0<\Delta\leq\xi$, then $\frac{d\mathrm{I}(\tilde{V};\tilde{P})_{\mathrm{int}}}{d\overline{\gamma}_1}<0$, $\frac{d\mathrm{I}(\tilde{V};\tilde{P})_{\mathrm{int}}}{d\overline{\gamma}_2}\geq 0$;

(3) If one asset is significantly more volatile such that $|\Delta|>\xi$, then $\frac{d\mathrm{I}(\tilde{V};\tilde{P})_{\mathrm{int}}}{d\overline{\gamma}_1}<0$, $\frac{d\mathrm{I}(\tilde{V};\tilde{P})_{\mathrm{int}}}{d\overline{\gamma}_2}<0$.

Proof. It follows directly from equation (48) and Proposition 6. □

By (43) and (53), the sign of $\frac{\mathrm{dI}(\tilde{V};\tilde{P})_{\mathrm{int}}}{\mathrm{d}\overline{\gamma}_1}$ is the same as that of $-\Delta(\Delta+\xi)$ and the sign of $\frac{\mathrm{dI}(\tilde{V};\tilde{P})_{\mathrm{int}}}{\mathrm{d}\overline{\gamma}_2}$ is the same as that of $-\Delta(\Delta-\xi)$. Thus, $\frac{\mathrm{dI}(\tilde{V};\tilde{P})_{\mathrm{int}}}{\mathrm{d}\overline{\gamma}_j}>0$ holds for asset 1 when $-\xi\leq\Delta<0$ and for asset 2 when $0<\Delta\leq\xi$. This sign pattern means that market efficiency can be improved over the range $|\Delta|\leq\xi$ when the more uncertain asset has an increase in its benchmarking level.

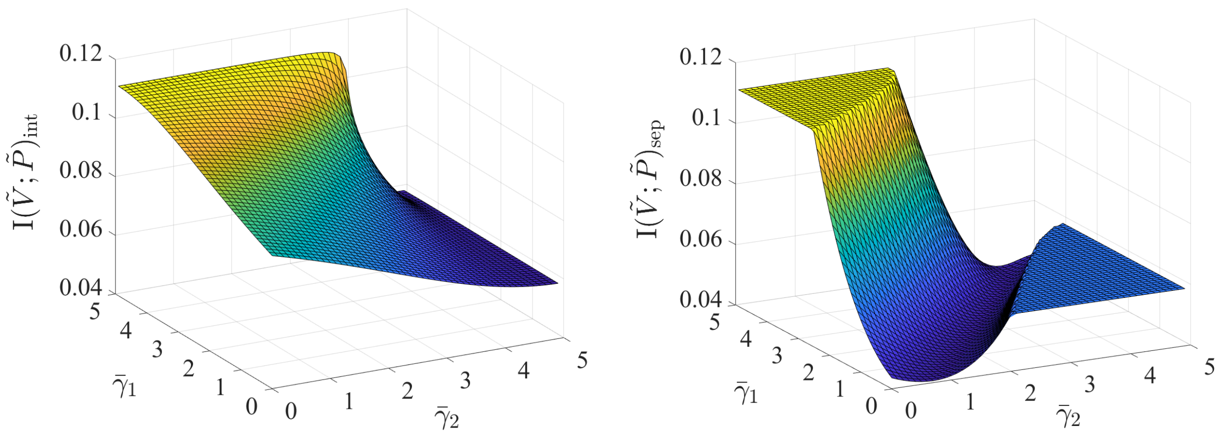

Figure 3. Market informational efficiency $\mathrm{I}(\tilde{V};\tilde{P})_{\mathrm{int}}$ and $\mathrm{I}(\tilde{V};\tilde{P})_{\mathrm{sep}}$ versus the benchmarking levels of two assets. Parameters: $\lambda = 1$, $K = 1.5$, $\tau_{v,1} = \tau_{v,2} = \tau_{z,2} = 1$, $\tau_{z,1} = 0.5$, $\overline{z}_1 = \overline{z}_2 = 5$.

Figure 3 gives a numerical example when asset 1 is more uncertain than asset 2 ($\Delta < 0$). The left panel shows that the market informational efficiency under integrative learning, $\mathrm{I}(\tilde{V};\tilde{P})_{\mathrm{int}}$, increases in $\overline{\gamma}_1$ and decreases in $\overline{\gamma}_2$. The impact of benchmarking on market efficiency is different under separative learning, as shown in the right panel of Figure 3. There are two plateaus with a large sink in the graph of $\mathrm{I}(\tilde{V};\tilde{P})_{\mathrm{sep}}$. For a wide range of benchmarking levels, integrative learning outperforms separative learning in terms of market efficiency, $\mathrm{I}(\tilde{V};\tilde{P})_{\mathrm{int}} > \mathrm{I}(\tilde{V};\tilde{P})_{\mathrm{sep}}$. This relationship is yet reversed near the corner when $\overline{\gamma}_1$ is sufficiently close to $\overline{z}_1$.

It is generally true that the more attention investors put to an asset, the more informative the asset price is. The implication is more involved for market efficiency in a multi-asset economy. For example, in the separative learning equilibrium, more attention to asset 1 implies higher price efficiency for asset 1, less attention to asset 2, and lower price efficiency of asset 2 (see Proposition 3). As a result, the net effect of investors' attention allocation on market efficiency is ambiguous. This explains the nonmonotonic pattern of $\mathrm{I}(\tilde{V};\tilde{P})_{\mathrm{sep}}$ in Figure 3 (right).

When two assets are equally uncertain ($\Delta = 0$), investors' optimal attention allocation under integrative learning is exactly the ratio of effective supplies, $\omega(\Delta = 0) = \frac{\overline{z}_2-\overline{\gamma}_2}{\overline{z}_1-\overline{\gamma}_1}$. The overall market efficiency becomes invariant with investors' benchmarking concerns or attention allocation since $\mathrm{I}(\tilde{V};\tilde{P})_{\mathrm{int},\Delta=0} = \frac{1}{2}\ln\left(1 + \left(\frac{K-1}{\lambda}\right)^2 \tau_{v,j}\tau_{z,j}\right)$ for $j = 1$ or 2. Aggregate benchmarking levels still affect investors' attention through changing the effective supplies of assets. This affects the price informativeness of each asset but does not change the overall market efficiency. For this reason, we may take the ratio $\frac{\overline{z}_2-\overline{\gamma}_2}{\overline{z}_1-\overline{\gamma}_1}$ as a nominal baseline for investors' attention allocation.

Figure 4 (left) confirms the non-monotonic dependence of $\mathrm{I}(\tilde{V};\tilde{P})_{\mathrm{int}}$ on asset benchmarking levels (Proposition 7). The economy moves from Case (1) to Case (3) when $\overline{\gamma}_1$ crosses a critical point γ_1^*. The market informational efficiency is maximized at this critical point which is deter-

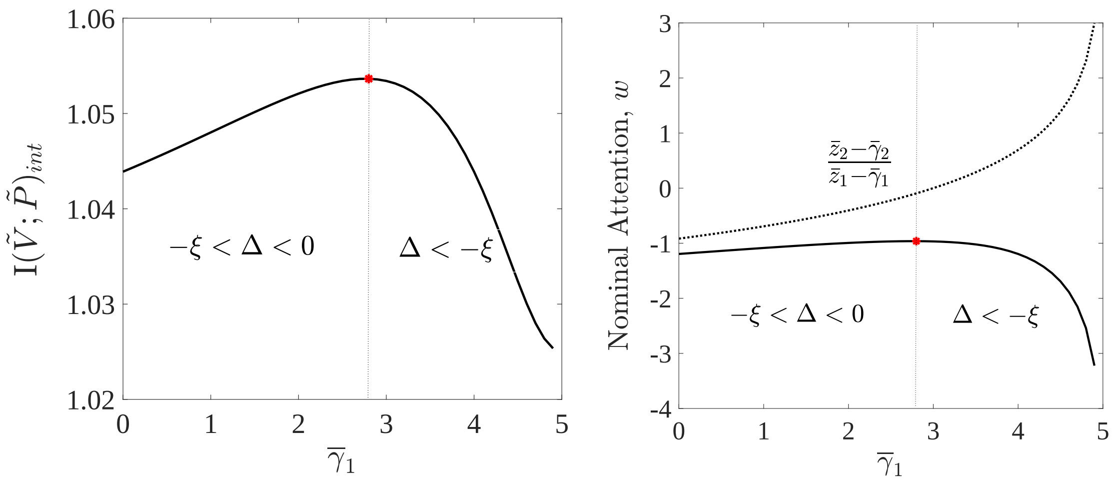

Figure 4. Market informational efficiency $\mathrm{I}(\tilde{V};\tilde{P})_{\mathrm{int}}$ and the logarithm of the optimal attention $\ln(\omega)$ versus $\overline{\gamma}_1$. Parameters: $\lambda = 1$, $K = 1.5$, $\tau_{v,1} = \tau_{v,2} = \tau_{z,2} = 1$, $\tau_{z,1} = 0.1$, $\overline{z}_1 = \overline{z}_2 = 5$, $\overline{\gamma}_2 = 3$.

mined by the first order condition $\Delta + \xi = 0$ and labeled by the "$*$" symbol. Figure 4 (right) plots the logarithm of investors' attention to asset 2. In Case (1) with $\Delta + \xi > 0$, investors shift moderately more attention to asset 2 (solid line) when the benchmarking level of asset 1 increases. Their attention weight on asset 2 moves in the same direction as the baseline ratio $\frac{\overline{z}_2-\overline{\gamma}_2}{\overline{z}_1-\overline{\gamma}_1}$ (dotted line) but to a lesser extent. In Case (3) with $\Delta + \xi < 0$, investors shift dramatically more attention to asset 1. This movement is in the opposite direction to that of the baseline ratio $\frac{\overline{z}_2-\overline{\gamma}_2}{\overline{z}_1-\overline{\gamma}_1}$.

We can think of ω in (37) as the *nominal* attention and define the *real* attention as

$$\omega^* := \frac{\omega(\Delta)}{\omega(\Delta = 0)} = \omega(\Delta) \cdot \frac{\overline{z}_1 - \overline{\gamma}_1}{\overline{z}_2 - \overline{\gamma}_2}. \tag{54}$$

This reflects the part of investors' attention change that has a real impact on market efficiency. The baseline case is that the two assets are equally uncertain ($\tau_{v,1}\tau_{z,1} = \tau_{v,2}\tau_{v,2}$) such that investors' optimal attention choice is $\omega(\Delta = 0) = \frac{\overline{z}_1-\overline{\gamma}_1}{\overline{z}_2-\overline{\gamma}_2}$. When the benchmarking level of an asset changes, this baseline attention ratio can shift in either direction without changing market efficiency because $\mathrm{I}(\tilde{V};\tilde{P})_{\mathrm{int}}$ is invariant with ω when $\Delta = 0$. Now if asset 1 becomes more volatile, the symmetry is broken with $\Delta < 0$. Relative to the baseline ratio $\omega(\Delta = 0)$, asset 1 attracts more attention than asset 2, as reflected by the real attention ω^* which is smaller than one for $\Delta < 0$. When the benchmarking level of an asset changes, investors' real attention also changes, with a real impact on the market informational efficiency. We have discussed before that the impact of benchmarking on asset-specific price informativeness is determined by investors' (nominal) attention choice ω. However, to study the implications for the market informational efficiency, it is helpful to examine how benchmarking affects investors' real attention ω^*.

Proposition 8. *Suppose the effective supply is positive for each asset. The real attention ω^* always shifts to the asset that has the higher uncertainty. Given the sign of $\Delta := \tau_{v,2}^{-1}\tau_{z,2}^{-1} - \tau_{v,1}^{-1}\tau_{z,1}^{-1}$, we have*

$$\omega^* = 1 \quad \text{if} \quad \Delta = 0, \qquad \omega^* > 1 \quad \text{if} \quad \Delta > 0, \qquad \omega^* < 1 \quad \text{if} \quad \Delta < 0. \tag{55}$$

The impact of benchmarking on investors' real attention always takes the same sign of Δ:

$$\frac{d\omega^*}{d\overline{\gamma}_j} = 0 \quad \text{if} \quad \Delta = 0, \qquad \frac{d\omega^*}{d\overline{\gamma}_j} > 0 \quad \text{if} \quad \Delta > 0, \qquad \frac{d\omega^*}{d\overline{\gamma}_j} < 0 \quad \text{if} \quad \Delta < 0. \tag{56}$$

Proof. See Appendix A.16. □

While ω can increase or decrease in $\overline{\gamma}_j$ on either side of Δ (Proposition 6), the dependence of real attention ω^* on the benchmarking level is always monotonic (Proposition 8). Investors' real attention always leans toward the more uncertain asset. This simple tendency can be seen, for example, from the widening gap between the two curves in Figure 4 (right).

Investors' optimal attention choice $(1,\omega)'$ targets a hypothetical portfolio that pays $\tilde{V}_1 + \omega\tilde{V}_2$. In general, this attention-implied portfolio $(1,\omega)'$ differs from the average speculative portfolio $\left(1, \frac{\overline{z}_2 - \overline{\gamma}_2}{\overline{z}_1 - \overline{\gamma}_1}\right)'$ implied by the effective asset supplies. When $|\Delta| < \xi$, these two portfolios move in the same direction in response to benchmarking variations; see Figure 4 (right) for example. As $\overline{\gamma}_1$ increases, investors' nominal attention ω moderately leans toward asset 2, while their real attention ω^* actually leans toward the more uncertain asset 1 ($\Delta < 0$). This real attention shift can effectively reduce the posterior uncertainty about the attention-implied portfolio $(1,\omega)'$. Thus, $\mathrm{I}(\tilde{V};\tilde{P})_{\text{int}} = \mathrm{I}(\tilde{V}_1 + \omega\tilde{V}_2;\tilde{P})_{\text{int}}$ increases in the benchmarking level of the riskier asset.

The positive effect of real attention shift to the more uncertain asset holds for a finite range. When $|\Delta| > \xi$, the attention-implied portfolio $(1,\omega)'$ can deviate far from the supply-implied portfolio $\left(1, \frac{\overline{z}_2 - \overline{\gamma}_2}{\overline{z}_1 - \overline{\gamma}_1}\right)'$. As illustrated in Figure 4, when $\overline{\gamma}_1$ exceeds γ_1^*, the economy is in the regime $-\Delta > \xi$ where investors start shifting their attention disproportionately to asset 1 despite its shrinking supply available for speculation. In this regime, the nominal attention ω and the real attention ω^* move in the same direction until asset 1 exhausts all investors' attention. In that limit ($\omega \to 0$), the market efficiency $\mathrm{I}(\tilde{V};\tilde{P})_{\text{int}} = \mathrm{I}(\tilde{V}_1 + \omega\tilde{V}_2;\tilde{P})_{\text{int}}$ hits its lower bound (Corollary 6). This negative boundary effect extends to dominate the positive real-attention effect if $|\Delta| > \xi$. In those regimes, market efficiency declines in the benchmarking level of either asset.

In summary, the conjecture that benchmarking always reduces market efficiency can fail in a multi-asset economy. This failure can occur under either separative or integrative learning.

4.3 Asset Returns and Comovements

Based on our equilibrium solutions, we now discuss various implication of benchmarking for asset prices and asset comovements under different learning technology.

Expected Prices and Risk Premium. The expected price of an asset tends to increase with the benchmarking level of the asset. A higher benchmarking level of an asset drives more hedging demand and less effective supply, resulting in a lower risk premium of the asset.

Corollary 7. *In the separative learning equilibrium, the expected price of each asset j is*

$$\mathrm{E}\left[\tilde{P}_j\right]_{\mathrm{sep}} = \overline{v}_j - \frac{\lambda \tau_{v,j}^{-1}(\overline{z}_j - \overline{\gamma}_j)}{1 + \Gamma_j(K-1) + \left(\frac{K-1}{\lambda}\Gamma_j\right)^2 \tau_{v,j}\tau_{z,j}}. \tag{57}$$

In the integrative learning equilibrium, when two assets are equally uncertain ($\Delta = 0$), the expected price of each asset j is independent of the other asset's characteristics:

$$\mathrm{E}\left[\tilde{P}_j\right]_{\mathrm{int},\Delta=0} = \overline{v}_j - \frac{\lambda \tau_{v,j}^{-1}(\overline{z}_j - \overline{\gamma}_j)}{K + \left(\frac{K-1}{\lambda}\right)^2 \tau_{v,j}\tau_{z,j}}. \tag{58}$$

Proof. Equation (57) is from equation (10), with $\overline{\tau}_j = \Gamma_j(K-1)\tau_{v,j}$ and $\tau_{p,j} = \left(\Gamma_j \tau_{v,j}\frac{K-1}{\lambda}\right)^2 \tau_{z,j}$. Equation (58) follows from the price equation (31) and the result $\omega(\Delta = 0) = \frac{\overline{z}_2 - \overline{\gamma}_2}{\overline{z}_1 - \overline{\gamma}_1}$. □

For integrative learning with $\Delta = 0$, the risk premium of an asset depends on its own characteristics and is independent of the other asset's characteristics. Their prices are still correlated because the noise terms are subject to the shocks $\tilde{V}_j - \overline{v}_j$ and $\tilde{Z}_j - \overline{z}_j$ of both assets. In general ($\Delta \neq 0$), an increase in the benchmarking level of an asset tends to increase the expected asset price. The reduction of an asset's effective supply has a dominating effect that lowers the risk premium of the asset, consistent with the empirical finding of Pavlova and Sikorskaya (2022).

When $\Delta < 0$, the equilibrium price of asset 1 is higher than its baseline price given by equation (58) for $j = 1$, whereas the equilibrium price of asset 2 is lower than its baseline price:

$$\mathrm{E}\left[\tilde{P}_1\right]_{\mathrm{int},\Delta<0} \geq \overline{v}_1 - \frac{\lambda \tau_{v,1}^{-1}(\overline{z}_1 - \overline{\gamma}_1)}{K + \left(\frac{K-1}{\lambda}\right)^2 \tau_{v,1}\tau_{z,1}}, \qquad \mathrm{E}\left[\tilde{P}_2\right]_{\mathrm{int},\Delta<0} \leq \overline{v}_2 - \frac{\lambda \tau_{v,2}^{-1}(\overline{z}_2 - \overline{\gamma}_2)}{K + \left(\frac{K-1}{\lambda}\right)^2 \tau_{v,2}\tau_{z,2}}. \tag{59}$$

When $\Delta > 0$, the above inequalities are reversed in direction. For any configuration of parameters $\{\tau_{v,1}, \tau_{z,1}, \tau_{v,2}, \tau_{z,2}\}$, we can find a baseline $\{\tau_{v,1}, \tau_{z,1}, \tau_{v,2}, \tau'_{z,2}\}$ that satisfies $\tau_{v,1}\tau_{z,1} = \tau_{v,2}\tau'_{z,2}$. Asset 1 is relatively more (resp. less) uncertain than asset 2 if $\tau_{z,2} > \tau'_{z,2}$ (resp. $\tau_{z,2} < \tau'_{z,2}$). The more (resp. less) uncertain asset is traded at a higher (resp. lower) price than the baseline price (58). Starting from the baseline with $\Delta = 0$, if the supply variance of asset 2 drops from $(\tau'_{z,2})^{-1}$ to

$\tau_{z,2}^{-1}$, investors will increase their attention to the more uncertain asset 1, making its price more informative and its risk premium lower. The price gap relative to the baseline (58) vanishes when asset 1's effective supply shrinks to zero. This is due to the boundary effect in Corollary 6: when $\overline{z}_j \to \overline{\gamma}_j$, the average prices must converge to the average payoffs, $\mathrm{E}[\tilde{P}_j] \to \overline{v}_j$.

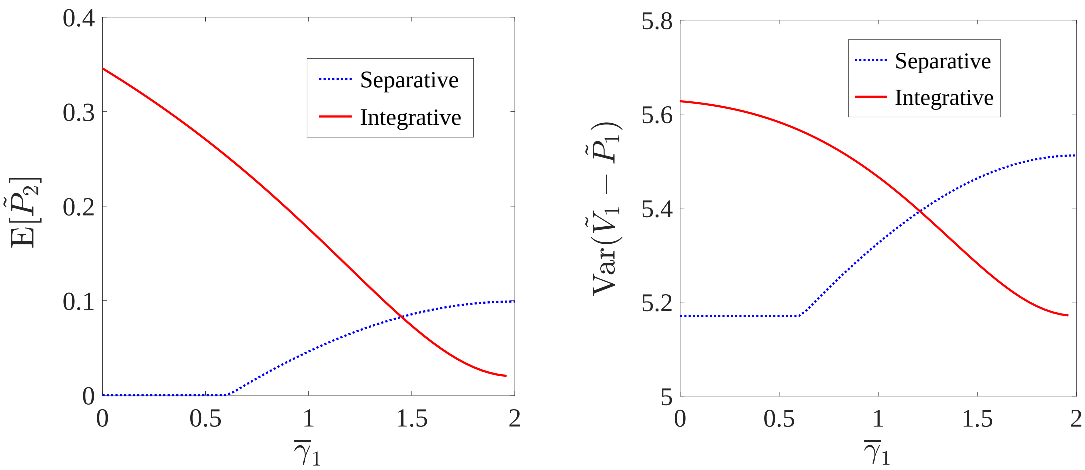

Figure 5. Expected price of asset 2 and return volatility of asset 1 versus asset 1's benchmarking level $\overline{\gamma}_1$. Parameters: $\lambda = 1$, $K = 1.5$, $\tau_{z,1} = 0.1$, $\tau_{v,1} = \tau_{v,2} = \tau_{z,2} = 1$, $\overline{z}_1 = \overline{z}_2 = 2$, and $\overline{\gamma}_2 = 0$.

The benchmarking level of an asset has a mild impact on the other asset price. If asset j is significantly more uncertain than the other asset such that $|\Delta| > \xi$, an increase in asset j's benchmarking level can decrease the average price of the other asset, that is, $\frac{\mathrm{dE}[\tilde{P}_{-j}]}{\mathrm{d}\overline{\gamma}_j} < 0$. In this case, a higher benchmarking level of asset j draw more attention to asset j and less to the other asset, leading to higher risk premium and lower price of the other asset. This pattern is opposite to the result in the separative learning equilibrium; see Figure 5 (left).

Asset Return Volatility and Portfolio Return Dispersion. In the separative learning case, an increase in the benchmarking level of an asset unambiguously increases the return volatility of this asset and decreases that of the other asset (Corollary 3). In the integrative learning case, if two assets have similar prior uncertainties, then increasing the benchmarking level of asset j will reduce investors' attention to asset j. This will increase the return volatility of asset j and decrease the volatility of the other asset, similar to the result under separative learning. However, if one asset is much more uncertain than the other ($|\Delta| > \xi$), then increasing the benchmarking level of the more uncertain asset can direct relatively more attention to this asset, and thus reduce the return volatility of this asset; see Figure 5 (right).

Following Kacperczyk et al. (2016), we can define the dispersion of portfolio excess returns as $D := \int_0^1 \mathrm{E}\left[\left((\theta^i - \tilde{Z})'(\tilde{V} - \tilde{P})\right)^2\right] di$. Conventional wisdom suggests that if institutional investors have more similar benchmarks, the dispersion of their portfolio returns would become smaller. It is unclear how benchmarking affects this measure under different learning technology.

Corollary 8. *Investors' portfolio return dispersion in the separative learning equilibrium is*

$$D_{\text{sep}} = \int_0^1 \sum_{j=1,2} A_{jj}(\gamma_j^i - \overline{\gamma}_j)^2 di + \frac{K-1}{\lambda^2}(\Gamma_1 \text{ASSR}_1 + (1-\Gamma_1)\text{ASSR}_2) \tag{60}$$

The dispersion of their portfolio excess returns in the integrative learning equilibrium is

$$D_{\text{int}} = \int_0^1 (\gamma^i - \overline{\gamma})' A (\gamma^i - \overline{\gamma}) di + \frac{K-1}{\lambda^2} \cdot \frac{\tau_{v,1}^{-1}\left(\text{ASSR}_1 + A_{12}\tau_{v,1}\omega\right) + \omega^2 \tau_{v,2}^{-1}\left(\text{ASSR}_2 + A_{12}\tau_{v,2}\omega^{-1}\right)}{\tau_{v,1}^{-1} + \omega^2 \tau_{v,2}^{-1}}. \tag{61}$$

Proof. See Appendix A.17. □

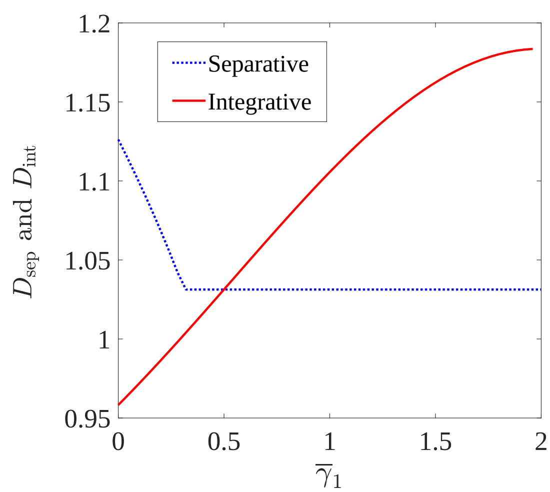

Figure 6. The dispersion of portfolio excess returns under different learning technology (D_{sep} and D_{int}) versus the benchmarking level of the less uncertain asset ($\overline{\gamma}_1$). Parameters: $\lambda = 1$, $K = 1.5$, $\tau_{v,1} = \tau_{z,1} = 3$, $\tau_{v,2} = \tau_{z,2} = 1$, $\overline{z}_1 = \overline{z}_2 = 2$, $\overline{\gamma}_2 = 0$.

In either type of equilibrium, the portfolio return dispersion contains two components: the first term is due to heterogeneous benchmarking compositions across investors, while the second term is affected by changes in the aggregate benchmarking levels since it is the weighted average of the marginal benefits of information about both assets. To focus on the second term, we consider the case that investors have identical benchmarks, that is, $\gamma_j^i = \overline{\gamma}_j$ for all i and j.

In the separative learning equilibrium, benchmarking always reduces the marginal value of information (measured by the ASSR) and thus reduces the dispersion of portfolio returns D_{sep}.

In the integrative learning equilibrium, we find that D_{int} can increase in the benchmarking level of the relatively less uncertain asset. For example, if asset 1 is less risky than asset 2, investors will significantly increase their attention (ω) to asset 2 when the benchmarking level of asset 1 increases. The process continues until the marginal benefits of private information become equalized across assets. Thus, in the new equilibrium the marginal benefit of information can be at a higher value than before. It then follows from (61) that the portfolio return dispersion D_{int} can increase in the benchmarking level of the relatively less uncertain asset (Figure 6).

Common Risk Factor and Asset Comovements. Prices are independent across assets in the separative learning equilibrium, while they are endogenously correlated in the integrative learning equilibrium. To better compare the impacts of benchmarking on asset comovements under different learning technologies, we take one asset in the original model as an aggregate common risk factor and treat the other asset as an idiosyncratic risk factor. These two factors are assumed to have independent payoffs, denoted $\tilde{V}_C$ and $\tilde{V}_I$. Now consider two risky assets, labeled by the subscripts "a" and "b". We assume their payoffs are given by

$$\tilde{V}_a = \tilde{V}_C + \tilde{V}_I, \qquad \tilde{V}_b = \tilde{V}_C. \tag{62}$$

With separative learning, investors only observe private signals about each risk factor separately. The correlation between asset returns is

$$\operatorname{Corr}(\tilde{V}_a - \tilde{P}_a, \tilde{V}_b - \tilde{P}_b)_{\text{sep}} = \left(1 + \operatorname{Var}(\tilde{V}_I - \tilde{P}_I)/\operatorname{Var}(\tilde{V}_C - \tilde{P}_C)\right)^{-1/2}. \tag{63}$$

We find that a higher benchmarking level of the common risk factor ($\tilde{V}_C$) always leads to a lower attention allocated to this factor. This increases the return volatility of the common risk factor $\operatorname{Var}(\tilde{V}_C - \tilde{P}_C)$ which further increases the correlation of asset returns, $\operatorname{Corr}(\tilde{V}_a - \tilde{P}_a, \tilde{V}_b - \tilde{P}_b)_{\text{sep}}$. In this case, benchmarking on the common risk factor always promotes asset comovements.

With integrative learning, each investor chooses to observe a scalar-valued signal about a linear combination of $\tilde{V}_C$ and $\tilde{V}_I$. The correlation between asset returns is found to be

$$\operatorname{Corr}(\tilde{V}_a - \tilde{P}_a, \tilde{V}_b - \tilde{P}_b)_{\text{int}} = \left(\sqrt{\operatorname{Var}(\tilde{V}_C - \tilde{P}_C)/\operatorname{Var}(\tilde{V}_I - \tilde{P}_I)} + \operatorname{Corr}(\tilde{V}_I - \tilde{P}_I, \tilde{V}_C - \tilde{P}_C)\right) \times$$
$$\left(\left(\sqrt{\operatorname{Var}(\tilde{V}_C - \tilde{P}_C)/\operatorname{Var}(\tilde{V}_I - \tilde{P}_I)} + 2\operatorname{Corr}(\tilde{V}_I - \tilde{P}_I, \tilde{V}_C - \tilde{P}_C)\right)\sqrt{\operatorname{Var}(\tilde{V}_C - \tilde{P}_C)/\operatorname{Var}(\tilde{V}_I - \tilde{P}_I)} + 1\right)^{-1/2}.$$

If the common risk factor become much more volatile than the idiosyncratic risk factor, then a higher benchmarking level of the common risk factor makes investors allocate more attention to this factor. This reduces the unconditional variance of the return from investing in the common risk factor, $\operatorname{Var}(\tilde{V}_C - \tilde{P}_C)$. So the variance ratio $\operatorname{Var}(\tilde{V}_C - \tilde{P}_C)/\operatorname{Var}(\tilde{V}_I - \tilde{P}_I)$ tends

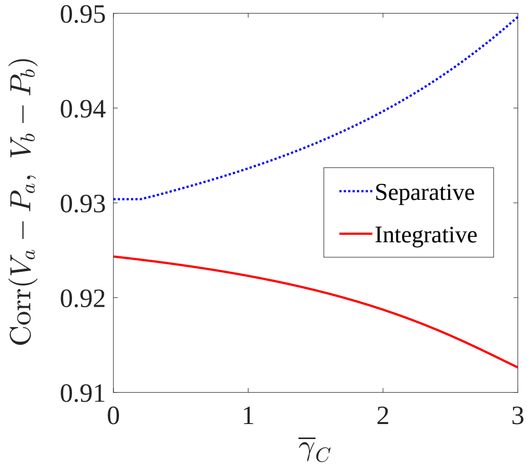

Figure 7. The asset return correlation versus the benchmarking level of the common risk factor $\overline{\gamma}_C$. Parameters: $\lambda = 1$, $K = 1.5$, $\tau_{v,C} = 0.5\ \tau_{v,I} = \tau_{z,C} = \tau_{z,I} = 5$, $\overline{z}_C = \overline{z}_I = 5$, and $\overline{\gamma}_I = 3$.

to decrease in the benchmarking level of the common risk factor $\overline{\gamma}_C$. For a reasonable range of parameter values, we find that the $\mathrm{Corr}(\tilde{V}_a - \tilde{P}_a, \tilde{V}_b - \tilde{P}_b)_{\mathrm{int}}$ tends to increase in the variance ratio $\mathrm{Var}(\tilde{V}_C - \tilde{P}_C)/\mathrm{Var}(\tilde{V}_I - \tilde{P}_I)$. A numerical example is shown in Figure 7. When the common risk factor becomes much more uncertain than the idiosyncratic risk factor, asset comovement can decrease in the benchmarking level of the common risk factor. This observation suggests that more benchmarking on the common risk factor may help dampen asset comovement when this factor becomes highly volatile, for example, during recessions.

5 Conclusion

This paper studies how asset managers' benchmarking concerns affect market efficiency and asset prices in a two-asset economy. We analyze both separative and integrative learning technologies when benchmarked asset managers have limited attention to learn about assets.

We find that in the separative learning case the price informativeness of an asset always decreases in the benchmarking level of this asset, whereas in the integrative learning case the price informativeness may increase in the benchmarking level of the more uncertain asset. Under either learning technology, the overall market informational efficiency can increase in the benchmarking level of an asset. This paper also shows that benchmarking can affect expected asset prices, return volatility, dispersion of portfolio returns, and asset comovements in opposite directions under different learning technology. Thus, we highlight the critical role of learning technology in studying the impact of benchmarking on market efficiency and asset pricing.

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Appendix

A.1 Proof of Proposition 1

Plugging the wealth W^i into investor i's compensation C^i, and substituting into the time-2 objective function $\lambda \mathrm{E}^i[C^i] - \frac{\lambda^2}{2}\mathrm{Var}^i(C^i)$ yields:

$$\lambda[W_0 + (\theta^i - \gamma^i)'(\hat{V}^i - \tilde{P})] - \frac{\lambda^2}{2}(\theta^i - \gamma^i)'\hat{\Sigma}^i(\theta^i - \gamma^i). \tag{A-1}$$

Thus, trader i's optimal stock holding is

$$\theta^i = \gamma^i + \frac{1}{\lambda}\left(\hat{\Sigma}_v^i\right)^{-1}\left(\hat{V}^i - \tilde{P}\right). \tag{A-2}$$

Substituting (A-2) into the market clearing condition, $\int_0^1 \theta^i = \tilde{Z}$, we get

$$\tilde{P} = \left[\int_0^1 (\hat{\Sigma}_v^i)^{-1} di\right]^{-1}\left[\int_0^1 (\hat{\Sigma}_v^i)^{-1}\hat{V}^i di - \lambda(\tilde{Z} - \overline{\gamma})\right], \tag{A-3}$$

where $\overline{\gamma} := (\overline{\gamma}_1, \overline{\gamma}_2)'$ and $\overline{\gamma}_j := \int_0^1 \gamma_j^i di$ is the aggregate benchmarking level of asset j. Observing prices is equal to observing $\tilde{s}_p = \tilde{V} + \tilde{\varepsilon}_p$, where $\tilde{\varepsilon}_p \sim \mathcal{N}(0, \Sigma_p)$, and $\Sigma_p = \begin{pmatrix} \tau_{p,1}^{-1} & 0 \\ 0 & \tau_{p,2}^{-1} \end{pmatrix}$ is a diagonal variance matrix. Then, investor i's posterior mean and variance about $\tilde{V}_j$ are

$$\hat{V}_j^i = \mathrm{E}[\tilde{V}_j \mid \tilde{Y}^i, \tilde{s}_p] = \frac{\tau_{v,j}\overline{v}_j + \tau_j^i \tilde{Y}_j^i + \tau_{p,j}\tilde{s}_{p,j}}{\tau_{v,j} + \tau_j^i + \tau_{p,j}}, \ \mathrm{Var}^i(\tilde{V}_j) = (\tau_{v,j} + \tau_j^i + \tau_{p,j})^{-1}, \ \mathrm{Cov}^i(\tilde{V}_1, \tilde{V}_2) = 0. \tag{A-4}$$

The two assets' payoffs remain uncorrelated ex-post, and we have diagonal matrix

$$\left(\hat{\Sigma}_V^i\right)^{-1}\hat{V}^i = \begin{pmatrix} \tau_{v,1}\overline{v}_1 + \tau_1^i \tilde{Y}_1^i + \tau_{p,1}\tilde{s}_{p,1} & 0 \\ 0 & \tau_{v,2}\overline{v}_2 + \tau_2^i \tilde{Y}_2^i + \tau_{p,2}\tilde{s}_{p,2} \end{pmatrix}. \tag{A-5}$$

We can integrate over all investors ($\int_0^1 \tilde{\varepsilon}^i di = 0$) and use the notation $\overline{\tau}_j = \int_0^1 \tau_j^i di$ to obtain

$$\int_0^1 \left(\hat{\Sigma}_V^i\right)^{-1} di = \begin{pmatrix} \tau_{v,1} + \overline{\tau}_1 + \tau_{p,1} & 0 \\ 0 & \tau_{v,2} + \overline{\tau}_2 + \tau_{p,2} \end{pmatrix}, \tag{A-6}$$

$$\int_0^1 \left(\hat{\Sigma}_V^i\right)^{-1}\hat{V}^i di = \begin{pmatrix} \tau_{v,1}\overline{v}_1 + \overline{\tau}_1\tilde{V}_1 + \tau_{p,1}\tilde{s}_{p,1} & 0 \\ 0 & \tau_{v,2}\overline{v}_2 + \overline{\tau}_2\tilde{V}_2 + \tau_{p,2}\tilde{s}_{p,2} \end{pmatrix}. \tag{A-7}$$

The market clearing condition determines the equilibrium price as stated in Proposition 1,

$$\tilde{P}_j = \left(\tau_{v,j} + \tau_{p,j} + \overline{\tau}_j\right)^{-1}\left[\tau_{v,j}\overline{v}_j + (\tau_{p,j} + \overline{\tau}_j)\tilde{s}_{p,j} - \lambda(\overline{z}_j - \overline{\gamma}_j)\right], \tag{A-8}$$

Note that $\tilde{P}_j$ is informationally equivalent to the signal $\tilde{s}_{p,j} = \tilde{V}_j + \tilde{\varepsilon}_{p,j}$ where

$$\tilde{\varepsilon}_{p,j} = -\tfrac{\lambda}{\overline{\tau}_j}(\tilde{Z}_j - \overline{z}_j) \sim \mathcal{N}(0, \tau_{p,j}^{-1}). \tag{A-9}$$

Thus, the precision of $\tilde{s}_{p,j}$ is $\tau_{p,j} = 1/\mathrm{Var}(\tilde{\varepsilon}_{p,j}) = \overline{\tau}_j^2 \tau_{z,j}/\lambda^2$.

A.2 Proof of Proposition 2

By standard results in statistics, if $\tilde{x} \sim \mathcal{N}(\mu, \sigma)$ is normal random vector, then $\tilde{q} = \tilde{x}' M \tilde{x}$ follows the non-central χ^2-distribution and its expected value is $\mathrm{E}(\tilde{q}) = \mathrm{Tr}[M\sigma] + \mu' M \mu$. Taking the expectation of U_2^i yields trader i's utility at $t = 1$ as follows:

$$U_1^i := \mathrm{E}[U_2^i] = \lambda W_0 + \frac{1}{2}\mathrm{Tr}\left[(\hat{\Sigma}_V^i)^{-1}\mathrm{Var}\left(\hat{V}^i - \tilde{P}\right)\right] + \frac{1}{2}\mathrm{E}\left(\hat{V}^i - \tilde{P}\right)'(\hat{\Sigma}_V^i)^{-1}\mathrm{E}\left(\hat{V}^i - \tilde{P}\right). \tag{A-10}$$

It is straightforward to compute the mean and the variance of $\hat{V}_j^i - \tilde{P}_j$ as follows

$$\begin{aligned}
&\mathrm{E}[\hat{V}_j^i - \tilde{P}_j] = (\tau_{v,j} + \tau_{p,j} + \overline{\tau}_j)^{-1}\lambda(\overline{z}_j - \overline{\gamma}_j), \qquad \mathrm{Cov}[\hat{V}_1^i - \tilde{P}_1, \hat{V}_2^i - \tilde{P}_2] = 0,\\
&\mathrm{Var}[\hat{V}_j^i - \tilde{P}_j] = -\left(\tau_{v,j} + \tau_{p,j} + \tau_j^i\right)^{-1} + \left(\tau_{v,j} + \tau_{p,j} + \overline{\tau}_j\right)^{-2}\left(\tau_{v,j} + (\tau_{p,j} + \overline{\tau}_j)^2 \tau_{p,j}^{-1}\right).
\end{aligned}$$

Substituting the above results into equation (A-10) yields

$$\begin{aligned}
U_1^i &= \lambda W_0 - \frac{1}{2} + \frac{1}{2}\sum_{j=1,2}\beta_j\left(\tau_{v,j} + \tau_{p,j} + \tau_j^i\right), \text{ where}\\
\beta_j &= \left(\mathrm{E}\left(\hat{V}_j^i - \tilde{P}_j\right)\right)^2 + \mathrm{Var}\left(\hat{V}_j^i - \tilde{P}_j\right) + \mathrm{Var}^i(\tilde{V}_j) = \mathrm{E}\left[(\tilde{V}_j - \tilde{P}_j)^2\right],\\
&= \left(\tau_{v,j} + \tau_{p,j} + \overline{\tau}_j\right)^{-2}\left[\lambda^2(\overline{z}_j - \overline{\gamma}_j)^2 + \lambda^2\tau_{z,j}^{-1} + \overline{\tau}_j\right] + \left(\tau_{v,j} + \tau_{p,j} + \overline{\tau}_j\right)^{-1}.
\end{aligned}$$

Therefore, investor i's information choice problem (15) is equivalent to solving

$$\max_{\tau_1^i, \tau_2^i} \sum_{j=1,2}\beta_j \tau_j^i, \qquad \text{s.t.} \quad \prod_{j=1,2}(\tau_{v,j} + \tau_j^i) \le K \prod_{j=1,2}\tau_{v,j} \quad \text{and} \quad \tau_j^i \ge 0. \tag{A-11}$$

For a continuum of investors, each of them takes β_j as given. The information choice problem (A-11) is not a concave objective function. Note that $\beta_j \tau_{v,j} = \mathrm{E}\left[(\tilde{V}_j - \tilde{P}_j)^2\right]/\mathrm{Var}(\tilde{V}_j)$. We thus have a corner solution, i.e., the best response of investor-i is given as Proposition 2.

A.3 Proof of Proposition 3

Equation (18) follows from the linear-Gaussian relationship of $\tilde{V}_j$ and $\tilde{P}_j$. The key calculation is, $\tau_{p,j} = \left(\Gamma_j \tau_{v,j} \frac{K-1}{\lambda}\right)^2 \tau_{z,j}$, which follows from (A-8) and (A-9).

From the proof of Proposition 2 and with the notation that $\overline{\tau}_j = \Gamma_j (K-1)\tau_{v,j}$, we have

$$\mathrm{ASSR}_j = \tau_{v,j}\left(\tau_{v,j} + \overline{\tau}_j^2 \tau_{z,j}/\lambda^2 + \overline{\tau}_j\right)^{-1}\left[\frac{\lambda^2(\overline{z}_j - \overline{\gamma}_j)^2 + \lambda^2 \tau_{z,j}^{-1} + \overline{\tau}_j}{\tau_{v,j} + \overline{\tau}_j^2 \tau_{z,j}/\lambda^2 + \overline{\tau}_j} + 1\right]. \tag{A-12}$$

Clearly, ASSR_j depends on the fraction of investors who choose to learn about asset 1 and on the benchmarking level of asset j, i.e., $\mathrm{ASSR}_j = \mathrm{ASSR}_j(\Gamma_1, \overline{\gamma}_j)$. Suppose $\overline{z}_j - \overline{\gamma}_j > 0$. We have

$$\frac{\partial \mathrm{ASSR}_j}{\partial \overline{\gamma}_j} < 0, \quad \frac{\partial \mathrm{ASSR}_1}{\partial \Gamma_1} < 0, \quad \frac{\partial \mathrm{ASSR}_2}{\partial \Gamma_1} > 0. \tag{A-13}$$

Equation (A-13) shows that the value of information about asset 1 decreases with the population of investors learning about asset 1. Since Γ_j is a function of $\overline{\gamma}_1$ and $\overline{\gamma}_2$, we examine how aggregate benchmarking levels affect Γ_j. In the interior equilibrium with $0 < \Gamma_j < 1$, we have

$$\mathrm{ASSR}_1(\Gamma_1, \overline{\gamma}_1) = \mathrm{ASSR}_2(\Gamma_2, \overline{\gamma}_2). \tag{A-14}$$

Take the total derivative of both sides with respect to $\overline{\gamma}_1$,

$$\frac{\mathrm{dASSR}_1}{\mathrm{d}\overline{\gamma}_1} = \frac{\mathrm{dASSR}_1}{\mathrm{d}\Gamma_1}\frac{\mathrm{d}\Gamma_1}{\mathrm{d}\overline{\gamma}_1} + \frac{\mathrm{dASSR}_1}{\mathrm{d}\overline{\gamma}_1} = \frac{\mathrm{dASSR}_2}{\mathrm{d}\Gamma_2}\frac{\mathrm{d}\Gamma_2}{\mathrm{d}\overline{\gamma}_1}. \tag{A-15}$$

Rearrange the equation,

$$\frac{\mathrm{d}\Gamma_1}{\mathrm{d}\overline{\gamma}_1} = -\Big(\underbrace{\frac{\partial \mathrm{ASSR}_1}{\partial \Gamma_1}}_{-} - \underbrace{\frac{\partial \mathrm{ASSR}_2}{\partial \Gamma_1}}_{+}\Big)^{-1} \underbrace{\frac{\partial \mathrm{ASSR}_1}{\partial \overline{\gamma}_1}}_{-} < 0. \tag{A-16}$$

Similarly, we can show that

$$\frac{\mathrm{d}\Gamma_1}{\mathrm{d}\overline{\gamma}_2} = \Big(\underbrace{\frac{\partial \mathrm{ASSR}_1}{\partial \Gamma_1}}_{-} - \underbrace{\frac{\partial \mathrm{ASSR}_2}{\partial \Gamma_1}}_{+}\Big)^{-1} \underbrace{\frac{\partial \mathrm{ASSR}_2}{\partial \overline{\gamma}_2}}_{-} > 0. \tag{A-17}$$

Combined with results in the corner equilibrium where $\Gamma_1 = 0$ or $\Gamma_1 = 1$, we can write $\frac{\mathrm{d}\Gamma_1}{\mathrm{d}\overline{\gamma}_1} \le 0$ and $\frac{\mathrm{d}\Gamma_1}{\mathrm{d}\overline{\gamma}_2} \ge 0$. Since $\tau_{p,1}$ is an increasing function of Γ_1, it follows that $\frac{\mathrm{d}\tau_{p,1}}{\mathrm{d}\overline{\gamma}_1} \le 0$ and $\frac{\mathrm{d}\tau_{p,1}}{\mathrm{d}\overline{\gamma}_2} \ge 0$. Similar results hold for the derivative of Γ_2 and $\tau_{p,2}$.

A.4 Proof of Corollary 1

The adjusted squared Sharpe ratio can be expressed as $\text{ASSR}_j = \text{ASSR}_j(\tau_{v,j}, \tau_{z,j}, \Gamma_j)$, where Γ_i is endogenously determined by $\tau_{v,j}$ and $\tau_{z,j}$. By equation (A-12),

$$\text{ASSR}_j = \frac{1}{1+\Gamma_j^2(K-1)^2\tau_{v,j}\tau_{z,j}/\lambda^2+\Gamma_j(K-1)}\left(1+\frac{\lambda^2(\overline{z}_j-\overline{\gamma}_j)^2/\tau_{v,j}+\lambda^2/(\tau_{v,j}\tau_{z,j})+\Gamma_j(K-1)}{1+\Gamma_j^2(K-1)^2\tau_{v,j}\tau_{z,j}/\lambda^2+\Gamma_j(K-1)}\right). \tag{A-18}$$

One can see that $\partial\text{ASSR}_j/\partial\tau_{v,j} < 0$ and $\partial\text{ASSR}_j/\partial\tau_{z,j} < 0$. In the interior equilibrium, we have

$$\text{ASSR}_1(\tau_{v,1}, \tau_{z,1}, \Gamma_1) = \text{ASSR}_2(\tau_{v,2}, \tau_{z,2}, \Gamma_2). \tag{A-19}$$

Taking derivative of both sides with respect to $\tau_{v,1}$ and $\tau_{z,1}$ yields

$$\frac{\mathrm{d}\Gamma_1}{\mathrm{d}\tau_{v,1}} = -\left(\frac{\partial\text{ASSR}_1}{\partial\Gamma_1}-\frac{\partial\text{ASSR}_1}{\partial\Gamma_1}\right)^{-1}\frac{\partial\text{ASSR}_1}{\partial\tau_{v,1}} < 0, \tag{A-20}$$

$$\frac{\mathrm{d}\Gamma_1}{\mathrm{d}\tau_{z,1}} = -\left(\frac{\partial\text{ASSR}_1}{\partial\Gamma_1}-\frac{\partial\text{ASSR}_1}{\partial\Gamma_1}\right)^{-1}\frac{\partial\text{ASSR}_1}{\partial\tau_{z,1}} < 0. \tag{A-21}$$

Since the "equality" holding in the corner solution, we have $\mathrm{d}\Gamma_1/\mathrm{d}\tau_{v,1} \le 0$ and $\mathrm{d}\Gamma_1/\mathrm{d}\tau_{z,1} \le 0$.

A.5 Proof of Corollary 3

The variance of asset price and the covariance between asset price and payoff are

$$\text{Var}(\tilde{P}_j) = \left(1-\frac{\tau_{v,j}}{\tau_{v,j}+\overline{\tau}_j+\tau_{p,j}}\right)^2\left(\tau_{v,j}^{-1}+\tau_{p,j}^{-1}\right), \tag{A-22}$$

$$\text{Cov}(\tilde{V}_j, \tilde{P}_j) = \left(1-\frac{\tau_{v,j}}{\tau_{v,j}+\overline{\tau}_j+\tau_{p,j}}\right)\tau_{v,j}^{-1} > 0. \tag{A-23}$$

Thus, the correlation between price and payoff of each asset is strictly positive,

$$\text{Corr}(\tilde{V}_j, \tilde{P}_j) = \sqrt{\tau_{v,j}^{-1}}\Big/\sqrt{\tau_{v,j}^{-1}+\tau_{p,j}^{-1}} > 0. \tag{A-24}$$

As $\overline{\gamma}_j$ increases, $\tau_{p,j}$ and $\overline{\tau}_j$ decrease, and $\tau_{p,-j}$ and $\overline{\tau}_{-j}$ increase, according to Proposition 3. Thus, the correlation between $\tilde{P}_j$ and $\tilde{V}_j$ decreases as $\overline{\gamma}_j$ increases. The variance of return is

$$\text{Var}(\tilde{V}_j-\tilde{P}_j) = \frac{1}{\tau_{v,j}+\overline{\tau}_j+\tau_{p,j}}+\frac{\overline{\tau}_j+\lambda^2\tau_{z,j}^{-1}}{(\tau_{v,j}+\overline{\tau}_j+\tau_{p,j})^2}, \tag{A-25}$$

which increases in $\overline{\gamma}_j$ and decreases in $\overline{\gamma}_{-j}$.

A.6 Proof of Proposition 4

Following Admati (1985), we can derive the market-clearing asset prices, conditional on investors' information choices denoted by $\{\omega^i, \tau^i\}$ for all $i \in [0,1]$. Each asset price is linear function of the asset payoff and the noisy supply. The price vector can be expressed as

$$\tilde{P} = C + B(\Omega\tilde{V} - \lambda(\tilde{Z} - \overline{z})), \qquad \Omega := \int \tau^i \begin{pmatrix} 1 & w^i \\ w^i & (w^i)^2 \end{pmatrix} di \tag{A-26}$$

where the constant vector C and the non-diagonal matrix B are given as

$$C := \left(\Sigma_v^{-1} + \frac{1}{\lambda^2}\Omega\Sigma_z^{-1}\Omega + \Omega\right)^{-1}\left(\Sigma_v^{-1}\overline{v} - \lambda(\overline{z} - \overline{\gamma})\right), \tag{A-27}$$

$$B := \left(\Sigma_v^{-1} + \frac{1}{\lambda^2}\Omega\Sigma_z^{-1}\Omega + \Omega\right)^{-1}\left(I_2 + \tfrac{1}{\lambda^2}\Omega\Sigma_z^{-1}\right). \tag{A-28}$$

Following the same steps in A.1, we obtain investor i's optimal asset holdings

$$\theta^i = \gamma^i + \frac{1}{\lambda}(\hat{\Sigma}_v^i)^{-1}(\hat{V}^i - \tilde{P}). \tag{A-29}$$

It is based on his posterior belief about the variance-covariance matrix and the mean of $\tilde{V}$:

$$\hat{\Sigma}_v^i = \left(\Sigma_v^{-1} + \frac{1}{\lambda^2}\Omega\Sigma_z^{-1}\Omega + (\Lambda^i)'(\Sigma^i)^{-1}\Lambda^i\right)^{-1}, \quad \text{where} \quad \Lambda^i = (1, \omega^i) \tag{A-30}$$

$$\hat{V}^i = \hat{\Sigma}_v^i\left(\left(I_2 - \tfrac{1}{\lambda^2}\Omega\Sigma_z^{-1}\left(I_2 + \tfrac{1}{\lambda^2}\Omega\Sigma_z^{-1}\right)^{-1}\right)\left(\Sigma_v^{-1}\overline{v} + \tfrac{1}{\lambda}\Omega\Sigma_z^{-1}\overline{z}\right) + (\Lambda^i)'(\Sigma^i)^{-1}\tilde{Y}^i + \tfrac{1}{\lambda^2}\Omega\Sigma_z^{-1}B^{-1}\tilde{P}\right). \tag{A-31}$$

A.7 Proof of Proposition 5

We compute the unconditional mean and variance of $\hat{V}^i - \tilde{P}$ as follows

$$\begin{aligned}
\mathrm{E}\left(\hat{V}^i - \tilde{P}\right) &:= R = (R_1, R_2)' = \left(\Sigma_v^{-1} + \lambda^{-2}\Omega\Sigma_z^{-1}\Omega + \Omega\right)^{-1}\lambda(\overline{z} - \overline{\gamma}), \\
\mathrm{Var}\left(\hat{V}^i - \tilde{P}\right) &= \mathrm{Var}\left(\tilde{V} - \tilde{P}\right) - \mathrm{E}\left(\mathrm{Var}^i(\tilde{V} - \tilde{P})\right) = \mathrm{Var}\left(\tilde{V} - \tilde{P}\right) - \hat{\Sigma}_v^i, \\
\mathrm{Var}\left(\tilde{V} - \tilde{P}\right) &:= Q = \begin{pmatrix} Q_{11} & Q_{12} \\ Q_{12} & Q_{22} \end{pmatrix} = (I_2 - B_1\Omega)\Sigma_v(I_2 - B_1\Omega)' + \lambda^2 B_1\Sigma_z B_1'.
\end{aligned} \tag{A-32}$$

For notation simplicity, we further define

$$\begin{aligned}
&A_{12} := R_1R_2 + Q_{12} = \mathrm{E}[(\tilde{V}_1 - \tilde{P}_1)(\tilde{V}_2 - \tilde{P}_2)], \quad A_{11} := R_1^2 + Q_{11} = \mathrm{E}[(\tilde{V}_1 - \tilde{P}_1)^2], \\
&A_{22} := R_2^2 + Q_{22} = \mathrm{E}[(\tilde{V}_2 - \tilde{P}_2)^2].
\end{aligned} \tag{A-33}$$

Thus, the squared Sharpe ratio of the two assets are $\text{ASSR}_1 = A_{11}\tau_{v,1}$ and $\text{ASSR}_2 = A_{22}\tau_{v,2}$. The investor i's utility at time-1 can be written as

$$U_1^i = \lambda W_0 + \frac{1}{2}\text{Tr}\left[(\hat{\Sigma}_V^i)^{-1}Q - I_2\right] + \frac{1}{2}R'(\hat{\Sigma}_V^i)^{-1}R = \lambda W_0 - 1 + \frac{1}{2}\text{Tr}\left[(\hat{\Sigma}_V^i)^{-1}Q\right] + \frac{1}{2}R'(\hat{\Sigma}_V^i)^{-1}R.$$

Obviously, for optimality, the information capacity constraint is always binding, i.e.,

$$\tau^i = \frac{K-1}{\tau_{v,1}^{-1} + (\omega^i)^2\tau_{v,2}^{-1}}. \tag{A-34}$$

Therefore, we can rewrite the investor i's information choice problem as

$$\begin{aligned} &\max_{\omega^i} \frac{1}{2}\left(\tau_{v,1}^{-1} + (\omega^i)^2\tau_{v,2}^{-1}\right)^{-1}\left[A_{22}(\omega^i)^2 + 2A_{12}\omega^i + A_{11}\right](K-1) + A_0, \\ &\text{where} \quad \text{A}_0 = \lambda W_0 - 1 + \frac{1}{2}\text{Tr}\left((\Sigma_v^{-1} + \lambda^{-2}\Omega\Sigma_z^{-1}\Omega)Q\right) + \frac{1}{2}R'(\Sigma_v^{-1} + \lambda^{-2}\Omega\Sigma_z^{-1}\Omega)R. \end{aligned} \tag{A-35}$$

The first order condition of the above problem is given by

$$\left[-A_{12}(\omega^i)^2 - \left(A_{11} - A_{22}\tau_{v,1}^{-1}\tau_{v,2}\right)\omega^i + A_{12}\tau_{v,1}^{-1}\tau_{v,2}\right]\left(\tau_{v,1}^{-1} + \tau_{v,2}^{-1}(\omega^i)^2\right)^{-2}\tau_{v,2}^{-1} = 0. \tag{A-36}$$

One can easily solve this equation and verify the second-order condition. If $A_{12} = R_1R_2 + Q_{12} \neq 0$, i.e., $\text{E}[(\tilde{V}_1 - \tilde{P}_1)(\tilde{V}_2 - \tilde{P}_2)] \neq 0$, then there are two solutions to (A-36). By checking the second order condition (SOC) at the optimal ω^i,

$$SOC(\omega^i) = \left(-(R_1R_2 + Q_{12})\omega^i - \left((R_1^2 + Q_{11}) - (R_2^2 + Q_{22})\tau_{v,1}^{-1}\tau_{v,2}\right)\right)\frac{\tau_{v,2}^{-1}}{\left(\tau_{v,1}^{-1} + \tau_{v,2}^{-1}(\omega^i)^2\right)^2}, \tag{A-37}$$

we can determine the optimal ω^i as

$$\omega^i = \frac{(R_2^2+Q_{22})\tau_{v,1}^{-1}\tau_{v,2} - (R_1^2+Q_{11}) + \sqrt{\left((R_1^2+Q_{11}) - (R_2^2+Q_{22})\tau_{v,1}^{-1}\tau_{v,2}\right)^2 + 4(R_1R_2+Q_{12})^2\tau_{v,1}^{-1}\tau_{v,2}}}{2(R_1R_2+Q_{12})}, \tag{A-38}$$

which is exactly equation (36) in Proposition 5. The SOC at this solution is indeed negative,

$$SOC(\omega^i) = -\sqrt{\left((R_1^2 + Q_{11}) - (R_2^2 + Q_{22})\tau_{v,1}^{-1}\tau_{v,2}\right)^2 + 4(R_1R_2 + Q_{12})^2\tau_{v,1}^{-1}\tau_{v,2}}\,\frac{\tau_{v,2}^{-1}}{\left(\tau_{v,1}^{-1} + \tau_{v,2}^{-1}(\omega^i)^2\right)^2} < 0.$$

Therefore, when $A_{12} := \text{E}[(\tilde{V}_1 - \tilde{P}_1)(\tilde{V}_2 - \tilde{P}_2)] \neq 0$, the best response for investor i is to observe a signal about the linear combination of both assets, with attention weight ω^i given by (36).

A.8 Proof of Corollary 4

If investors specialized in learning either asset 1 or asset 2, then the two assets' prices would be uncorrelated. Therefore, the two assets' returns would be uncorrelated, ex-ante and ex-post, so $A_{12} = \mathrm{E}[(\tilde{V}_1 - \tilde{P}_1)(\tilde{V}_2 - \tilde{P}_2)] = \mathrm{E}[\tilde{V}_1 - \tilde{P}_1]\mathrm{E}[\tilde{V}_2 - \tilde{P}_2]$. Suppose that the effective supply is positive for each asset, $\overline{z}_j - \overline{\gamma}_j > 0$, then $\mathrm{E}[\tilde{V}_j - \tilde{P}_j] > 0$ and $A_{12} > 0$. By Proposition 5, the best response of each investor i is to directly learn about a linear combination of the two assets, with $\omega^i \in (0, \infty)$ given by equation (36), rather than to learn about one asset at a time. Thus, each investor would have an incentive to deviate from the conjectured case where they specialized in learning about only one asset. In the general specification where Λ^i is not restricted to be diagonal, a separative learning equilibrium where investors specialize in learning about one asset does not exist.

A.9 Proof of Theorem 2

Conjecture a symmetric equilibrium where all investors choose the same attention weight in the private signal, $\omega^i = \omega$. Then they must choose the same signal precision,

$$\tau^i = \tau = \frac{K-1}{\tau_{v,1}^{-1} + \omega^2 \tau_{v,2}^{-1}} \tag{A-39}$$

If such an ω exists, then Ω is a singular matrix and we need to rewrite equation (A-32). One can substitute the symmetric ω into R and Q in the first order condition (A-36), and obtain

$$\frac{\lambda^4(\tau_{v,1}^{-1} + \omega^2\tau_{v,2}^{-1})\left[\omega(\tau_{v,1}^{-1}\tau_{z,1}^{-1} - \tau_{v,2}^{-1}\tau_{z,2}^{-1}) + \left(\omega(\overline{z}_1 - \overline{\gamma}_1) - (\overline{z}_2 - \overline{\gamma}_2)\right)\left(\omega(\overline{z}_2 - \overline{\gamma}_2)\tau_{v,2}^{-1} + (\overline{z}_1 - \overline{\gamma}_1)\tau_{v,1}^{-1}\right)\right]}{\tau_{v,1}\left(\lambda^2 K\left(\tau_{v,1}^{-1} + \omega^2\tau_{v,2}^{-1}\right) + (K-1)^2(\tau_{z,1} + \omega^2\tau_{z,2})\right)} = 0. \tag{A-40}$$

There is only one real solution that satisfies the second order condition (A-37):

$$\omega = \alpha + \sqrt{\alpha^2 + \frac{\tau_{v,2}}{\tau_{v,1}}}, \qquad \text{where} \tag{A-41}$$

$$\alpha = \frac{\tau_{v,2}^{-1}\tau_{z,2}^{-1} + \tau_{v,2}^{-1}(\overline{z}_2 - \overline{\gamma}_2)^2 - \tau_{v,1}^{-1}\tau_{z,1}^{-1} - \tau_{v,1}^{-1}(\overline{z}_1 - \overline{\gamma}_1)^2}{2(\overline{z}_1 - \overline{\gamma}_1)(\overline{z}_2 - \overline{\gamma}_2)\tau_{v,2}^{-1}}. \tag{A-42}$$

Since ω has to be positive by equation (A-41), it implies that $\mathrm{E}[(\tilde{V}_1 - \tilde{P}_1)(\tilde{V}_2 - \tilde{P}_2)] \neq 0$ is necessary but not sufficient for the existence of the conjectured symmetric equilibrium. We also need

$$A_{12}(\omega) := R_1(\omega)R_2(\omega) + Q_{12}(\omega) = \mathrm{E}[(\tilde{V}_1 - \tilde{P}_1)(\tilde{V}_2 - \tilde{P}_2)] > 0, \tag{A-43}$$

to ensure that the solution (A-38) is positive. Therefore, the sufficient and necessary condition for the existence and uniqueness of ω is that $A_{12} = R_1(\omega)R_2(\omega) + Q_{12}(\omega) > 0$. Note that both $R(\omega)$ and $Q(\omega)$ depend on the optimal ω through Ω,

$$\Omega(\omega) = \begin{pmatrix} \Omega_{11} & \Omega_{12} \\ \Omega_{21} & \Omega_{22} \end{pmatrix} = \int \tau \begin{pmatrix} 1 & \omega \\ \omega & \omega^2 \end{pmatrix} di = \frac{K-1}{\tau_{v,1}^{-1} + \omega^2 \tau_{v,2}^{-1}} \begin{pmatrix} 1 & \omega \\ \omega & \omega^2 \end{pmatrix}. \tag{A-44}$$

Let's define the following variables,

$$\begin{aligned} b_1 &:= \tau + \tau_{v,1} + \tfrac{\tau^2}{\lambda^2}(\tau_{z,1} + \omega^2 \tau_{z,2}), \\ b_2 &:= \omega\tau + \omega \tfrac{\tau^2}{\lambda^2}(\tau_{z,1} + \omega^2 \tau_{z,2}), \\ b_3 &:= \tau_{v,2} + \omega^2 \tau + \omega^2 \tfrac{\tau^2}{\lambda^2}(\tau_{z,1} + \omega^2 \tau_{z,2}), \\ \text{Det} &:= (\tau + \tau_{v,1})\tau_{v,2} + \tfrac{\tau^2}{\lambda^2}\left(\tau_{v,2}\tau_{z,1} + \omega^4 \tau_{v,1}\tau_{z,2}\right) + \omega^2 \tau \left(\tau_{v,1} + \tfrac{\tau}{\lambda^2}\left(\tau_{v,1}\tau_{z,1} + \tau_{v,2}\tau_{z,2}\right)\right), \end{aligned} \tag{A-45}$$

where τ is given by equation (A-39). Let $\beta = \tau(\tau_{z,1} + \omega^2 \tau_{z,2})$, we have

$$\begin{aligned} R_1 R_2 = & \frac{\lambda^2}{\text{Det}}(\bar{z}_1 - \bar{\gamma}_1)(\bar{z}_2 - \bar{\gamma}_2) - \frac{\lambda^2 \Omega_{12}}{\text{Det}^2}\left(1 + \frac{\beta}{\lambda^2}\right)\left[\tau_{v,2}(\bar{z}_1 - \bar{\gamma}_1)^2 + \tau_{v,1}(\bar{z}_2 - \bar{\gamma}_2)^2\right] \\ & - \frac{\lambda^2 \Omega_{12}}{\text{Det}^2}\left(1 + \frac{\beta}{\lambda^2}\right)^2 \left[\sqrt{\Omega_{22}}(\bar{z}_1 - \bar{\gamma}_1) - \sqrt{\Omega_{11}}(\bar{z}_2 - \bar{\gamma}_2)\right]^2, \end{aligned}$$

where Ω_{11}, Ω_{12}, and Ω_{22} are defined in equation (A-44). Since $Q_{12} = \text{Cov}(\tilde{P}_1, \tilde{P}_2) - \text{Cov}(\tilde{V}_1, \tilde{P}_2) - \text{Cov}(\tilde{V}_2, \tilde{P}_1)$, a sufficient condition for $A_{12} = R_1 R_2 + Q_{12} > 0$ is that

$$\text{Cov}(\tilde{P}_1, \tilde{P}_2) > 0, \text{ and } R_1 R_2 > \text{Cov}(\tilde{V}_1, \tilde{P}_2) + \text{Cov}(\tilde{V}_2, \tilde{P}_1). \tag{A-46}$$

First, a sufficient condition for $R_1 R_2 > \text{Cov}(\tilde{V}_1, \tilde{P}_2) + \text{Cov}(\tilde{V}_2, \tilde{P}_1)$ is that

$$\frac{1}{\lambda^2} < \frac{(\bar{z}_1 - \bar{\gamma}_1)(\bar{z}_2 - \bar{\gamma}_2)}{\sqrt{\tau_{v,1}\tau_{v,2}}} \sqrt{\frac{2}{(1 + \beta/\lambda^2)\left(1 + (K-1)(1 + \beta/\lambda^2)\right)}} - \frac{1}{2}\left[\frac{(\bar{z}_1 - \bar{\gamma}_1)^2}{\tau_{v,1}} + \frac{(\bar{z}_2 - \bar{\gamma}_2)^2}{\tau_{v,2}}\right],$$

Second, a sufficient condition for positive price covariance is given by $\lambda^{-2} > \tau_{v,1}^{-1}\tau_{z,1}^{-1} + \tau_{v,2}^{-1}\tau_{z,2}^{-1}$. In sum, a sufficient condition for $A_{12}(\omega) > 0$ or the existence of a symmetric equilibrium is $\lambda_l < \lambda < \lambda_h$, where

$$\begin{aligned} \lambda_h &:= \left(\frac{1}{\tau_{v,1}\tau_{z,1}} + \frac{1}{\tau_{v,2}\tau_{z,2}}\right)^{1/2}, \qquad \text{(A-47)} \\ \lambda_l &:= \tfrac{1}{\sqrt{2\beta}}\left(-1 - \tfrac{\beta}{2}\left(\tfrac{(\bar{z}_1 - \bar{\gamma}_1)^2}{\tau_{v,1}} + \tfrac{(\bar{z}_2 - \bar{\gamma}_2)^2}{\tau_{v,2}}\right) + \sqrt{\left(1 - \tfrac{\beta}{2}\left(\tfrac{(\bar{z}_1 - \bar{\gamma}_1)^2}{\tau_{v,1}} + \tfrac{(\bar{z}_2 - \bar{\gamma}_2)^2}{\tau_{v,2}}\right)\right)^2 + 4\beta\sqrt{\tfrac{2}{K}}\tfrac{(\bar{z}_1 - \bar{\gamma}_1)(\bar{z}_2 - \bar{\gamma}_2)}{\sqrt{\tau_{v,1}\tau_{v,2}}}}\right)^{1/2}. \end{aligned}$$

A.10 Proof of Proposition 6

When $\Delta = 0$, it is straightforward to calculate that $\omega = \frac{\overline{z}_2-\overline{\gamma}_2}{\overline{z}_1-\overline{\gamma}_1}$ based on equations (37) and (38). Moreover, from equation (38), one can see that α is an increasing function of $\tau_{z,1}$ and a decreasing function of $\tau_{z,2}$. By equation (37), ω is an increasing function of α and depends on $\tau_{z,j}$ only through α. It follows that $\frac{\mathrm{d}\omega}{\mathrm{d}\tau_{z,1}} > 0$ and $\frac{\mathrm{d}\omega}{\mathrm{d}\tau_{z,2}} < 0$. Given an arbitrary set of parameters $\{\tau_{v,1}, \tau_{v,2}, \tau_{z,1}, \tau_{z,2}\}$, we can always define a "break-even" $\tau^*_{z,1} := \tau_{v,2}\tau_{z,2}/\tau_{z,1}$ and compare it with the actual $\tau_{z,1}$. When $\Delta > 0$, we have $\tau^{-1}_{v,2}\tau^{-1}_{z,2} > \tau^{-1}_{v,1}\tau^{-1}_{z,1}$ which implies $\tau_{z,1} > \tau^*_{z,1}$. Since ω increases in $\tau_{z,1}$, we must have $\omega(\tau_{v,1}, \tau_{v,2}, \tau_{z,1}, \tau_{z,2}) > \omega(\tau_{v,1}, \tau_{v,2}, \tau^*_{z,1}, \tau_{z,2})$, that is, $\omega > \frac{\overline{z}_2-\overline{\gamma}_2}{\overline{z}_1-\overline{\gamma}_1}$. When $\Delta < 0$, we must have $\tau_{z,1} < \tau^*_{z,1}$ and $\omega < \frac{\overline{z}_2-\overline{\gamma}_2}{\overline{z}_1-\overline{\gamma}_1}$. The above results are summarized by (42).

Taking derivative of equation (37) with respect to $\overline{\gamma}_j$ yields

$$\frac{\mathrm{d}\omega}{\mathrm{d}\overline{\gamma}_j} = \left[1 + \alpha\left(\alpha^2 + \frac{\tau_{v,2}}{\tau_{v,1}}\right)^{-\frac{1}{2}}\right]\frac{\mathrm{d}\alpha}{\mathrm{d}\overline{\gamma}_j} = \frac{\omega}{\omega - \alpha}\frac{\mathrm{d}\alpha}{\mathrm{d}\overline{\gamma}_j}. \tag{A-48}$$

Since $\omega \geq \alpha$ and $\omega > 0$, the sign of $\frac{\mathrm{d}\omega}{\mathrm{d}\overline{\gamma}_j}$ should be the same as that of $\frac{\mathrm{d}\alpha}{\mathrm{d}\overline{\gamma}_j}$. Direct calculations yield

$$\begin{aligned}
\frac{\mathrm{d}\alpha}{\mathrm{d}\overline{\gamma}_1} &= \frac{\tau_{v,1}\tau_{z,1}\left(1 + (\overline{z}_2-\overline{\gamma}_2)^2\tau_{z,2}\right) - \tau_{v,2}\tau_{z,2}\left(1 - (\overline{z}_1-\overline{\gamma}_1)^2\tau_{z,1}\right)}{2(\overline{z}_1-\overline{\gamma}_1)^2(\overline{z}_2-\overline{\gamma}_2)\tau_{v,1}\tau_{z,1}\tau_{z,2}} = \frac{\Delta+\xi}{2(\overline{z}_1-\overline{\gamma}_1)^2(\overline{z}_2-\overline{\gamma}_2)\tau^{-1}_{v,2}}, \\
\frac{\mathrm{d}\alpha}{\mathrm{d}\overline{\gamma}_2} &= \frac{\tau_{v,1}\tau_{z,1}\left(1 - (\overline{z}_2-\overline{\gamma}_2)^2\tau_{z,2}\right) - \tau_{v,2}\tau_{z,2}\left(1 + (\overline{z}_1-\overline{\gamma}_1)^2\tau_{z,1}\right)}{2(\overline{z}_1-\overline{\gamma}_1)(\overline{z}_2-\overline{\gamma}_2)^2\tau_{v,1}\tau_{z,1}\tau_{z,2}} = \frac{\Delta-\xi}{2(\overline{z}_1-\overline{\gamma}_1)(\overline{z}_2-\overline{\gamma}_2)^2\tau^{-1}_{v,2}}.
\end{aligned} \tag{A-49}$$

Thus, the sign pattern of $\frac{\mathrm{d}\omega}{\mathrm{d}\overline{\gamma}_j}$ in Proposition 6 follows from equations (A-48) and (A-49).

Case (1): If $\Delta \geq \xi$, then $\frac{\mathrm{d}\omega}{\mathrm{d}\overline{\gamma}_1} > 0$, $\frac{\mathrm{d}\omega}{\mathrm{d}\overline{\gamma}_2} \geq 0$;

Case (2): If $\Delta \leq -\xi$, then $\frac{\mathrm{d}\omega}{\mathrm{d}\overline{\gamma}_1} \leq 0$, $\frac{\mathrm{d}\omega}{\mathrm{d}\overline{\gamma}_2} < 0$;

Case (3): If $|\Delta| < \xi$, then $\frac{\mathrm{d}\omega}{\mathrm{d}\overline{\gamma}_1} > 0$, $\frac{\mathrm{d}\omega}{\mathrm{d}\overline{\gamma}_2} < 0$.

A.11 The Impact of Benchmarking on the Marginal Value of Information

Since each individual investor is atomic, $\frac{\partial\Omega}{\partial\omega^i} = 0$, even though $\overline{\gamma}_j$ affects the symmetric equilibrium ω, it does not affect Ω through individual trader's ω^i. Hence, by equation (A-32), $\frac{\partial Q(\Omega)}{\partial\overline{\gamma}_j} = Q'(\Omega)\frac{\partial\Omega}{\partial\overline{\gamma}_j} = 0$. It can be shown that

$$\left(\Sigma^{-1}_v + \frac{1}{\lambda^2}\Omega\Sigma^{-1}_z\Omega + \Omega\right)^{-1} = \frac{1}{\mathrm{Det}}\begin{pmatrix} b_3 & -b_2 \\ -b_2 & b_1 \end{pmatrix},$$

where b_1, b_2, b_3, and Det are defined in equation (A-45). Then, we can derive that

$$R = \left(\Sigma_v^{-1} + \frac{1}{\lambda^2}\Omega\Sigma_z^{-1}\Omega + \Omega\right)^{-1} \lambda(\overline{z} - \overline{\gamma}) = \frac{\lambda}{\text{Det}} \begin{pmatrix} b_3(\overline{z}_1 - \overline{\gamma}_1) - b_2(\overline{z}_2 - \overline{\gamma}_2) \\ -b_2(\overline{z}_1 - \overline{\gamma}_1) + b_1(\overline{z}_2 - \overline{\gamma}_2) \end{pmatrix}, \tag{A-50}$$

$$\frac{\partial R_1}{\partial \overline{\gamma}_1} = -\frac{\lambda b_3}{\text{Det}} < 0, \quad \frac{\partial R_1}{\partial \overline{\gamma}_2} = \frac{\lambda b_2}{\text{Det}} > 0, \quad \frac{\partial R_2}{\partial \overline{\gamma}_1} = \frac{\lambda b_2}{\text{Det}} > 0, \quad \frac{\partial R_2}{\partial \overline{\gamma}_2} = -\frac{\lambda b_1}{\text{Det}} < 0. \tag{A-51}$$

From equation (A-33), $\text{ASSR}_j = \left(\text{Var}\left(\tilde{V}_j - \tilde{P}_j\right) + R_j^2\right)\tau_{v,j}$, which implies

$$\frac{\partial \text{ASSR}_j}{\partial \overline{\gamma}_j} = 2\tau_{v,j} R_j \frac{\partial R_j}{\partial \overline{\gamma}_j} < 0, \qquad \frac{\partial \text{ASSR}_j}{\partial \overline{\gamma}_{-j}} = 2\tau_{v,j} R_j \frac{\partial R_j}{\partial \overline{\gamma}_{-j}} > 0, \tag{A-52}$$

$$\frac{\partial(\text{ASSR}_1 - \text{ASSR}_2)}{\partial \overline{\gamma}_1} < 0, \qquad \frac{\partial(\text{ASSR}_1 - \text{ASSR}_2)}{\partial \overline{\gamma}_2} > 0. \tag{A-53}$$

By equation (A-33), we have

$$\frac{\mathrm{d}A_{12}}{\mathrm{d}\overline{\gamma}_1} = R_1 \frac{\partial R_2}{\partial \overline{\gamma}_1} + \frac{\partial R_1}{\partial \overline{\gamma}_1} R_2 = \frac{\lambda^2}{\text{Det}^2}\left[2b_2 b_3(\overline{z}_1 - \overline{\gamma}_1) - (b_1 b_3 + b_2^2)(\overline{z}_2 - \overline{\gamma}_2)\right], \tag{A-54}$$

$$\frac{\mathrm{d}A_{12}}{\mathrm{d}\overline{\gamma}_2} = R_1 \frac{\partial R_2}{\partial \overline{\gamma}_2} + \frac{\partial R_1}{\partial \overline{\gamma}_2} R_2 = \frac{\lambda^2}{\text{Det}^2}\left[-(b_1 b_3 + b_2^2)(\overline{z}_1 - \overline{\gamma}_1) + 2b_1 b_2(\overline{z}_2 - \overline{\gamma}_2)\right]. \tag{A-55}$$

Unlike the separative learning case, the benchmarking of one asset has not only a negative impact on its own expected return but also a positive impact on the other asset's expected return. Thus, the net effect of benchmarking on A_{12} depends on which effect dominates. For the extreme cases where $\omega \to 0$ or $\omega \to +\infty$, we can show that $b_2 \to 0$, $b_1 > 0$ and $b_3 > 0$ and thus

$$\lim_{\omega \to 0 \text{ or } \omega \to +\infty} \frac{\mathrm{d}A_{12}}{\mathrm{d}\overline{\gamma}_1} = -\frac{\lambda^2 b_1 b_3}{\text{Det}^2}(\overline{z}_2 - \overline{\gamma}_2) < 0, \qquad \lim_{\omega \to 0 \text{ or } \omega \to +\infty} \frac{\mathrm{d}A_{12}}{\mathrm{d}\overline{\gamma}_2} = -\frac{\lambda^2 b_1 b_3}{\text{Det}^2}(\overline{z}_1 - \overline{\gamma}_1) < 0.$$

Consider another special case where the two assets have equal characteristics: $\tau_{v,1} = \tau_{v,2}$, $\tau_{z,1} = \tau_{z,2}$, and $\overline{z}_1 - \overline{\gamma}_1 = \overline{z}_2 - \overline{\gamma}_2$. In this case, we have $\omega = 1$, $b_1 = b_3$, and then

$$\frac{\mathrm{d}A_{12}}{\mathrm{d}\overline{\gamma}_1} = \frac{\mathrm{d}A_{12}}{\mathrm{d}\overline{\gamma}_2} = -\frac{\lambda^2(\overline{z}_1 - \overline{\gamma}_1)}{\text{Det}^2}(b_1 - b_2)^2 < 0.$$

In several special cases, we can prove that the negative effect of benchmarking on its own expected return dominates the positive effect of benchmarking on the other asset's expected return. In general, the endogenous parameters b_1, b_2, and b_3 depend on $\overline{\gamma}_1$, $\overline{\gamma}_2$, $\overline{z}_1$, and $\overline{z}_2$. We have run extensive numerical experiments. The results suggest that the negative effect of benchmarking on its own expected return dominates, and $\frac{\mathrm{d}A_{12}}{\mathrm{d}\overline{\gamma}_j} < 0$ holds in general.

A.12 Proof of Corollary 5

When $\mathrm{ASSR}_2 > \mathrm{ASSR}_1$, the cross-learning effect (the second term) dominates in (46) if

$$-\frac{\mathrm{ASSR}_2 - \mathrm{ASSR}_1}{A_{12}^2}\frac{\mathrm{d}A_{12}}{\mathrm{d}\overline{\gamma}_2} > -\frac{1}{A_{12}}\frac{\mathrm{d}(\mathrm{ASSR}_2 - \mathrm{ASSR}_1)}{\mathrm{d}\overline{\gamma}_2}, \tag{A-56}$$

which is equivalent to

$$\frac{\mathrm{d}\ln(\mathrm{ASSR}_2 - \mathrm{ASSR}_1)}{\mathrm{d}\overline{\gamma}_2} - \frac{\mathrm{d}\ln(A_{12})}{\mathrm{d}\overline{\gamma}_2} > 0. \tag{A-57}$$

By equation (38), the above inequality is equivalent to

$$\frac{\mathrm{d}\ln(\alpha)}{\mathrm{d}\overline{\gamma}_2} = \frac{1}{\alpha}\frac{\mathrm{d}\alpha}{\mathrm{d}\overline{\gamma}_2} > 0 \tag{A-58}$$

By equation (A-49), this occurs when $\Delta > \xi$. From equations (38), (40), and (41), we can write

$$\alpha = \frac{\Delta - \xi}{2\left(\overline{z}_1 - \overline{\gamma}_1\right)\left(\overline{z}_2 - \overline{\gamma}_2\right)\tau_{v,2}^{-1}} + \frac{\overline{z}_2 - \overline{\gamma}_2}{\overline{z}_1 - \overline{\gamma}_1}. \tag{A-59}$$

Thus, the cross-learning effect dominates when

$$\alpha := \frac{\mathrm{ASSR}_2 - \mathrm{ASSR}_1}{2A_{12}\tau_{v,1}} > \alpha(\Delta = \xi) = \frac{\overline{z}_2 - \overline{\gamma}_2}{\overline{z}_1 - \overline{\gamma}_1}. \tag{A-60}$$

The left-hand side reflects the strength of cross-learning effect, while the right-hand side reflects the impact of benchmarking on the effective supplies. Since $\omega = \alpha + \sqrt{\alpha^2 + \tau_{v,2}/\tau_{v,1}}$ is an increasing function of α. The above condition (A-60) is equivalent to

$$\omega > \omega(\Delta = \xi) = \frac{\overline{z}_2 - \overline{\gamma}_2}{\overline{z}_1 - \overline{\gamma}_1}\left(1 + \sqrt{1 + \frac{\tau_{v,1}^{-1}(\overline{z}_1 - \overline{\gamma}_1)^2}{\tau_{v,2}^{-1}(\overline{z}_2 - \overline{\gamma}_2)^2}}\right) = \frac{\overline{z}_2 - \overline{\gamma}_2 + \sqrt{\tau_{v,2}\xi}}{\overline{z}_1 - \overline{\gamma}_1}. \tag{A-61}$$

When $\mathrm{ASSR}_1 > \mathrm{ASSR}_2$, the cross-learning effect dominates in $\frac{\mathrm{d}\omega}{\mathrm{d}\overline{\gamma}_1}$ if

$$\frac{\mathrm{ASSR}_1 - \mathrm{ASSR}_2}{A_{12}^2}\frac{\mathrm{d}A_{12}}{\mathrm{d}\overline{\gamma}_1} < \frac{1}{A_{12}}\frac{\mathrm{d}(\mathrm{ASSR}_1 - \mathrm{ASSR}_2)}{\mathrm{d}\overline{\gamma}_1}, \tag{A-62}$$

which is equivalent to

$$\frac{\mathrm{d}\ln(\mathrm{ASSR}_1 - \mathrm{ASSR}_2)}{\mathrm{d}\overline{\gamma}_1} - \frac{\mathrm{d}\ln(A_{12})}{\mathrm{d}\overline{\gamma}_1} < 0. \tag{A-63}$$

By equation (38), the above inequality is equivalent to

$$\frac{\mathrm{d}\ln(-\alpha)}{\mathrm{d}\overline{\gamma}_1} = \frac{1}{\alpha}\frac{\mathrm{d}\alpha}{\mathrm{d}\overline{\gamma}_1} < 0 \tag{A-64}$$

By equation (A-49), this occurs when $\Delta < -\xi$. From equations (38), (40), and (41), we can write

$$\alpha = \frac{\Delta + \xi}{2\left(\overline{z}_1 - \overline{\gamma}_1\right)\left(\overline{z}_2 - \overline{\gamma}_2\right)\tau_{v,2}^{-1}} - \frac{\tau_{v,2}}{\tau_{v,1}}\left(\frac{\overline{z}_1 - \overline{\gamma}_1}{\overline{z}_2 - \overline{\gamma}_2}\right). \tag{A-65}$$

Thus, in this case, the cross-learning effect dominates when the following holds

$$|\alpha| = \frac{\mathrm{ASSR}_1 - \mathrm{ASSR}_2}{2A_{12}\tau_{v,1}} > |\alpha(\Delta = -\xi)| = \frac{\tau_{v,2}}{\tau_{v,1}}\left(\frac{\overline{z}_1 - \overline{\gamma}_1}{\overline{z}_2 - \overline{\gamma}_2}\right), \tag{A-66}$$

which is equivalent to

$$\frac{\mathrm{ASSR}_1 - \mathrm{ASSR}_2}{2A_{12}\tau_{v,2}} > \frac{\overline{z}_1 - \overline{\gamma}_1}{\overline{z}_2 - \overline{\gamma}_2}. \tag{A-67}$$

Again, the left-hand side reflects the strength of cross-learning effect, while the right-hand side reflects the impact of benchmarking on the effective supplies. An equivalent condition is

$$\frac{1}{\omega} > \frac{1}{\omega(\Delta = -\xi)} = \frac{\overline{z}_1 - \overline{\gamma}_1}{\overline{z}_2 - \overline{\gamma}_2}\left(1 + \sqrt{1 + \frac{\tau_{v,2}^{-1}(\overline{z}_2 - \overline{\gamma}_2)^2}{\tau_{v,1}^{-1}(\overline{z}_1 - \overline{\gamma}_1)^2}}\right) = \frac{\overline{z}_1 - \overline{\gamma}_1 + \sqrt{\tau_{v,1}\xi}}{\overline{z}_2 - \overline{\gamma}_2}. \tag{A-68}$$

It is easy to verify the following limits:

$$\lim_{\overline{\gamma}_2 \to \overline{z}_2} \omega(\Delta = +\xi) = \lim_{\overline{\gamma}_1 \to \overline{z}_1} \omega(\Delta = -\xi) = \sqrt{\frac{\tau_{v,2}}{\tau_{v,1}}}. \tag{A-69}$$

A.13 More on Price Informativeness

We discuss two measures for the price informativeness of individual assets. Due to investors' integrative learning, each asset price incorporates private information about both $\tilde{V}_1$ and $\tilde{V}_2$. The noise terms are also correlated. This follows from (31) which can be explicitly written as

$$\tilde{P}_1 = C_1 + \tau(B_{11} + B_{12}\omega)(\tilde{V}_1 + \omega\tilde{V}_2) - \lambda\left(B_{11}(\tilde{Z}_1 - \overline{z}_1) + B_{12}(\tilde{Z}_2 - \overline{z}_2)\right), \tag{A-70}$$

$$\tilde{P}_2 = C_2 + \tau(B_{21} + B_{22}\omega)(\tilde{V}_1 + \omega\tilde{V}_2) - \lambda\left(B_{21}(\tilde{Z}_1 - \overline{z}_1) + B_{22}(\tilde{Z}_2 - \overline{z}_2)\right). \tag{A-71}$$

The four elements of the matrix B have full expressions below:

$$
\begin{aligned}
B_{11} &= \frac{\omega^2}{\tau_{v,2}+\omega^2\tau_{v,1}} + \frac{\tau_{v,2}(\lambda^2+(K-1)\tau_{v,1}\tau_{z,1})}{\tau_{v,1}(K\lambda^2(\tau_{v,2}+\omega^2\tau_{v,1})+(K-1)^2\tau_{v,1}\tau_{v,2}(\tau_{z,1}+\omega^2\tau_{z,2})}, \\
B_{12} &= -\frac{(K-1)\omega((\lambda^2-\tau_{v,2}\tau_{z,2})(\tau_{v,2}+\omega^2\tau_{v,1})+(K-1)\tau_{v,1}\tau_{v,2}(\tau_{z,1}+\omega^2\tau_{z,2}))}{(\tau_{v,2}+\omega^2\tau_{v,1})(K\lambda^2(\tau_{v,2}+\omega^2\tau_{v,1})+(K-1)^2\tau_{v,1}\tau_{v,2}(\tau_{z,1}+\omega^2\tau_{z,2}))}, \\
B_{21} &= -\frac{\omega}{\tau_{v,2}+\omega^2\tau_{v,1}} + \frac{\omega(\lambda^2+(K-1)\tau_{v,1}\tau_{z,1})}{K\lambda^2(\tau_{v,2}+\omega^2\tau_{v,1})+(K-1)^2\tau_{v,1}\tau_{v,2}(\tau_{z,1}+\omega^2\tau_{z,2})}, \\
B_{22} &= \frac{\lambda^2(\tau_{v,2}+\omega^2\tau_{v,1})(\omega^2\tau_{v,1}+K\tau_{v,2})+(K-1)\tau_{v,1}\tau_{v,2}((K-1)\tau_{z,1}\tau_{v,2}+\omega^2\tau_{z,2}(\omega^2\tau_{v,1}+K\tau_{v,2}))}{\tau_{v,2}(\tau_{v,2}+\omega^2\tau_{v,1})(K\lambda^2(\tau_{v,2}+\omega^2\tau_{v,1})+(K-1)^2\tau_{v,1}\tau_{v,2}(\tau_{z,1}+\omega^2\tau_{z,2})}.
\end{aligned}
$$

One can see from (A-70) and (A-71) that both asset prices contain information about the linear combination of asset payoffs, $\tilde{V}_1+\omega\tilde{V}_2$. In the integrative learning equilibrium of Theorem 2, the price informativeness for asset j can be derived from equations (A-70) and (A-71):

$$
\begin{aligned}
\mathrm{I}(\tilde{V}_1;\tilde{P}_1) &= \frac{1}{2}\ln\left(1+\frac{\tau^2\tau_{v,1}^{-1}(B_{11}+\omega B_{12})^2}{\omega^2\tau^2(B_{11}+\omega B_{12})^2\tau_{v,2}^{-1}+\lambda^2\left(B_{11}^2\tau_{z,1}^{-1}+B_{12}^2\tau_{z,2}^{-1}\right)}\right), \qquad \text{(A-72)} \\
\mathrm{I}(\tilde{V}_2;\tilde{P}_2) &= \frac{1}{2}\ln\left(1+\frac{\omega^2\tau^2\tau_{v,2}^{-1}(B_{21}+\omega B_{22})^2}{\tau^2(B_{21}+\omega B_{22})^2\tau_{v,1}^{-1}+\lambda^2\left(B_{21}^2\tau_{z,1}^{-1}+B_{22}^2\tau_{z,2}^{-1}\right)}\right), \qquad \text{(A-73)}
\end{aligned}
$$

where ω and τ are given by (37). This measure $\mathrm{I}(\tilde{V}_j;\tilde{P}_j)$ misses the information about $\tilde{V}_j$ from the other asset price. In the integrative learning caes, we may refer to $\mathrm{I}(\tilde{V}_j;\tilde{P}_j)$ as the *partial* price informativeness and define the *total* price informativeness by the more inclusive measure

$$\mathrm{I}(\tilde{V}_j;\tilde{P}) = \mathrm{I}(\tilde{V}_j;\tilde{P}_1,\tilde{P}_2) = \mathrm{H}(\tilde{V}_j) - \mathrm{H}(\tilde{V}_j|\tilde{P}_1,\tilde{P}_2). \qquad \text{(A-74)}$$

To calculate $\mathrm{I}(\tilde{V}_j;\tilde{P})$, we need the result of variance reduction by standard Kalman filtering:

$$\Theta := \mathrm{Var}(\tilde{V}) - \mathrm{Var}(\tilde{V}|\tilde{P}) = B\Omega\Sigma_v(B\Omega\Sigma_v\Omega B' + \lambda^2 B\Sigma_z B')^{-1}\Sigma_v\Omega B'. \qquad \text{(A-75)}$$

This matrix Θ is symmetric and full-rank. So the condintional variance for asset $j=\{1,2\}$ is

$$\mathrm{Var}(\tilde{V}_1 \mid \tilde{P}_1,\tilde{P}_2) = \tau_{v,1}^{-1} - \Theta_{11}, \qquad \mathrm{Var}(\tilde{V}_1 \mid \tilde{P}_1,\tilde{P}_2) = \tau_{v,2}^{-1} - \Theta_{22}. \qquad \text{(A-76)}$$

After some algebra, we find a concise expression of the total price informativeness:

$$\mathrm{I}(\tilde{V}_j;\tilde{P}) = -\frac{1}{2}\ln\left(1-\frac{\omega^{2(j-1)}\tau_{v,j}^{-1}(\tau_{z,1}+\omega^2\tau_{z,2})}{\frac{\lambda^2}{(K-1)^2}\left(\tau_{v,1}^{-1}+\omega^2\tau_{v,2}^{-1}\right)^2+(\tau_{z,1}+\omega^2\tau_{z,2})\left(\tau_{v,1}^{-1}+\omega^2\tau_{v,2}^{-1}\right)}\right). \qquad \text{(A-77)}$$

We find that both $\mathrm{I}(\tilde{V}_j;\tilde{P}_j)$ and $\mathrm{I}(\tilde{V}_j;\tilde{P})$ are generally non-monotonic functions of the benchmarking levels (Figure 2).

Information theory provides a general relationship: $\mathrm{I}(\tilde{V}_j;\tilde{P}) \geq \mathrm{I}(\tilde{V}_j;\tilde{P}_j)$. The equality holds in the separative learning equilibrium where only asset j's price is informative about its own payoff $\tilde{V}_j$. The inequality holds in the integrative learning equilibrium, because the price vector $\tilde{P}$ contains more information than its element $\tilde{P}_j$. The gap between these two measures, $\mathrm{I}(\tilde{V}_j;\tilde{P}) - \mathrm{I}(\tilde{V}_j;\tilde{P}_j) \geq 0$, reflects the information loss if one only considers asset j's price or the information gain if one considers the other asset price. The significant gap in Figure 2 indicates that *measuring price informativeness solely based on a specific stock price can miss valuable information embedded in the price of some seemingly unrelated stock.*

Thus, in the integrative learning equilibrium, it is most reasonable to use the inclusive measure $\mathrm{I}(\tilde{V}_j;\tilde{P})$ to quantify the overall price informativeness for an asset. The empirical implication is to use multiple stocks to predict the payoff of a single stock. Without knowing investors' learning technology, an econometrician in this economy may view price correlations as spurious and exclude the seemingly unrelated asset from his analysis. In the separative learning equilibrium, the implication is trivial because the two measures become identical, $\mathrm{I}(\tilde{V}_j;\tilde{P}) = \mathrm{I}(\tilde{V}_j;\tilde{P}_j)$, and they always (weakly) decrease in the benchmarking level of the same asset (Proposition 3).

A.14 Proof of Theorem 3

Since both $\tilde{V}$ and $\tilde{P}$ are vectors of Gaussian random variables, the entropy measure of informational efficiency in the integrative learning case is given by

$$\mathrm{I}(\tilde{V};\tilde{P})_{\text{int}} = \frac{1}{2}\ln\left(\frac{|\mathrm{Var}(\tilde{V})|}{|\mathrm{Var}(\tilde{V}|\tilde{P})|}\right) = \frac{1}{2}\ln\left(\frac{|\Sigma_v|}{|\Sigma_v - \Theta|}\right) \tag{A-78}$$

where $\Theta := \mathrm{Var}(\tilde{V}) - \mathrm{Var}(\tilde{V}|\tilde{P})$ is the variance reduction for the payoff vector $\tilde{V}$ conditional on the price vector $\tilde{P}$, as given in equation (A-75). We derive and find a simple expression for the determinant of variance-covariance matrix $\mathrm{Var}(\tilde{V}|\tilde{P})$, i.e.,

$$|\Sigma_v - \Theta| = \left(\tau_{v,1}^{-1} - \Theta_{11}\right)\left(\tau_{v,2}^{-1} - \Theta_{22}\right) - \Theta_{12}\Theta_{21} = \frac{1}{\tau_{v,1}\tau_{v,2}}\left(1 + \left(\frac{K-1}{\lambda}\right)^2 \frac{\tau_{z,1} + \omega^2\tau_{z,2}}{\tau_{v,1}^{-1} + \omega^2\tau_{v,2}^{-1}}\right)^{-1}. \tag{A-79}$$

This leads to equation (48). Using equations (A-76) and (A-79), we can prove the inequality

$$\begin{aligned}
& \mathrm{I}(\tilde{V};\tilde{P})_{\text{int}} - \mathrm{I}(\tilde{V}_1;\tilde{P})_{\text{int}} - \mathrm{I}(\tilde{V}_2;\tilde{P})_{\text{int}} \\
= \; & H(\tilde{V}_1|\tilde{P}_1,\tilde{P}_2)_{\text{int}} + H(\tilde{V}_2|\tilde{P}_1,\tilde{P}_2)_{\text{int}} - H(\tilde{V}|\tilde{P})_{\text{int}} \\
= \; & \frac{1}{2}\ln\left(\mathrm{Var}(\tilde{V}_1|\tilde{P}_1,\tilde{P}_2)\mathrm{Var}(\tilde{V}_2|\tilde{P}_1,\tilde{P}_2)\right) - \frac{1}{2}\ln\left(|\mathrm{Var}(\tilde{V}|\tilde{P})|\right) \\
= \; & \frac{1}{2}\ln\left((\tau_{v,1}^{-1}-\Theta_{11})(\tau_{v,2}^{-1}-\Theta_{22})\right) - \frac{1}{2}\ln\left((\tau_{v,1}^{-1}-\Theta_{11})(\tau_{v,2}^{-1}-\Theta_{22}) - \Theta_{12}\Theta_{21}\right) \\
= \; & -\frac{1}{2}\ln\left(1 - \frac{\Theta_{12}^2}{(\tau_{v,1}^{-1}-\Theta_{11})(\tau_{v,2}^{-1}-\Theta_{22})}\right) > 0, \qquad \text{(A-80)}
\end{aligned}$$

where the last step follows from the fact that Θ is a symmetric non-diagonal matrix such that $\Theta_{12}\Theta_{21} = \Theta_{12}^2 > 0$. We give the explicit expression below which is bounded between 0 and 1:

$$\frac{\Theta_{12}\Theta_{21}}{(\tau_{v,1}^{-1}-\Theta_{11})(\tau_{v,2}^{-1}-\Theta_{22})} = \left(1 + \frac{\lambda^4(\tau_{v,1}^{-1}+\omega^2\tau_{v,2}^{-1})^4 + (K-1)^2\lambda^2(\tau_{v,1}^{-1}+\omega^2\tau_{v,2}^{-1})^3(\tau_{z,1}+\omega^2\tau_{z,2})}{(K-1)^4\omega^2\tau_{v,1}^{-1}\tau_{v,2}^{-1}(\tau_{z,1}+\omega^2\tau_{z,2})^2}\right)^{-1}$$

Since $\mathrm{Var}(\tilde{V}_j|\tilde{P}_1,\tilde{P}_2) < \mathrm{Var}(\tilde{V}_j|\tilde{P}_j)$ for each asset j, it follows that $\mathrm{I}(\tilde{V}_j;\tilde{P})_{\text{int}} > \mathrm{I}(\tilde{V}_j;\tilde{P}_j)_{\text{int}}$ and thus

$$\mathrm{I}(\tilde{V};\tilde{P})_{\text{int}} > \mathrm{I}(\tilde{V}_1;\tilde{P})_{\text{int}} + \mathrm{I}(\tilde{V}_2;\tilde{P})_{\text{int}} > \mathrm{I}(\tilde{V}_1;\tilde{P}_1)_{\text{int}} + \mathrm{I}(\tilde{V}_2;\tilde{P}_2)_{\text{int}} \qquad \text{(A-81)}$$

Next, we prove the following mutual-information identity: $\mathrm{I}(\tilde{V};\tilde{P})_{\text{int}} = \mathrm{I}(\tilde{V}_1+\omega\tilde{V}_2;\tilde{P})_{\text{int}}$. In the integrative learning equilibrium, equation (31) can be written as

$$\tilde{P} = C + B\left(\Omega\tilde{V} - \lambda(\tilde{Z}-\overline{z})\right) = C + B\left(\tau\Lambda(\tilde{V}_1+\omega\tilde{V}_2) - \lambda(\tilde{Z}-\overline{z})\right), \qquad \Lambda = (1,\omega)'. \qquad \text{(A-82)}$$

Thus, the price $\tilde{P} = (\tilde{P}_1,\tilde{P}_2)'$ is informational equivalent to a signal vector $\tilde{s}_p = (\tilde{s}_{p,1},\tilde{s}_{p,2})'$ where

$$\begin{aligned}
\tilde{s}_{p,1} &= \tilde{V}_1 + \omega\tilde{V}_2 - \frac{\lambda}{\tau}(\tilde{Z}_1-\overline{z}_1) = \tilde{V}_\omega - \frac{\lambda}{\tau}(\tilde{Z}_1-\overline{z}_1), \qquad \text{(A-83)} \\
\tilde{s}_{p,2} &= \tilde{V}_1 + \omega\tilde{V}_2 - \frac{\lambda}{\omega\tau}(\tilde{Z}_2-\overline{z}_2) = \tilde{V}_\omega - \frac{\lambda}{\omega\tau}(\tilde{Z}_2-\overline{z}_2). \qquad \text{(A-84)}
\end{aligned}$$

Using the notation $\tilde{V}_\omega := \tilde{V}_1 + \omega\tilde{V}_2 \sim \mathcal{N}\left(\overline{v}_1+\omega\overline{v}_2, \tau_{v,\omega}^{-1}\right)$ and $\tau_{v,\omega}^{-1} := \tau_{v,1}^{-1} + \omega^2\tau_{v,2}^{-1}$, we can derive

$$\begin{aligned}
\mathrm{I}(\tilde{V}_1+\omega\tilde{V}_2;\tilde{P})_{\text{int}} &= \mathrm{I}(\tilde{V}_\omega;\tilde{s}_p)_{\text{int}} = \frac{1}{2}\ln\left(\frac{\mathrm{Var}(\tilde{V}_\omega)}{\mathrm{Var}(\tilde{V}_\omega|\tilde{s}_{p,1},\tilde{s}_{p,2})}\right) \\
&= \frac{1}{2}\ln\left(\tau_{v,\omega}^{-1}(\tau_{v,\omega} + \tau^2\tau_{z,1}/\lambda^2 + \omega^2\tau^2\tau_{z,2}/\lambda^2)\right) \\
&= \frac{1}{2}\ln\left(1 + \left(\frac{K-1}{\lambda}\right)^2 \frac{\tau_{z,1}+\omega^2\tau_{z,2}}{\tau_{v,1}^{-1}+\omega^2\tau_{v,2}^{-1}}\right) = \mathrm{I}(\tilde{V};\tilde{P})_{\text{int}} \qquad \text{(A-85)}
\end{aligned}$$

It is straightforward to derive equation (50), the entropy measure of informational efficiency in the separative learning case. This follows from the independence of two assets and equation (18) in Proposition 3. The identity below follows from the fact that $\mathrm{Var}(\tilde{V}|\tilde{P})$ is a diagonal matrix:

$$\mathrm{I}(\tilde{V};\tilde{P})_{\text{sep}} = \mathrm{I}(\tilde{V}_1;\tilde{P})_{\text{sep}} + \mathrm{I}(\tilde{V}_2;\tilde{P})_{\text{sep}} = \mathrm{I}(\tilde{V}_1;\tilde{P}_1)_{\text{sep}} + \mathrm{I}(\tilde{V}_2;\tilde{P}_2)_{\text{sep}} \tag{A-86}$$

A.15 Informational Efficiency Under Different Learning Technology

We have the following two observations:

(1) When $\Delta = 0$ and if the condition $\tau_{v,j}^{-1}\tau_{z,j}^{-1} \leq \frac{1}{8}\left(\frac{K-1}{\lambda}\right)^2$ holds, there will be a space of benchmarking levels, $\overline{\gamma} = (\overline{\gamma}_1, \overline{\gamma}_2)' \in \mathbb{R}^2$, within which the informational efficiency under separative learning is higher than that under integrative learning: $\mathrm{I}(\tilde{V};\tilde{P})_{\text{sep},\Delta=0} > \mathrm{I}(\tilde{V};\tilde{P})_{\text{int},\Delta=0}$.

(2) When $\Delta \neq 0$ and suppose asset j is the more volatile asset, separative learning with the corner solution ($\Gamma_j = 1$) can generate higher informational efficiency than integrative learning, that is, $\mathrm{I}(\tilde{V};\tilde{P})_{\text{sep},\Gamma_j=1} \geq \mathrm{I}(\tilde{V};\tilde{P})_{\text{int}}$, where the equality holds when $\omega \to 0$ if $j = 1$ or $\omega \to \infty$ if $j = 2$.

When $\Delta = 0$, the informational efficiency under integrative learning is

$$\begin{aligned}
\mathrm{I}(\tilde{V};\tilde{P})_{\text{int},\Delta=0} &= \tfrac{1}{2}\ln\left(1 + \left(\tfrac{K-1}{\lambda}\right)^2 \tau_{v,1}\tau_{z,1}\right) = \tfrac{1}{2}\ln\left(1 + (\Gamma_1+\Gamma_2)^2\left(\tfrac{K-1}{\lambda}\right)^2 \tau_{v,1}\tau_{z,1}\right) \\
&= \tfrac{1}{2}\ln\left(1 + (\Gamma_1^2+\Gamma_2^2)\left(\tfrac{K-1}{\lambda}\right)^2 \tau_{v,1}\tau_{z,1} + 2\Gamma_1\Gamma_2\left(\tfrac{K-1}{\lambda}\right)^2 \tau_{v,1}\tau_{z,1}\right).
\end{aligned} \tag{A-87}$$

In contrast, the informational efficiency under separative learning is given by

$$\begin{aligned}
\mathrm{I}(\tilde{V};\tilde{P})_{\text{sep},\Delta=0} &= \tfrac{1}{2}\ln\left(1 + \Gamma_1^2\left(\tfrac{K-1}{\lambda}\right)^2 \tau_{v,1}\tau_{z,1}\right) + \tfrac{1}{2}\ln\left(1 + \Gamma_2^2\left(\tfrac{K-1}{\lambda}\right)^2 \tau_{v,2}\tau_{z,2}\right) \\
&= \tfrac{1}{2}\ln\left(1 + (\Gamma_1^2+\Gamma_2^2)\left(\tfrac{K-1}{\lambda}\right)^2 \tau_{v,1}\tau_{z,1} + \Gamma_1^2\Gamma_2^2\left(\tfrac{K-1}{\lambda}\right)^4 \tau_{v,1}^2\tau_{z,1}^2\right).
\end{aligned} \tag{A-88}$$

Comparing the last expressions of (A-87) and (A-88), we find that for any benchmarking levels,

$$\mathrm{I}(\tilde{V};\tilde{P})_{\text{int},\Delta=0} \geq \mathrm{I}(\tilde{V};\tilde{P})_{\text{sep},\Delta=0} \qquad \text{if} \quad \tau_{v,1}^{-1}\tau_{z,1}^{-1} \geq \frac{1}{8}\left(\frac{K-1}{\lambda}\right)^2 \geq \frac{\Gamma_1(1-\Gamma_1)}{2}\left(\frac{K-1}{\lambda}\right)^2. \tag{A-89}$$

When the opposite condition $\tau_{v,1}^{-1}\tau_{z,1}^{-1} < \frac{1}{8}\left(\frac{K-1}{\lambda}\right)^2$ holds, there are four solutions to the equation, $\mathrm{I}(\tilde{V};\tilde{P})_{\text{int},\Delta=0} = \mathrm{I}(\tilde{V};\tilde{P})_{\text{sep},\Delta=0}$, which can be reduced to

$$\Gamma_1^2 + (1-\Gamma_1)^2 + \Gamma_1^2(1-\Gamma_1)^2\left(\frac{K-1}{\lambda}\right)^2 \tau_{v,1}\tau_{z,1} = 1. \tag{A-90}$$

These four solutions are

$$\Gamma_1 = 0, \qquad \Gamma_1 = 1,$$
$$\Gamma_1 = \frac{1}{2} + \frac{1}{2}\sqrt{1 - \left(\frac{\lambda}{K-1}\right)^2 \frac{8}{\tau_{v,1}\tau_{z,1}}}, \qquad \Gamma_1 = \frac{1}{2} - \frac{1}{2}\sqrt{1 - \left(\frac{\lambda}{K-1}\right)^2 \frac{8}{\tau_{v,1}\tau_{z,1}}}.$$

The two corner solutions, $\Gamma_1 = 0$ and $\Gamma_1 = 1$, are obvious cases where the equality holds. The other two solutions indicate a positive-measure space of benchmarking levels $(\overline{\gamma}_1, \overline{\gamma}_2) \in \Phi \subset \mathbb{R}^2$, within which the inequality $\mathrm{I}(\tilde{V};\tilde{P})_{\text{sep},\Delta=0} > \mathrm{I}(\tilde{V};\tilde{P})_{\text{int},\Delta=0}$ holds (Figure xxx). When $\overline{z}_1$ and $\overline{z}_2$ are sufficiently large, the boundary of this space Φ can be determined by

$$\Gamma_1(\overline{\gamma}_1, \overline{\gamma}_2) = \frac{1}{2} \pm \frac{1}{2}\sqrt{1 - \left(\frac{\lambda}{K-1}\right)^2 \frac{8}{\tau_{v,1}\tau_{z,1}}}. \tag{A-91}$$

When asset j is more volatile than the other ($\Delta \neq 0$), the informational efficiency in a corner equilibrium ($\Gamma_j = 1$) under separative learning is no less than that under integrative learning:

$$\mathrm{I}(\tilde{V};\tilde{P})_{\text{sep},\Gamma_j=1} = \tfrac{1}{2}\ln\left(1 + \left(\frac{K-1}{\lambda}\right)^2 \tau_{v,j}\tau_{z,j}\right) \geq \mathrm{I}(\tilde{V};\tilde{P})_{\text{int}}. \tag{A-92}$$

where the equality holds only when $\omega \to 0$ if $j = 1$ or when $\omega \to \infty$ if $j = 2$.

A.16 Proof of Proposition 8

In general, when $\Delta \neq 0$, the marginal benefit of private information is different between the two assets, and investors will tilt their attention weight toward the asset with higher total payoff uncertainty. This attention shift is captured by the real attention measure. This makes investors' optimal attention weight mismatch with the aggregate speculative portfolio weight $\frac{\overline{z}_2 - \overline{\gamma}_2}{\overline{z}_1 - \overline{\gamma}_1}$.

From equation (37) and the definition of real attention weight (54), we can derive

$$\begin{aligned}
\frac{d\omega^*}{d\overline{\gamma}_1} &= \frac{\Delta - \xi_+ + \sqrt{4\tau_{v,1}^{-1}\tau_{v,2}^{-1}(\overline{z}_1 - \overline{\gamma}_1)^2(\overline{z}_2 - \overline{\gamma}_2)^2 + (\Delta + \xi_-)^2}}{\sqrt{4\tau_{v,1}^{-1}\tau_{v,2}^{-1}(\overline{z}_1 - \overline{\gamma}_1)^2(\overline{z}_2 - \overline{\gamma}_2)^2 + (\Delta + \xi_-)^2}} \frac{\tau_{v,1}^{-1}(\overline{z}_1 - \overline{\gamma}_1)}{\tau_{v,2}^{-1}(\overline{z}_2 - \overline{\gamma}_2)^2} \\
&= \left(1 + \frac{\Delta - \xi_+}{\sqrt{\xi_+^2 + 2\Delta\xi_- + \Delta^2}}\right) \frac{\tau_{v,1}^{-1}(\overline{z}_1 - \overline{\gamma}_1)}{\tau_{v,2}^{-1}(\overline{z}_2 - \overline{\gamma}_2)^2},
\end{aligned} \tag{A-93}$$

where $\xi_\pm := \tau_{v,1}^{-1}(\overline{z}_1 - \overline{\gamma}_1)^2 \pm \tau_{v,2}^{-1}(\overline{z}_2 - \overline{\gamma}_2)^2$. From equation (A-93), it is easy to see that $\frac{d\omega^*}{d\overline{\gamma}_1} = 0$

when $\Delta = 0$ (and $\omega^* = 1$ in this case). The following result is useful

$$(\Delta - \xi_+)^2 - (\xi_+^2 + 2\Delta\xi_- + \Delta^2) = -2\Delta(\xi_+ + \xi_-) = -4\Delta\tau_{v,1}^{-1}(\overline{z}_1 - \overline{\gamma}_1)^2, \tag{A-94}$$

Therefore, we have $|\Delta - \xi_+| < \sqrt{\xi_+^2 + 2\Delta\xi_- + \Delta^2}$ if $\Delta > 0$ and $|\Delta - \xi_+| > \sqrt{\xi_+^2 + 2\Delta\xi_- + \Delta^2}$ if $\Delta < 0$. Since $\xi_+ > 0$, one can see that

$$-1 < \frac{\Delta - \xi_+}{\sqrt{\xi_+^2 + 2\Delta\xi_- + \Delta^2}} < 1 \quad \text{if} \quad \Delta > 0, \qquad \frac{\Delta - \xi_+}{\sqrt{\xi_+^2 + 2\Delta\xi_- + \Delta^2}} < -1 \quad \text{if} \quad \Delta < 0. \tag{A-95}$$

With positive effective supplies, we thus obtain the monotonic dependence of ω^* on $\overline{\gamma}_1$,

$$\frac{\mathrm{d}\omega^*}{\mathrm{d}\overline{\gamma}_1} > 0 \quad \text{if} \quad \Delta > 0, \qquad \frac{\mathrm{d}\omega^*}{\mathrm{d}\overline{\gamma}_1} < 0 \quad \text{if} \quad \Delta < 0. \tag{A-96}$$

Similar arguments can be applied to asset 2 and lead to the following result

$$\frac{\mathrm{d}\omega^*}{\mathrm{d}\overline{\gamma}_2} > 0 \quad \text{if} \quad \Delta > 0, \qquad \frac{\mathrm{d}\omega^*}{\mathrm{d}\overline{\gamma}_2} < 0 \quad \text{if} \quad \Delta < 0. \tag{A-97}$$

$\mathrm{I}(\tilde{V};\tilde{P})_{\mathrm{int}}$ increases in the benchmarking level $\overline{\gamma}_j$ if ω and ω^* move to opposite directions. $\mathrm{I}(\tilde{V};\tilde{P})_{\mathrm{int}}$ decreases in $\overline{\gamma}_j$ if ω and ω^* move in the same direction. This is implied by Proposition 6, Theorem 3, and Proposition 8.

A.17 Proof of Corollary 8

We first study the integrative learning case. Investor i's demand is

$$\theta^i = \gamma^i + \frac{1}{\lambda}(\hat{\Sigma}_v^i)^{-1}(\hat{V}^i - \tilde{P}), \tag{A-98}$$

where, in the symmetric equilibrium, $\Lambda^i = (1, \omega)$, $\tau^i = \tau$, and $\hat{\Sigma}_v^i = \hat{\Sigma}_V$, so that

$$\tilde{P} = \int_0^1 \hat{V}^i di - \lambda\hat{\Sigma}_V(\tilde{Z} - \overline{\gamma}). \tag{A-99}$$

From Proposition 4, we obtain that the equilibrium demand is

$$\theta^i = \gamma^i + \frac{1}{\lambda}(\Lambda)'\tau\tilde{\varepsilon}^i + \tilde{Z} - \overline{\gamma}. \tag{A-100}$$

More explicitly, we can write

$$\theta_1^i = \tilde{Z}_1 + \gamma_1^i - \overline{\gamma}_1 + \frac{\tilde{\varepsilon}^i}{\lambda}\frac{K-1}{\tau_{v,1}^{-1}+\omega^2\tau_{v,2}^{-1}}, \quad \theta_2^i = \tilde{Z}_2 + \gamma_2^i - \overline{\gamma}_2 + \frac{\tilde{\varepsilon}^i}{\lambda}\frac{(K-1)\omega}{\tau_{v,1}^{-1}+\omega^2\tau_{v,2}^{-1}}. \tag{A-101}$$

By definitions the dispersion of portfolio holdings and the dispersion of the portfolio excess return, it is straightforward to show that that the dispersions of portfolio holdings and the dispersion of the portfolio excess return are given in Corollary 8 in both learning models.

$$D_r = \int_0^1 (\gamma^i - \overline{\gamma})' \begin{pmatrix} A_{11} & A_{12} \\ A_{12} & A_{22} \end{pmatrix} (\gamma^i - \overline{\gamma}) di + \frac{K-1}{\lambda^2}\frac{A_{11}+2\omega A_{12}+\omega^2 A_{22}}{\tau_{v,1}^{-1}+\omega^2\tau_{v,2}^{-1}}, \tag{A-102}$$

this can be rewritten as equation (61) in Corollary 8.